UVA DATA SCIENCE

DS 6050 Deep Learning

3. Convolutional Neural Networks

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School of Data Science
University of Virginia

Recap

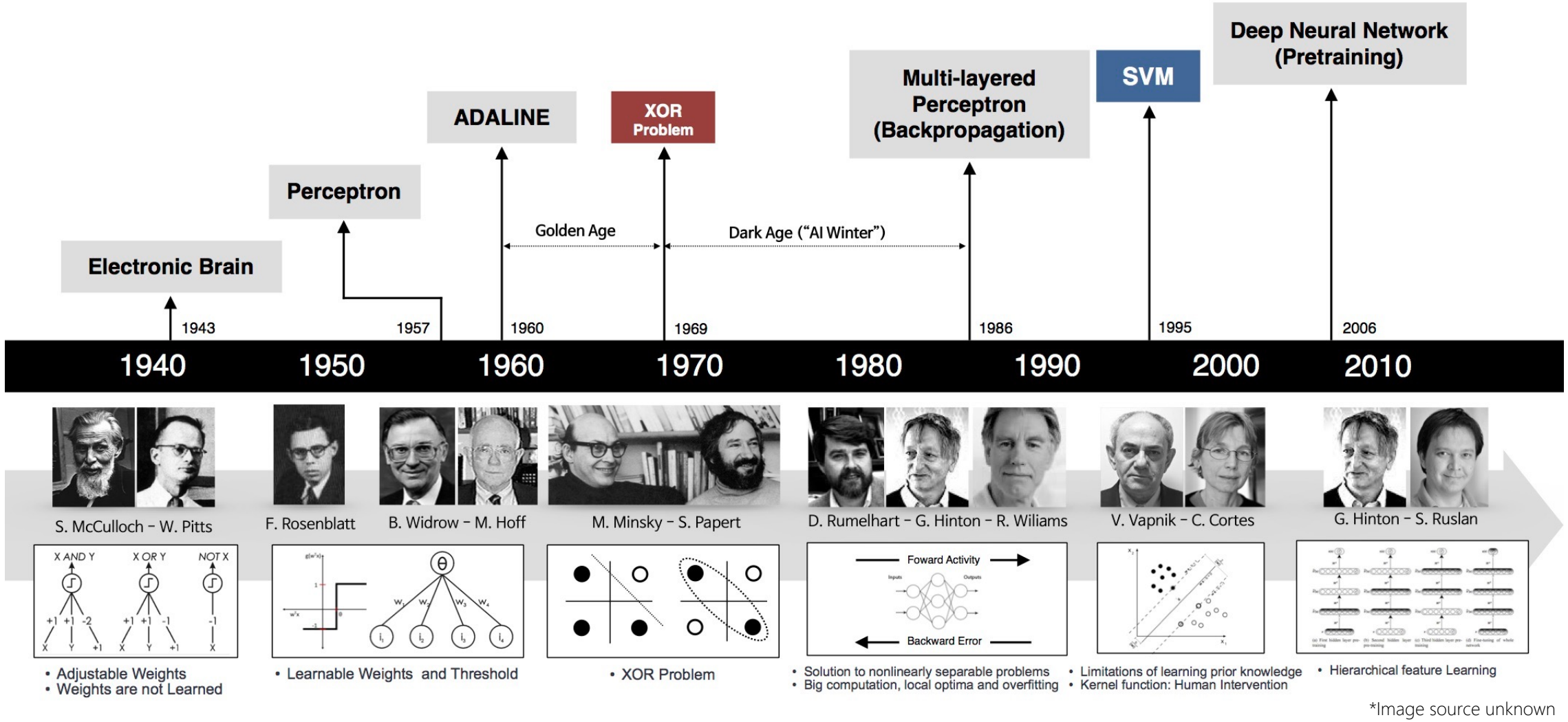
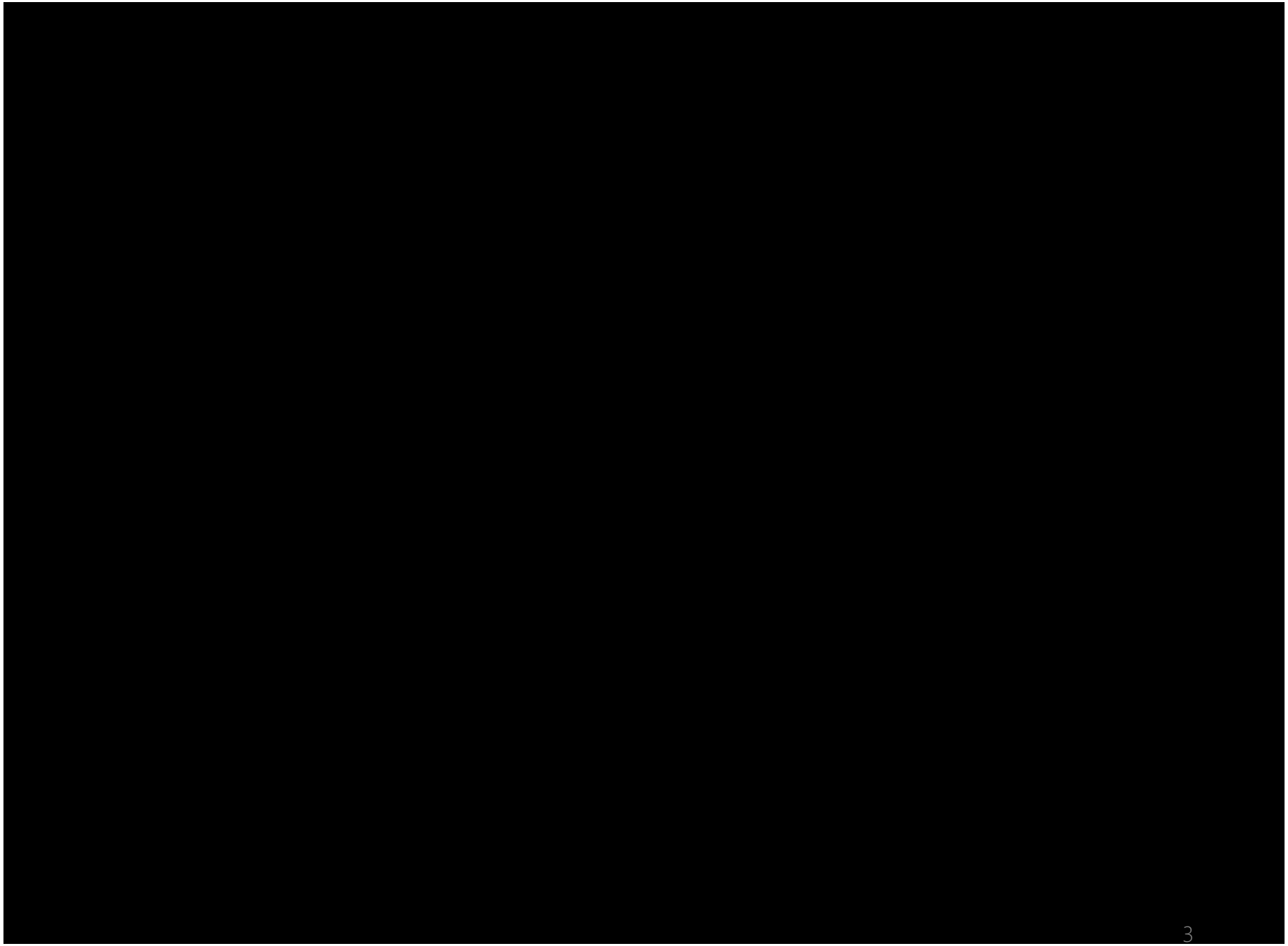


*Image source unknown

LeNet

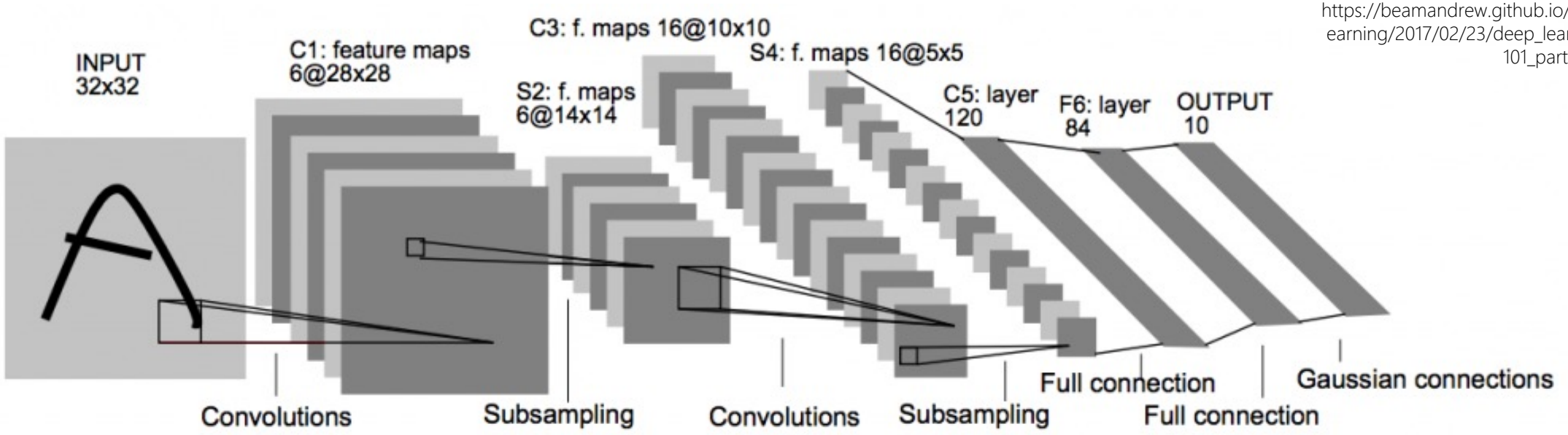


LeCun et al. (1989). Handwritten digit recognition with a back-propagation network. NuerIPS.

https://www.youtube.com/watch?v=FwFduRA_L6Q

Convolutional Neural Networks

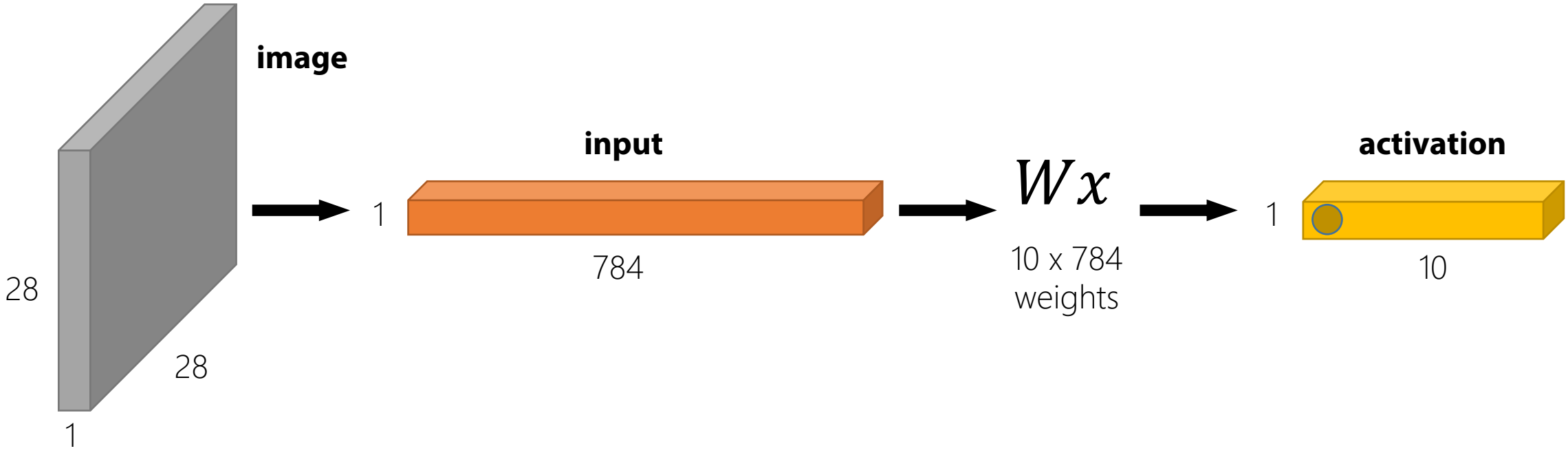
https://beamandrew.github.io/deeplearning/2017/02/23/deep_learning_101_part1.html



LeCun, Bottou, Bengio, & Haffner. (1998). Gradient-based learning applied to document recognition. Proc. IEEE.

Fully Connected Layer

- 28 x 28 image → stretch to 784 x 1
- 64 x 64 x 3 image → stretch to 12288 x 1
- …



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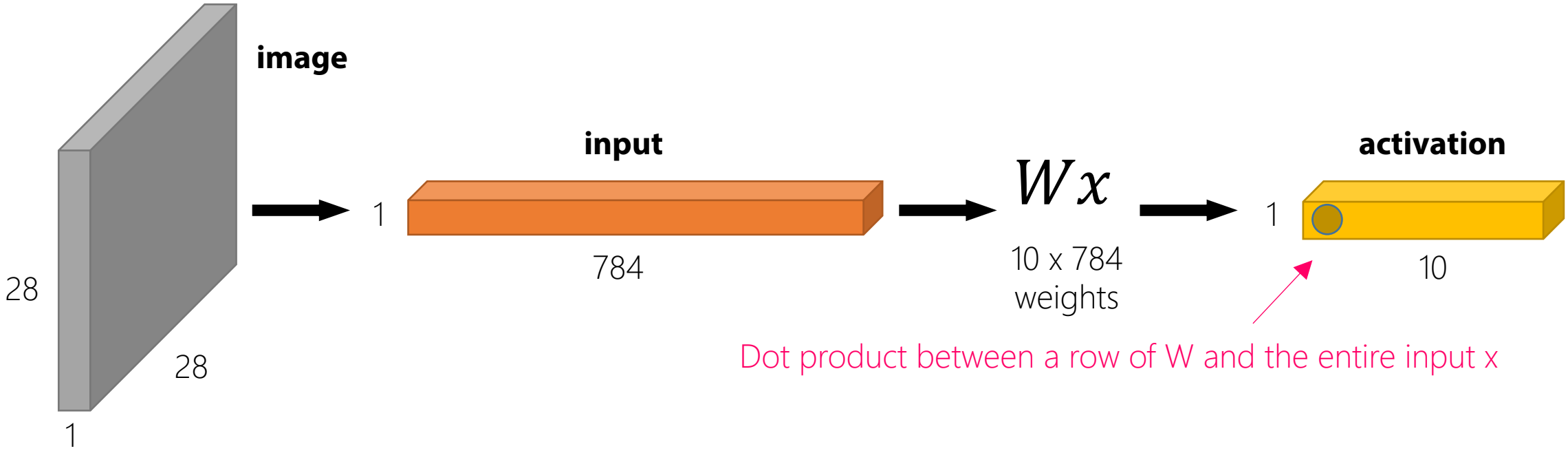
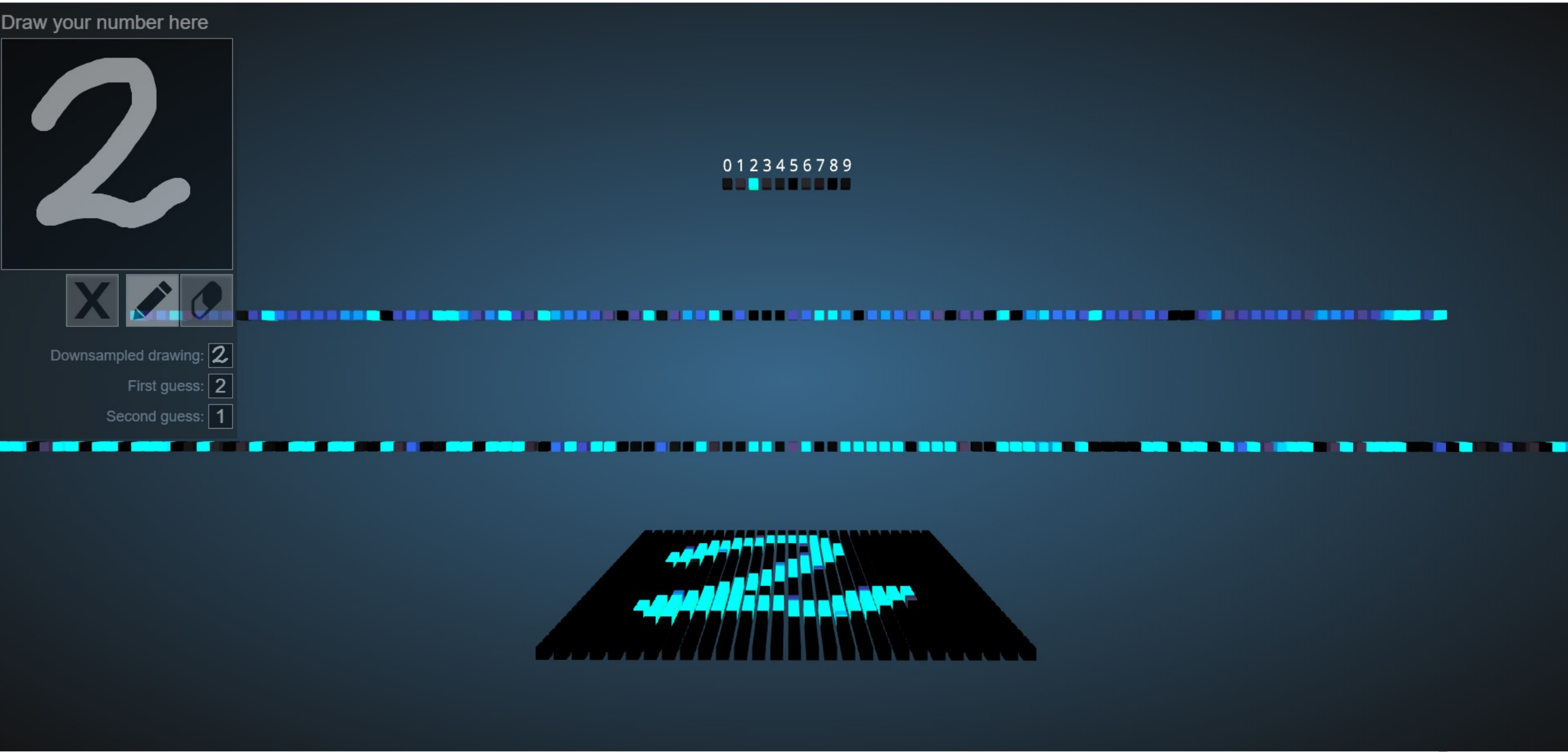
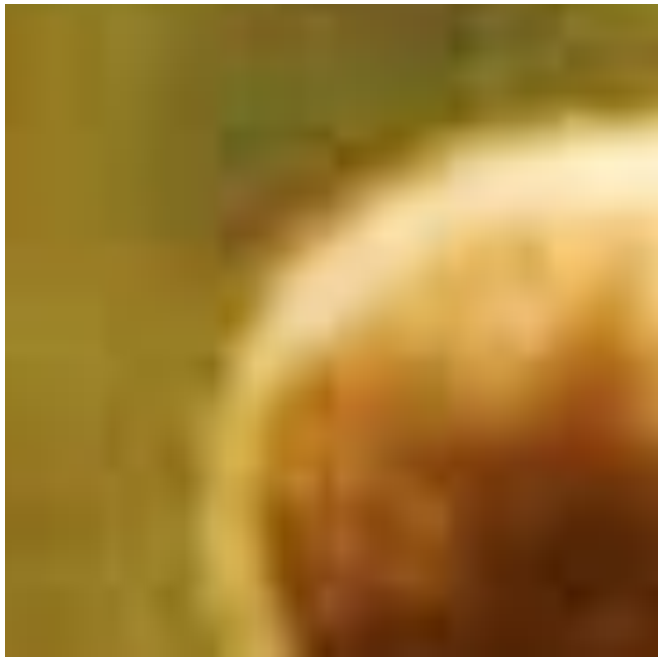
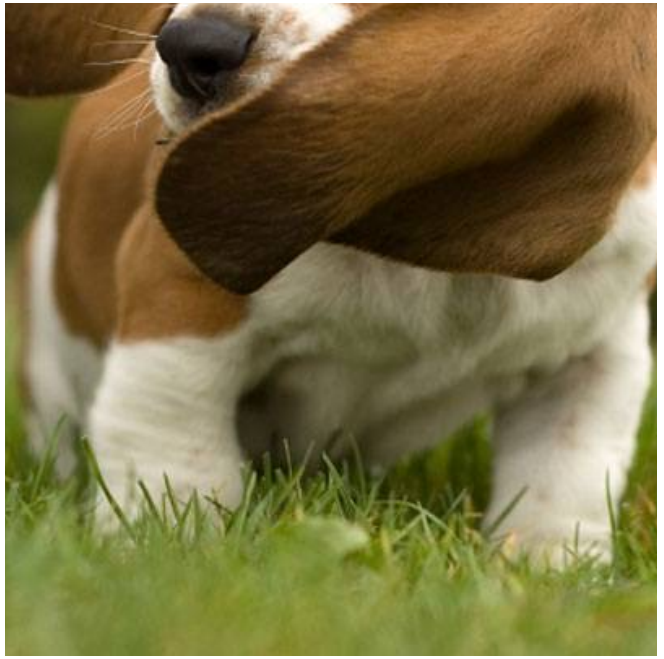




https://adamharley.com/nn_vis/





The human visual system

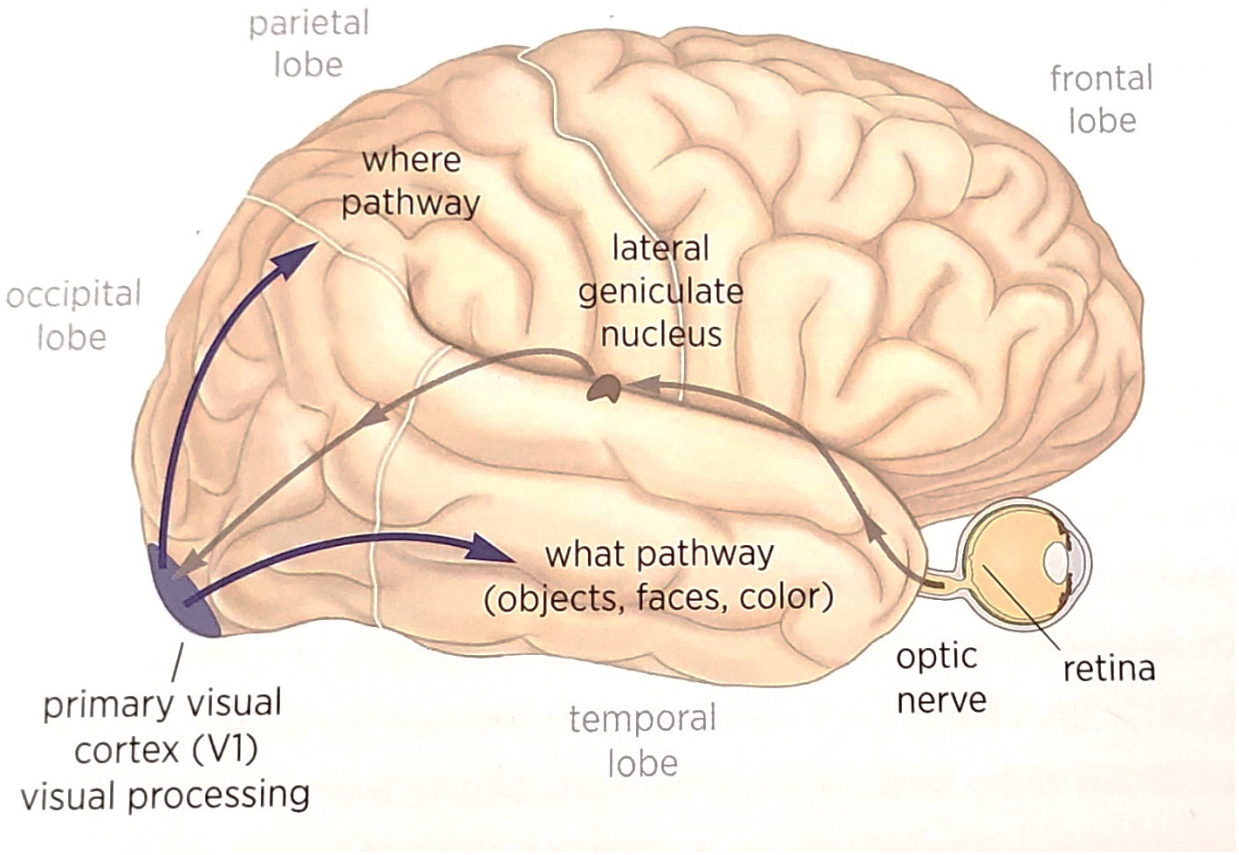
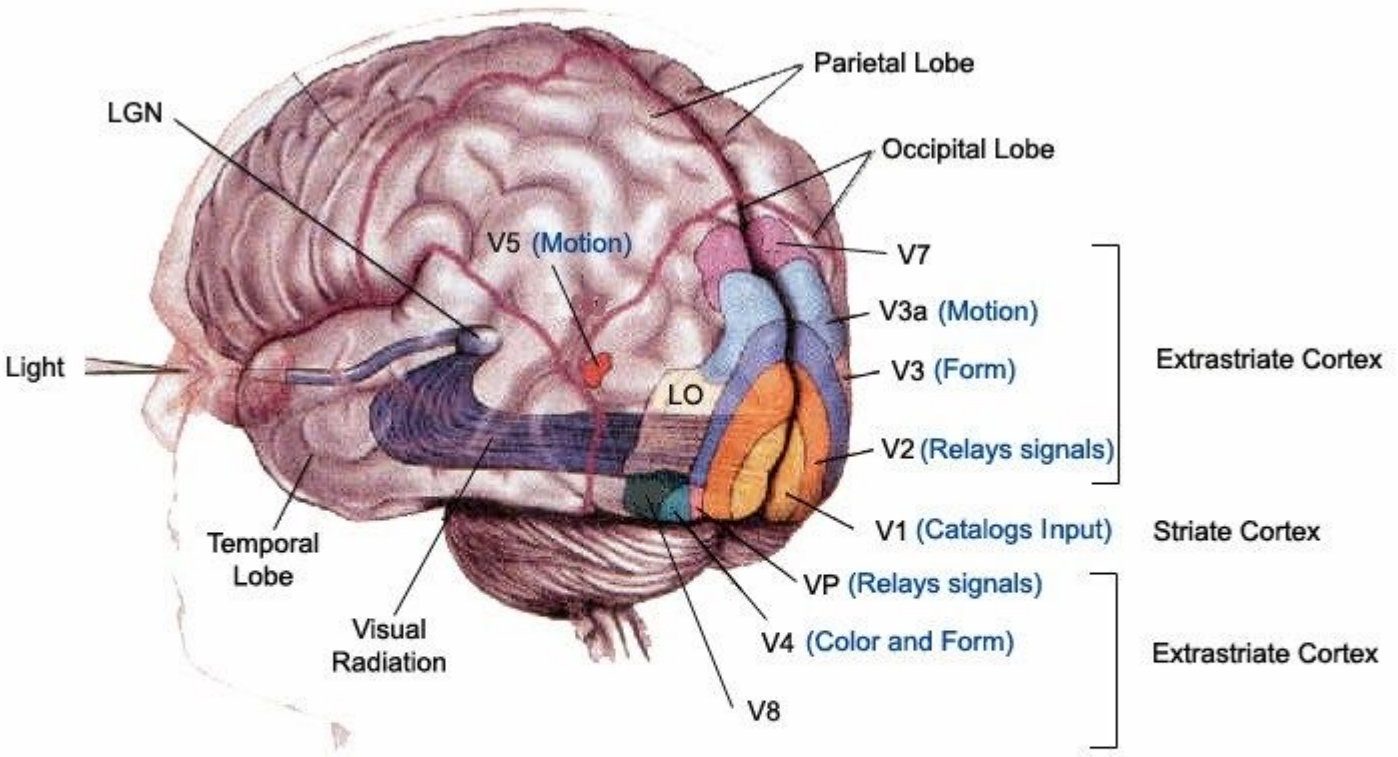


