UVA DATA SCIENCE

DS 6050 Deep Learning

5. How to Train Convolutional Neural Networks Part II

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Regularization

Preventing weights going crazy…

- Too flexible weights → Too flexible models → Overfitting!

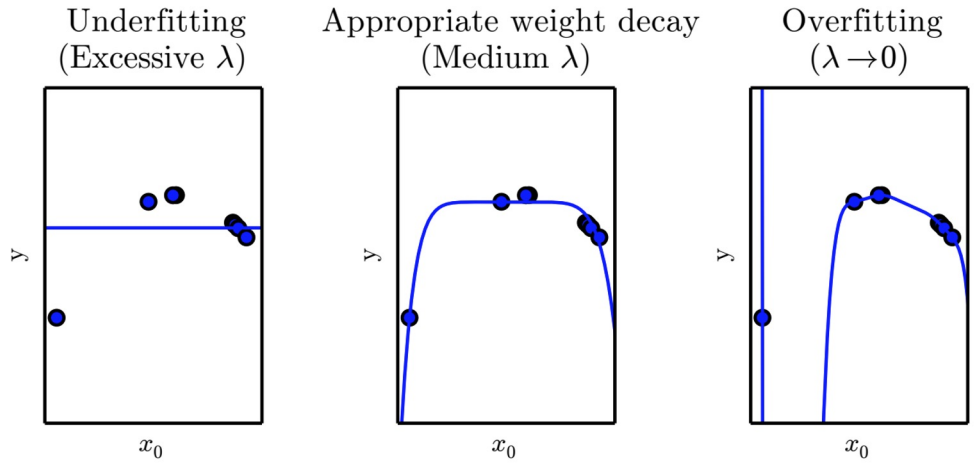
- You can *regularize* (meaning, prevent the size of weights going too big) by adding a regularization term in the loss function.

$$L(W,b) = \frac{1}{N}\sum_{i=1}^{N}\left\|y^{(i)} - f\left(x^{(i)} \mid W,b\right)\right\|^2 + \lambda R(W,b)$$

Regularization

- Regularization is the modification made to a learning algorithm that is intended to reduce its generalization error but not its training error.
- For example, we can modify the loss function for linear regression to include a preference for the weights to have smaller L2 norm as a regularizer:

$$J(\mathbf{w}) = \mathcal{L}_{\text{mse}}(\mathbf{X}_{\text{train}}, \mathbf{y}_{\text{train}}) + \lambda \mathbf{w}^\top \mathbf{w}$$



Parameter Norm Penalties

- Many regularization approaches are based on limiting the model capacity by adding a parameter norm penalty to the objective (loss) function:

$$J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda \Omega(\mathbf{w})$$

Data Loss Norm Penalty

where $\boldsymbol{\lambda}$ is a hyperparameter that controls the contribution of the norm penalty term, $\boldsymbol{\Omega}$, relative to the standard data loss function L

- Larger value of $\boldsymbol{\lambda}$ correspond to more regularization
- Setting $\boldsymbol{\lambda}$ to 0 results in no regularization
- Norm penalty $\boldsymbol{\Omega}$ penalizes only the weights of the affine transformation
- Different choice of $\boldsymbol{\Omega}$ can result in different solutions being preferred.

L2 Parameter Regularization

- L2 regularization is aka Ridge Regression or Tikhonov regularization
- The L2 norm penalty commonly known as weight decay

$$J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \frac{\lambda}{2}||\mathbf{w}||^2$$

$$\nabla_{\mathbf{w}} J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \nabla_{\mathbf{w}} \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda \mathbf{w}$$

To take a single **Gradient Descent** step to update the weights:

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha \nabla_{\mathbf{w}} J(\mathbf{w}; \mathbf{X}, \mathbf{y})$$

$$\mathbf{w} \leftarrow \mathbf{w} - \alpha(\nabla_{\mathbf{w}} \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda \mathbf{w})$$

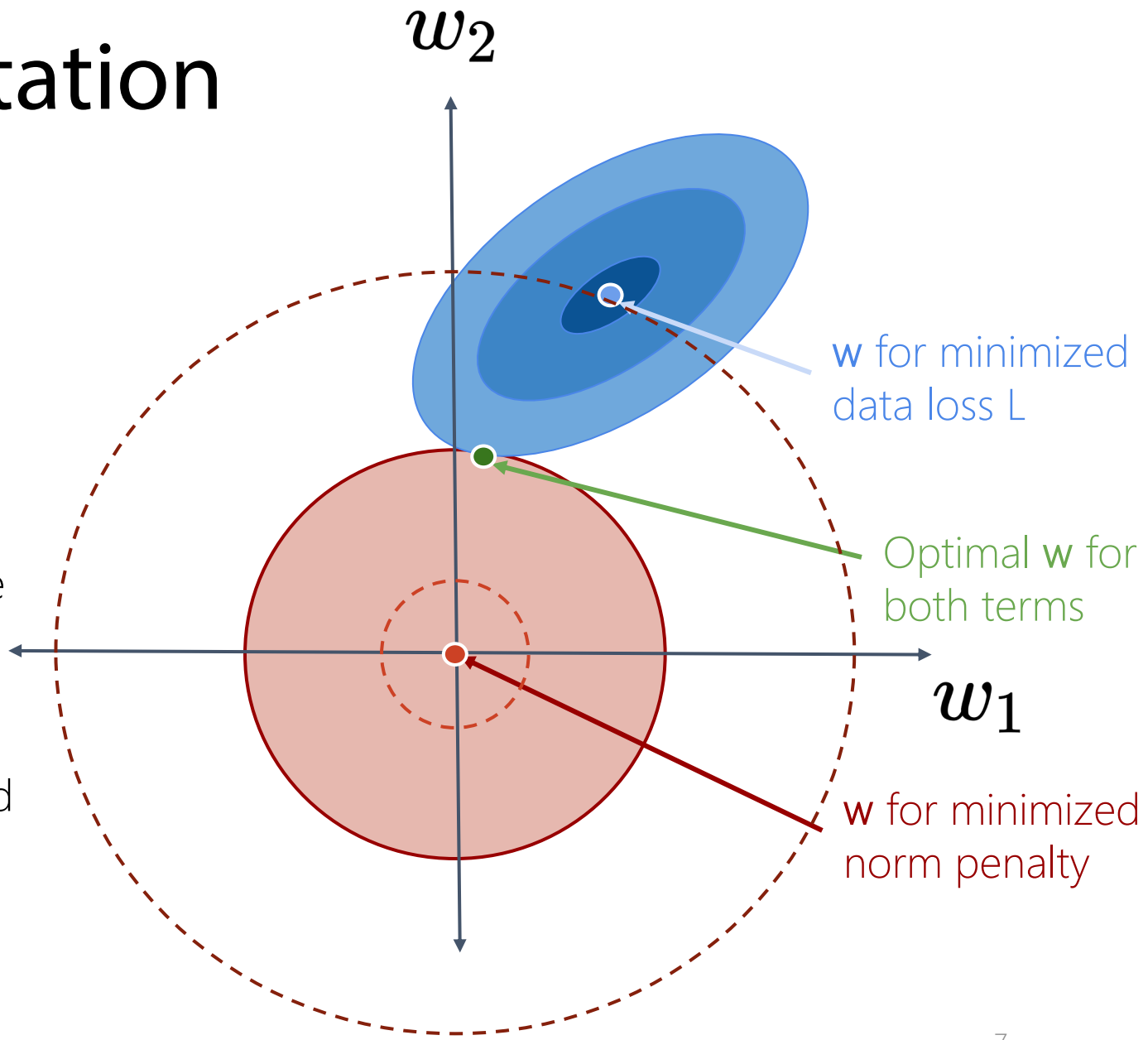
$$\mathbf{w} \leftarrow (1 - \alpha\lambda)\mathbf{w} - \alpha \nabla_{\mathbf{w}} \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y})$$

< 1

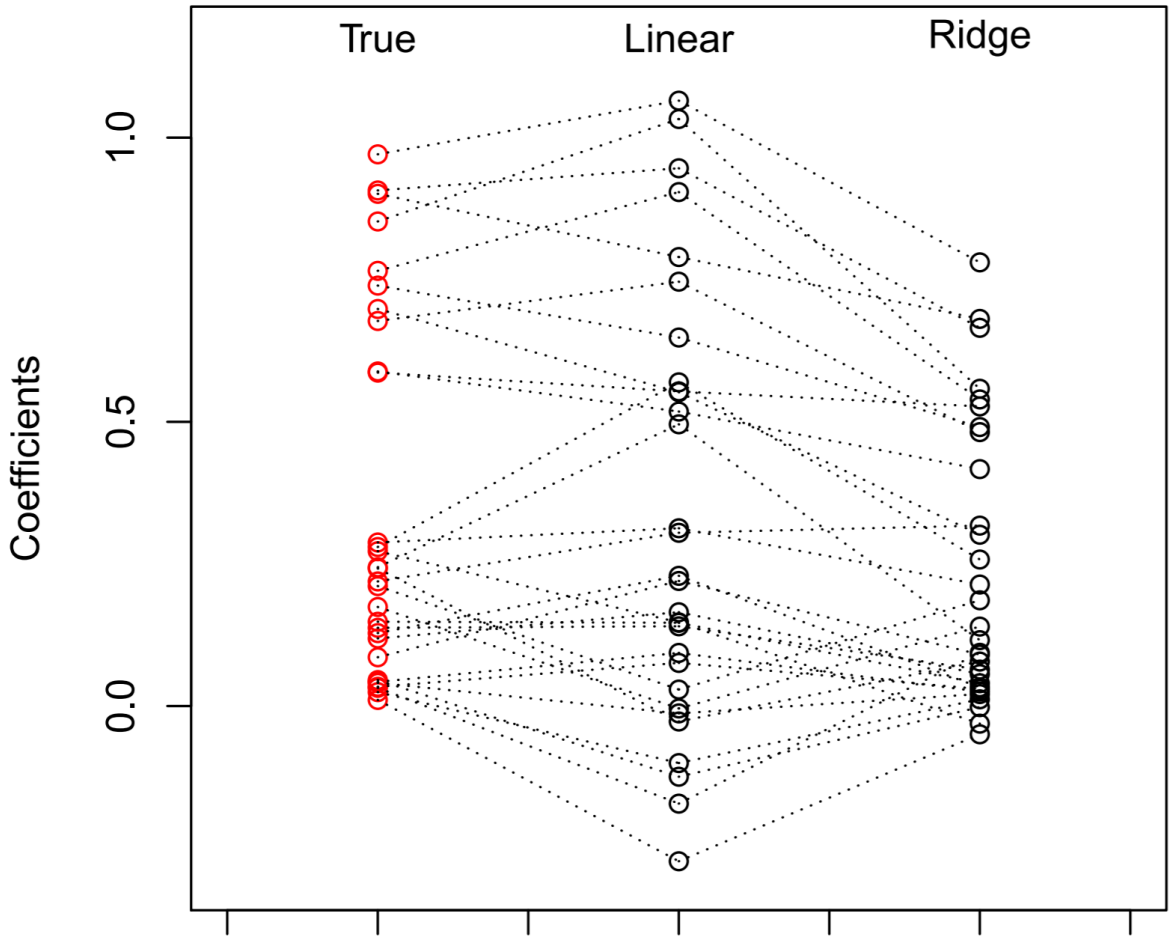
Shrink the weight vector before gradient update

Geometric Interpretation

- L2 regularizations $||w||^2$ can be geometrically represented as concentric (red) circles.
- When circle is too small, the params are not useful to the model.
- When the circle region (in red) grows, due to its shape the region intersects the data contour closer to the origin, L2 makes both parameters shrink and w1 near zero.
- When the circle grows too large, you end up with a similar params as data loss.



Visual Representation of Coefficients



L1 Parameter Regularization

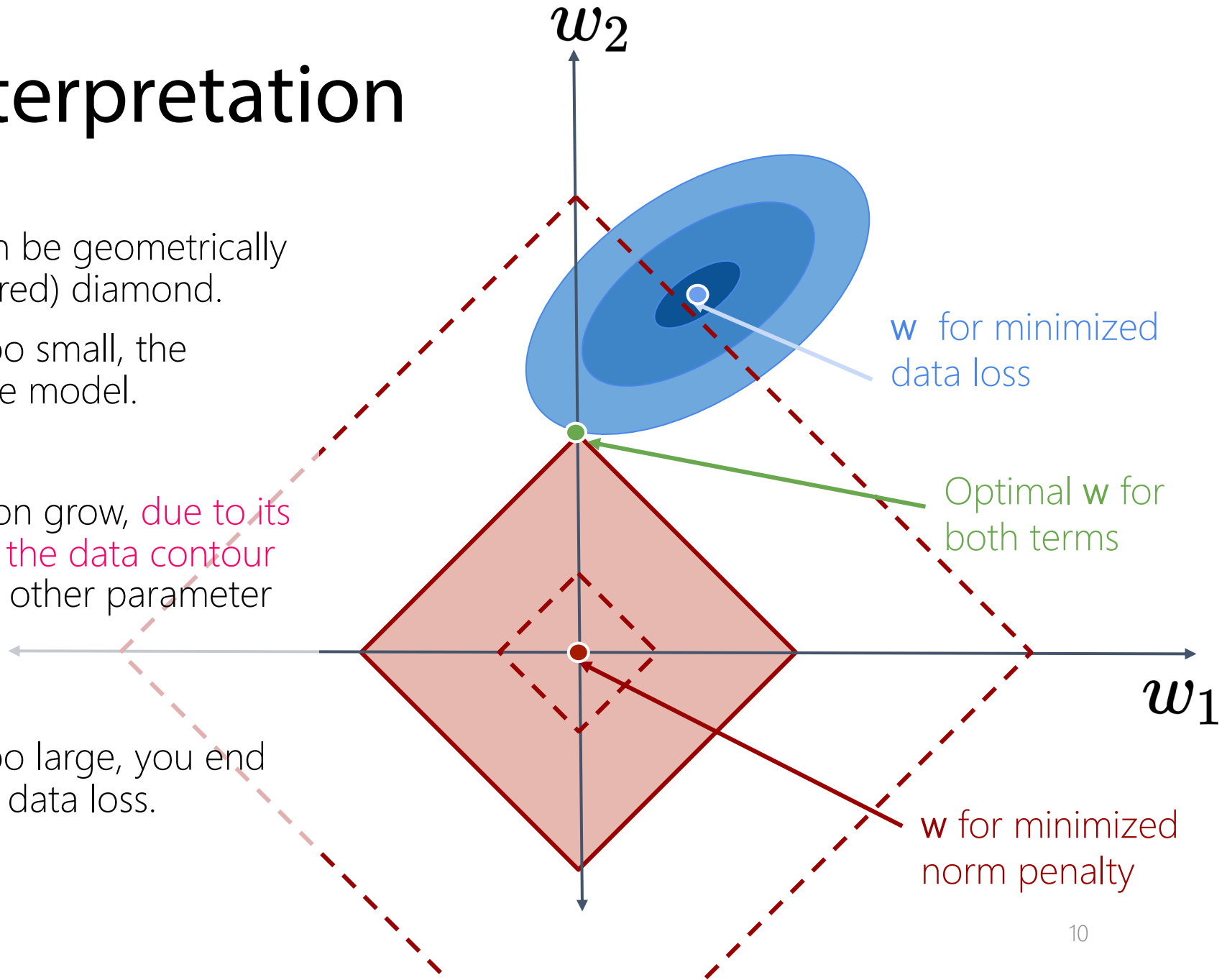
- L1 regularization is aka LASSO (least absolute shrinkage and selection operator)
- L1 norm commonly is known as the Manhattan Distance.

$$J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda ||\mathbf{w}||_1$$
$$\nabla_{\mathbf{w}} J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \nabla_{\mathbf{w}} \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda \mathbf{sign}(\mathbf{w})$$

- The regularization to the gradient no longer scale linearly with each $\mathbf{w}$, instead it is a constant factor with a sign equal to $\mathbf{sign}(\mathbf{w})$. Thus, there is no clean algebraic solution to approximates J as we have just seen in L2 regularization
- L1 norm makes the parameters to become sparse (contains lots of zeros)
- L1 norm tends to cause a subset of the network weights to become zero, suggesting that the corresponding signal may safely be discarded (dead neuron).

