

CAMBRIDGE INTERNATIONAL EXAMINATIONS

Cambridge Ordinary Level

MARK SCHEME for the May/June 2015 series

4037 ADDITIONAL MATHEMATICS

4037/11 Paper 1, maximum raw mark 80

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Abbreviations

awrt	answers which round to
cao	correct answer only
dep	dependent
FT	follow through after error
isw	ignore subsequent working
oe	or equivalent
rot	rounded or truncated
SC	Special Case
soi	seen or implied
www	without wrong working

1 (i)	180° or π radians or 3.14 radians (or better)	B1	
(ii)	2	B1	
(iii) (a)		B1	$y = \sin 2x$ all correct
(b)		B1	for either ↑↓ starting at their highest value and ending at their lowest value Or a curve with highest value at $y = 3$ and lowest value at $y = -1$ completely correct graph
(iv)	3	B1	
2 (i)	$\tan \theta = \frac{(8+5\sqrt{2})(4-3\sqrt{2})}{(4+3\sqrt{2})(4-3\sqrt{2})}$ $= \frac{32 - 24\sqrt{2} + 20\sqrt{2} - 30}{16 - 18}$ $= 1 + 2\sqrt{2} \text{ cao}$	M1	attempt to obtain $\tan \theta$ and rationalise. Must be convinced that no calculators are being used
		A1	

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(ii)	$\begin{aligned}\sec^2 \theta &= 1 + \tan^2 \theta \\ &= 1 + (-1 + 2\sqrt{2})^2 \\ &= 1 + 1 - 4\sqrt{2} + 8 \\ &= 10 - 4\sqrt{2}\end{aligned}$ <p>Alternative solution:</p> $\begin{aligned}AC^2 &= (4 + 3\sqrt{2})^2 + (8 + 5\sqrt{2})^2 \\ &= 148 + 104\sqrt{2} \\ \sec^2 \theta &= \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})^2} \\ &= \frac{148 + 104\sqrt{2}}{(4 + 3\sqrt{2})^2} \times \frac{34 - 24\sqrt{2}}{34 - 24\sqrt{2}} \\ &= 10 - 4\sqrt{2}\end{aligned}$	M1 DM1 A1	attempt to use $\sec^2 \theta = 1 + \tan^2 \theta$, with <i>their</i> answer to (i) attempt to simplify, must be convinced no calculators are being used. Need to expand $(-1 + 2\sqrt{2})^2$ as 3 terms
3 (i)	$64 + 192x^2 + 240x^4 + 160x^6$	B3,2,1,0	-1 each error
(ii)	$(64 + 192x^2 + 240x^4) \left(1 - \frac{6}{x^2} + \frac{9}{x^4}\right)$	B1	expansion of $\left(1 - \frac{3}{x^2}\right)^2$
	Terms needed $64 - (192 \times 6) + (240 \times 9)$ $= 1072$	M1 A1	attempt to obtain 2 or 3 terms using <i>their</i> (i)

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4	(a) $\mathbf{X}^2 = \begin{pmatrix} 4-4k & -8 \\ 2k & -4k \end{pmatrix}$	B2,1,0	-1 each incorrect element
	(b) Use of $\mathbf{AA}^{-1} = \mathbf{I}$ $\begin{pmatrix} a & 1 \\ b & 5 \end{pmatrix} \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ <p>Any 2 equations will give $a = 2, b = 4$</p>	M1 A1,A1	use of $\mathbf{AA}^{-1} = \mathbf{I}$ and an attempt to obtain at least one equation.
	Alternative method 1: $\frac{1}{5a-b} \begin{pmatrix} 5 & -1 \\ b & a \end{pmatrix} = \begin{pmatrix} \frac{5}{6} & -\frac{1}{6} \\ -\frac{2}{3} & \frac{1}{3} \end{pmatrix}$ <p>Compare any 2 terms to give $a = 2, b = 4$</p>	M1 A1,A1	correct attempt to obtain \mathbf{A}^{-1} and comparison of at least one term.
	Alternative method 2: Inverse of $\frac{1}{6} \begin{pmatrix} 5 & -1 \\ -4 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 1 \\ 4 & 5 \end{pmatrix}$	M1 A1,A1	reasoning and attempt at inverse
	5 $3x - 1 = x(3x - 1) + x^2 - 4$ or $y = \left(\frac{y+1}{3}\right)y + \left(\frac{y+1}{3}\right)^2 - 4$ $4x^2 - 4x - 3 = 0$ or $4y^2 - 4y - 35 = 0$ $(2x - 3)(2x + 1) = 0$ or $(2y - 7)(2y + 5) = 0$ leading to $x = \frac{3}{2}, x = -\frac{1}{2}$ and $y = \frac{7}{2}, y = -\frac{5}{2}$ Midpoint $\left(\frac{1}{2}, \frac{1}{2}\right)$ Perpendicular gradient = $-\frac{1}{3}$ Perp bisector: $y - \frac{1}{2} = -\frac{1}{3}\left(x - \frac{1}{2}\right)$ $(3y + x - 2 = 0)$	M1 DM1 A1 A1 B1 M1 M1 A1	equate and attempt to obtain an equation in 1 variable forming a 3 term quadratic equation and attempt to solve x values y values for midpoint, allow anywhere correct attempt to obtain the gradient of the perpendicular, using AB straight line equation through the midpoint; must be convinced it is a perpendicular gradient. allow unsimplified