Image Courtesy: Kandel, "Reductionism in Art and Brain Science," 2016

- Retina: visual input
- Retina → Lateral Geniculate Nucleus
  - Visual information flows through the optic nerve
- Lateral Geniculate Nucleus (LGN):
  - A small, ovoid object at the end of the optic tract
  - One on each side of the brain
  - In humans, each LGN has six layers of neurons
  - Sends information to the primary visual cortex (V1)

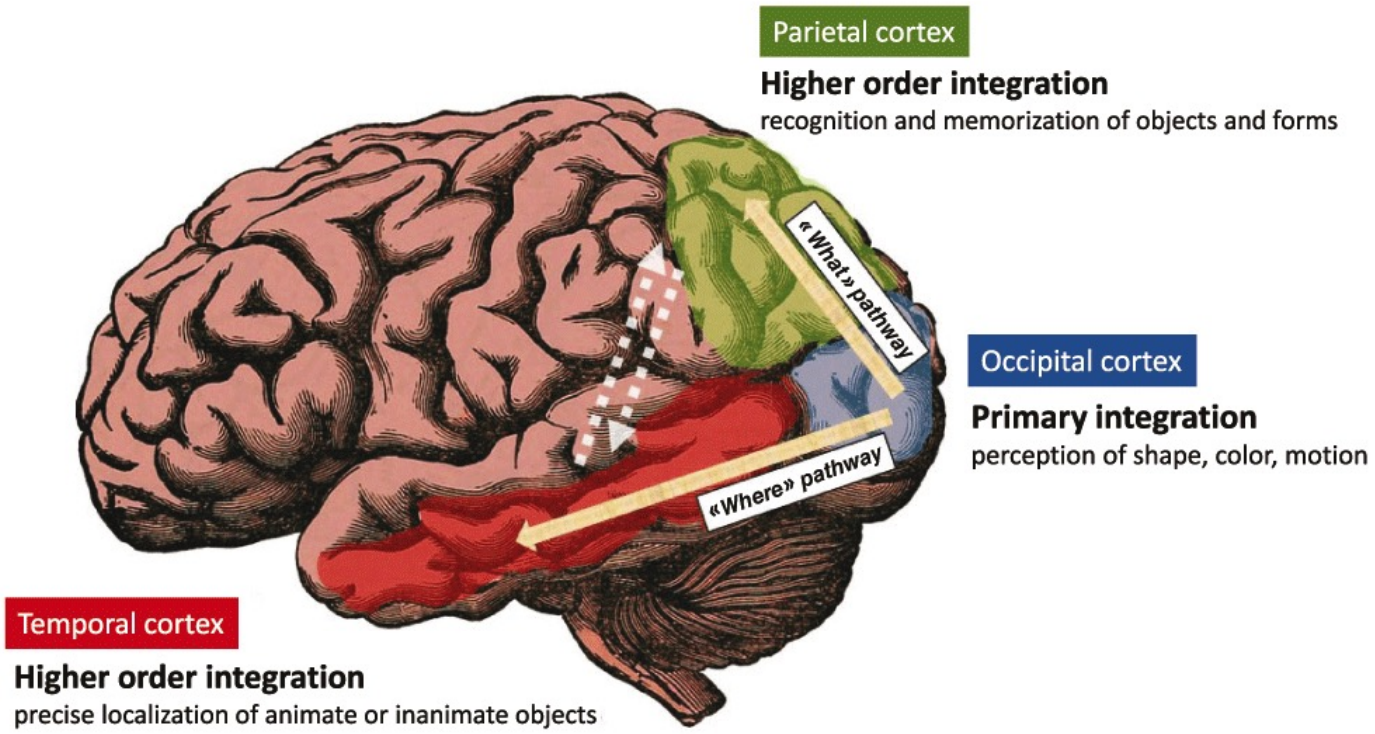
The human visual system



- Primary Visual Cortex (V1):
  - Highly specialized for processing visual information about static/moving objects
  - Pattern recognition happens
  - In average, the adult human V1 in each hemisphere has around 140 million neurons*
  - Transforms visual inputs to neural firing rates of those 140 million neurons → saliency determined by the visual location signaled by the highest firing neuron ("saliency map")

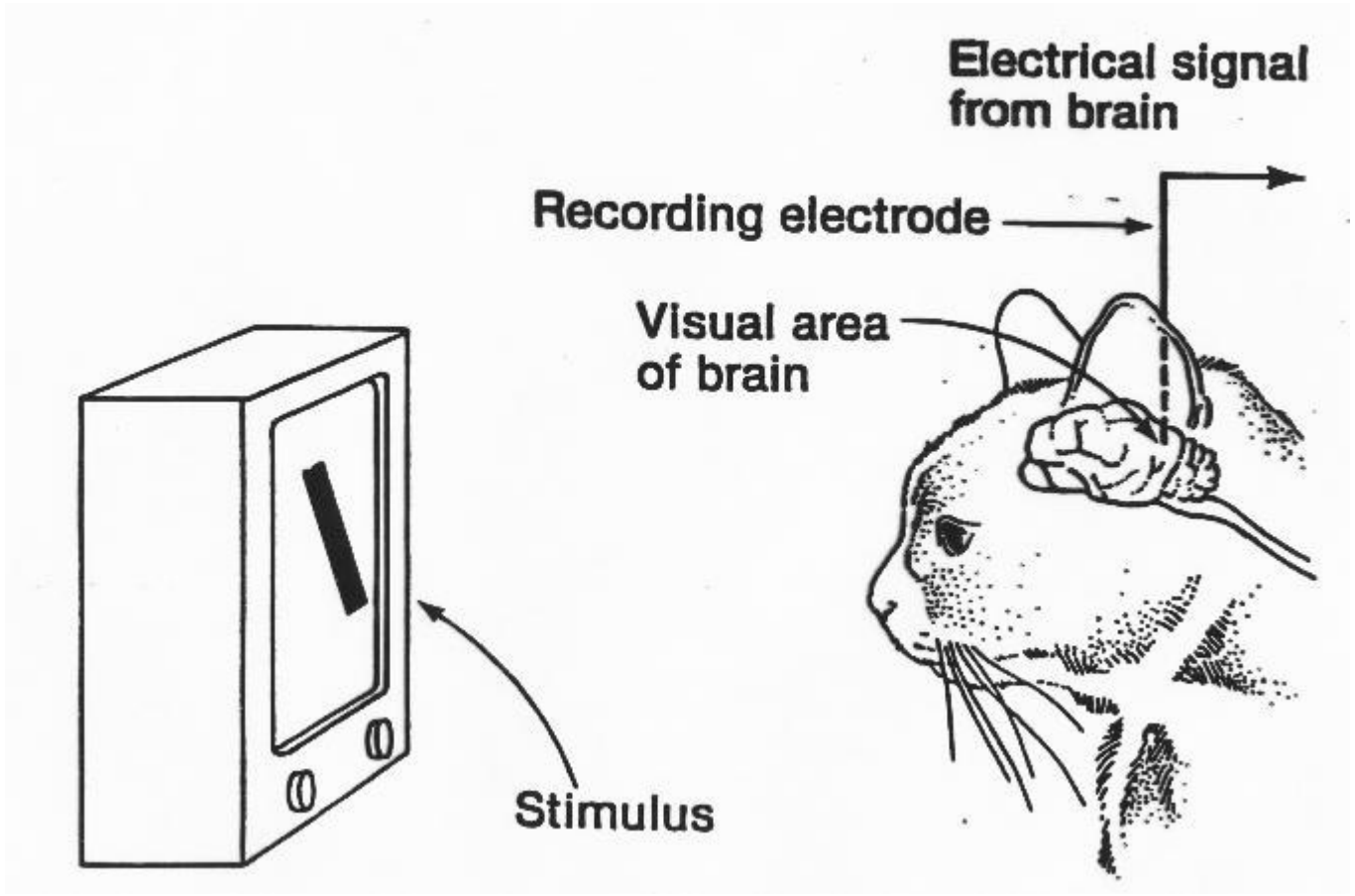
* Leuba G; Kraftsik R (1994). "Changes in volume, surface estimate, three-dimensional shape and total number of neurons of the human primary visual cortex from midgestation until old age". Anatomy and Embryology. 190 (4): 351–366.

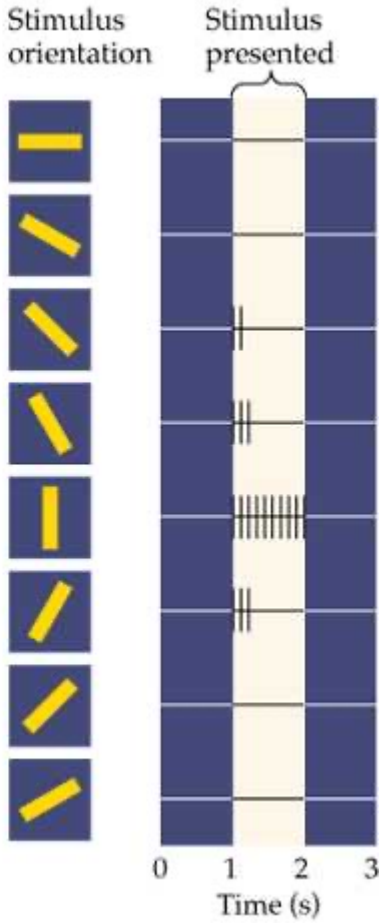
The human visual system



- Visual pathways
  - V1 gives rise to two major pathways: the *where* and *what* pathways
- The "where" pathway:
  - Concerned with where an object or a person is located
- The "what" pathway:
  - Concerned with what an object is or who a person is.

Hubel & Wiesel (1959 ~)





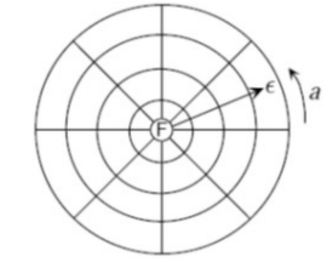
Neurons in the visual cortex respond selectively to oriented edges. Neurons in visual cortex typically respond vigorously to a bar of light oriented at a particular angle and weakly or not at all to other orientations.

Hubel & Wiesel (1959 ~)

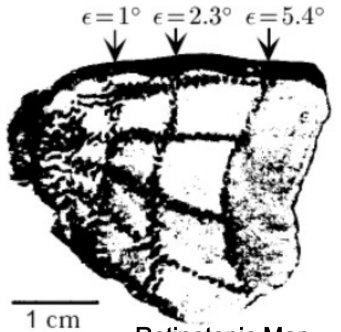
Warning: DISTURBING CONTENT

https://www.youtube.com/watch?v=IOHayh06LJ4

Topographic Maps in Cortex

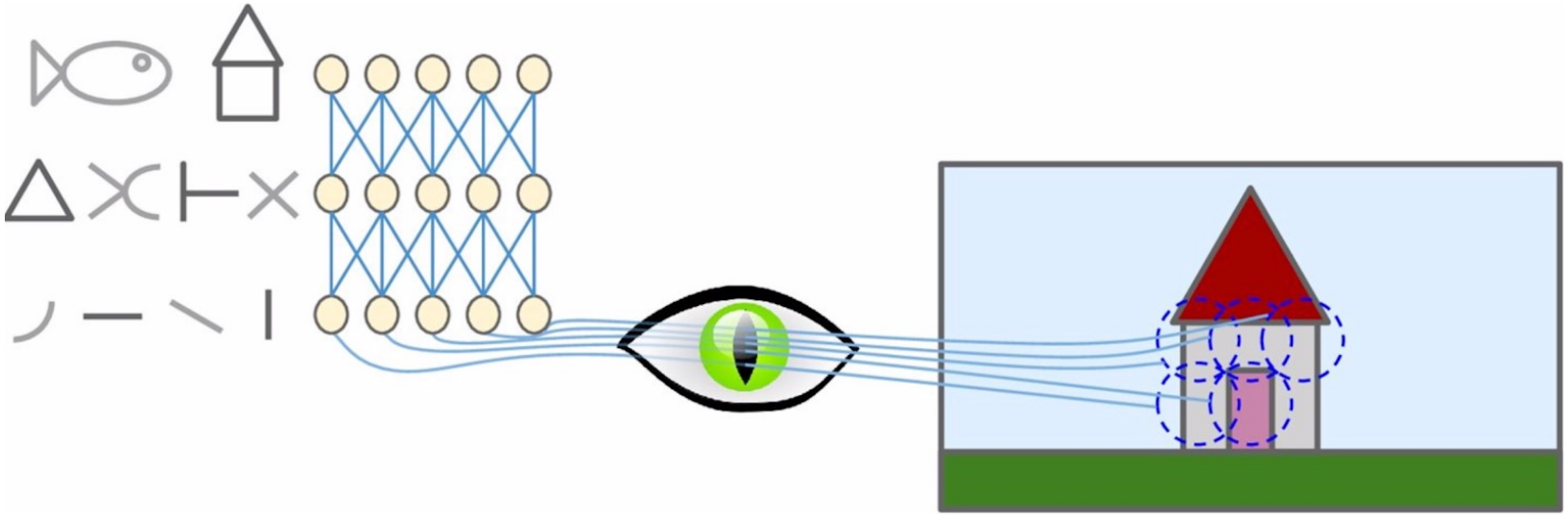


(Dayan and Abbott 2001)

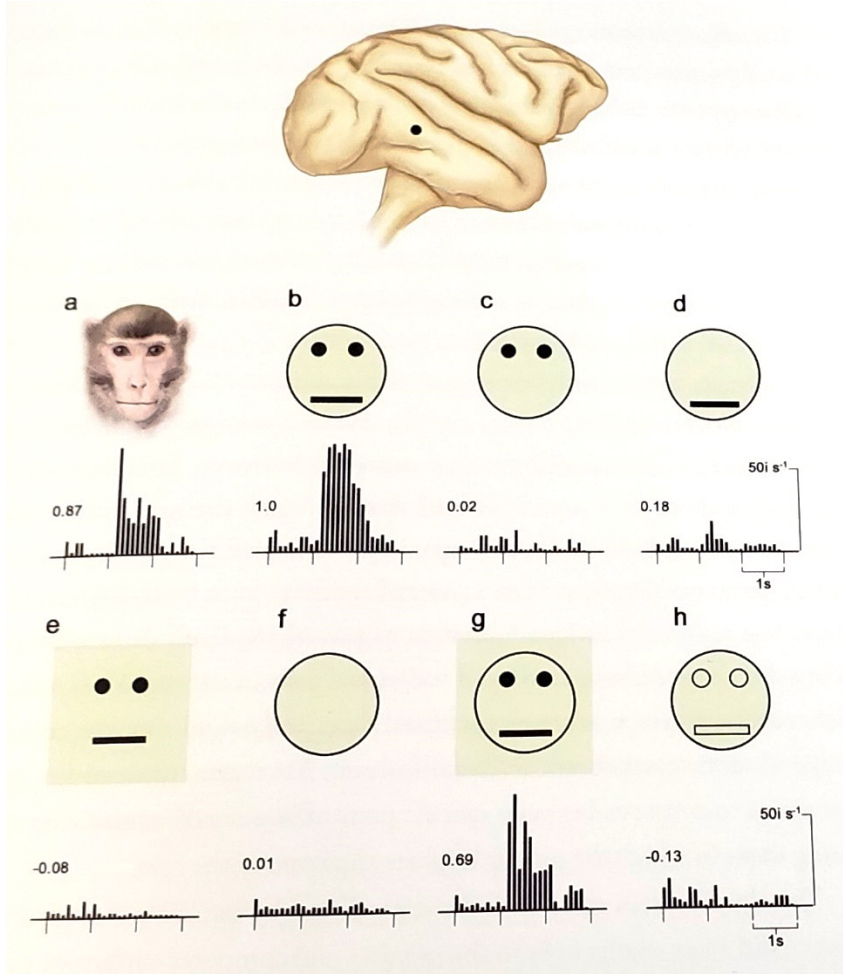


Retinotopic Map

- Each visual sensitive cell only responses to stimuli of a limited region (receptive field)
- Neighboring cells have partially overlapping receptive fields
- Neighboring points in a visual image evoke activity in neighboring regions of visual cortex
- In this manner, the visual system easily maintain the information of the spatial location of stimulus



e.g. Facial Recognition



- Holistic face detection
  - "Face cell" in the inferior temporal cortex
  - Fires when there is a face-like object

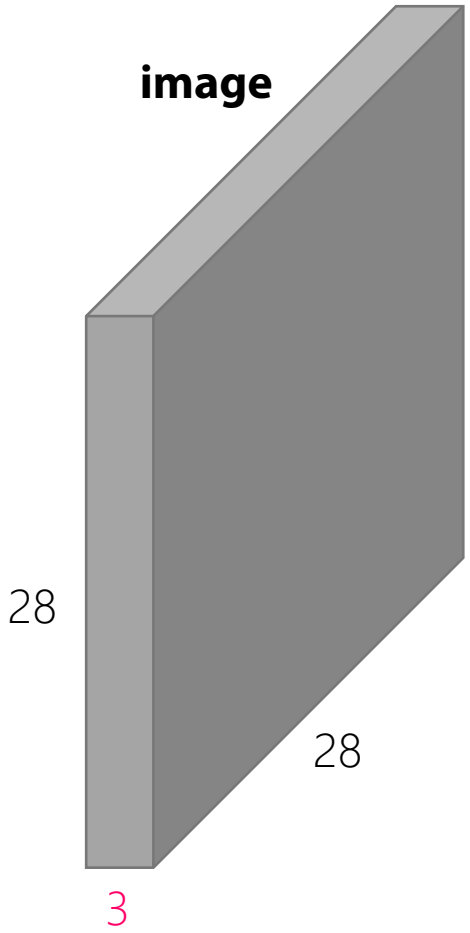
Image Courtesy: Kandel, "Reductionism in Art and Brain Science," 2016

Convolutional Neural Networks

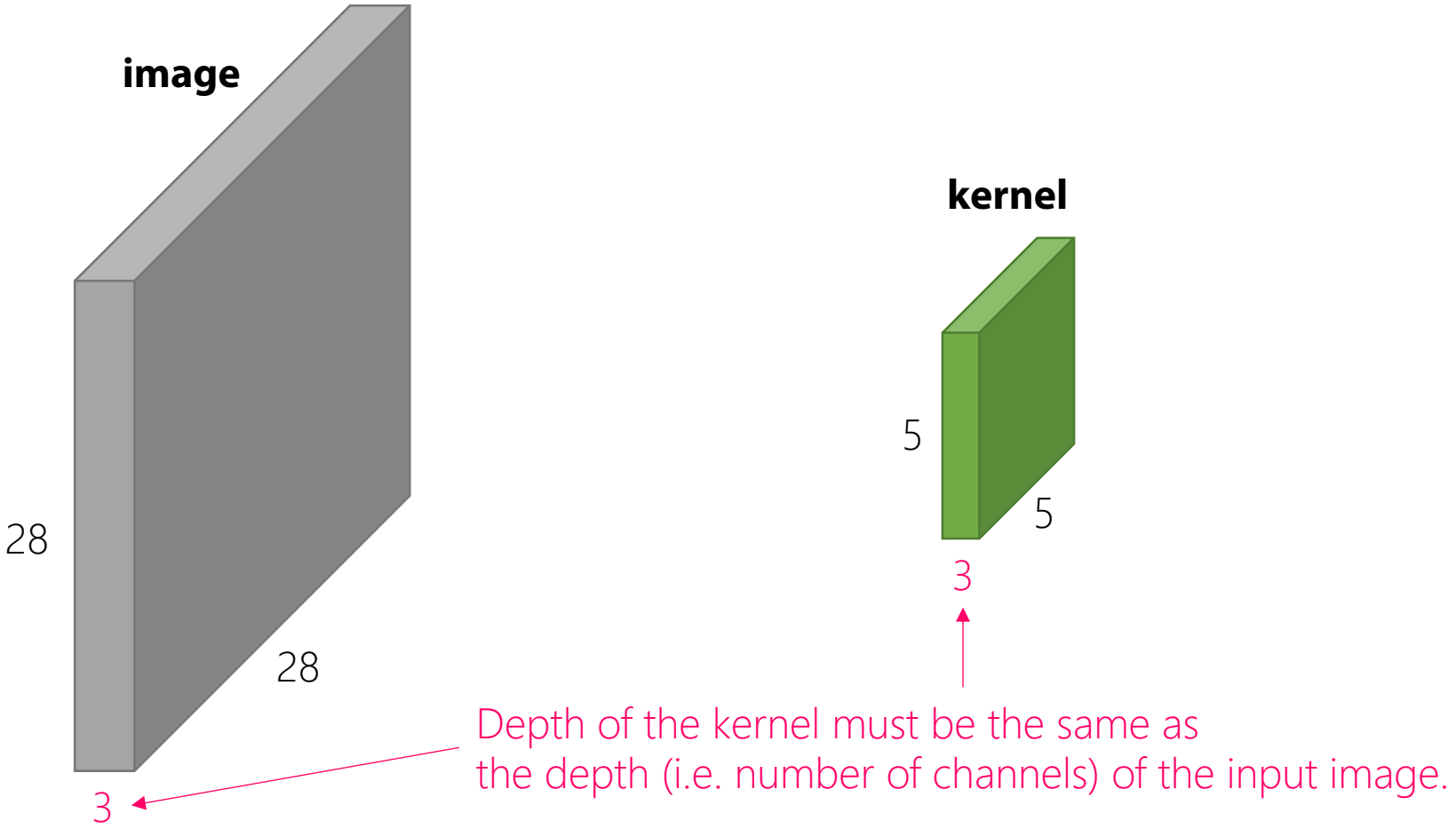
- Key idea:
  - As like how we (humans) understand a visual scene, if neural nets could see small pieces, understand patterns and textures, combine the pieces to see a bigger picture, computers should be able to recognize images.

- A bonus:
  - Typical neural networks are “fully connected”.
  - In an image domain, this means all the pixels are interconnected.
  - However, pixels far apart have no significant meaning…
  - By connecting only the nearing neighbors, computational load could be much lower.

