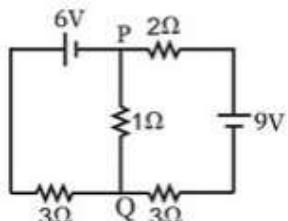


1.

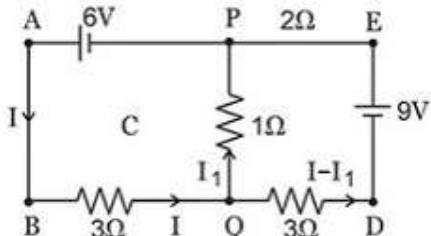


In the circuit shown, the current in the 1Ω resistor is:

- (A) 0.13 A, from P to Q
- (B) 1.3 A, from P to Q
- (C) 0A
- (D) 0.13 A, from Q to P**

Answer: (D)

Solution:



The distribution of current according to Kirchhoff's first law is as shown in the circuit.
By Kirchoff's second law (voltage rule)

In loop APQBA using sign curve line

$$6 - 3I - I_1 = 0$$

$$\Rightarrow 3I + I_1 = 6 \quad \dots(i)$$

In loop QD & PQ

$$\Rightarrow -3(I - I_1) + 9 - 2(I - I_1) + 1 \times I_1 = 0$$

$$\Rightarrow 9 - 5(I - I_1) + I_1 = 0$$

$$\Rightarrow 9 + 6I_1 - 5I = 0$$

$$\Rightarrow 5I - 6I_1 = 9 \quad \dots \text{(ii)}$$

(Multiplying (i) by 5) - (Multiplying (ii) by 3)

$$\Rightarrow 15I + 5I_1 = 30$$

$$15I - 18I_1 = 27$$

$$- \quad + \quad -$$

$$23I_1 = 3 \Rightarrow I_1 = \frac{3}{23} \text{ A} = 0.13 \text{ A}$$

+ve sign of I_1 shows that current 0.13 A flows from Q to P.

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 90

2. Distance of the centre of mass of a solid uniform cone its vertex is z_0 . If the radius of its base is R and its height is h then z_0 is equal to:

(A) $\frac{3h^2}{8R}$

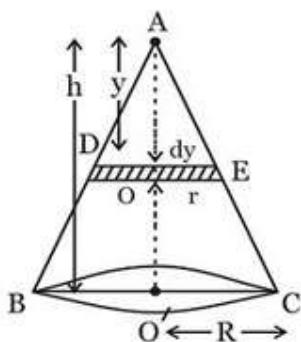
(B) $\frac{h^2}{4R}$

(C) $\frac{3h}{4}$

(D) $\frac{5h}{8}$

Answer: (C)

Solution:



Consider an elementary disc of radius r and thickness dy .

If total mass of cone = M and density = ρ

Then mass of elementary disc is $dm = \rho dv = \rho \times \pi r^2 dy$ (1)

In similar Δ 's AOE and $AO'C$

$$\frac{y}{h} = \frac{r}{R} \Rightarrow r = \frac{y}{h} R \quad \dots\dots (2)$$

Put (2) in (1)

$$dm = \rho(\pi) \left(\frac{y}{h} R \right)^2 dy$$

$$dm = \rho \times \frac{\pi R^2}{h^2} y^2 dy$$

\therefore The centre of mass of cone lying on the line AO' at a distance y_{cm} from A can be calculated as

$$y_{cm} = \frac{\int(dm)y}{\int dm} = \frac{\int \rho \pi R^2 \frac{y^3}{h^2} dy}{\int dm}$$

$$= \frac{\rho \pi R^2}{h^2 M} \int_0^h y^3 dy$$

$$\therefore M = \rho \times \frac{1}{3} \pi R^2 h$$

$$\Rightarrow y_{cm} = \frac{\rho \pi R^2}{h^2 \rho \times \pi R^2 h} \times \frac{h^4}{4} = \frac{3h}{4}$$

Topic: Centre of Mass

Difficulty: Easy (embibe predicted Low Weightage)

Ideal time: 30

3. Match list-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

	List-I		List-II
A	Franck-Hertz Experiment	(i)	Particle nature of light
B	Photo-electric experiment	(ii)	Discrete energy levels of atom
C	Davison-Germer Experiment	(iii)	Wave nature of electron
D		(iv)	Structure of atom

- (A) A – (iv); B – (iii); C – (ii)
 (B) A – (i); B – (iv); C – (iii)
 (C) A – (ii); B – (iv); C – (iii)
 (D) A – (ii); B – (i); C – (iii)

Answer: (D)

Solution: Frank-Hertz experiment demonstrated the existence of excited states in mercury atoms helping to confirm the quantum theory which predicted that electrons occupied only discrete quantized energy states.