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6	(i)	$f\left(\frac{1}{2}\right) = \frac{a}{8} - \frac{15}{4} + \frac{b}{2} - 2 = 0$ leading to $a + 4b = 46$ $f(1) = a - 15 + b - 2 = 5$ leading to $a + b = 22$ giving $b = 8$ (AG), $a = 14$	M1	correct use of either $f\left(\frac{1}{2}\right)$ or $f(1)$ paired correctly
	(ii)	$(2x-1)(7x^2 - 4x + 2)$	M1,A1	both equations correct (allow unsimplified)
	(iii)	$7x^2 - 4x + 2 = 0$ has no real solutions as $b^2 < 4ac$ $16 < 56$	M1	M1 for solution of equations A1 for both a and b . AG for b .
7	(i)	$\frac{dy}{dx} = \frac{(x-1)\left(8x\right)}{\left(4x^2+2\right)} - \ln(4x^2+3)$ When $x = 0$, $y = -\ln 3$ oe $\frac{dy}{dx} = -\ln 3$ so gradient of normal is $\frac{1}{\ln 3}$ (allow numerical equivalent) normal equation $y + \ln 3 = \frac{1}{\ln 3}x$ or $y = 0.910x - 1.10$, or $y = \frac{10}{11}x - \frac{11}{10}$ cao (Allow $y = 0.91x - 1.1$)	M1 B1 A1 B1 M1 A1	differentiation of a quotient (or product) correct differentiation of $\ln(4x^2+3)$ all else correct for y value valid attempt to obtain gradient of the normal attempt at normal equation must be using a perpendicular
	(ii)	when $x = 0$, $y = -\ln 3$ when $y = 0$, $x = (\ln 3)^2$ Area = ± 0.66 or ± 0.67 or awrt these or $\frac{1}{2}(\ln 3)^3$	M1 A1	valid attempt at area

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8	(i) Range for f: $y \geq 3$ Range for g: $y \geq 9$	B1 B1	
	(ii) $x = -2 + \sqrt{y-5}$	M1	attempt to obtain the inverse function
	$g^{-1}(x) = -2 + \sqrt{x-5}$ Domain of g^{-1} : $x \geq 9$	A1 B1	Must be correct form for domain
	Alternative method: $y^2 + 4y + 9 - x = 0$ $y = \frac{-4 + \sqrt{16 - 4(9-x)}}{2}$	M1 A1	attempt to use quadratic formula and find inverse must have + not \pm
(iii)	Need $g(3e^{2x})$ $(3e^{2x} + 2)^2 + 5 = 41$ or $9e^{4x} + 12e^{2x} - 32 = 0$ $(3e^{2x} - 4)(3e^{2x} + 8) = 0$	M1 DM1	correct order correct attempt to solve the equation
	leading to $3e^{2x} + 2 = \pm 6$ so $x = \frac{1}{2} \ln \frac{4}{3}$	M1	dealing with the exponential correctly in order to reach a solution for x
	or $e^{2x} = \frac{4}{3}$ so $x = \frac{1}{2} \ln \frac{4}{3}$	A1	Allow equivalent logarithmic forms
	Alternative method: Using $f(x) = g^{-1}(41)$, $g^{-1}(41) = 4$ leading to $3e^{2x} = 4$, so $x = \frac{1}{2} \ln \frac{4}{3}$	M1 DM1 M1 A1	correct use of g^{-1} dealing with $g^{-1}(41)$ to obtain an equation in terms of e^{2x} dealing with the exponential correctly in order to reach a solution for x Allow equivalent logarithmic forms
(iv)	$g'(x) = 6e^{2x}$ $g'(\ln 4) = 96$	B1 B1	B1 for each

