UVA DATA SCIENCE

DS 6050 Deep Learning

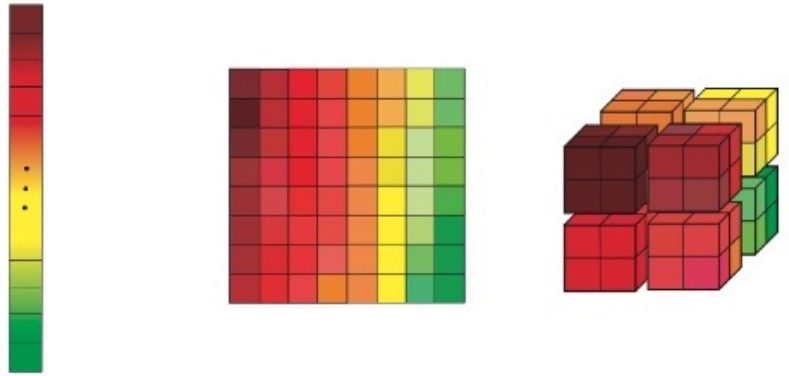
1. Machine Learning Recap

Sheng Li
Associate Professor
School of Data Science
University of Virginia

Matrices and Tensors

Tensors

- What is a tensor?
  - Casual definitions:
    - A "tensor" is like a matrix but with an arbitrary number of dimensions. A 1-dimensional tensor is a vector. A 2-dimensional tensor is a matrix. And then you can have tensors with 3, 4, 5 or more dimensions.
    - A tensor is a container for numerical data.
    - An arbitrary dimensional generalization of matrices.
    - *note: dimension == axis



$\mathbf{v} \in \mathbb{R}^{64}$ $X \in \mathbb{R}^{8x8}$ $\mathcal{X} \in \mathbb{R}^{4x4x4}$

Scalars (0D Tensors)

- A tensor that contains only one number is called a scalar
  - or scalar tensor; or zero-dimensional tensor; or 0D tensor.

- We will write scalars in italics (e.g. $a, b, c, d, e, x, y, z, \alpha, \beta, \gamma, \varphi, \phi, \psi$)

- Scalars can be real-valued, complex-valued, etc. We will always specify what kind of number they are when we introduce them.
  - e.g. we might say "Let $s \in \mathbb{R}$ be the slope of the line," while defining a real-valued scalar
  - or, we might say "Let $n \in \mathbb{N}$ be the number of units," while defining a natural number scalar.

Vectors (1D Tensors)

- An array of numbers is called a vector, or 1D tensor.
- A vector that has n-entities/elements is called n-dimensional vector.
  - Don't get confused with nD Tensor!

number of axis vs
number of rows/cols

- We will write vectors in bold (non-italic) typeface (e.g. $\mathbf{x}, \mathbf{y}, \mathbf{z}, \mathbf{u}, \mathbf{v}, \mathbf{w}, \boldsymbol{\phi}, \boldsymbol{\varphi}, \boldsymbol{\psi}$)

- The elements of a vector are identified by writing its name in italic with a subscript:

$$\mathbf{x} = [x_1, x_2, \cdots, x_n]$$

Matrices (2D Tensors)

- An array of vectors is called a matrix, or 2D tensor.
  - Visually, a matrix is a rectangular grid of numbers.
  - A matrix has two axes—rows and columns.
- We will write matrices in bold uppercase letters (e.g. $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{X}, \mathbf{Y}, \mathbf{Z}, \mathbf{\Phi}, \mathbf{\Psi}, \mathbf{\Omega}$)
- The elements of a matrix are identified by writing its name in italic with two subscript indices:

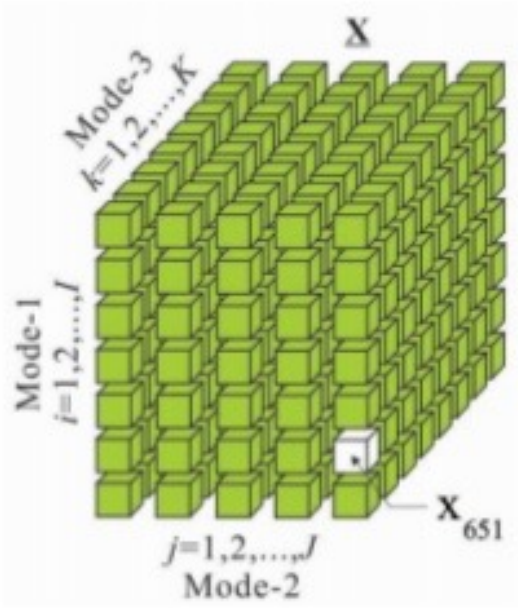
row, col

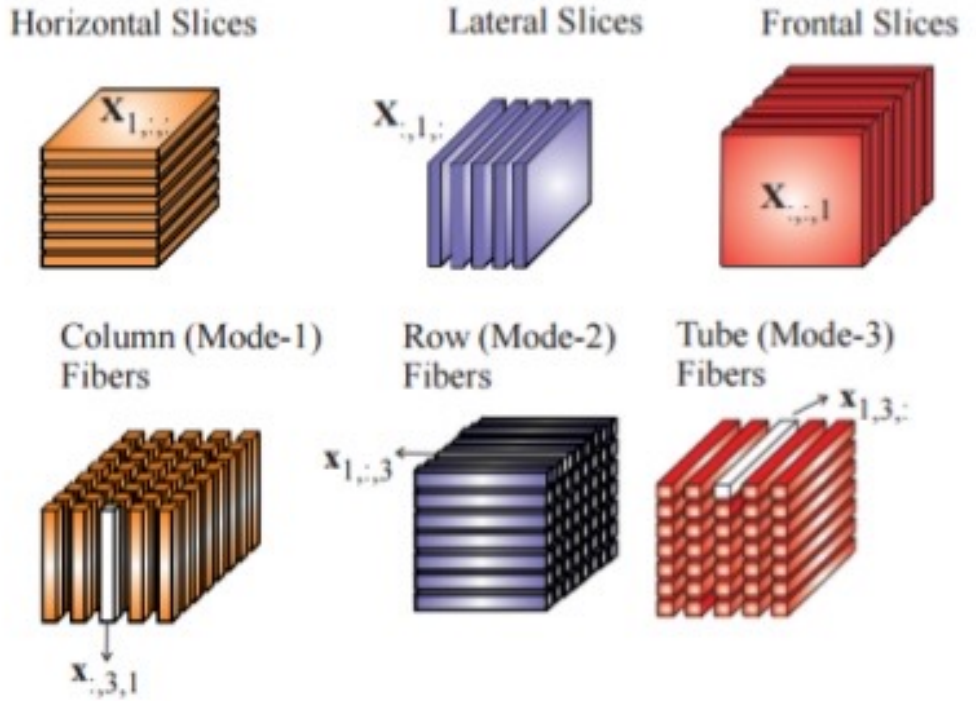
$$\mathbf{A} = \begin{bmatrix} A_{1,1} & A_{1,2} & \cdots & A_{1,\mathrm{n}} \\ A_{2,1} & A_{2,2} & \cdots & A_{2,\mathrm{n}} \\ \vdots & \vdots & \ddots & \vdots \\ A_{\mathrm{m},1} & A_{\mathrm{m},2} & \cdots & A_{\mathrm{m,n}} \end{bmatrix}$$

- We can identify all the numbers with vertical coordinate $\boldsymbol{i}$ by writing ':' for the horizontal coordinate: $\boldsymbol{A_{i,:}}$ (same goes for the horizontal cross-section)

3D Tensors and Higher

- A stack of matrices is called 3D tensor.
  - 3D tensor is like a cube of numbers.

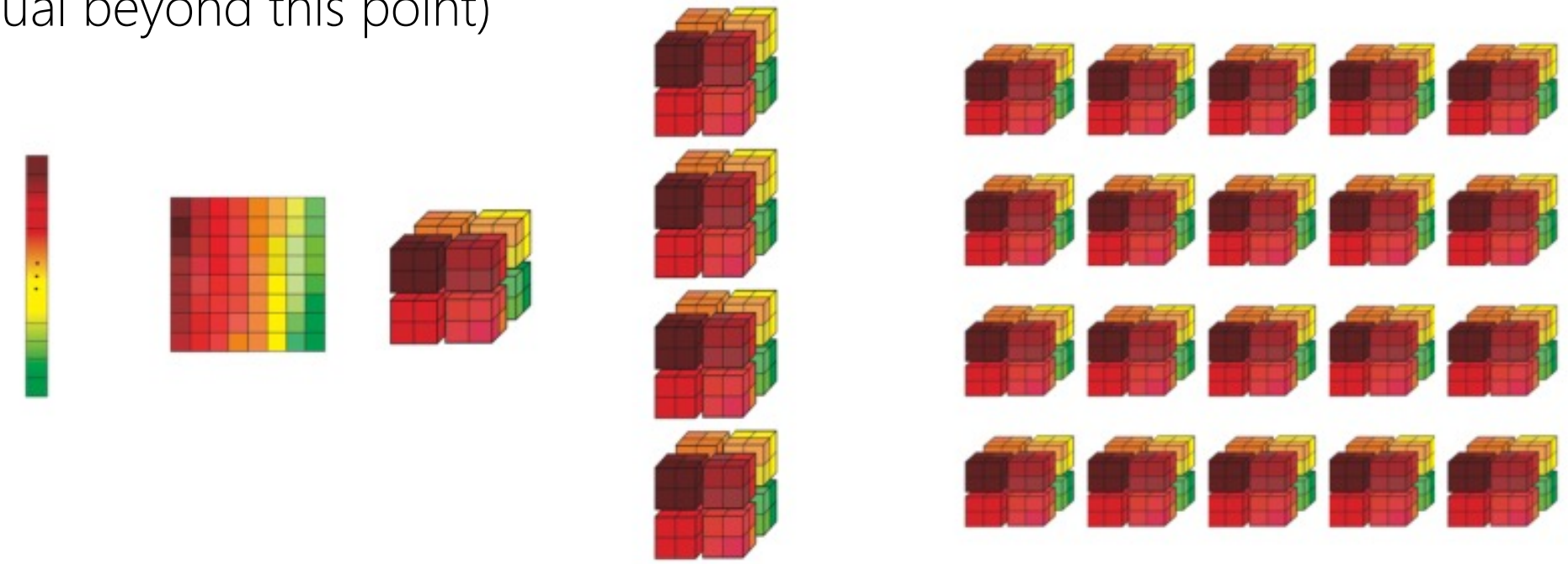




$$\mathcal{X} \in \mathbb{R}^{7x5x8}$$

3D Tensors and Higher

- A stack of matrices is called 3D tensor.
  - 3D tensor is like a cube of numbers.
- A stack of 3D tensors along the fourth axis is called 4D tensor. (Things become virtual beyond this point)



ex) An Avocado Price Data is a Vector

Date	Average Price	Total Volume	Organic?	Region
12/18/2016	1.18	631117.18	conventional	Chicago

$$\mathbf{x} = \begin{bmatrix} 2016 \\ 12 \\ 18 \\ 1.18 \\ 631117.18 \\ 0 \\ 3 \end{bmatrix}$$