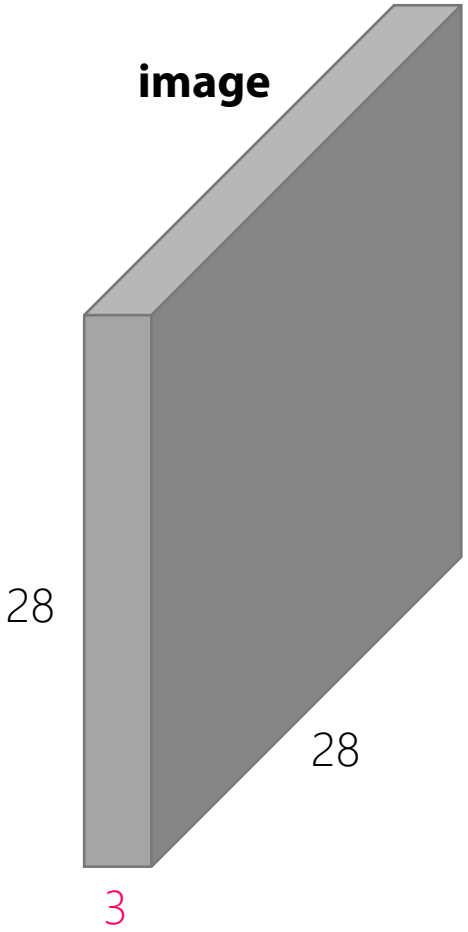
Convolution Layer

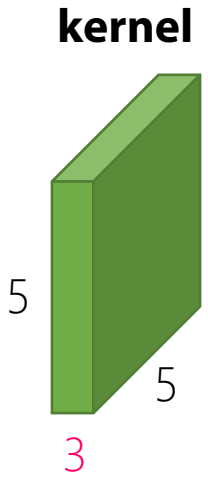


Convolution Layer



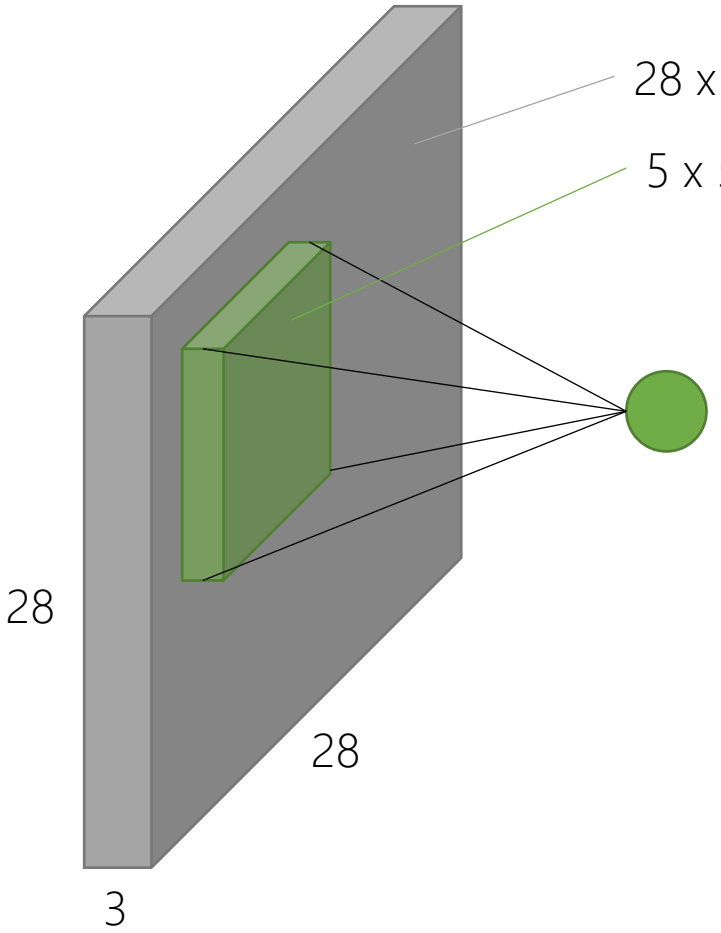
Convolution Layer





Convolve the kernel with the image!
In other words…
"slide the kernel over the image, compute dot products each time."

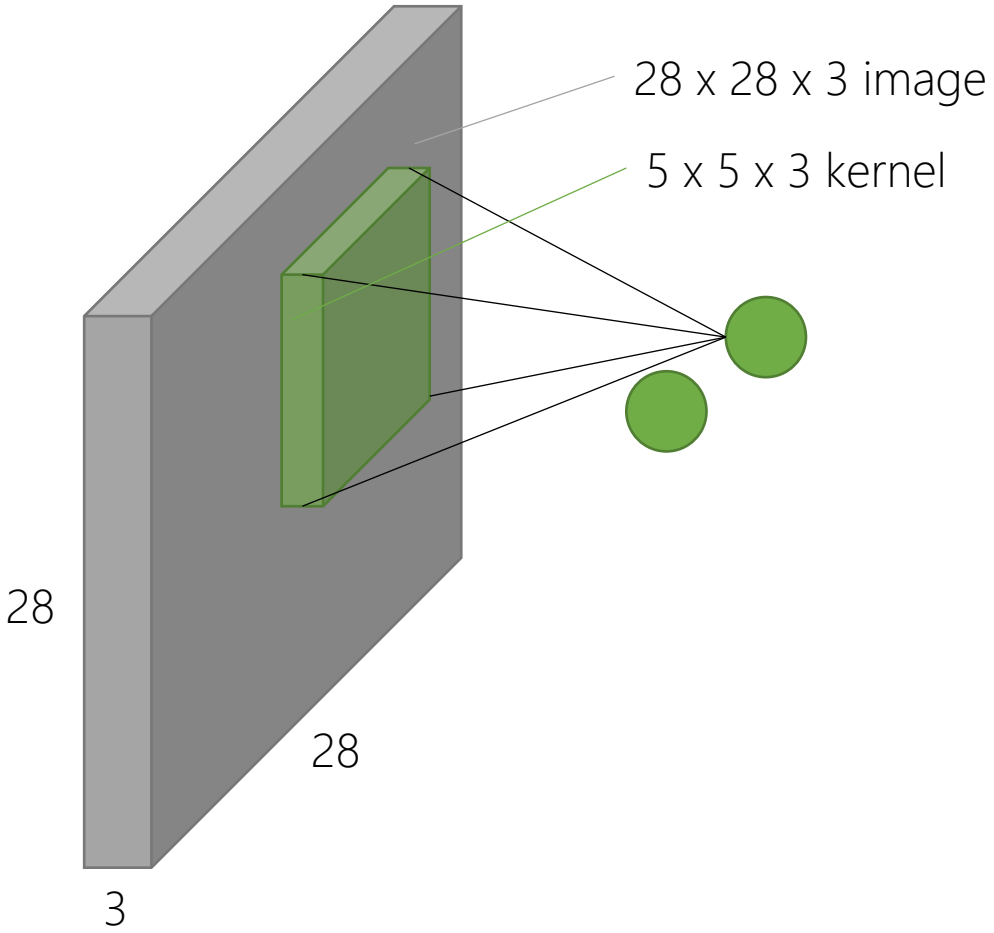
Convolution Layer



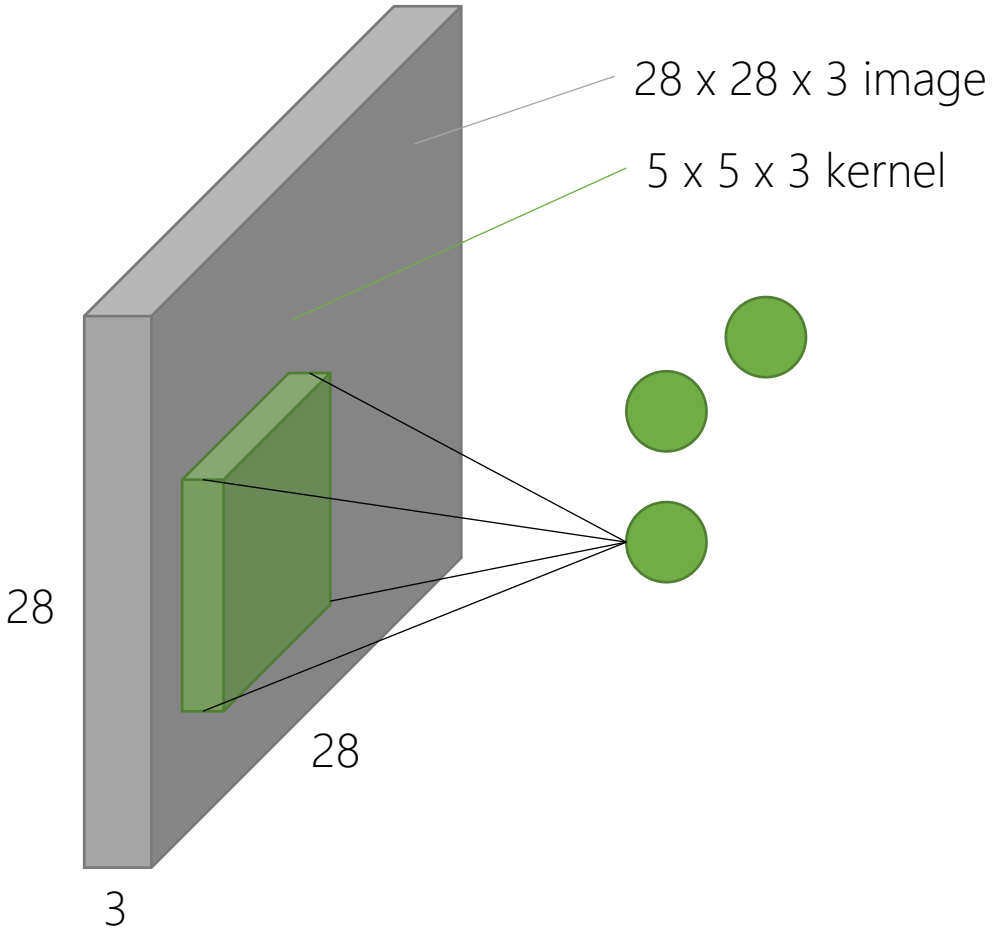
1 number as a result of taking dot product between the kernel and a small 5 x 5 x 3 chunk of the image, plus bias:

$$w^T x + b$$

Convolution Layer



Convolution Layer



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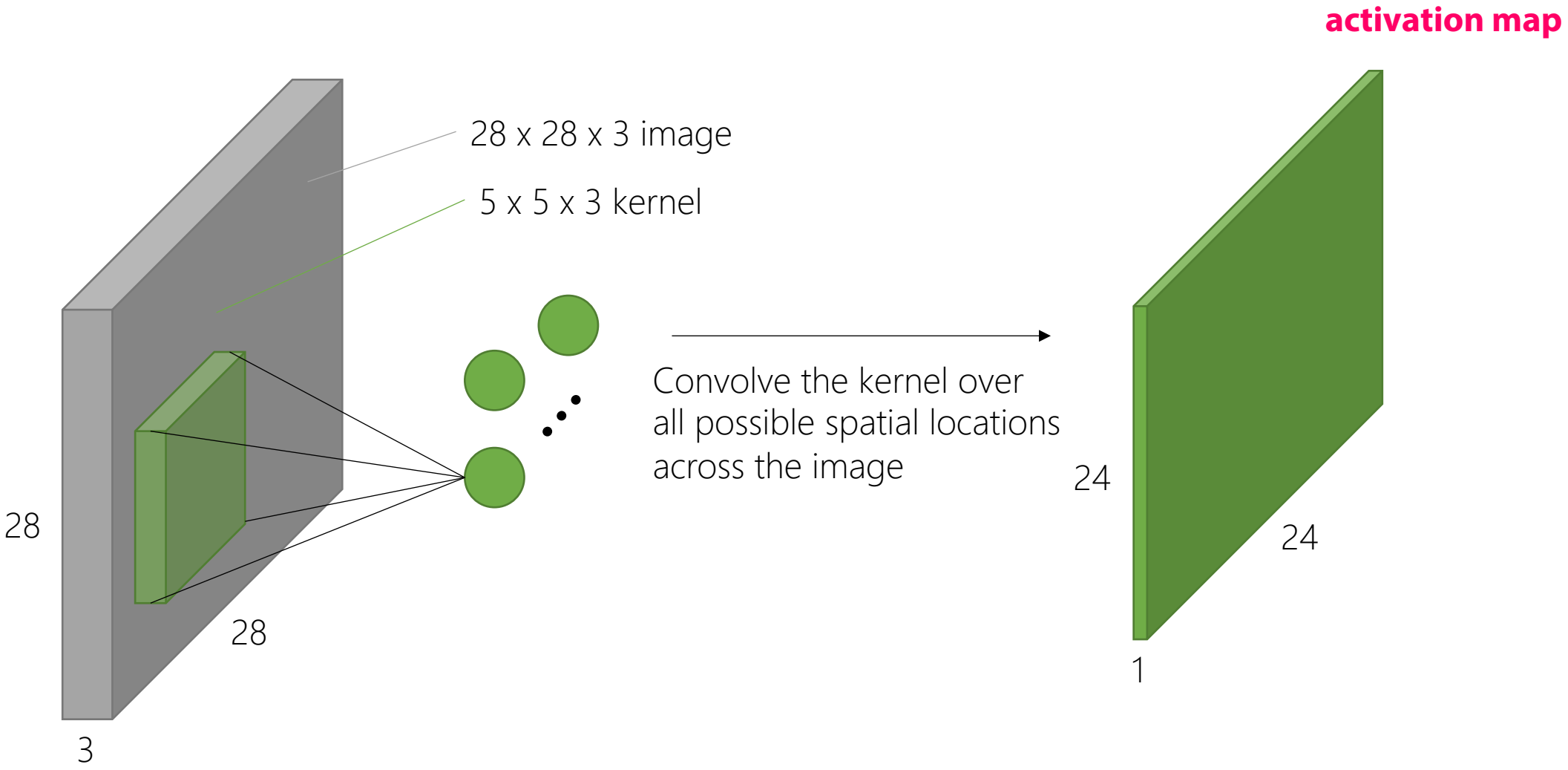
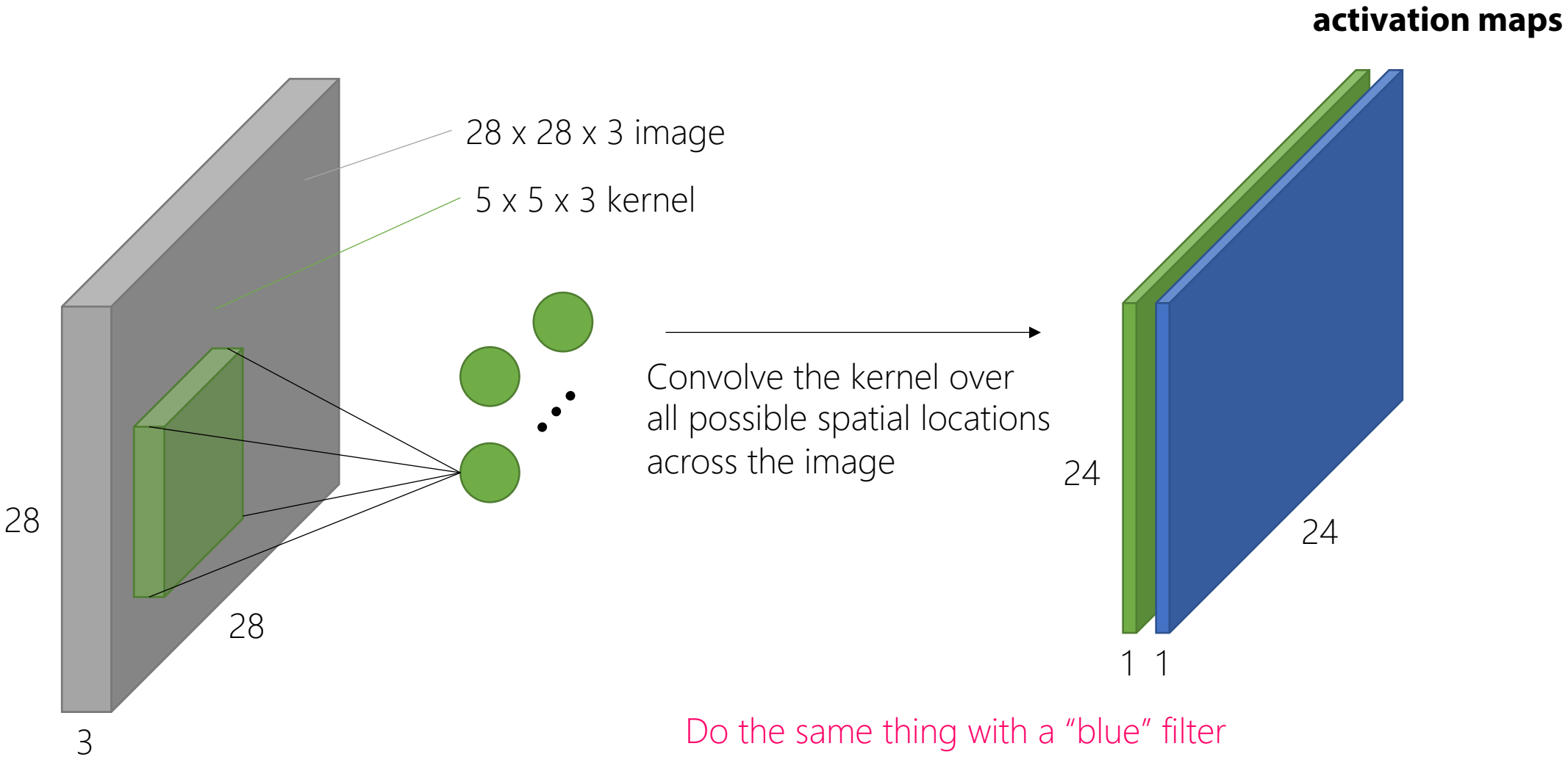
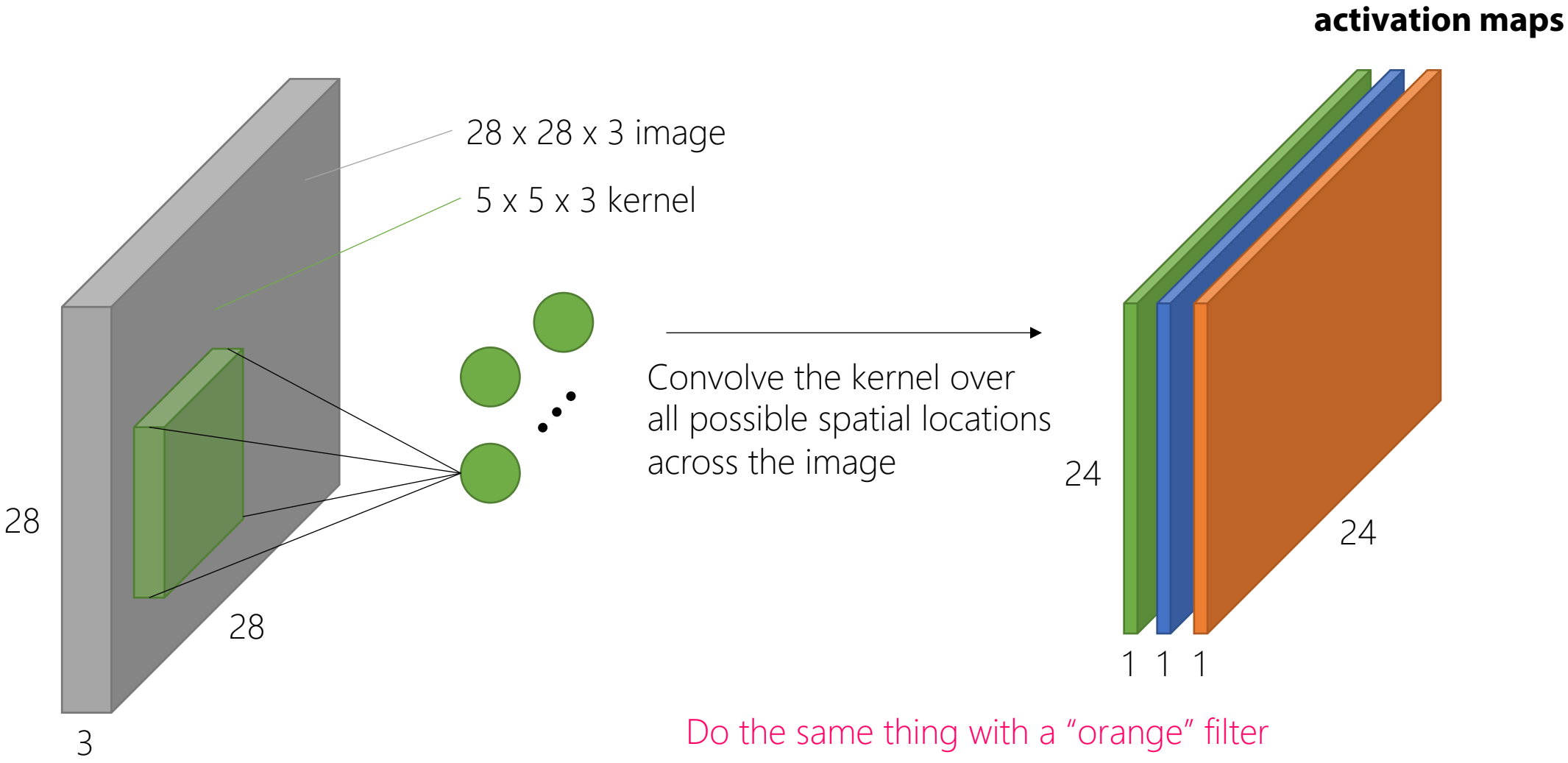