Geometric Interpretation

- L1 regularizations $||w||^1$ can be geometrically represented as concentric (red) diamond.
- When diamond region is too small, the params are not useful to the model.
- When as the diamond region grow, due to its shape the region intersects the data contour on one of the axis, thus the other parameter is zero out.
- When diamond region is too large, you end up with a similar params as data loss.



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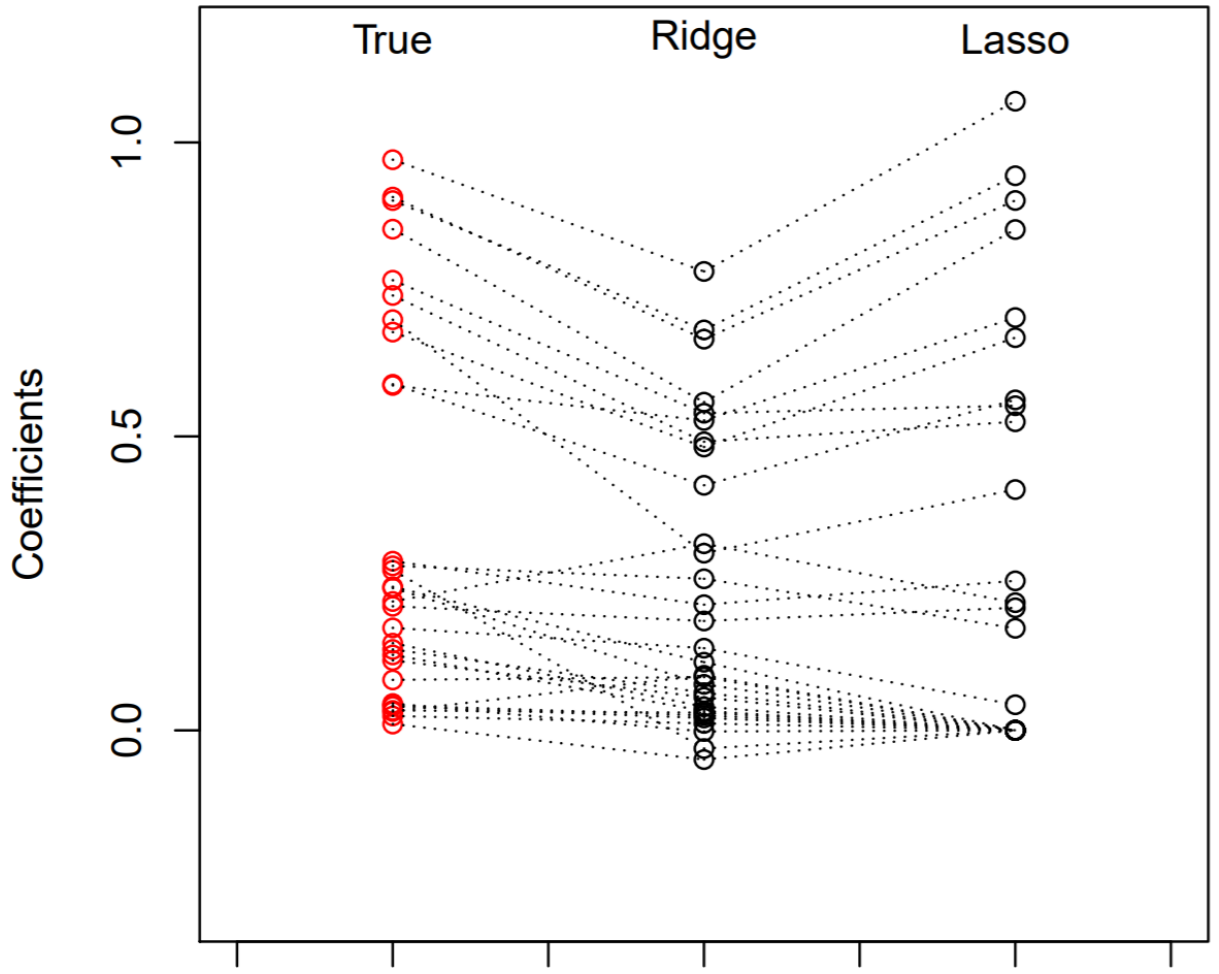
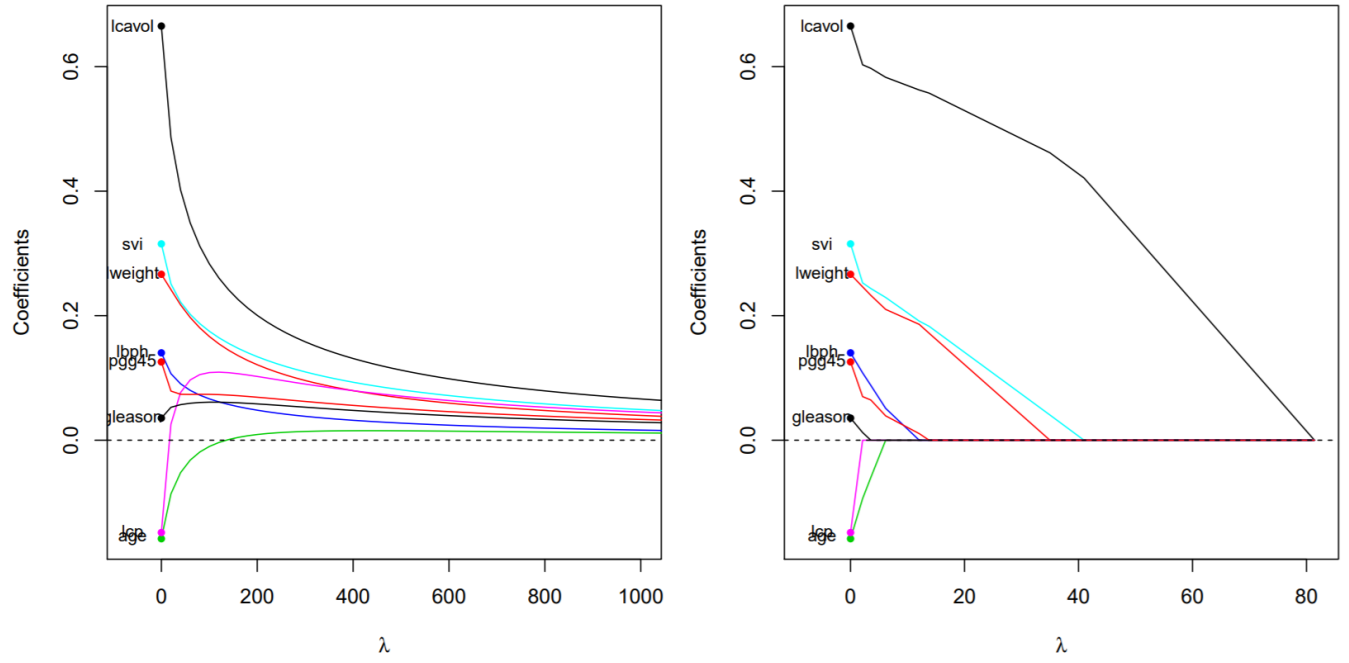


Example of Variable Selection

- Example: suppose that we are studying the level of prostate-specific antigen (PSA), which is often elevated in men who have prostate cancer. We look at n = 97 men with prostate cancer, and p = 8 clinical measurements. We are interested in identifying a small number of predictors, say 2 or 3, that drive PSA



Ridge Regression

LASSO

Elastic Net

- Is a middle ground between Ridge (L2) and Lasso (L1) with a ratio r.

$$J(\mathbf{w};\mathbf{X},\mathbf{y}) = \mathcal{L}(\hat{\mathbf{y}},\mathbf{y}) + r\lambda||\mathbf{w}||_1 + (1-r)\lambda||\mathbf{w}||_2^2$$

LASSO

RIDGE

Geometric Interpretation

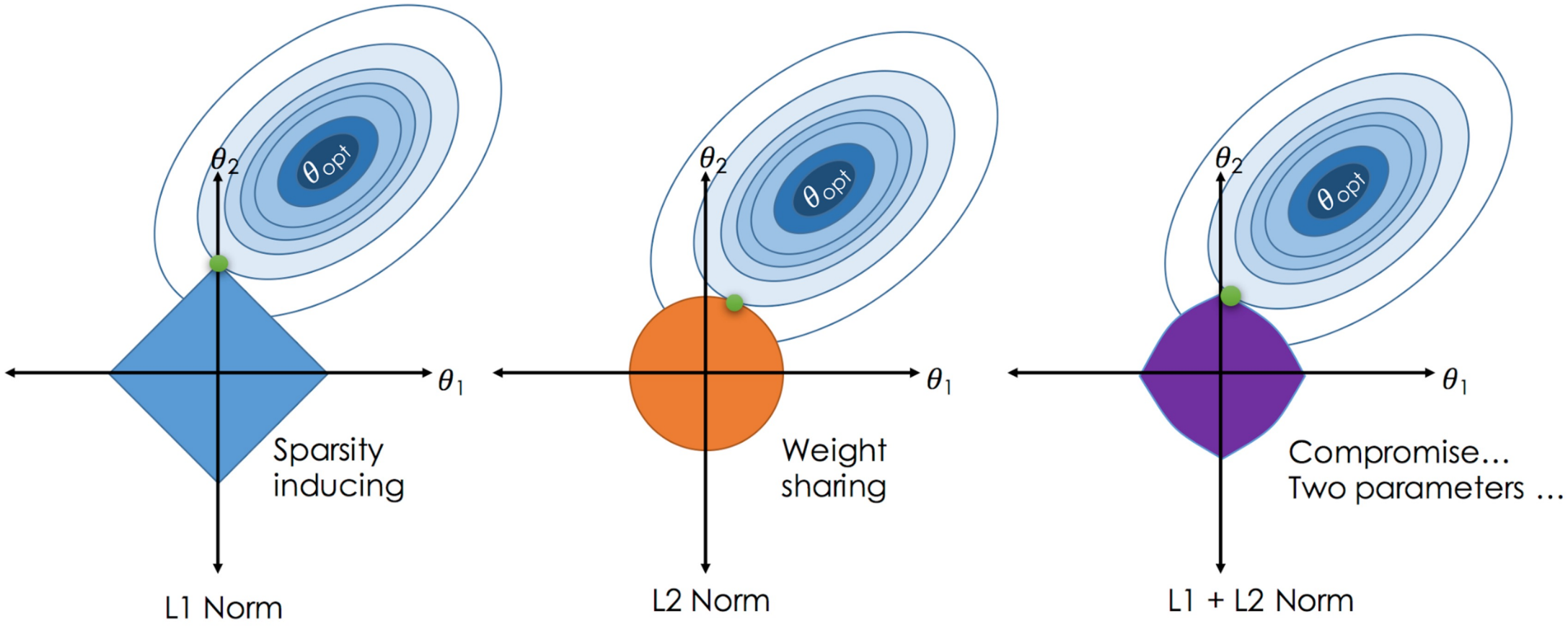
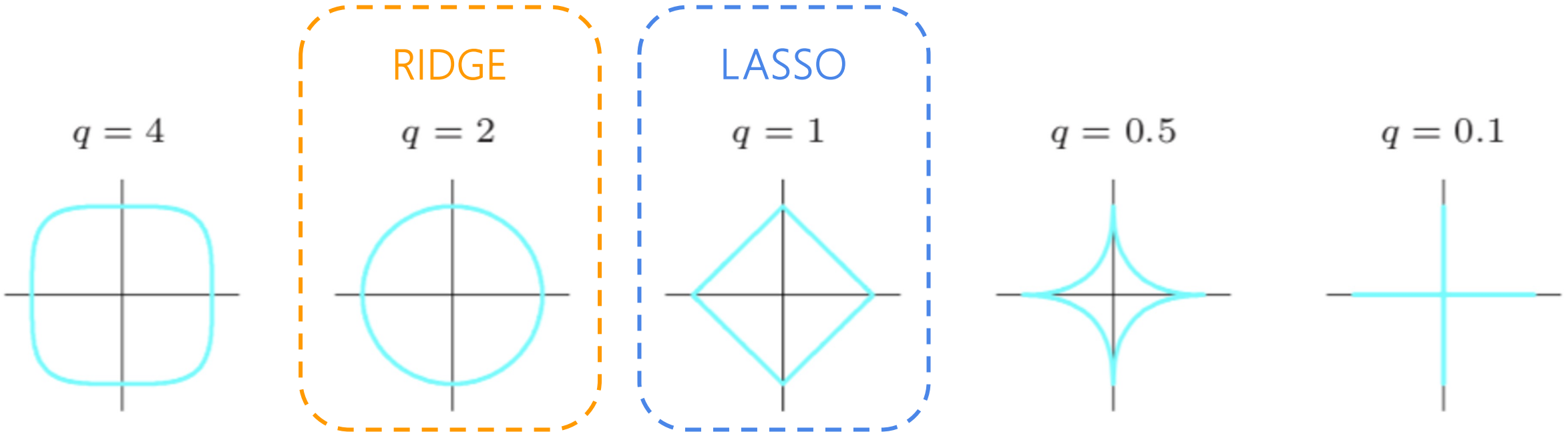


*Figure from Yin Hu, Medium

Family of Parameter Norm Models

$$J(\mathbf{w}; \mathbf{X}, \mathbf{y}) = \mathcal{L}(\hat{\mathbf{y}}, \mathbf{y}) + \lambda ||\mathbf{w}||_q$$