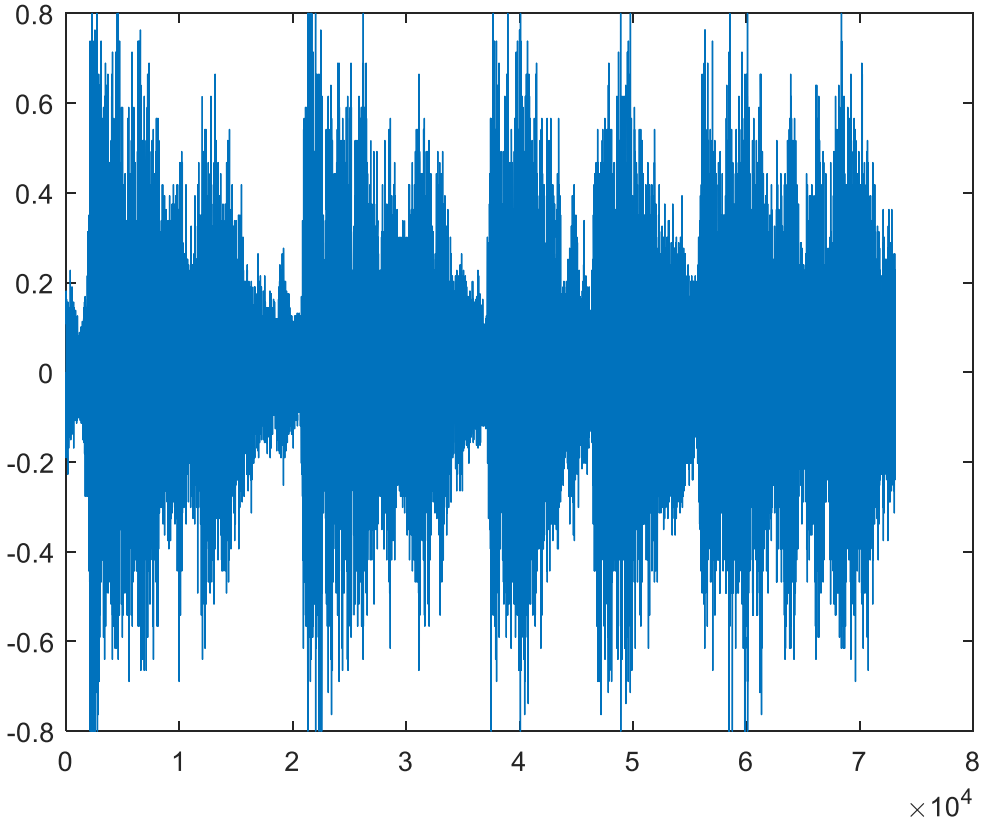
→ If organic 1, otherwise 0.

→ Region code.

ex) An Avocado Price Dataset is a Matrix

Date	Average Price	Total Volume	Organic?	Region
12/18/2016	1.18	631117.18	conventional	Chicago
12/11/2016	1.09	734954.21	conventional	Chicago
12/4/2016	1.27	624751.15	conventional	Chicago
11/27/2016	1.1	581638.75	conventional	Chicago
11/20/2016	1.13	694873.06	conventional	Chicago
11/13/2016	1.82	436710.91	conventional	Chicago
11/6/2016	2.07	376476.71	conventional	Chicago
10/30/2016	2.07	375213.57	conventional	Chicago
10/23/2016	1.84	431274.2	conventional	Chicago
10/16/2016	1.8	523682.5	conventional	Chicago
10/9/2016	1.82	515146.04	conventional	Chicago
10/2/2016	1.83	509215.79	conventional	Chicago
9/25/2016	1.8	553863.82	conventional	Chicago
9/18/2016	1.7	600389.56	conventional	Chicago

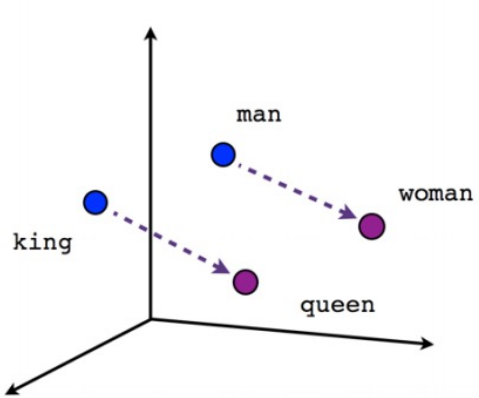
ex) A Mono Audio Signal is a Vector



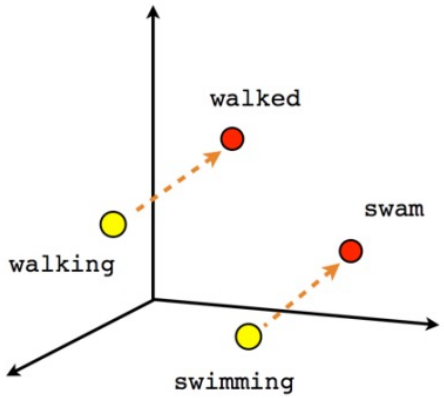
73113x1 double

	1
1	0
2	-0.0062
3	-0.0750
4	-0.0312
5	0.0062
6	0.0381
7	0.0189
8	-0.0250
9	-0.0312
10	-0.0750
11	-0.1258
12	-0.1443
13	-0.1812
14	-0.1905
15	-0.0750
16	-0.0127
17	-0.0381
18	-0.0750
19	0
20	0.0750
21	0.1258
22	0.1812
23	0.1505
24	0.1258
25	0.0812
26	0.0566
27	0
28	-0.0627
29	-0.0950
30	-0.0381

ex) A Word is a Vector

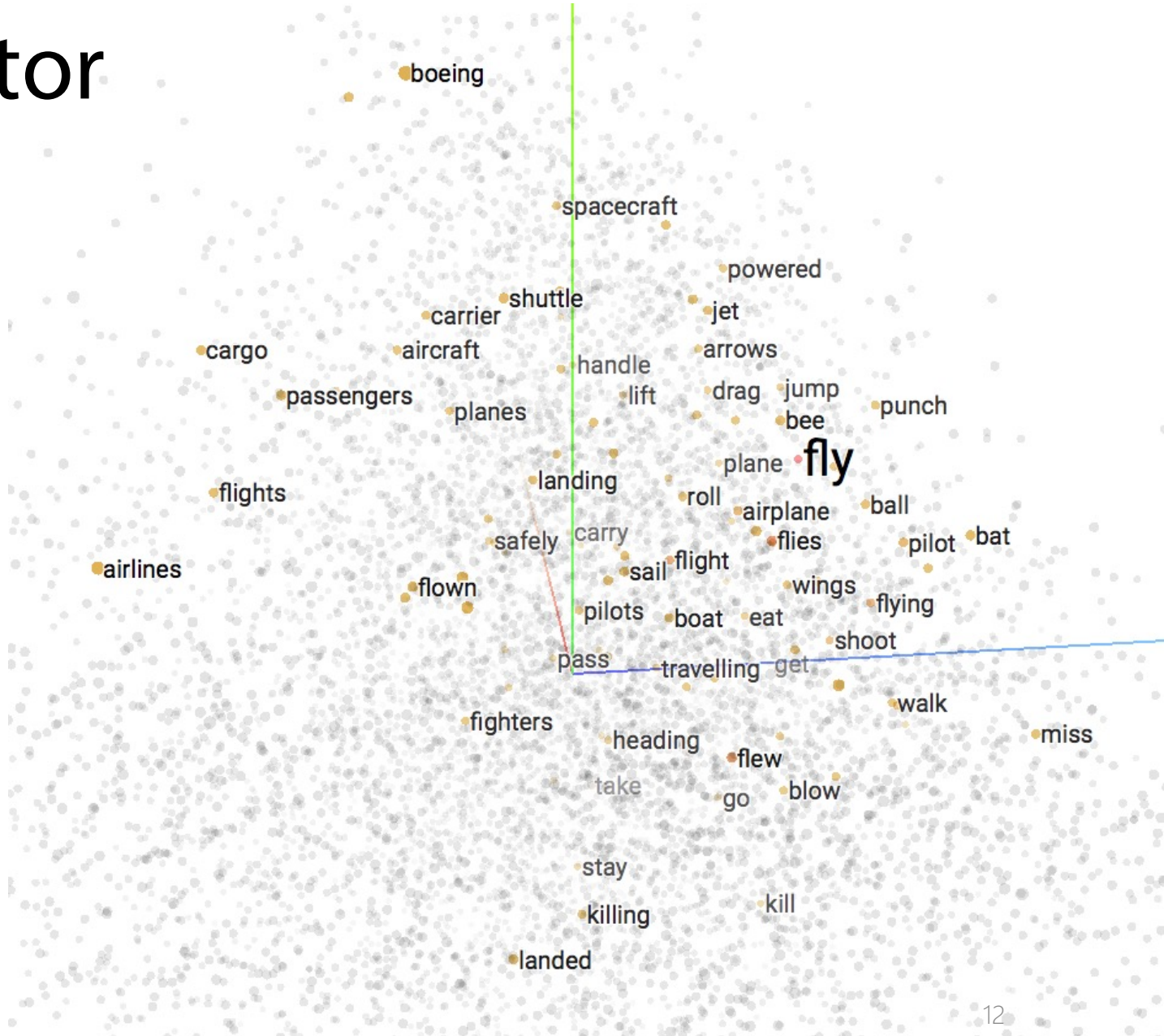


Male-Female



Verb tense

https://www.tensorflow.org/tutorials/representation/word2vec

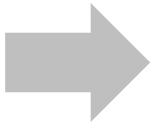


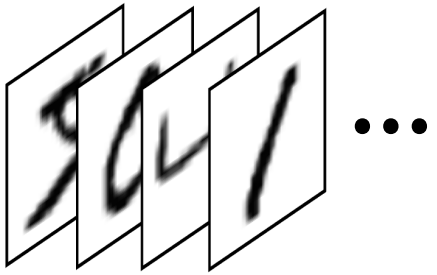
http://www.mattrajca.com/2016/12/30/performing-reverse-dictionary-searches-with-word2vec-models.html

ex) A Gray Image is a Matrix

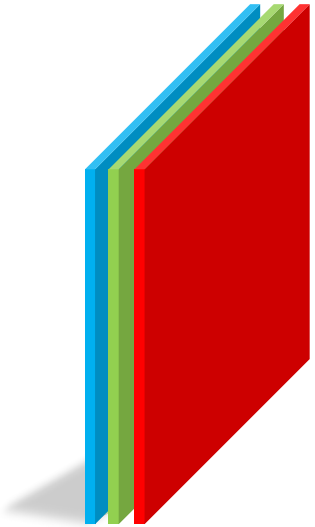
127	123	101	40	12	13	14	22	26	18	13	13	13	13	13	15
114	81	98	38	13	14	28	36	34	29	17	13	13	13	13	15
142	102	87	33	13	15	31	38	34	36	22	13	13	14	15	17
132	112	87	24	14	22	34	52	46	37	26	20	14	15	16	18
120	103	83	19	15	18	57	158	161	131	24	19	14	15	15	17
117	110	84	18	14	21	114	163	171	128	49	17	15	15	15	17
117	112	89	21	14	26	155	163	174	157	117	15	16	16	16	17
117	103	84	21	15	13	120	174	106	182	87	11	15	15	16	17
117	103	86	21	16	16	56	158	125	156	55	19	21	16	16	17
112	102	94	23	17	29	33	86	97	80	44	38	40	26	15	16
107	91	95	25	19	33	39	98	77	105	65	42	35	40	17	16
98	95	97	26	25	29	39	101	108	70	70	39	29	46	20	16
111	92	91	34	34	23	27	64	144	89	54	23	30	42	31	16
113	96	90	36	32	28	21	31	92	83	47	21	35	42	42	17
119	109	87	34	31	33	45	51	45	134	128	36	33	40	35	17
98	90	78	29	28	34	28	29	39	70	51	65	64	35	23	16

