Investigating Photoelectric Effect

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1 Abstract

This study investigates the photoelectric effect through three experimental exercises. In **Exercise 1**, Planck's constant was experimentally determined to be $h = (3.51 \pm 0.55) \times 10^{-34}$ J·s, with a 47.0% error from the theoretical value. The cutoff frequency was found to be $(2.51 \pm 0.33) \times 10^{14}$ Hz, and the work function was calculated as 0.552 eV. **Exercise 2** looked at the relationship between photocurrent and light intensity, which had a strong correlation ($R^2 = 0.961$), while the stopping voltage was found to be independent of intensity ($R^2 = 0.982$). In **Exercise 3**, the transient photocurrent response was measured, resulting in an delay time of 240.0 ± 2.5 ns, which is shorter than the theoretical estimate of 528.5 ± 37.7 ns. Sources of error, like the effects of LED bandwidths, external lighting, and measurement uncertainties, were discussed. Despite discrepancies, the results nonetheless align with the expected behavior of the photoelectric effect.

2 Introduction

The photoelectric effect, first discovered by J.J. Thomson, describes the emission of photoelectrons from a metal surface when exposed to light. It was initially proposed that the energy in the oscillating electric field of light would enable electrons to overcome the metal's work function (E_0). However, experiments revealed that electron emission occurs only if the light's frequency exceeds a threshold, contradicting classical wave theory predictions.

Albert Einstein later proposed the concept of energy quantization for electrons, suggesting that the energy of a photon is given by:

$$E = hf \tag{1}$$

where $h = 6.626 \times 10^{-34}$ J · s is Planck's constant and f is the frequency of the incident light. The stopping voltage, defined as the voltage required to halt the movement of photoelectrons between plates and prevent a current, is expressed by:

$$eV_{\text{stop}} = hf - E_0 \tag{2}$$

Here, $e = 1.602 \times 10^{-19}$ C is the elementary charge, and E_0 is the work function, representing the minimum energy needed to eject an electron. This leads to a linear relationship between the stopping voltage and the frequency:

$$V_{\text{stop}} = \frac{h}{e}(f - f_0) \tag{3}$$

where $f_0 = \frac{E_0}{h}$ is the threshold frequency, below which no electrons are emitted regardless of light intensity. This explanation not only accounted for the threshold frequency but also confirmed the quantized nature of light energy. Furthermore, to calculate the energy absorbed by each electron in this experiment, the following equation is used:

$$P_e = \frac{A_e}{A_{\text{PC}}} P_{\text{LED}} \tag{4}$$

which will be explained in Exercise 3. This can then directly calculate the theoretical delay time for an electron to absorb enough energy to escape the photocathode:

$$t_{\text{delay}} = \frac{E_0}{P_e} \tag{5}$$

where P_{LED} is the power output of the LED, A_{PC} is the area of the photocathode, and A_e is the area associated with each electron.

3 Methods

3.1 Materials

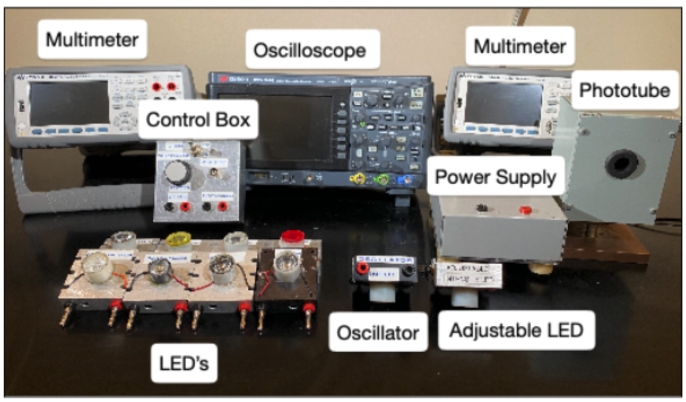

Figure 1: Photo of the experimental materials

The following materials are required for this lab:

Equipment
Phototube
8 monochromatic LEDs of various wavelengths
Variable intensity LED
Oscillator-driven LED
Stopping voltage multimeter
Photocurrent multimeter
Digital oscilloscope (Keysight DSOX1202G)
Control box
Power supply
Banana wires
BNC cables

Table 1: List of equipment used in the experiment.