Convolution Layer



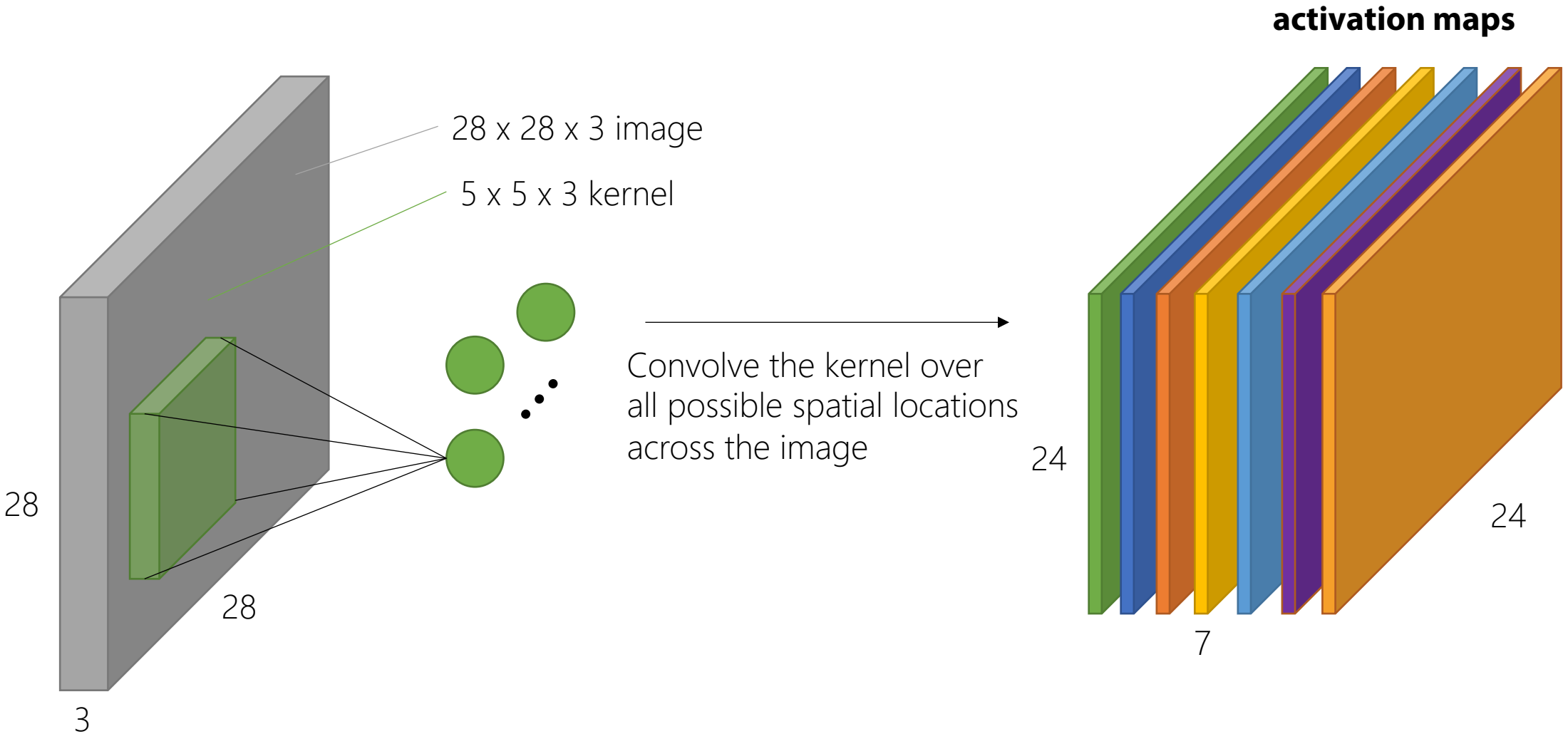
Do the same thing with a “blue” filter

Convolution Layer

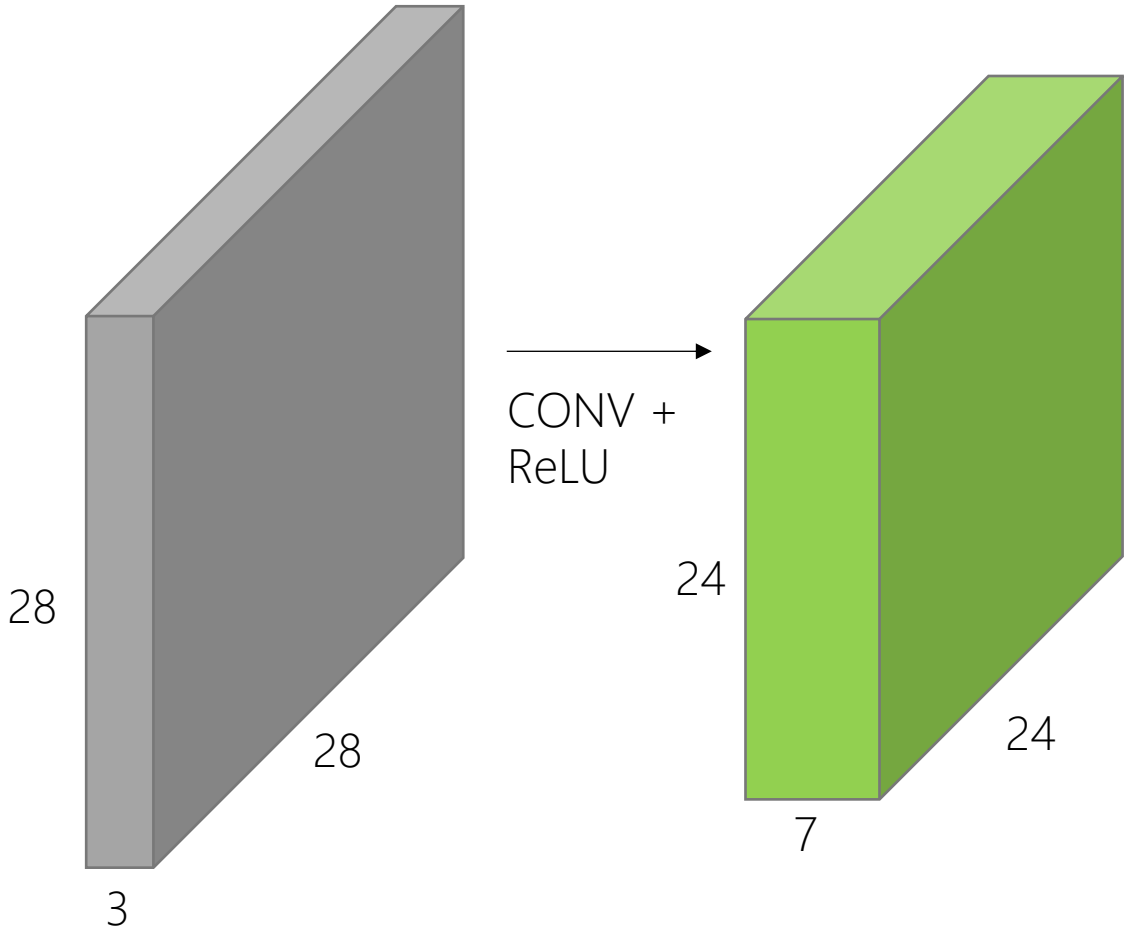


Do the same thing with a "orange" filter

Convolution Layer



ConvNet is a sequence of convolution layers



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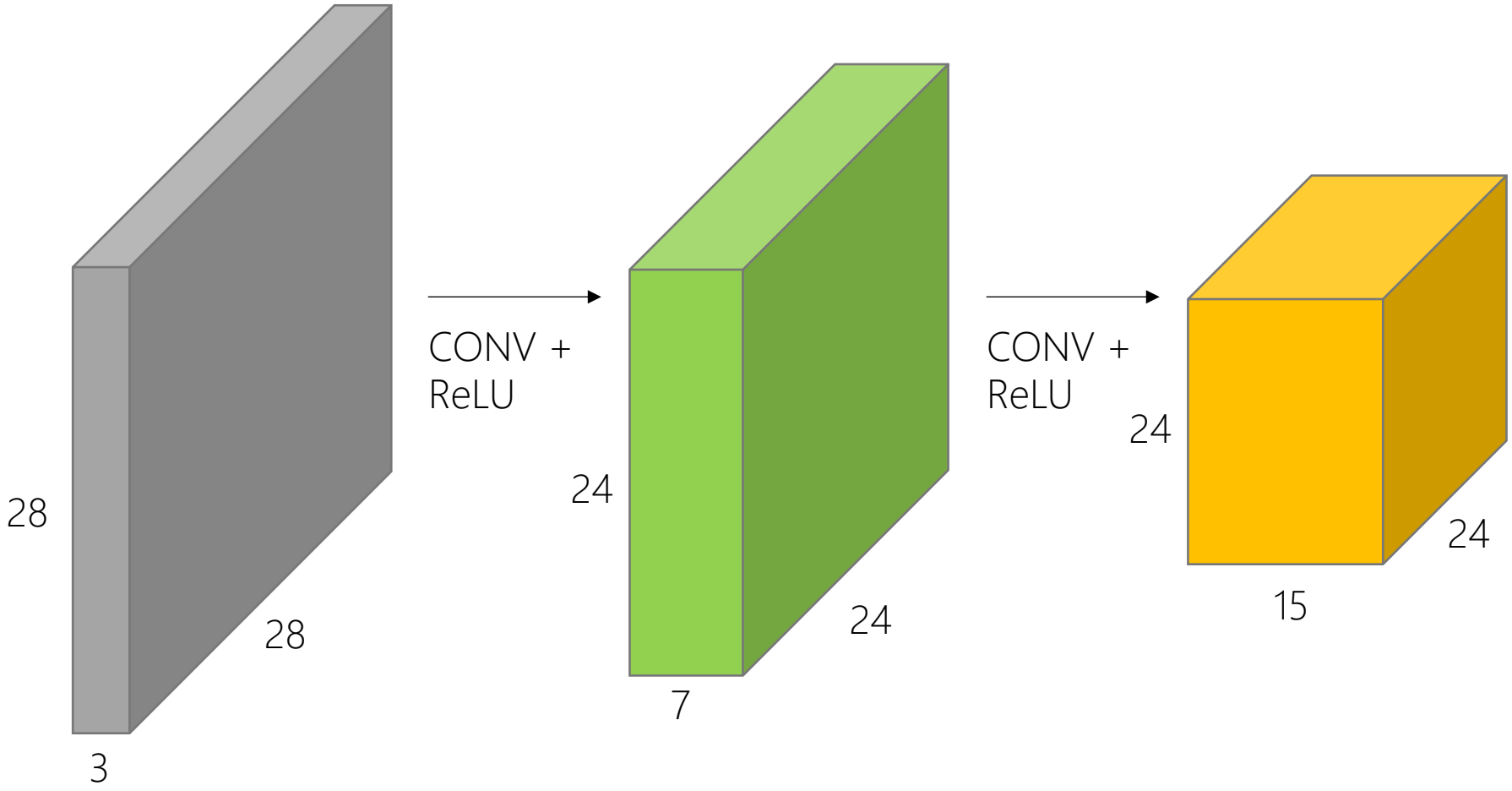

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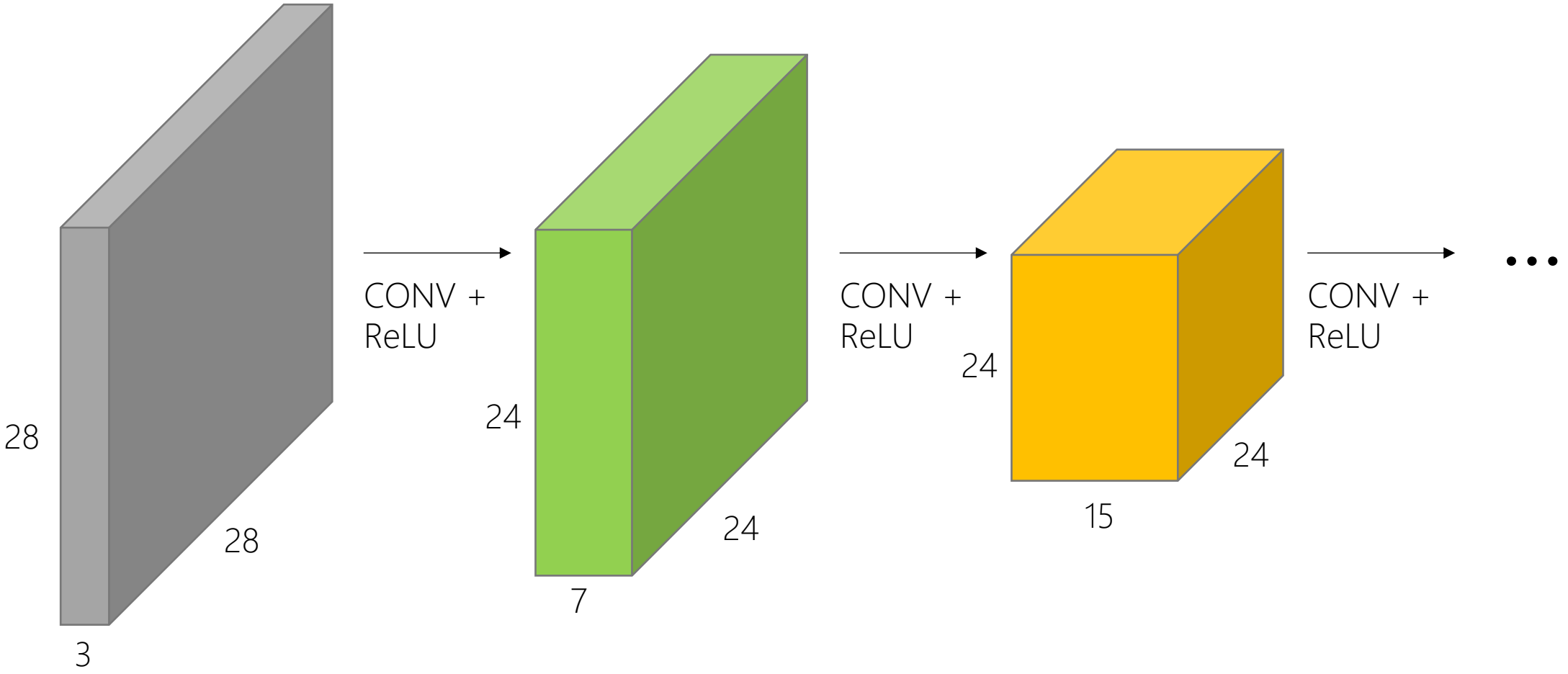
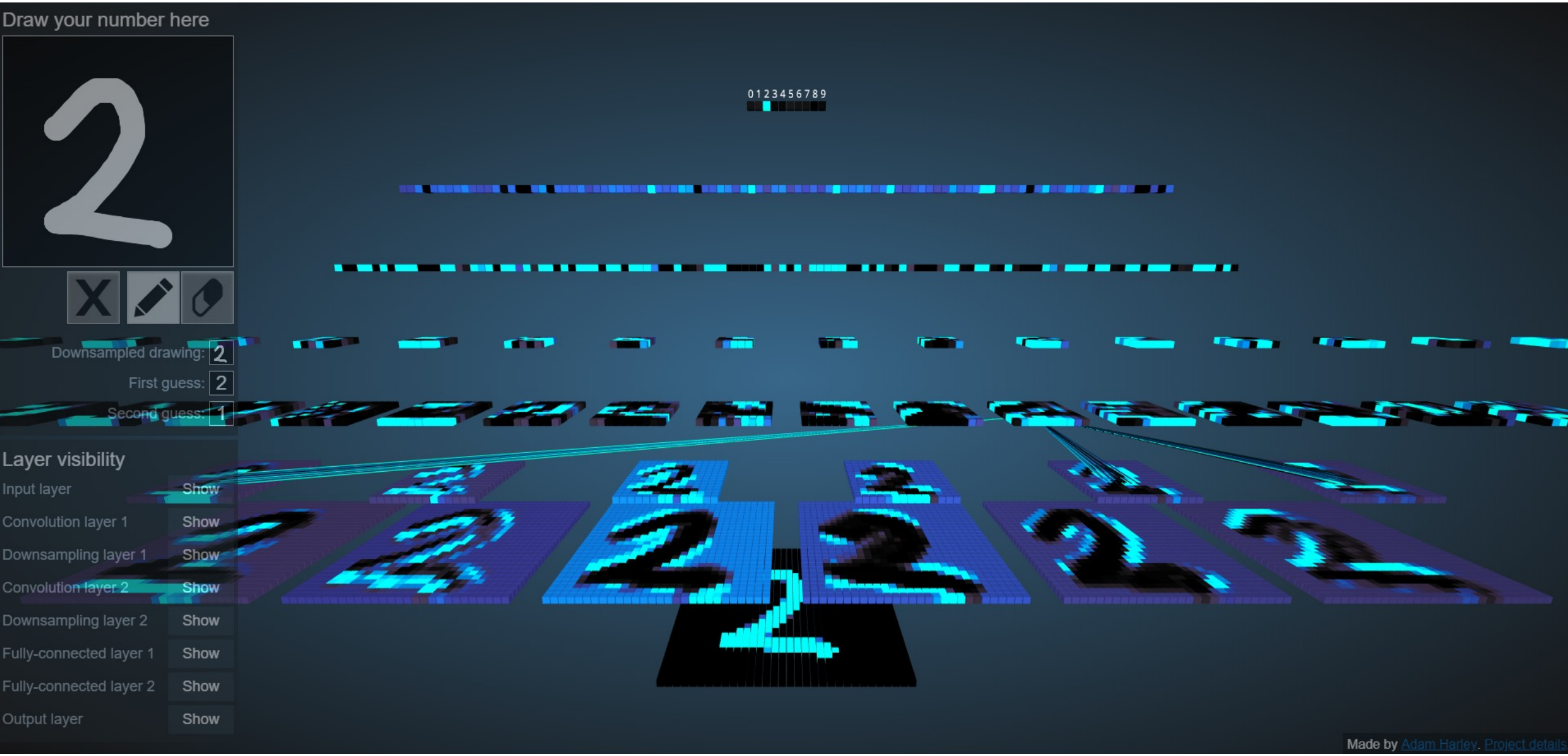
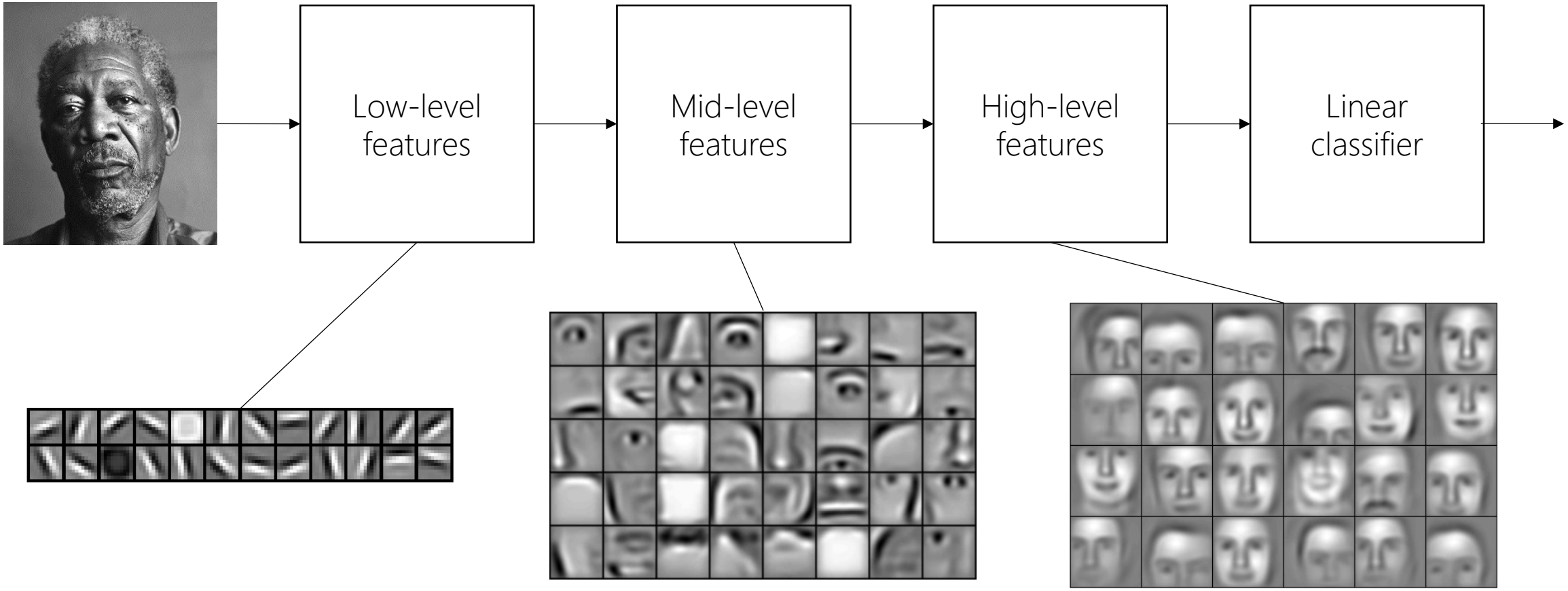




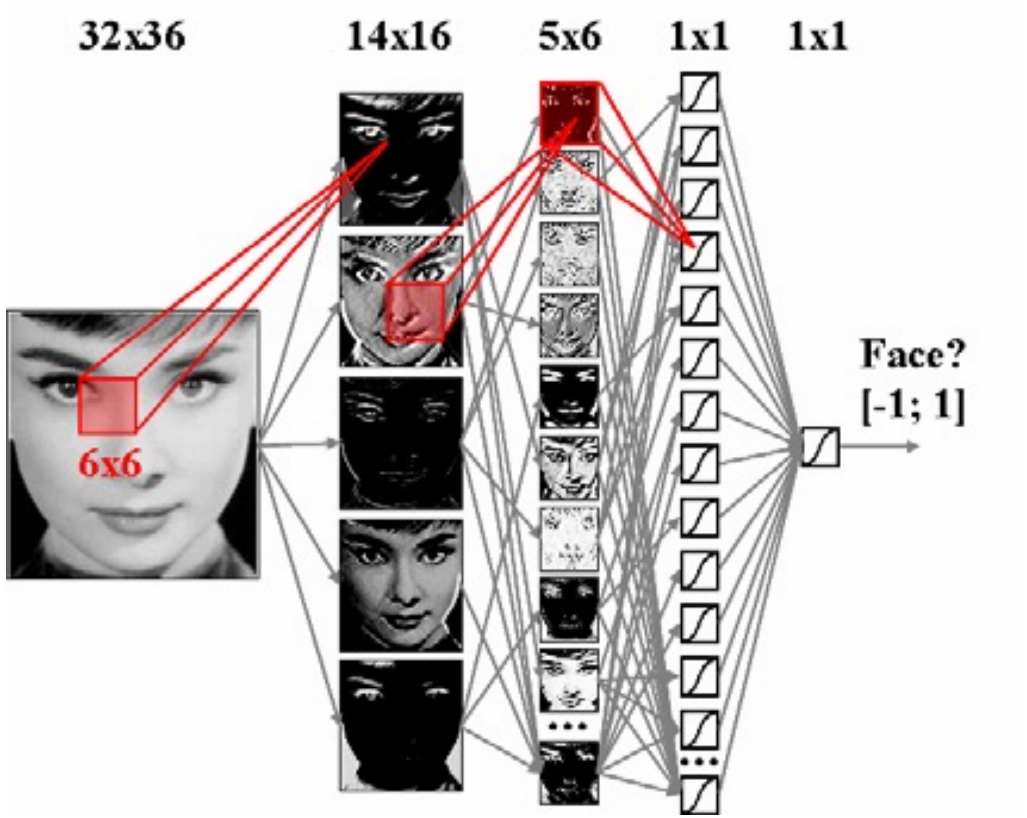
https://adamharley.com/nn_vis/

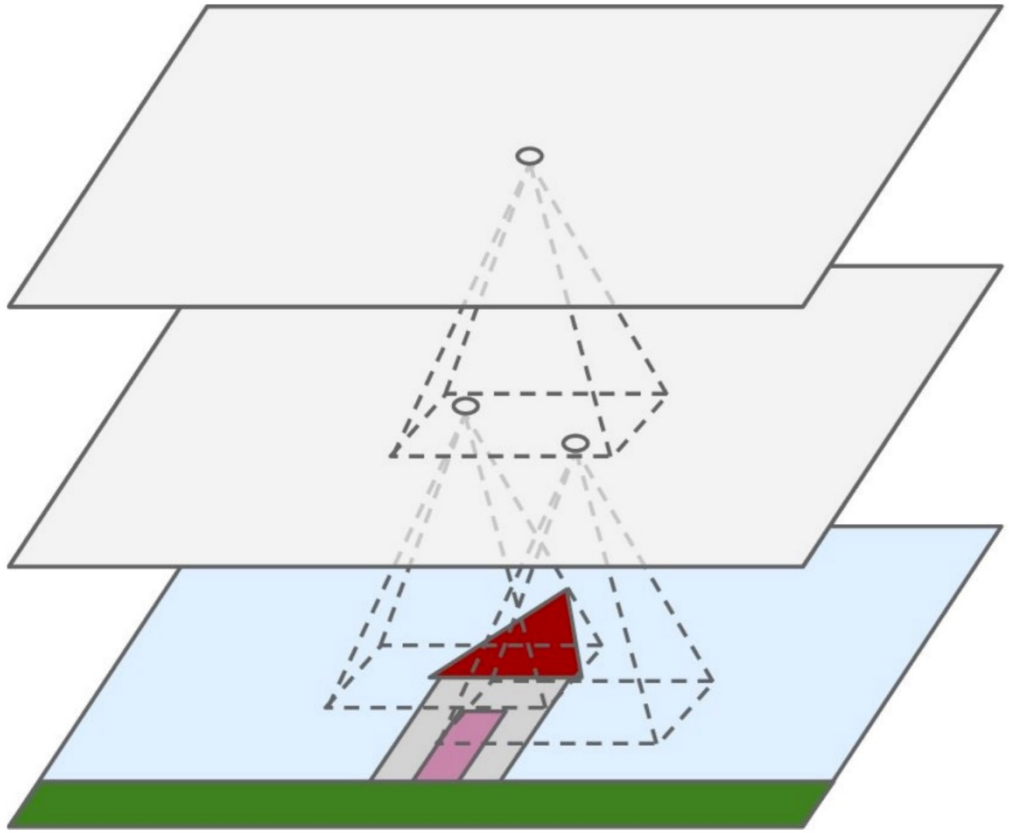
ConvNet is a sequence of convolution layers



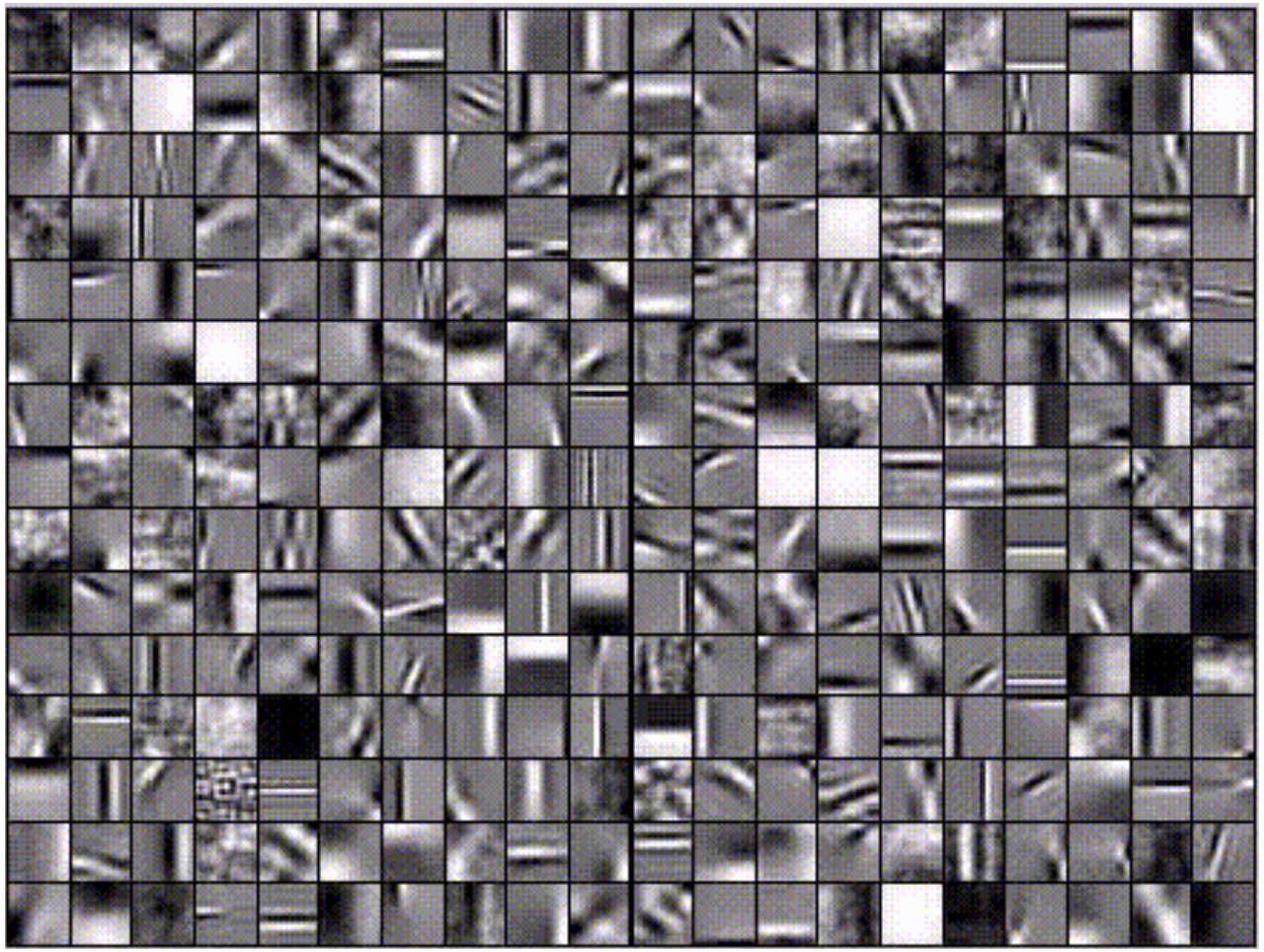
Lee *et al.* (2009)