Phot-electric experiment = Demonstrate that photon is the field particle of light which can transfer momentum and energy due to collision.

Davisson-Germer experiment = this experiment shows the wave nature of electron.

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

4. The period of oscillation of a simple pendulum is $T = 2\pi \sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is:

- (A) 5%
 (B) 2%
 (C) 3%
 (D) 1%

Answer: (C)

Solution: $\therefore T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = 4\pi^2 \frac{L}{T^2}$

\therefore Error in g can be calculated as



JEE MAIN 2015

Detailed Solution - Offline 4th April



$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

∴ Total time for n oscillation is $t = nT$ where T = time for oscillation.

$$\Rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{T}$$

$$\Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t}$$

Given that $\Delta L = 1mm = 10^{-3}m, L = 20 \times 10^{-2}m$

$$\Delta t = 1s, t = 90s$$

$$error \in g$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t} \right) \times 100$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t} \right) \times 100$$

$$\left(\frac{10^{-3}}{20 \times 10^{-2}} + \frac{2 \times 1}{90} \right) \times 100$$

$$\frac{1}{2} + \frac{20}{9}$$

$$0.5 + 2.22$$

$$2.72$$

$$\cong 3$$

Topic: Unit & Dimensions

Difficulty: level: Easy (embibe predicted easy to score)

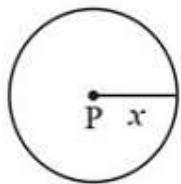
Ideal time: 90

5. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:

- (A) 7.75 V/m
- (B) 1.73 V/m
- (C) 2.45 V/m
- (D) 5.48 V/m

Answer: (C)

Solution:



For a point source of power = P , then intensity at a point at a separation x from the source is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi x^2}$$

∴ Average intensity of EM wave is given by

$$I = \frac{1}{2} C \epsilon_0 E_0^2$$

$$\Rightarrow E_0 = \sqrt{\frac{2P}{4\pi \epsilon_0 C x^2}}$$

$$\therefore \frac{1}{4\pi \epsilon_0 C} = 9 \times 10^9, P = 0.1 \text{ W}, x = 1 \text{ m}$$

$$C = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

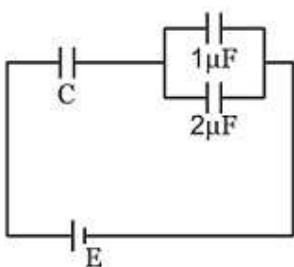
$$\Rightarrow E_0 = \sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{3 \times 10^8 \times 1^2}} = \sqrt{6} = 2.45 \text{ V/m}$$

Topic: Optics

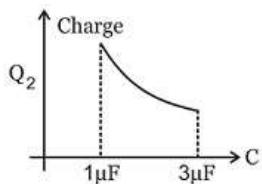
Difficulty: Easy (embibe predicted high weightage)

Ideal time: 120

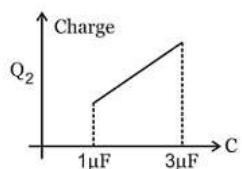
6. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of ' C ' is given properly by: (figure are drawn schematically and are not to scale)



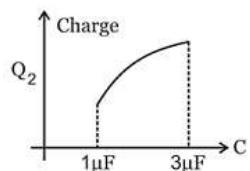
(A)



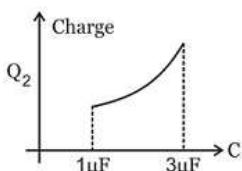
(B)



(C)

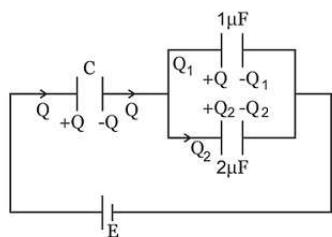


(D)

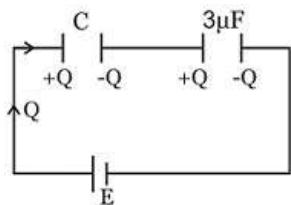


Answer: (C)

Solution:



$\because 1 \text{ } \mu\text{F}$ and $2 \text{ } \mu\text{F}$ are in parallel.