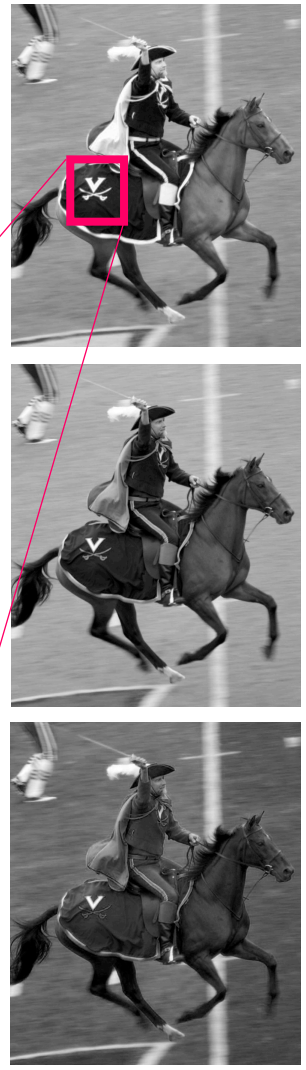
ex) MNIST Hand-written Digit Dataset = 3D





ex) A Color Image is a 3D Tensor





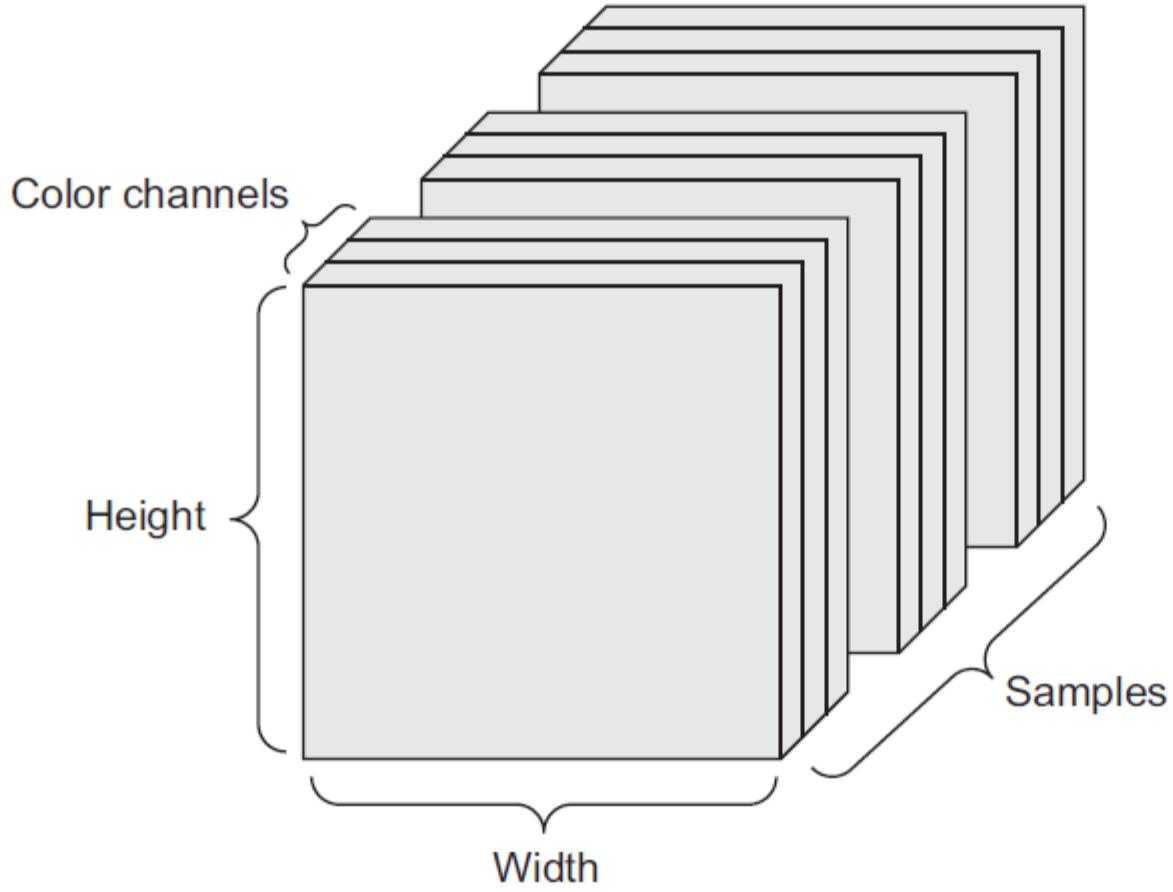
R

G

B

118	118	117	120	134	118	123	123	133	134	122	134
116	121	112	119	141	131	135	133	134	133	105	116
116	120	104	114	131	127	130	128	121	115	112	116
120	127	108	118	130	136	136	143	129	123	129	124
121	133	95	109	122	136	133	142	116	107	144	133
112	103	115	131	138	143	141	126	120	119	130	129
119	113	118	130	132	139	141	129	123	117	120	111
131	124	108	115	113	122	134	136	140	141	128	120
130	125	97	107	108	120	130	130	130	130	112	113
134	133	109	120	118	119	123	118	122	125	114	117
117	118	111	115	107	100	98	98	109	120	119	114
115	112	121	127	118	113	108	101	106	114	118	115
112	113	121	125	113	106	102	96	101	105	124	128
114	120	119	109	106	113	117	115	115	116	127	127
120	129	135	123	114	117	119	116	114	114	121	121
129	140	159	145	131	129	134	135	139	143	149	149
141	156	144	140	139	141	146	144	141	149	150	150
128	132	127	132	135	138	138	129	127	138	123	123

ex) Color Image Dataset is a 4D Tensor



ex) A Color Video is a 4D Tensor

Linear Algebra for ML

I assume you are comfortable with…

- Matrix Transpose
- Matrix Multiplication
- Identity and Inverse
- Linear Dependence and Span
- Norms
- Determinant
- Trace
- Eigendecomposition

Hadamard Product

look over LA notes again ajfkldsjfkl

This guy!

- a.k.a. Element-wise matrix multiplication

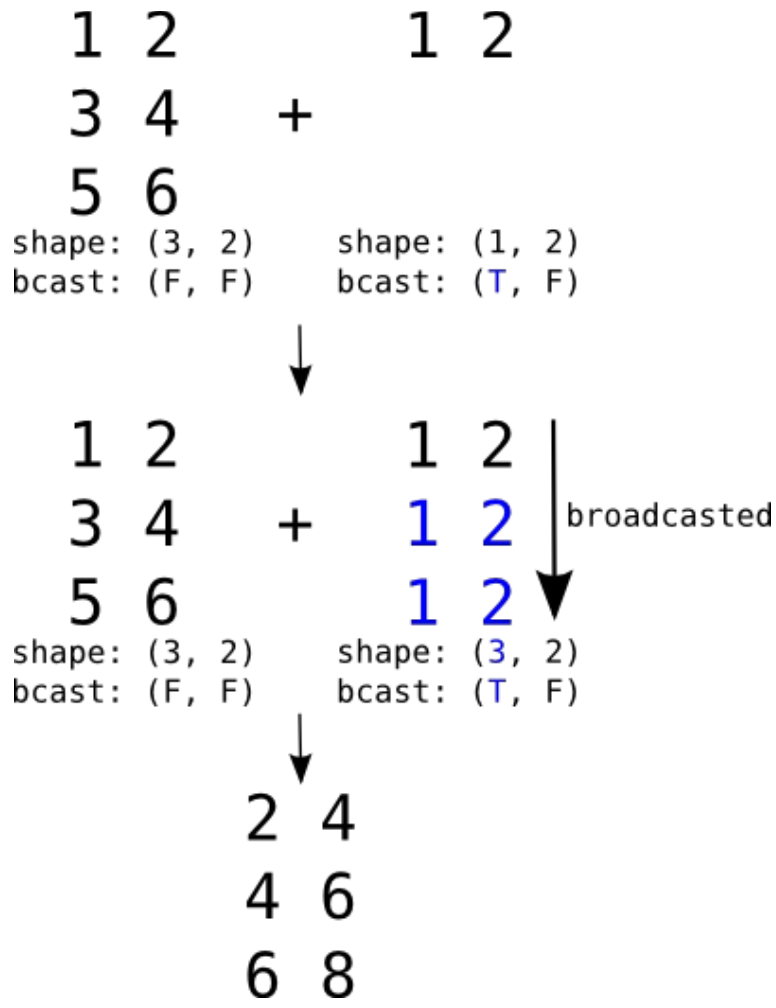
$$\mathbf{A} \odot \mathbf{B} \coloneqq [a_{ij} b_{ij}]$$

- Hadamard product is commutative (unlike ordinary matmul)

$$\mathbf{A} \odot \mathbf{B} = \mathbf{B} \odot \mathbf{A}$$

Broadcasting

- A shorthand notation commonly used in ML.
- Given $\mathbf{A} \in \mathbb{R}^{m \times n}$ and $\mathbf{x} \in \mathbb{R}^{m}$, under broadcasting convention, the expression $\mathbf{B} = \mathbf{A} + \mathbf{x}$ is assumed to be equivalent to $b_{ij} = a_{ij} + x_j$.



http://deeplearning.net/software/theano/tutorial/broadcasting.html

Tensor Dot Product

- Most common, most useful tensor operation.
- Not to be confused with an element-wise (Hadamard) product.

$$\mathbf{x} \cdot \mathbf{y} = \mathbf{y} \cdot \mathbf{x} = \sum_i x_i y_i \quad \text{(vector dot)}$$

$$\mathbf{A} \cdot \mathbf{y} = \sum_j A_{i,j} y_j \quad \text{(matrix-vector dot)}$$

$$\mathbf{A} \cdot \mathbf{B} = \sum_j A_{i,j} B_{j,k} \quad \text{(matrix-matrix dot)}$$

$$\mathbf{X} \cdot \mathbf{Y} = \sum_l X_{i,j,k,l} Y_{l,m,n} \quad \text{(tensor-tensor dot)}$$

Moore-Penrose Pseudoinverse

- Consider a linear system: $\mathbf{XW} = \mathbf{Y}$, where $\mathbf{X} \in \mathbb{R}^{N \times d}$, $\mathbf{W} \in \mathbb{R}^{d \times 1}$, and $\mathbf{Y} \in \mathbb{R}^{N \times 1}$
- Find $\mathbf{W}$.
  - If $N = d$, $\exists \mathbf{X}$: ** if sq-matrix...
    - $\mathbf{W} = \boldsymbol{X}^{-1}\mathbf{Y}$
  - If $N > d$?
    - Least-square solution: $\min_{\mathbf{W}} \|\mathbf{Y} - \mathbf{XW}\|^2$
    - $\mathcal{L}(\mathbf{W}) = \|\mathbf{Y} - \mathbf{XW}\|^2 = (\mathbf{Y} - \mathbf{XW})'(\mathbf{Y} - \mathbf{XW})$
      $= \mathbf{Y}'\mathbf{Y} - \mathbf{Y}'\mathbf{XW} - (\mathbf{XW})'\mathbf{Y} + (\mathbf{XW})'(\mathbf{XW})$ ** (AB)^T==B^T A^T
      $= \mathbf{Y}'\mathbf{Y} - \mathbf{Y}'\mathbf{XW} - \mathbf{Y}'\mathbf{XW} + \mathbf{W}'\mathbf{X}'\mathbf{XW}$ (why?)
    - $\frac{\partial \mathcal{L}}{\partial \mathbf{W}}(\mathbf{W}) = -2\mathbf{Y}'\mathbf{X} + 2\mathbf{W}'\mathbf{X}'\mathbf{X} \equiv \mathbf{0}$ (if you're not comfortable with vector calculus: http://matrixcookbook.com/)
    - $\mathbf{Y}'\mathbf{X} = \mathbf{W}'\mathbf{X}'\mathbf{X}$
    - $\mathbf{X}'\mathbf{XW} = \mathbf{X}'\mathbf{Y}$
    - $\mathbf{W} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{Y}$ (if $(\mathbf{X}'\mathbf{X})$ non-singular)

Moore-Penrose Pseudoinverse:

$$\mathbf{X}^{+} := (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'$$

Linear Regression

$$\mathbf{x}^{(t)} \in \mathbb{R}^{1\times d}, \qquad 1 \le t \le N$$

$$r^{(t)} \in \mathbb{R}, \qquad 1 \le t \le N$$

$$\mathbf{X} = \begin{bmatrix} \vdots \\ - \ \mathbf{x}^{(t)} \ - \\ \vdots \end{bmatrix} \in \mathbb{R}^{N\times d} \qquad \mathbf{R} = \begin{bmatrix} \vdots \\ r^{(t)} \\ \vdots \end{bmatrix} \in \mathbb{R}^{N\times 1}$$

Linear Regression

Model Parameters: $\mathbf{W} \in \mathbb{R}^{d \times 1}$, $\mathbf{b} \in \mathbb{R}$

Linear Model: $$\mathbf{R} = \mathbf{X}\mathbf{W} + b$$

Broadcasting adding scalar to vector...