ConvNet is a sequence of convolution layers





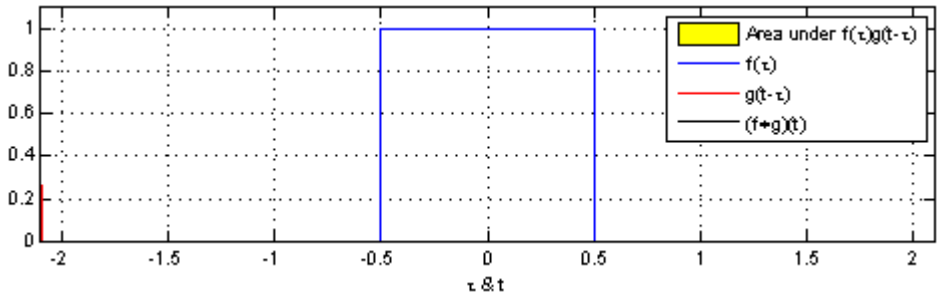
Conv kernels are trainable

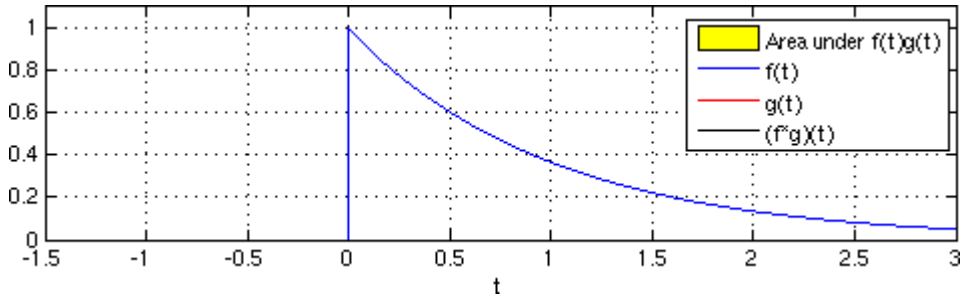


A closer look…

- Convolution

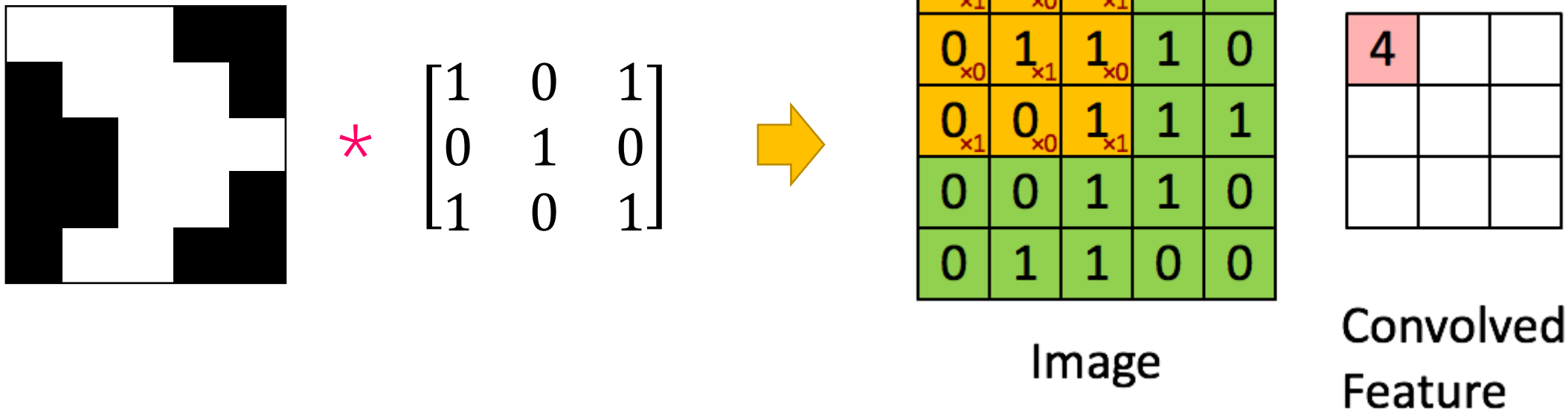
$$(f * g)(t) := \int_{-\infty}^{\infty} f(\tau) g(t - \tau)\, d\tau$$





A closer look…

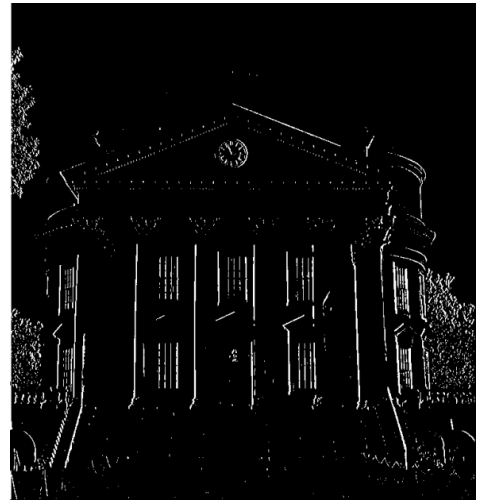
- 2D Discrete Convolution

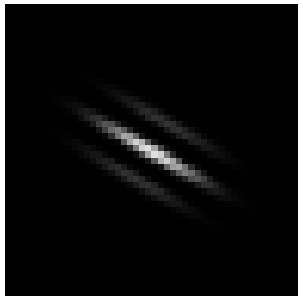


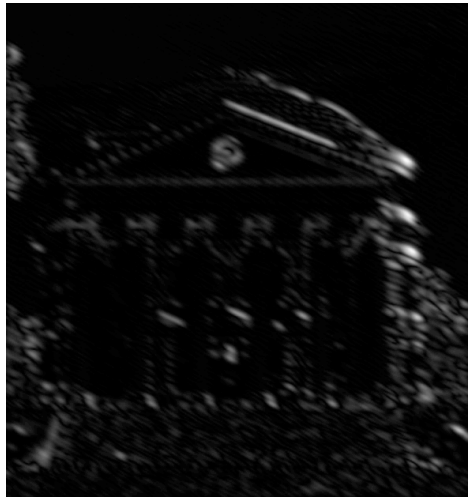
A closer look…

- 2D Discrete Convolution

$$* \begin{bmatrix} 1 & 0 & -1 \\ 2 & 0 & -2 \\ 1 & 0 & -1 \end{bmatrix} =$$



$*$  $=$



Case Study of Convolution

X or X?

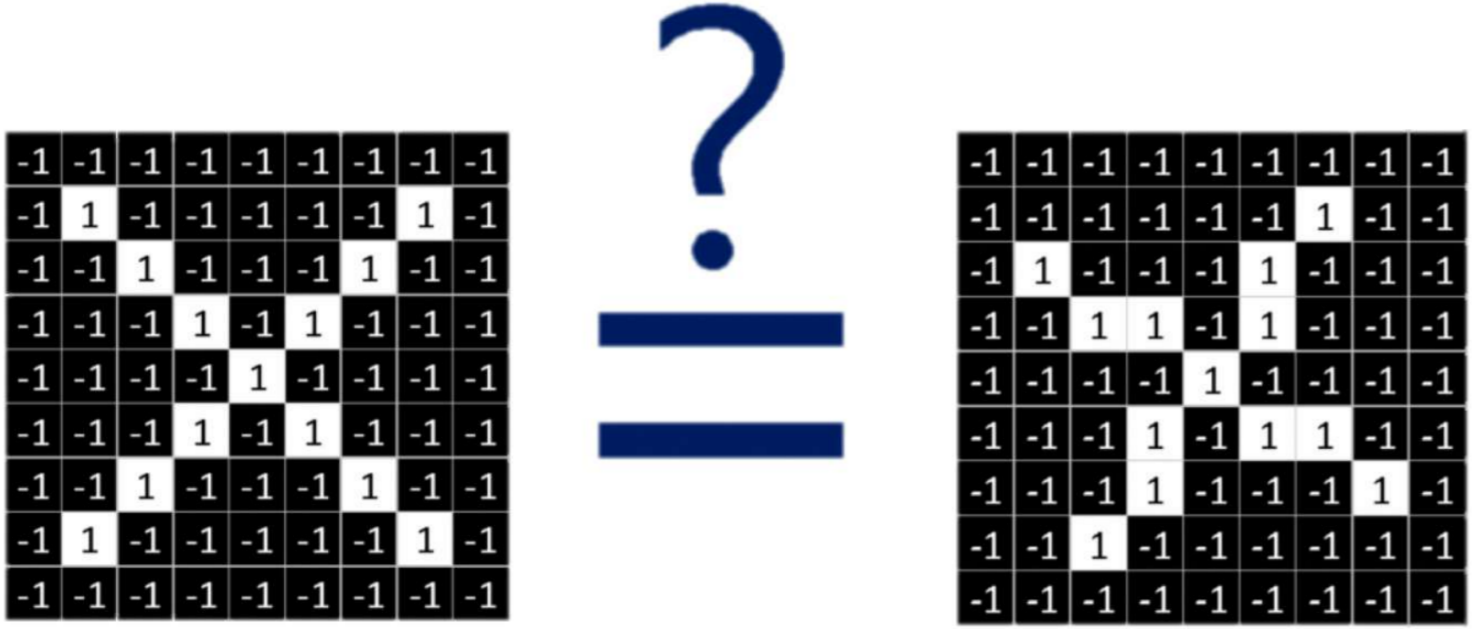
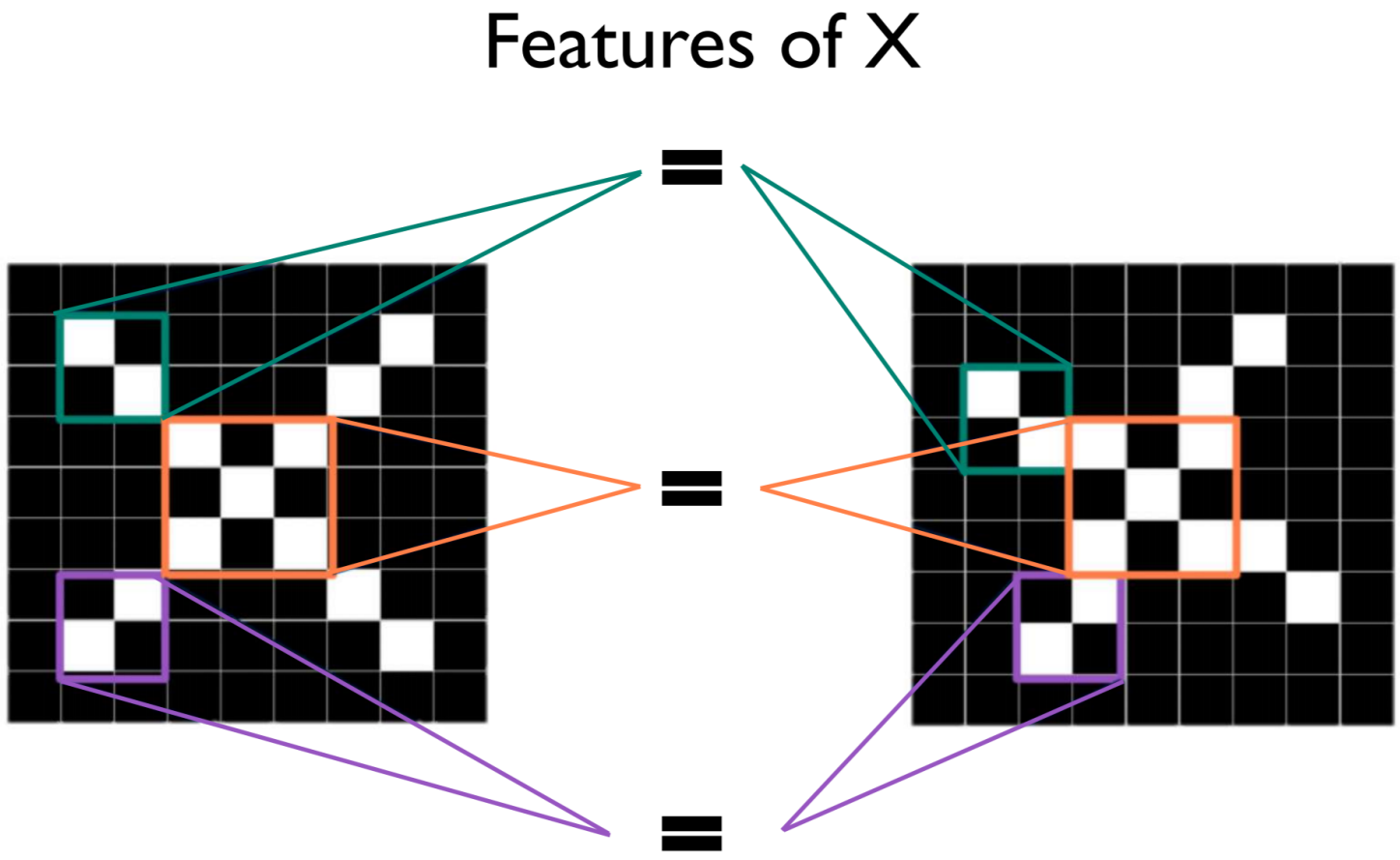
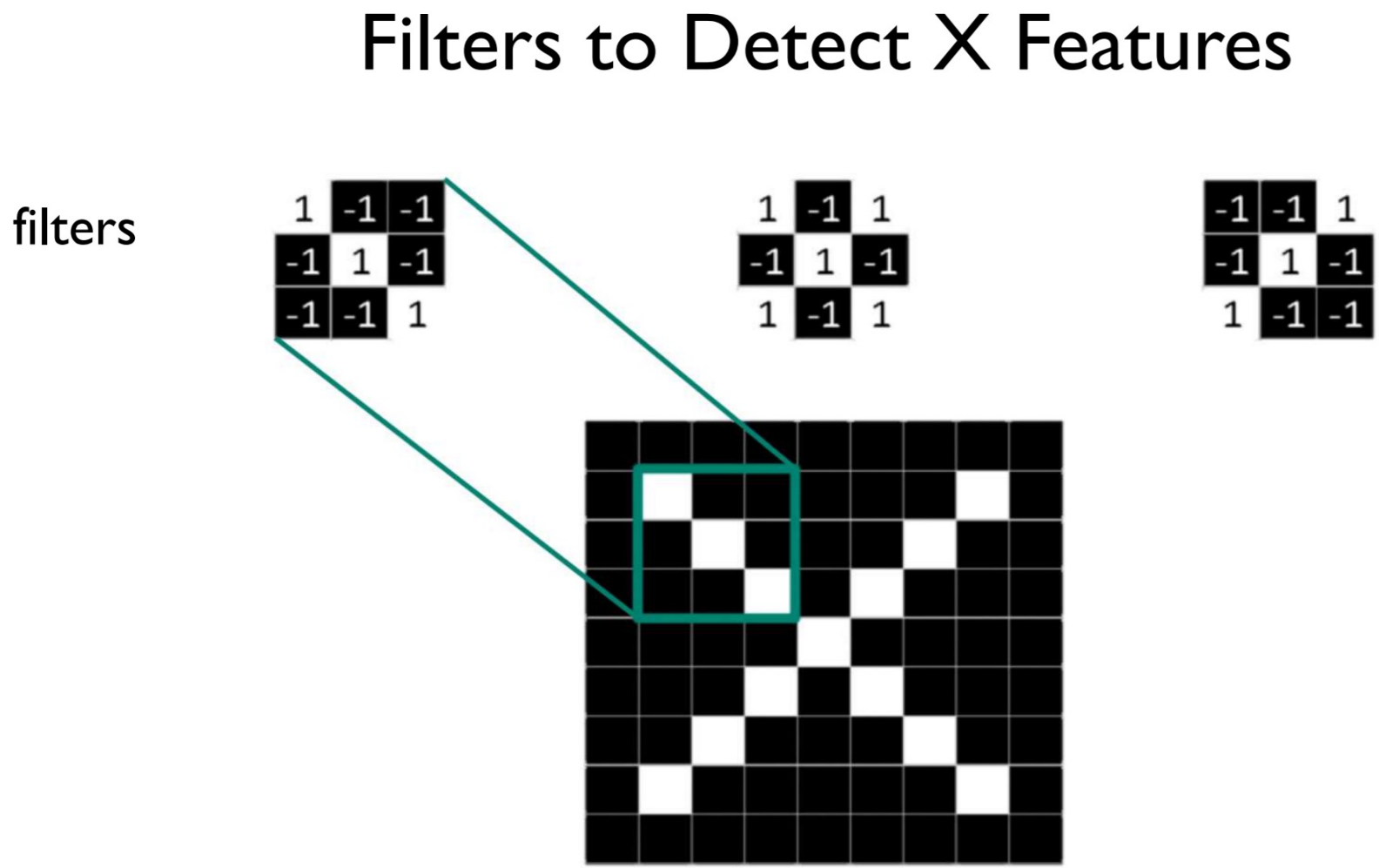


Image is represented as matrix of pixel values… and computers are literal!
We want to be able to classify an X as an X even if it's shifted, shrunk, rotated, deformed.