∴ Equivalent capacitance of the series combination is

$$C_{eq} \text{ is } \frac{3c}{c+3}$$

$$\text{So total charge supplied by battery is } Q = C_{eq} = \frac{3CE}{c+3}$$

∴ Potential difference across parallel combination of $1\mu F$ ad $2\mu F$ is

$$\Delta V = \frac{Q}{3} = \frac{CE}{c+3}$$

So charge on $2\mu F$ capacitor is

$$Q_2 = C_2 \Delta V = \frac{2CE}{c+3}$$

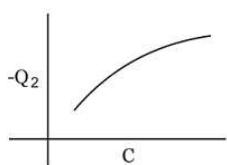
$$\Rightarrow \frac{Q_2}{2E} = \frac{C}{c+3} \Rightarrow \frac{Q_2}{2E} = \frac{c+3-3}{c+3}$$

$$\Rightarrow \frac{Q_2}{2E} = 1 - \frac{3}{c+3} \Rightarrow \left(\frac{Q_2}{2E} - 1 \right) = \frac{-3}{c+3}$$

$$\Rightarrow (Q_2 - 2E)(C + 3) = -6E$$

Which is of the form $(y - \alpha)(x + \beta) < 0$

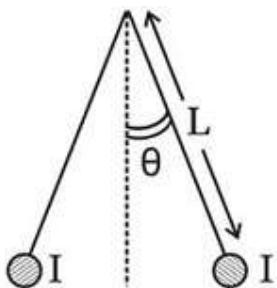
So the graph in hyperbola. With down ward curve line. i.e



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

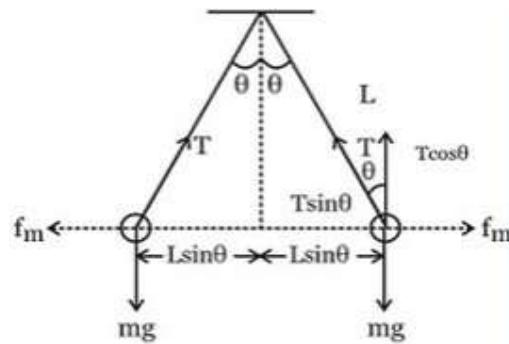


7. Two long current carrying thin wires, both with current I , are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle ' θ ' with the vertical. If wires have mass λ per unit length then the value of I is:
(g = gravitational acceleration)

- (A) $\sqrt{\frac{\pi\lambda g L}{\mu_0} \tan\theta}$
- (B) $\sin\theta \sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$
- (C) $2\sin\theta \sqrt{\frac{\pi\lambda g L}{\mu_0 \cos\theta}}$
- (D) $2 \sqrt{\frac{\pi g L}{\mu_0} \tan\theta}$

Answer: (C)

Solution



Two wires will repel each other due to current in the same direction and due to magnetic force.

∴ magnetic force per unit length is

$$\frac{df}{dl} = \frac{\mu_0 I^2}{2\pi(2L \sin\theta)} = \frac{\mu_0 I^2}{4\pi L \sin\theta}$$

and mass per unit length of each mix = $\frac{dm}{dl} = \lambda$

So, magnetic force on total length ℓ' of the mix is $f_m = \frac{\mu_0 I^2 \ell'}{4\pi L \sin\theta}$

and weight = $\lambda \ell' g$

By equilibrium of mix,

$$T \sin\theta = f_m \text{ and } T \cos\theta = mg$$

$$\Rightarrow \frac{T \sin\theta}{T \cos\theta} = \frac{f_m}{mg} \Rightarrow f_m = mg \tan\theta$$

$$\Rightarrow \frac{\mu_0 I^2}{4\pi L \sin\theta} \ell' = mg \frac{\sin\theta}{\cos\theta} = \lambda \ell' g \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{\mu_0 I^2}{4\pi L \sin\theta} \ell' = \lambda \ell' g \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow I^2 = \frac{\lambda g \pi L}{\mu_0 \cos\theta} 4 \sin^2\theta$$

$$I = 2 \sin\theta \sqrt{\frac{\lambda g \pi L}{\mu_0 \cos\theta}}$$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

8. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v . If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

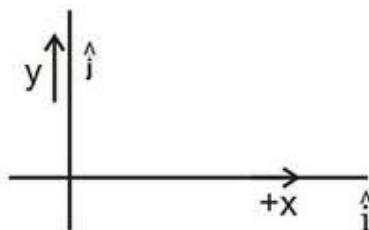
- (A) 62%
- (B) 44%
- (C) 50%
- (D) 56%**