3.2 Methods

3.2.1 Exercise 1: Stopping Voltage

1. Insert LED into the power supply socket and turn it on.
2. Align the LED's diode head with the phototube.
3. Set up the two multimeters as voltmeters since the photocurrent is measured as a potential drop across a 100 kΩ resistor, and the same setup applies for measuring the stopping voltage.
4. Turn on the potentiometer to measure the stopping voltage for the 8 wavelengths provided (see Appendix 1 for wavelengths and bandwidths).
5. Set photocurrent multimeter to zero, with a ± 0.0100 buffer zone, as it is impossible to set the voltage to exactly zero due to noise.
6. Steps 1-5 are repeated to measure the stopping voltage for the seven other LEDs.

3.2.2 Exercise 2: Intensity

1. Insert the variable intensity LED into the power supply socket and align it with the phototube.
2. For each of the four intensities on the LED, record the corresponding stopping voltage and photocurrent.
3. Find the stopping voltage with the process in Exercise 1, and determine the photocurrent keeping the potentiometer constant (initial threshold) while increasing the LED's intensity level.

3.2.3 Exercise 3: Delay Time

1. Turn off the phototube power supply and connect the phototube to Ch1 of the oscilloscope through the rectifying adapter.
2. Connect the wave generator to the oscillator-driven LED and to Ch2 of the oscilloscope.

3. Turn on the function generator and adjust the output to generate a square wave with a frequency around 1-2 kHz.
4. Set the oscilloscope to activate Ch2 and adjust the settings to capture 1-4 full periods of oscillation within the viewing window.
5. Measure the transient photocurrent over time.
6. To estimate the delay between the incident light and the response in the photocurrent, record the difference between the rising edges of the waves using the oscilloscope's cursor. Align the vertical cursor with the starting points of each pulse. The rising edges represent the point of maximum signal change and are less affected by noise due to the transition from low to high voltage (0V to peak), making them the most reliable feature for time measurement.

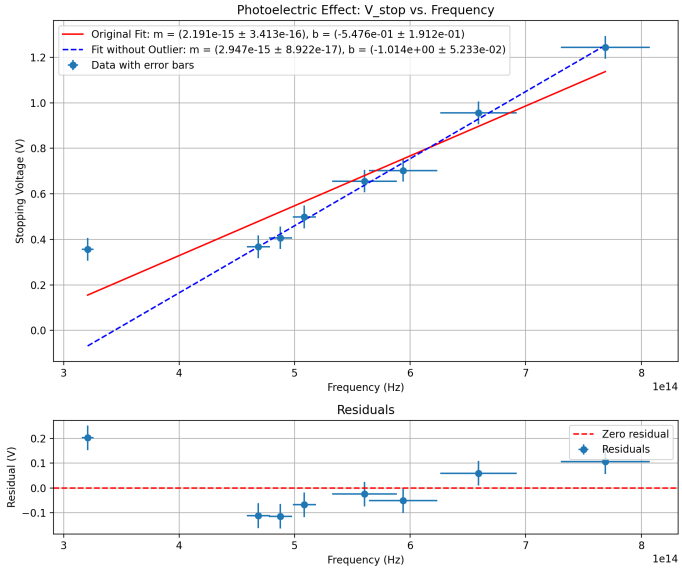

Figure 2: Linear relationship between V_{stop} and f with best-fit lines and corresponding error bars. The original fit has an R^2 value of 0.893. The stopping voltage (V_{stop}) for infrared light was identified as an outlier; thus, an additional best-fit line (dotted line) was calculated with the outlier removed to assess its impact on the correlation.

4 Data and Analysis

4.1 Exercise 1: Stopping Voltage vs. Frequency Analysis

By adjusting the photocurrent to zero across the $100\,k\Omega$ resistor, the stopping voltage V_{stop} was measured for various wavelengths using LEDs ranging from ultraviolet to infrared.