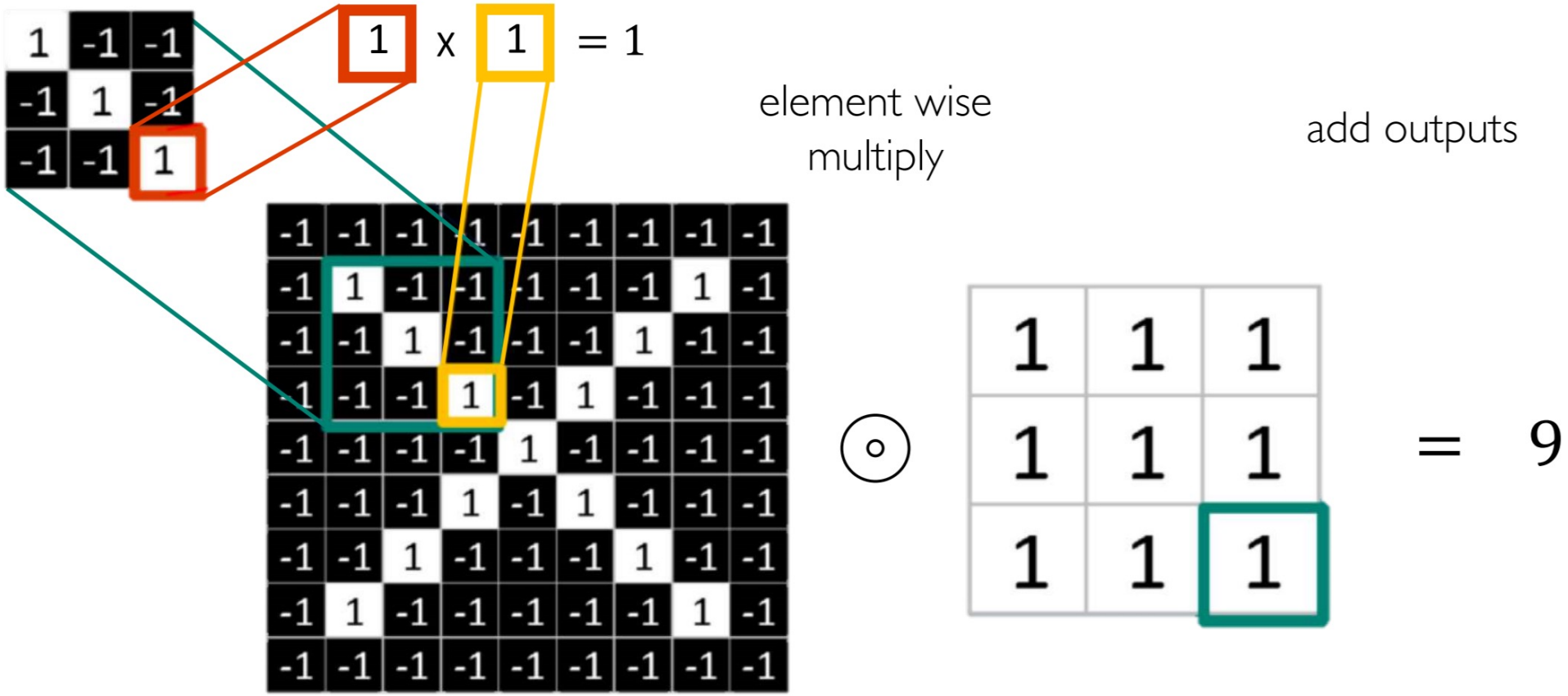
Case Study of Convolution



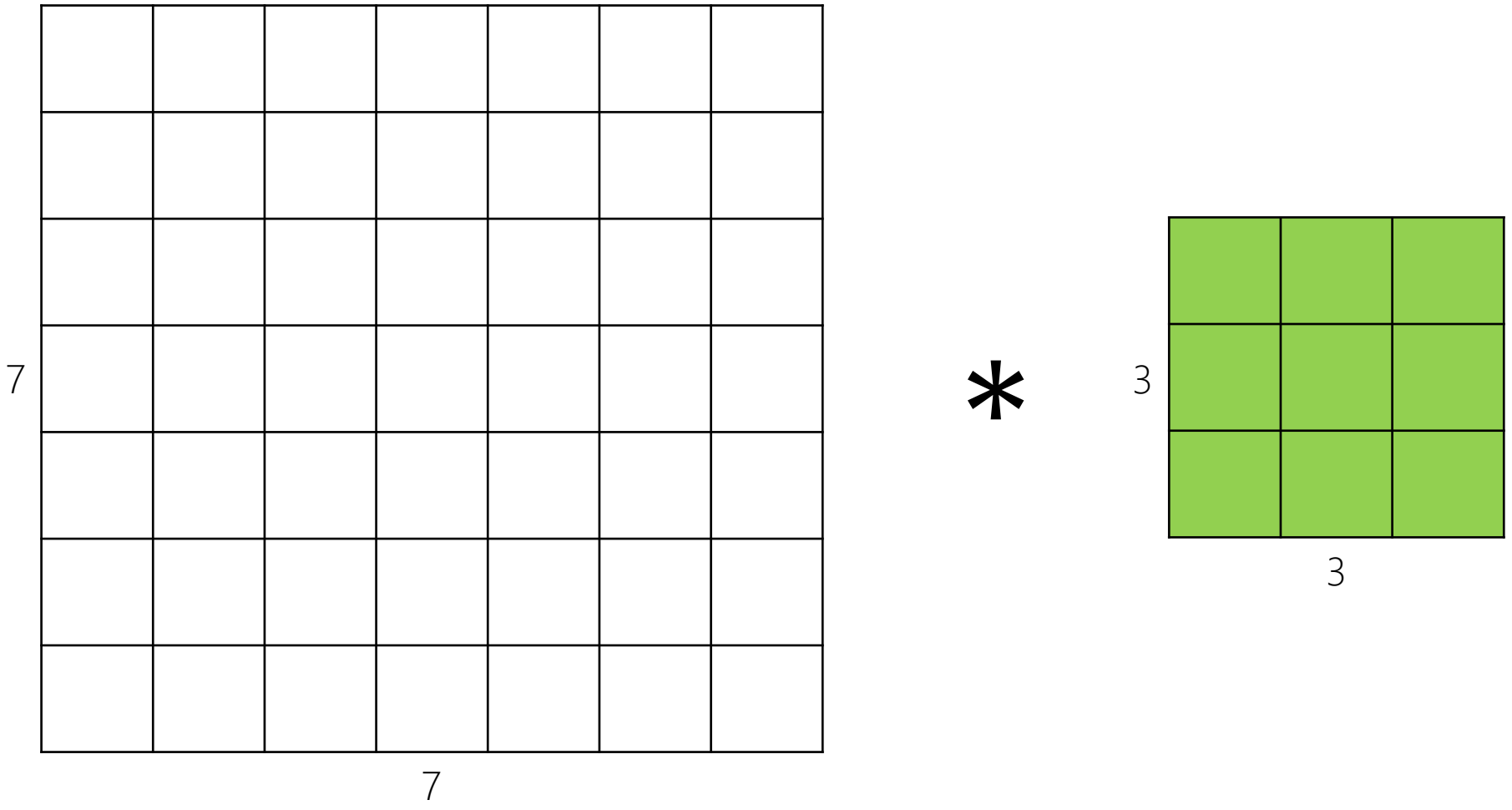
Case Study of Convolution



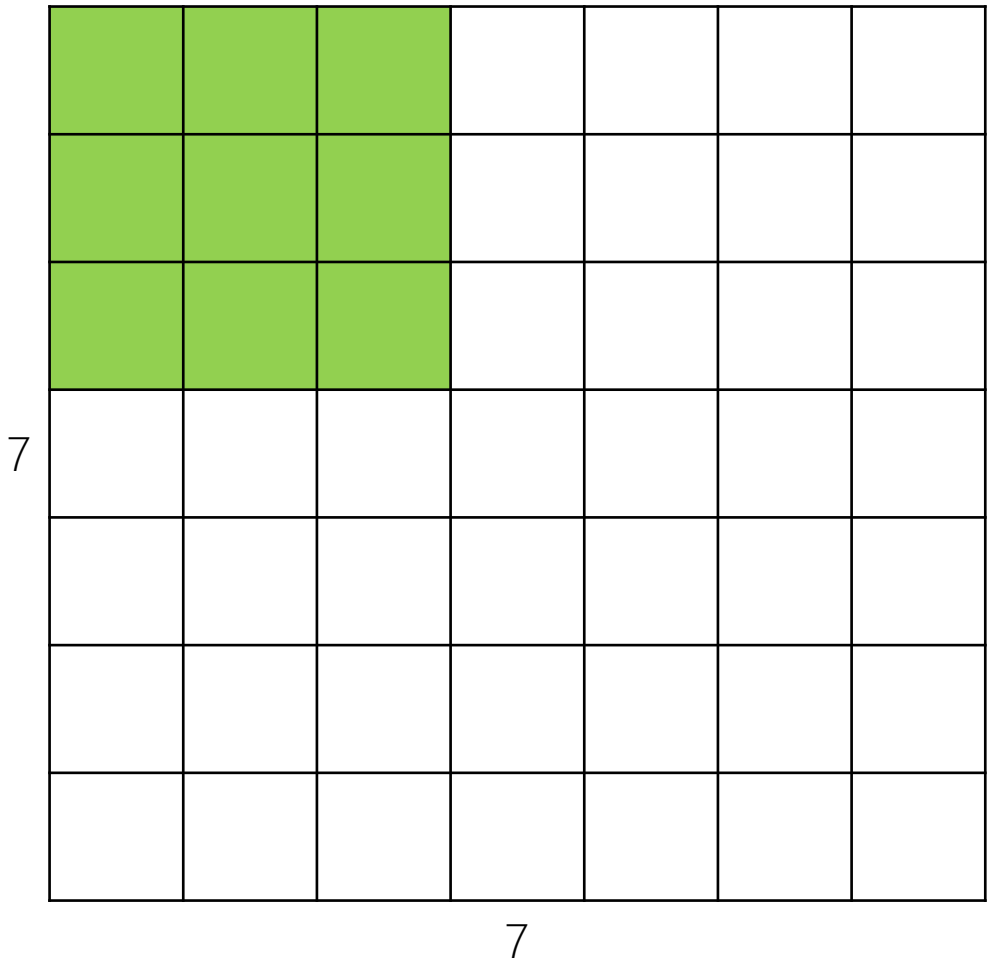
Case Study of Convolution



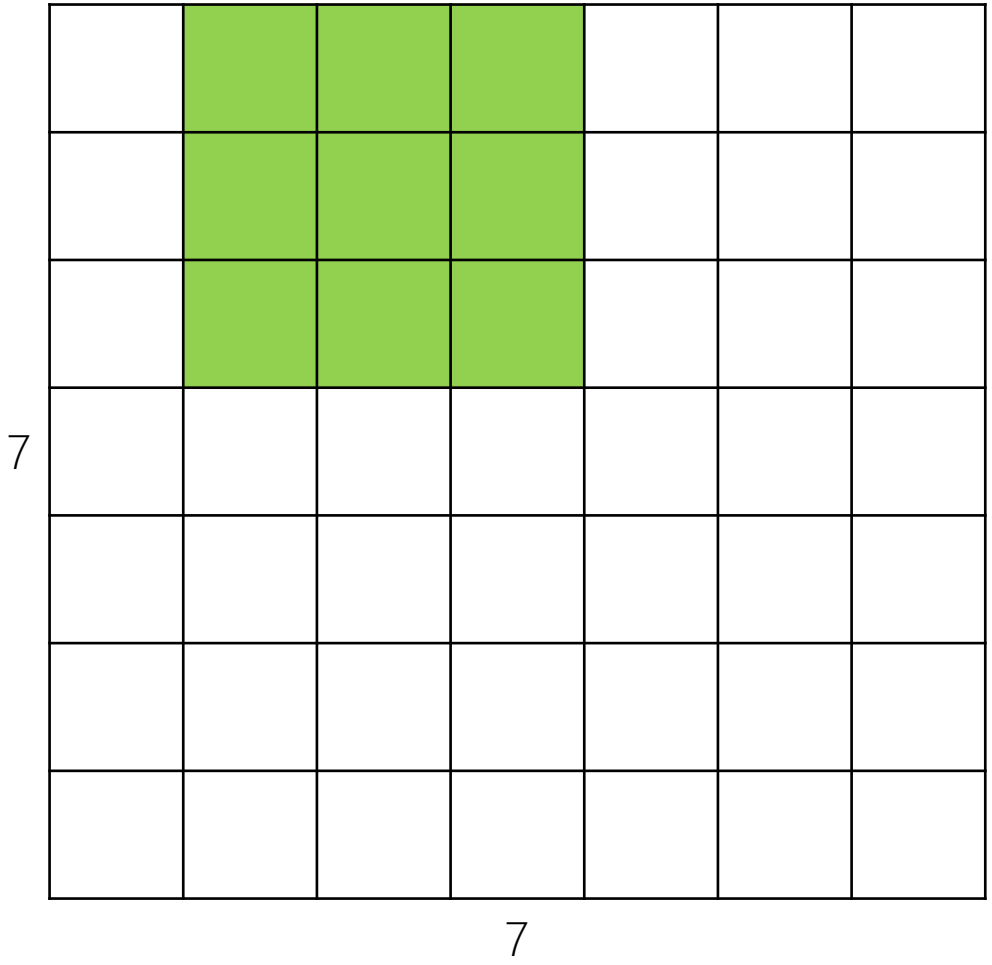
Stride & Padding



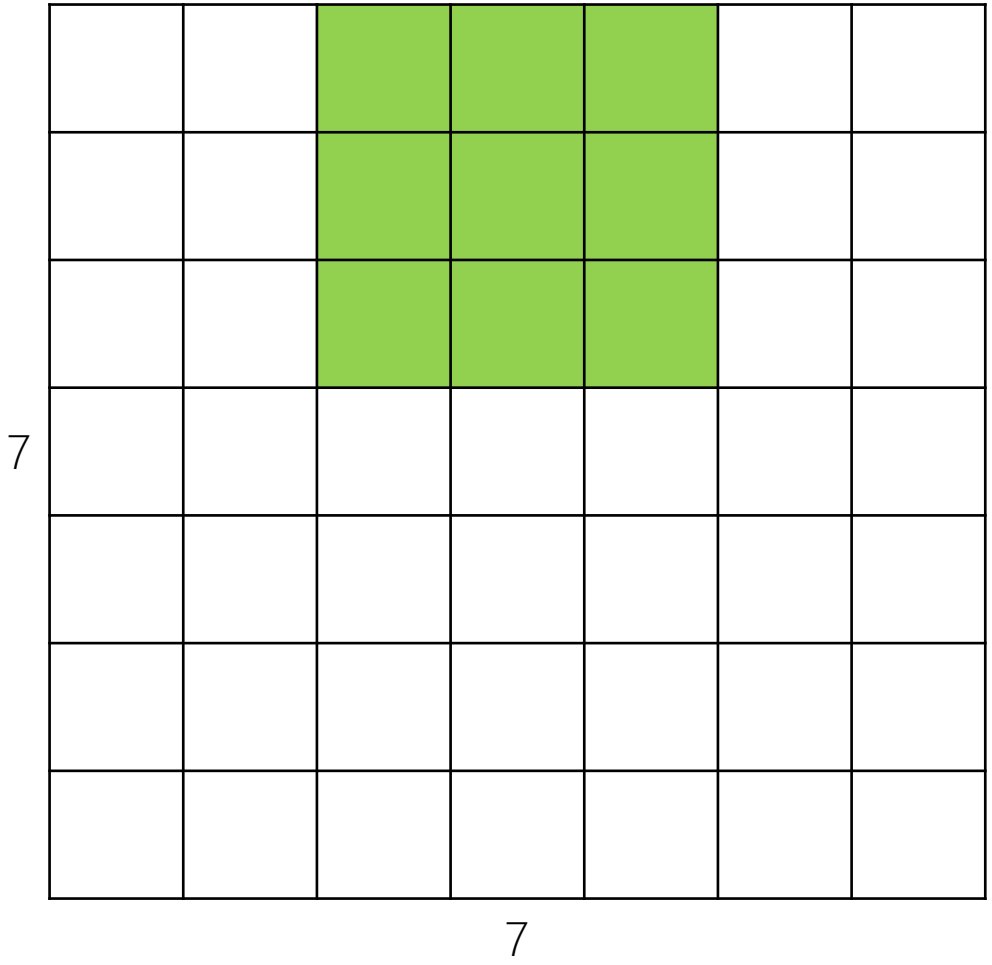
Stride & Padding



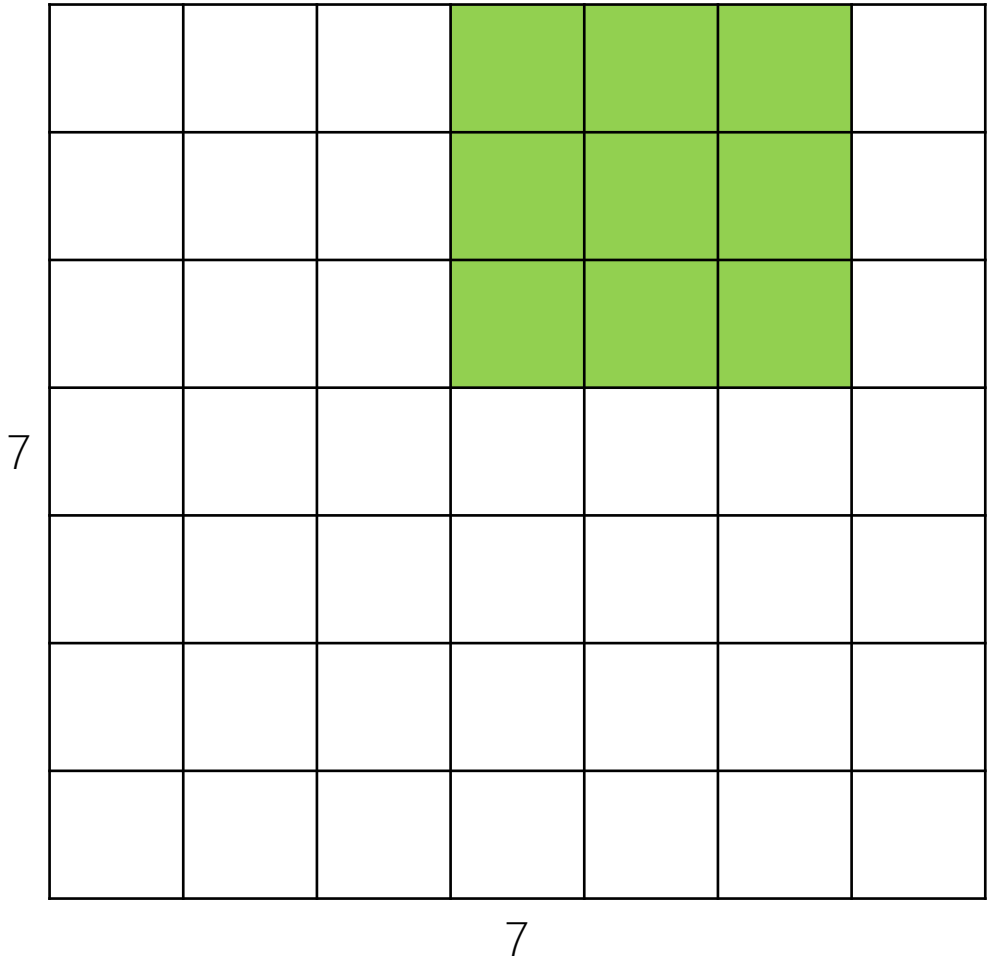
Stride & Padding



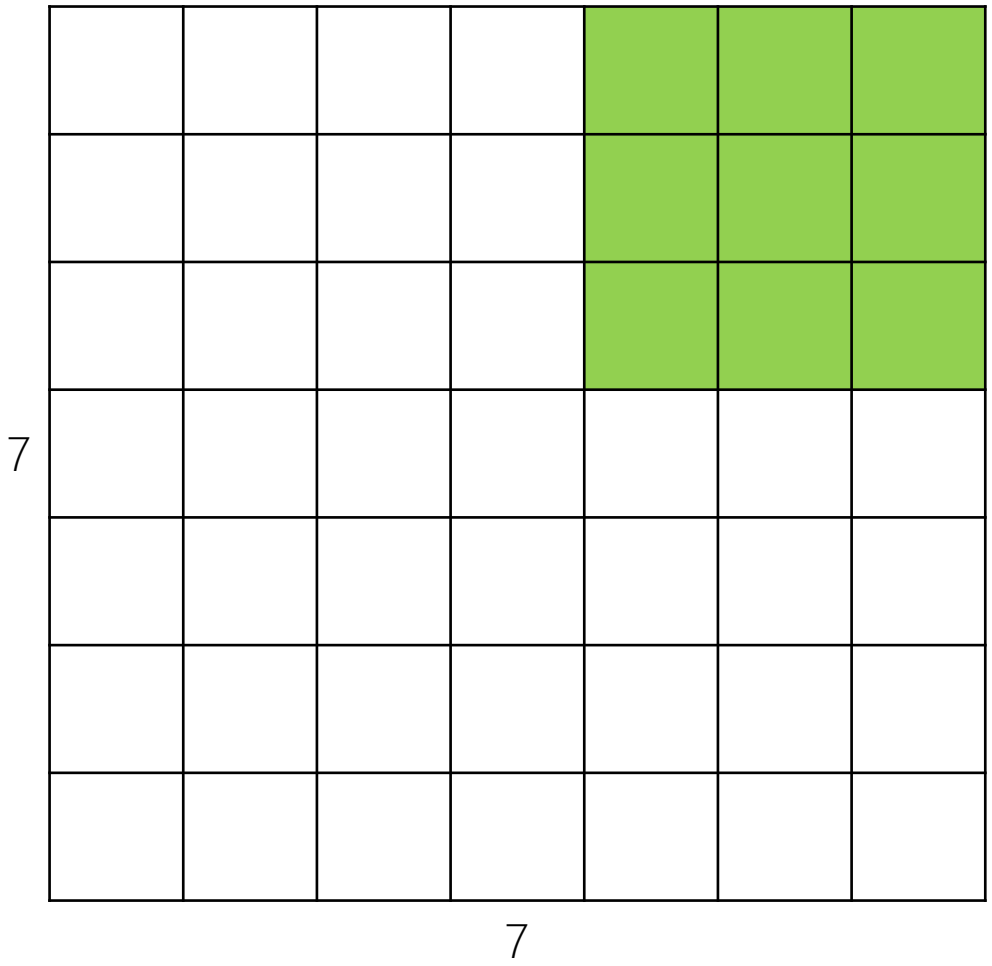
Stride & Padding



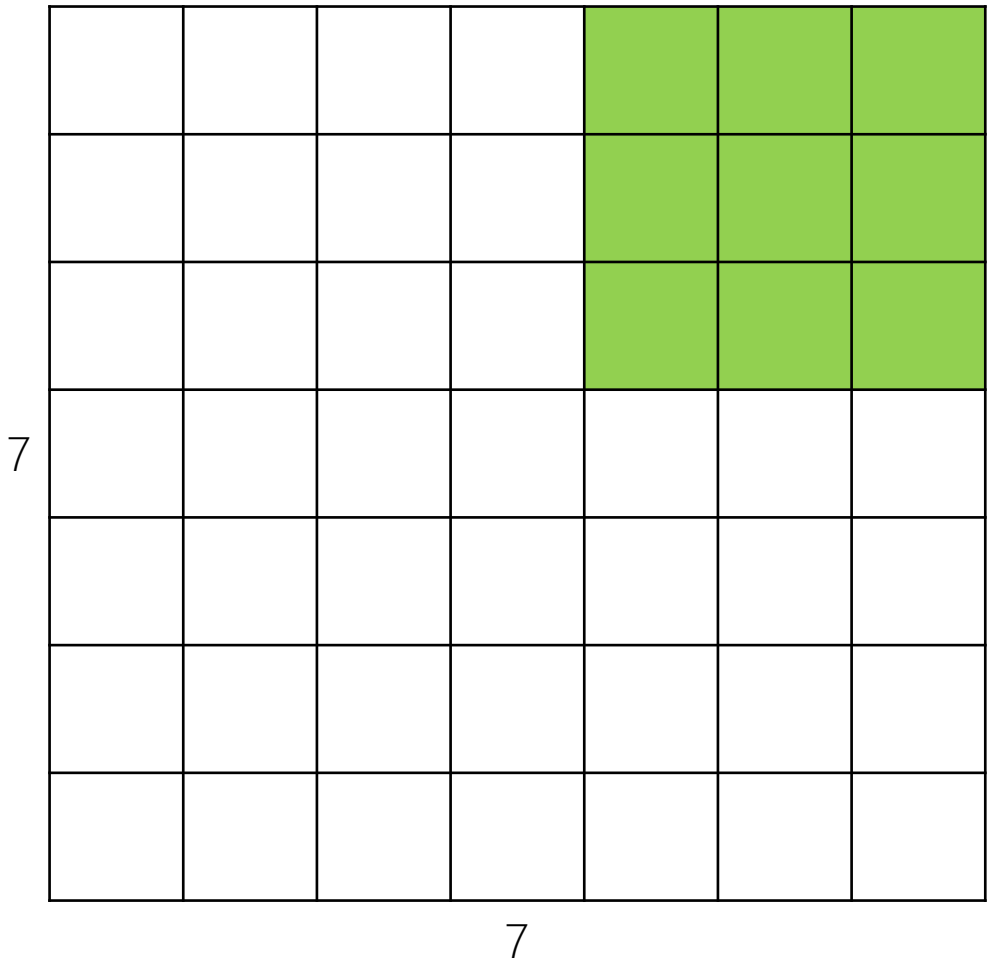
Stride & Padding



Stride & Padding

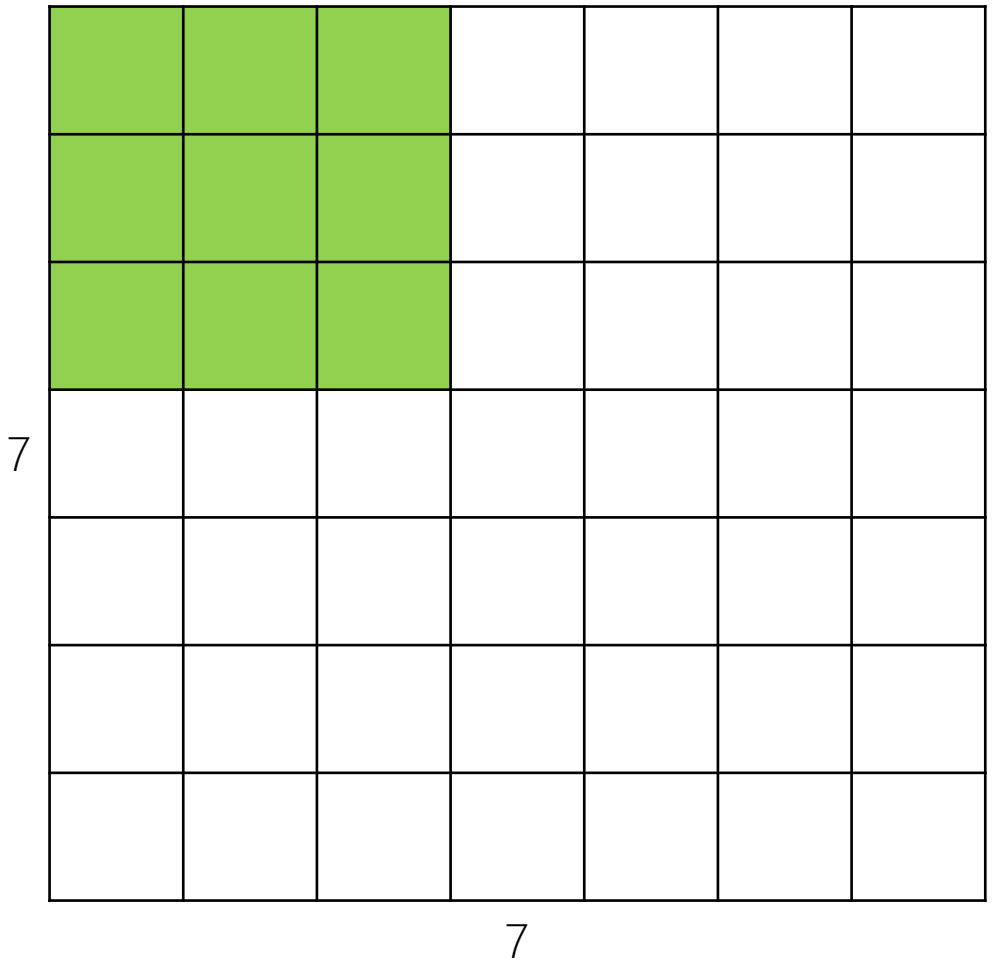


Stride & Padding



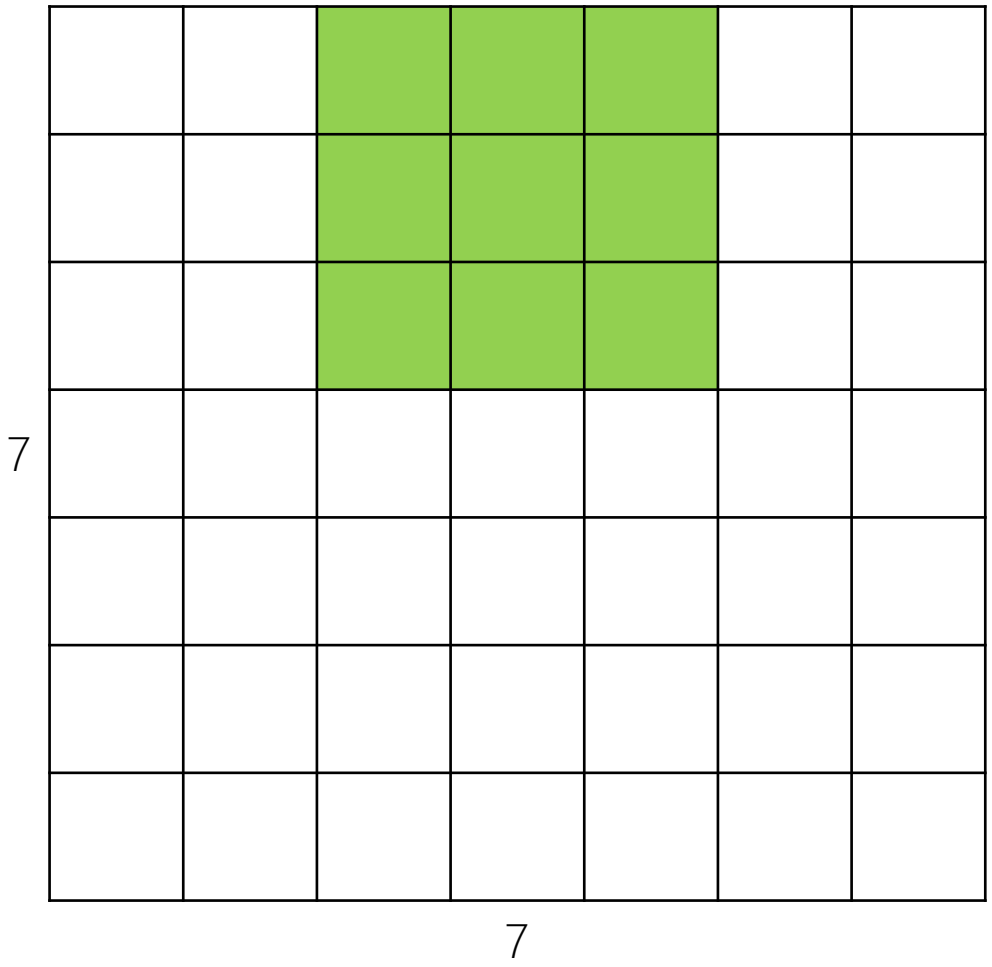
→ 5x5 output

Stride & Padding



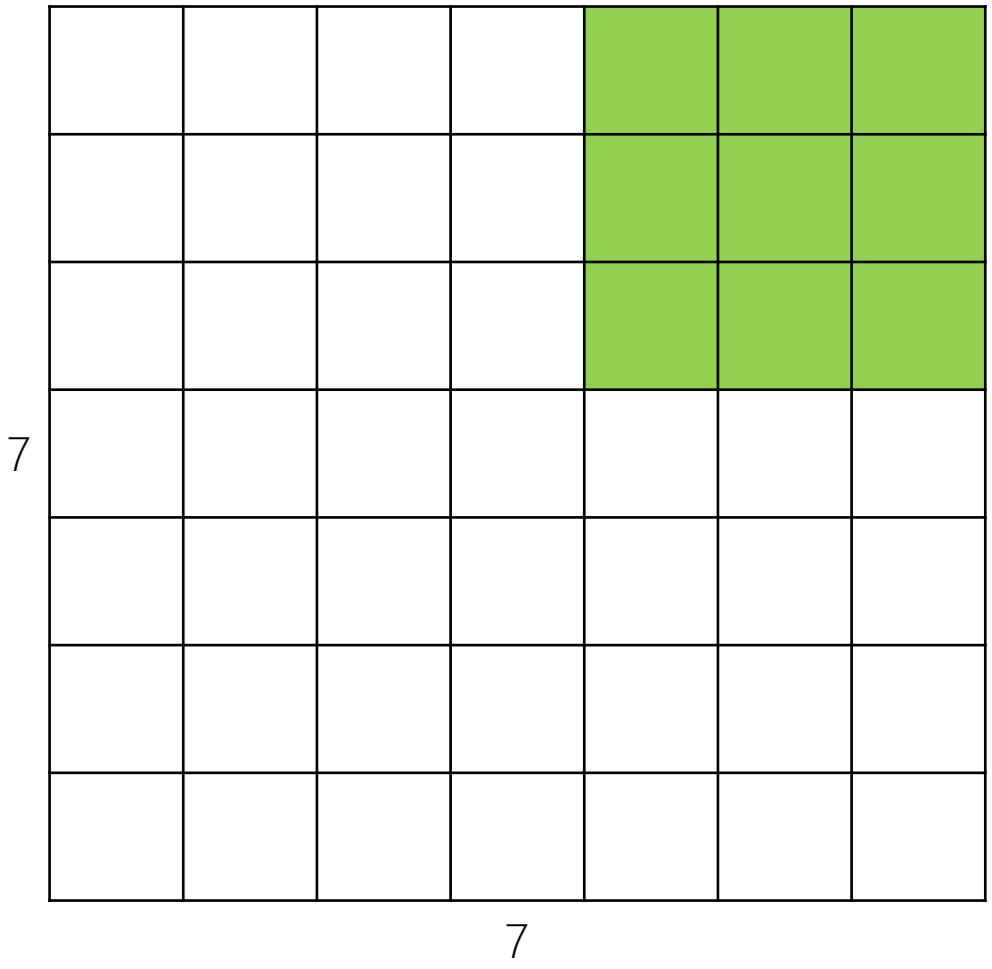