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9 (i)	$\frac{dy}{dx} = 3x^2 - 10x + 3$	M1	for differentiation
	When $x = 0$, for curve $\frac{dy}{dx} = 3$, gradient of line also 3 so line is a tangent.	A1	comparing both gradients
	Alternate method: $3x + 10 = x^3 - 5x^2 + 3x + 10$ leading to $x^2 = 0$, so tangent at $x = 0$	M1 A1	attempt to deal with simultaneous equations obtaining $x = 0$
(ii)	When $\frac{dy}{dx} = 0$, $(3x-1)(x-3) = 0$ $x = \frac{1}{3}$, $x = 3$	M1 A1,A1	equating gradient to zero and valid attempt to solve A1 for each
	$\text{Area} = \frac{1}{2}(10+19)3 - \int_0^3 x^3 - 5x^2 + 3x + 10 dx$ $= \frac{87}{2} - \left[\frac{x^4}{4} - \frac{5x^3}{3} + \frac{3x^2}{2} + 10x \right]_0^3$ $= \frac{87}{2} - \left(\frac{81}{4} - 45 + \frac{27}{2} + 30 \right)$ $= 24.7 \text{ or } 24.8$	B1 M1 A1 DM1 A1	area of the trapezium attempt to obtain the area enclosed by the curve and the coordinate axes, by integration integration all correct correct application of limits (must be using <i>their</i> 3 from (ii) and 0)
	Alternative method: $\text{Area} = \int_0^3 (3x+10) - (x^3 - 5x^2 + 3x + 10) dx$ $= \int_0^3 -x^3 + 5x^2 dx$ $= \left[-\frac{x^4}{4} + \frac{5x^3}{3} \right]_0^3 = \frac{99}{4}$	B1 M1 A1 DM1 A1	correct use of ' $Y-y$ ' attempt to integrate integration all correct correct application of limits
10 (a)	$\sin^2 x = \frac{1}{4}$ $\sin x = (\pm)\frac{1}{2}$ $x = 30^\circ, 150^\circ, 210^\circ, 330^\circ$	M1 A1,A1	using $\operatorname{cosec} x = \frac{1}{\sin x}$ and obtaining $\sin x = \dots$ A1 for one correct pair, A1 for another correct pair with no extra solutions

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(b) $(\sec^2 3y - 1) - 2\sec 3y - 2 = 0$ $\sec^2 3y - 2\sec 3y - 3 = 0$ $(\sec 3y + 1)(\sec 3y - 3) = 0$ leading to $\cos 3y = -1$, $\cos 3y = \frac{1}{3}$ $3y = 180^\circ, 540^\circ \quad 3y = 70.5^\circ, 289.5^\circ, 430.5^\circ$ $y = 60^\circ, 180^\circ, 23.5^\circ, 96.5^\circ, 143.5^\circ$ Alternative 1: $\sec^2 3y - 2\sec 3y - 3 = 0$ leading to $3\cos^2 3y + 2\cos 3y - 1$ $(3\cos y - 1)(\cos y + 1) = 0$ Alternative 2: $\frac{\sin^2 y}{\cos^2 y} - \frac{2}{\cos y} - 2 = 0$ $(1 - \cos^2 x) - 2\cos x - 2\cos^2 x = 0$ (c) $z - \frac{\pi}{3} = \frac{\pi}{3}, \frac{4\pi}{3}$ $z = \frac{2\pi}{3}, \frac{5\pi}{3}$ or 2.09 or 2.1, 5.24	M1 M1 M1 A1,A1 A1	use of the correct identity attempt to obtain a 3 term quadratic equation in $\sec 3y$ and attempt to solve dealing with \sec and $3y$ correctly A1 for a correct pair, A1 for a second correct pair, A1 for correct 5 th solution and no other within the range M1 M1 M1 A1 M1 M1 M1 M1 A1,A1	use of the correct identity attempt to obtain a quadratic equation in $\cos 3y$ and attempt to solve dealing with $3y$ correctly A marks as above use of the correct identity, $\tan y = \frac{\sin y}{\cos y}$ and $\sec y = \frac{1}{\cos y}$, then as before correct order of operations A1 for a correct solution A1 for a second correct solution and no other within the range
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