Each individual measurement was conducted for three times and the average values were taken. The resulting plot of V_{stop} versus frequency is shown below:
The best-fit line for the data was determined to be:

$$V_{\text{stop}} = (2.19 \pm 0.34) \times 10^{-15} \frac{V}{Hz} \cdot (f - (2.5 \pm 0.3) \times 10^{14} \text{ Hz}) \quad (6)$$

According to Equation 3, the experimental values are summarized:

Parameter	Experimental Value
Planck's constant h	$(3.51 \pm 0.55) \times 10^{-34}\text{J} \cdot \text{s}$
Cutoff frequency f_0	$(2.51 \pm 0.33) \times 10^{14}\text{Hz}$
Work function E_0	0.552eV

Table 2: Summary of experimental results with outlier value retained

The experimental value of Planck's constant, when compared to the accepted value of $h_{\text{theoretical}} = 6.626 \times 10^{-34}$ J$\cdot$s, shows a percentage error of 47.0%. This error was initially likely attributed to the outlier in the plot. However, after removing the outlier, the percentage error increased to 48.4%, suggesting that the outlier may not be the primary cause of the discrepancy.

The goodness of fit was evaluated using two methods: residual analysis and the reduced chi-squared test. As shown in Figure 2, the residuals are mostly centered around zero with small deviations, suggesting that the observed data aligns reasonably well with the predicted values. However, the stopping voltage for infrared light ($f = 3.21 \times 10^{14}$ Hz) was identified as an outlier. The reduced Chi-squared χ^2_ν was found to be 4.603 (calculation see Appendix A.2), which is significantly higher than the ideal value of 1. This suggests that either the uncertainties were underestimated or additional sources of error, such as LED bandwidth effects or external lighting, may have influenced the results. Moreover, uncertainty and error propagation details are in Appendix A.

4.2 Exercise 2: Photocurrent and Stopping Voltage vs. Light Intensity

The photocurrent and stopping voltage V_{stop} were evaluated at four discrete light intensity levels using a variable-intensity LED, as illustrated in Figure 3 and 4.

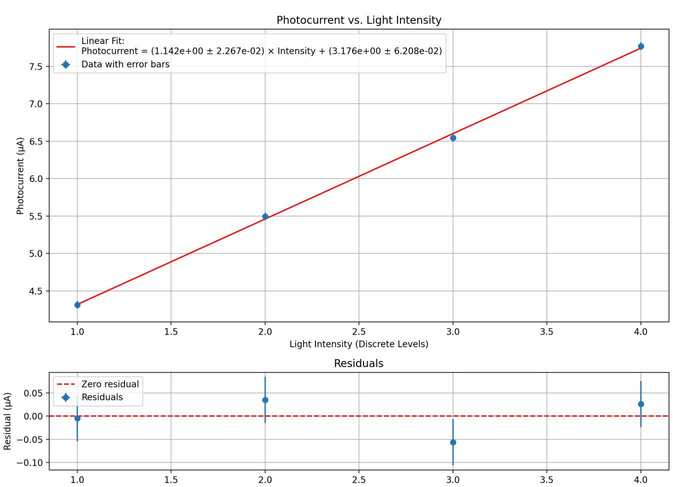

Figure 3: Photocurrent vs. light intensity with the best-fit line. The fit has $R^2 = 0.961$. No uncertainty is assigned to discrete intensity levels.

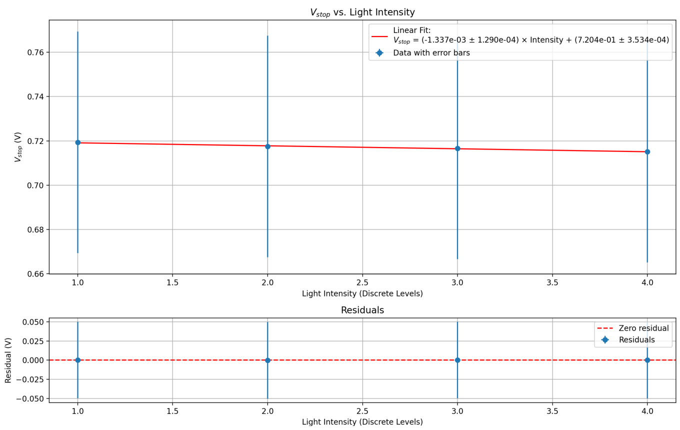

Figure 4: Stopping voltage V_{stop} vs. light intensity with the best-fit line. The fit has $R^2 = 0.982$. No uncertainty is assigned to discrete intensity levels. The slope is close to zero, indicating independence between V_{stop} and intensity.