Now with stride 2

Stride & Padding



Now with stride 2

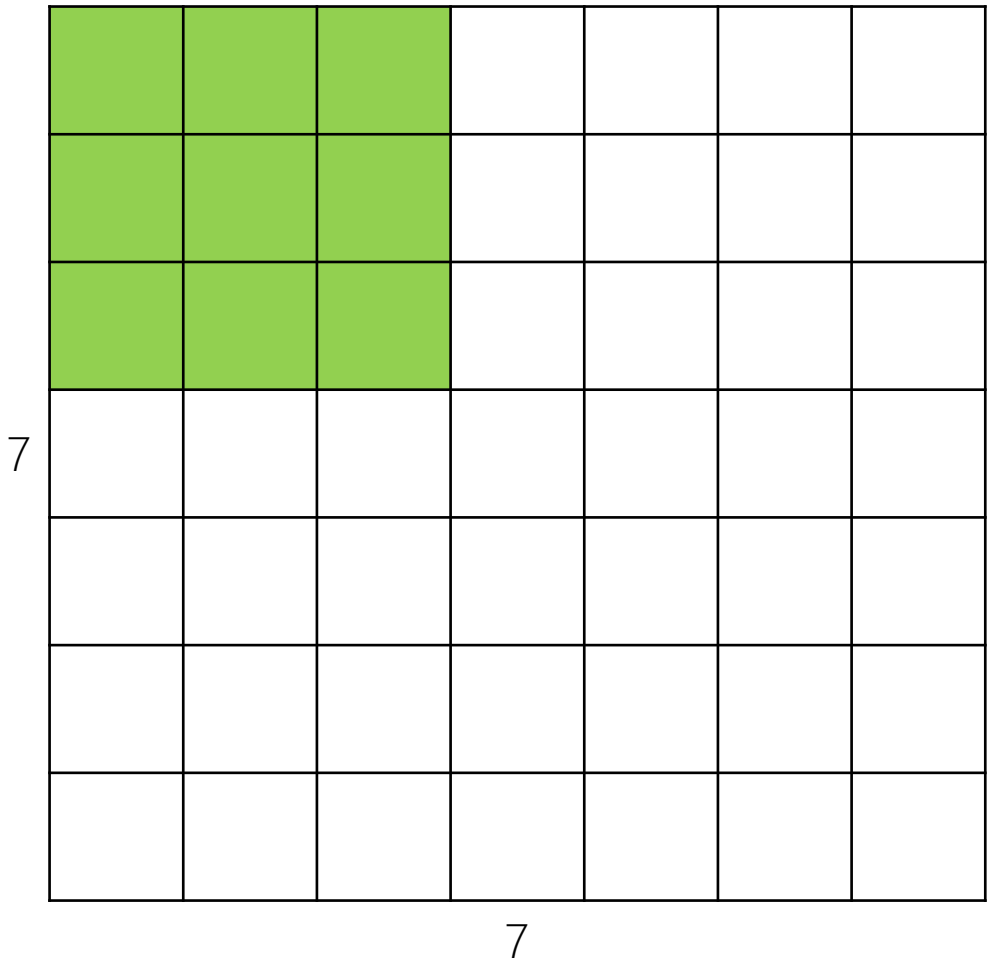
Stride & Padding



Now with stride 2

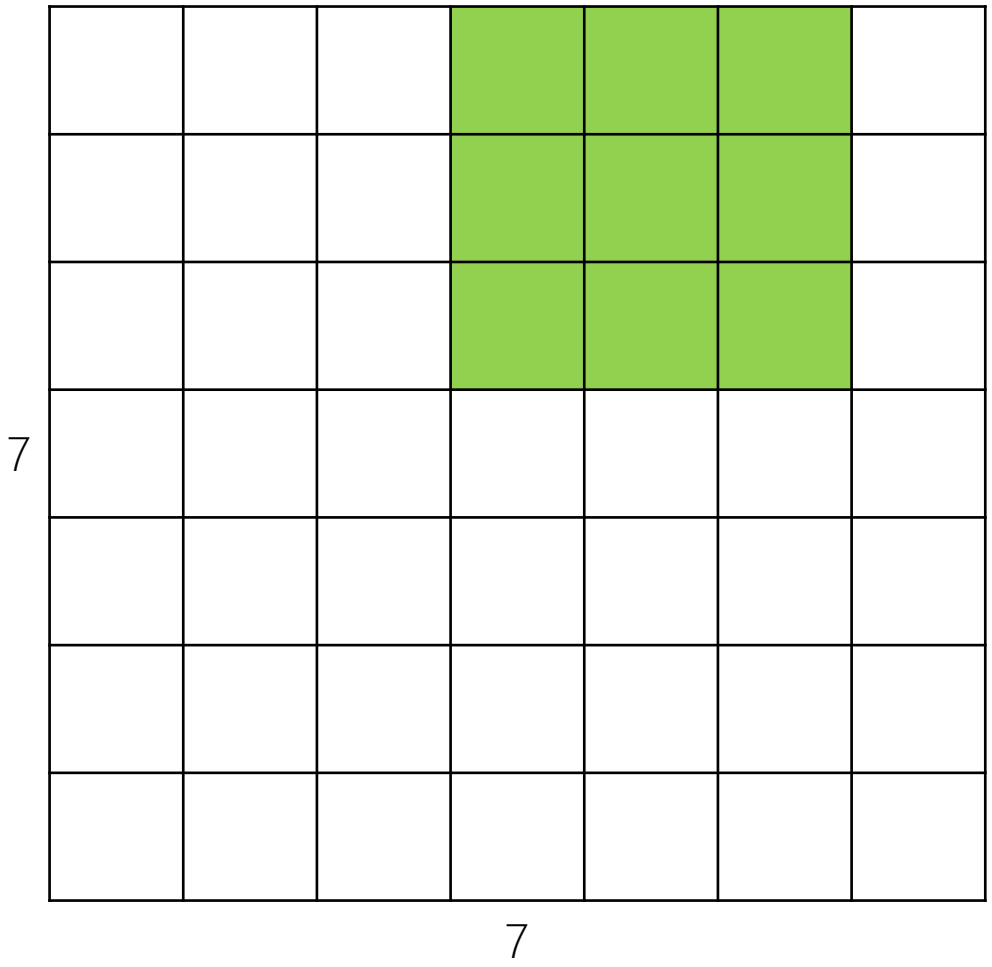
→ 3x3 output

Stride & Padding



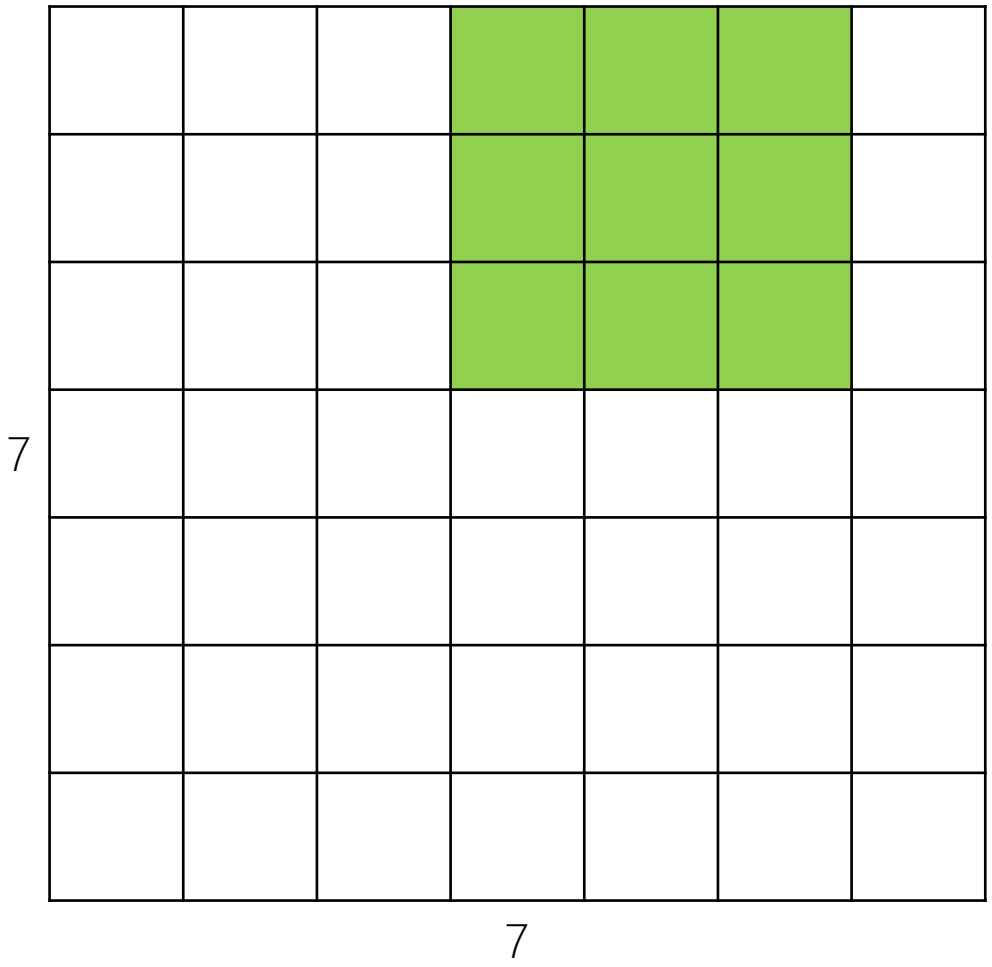
How about with stride 3?

Stride & Padding



How about with stride 3?

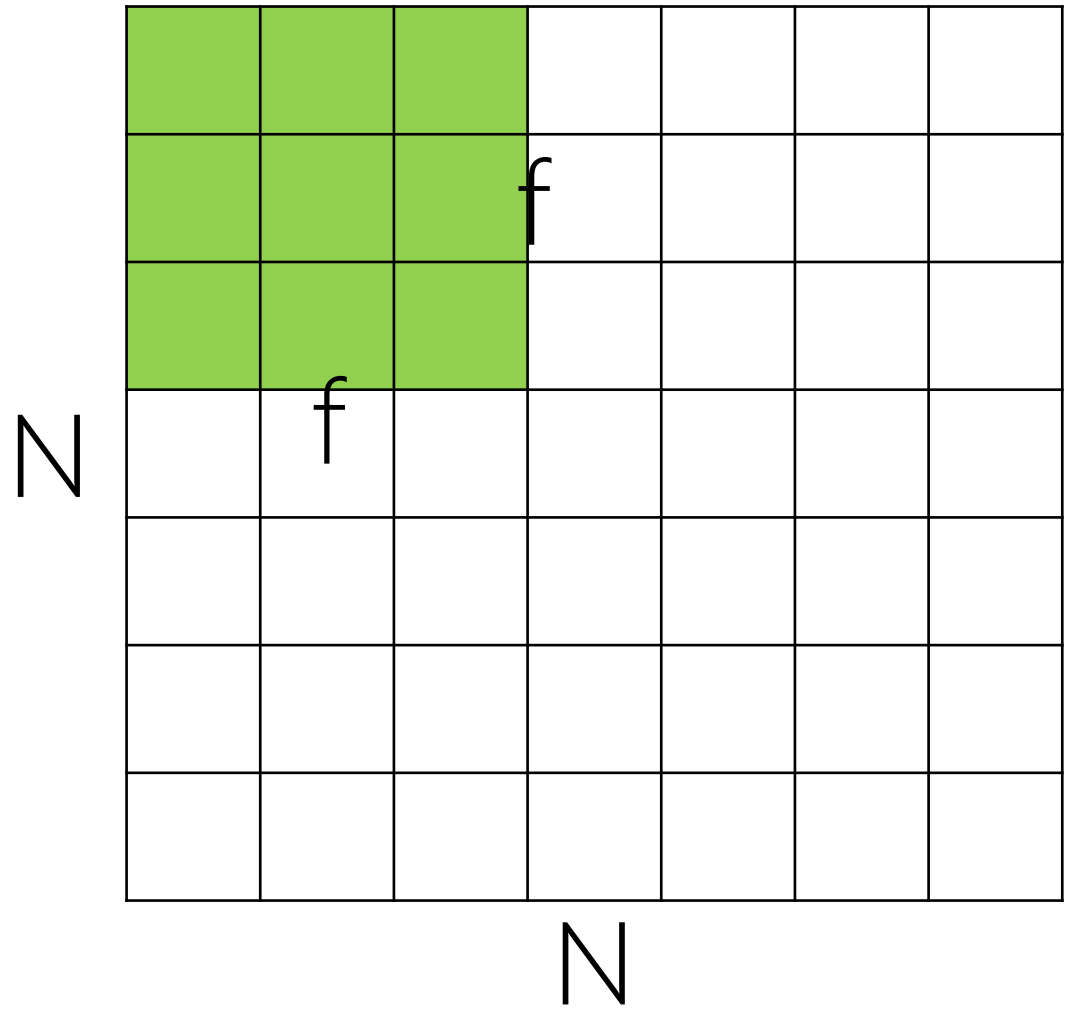
Stride & Padding



How about with stride 3?

→ …?

Stride & Padding

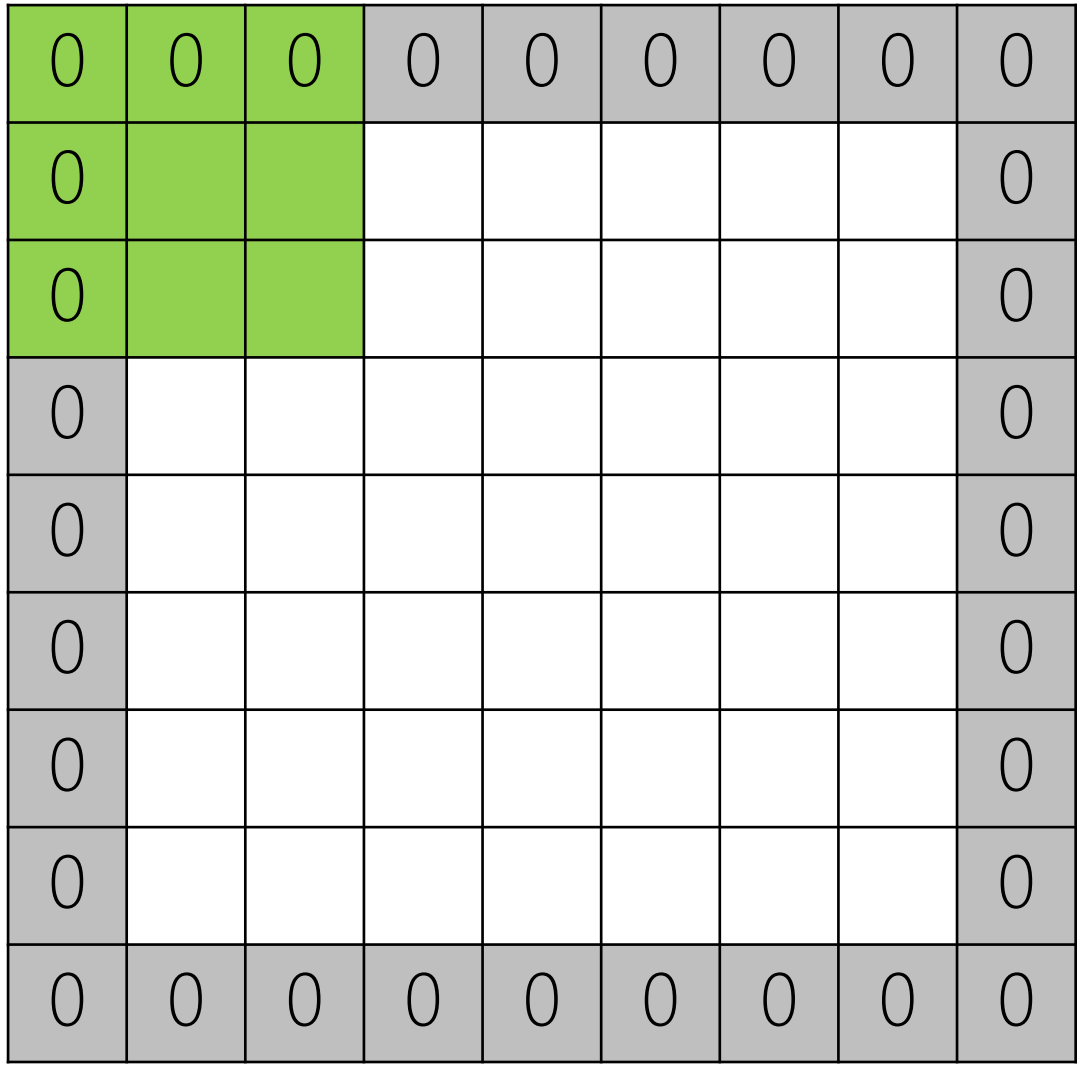


Output size:
(N-f)/stride + 1

Example, N=7, f=3:

Stride 1: (7-3)/1 + 1 = 5
Stride 2: (7-3)/2 + 1 = 3
Stride 3: (7-3)/3 + 1 = 2.3333

Stride & Padding

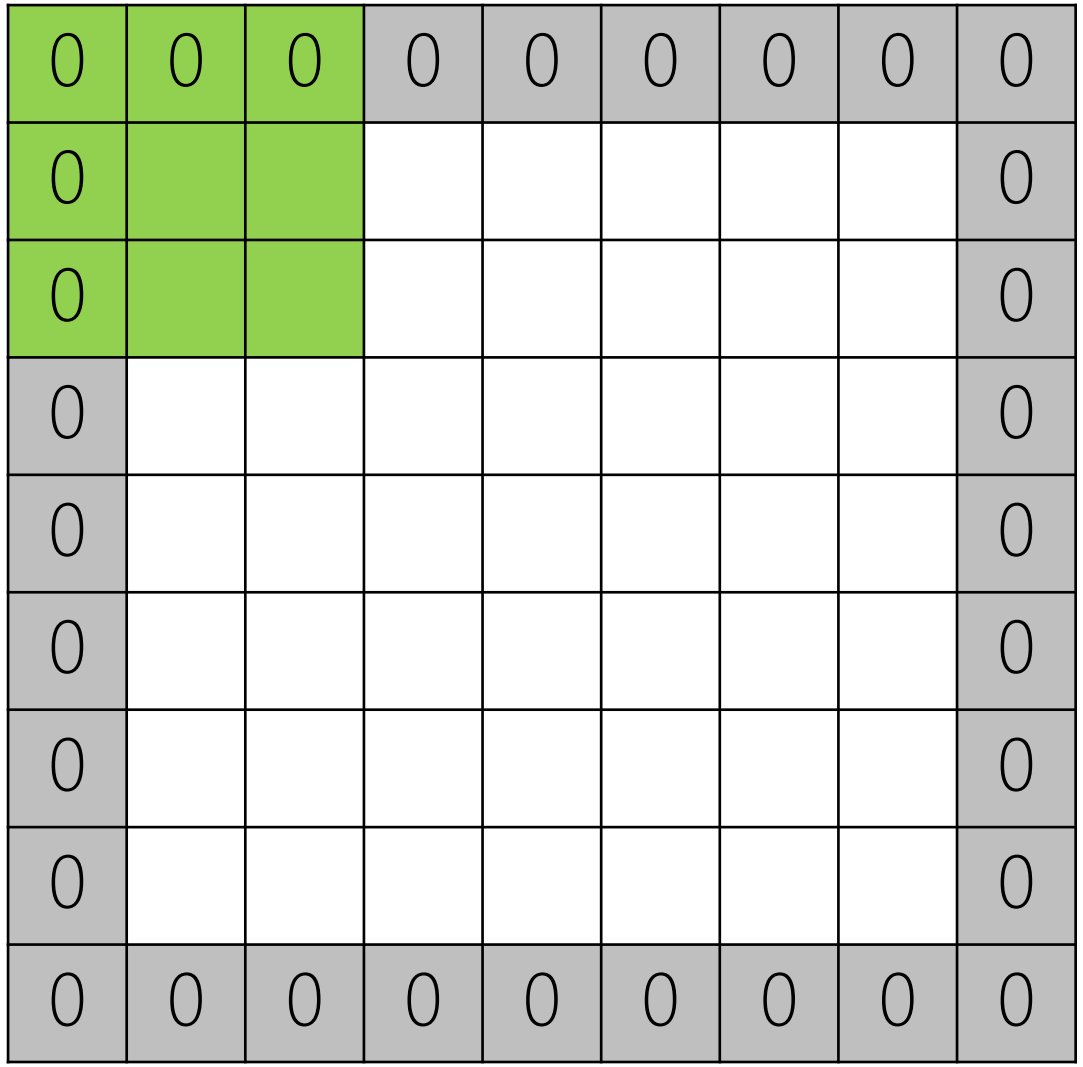


Output size:

(N-f)/stride + 1

Q. 7x7 input, 3x3 kernel, with stride 1, padded with 1 pixel. Output size?

Stride & Padding



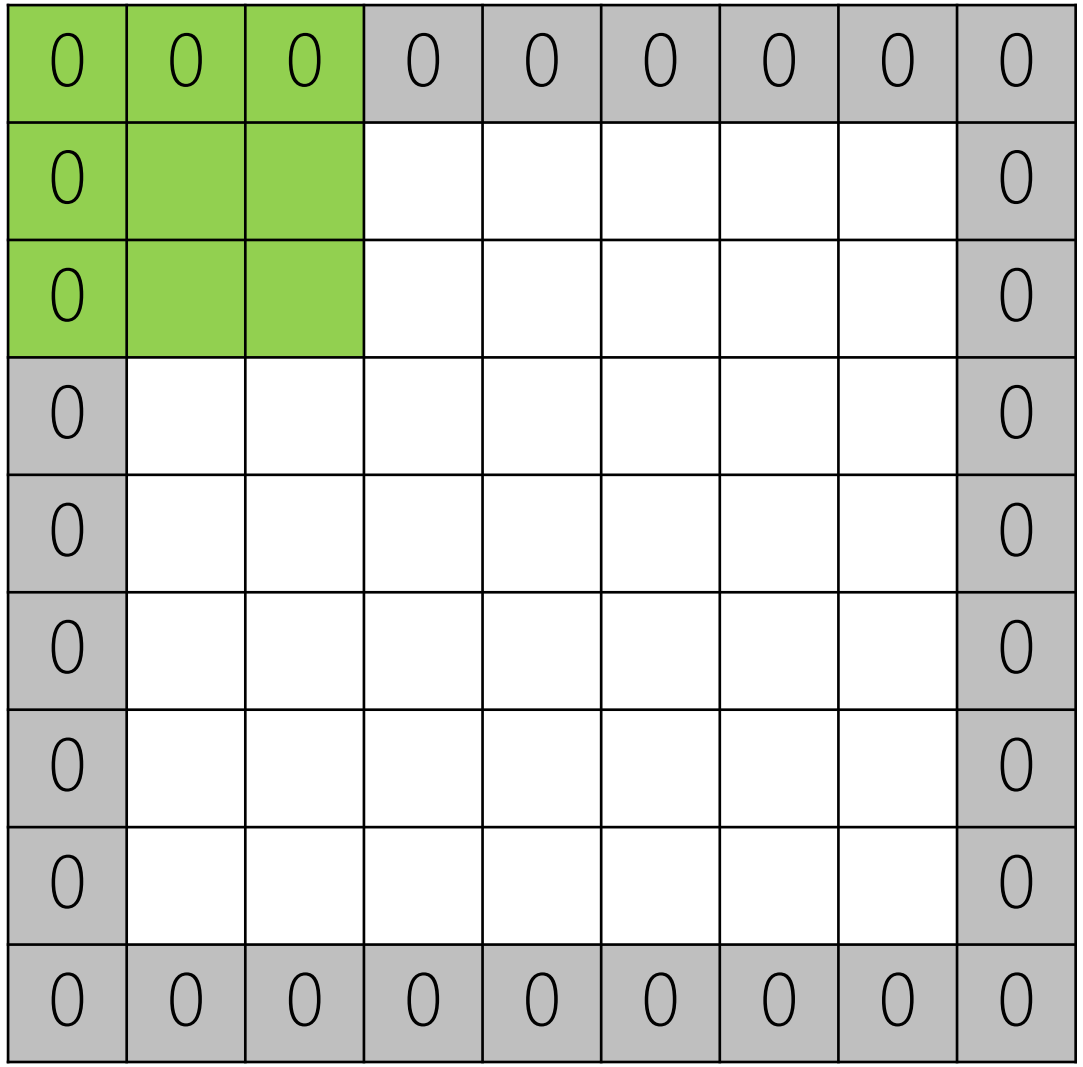
Output size:

(N-f)/stride + 1

Q. 7x7 input, 3x3 kernel, with stride 1, padded with 1 pixel. Output size?

→ 7x7

Stride & Padding



Output size:

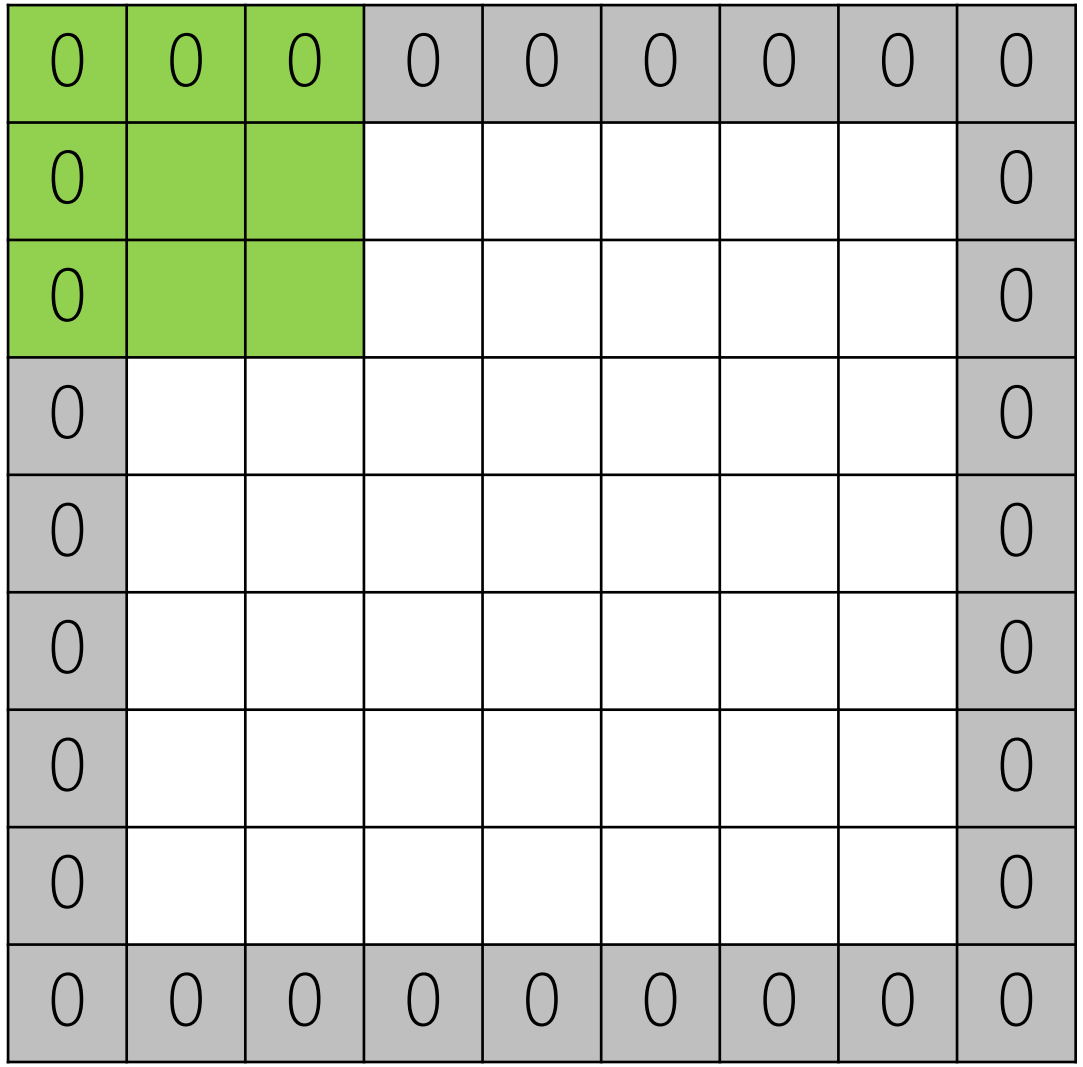
(N-f)/stride + 1

Q. 7x7 input, 3x3 kernel, with stride 1, padded with 1 pixel. Output size?

→ 7x7

Q. 7x7 input, 3x3 kernel, with stride 3. You want to make 3x3 output, padding?

Stride & Padding



Output size:

(N-f)/stride + 1

Q. 7x7 input, 3x3 kernel, with stride 1, padded with 1 pixel. Output size?

→ 7x7

Q. 7x7 input, 3x3 kernel, with stride 3. You want to make 3x3 output, padding?

→ 1

More questions:

- Input volume: 50x50x3
- 10 5x5x3 filters with stride 1, pad 2

- Output size?
  - (50 + 2*2 - 5)/1+1 = 50
  - Therefore, 50x50x10

- Total number of parameters in this layer?
  - Each filter: 5*5*3+1 = 76 parameters
  - Therefore, 76*10=760

Pooling Layers

- Pooling == ConvNet way of downsampling
- “Reduce the image size” to obtain smaller but more manageable features
- Receptive fields becomes relatively larger as the image size gets smaller
- Operates over each activation map independently

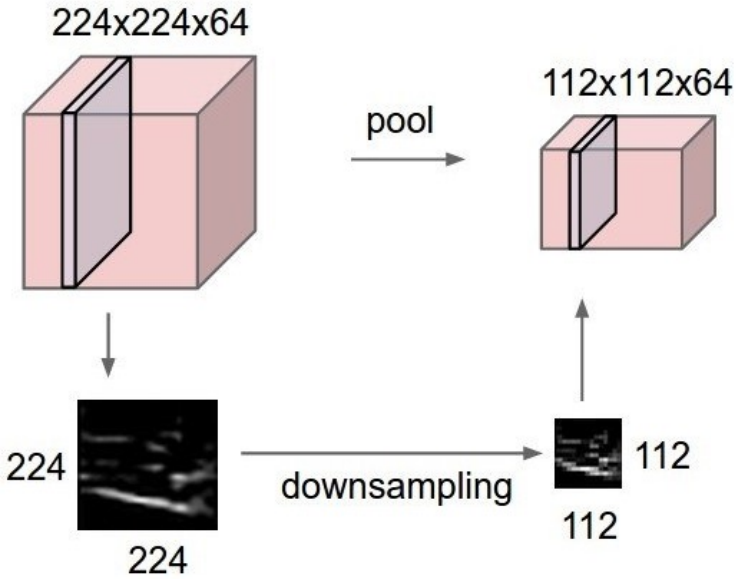
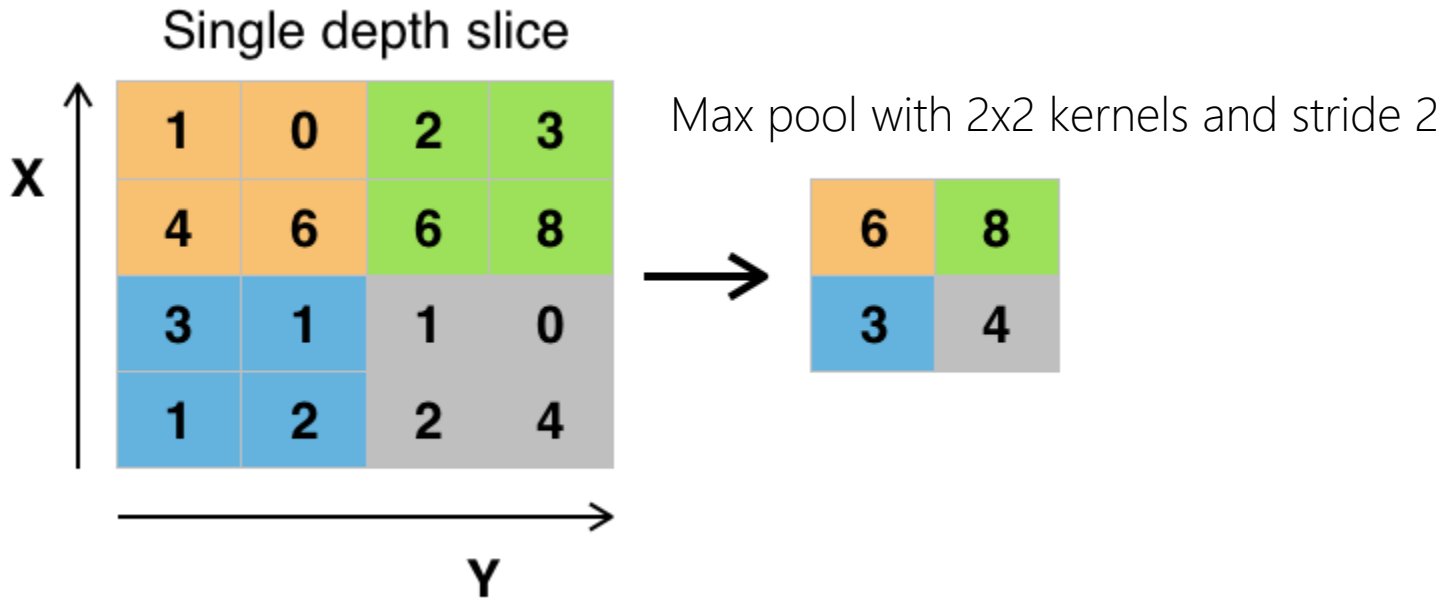


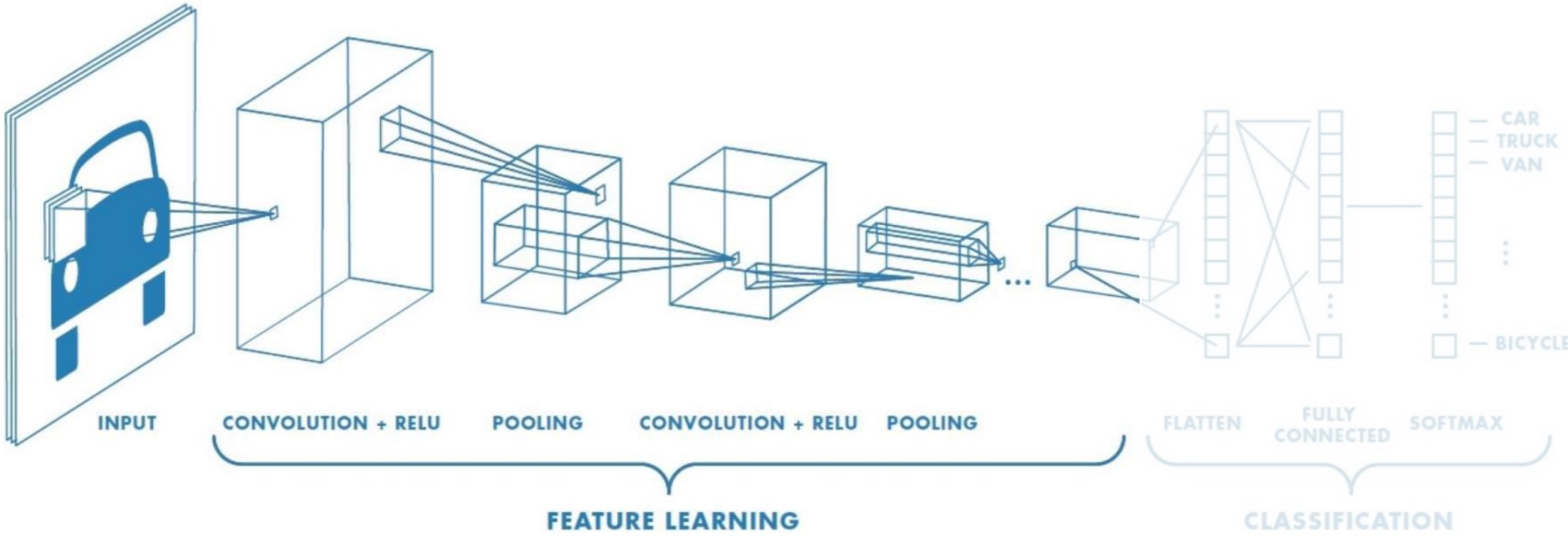
Image source: Stanford CS231n Convolutional Neural Networks for Visual Recognition

Max Pooling

- A non-linear down-sampling method
- An image is partitioned into a set of (non-overlapping) rectangles.
- The maximum of each such sub-region is sampled.

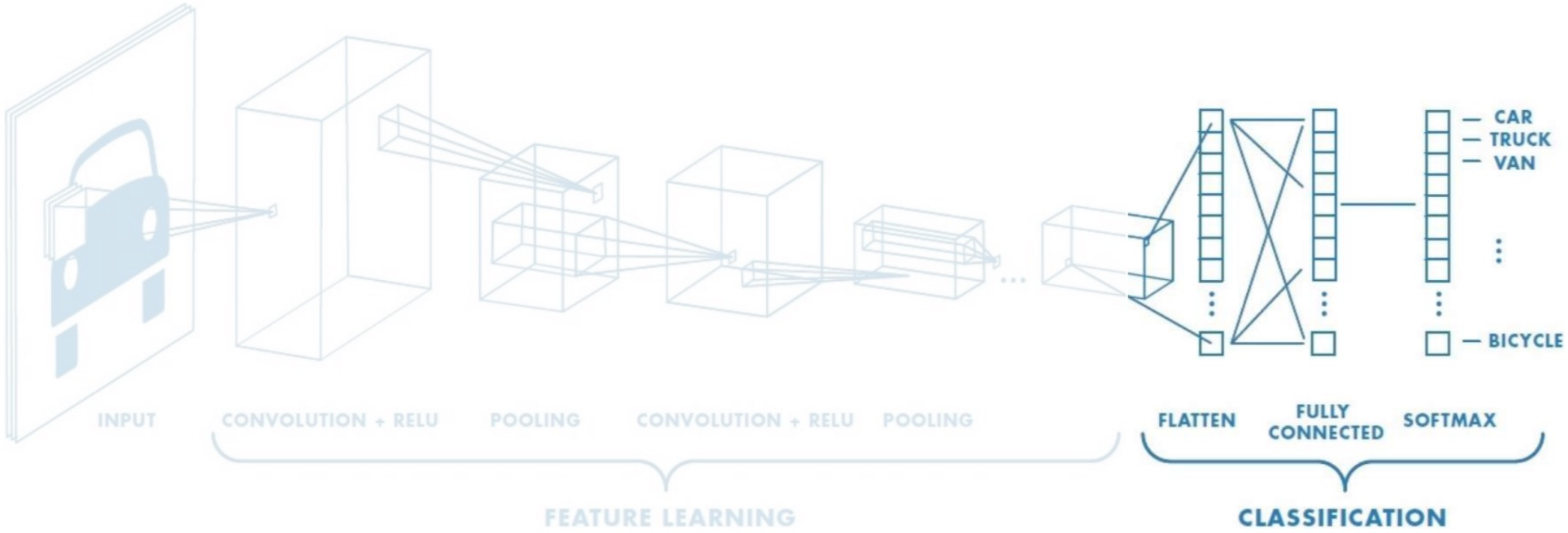


CNN for Classification: Feature Learning



1. Learn features in input image through **convolution**
2. Introduce **non-linearity** through activation function (real-world data is non-linear!)
3. Reduce dimensionality and preserve spatial invariance with **pooling**

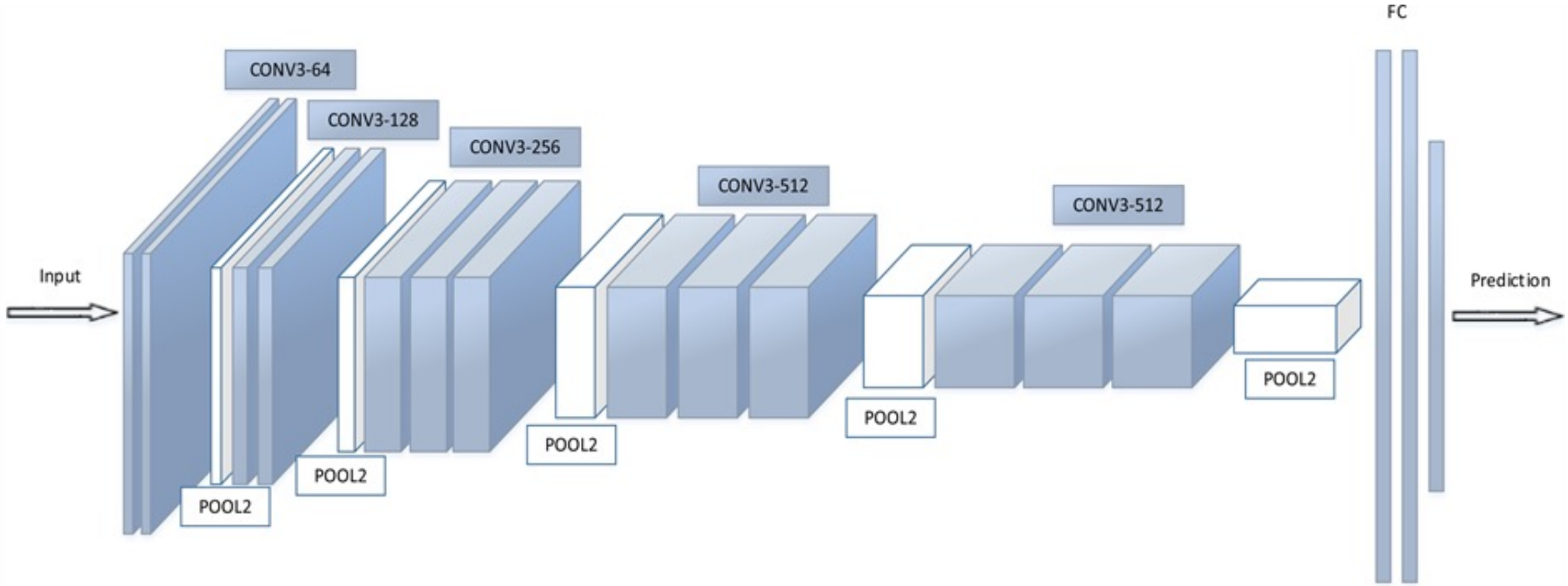
CNN for Classification: Prediction



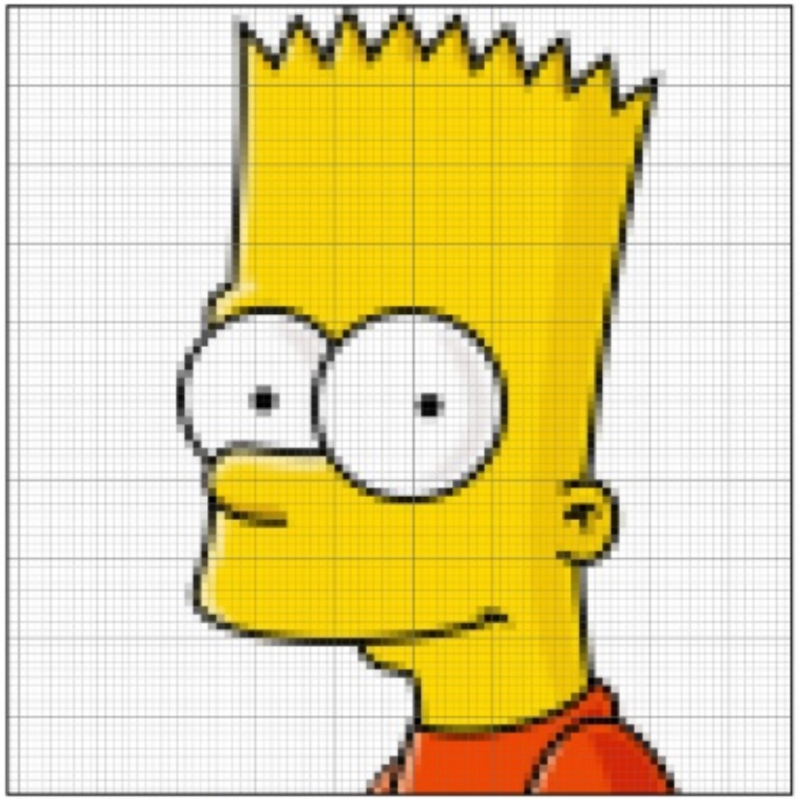
- CONV and POOL layers output high-level features of input
- Fully connected layer uses these features for classifying input image
- Express output as **probability** of image belonging to a particular class

$$\text{softmax}(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$$

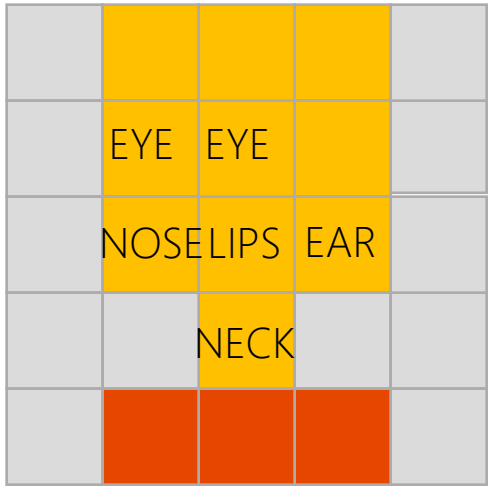
VGG Networks



Abstraction of an Image

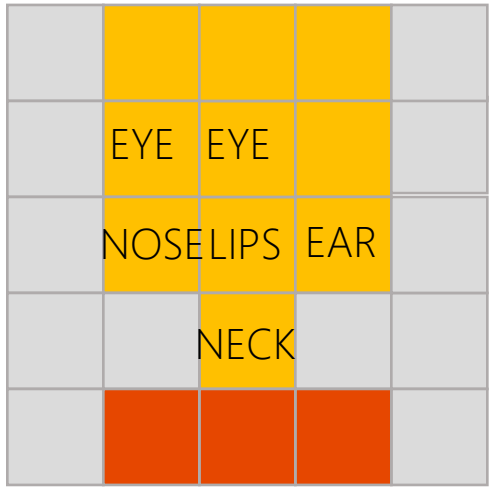


Conv2D + MaxPooling
+ Conv2D + MaxPooling
+ Conv2D + MaxPooling
+ ...



Abstraction of an Image

- Final decision is made by the fully-connected layers:



"Has two eyes, a nose, lips, an ear, and a neck and wears t-shirts"

Cat	X
Dog	X
Person	O
Orange	X