A "trick": $$\mathbf{X} \rightarrow \begin{bmatrix} \mathbf{X} & \mathbf{1} \end{bmatrix}$$

$$\mathbf{\Theta} := \begin{bmatrix} \mathbf{W} \\ b \end{bmatrix}$$

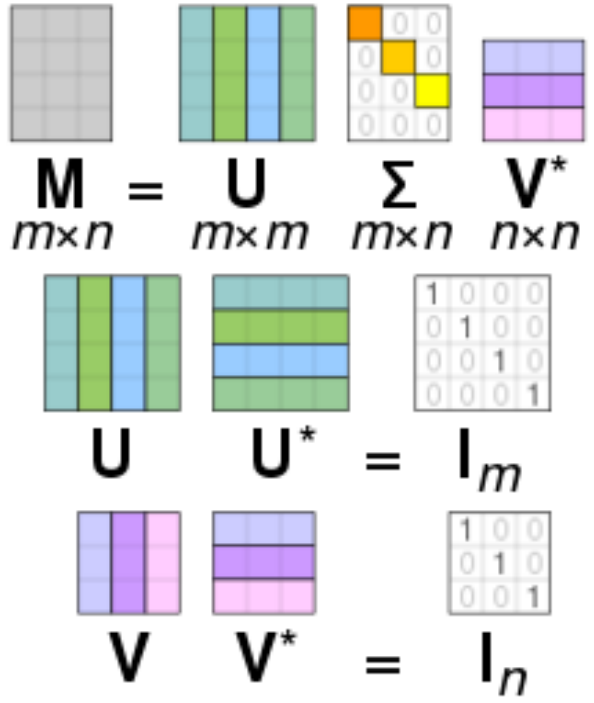
$$\mathbf{R} = \mathbf{X}\mathbf{W} + b \quad \rightarrow \quad \mathbf{R} = \mathbf{X}\mathbf{\Theta}$$

$$\mathbf{\Theta} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{R}$$

Singular Value Decomposition (SVD)

- $\mathbf{X} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}'$ samples x attributes, N x d
  - $\boldsymbol{\Sigma} \in \mathbb{R}^{N \times d}$: a diagonal matrix (singular values)
  - $\boldsymbol{U} \in \mathbb{R}^{N \times N}$: an orthogonal matrix (left singular vectors)
    - Eigenvectors of $\mathbf{X}\mathbf{X}'$
  - $\boldsymbol{V} \in \mathbb{R}^{d \times d}$: an orthogonal matrix (right singular vectors)
    - Eigenvectors of $\mathbf{X}'\mathbf{X}$

- $\mathbf{X}^{+} = (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}' = \mathbf{V}\boldsymbol{\Sigma}^{+}\mathbf{U}'$
  - $\boldsymbol{\Sigma}^{+}$: pseudoinverse of a diagonal matrix is calculated as…?



Singular Value Decomposition (SVD)

- $\mathbf{X} = \mathbf{U\Sigma V'}$

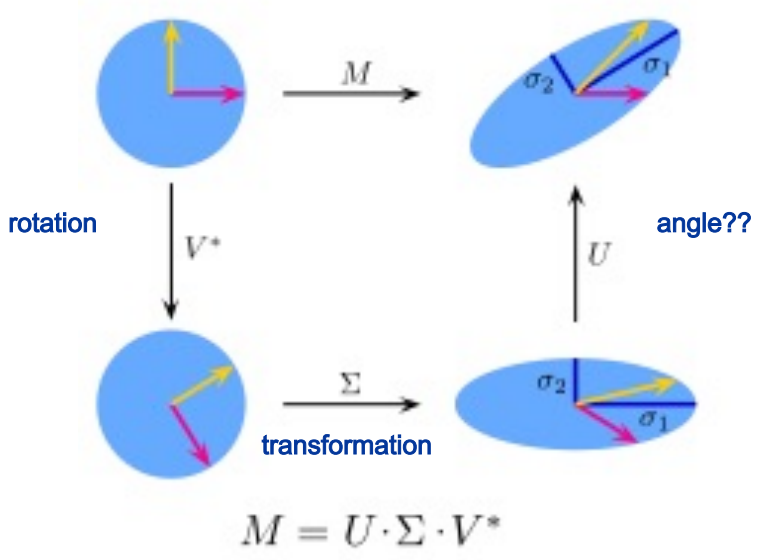


Image Classification

Why Image Classification?

- Images are more straightforward and intuitive subject to study deep networks
- Easy to generalize to other forms of data
- Major breakthroughs in deep learning research were in computer vision
- Image classification = core task in computer vision

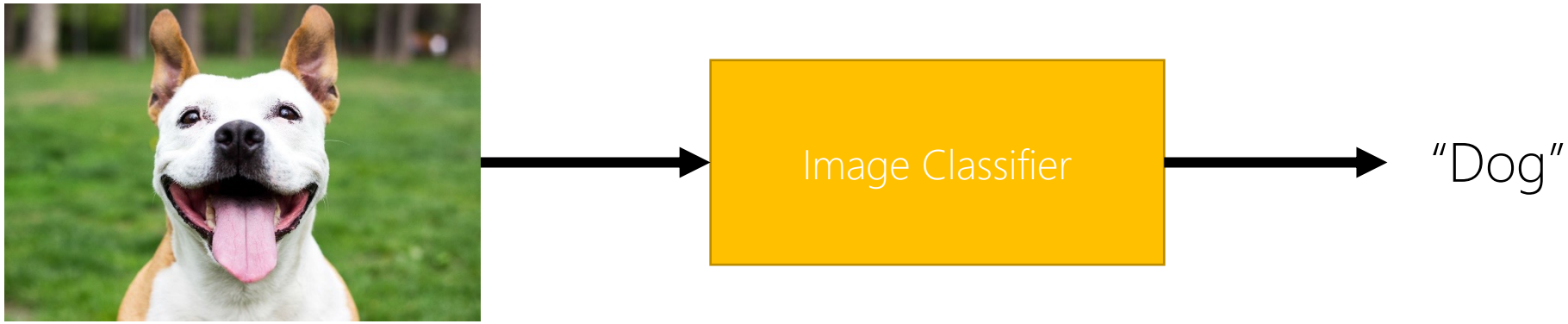


Image Classification is Challenging!

- Problem 1. Semantic Challenge

What we see

127	123	101	40	12	13	14	22	26	18	13	13	13	13	13	15
114	81	98	38	13	14	28	36	34	29	17	13	13	13	13	15
142	102	87	33	13	15	31	38	34	36	22	13	13	14	15	17
132	112	87	24	14	22	34	52	46	37	26	20	14	15	16	18
120	103	83	19	15	18	57	158	161	131	24	19	14	15	15	17
117	110	84	18	14	21	114	163	171	128	49	17	15	15	15	17
117	112	89	21	14	26	155	163	174	157	117	15	16	16	16	17
117	103	84	21	15	13	120	174	106	182	87	11	15	15	16	17
117	103	86	21	16	16	56	158	125	156	55	19	21	16	16	17
112	102	94	23	17	29	33	86	97	80	44	38	40	26	15	16
107	91	95	25	19	33	39	98	77	105	65	42	35	40	17	16
98	95	97	26	25	29	39	101	108	70	70	39	29	46	20	16
111	92	91	34	34	23	27	64	144	89	54	23	30	42	31	16
113	96	90	36	32	28	21	31	92	83	47	21	35	42	42	17
119	109	87	34	31	33	45	51	45	134	128	36	33	40	35	17
98	90	78	29	28	34	28	29	39	70	51	65	64	35	23	16

What computers see
= just a grid of numbers (e.g. 1024 x 768 x 3)

Image Classification is Challenging!

- Problem 2. Varying Perspectives

Image Classification is Challenging!

- Problem 3. Illumination

http://www.ifp.illinois.edu/~tchen5/vlfr.html

Image Classification is Challenging!

- Problem 4. Occlusion

Image Classification is Challenging!

- Problem 5. Deformation, Shape & Posture Change

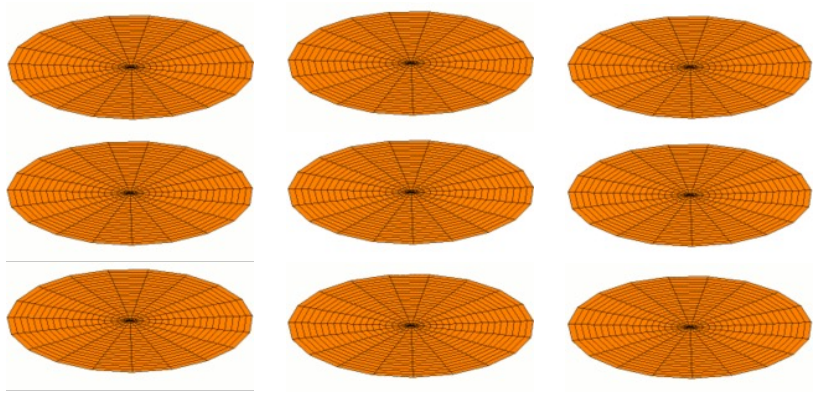
Image Classification is Challenging!