The correlation between photocurrent I and light intensity was determined to be:

$$I = (1.142 \pm 0.023)\mu A + (3.175 \pm 0.062)\mu A$$

While the error bar indicates a good fit, the reduced chi-square value of $\chi^2_\nu = 2.056$ suggests a moderate fit, possibly indicating minor underestimation of uncertainties or minor experimental inconsistencies.
In contrast, the stopping voltage V_{stop} was found to be nearly independent of light intensity, as shown by the following relation:

$$V_{\text{stop}} = (-1.34 \pm 0.13) \times 10^{-3} \cdot I + (7.20450 \pm 0.00053) \times 10^{-1}$$

The reduced chi-square value of $\chi^2_\nu = 0.003$ is significantly less than 1, indicating potential overfitting, likely due to the limited number of data points.

4.3 Exercise 3: Delay Time

The transient photocurrent was examined over time to determine the time delay between the incident light and the system's response. This delay was identified as the point just before the photocurrent stabilized. The time delay was measured to be $t_{\text{exp}} = 240\,\text{ns} \pm 2.56\,\text{ns}$, with the transient photocurrent at that instant recorded as 707.5 mV. The uncertainty in the experimental time delay is determined based on half of the smallest scale division on the oscilloscope window. The oscilloscope was zoomed in to fit 250 ns within the window, the uncertainty was found to be $\Delta t_{\text{exp}} = 2.5\,\text{ns} = 2.5 \times 10^{-9}\,\text{s}$.

The oscillator-driven LED consumes 60 mW of electric power, and it is assumed that all of this power is converted into light. Using Equation 4, we can calculate the energy absorbed by each electron per second given: $P_{\text{LED}} = 60\,\text{mW} = 6 \times 10^{-2}\,\text{W}$, $A_e = (0.3\,\text{nm} \times 0.3\,\text{nm}) = 0.09\,\text{nm}^2 = 9 \times 10^{-20}\,\text{m}^2$, and $A_{\text{PC}} = 3.23\,\text{cm}^2 = 3.23 \times 10^{-4}\,\text{m}^2$.

$$P_e = 6 \times 10^{-2}\,W \times \left(\frac{9 \times 10^{-20}\,m^2}{3.23 \times 10^{-4}\,m^2}\right)$$
$$= 1.670 \times 10^{-17} \pm 1.19 \times 10^{-18}\,\text{W}$$

Theoretical time delay t_{delay} is calculated to related how long it takes for an electron to absorb enough energy to escape the photocathode. This delay can be calculated using Equation 5, where the work function E_0 determined from Exercise 1 is given as: $E_0 = 0.55\,\text{eV} = 8.81 \times 10^{-20}\,\text{J}$.

$$t = \frac{8.81 \times 10^{-20}\,\text{J}}{1.67 \times 10^{-17}\,\text{W}} = 5.28 \times 10^{-4} \pm 3.77 \times 10^{-8}\,\text{s}$$

The calculation of these uncertainties can be found in Appendix A.

5 Discussion and Conclusion

5.1 Exercise 1

5.1.1 Infrared Light as an Outlier

In the photoelectric effect, the stopping voltage V_{stop} is directly related to the maximum kinetic energy of the emitted photoelectrons, which depends solely on the frequency of the incident light. Infrared photons have a longer wavelength and possess relatively low energy. It results in minimal kinetic energy for the emitted electrons and, consequently, a low stopping voltage. However, the high resistance $(100, k\Omega)$ used in the experimental setup can significantly influence the measured V_{stop}. According to Ohm's Law, $V = IR$. Even small photocurrents or minor fluctuations in potentiometer readings can cause relatively large voltage variations across the resistor. These voltage variations may either increase or decrease the measured stopping voltage, leading to inconsistencies. This sensitivity to small current fluctuations makes the stopping voltage for infrared light particularly unreliable, causing it to appear as an outlier that deviates from the expected linear trend.

5.1.2 Sources of Experimental Error

Several potential sources of experimental error could account for the large discrepancy between the experimental and theoretical values of Planck's constant (47% percentage error compared to $h_{theoretrical}$. Although the experiment was conducted in a dark room, the setup's proximity to a door could have allowed ambient light to influence the phototube, which is highly sensitive to even minimal light exposure. Additionally, the potentiometer used to measure V_{stop} exhibited constant fluctuations, potentially introducing variability in the readings despite averaging over three trials per data point. Furthermore, the LEDs in the setup emitted a spectrum of wavelengths rather than a single monochromatic value, which could affect the accuracy of the stopping voltage measurements.