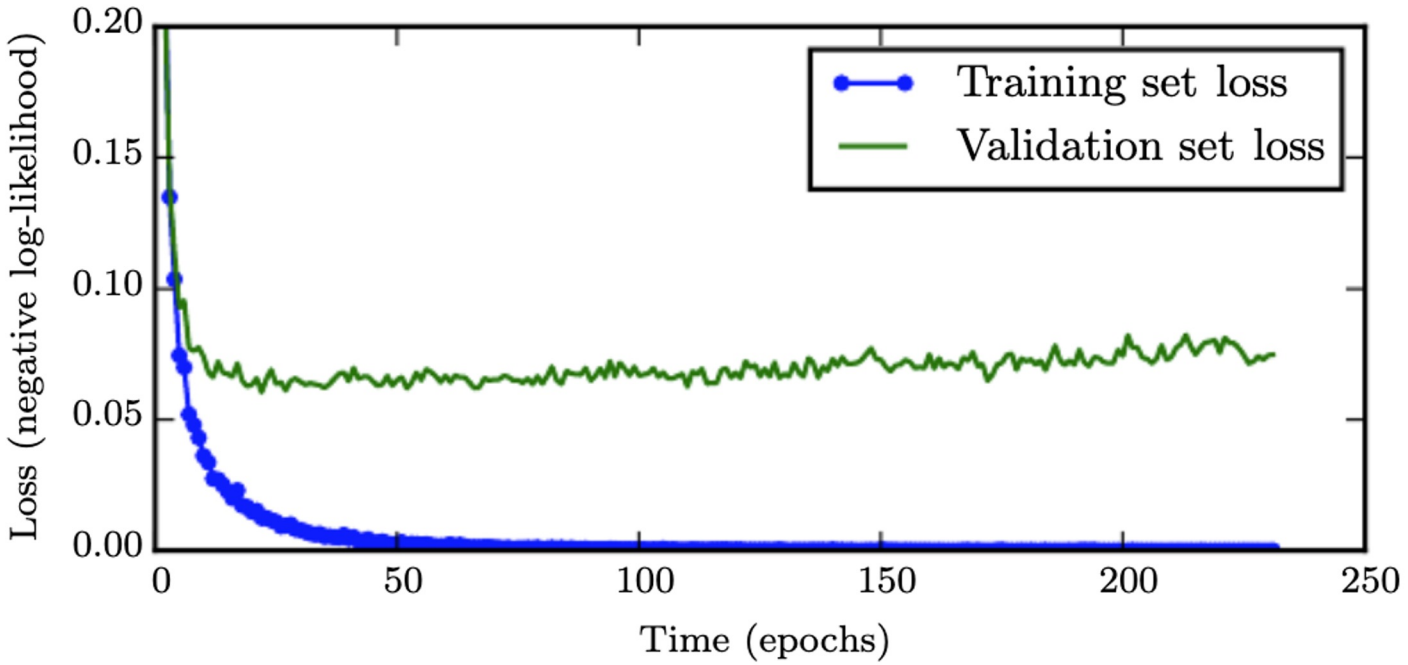
TL;DR

- It is preferable to have at least a little bit of regularization. Ridge (L2) is a good default.
- If you suspect that only a few features are actually useful, use Lasso (L1)
- Lasso may behave erratically when # features > # examples → use Elastic Net

Early Stopping

Early Stopping

- When training large models with sufficient capacity to overfit the task, we often observe reliably that training error decreases steadily over time, but validation error begins to rise again → we can stop at the lowest validation error!
- The most commonly used form of regularization in deep learning: simple yet effective!



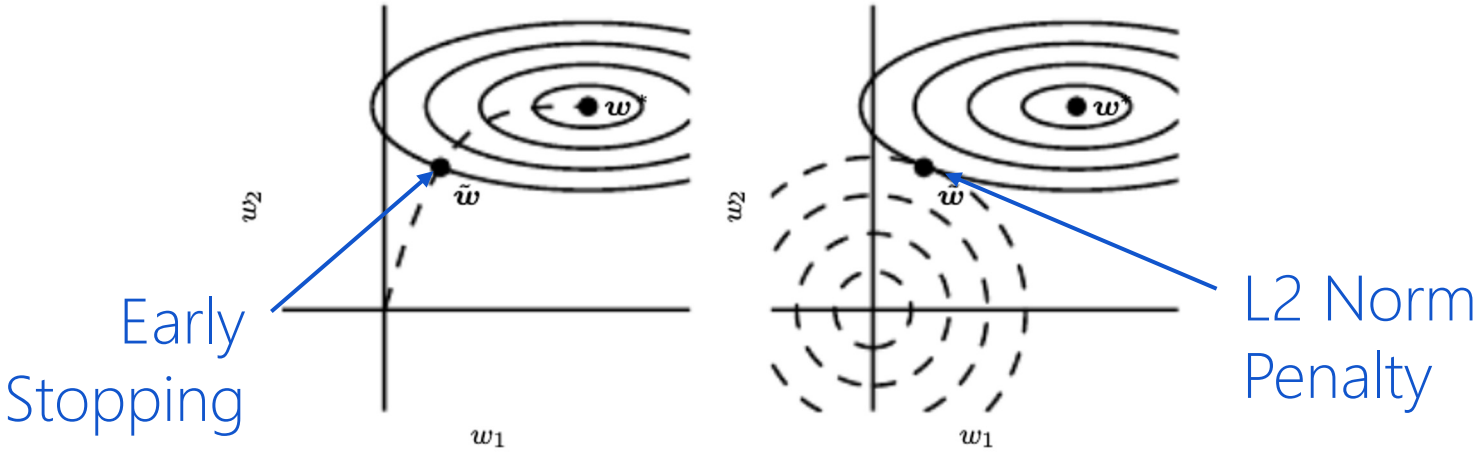
Early Stopping: Algorithm

Algorithm 7.1 The early stopping meta-algorithm for determining the best amount of time to train. This meta-algorithm is a general strategy that works well with a variety of training algorithms and ways of quantifying error on the validation set.

Let n be the number of steps between evaluations.
Let p be the "patience," the number of times to observe worsening validation set error before giving up.
Let $\boldsymbol{\theta}_o$ be the initial parameters.
$\boldsymbol{\theta} \leftarrow \boldsymbol{\theta}_o$
$i \leftarrow 0$
$j \leftarrow 0$
$v \leftarrow \infty$
$\boldsymbol{\theta}^* \leftarrow \boldsymbol{\theta}$
$i^* \leftarrow i$
while $j < p$ **do**
  Update $\boldsymbol{\theta}$ by running the training algorithm for n steps.
  $i \leftarrow i + n$
  $v' \leftarrow \text{ValidationSetError}(\boldsymbol{\theta})$
  if $v' < v$ **then**
    $j \leftarrow 0$
    $\boldsymbol{\theta}^* \leftarrow \boldsymbol{\theta}$
    $i^* \leftarrow i$
    $v \leftarrow v'$
  else
    $j \leftarrow j + 1$
  end if
end while
Best parameters are $\boldsymbol{\theta}^*$, best number of training steps is i^*.

Early Stopping: Properties

- Is a very efficient hyperparameter selection method (saves the best set).
- Controls the model capacity by determining how many steps to fit the training data (and reduce the computational cost of the training procedure)
- Unobtrusive form of regularization as almost no change in training procedure
- Can be used either alone or with other regularization strategies
- Is equivalent to L2 regularization, and better because it automatically determines the correct amount of regularization while L2 requires the tuning of hyperparameter $\boldsymbol{\lambda}$



Loss Functions

Lots of Puzzling Terms!

- L1 Loss: `torch.nn.L1Loss(y_true, y_pred)`
- Mean Squared Error (MSE): `torch.nn.MSELoss(y_true, y_pred)`
- Cross Entropy: `torch.nn.CrossEntropyLoss(y_true, y_pred)`
- Kullback-Leibler Divergence: `torch.nn.KLDivLoss(y_true, y_pred)`
- Hinge Embdding Loss: `torch.nn.HingeEmbeddingLoss(y_true, y_pred)`
- Huber Loss: `torch.nn.HuberLoss(y_true, y_pred)`
- Margin Ranking Loss: `torch.nn.MarginRankingLoss(y_true, y_pred)`
- Multi-label Margin Loss: `torch.nn.MultiLabelMarginLoss(y_true, y_pred)`
- Cosine Embedding Loss: `torch.nn.CosineEmbeddingLoss(y_true, y_pred)`
- Triplet Margin Loss: `torch.nn.TripletMarginLoss(y_true, y_pred)`
- Negative Log-likelihood Loss: `torch.nn.NLLLoss(y_true, y_pred)`
- Gaussian Negative Log-likelihood Loss: `torch.nn.GaussianNLLLoss(y_true, y_pred)`

Information Theory (Claude Shannon, 1948)

- Digital information: series of bits (either 0 or 1)
- Sending a single bit (useful) of information = reducing receiver's uncertainty into half
- For example, who would win the Virginia–Virginia Tech game, assuming the both team have the equal chance of winning (50%)?
  - With no information, the uncertainty is 50-50.
  - One bit of information (Cavaliers won!) → resolves the uncertainty.
- Another example, imagine a league of 8 teams. Who is the winner?
  - With no information, the uncertainty is 12.5% each.
  - How many bits of information do you need?
  - 3 bits! ($2^3 = 8$, or $\log_2(8)=3$)

Information Theory (Claude Shannon, 1948)

- What if the chance of winning is not equal?
  - Say, Cavaliers (75% of winning) and Hokies (25% of winning).
  - If a sender says, Hokies won the game, the uncertainty drops by the factor of 4.
    - Uncertainty reduction = $-\log_2(1/4) = 2$
  - If a sender says, Cavaliers won the game,
    - Uncertainty reduction = $-\log_2(3/4) = 0.42$
  - Therefore, the expected number of bits to resolve the uncertainty:
    - $-0.75*\log_2(3/4) - 0.25*\log_2(1/4) = 0.75*0.42 + 0.25*2 = 0.82$ bits

- Entropy: $H(p) = -\sum_i p_i \log_2(p_i)$
  - Average amount of information that can be derived from one sample drawn from a given probability distribution
  - Indicator of how unpredictable the probability distribution is.
  - More variation in the data → larger entropy.

Cross Entropy

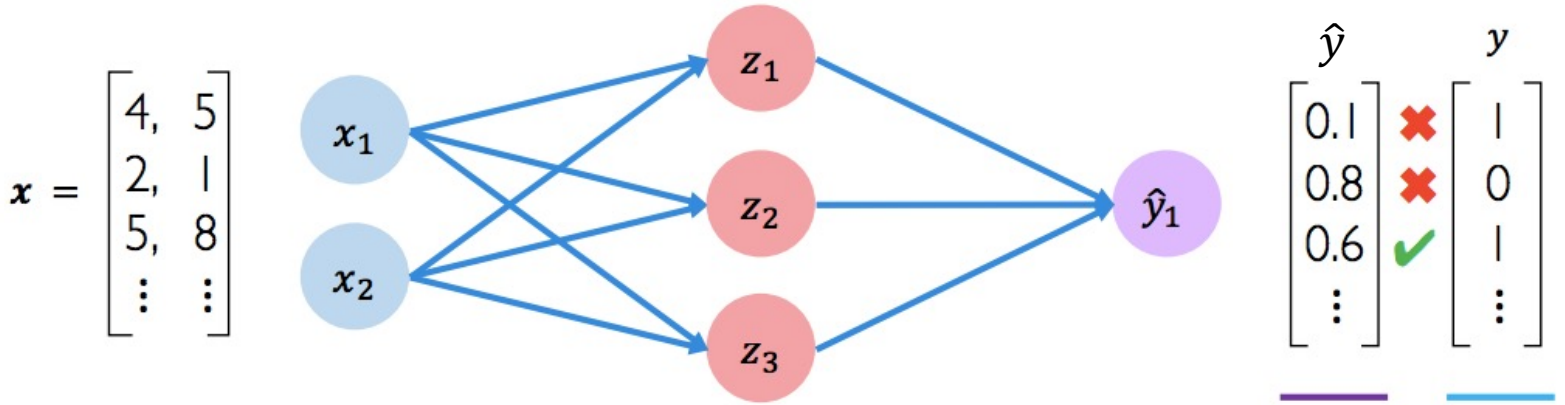
- Entropy: $H(p) = -\sum_i p_i \log_2(p_i)$
- Cross Entropy: $H(p, q) = -\sum_i p_i \log_2(q_i)$
  - Think of p as a true distribution and q as a predicted distribution.
  - If $q = p$ (correct prediction), cross entropy equals to entropy.
  - If $q \neq p$ (erroneous prediction), cross entropy gets greater (why?) than entropy by some number of bits

https://en.wikipedia.org/wiki/Gibbs'_inequality

- Kullback-Leibler (KL) Divergence: $D_{KL}(p, q) = H(p, q) - H(p)$
  - The amount of difference between cross entropy and entropy.
  - a.k.a, relative entropy

Cross Entropy Loss

- Binary Cross Entropy Loss:



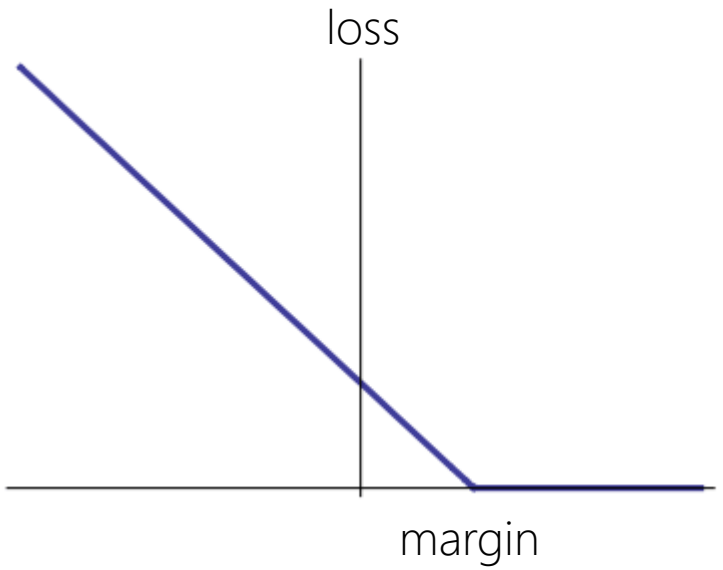
$$L_{BCE} = -\frac{1}{n}\sum_{i=1}^{n}(Y_i \cdot \log \hat{Y}_i + (1 - Y_i) \cdot \log (1 - \hat{Y}_i))$$

- Multiclass Cross Entropy Loss:

$$L(\hat{y}, y) = -\sum_{k}^{K} y^{(k)} \log \hat{y}^{(k)}$$

Hinge Loss

- Penalizes incorrectly classified examples + correctly classified examples that lie within the margin
- Hinge loss is generally faster than cross entropy but less accurate.