- Problem 6. Intraclass Variation

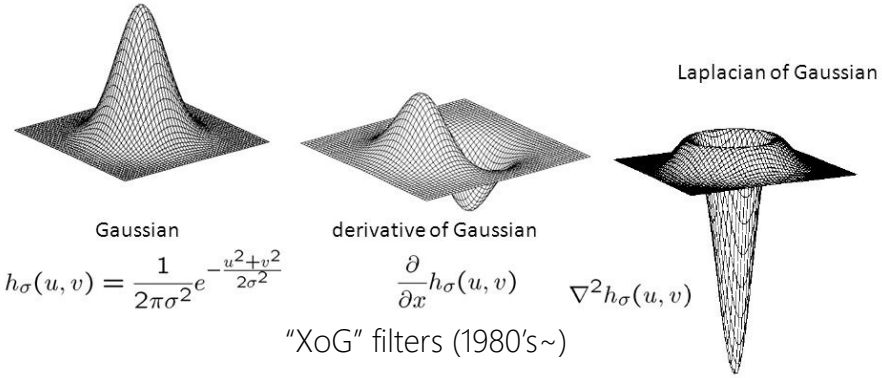
Arman, Alarm Clocks (Reveils), 1960, MCA Chicago

Conventional Approach (Model-Driven)

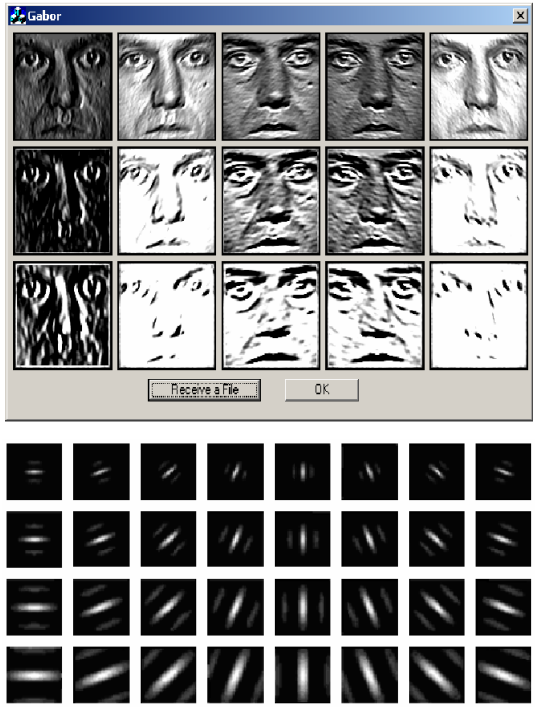
manually finding f(x)



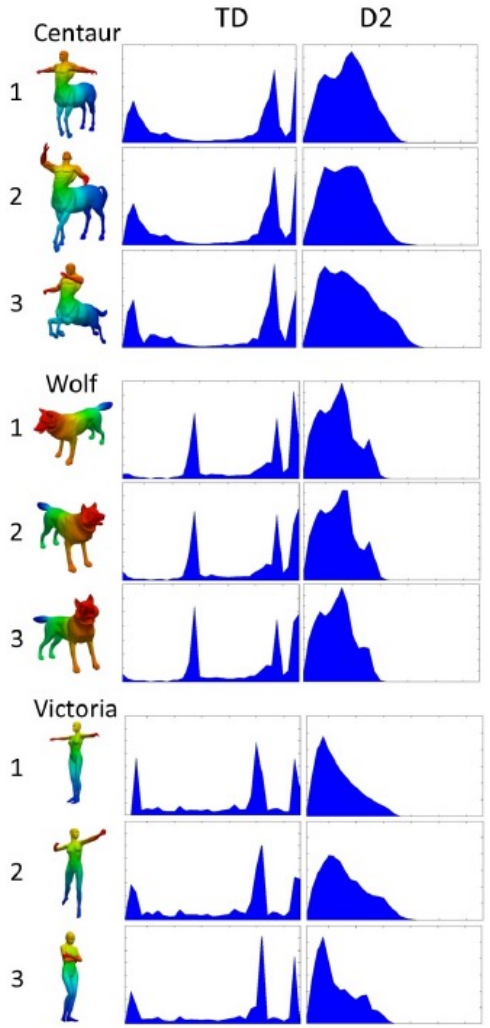
Laplace-Beltrami Spectra (2000's~)



"XoG" filters (1980's~)



Gabor Filter (1998~)

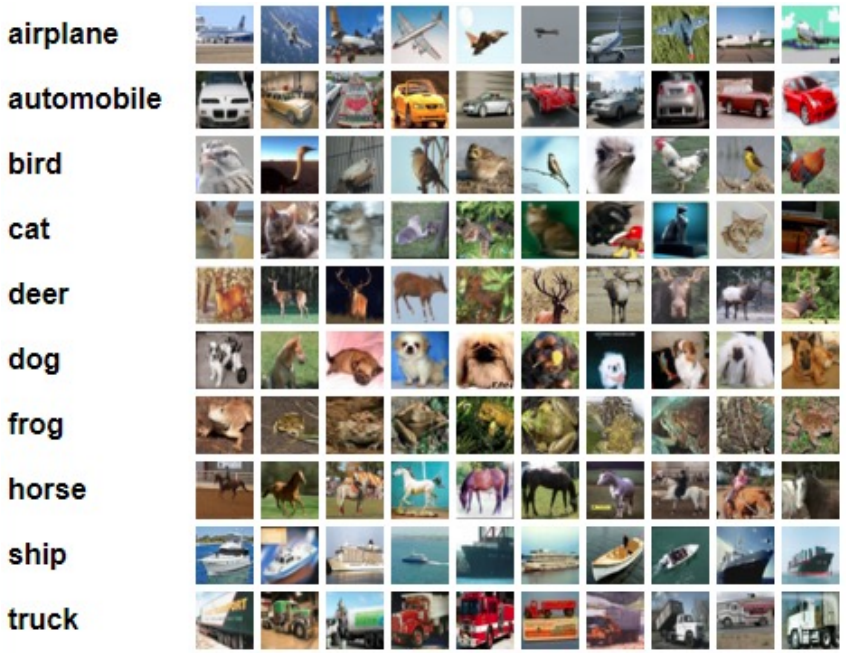


Distance Histograms (2002~)

Machine Learning (Data-Driven)

don't need to
manually find f(x),
instead use dataS

Data with Labels



training

Image Classifier

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{Wx} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{W}\mathbf{x} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

- Our machine learning model f is then defined as $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$. $\mathbf{W}$ and $\mathbf{b}$ are called model parameters.

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{W}\mathbf{x} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

- Our machine learning model f is then defined as $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$. $\mathbf{W}$ and $\mathbf{b}$ are called model parameters.

- Goal: Find $\mathbf{W}$ and $\mathbf{b}$ that minimize $|y - f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})|^2$

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{W}\mathbf{x} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

- Our machine learning model f is then defined as $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$. $\mathbf{W}$ and $\mathbf{b}$ are called model parameters.

- Goal: Find $\mathbf{W}$ and $\mathbf{b}$ that minimize $|y - f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})|^2$
  - In English: I want the predicted label of my model $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$ to be as close as possible to the true label y and I'm going to accomplish that by tuning $\mathbf{W}$ and $\mathbf{b}$.

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{W}\mathbf{x} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

- Our machine learning model f is then defined as $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$. $\mathbf{W}$ and $\mathbf{b}$ are called model parameters.

- Goal: Find $\mathbf{W}$ and $\mathbf{b}$ that minimize $|y - f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})|^2$
  - In English: I want the predicted label of my model $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$ to be as close as possible to the true label y and I'm going to accomplish that by tuning $\mathbf{W}$ and $\mathbf{b}$.
  - Based on what, though?

Simplest ML Model: Linear Classifier

- Idea: Formulate the image classification problem as $y = \mathbf{W}\mathbf{x} + \mathbf{b}$ where $\mathbf{x}$ is an image and y is the image label.

- Our machine learning model f is then defined as $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$. $\mathbf{W}$ and $\mathbf{b}$ are called model parameters.

- Goal: Find $\mathbf{W}$ and $\mathbf{b}$ that minimize $|y - f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})|^2$
  - In English: I want the predicted label of my model $f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$ to be as close as possible to the true label y and I'm going to accomplish that by tuning $\mathbf{W}$ and $\mathbf{b}$.
  - Based on what, though? Lots of labeled data! $(\mathbf{x}_i, y_i)_{i=1,\cdots,N}$

Simplest ML Model: Linear Classifier

- Formulation:

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |y_i - f(\mathbf{x}_i \mid \mathbf{W}, \mathbf{b})|^2$$

Simplest ML Model: Linear Classifier

- Formulation:

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |y_i - f(\mathbf{x}_i \mid \mathbf{W}, \mathbf{b})|^2$$

- Solution?

Simplest ML Model: Linear Classifier

- Formulation:

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |y_i - f(\mathbf{x}_i \mid \mathbf{W}, \mathbf{b})|^2$$

- Solution?
  - Linear least squares/Moore-Penrose Pseudoinverse
  - Iterative optimization
    - 1st order/gradient-based methods
    - 2nd order/Hassian-based/(quasi)Newton methods
    - …
  - …

Simplest ML Model: Linear Classifier

- Formulation:

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |y_i - f(\mathbf{x}_i \mid \mathbf{W}, \mathbf{b})|^2$$

- Solution?
  - Linear least squares/Moore-Penrose Pseudoinverse
  - Iterative optimization
    - 1st order/gradient-based methods
    - 2nd order/Hassian-based/(quasi)Newton methods
    - …
  - …
  - We are not going to worry about this now. (We'll re-visit this later.)

Linear Classifier: Step-by-Step

1. Collect a bunch of data $(\mathbf{x}_i, y_i)_{i=1,\cdots,N}$
2. Split them into two sets: training set and test set
3. Solve $\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N_{\text{train}}} |y_i - f(\mathbf{x}_i \mid \mathbf{W}, \mathbf{b})|^2$
4. See if $y - f(\mathbf{x} \mid \mathbf{W}, \mathbf{b})$ holds true for the test set
   - If it doesn't: sit and cry (or build a better model than the linear classifier)
   - If it does: profit!

Example: Cat/Dog Classifier

- The cats and dogs dataset: https://www.microsoft.com/en-us/download/details.aspx?id=54765
- 12,500 cat images & 12,500 dog images from internet





Example: Cat/Dog Classifier

( , −1)

( , −1)

( , −1)

. . .

( , 1)

(

, 1)

( , 1)

. . .