5.2 Exercise 2

The investigation of the relationship between light intensity, photocurrent, and stopping voltage (V_{stop}) in exercise align with Einstein's explanation of the photoelectric effect.

The original theory, which treats light as a continuous wave, predicted that V_{stop} would depend on light intensity, assuming higher intensity would transfer more energy to electrons, enabling them to overcome the work function. However, the experimentally determined slope of -1.34×10^{-3} suggests that V_{stop} remains nearly constant, aligning with Einstein's explanation that V_{stop} depends only on the frequency of light, not its intensity.

Photocurrent, on the other hand, is proportional to the number of photons striking the cathode per second and thus to the number of emitted photoelectrons. It increases linearly with light intensity. This observation supports the linear relationship found in Section 4.2. Since higher intensity means more photons per unit time but not higher photon energy, it increases photocurrent without affecting V_{stop}. Additionally, the existence of a threshold frequency for electron emission, as demonstrated in Exercise 1 and supported by Einstein's equation, contradicts the wave theory's prediction that sufficient intensity alone could cause electron emission regardless of frequency.

5.2.1 Sources of Experimental Error

A decrease in LED intensity was observed over time and is a source of systematic error in this experiment. As the LED remains on, it heats up, leading to a reduction in its efficiency and a corresponding decrease in the emitted light intensity. This, in turn, results in lower photon energy, which affects both the stopping voltage (V_{stop}) and photocurrent measurements, introducing a bias toward lower values. This error can be classified as an instrumental error, as it originates from the LED itself and may be due to its inability to dissipate heat. Additionally, environmental factors such as heat in the surrounding area could also contribute to this error. To mitigate this issue, allowing the LED to cool down between measurements would help maintain a stable intensity.

5.3 Exercise 3

The theoretical delay time, calculated using the power absorbed by each electron (P_e) and the work function (E_0), was found to be 5.28×10^{-4} s. However, the experimental delay time between the incident light and the photocurrent response was only 2.40×10^{-7} s. This significant discrepancy highlights a fundamental flaw in the classical wave theory, which assumes that electrons gradually absorb energy over time until they accumulate enough to escape the material. If this were true, the delay would be much longer.

Instead, our results support Einstein's photoelectric theory, which states that light consists of quantized energy packets (photons). According to this model, if a photon carries enough energy to overcome the material's work function, an electron can be emitted almost instantly. The fact that our observed delay is magnitudes shorter than the predicted value reinforces this idea, showing that energy transfer occurs in discrete quanta rather than through continuous accumulation. This experiment provides strong evidence for the quantization of light and validates Einstein's explanation of the photoelectric effect.

Appendix

A Uncertainty and Error Propagation

A.1 Exercise 1

Since each LED emits a range of wavelengths rather than a single monochromatic value, the uncertainty in wavelength, $\Delta\lambda$, was taken as half of the LED bandwidth. This was propagated to frequency uncertainty using:

$$\Delta f = f\frac{\Delta\lambda}{\lambda} \tag{7}$$

The uncertainty in the potentiometer reading was estimated to be $\pm 0.05\,V$, accounting for both resolution and potential fluctuations during measurement.

Uncertainty in Plank's Constant

Planck's constant was determined from the slope m of the best-fit line using:

$$h = em \tag{8}$$

Propagating the uncertainty:

$$(\Delta h)^2 = \left(\frac{\partial h}{\partial m}\Delta m\right)^2 \tag{9}$$

Since $h = em$, we obtain:

$$\Delta h = e\Delta m \tag{10}$$

Uncertainty in Cutoff Frequency (f_0)

The cutoff frequency was obtained from the x-intercept of the best-fit equation:

$$f_0 = \frac{b}{m} \tag{11}$$

Using propagation of uncertainty:

$$(\Delta f_0)^2 = \left(\frac{\partial f_0}{\partial b}\Delta b\right)^2 + \left(\frac{\partial f_0}{\partial m}\Delta m\right)^2 \tag{12}$$