$L = \max(0, 1 - ty)$

Regression: L_p Distances

- L_p norm or Minkowski distance:

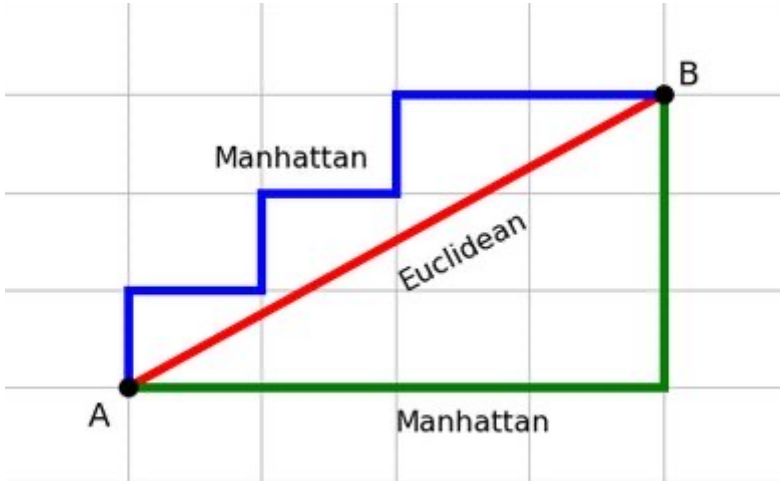
$$L_p(x, y) = \left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{\frac{1}{p}}$$

- For $p = 1$, Manhattan (or city-block) distance:

$$L_1(x, y) = \sum_{i=1}^{n} |x_i - y_i|$$

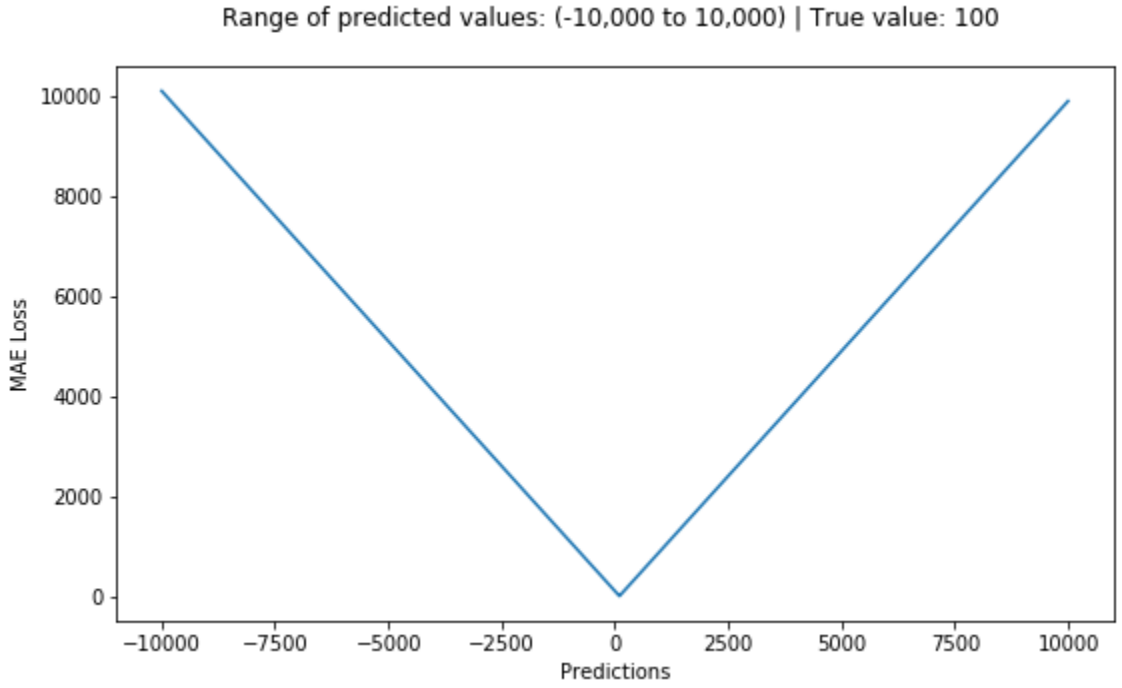
- For $p = 2$, Euclidean distance:

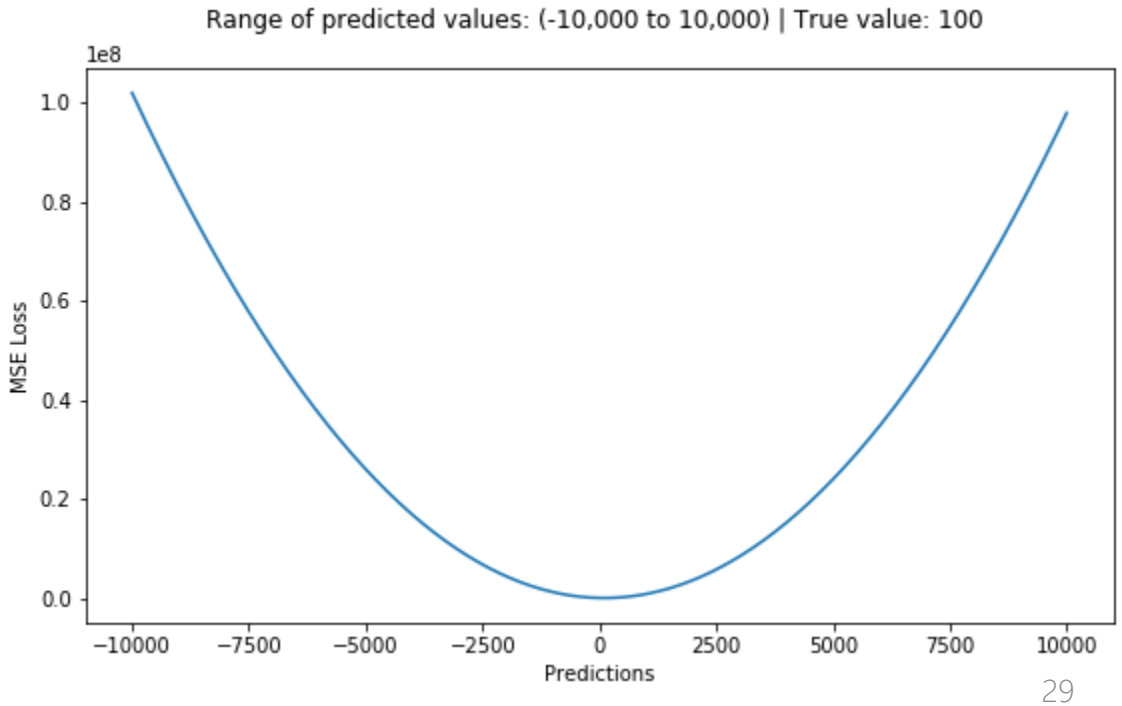
$$L_2(x, y) = \sqrt{\sum_{i=1}^{n} |x_i - y_i|^2}$$



L1 and L2 Loss

- Mean Absolute Error (MAE): $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|$
- Mean Squared Error (MSE): $\frac{1}{n}\sum_{i=1}^{n}|y_i - \hat{y}_i|^2$





https://heartbeat.fritz.ai/5-regression-loss-functions-all-machine-learners-should-know-4fb140e9d4b0

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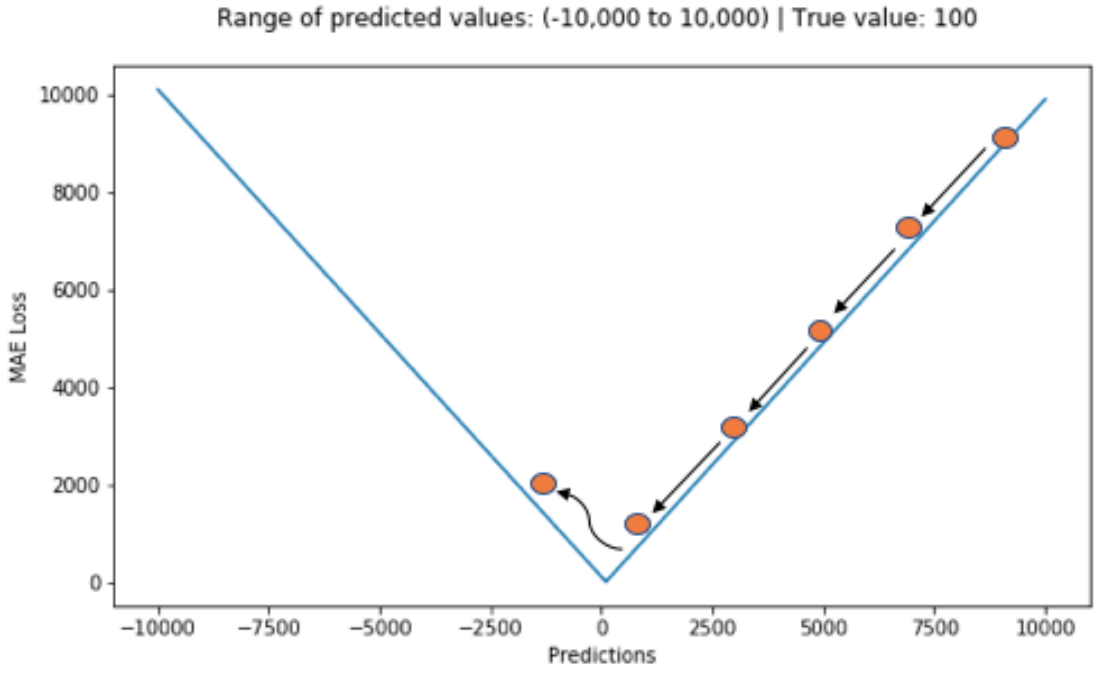

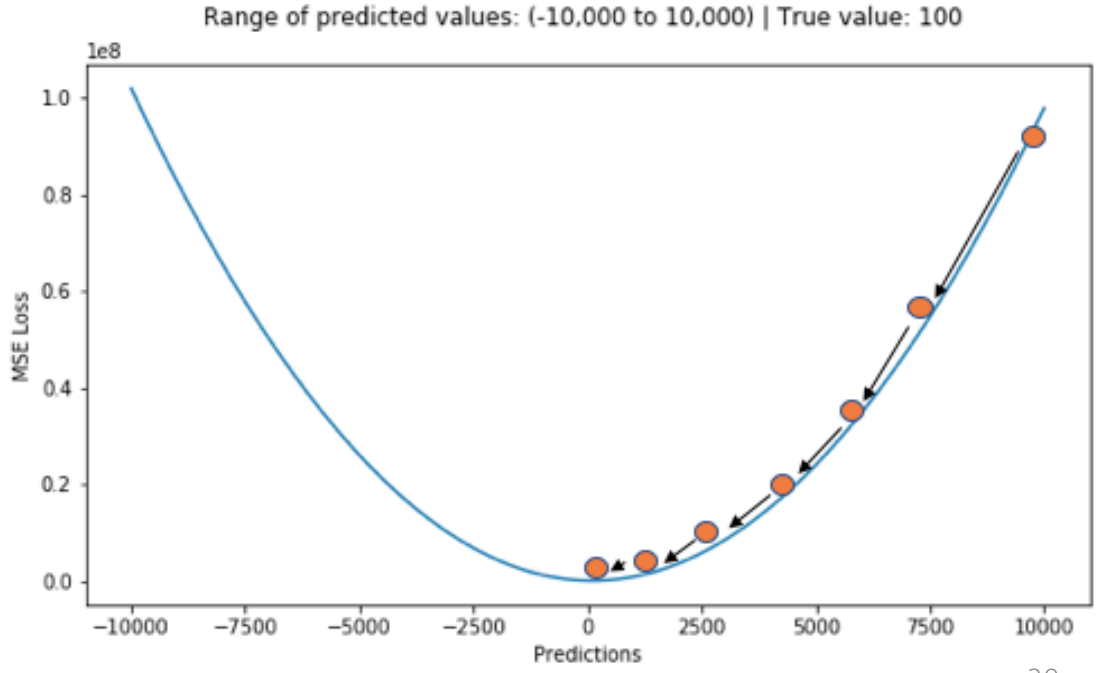

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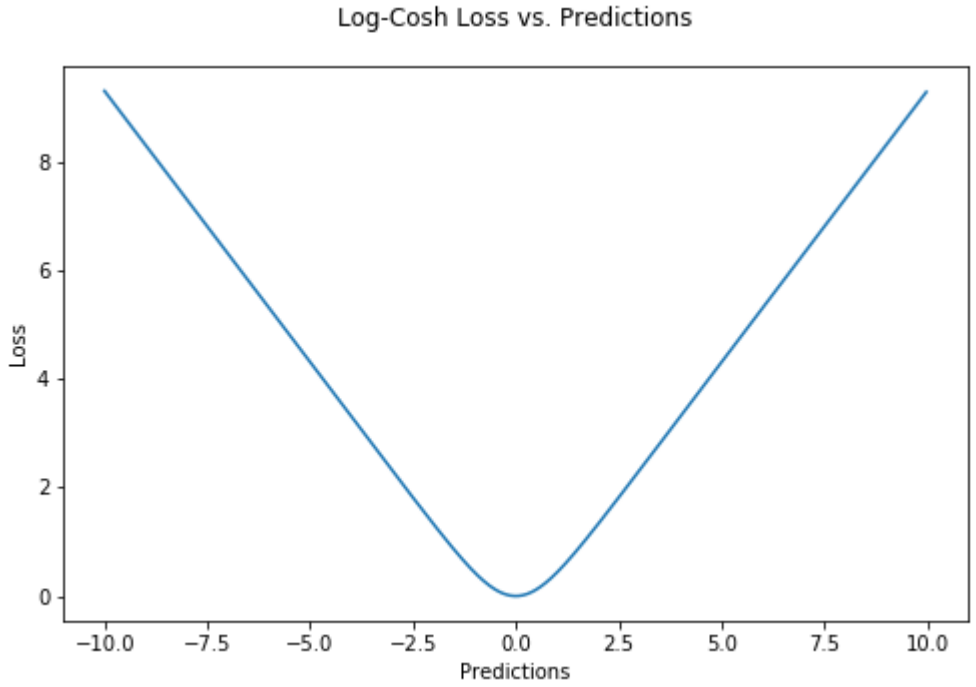
Logarithm of Hyperbolic Cosine (logcosh)

- Logcosh: $\frac{1}{n}\sum_{i=1}^{n} \log\cos(\hat{y}_i - y_i)$



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Beyond SGD:
Fancier Optimization Methods

Thought Experiment on Gradient Descent

- Q. What does it take to evaluate loss?

$$\mathcal{L}(\boldsymbol{\theta}) = \sum_{i=1}^{N} \left\| \mathbf{y}^{(i)} - f\left(\mathbf{x}^{(i)} | \, \boldsymbol{\theta}\right) \right\|^2$$

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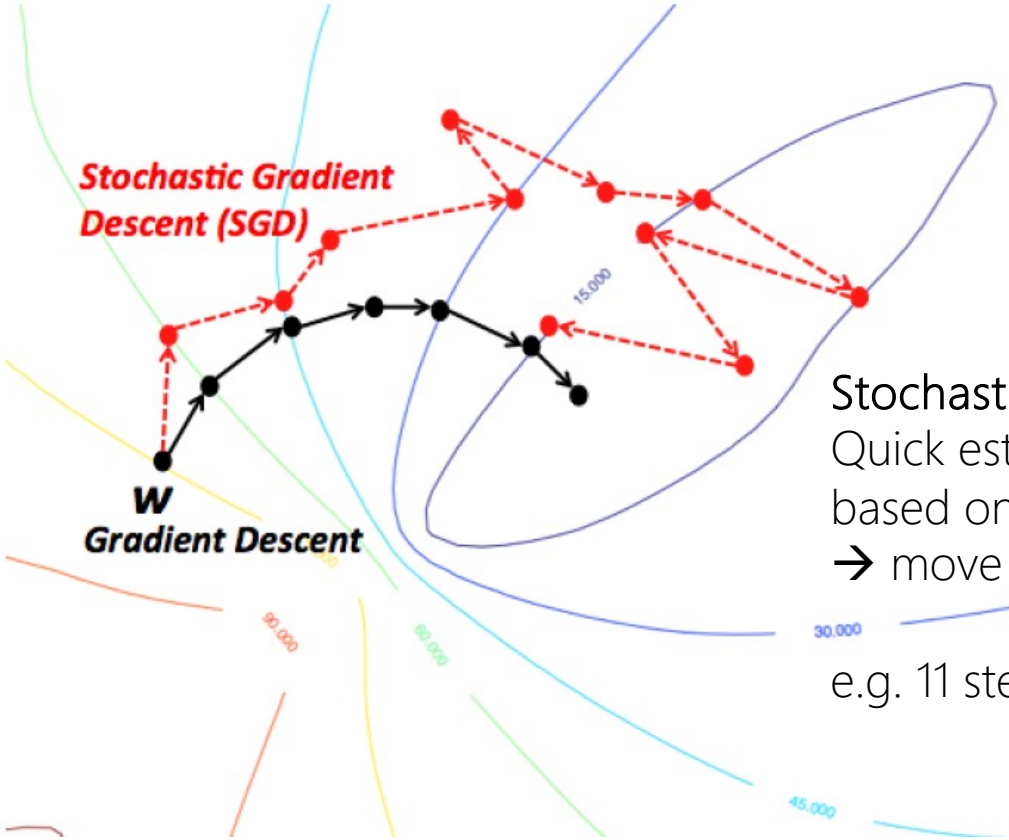
- Q. Then what happens to the gradient descent algorithm?
- A. Computational time increases exponentially as N goes up.