Example: Cat/Dog Classifier

width = 400
height = 500
channels = 3

Example: Cat/Dog Classifier

width = 400
height = 500
channels = 3

gray + resize

64 x 64

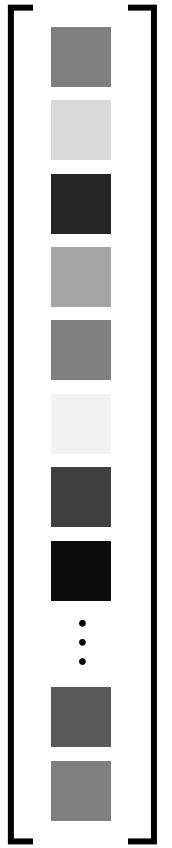
Example: Cat/Dog Classifier

width = 400
height = 500
channels = 3

gray + resize

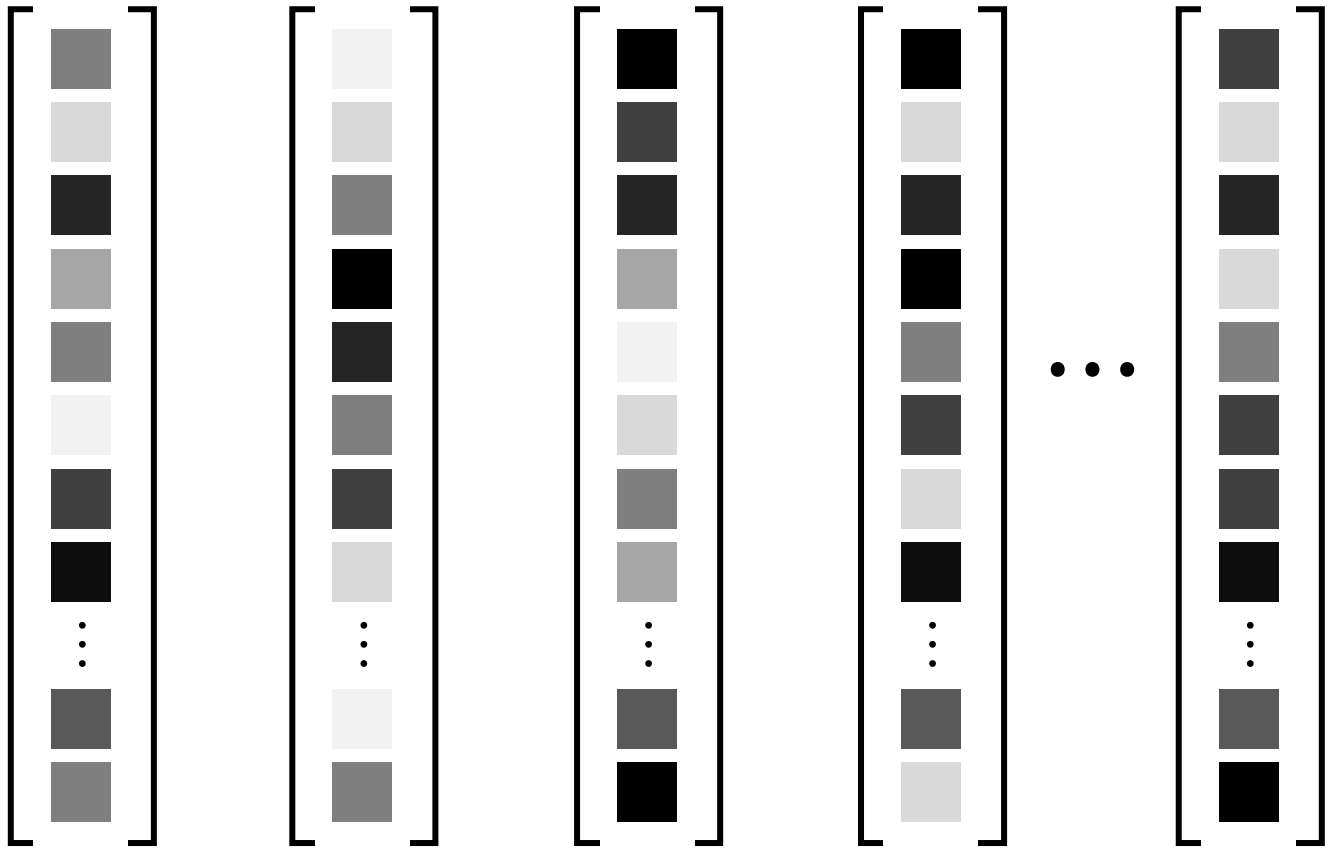
64 x 64

flatten



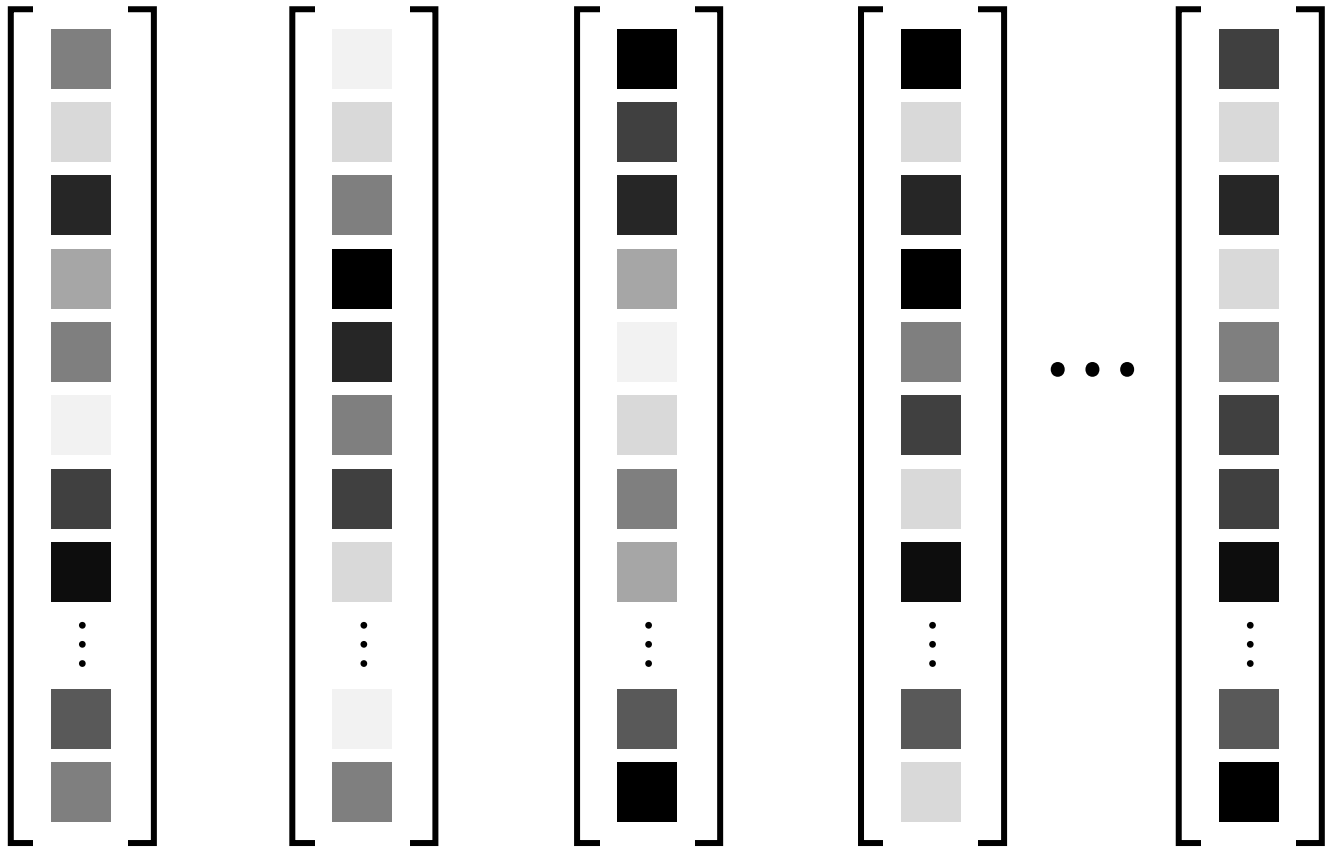
4096 x 1

Example: Cat/Dog Classifier

$\mathbf{x}$: 

y: -1 $\quad$ 1 $\quad$ 1 $\quad$ -1 $\quad$ -1

Example: Cat/Dog Classifier



$\mathbf{x}$:

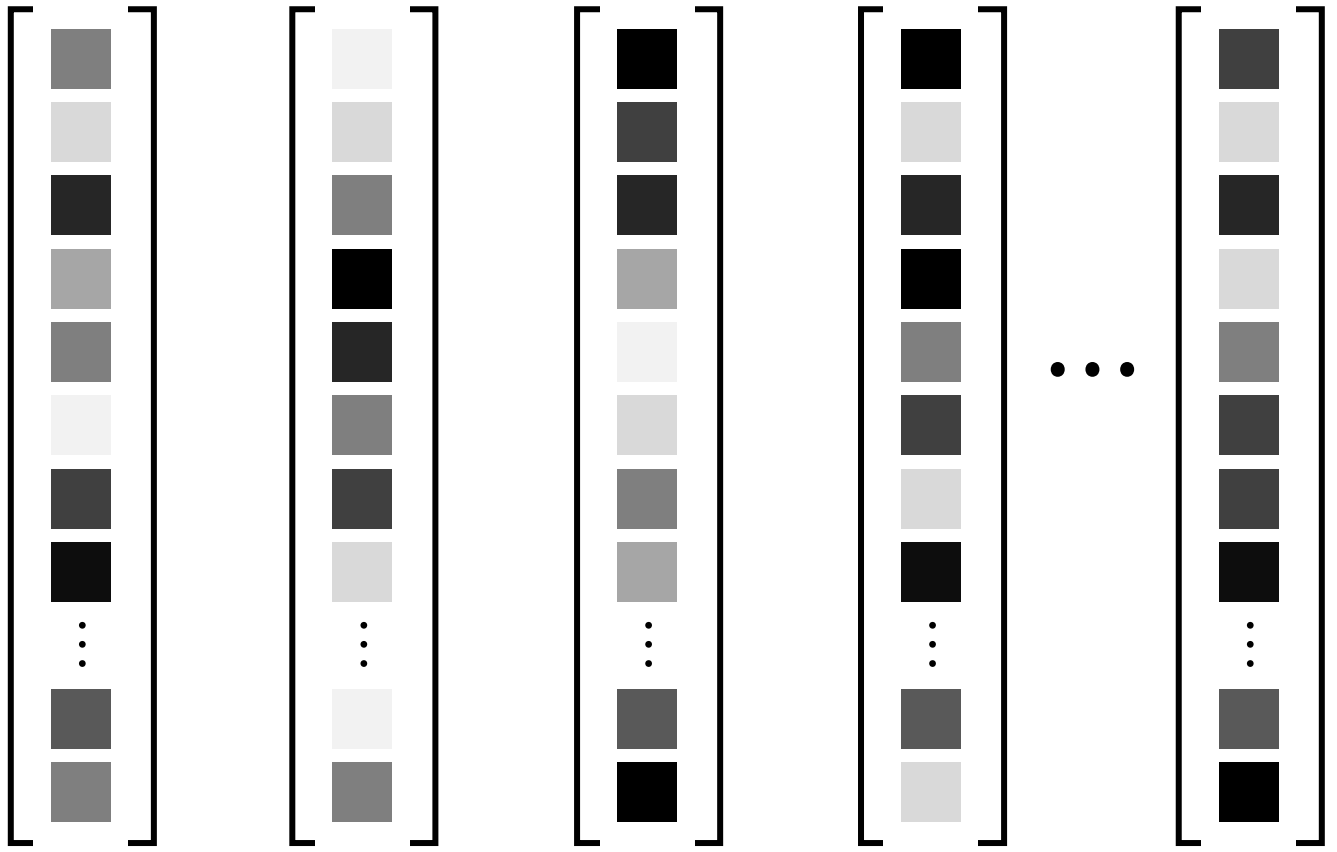
y: -1 1 1 -1 -1

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |\mathbf{W}\mathbf{x}_i + \mathbf{b} - y_i|^2$$

1 x 4096

1 x 1

Example: Cat/Dog Classifier

$\mathbf{x}$: 

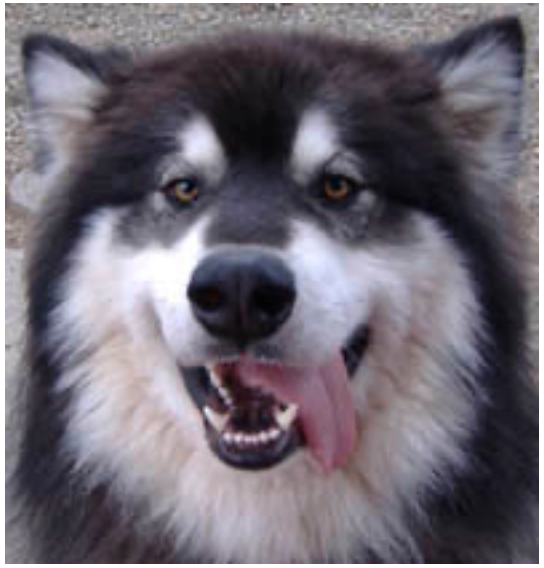
y: -1 $\quad 1 \quad 1 \quad -1 \quad -1$

$$\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N} |\mathbf{W}\mathbf{x}_i + \mathbf{b} - y_i|^2$$

$$\mathbf{W}^*, \mathbf{b}^*$$

Example: Cat/Dog Classifier

(Online)

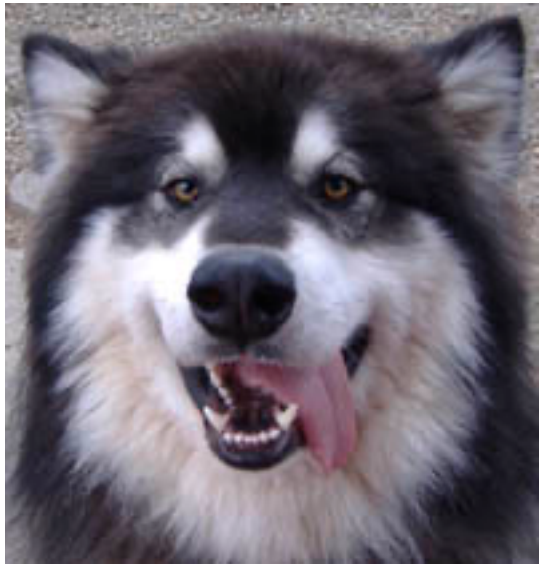


Query

(*Never seen by the model)