Taking derivatives:

$$\frac{\partial f_0}{\partial b} = \frac{1}{m}, \quad \frac{\partial f_0}{\partial m} = -\frac{b}{m^2} \tag{13}$$

Thus, the propagated uncertainty in f_0 is:

$$\Delta f_0 = \sqrt{\left(\frac{\Delta b}{m}\right)^2 + \left(\frac{b\Delta m}{m^2}\right)^2} \tag{14}$$

Uncertainty in Work Function (E_0)

The work function was calculated as:

$$E_0 = hf_0 \tag{15}$$

Applying uncertainty propagation:

$$(\Delta E_0)^2 = \left(\frac{\partial E_0}{\partial h}\Delta h\right)^2 + \left(\frac{\partial E_0}{\partial f_0}\Delta f_0\right)^2 \tag{16}$$

Since:

$$\frac{\partial E_0}{\partial h} = f_0, \quad \frac{\partial E_0}{\partial f_0} = h \tag{17}$$

The final uncertainty expression for E_0 is:

$$\Delta E_0 = \sqrt{(f_0\Delta h)^2 + (h\Delta f_0)^2} \tag{18}$$

A.2 Exercise 2

Reduced Chi-squared

The reduced chi-squared value was calculated using equation

$$\chi^2_\nu = \frac{\chi^2}{v}, \text{where } \chi^2 = \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma^2_{y_i}}$$

and v is the degree of freedom.

A.3 Exercise 3

Theoretical Delay Time

The uncertainty in the theoretical delay time was calculated using the following equation:

$$\frac{\Delta t_{\text{theory}}}{t_{\text{theory}}} = \sqrt{\left(\frac{\Delta E_0}{E_0}\right)^2 + \left(\frac{\Delta P_e}{P_e}\right)^2}$$

The uncertainty in P_e comes from the uncertainties in the areas A_e and A_{PC} as well as the power P_{LED}.

Using propagation of uncertainty:

$$\frac{\Delta P_e}{P_e} = \sqrt{\left(\frac{\Delta P_{\text{LED}}}{P_{\text{LED}}}\right)^2 + \left(\frac{\Delta A_e}{A_e}\right)^2 + \left(\frac{\Delta A_{\text{PC}}}{A_{\text{PC}}}\right)^2} = \sqrt{\left(\frac{6 \times 10^{-4}}{6 \times 10^{-2}}\right)^2 + \left(\frac{0.05}{1}\right)^2 + \left(\frac{0.05}{1}\right)^2}$$

B Delay Time Measurement

Color	Wavelength (nm)	LED Bandwidth (nm)
UV	390	40
Blue	455	40
Cyan	505	30
Green	535	30
Amber	590	10
Orange Red	615	10
Red	640	10
Infrared	935	10

Figure 5: Bandwidths (nm) and wavelengths (nm) for all eight LEDs

C Delay Time Measurement

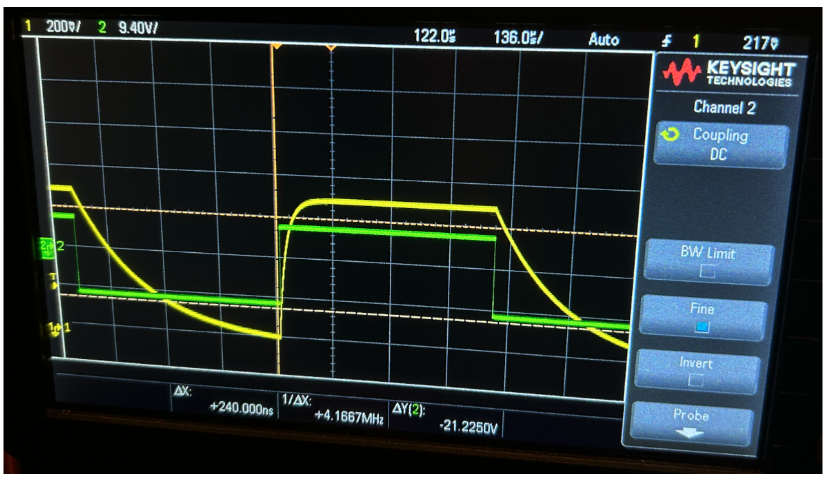

Figure 6: Photo of the oscilloscope window to measure the delay time