Stochastic Gradient Descent (SGD)

- Idea:

Gradient Descent
Compute everything
→ make the optimal one step

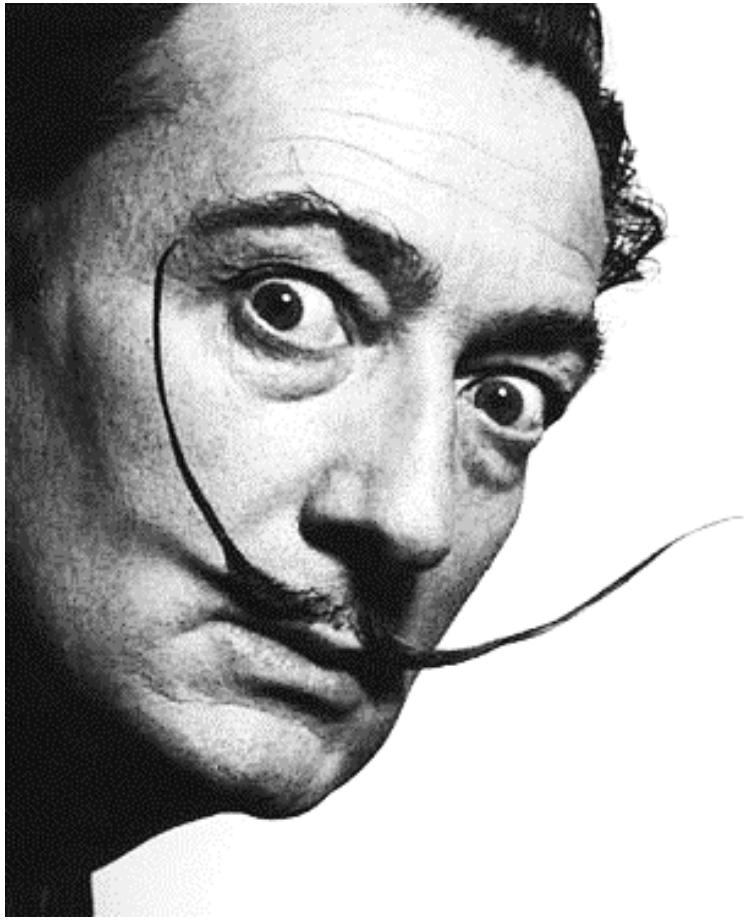
e.g. 6 steps * 1 hr/step = 6 hrs



Stochastic Gradient Descent
Quick estimation of the gradient
based on a sample (or a small batch)
→ move quickly even if it's not the optimal

e.g. 11 steps * 5 min/step = 55 min

Stochastic Gradient Descent (SGD)



"Have no fear of perfection,
you'll never reach it"

- Salvador Dali

TwistedSifter.com

Stochastic Gradient Descent (SGD)

1. Randomly shuffle dataset
2. Repeat until converge {
   randomly pick one sample
   compute gradient with the selected sample
   update weights
   }

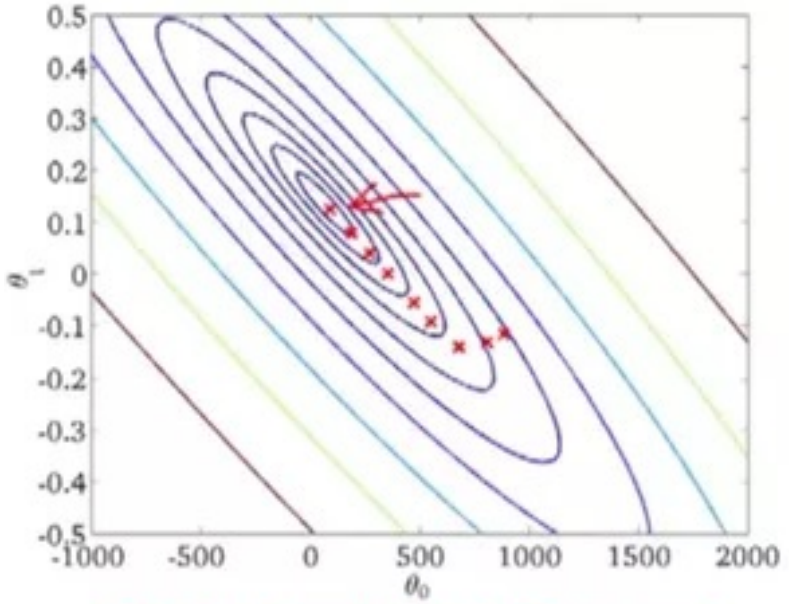
Mini-Batch SGD

1. Randomly shuffle dataset
2. Repeat until converge {
    for a mini-batch {
        compute gradient only with the mini-batch
        update weights
    }
}

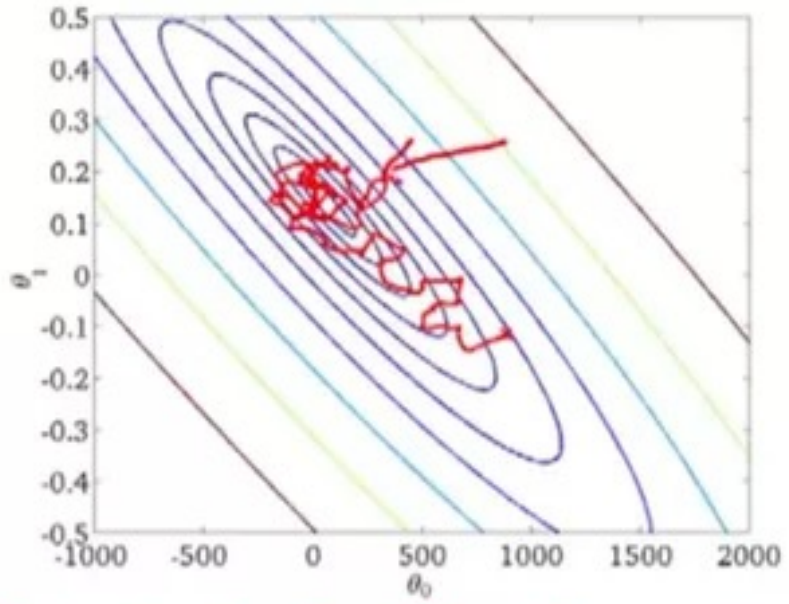
Note: In deep learning literature, we often use **mini-batch SGD** and **SGD** interchangeably.

Stochastic Gradient Descent (SGD)

- Gradients come from single samples, so they can be noisy and inaccurate!



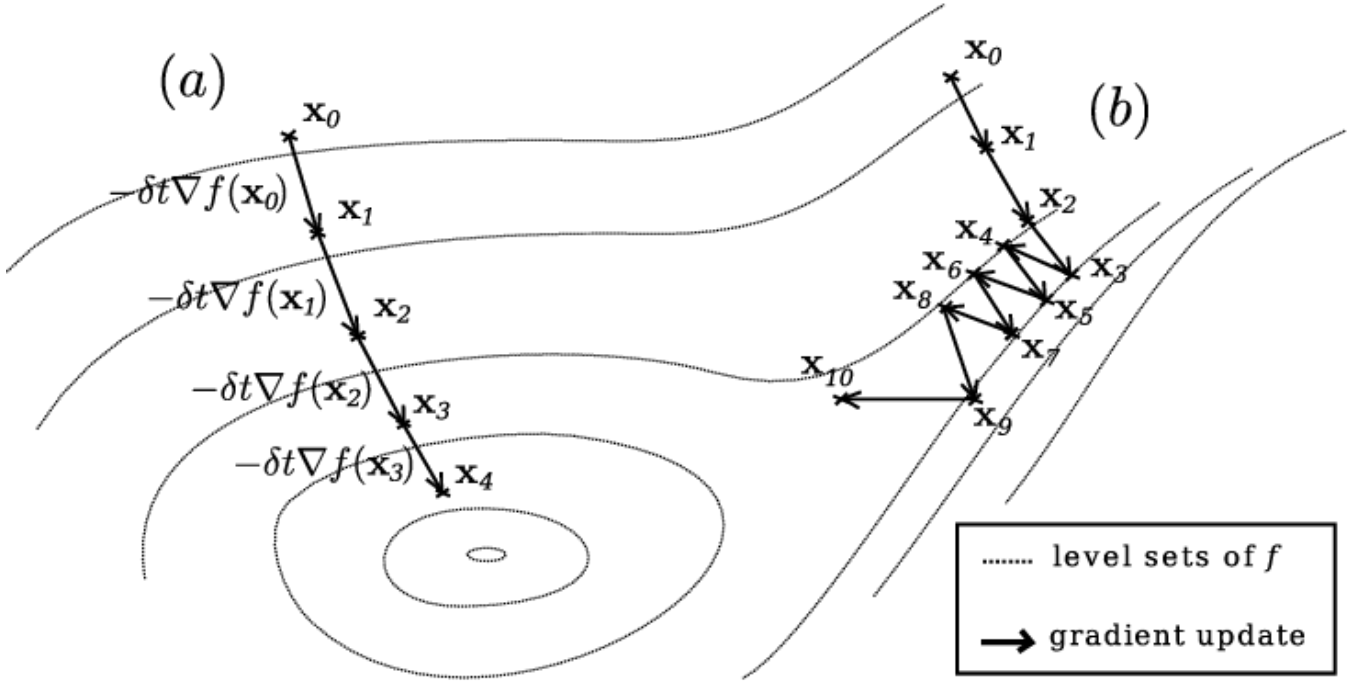
Batch Gradient Descent



Stochastic Gradient Descent

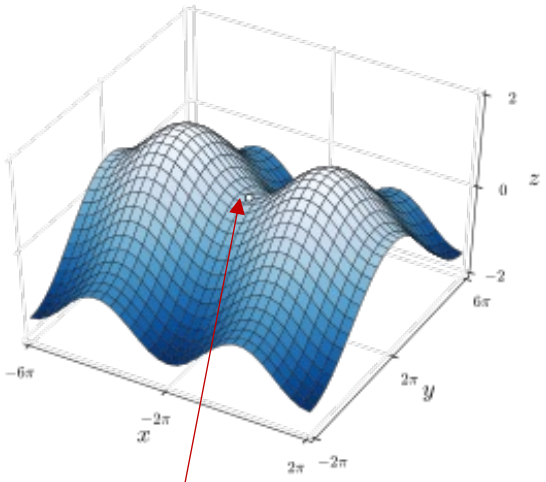
Problems of SGD

- "long-narrow valley"
  - What if your loss function happens to be steep in one direction but "flat" in another…
  - Condition number: ratio of largest to smallest singular value of the Hessian.
  - Large condition number → long-narrow valley

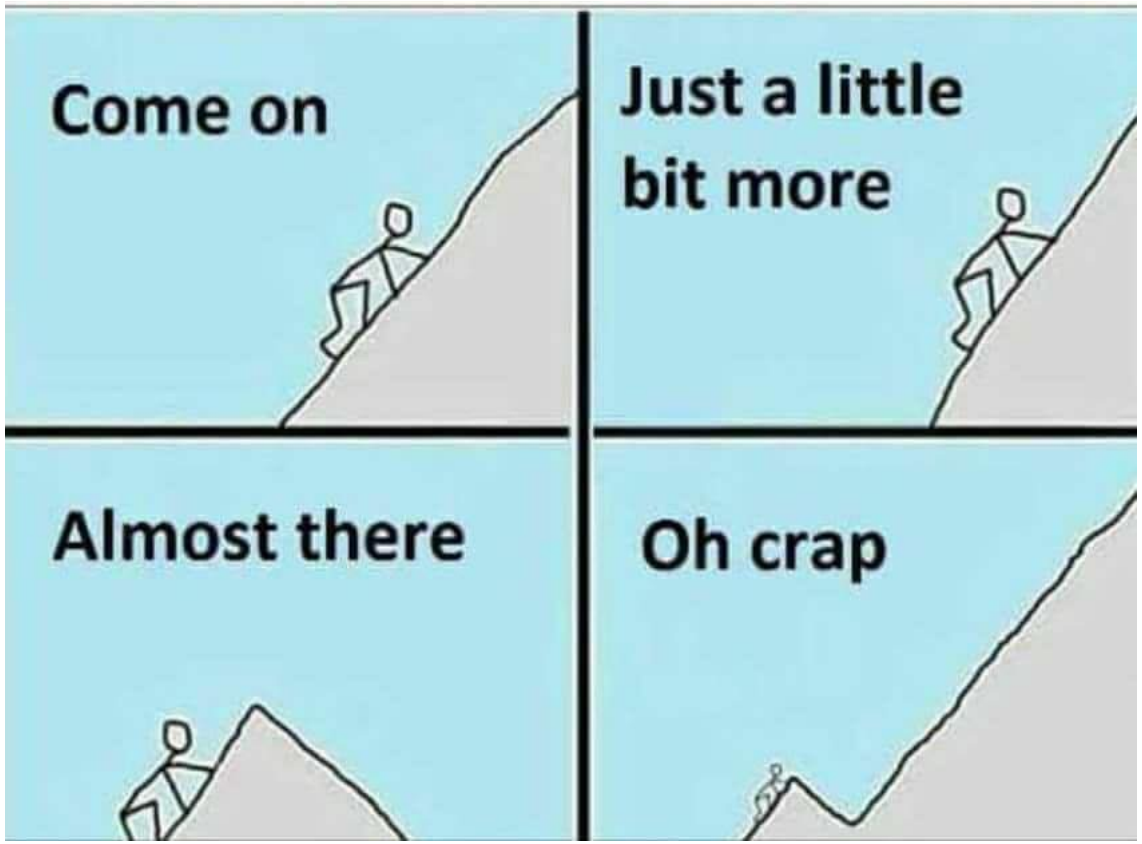


Problems of SGD

- Local minima or Saddle points → zero gradient! → No update (gets stuck)