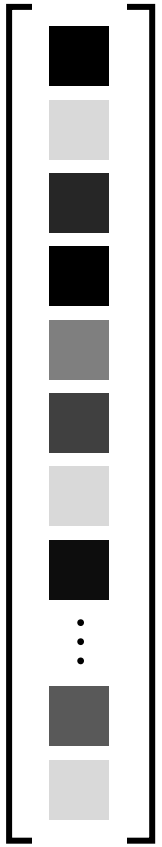
Example: Cat/Dog Classifier

(Online)



Query

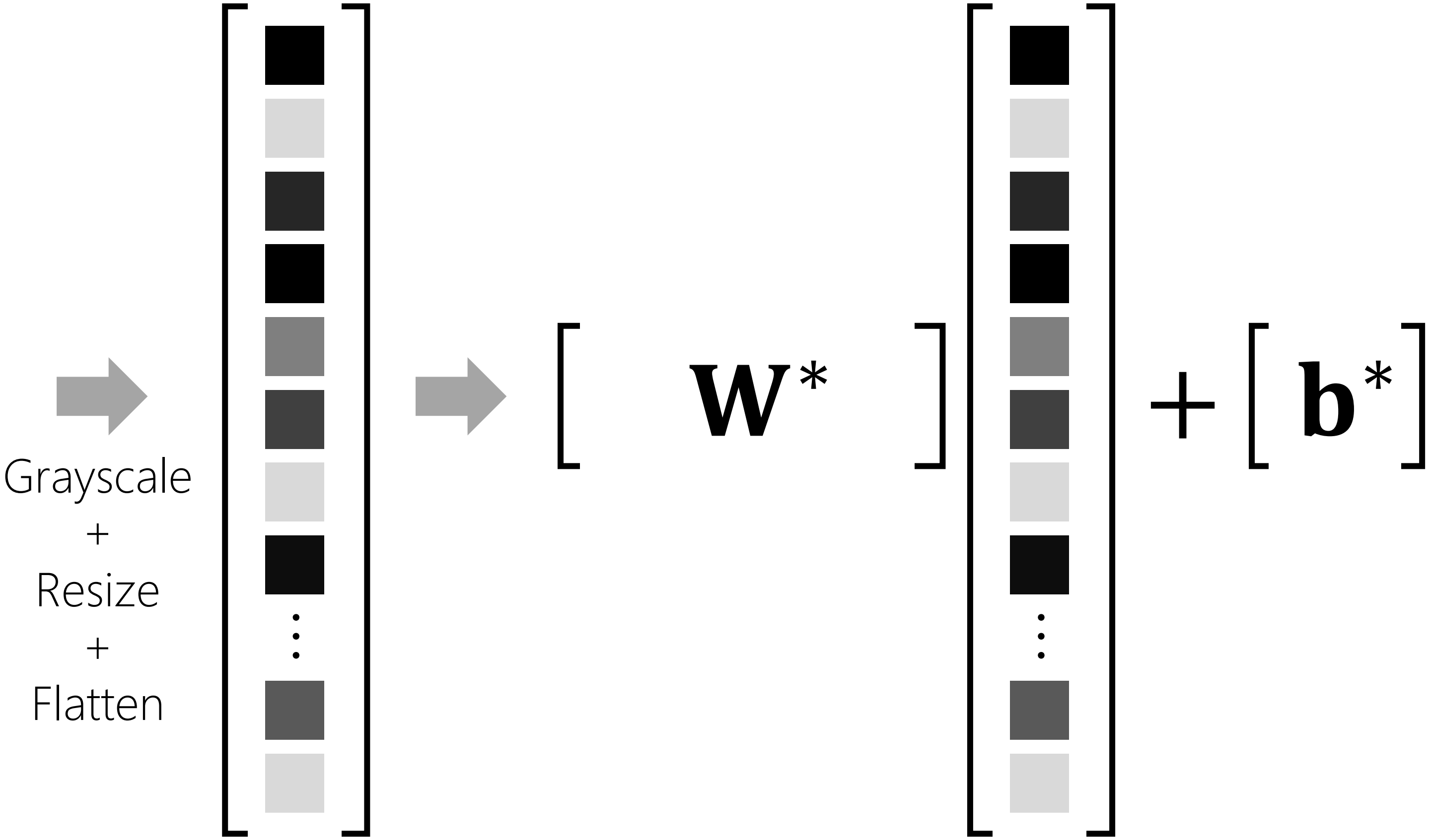
Grayscale
+
Resize
+
Flatten



Example: Cat/Dog Classifier

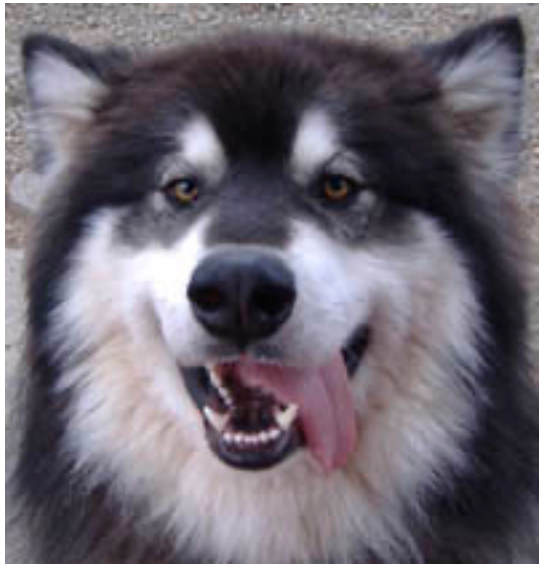
(Online)

Query



Example: Cat/Dog Classifier

(Online)



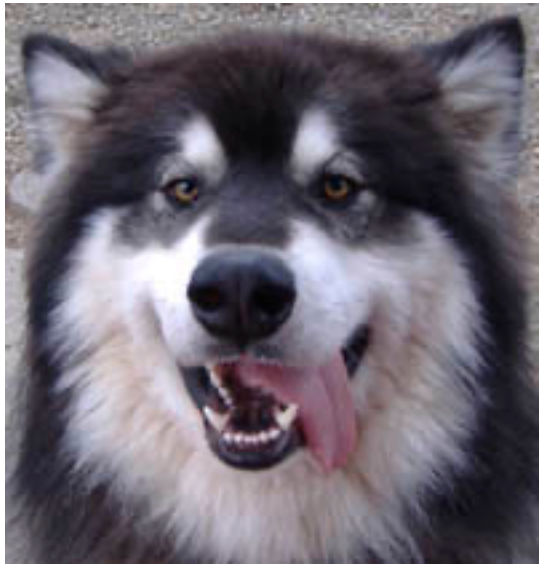
Query

Grayscale
+
Resize
+
Flatten

$$\begin{bmatrix} \\ \mathbf{W}^* \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix} \mathbf{b}^* \end{bmatrix} \rightarrow 0.47$$

Example: Cat/Dog Classifier

(Online)



Query

Grayscale
+
Resize
+
Flatten

$$\begin{bmatrix} \\ \mathbf{W}^* \\ \\ \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix} \\ \mathbf{b}^* \\ \\ \end{bmatrix} \rightarrow 0.47$$

> 0

$\therefore$ dog

Example: Cat/Dog Classifier

(Online)

Query

Grayscale
+
Resize
+
Flatten

$$\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \\ \end{bmatrix} \rightarrow \begin{bmatrix} & \mathbf{w}^* & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \\ \end{bmatrix} + \begin{bmatrix} \mathbf{b}^* \end{bmatrix} \rightarrow -0.7$$

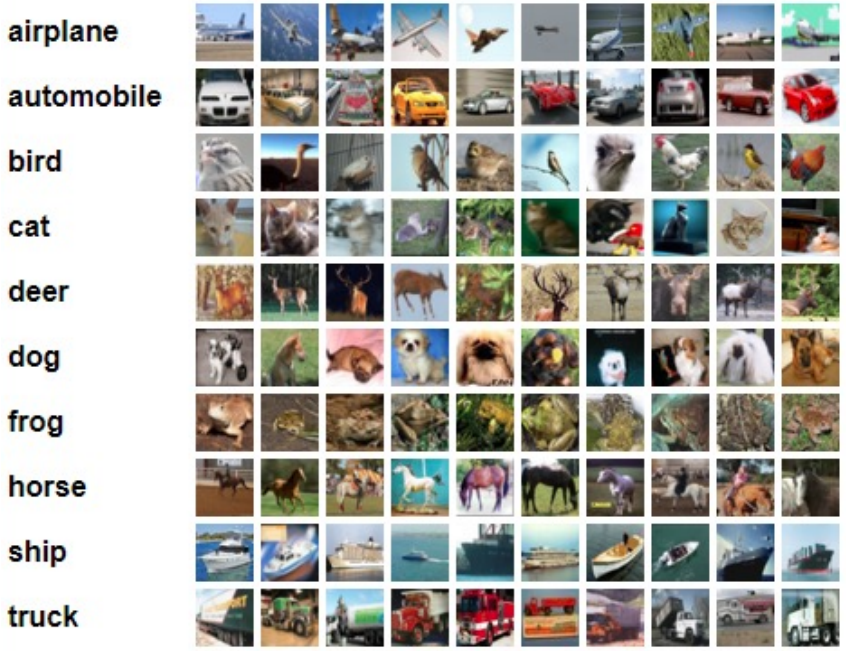
< 0

$\therefore$ cat

Example: 10-way Classifier (CIFAR-10 Dataset)

- What if there were more than two classes (e.g. CIFAR, ImageNet)?