Answer: (D)

Solution:

The initial momentum of system is $\vec{P}_i = m(2V)\hat{i} + (2m)v\hat{j}$

According to question as



On perfectly inelastic collision the particles stick to each other so.

$$\vec{P}_f = 3m\vec{V}_f$$

By conservation of linear momentum principle

$$\vec{P}_f = \vec{P}_i \Rightarrow 3m\vec{V}_f = m2V\hat{i} + 2mV\hat{j}$$

$$\Rightarrow \vec{V}_f = \frac{2V}{3}(\hat{i} + \hat{j}) \Rightarrow V_f = \frac{2\sqrt{2}}{3}V$$

\therefore loss in KE of system $K_{initial} - K_{final}$

$$\begin{aligned} & \frac{1}{2}m(2V)^2 + \frac{1}{2}(2m)V^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}V}{3}\right)^2 \\ & 2mV^2 + mV^2 - \frac{4}{3}mV^2 = 3mV^2 - \frac{4mV^2}{3} \\ & \frac{5}{3}mV^2 \end{aligned}$$

$$\% \text{ change in KE } 100 \times \frac{\Delta K}{K_i} = \frac{\frac{5}{3}mV^2}{3mV^2} = \frac{5}{9} \times 100$$

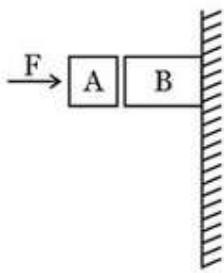
$$\frac{500}{9} = 56$$

Topic: Conservation of Momentum

Difficulty: level: Moderate (embibe predicted easy to score (Must Do))

Ideal time: 90

9.

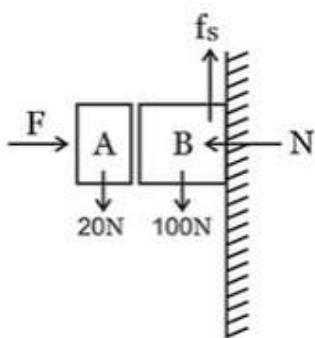


Given in the figure are two blocks A and B of weight 20N and 100N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is:

- (A) 150 N
- (B) 100 N
- (C) 80 N
- (D) 120 N**

Answer: (D)

Solution:



For complete state equilibrium of the system. The static friction on the block B by wall will balance the total weight 120 N of the system.

Topic: Laws of Motion

Difficulty: level: Moderate (embibe predicted Low Weightage)

Ideal time: 60

10. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increase as V^q , where V is the volume of the gas. The value of q is:

$$\left(\gamma = \frac{c_p}{c_v} \right)$$

- (A) $\frac{\gamma-1}{2}$
 (B) $\frac{3\gamma+5}{6}$
 (C) $\frac{3\gamma-5}{6}$
 (D) $\frac{\gamma+1}{2}$

Answer: (D)

Solution:

Average time of collision

$$t = \frac{\text{mean free path}(\lambda)}{\text{average speed } (v)}$$

$$t \propto \frac{\lambda}{v}$$

$$\therefore \lambda \propto \frac{1}{\text{no. of molecules per unit volume}}$$

$$\lambda \propto \frac{1}{\left(\frac{N}{V}\right)}$$

$$\Rightarrow \lambda \propto V$$

$$\text{And } \bar{v} \propto \sqrt{T}$$

$$\Rightarrow \bar{v} \propto \sqrt{PV}$$

$$\therefore P \propto V^{-\gamma}$$

for adiabatic process where $\gamma = \text{adiabatic coefficient}$

$$\Rightarrow \bar{v} \propto \sqrt{V^{-\gamma} V}$$

$$\Rightarrow \bar{v} \propto V^{\frac{1-\gamma}{2}}$$

So average time

$$\therefore t_{avg} \propto \frac{V}{V^{\frac{1-\gamma}{2}}}$$

$$t_{avg} \propto V^{1 - \left(\frac{1-\gamma}{2}\right)}$$

$$t_{avg} \propto V^{\frac{1+\gamma}{2}}$$

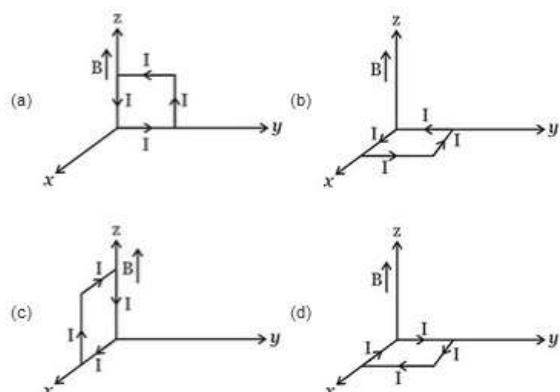
$$\therefore q = \frac{1+\gamma}{2}$$

Topic: Heat & Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

11. A rectangular loop of sides 10 cm and 5cm carrying a current I of 12 A is placed in different orientations as shown in the figure below:



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

- (A) (b) and (c), respectively
 (B) (a) and (b), respectively



JEE MAIN 2015

Detailed Solution - Offline 4th April



- (C) (a) and (c), respectively
(D) (b) and (d), respectively

Answer: (D)

Solution: For a magnetic dipole placed in a uniform magnetic field the torque is given by $\vec{\tau} = \vec{M} \times \vec{B}$ and potential energy U is given as

$$U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$$

When \vec{M} is in the same direction as \vec{B} then $\vec{\tau} = 0$ and U is min = - MB as $\theta = 0^\circ$

\Rightarrow Stable equilibrium is (b). and when \vec{M} then $\vec{\tau} = 0$ and U is max = + MB

As $\theta = 180^\circ$

Unstable equilibrium in (d).

Topic: Magnetism

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

12. Consider a spherical shell of radius R at temperature T. the black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume $u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

- (A) $T \propto \frac{1}{R^3}$
(B) $T \propto e^{-R}$
(C) $T \propto e^{-3R}$
(D) $T \propto \frac{1}{R}$

Answer: (D)

Solution: \because in an adiabatic process.

$$dQ = 0$$

So by first law of thermodynamics

$$dQ = dU + dW$$

$$\Rightarrow 0 = dU + dW$$

$$\Rightarrow dW = -d$$

$$\therefore dW = PdV$$

$$\Rightarrow PdV = -dU \dots(i)$$

$$\text{Given that } \frac{U}{V} \propto T^4 \Rightarrow U = kVT^4$$

$$\Rightarrow dU = kd(VT^4) = K(T^4dV + 4T^3VdT)$$

$$\text{Also, } P = \frac{1}{3} \frac{U}{V} = \frac{1}{3} \frac{kVT^4}{V} = \frac{KT^4}{3}$$

Putting these values in equation

$$\Rightarrow \frac{KT^4}{3} dV = -k(T^4dV + 4T^3VdT)$$

$$\Rightarrow \frac{TdV}{3} = -TdV - 4VdT$$

$$\Rightarrow \frac{4T}{3} dV = -4VdT$$

$$\Rightarrow \frac{\frac{1}{3} dV}{V} = \frac{-dT}{T}$$

$$\Rightarrow \frac{1}{3} \ln V = -\ln T \Rightarrow \ln V = l^{-3}$$

$$\Rightarrow VT^3 = \text{constant}$$

$$\frac{4}{3} \pi R^3 T^3 = \text{constant}$$

$$RT = \text{constant}$$

$$\Rightarrow T \propto \frac{1}{R}$$

Topic: Heat & Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

13. As an electron makes transition from an excited state to the ground state of a hydrogen like atom/ion:

- (A) Kinetic energy and total energy decreases but potential energy increases
- (B) Its kinetic energy increases but potential energy and total energy decrease
- (C) Kinetic energy, potential energy and total energy decrease
- (D) Kinetic energy decreases, potential energy increases but total energy remains same

Answer: (B)

Solution: $U = \frac{-e^2}{4\pi\epsilon_0 r}$

U = potential energy

$$k = \frac{e^2}{8\pi\epsilon_0} r$$

K = kinetic energy

$$E = U + k = \frac{-e^2}{8\pi\epsilon_0 r}$$

E = Total energy

∴ as electron de-excites from excited state to ground state k increases, U & E decreases

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

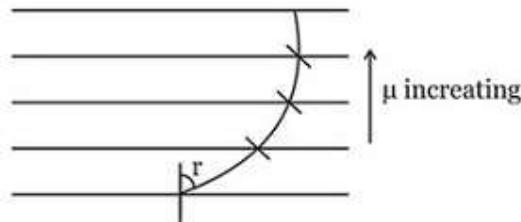
Ideal time: 30

14. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:

- (A) Bends upwards
- (B) Becomes narrower
- (C) Goes horizontally without any deflection
- (D) Bends downwards

Answer: (A)

Solution: Consider air layers with increasing refractive index.