Saddle Point



Momentum

- Idea: let's build up a velocity (momentum)!
- SGD

$$x_{t+1} = x_t - \alpha \nabla f(x_t)$$

- SGD + Momentum

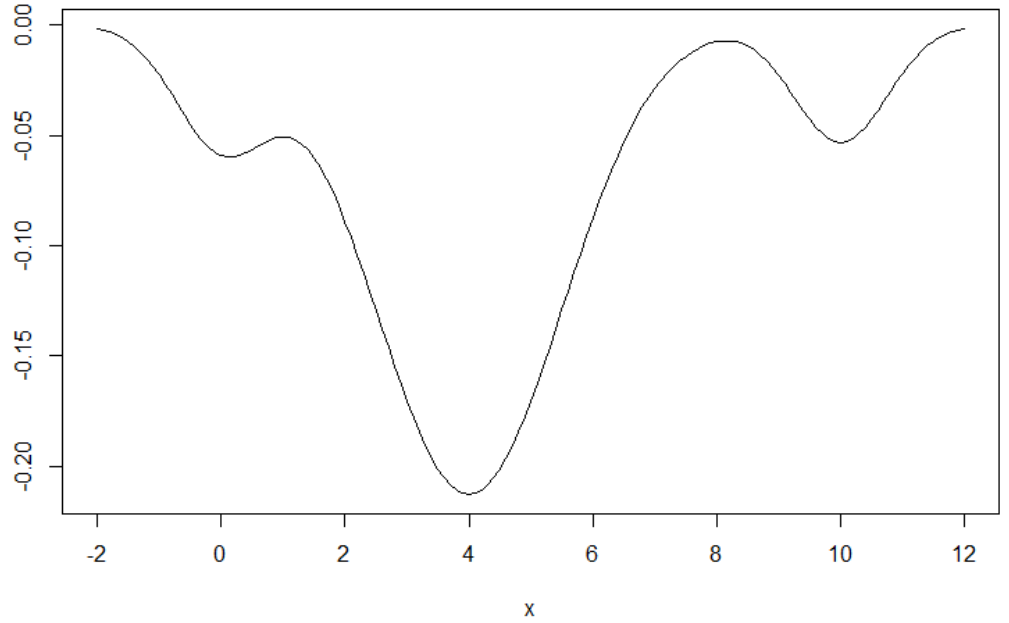
$$v_{t+1} = \rho v_t + \nabla f(x_t)$$

$$x_{t+1} = x_t - \alpha v_{t+1}$$

  - ρ: "friction" or "drag". Causes decrease of velocity. Typically, 0.9 or 0.99

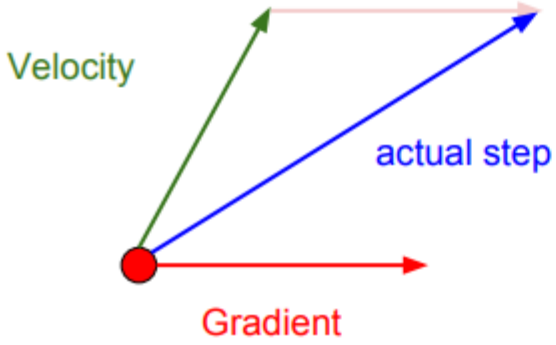
SGD + Momentum

- Discuss:
  - High condition number (long-narrow valley)
  - Local minima and saddle points
  - Noisy gradient



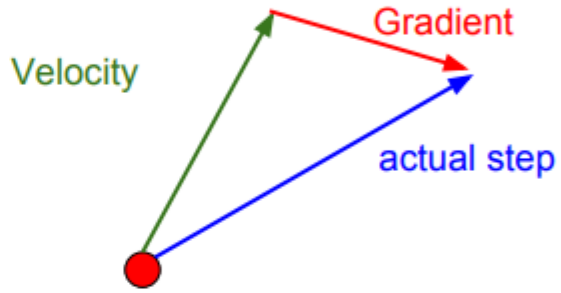
Nesterov Momentum

- Vanilla momentum method: Current gradient + Current velocity.



$$v_{t+1} = \rho v_t + \nabla f(x_t)$$
$$x_{t+1} = x_t - \alpha v_{t+1}$$

- Nesterov Version: Look ahead to the point where the current velocity would take us. Take the gradient there and perform the update.



☹

$$v_{t+1} = \rho v_t + \alpha \nabla f(x_t + \rho v_t)$$
$$x_{t+1} = x_t - v_{t+1}$$

AdaGrad

$$G(t) = \sum_{t=1}^{t} g(t)g^T(t)$$

$$\Delta W(t) = -\frac{\eta}{\sqrt{diag(G(t)) + \epsilon I}} * g(t)$$

- Perform element-wise scaling of the gradient
  - Scale factors determined based on the historical sum of squares…

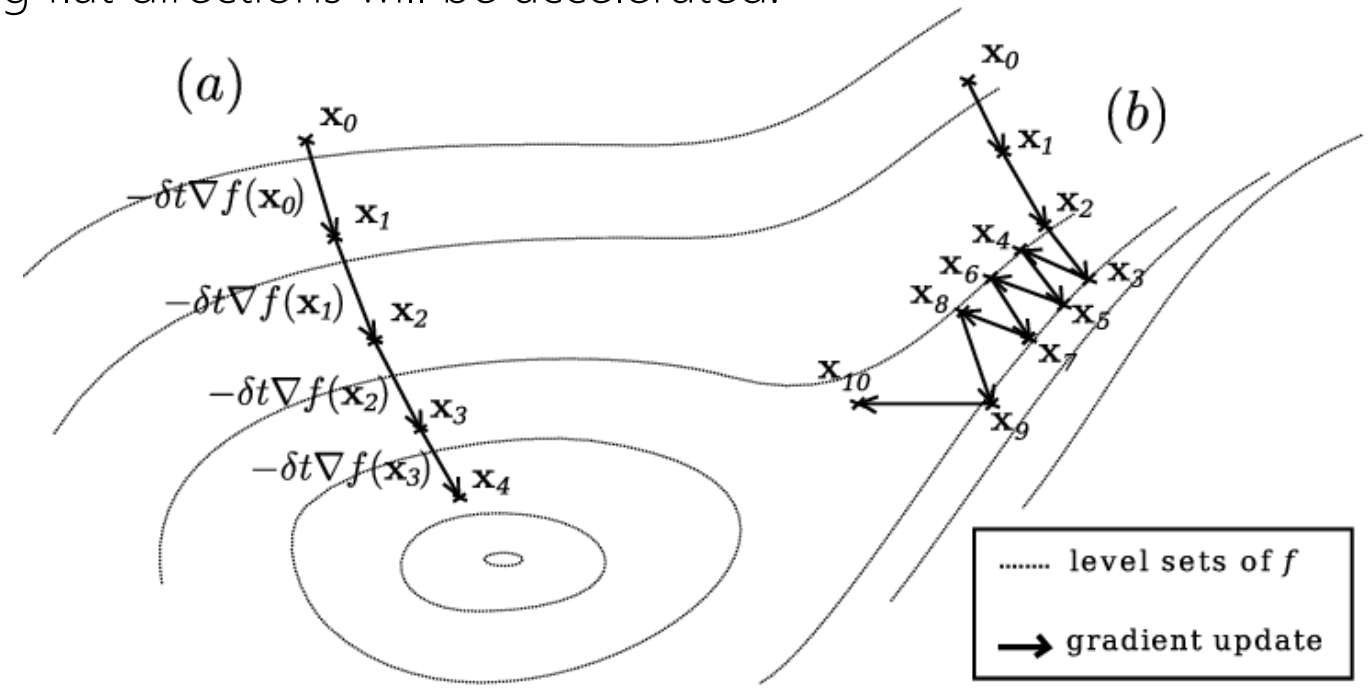
```
scale_factor = 0

for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    scale_factor += dx*dx
    x -= learning_rate * dx / (np.sqrt(scale_factor) + epsilon)
```

- The element-wise scaling has an effect of "per-parameter learning rates" or "adaptive learning rates," thus, the name Adaptive Gradient.

AdaGrad

- Long narrow valley: what happens with AdaGrad?
  - Step size along steep directions will be damped.
  - Step size along flat directions will be accelerated.



AdaGrad

- Historical sum: what happens with AdaGrad after many iterations?
  - Step size decays to zero… ☹

```
scale_factor = 0

for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    scale_factor += dx*dx
    x -= learning_rate * dx / (np.sqrt(scale_factor) + epsilon)
```

RMSProp

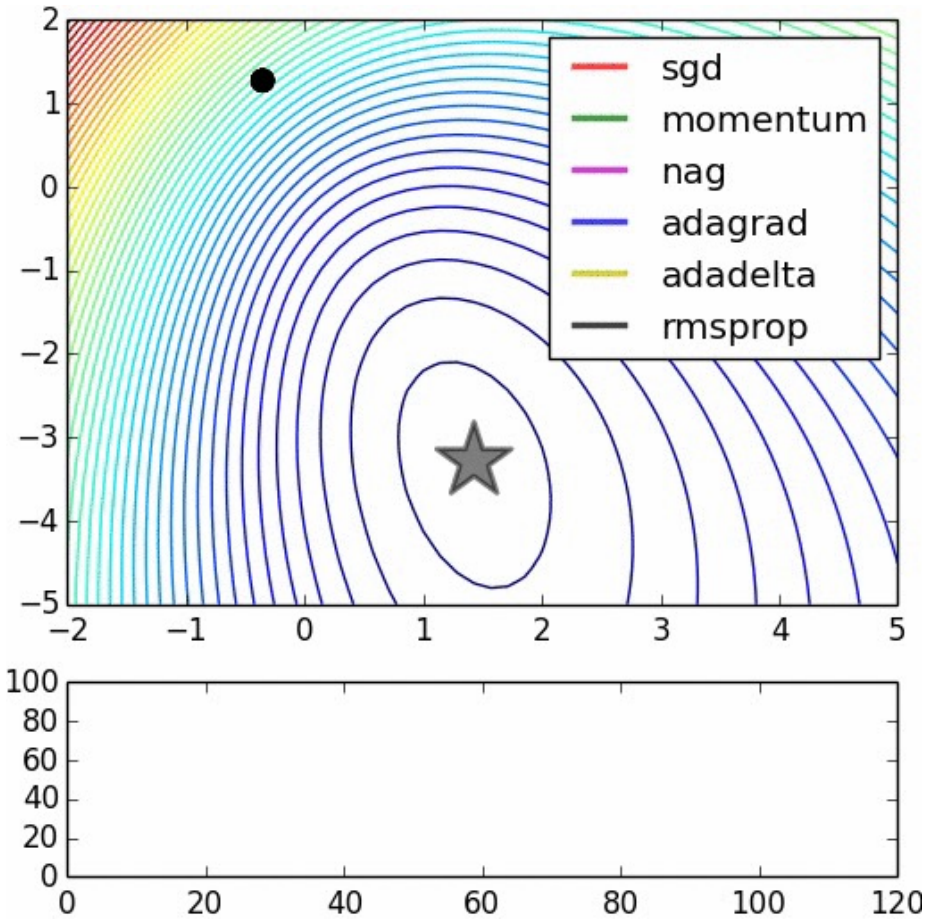
- AdaGrad (step size decays to zero):

```
scale_factor = 0
for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    scale_factor += dx*dx
    x -= learning_rate * dx / (np.sqrt(scale_factor) + epsilon)
```

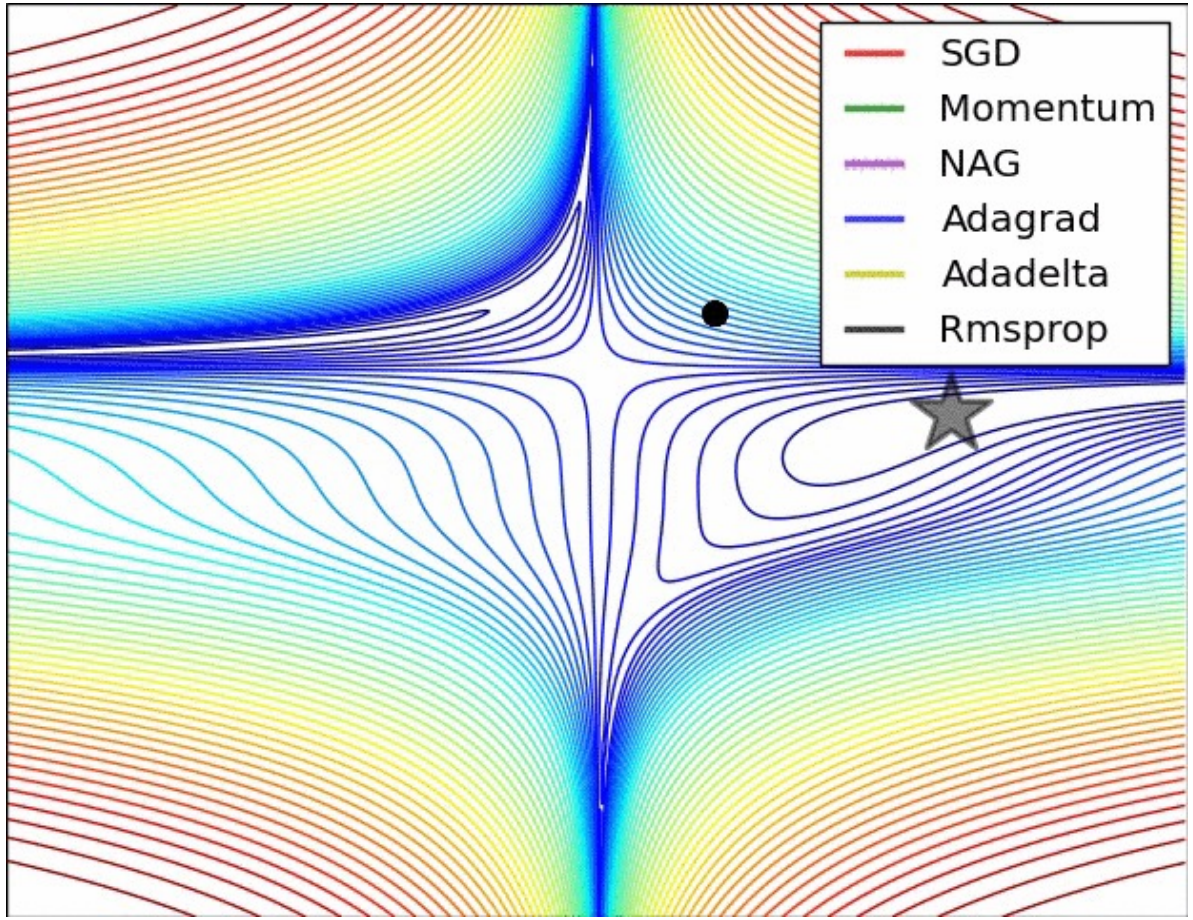
- RMSProp (problem solved ☺)

```
scale_factor = 0
for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    scale_factor = decay_rate*scale_factor + (1-decay_rate)*dx*dx
    x -= learning_rate * dx / (np.sqrt(scale_factor) + epsilon)
```