CIFAR-10: 60,000 images in 10 classes



ImageNet: 14,197,122 images in 21,841 classes
ILSVRC: 1.4M images in 1,000 classes

Probability Mass Function

$$y = \begin{bmatrix} P_{airplane} \\ P_{automobile} \\ P_{bird} \\ P_{cat} \\ P_{deer} \\ P_{dog} \\ P_{frog} \\ P_{horse} \\ P_{ship} \\ P_{truck} \end{bmatrix}$$

$$0 \leq P_i \leq 1, \forall i$$

$$\sum_i P_i = 1$$

“One-hot” Encoding

True PMF

$$y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

airplane
automobile
bird
cat
deer
dog
frog
horse
ship
truck

“One-hot” Encoding

True PMF

$$y = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

airplane
automobile
bird
cat
deer
dog
frog
horse
ship
truck

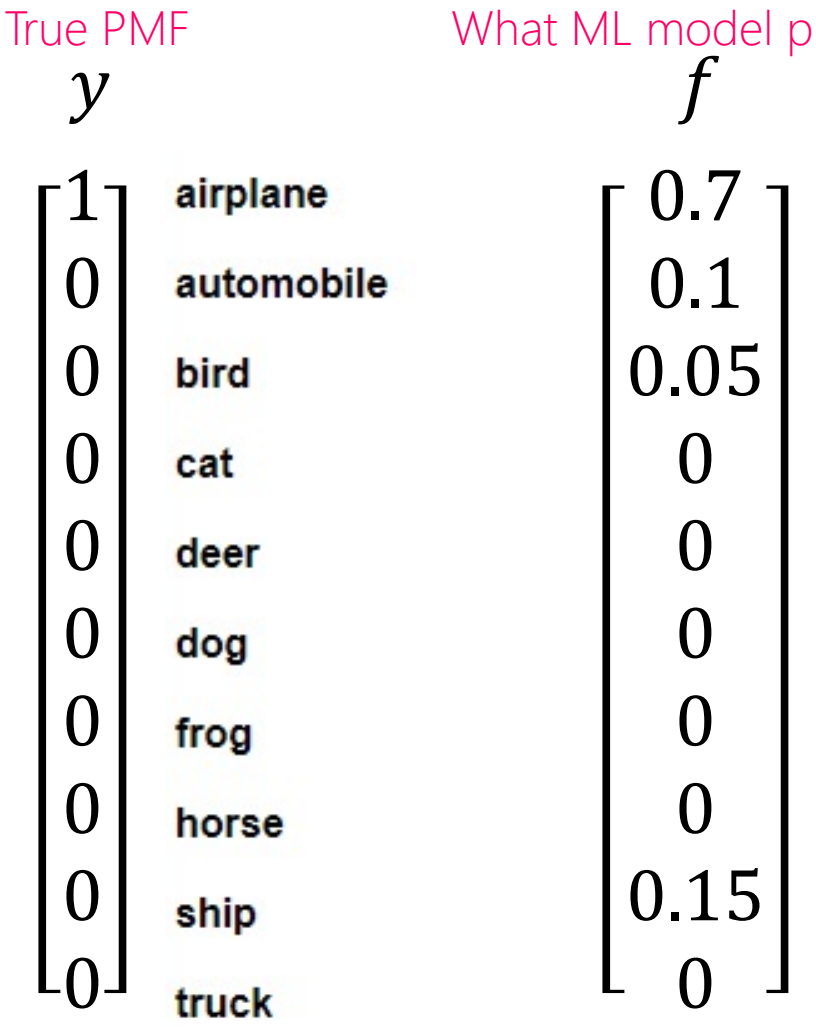
What ML model predicts

$$f = \begin{bmatrix} 0.7 \\ 0.1 \\ 0.05 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0.15 \\ 0 \end{bmatrix}$$

Cross-entropy

$$-\sum_i y_i \log(f_i)$$

In English: How similar are the two distributions?
(We will re-visit this later to learn how and why.)



Example: 10-way Classifier (CIFAR-10 Dataset)

$\mathbf{x}_1 =$ 

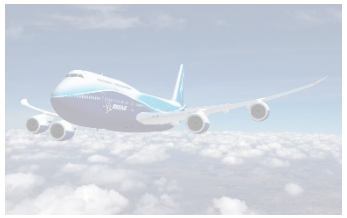
airplane automobile bird cat deer dog frog horse ship truck

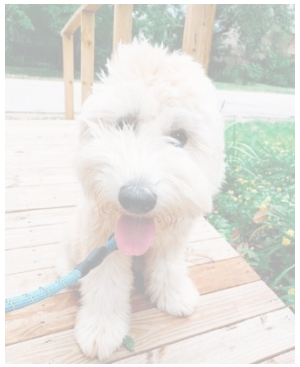
$y_1 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

$\mathbf{x}_2 =$ 

$y_2 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

Example: 10-way Classifier (CIFAR-10 Dataset)





airplane automobile bird cat deer dog frog horse ship truck

$\mathbf{x}_1 =$ ⋯ $y_1 = [1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0]$

Minimize $-\sum_i y_i \log(f(\mathbf{x}_i|\mathbf{W}, \mathbf{b}))$ ➡ $\mathbf{W}^*, \mathbf{b}^*$

$\mathbf{x}_2 =$ ⋯ $y_2 = [0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0]$

Example: 10-way Classifier (CIFAR-10 Dataset)

(online)

Query

$$\begin{bmatrix} & 10 \times 4096 & \\ & \mathbf{W}^* & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix} + \begin{bmatrix} 10 \times 1 \\ \mathbf{b}^* \end{bmatrix}$$

4096 x 1

Example: 10-way Classifier (CIFAR-10 Dataset)

(online)

Query

$$\begin{bmatrix} & 10 \times 4096 & \\ & \mathbf{W}^* & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \end{bmatrix} + \begin{bmatrix} 10 \times 1 \\ \mathbf{b}^* \end{bmatrix} \Rightarrow \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0.05 \\ 0.9 \\ 0.1 \\ 0 \\ 0.25 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \text{airplane} \\ \text{automobile} \\ \text{bird} \\ \text{cat} \\ \text{deer} \\ \text{dog} \\ \text{frog} \\ \text{horse} \\ \text{ship} \\ \text{truck} \end{matrix}$$

4096 x 1

Example: 10-way Classifier (CIFAR-10 Dataset)

(online)

Query

$$\begin{bmatrix} & \overset{10 \times 4096}{\mathbf{W}^*} & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \end{bmatrix} + \begin{bmatrix} \overset{10 \times 1}{\mathbf{b}^*} \end{bmatrix} \Rightarrow \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0.05 \\ 0.9 \\ 0.1 \\ 0 \\ 0.25 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \text{airplane} \\ \text{automobile} \\ \text{bird} \\ \text{cat} \\ \text{deer} \\ \text{dog} \\ \text{frog} \\ \text{horse} \\ \text{ship} \\ \text{truck} \end{matrix} \Rightarrow \text{“deer”}$$

4096×1

Example: 10-way Classifier (CIFAR-10 Dataset)

(online)

Query

$$\begin{bmatrix} & \overset{10 \times 4096}{\mathbf{W}^*} & \end{bmatrix} \begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \vdots \\ \\ \end{bmatrix} + \begin{bmatrix} \overset{10 \times 1}{\mathbf{b}^*} \end{bmatrix} \Rightarrow \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0.05 \\ 0.9 \\ 0.1 \\ 0 \\ 0.25 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \text{airplane} \\ \text{automobile} \\ \text{bird} \\ \text{cat} \\ \text{deer} \\ \text{dog} \\ \text{frog} \\ \text{horse} \\ \text{ship} \\ \text{truck} \end{matrix} \Rightarrow \text{“deer”}$$

4096×1

Wait a second, anything wrong?

Example: 10-way Classifier (CIFAR-10 Dataset)

(online)

Query

$$0 \leq P_i \leq 1, \forall i$$

$$\sum_i P_i = 1$$

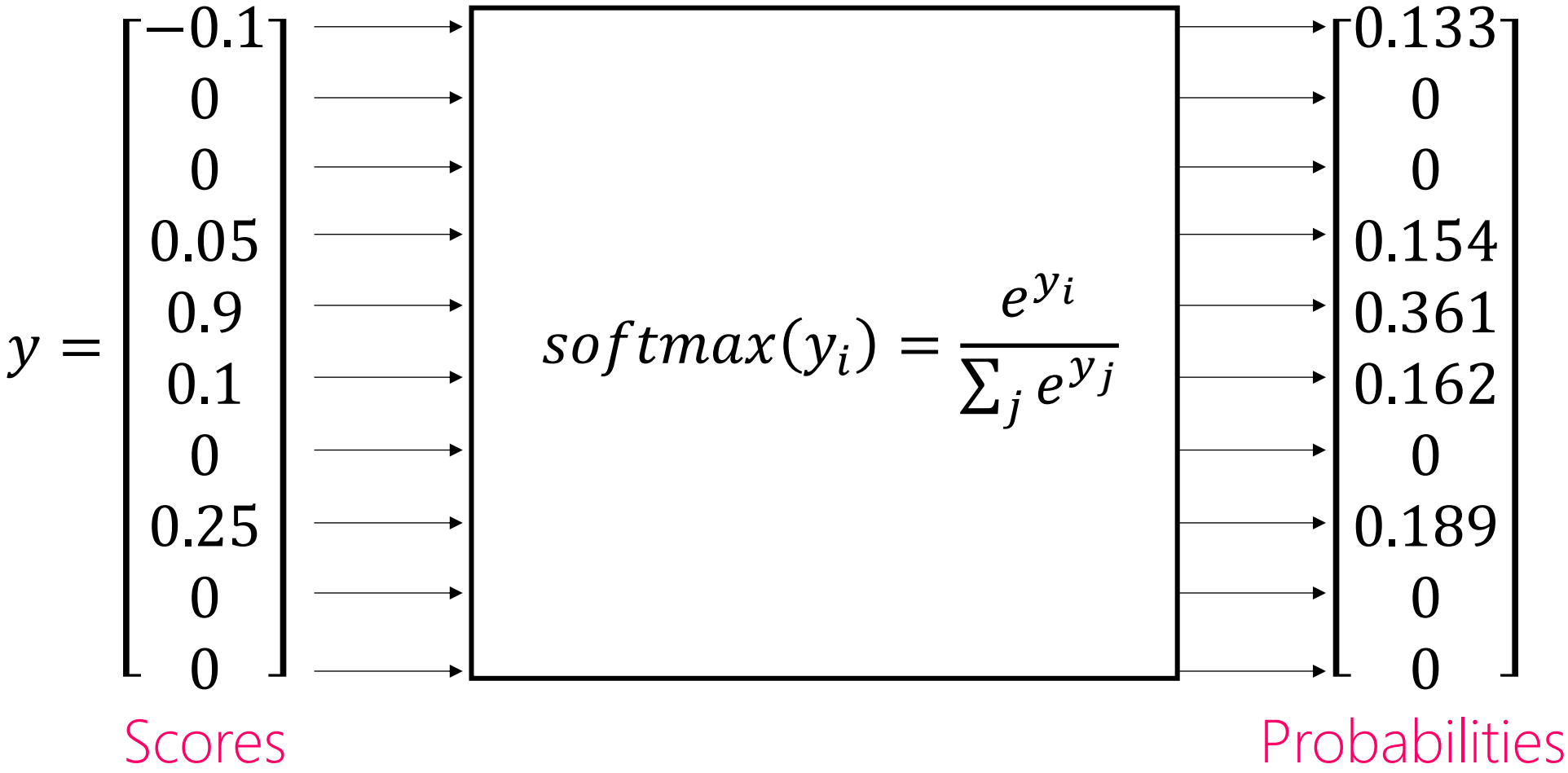
$$\begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0.05 \\ 0.9 \\ 0.1 \\ 0 \\ 0.25 \\ 0 \\ 0 \end{bmatrix}$$

airplane
automobile
bird
cat
deer
dog
frog
horse
ship
truck

"deer"

Wait a second, anything wrong?

Softmax!


$y = \begin{bmatrix} -0.1 \\ 0 \\ 0 \\ 0.05 \\ 0.9 \\ 0.1 \\ 0 \\ 0.25 \\ 0 \\ 0 \end{bmatrix}$
$softmax(y_i) = \frac{e^{y_i}}{\sum_j e^{y_j}}$
$\begin{bmatrix} 0.133 \\ 0 \\ 0 \\ 0.154 \\ 0.361 \\ 0.162 \\ 0 \\ 0.189 \\ 0 \\ 0 \end{bmatrix}$
Scores
Probabilities

n-way Softmax Classifier: Step-by-Step

1. Collect a bunch of data $(\mathbf{x}_i, y_i)_{i=1,\cdots,N}$
2. Encode y_i in one-hot encoding
3. Split the data into two sets: training set and test set
4. Solve $\min_{\mathbf{W},\mathbf{b}} \sum_{i=1}^{N_{\mathbf{train}}} -y_i \log(f_i)$ where $f_i = softmax(\mathbf{W}\mathbf{x}_i + \mathbf{b})$
5. Test the trained model $f(\mathbf{x} \mid \mathbf{W}^*, \mathbf{b}^*) = softmax(\mathbf{W}^*\mathbf{x} + \mathbf{b}^*)$ on the test set