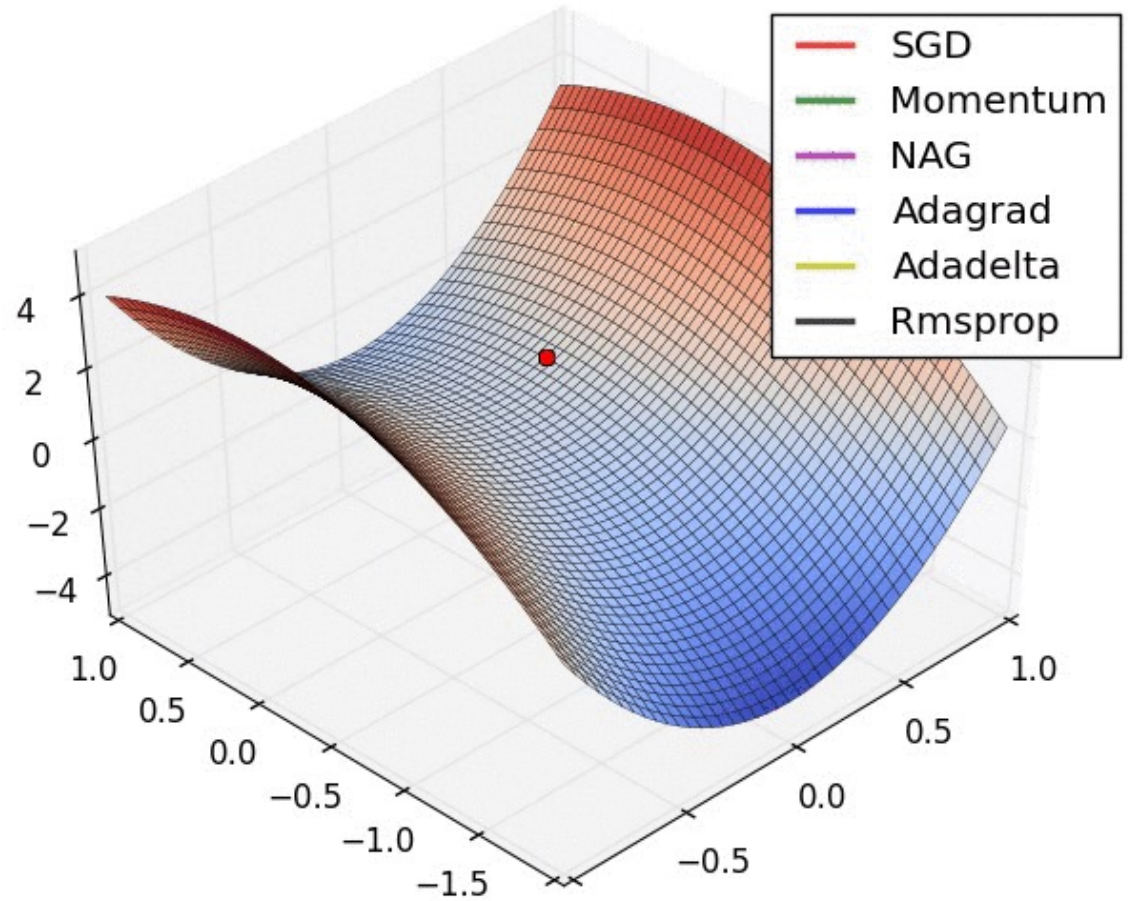
Comparison of Optimization Methods



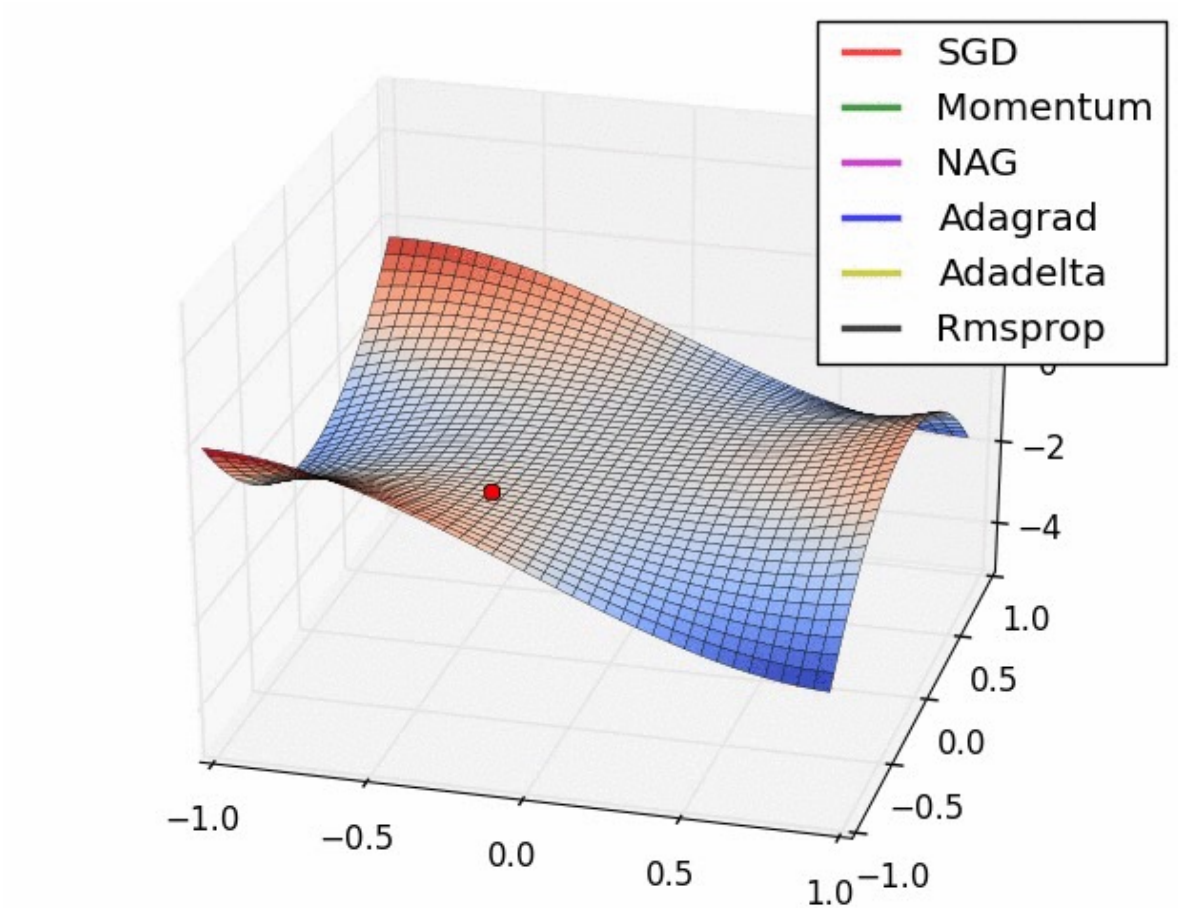
Comparison of Optimization Methods



Comparison of Optimization Methods



Comparison of Optimization Methods



Adam (All of the above!)

- Why not take the advantage of both momentum and adaptive gradient methods?

```
moment = [0, 0]
for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    moment[0] = beta[0]*moment[0] + (1-beta[0])*dx                    # momentum
    moment[1] = beta[1]*moment[1] + (1-beta[1])*dx*dx                 # RMSProp
    x -= learning_rate * moment[0] / (np.sqrt(moment[1]) + epsilon)
```

- Problem with the idea: what happens when iter = 0?
  - moments = 0 → bias!

Adam (All of the above!)

```
moment = [0, 0]
for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    moment[0] = beta[0]*moment[0] + (1-beta[0])*dx                  # momentum
    moment[1] = beta[1]*moment[1] + (1-beta[1])*dx*dx               # RMSProp
    x -= learning_rate * moment[0] / (np.sqrt(moment[1]) + epsilon)
```

- Modified version:

In typical scenarios, a good starting point:

- beta = some big number close to 1 (e.g. 0.9, 0.99)
- learning_rate = 1e-3

```
moment = [0, 0]
for iter in range(0, MAX_ITER):
    dx = backpropagate(x)   # compute gradient
    moment[0] = beta[0]*moment[0] + (1-beta[0])*dx                  # momentum
    moment[1] = beta[1]*moment[1] + (1-beta[1])*dx*dx               # RMSProp
    unbiased[0] = moment[0] / (1 - beta[0]**iter)                   # bias correction
    unbiased[1] = moment[1] / (1 - beta[1]**iter)
    x -= learning_rate * unbiased[0] / (np.sqrt(unbiased[1]) + epsilon)
```

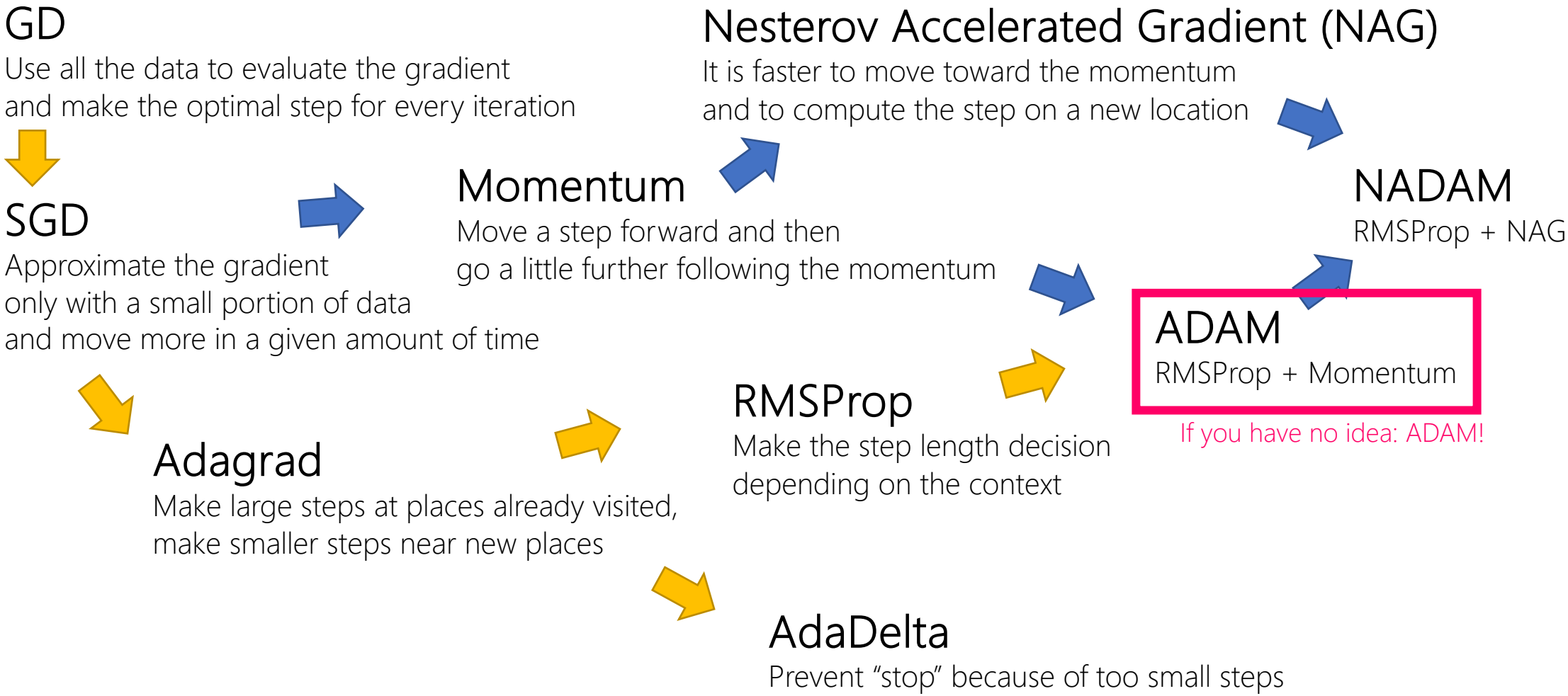
Adam (All of the above)

Algorithm 1: *Adam*, our proposed algorithm for stochastic optimization. See section 2 for details, and for a slightly more efficient (but less clear) order of computation. g_t^2 indicates the elementwise square $g_t \odot g_t$. Good default settings for the tested machine learning problems are $\alpha = 0.001$, $\beta_1 = 0.9$, $\beta_2 = 0.999$ and $\epsilon = 10^{-8}$. All operations on vectors are element-wise. With β_1^t and β_2^t we denote β_1 and β_2 to the power t.

Require: α: Stepsize
Require: $\beta_1, \beta_2 \in [0, 1)$: Exponential decay rates for the moment estimates
Require: $f(\theta)$: Stochastic objective function with parameters θ
Require: θ_0: Initial parameter vector
$m_0 \leftarrow 0$ (Initialize 1st moment vector)
$v_0 \leftarrow 0$ (Initialize 2nd moment vector)
$t \leftarrow 0$ (Initialize timestep)
while θ_t not converged **do**
$t \leftarrow t + 1$
$g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$ (Get gradients w.r.t. stochastic objective at timestep t)
$m_t \leftarrow \beta_1 \cdot m_{t-1} + (1 - \beta_1) \cdot g_t$ (Update biased first moment estimate)
$v_t \leftarrow \beta_2 \cdot v_{t-1} + (1 - \beta_2) \cdot g_t^2$ (Update biased second raw moment estimate)
$\widehat{m}_t \leftarrow m_t / (1 - \beta_1^t)$ (Compute bias-corrected first moment estimate)
$\widehat{v}_t \leftarrow v_t / (1 - \beta_2^t)$ (Compute bias-corrected second raw moment estimate)
$\theta_t \leftarrow \theta_{t-1} - \alpha \cdot \widehat{m}_t / (\sqrt{\widehat{v}_t} + \epsilon)$ (Update parameters)
end while
return θ_t (Resulting parameters)

https://arxiv.org/pdf/1412.6980.pdf

History of NN Optimizers



Tips on Learning Process & Hyperparameter Tuning

Training Process, Step-by-step

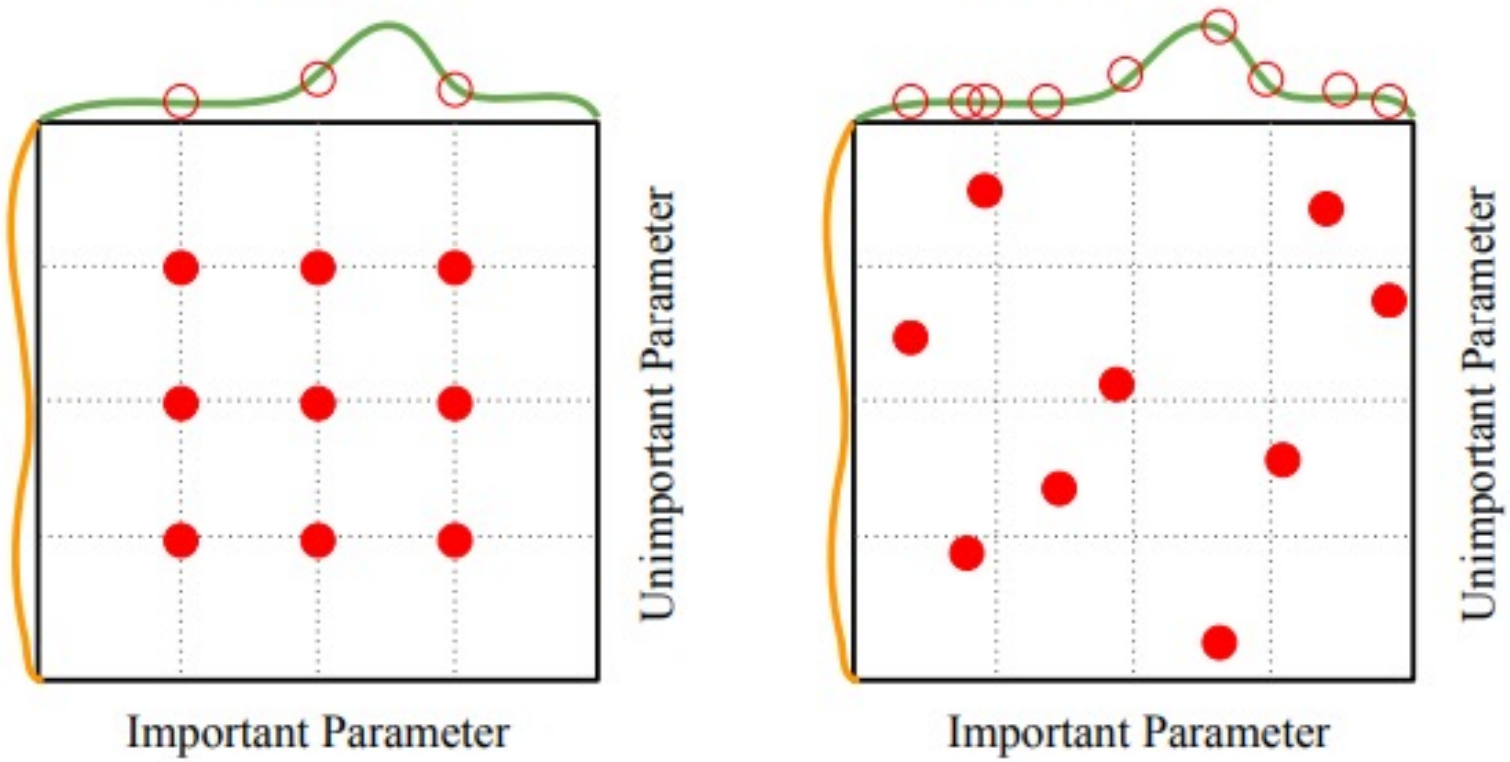
1. Preprocess the data (normalize/whiten, or at least zero-center them)
2. Design the network architecture (start with a simple one if possible)
3. Train with a small amount of data (a subset of your data). Make sure the model can overfit (i.e., loss go down to 0 and accuracy goes 1).
4. Now, the actual training, find the learning rate
   - LR too low: loss won't go down, LR too high: loss will explode.
   - Start with a small number and find the learning rate that makes some observable change.
5. Hyperparameter Tuning!

Hyperparameters Tuning

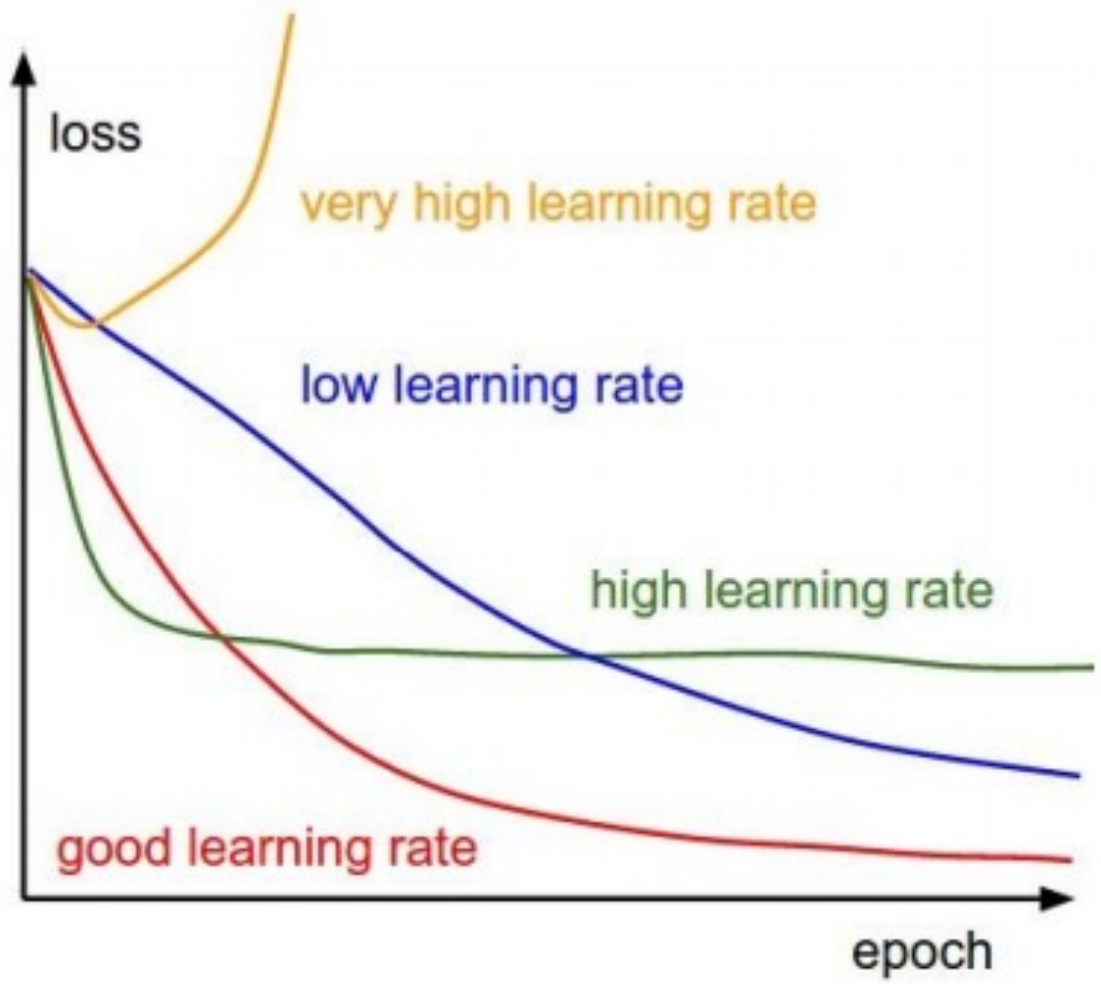
- Major hyperparameters: depth and breadth (or network architecture)
- Others: regularization terms, learning rate and its decay schedule, etc.

- Coarse-to-fine Strategy:
  - First, only a few epochs to get a rough idea of how parameters play
  - Second, longer epochs, do finer search
  - Repeat this as architecture changes.

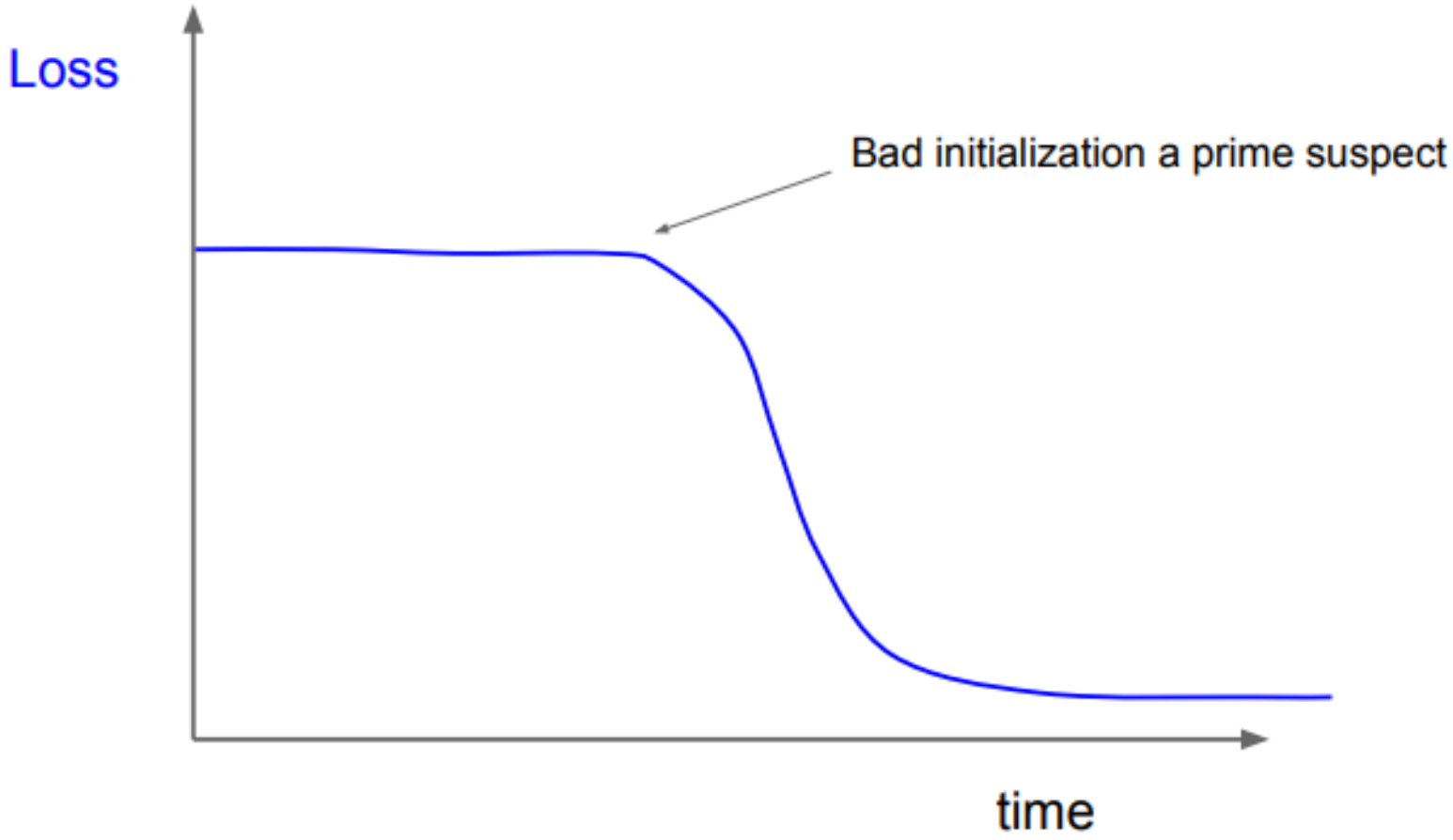
Grid Search vs. Random Search



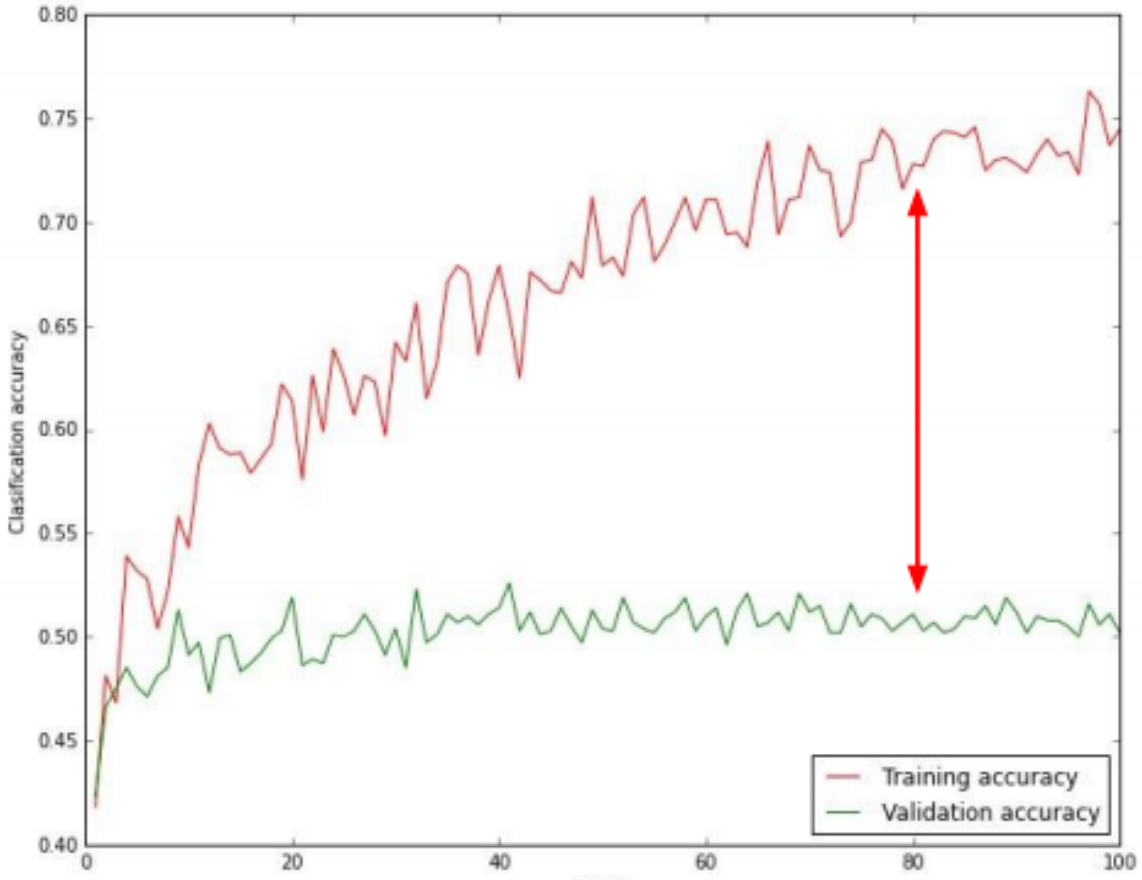
Always monitor and visualize the loss/accuracy!



Always monitor and visualize the loss/accuracy!



Always monitor and visualize the loss/accuracy!



big gap = overfitting

=> increase regularization strength?

no gap

=> increase model capacity?

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Tracking the weights also helps

- Percentage of weight update over weight magnitudes
  - Around 0.001 is ideal
  - Around 0.01 is about okay

```
# assume parameter vector W and its gradient vector dW
param_scale = np.linalg.norm(W.ravel())
update = -learning_rate*dW # simple SGD update
update_scale = np.linalg.norm(update.ravel())
W += update # the actual update
print update_scale / param_scale # want ~1e-3
```

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Model Ensemble

- Train multiple models independently.
- Throw those models to the test data points and take the average.
- Enjoy some extra accuracy (1~3%)
- (kind of regularization in a funny way)