At critical angle it will bend upwards at interface. This process continues at each layer, and light ray bends upwards continuously.

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 60

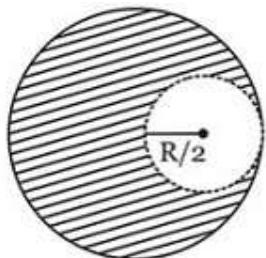
15. From a solid sphere of mass M and radius R , a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ and $r = \infty$, the potential at the center of the cavity thus formed is:
 (G = gravitational constant)



- (A) $\frac{-2GM}{R}$
- (B) $\frac{-G}{2R}$
- (C) $\frac{-GM}{R}$**
- (D) $\frac{-2GM}{3R}$

Answer: (C)

Solution:



Potential due to whole sphere if cavity is not there at distance $\frac{R}{2}$ from centre

$$= \frac{-GM}{R^3} \left(\frac{3}{2} R^2 - 0.5 r^2 \right)_{r=\left(\frac{R}{2}\right)}$$

$$= \frac{-GM}{R^3} \left(\frac{3}{2} R^2 - \frac{R^2}{8} \right)$$

$$= \frac{-GM}{R^3} \left(\frac{12 R^2 - R^2}{8} \right)$$

$$= \frac{-11GM}{8R} \quad \text{--- (1)}$$

Potential due to sphere of radius $\frac{R}{2}$ at its centre let M' be mass of this sphere (equating densities)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M'}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$M' = \frac{M}{8}$$

Potential due to the sphere of $\frac{R}{2}$ radius at its centre is

$$= \frac{3}{2} \frac{GM'}{\frac{R}{2}}$$

$$= \frac{3}{2} \frac{GM \times 2}{8R}$$

$$= \frac{-3}{8} \frac{GM}{R} \quad \text{--- (2)}$$

\therefore Potential at $r = \frac{R}{2}$ is $= (1) - (2)$

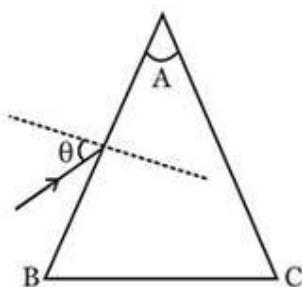
$$= \frac{-11}{8} \frac{GM}{R} + \frac{3}{8} \frac{GM}{R} = \frac{-GM}{R}$$

Topic: Gravitation

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 360

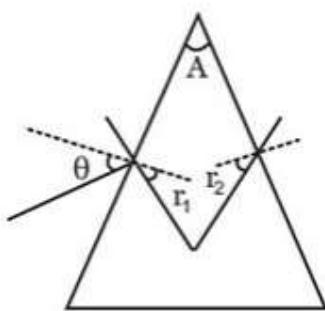
16. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ , a ray, incident at an angle θ , on the face AB would get transmitted through the face AC of the prism provided:



- (A) $\theta < \cos^{-1} \left[\mu \sin \left(A + \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (B)** $\theta > \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (C) $\theta < \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (D) $\theta > \cos^{-1} \left[\mu \sin \left(A + \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$

Answer: (B)

Solution:



For emergence $r_2 <$ critical angle

$$\Rightarrow r_2 < \sin^{-1} \left(\frac{1}{\mu} \right)$$

$$A = r_1 + r_2$$

$$\Rightarrow A - r_1 = r_2$$

$$\Rightarrow A - r_1 < \sin^{-1} \left(\frac{1}{\mu} \right)$$

$$\Rightarrow A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < r_1$$

\therefore By shells law

$$\sin \theta = \mu \sin r_1$$

$$\Rightarrow r_1 = \sin^{-1}\left(\frac{\sin \theta}{\mu}\right)$$

$$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin \theta}{\mu}\right)$$

$$\Rightarrow \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right)) < \frac{\sin \theta}{\mu}$$

$$\mu \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right)) < \sin \theta$$

$$\Rightarrow \theta > \sin^{-1}(\mu \sin(A - \sin^{-1}\left(\frac{1}{\mu}\right)))$$

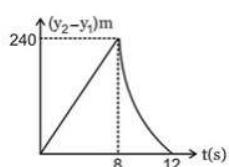
Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

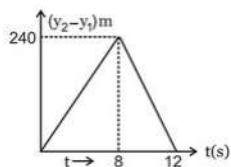
Ideal time: 240

17. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?
 (Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 \text{ ms}^{-2}$)
 (The figures are schematic and not drawn to scale)

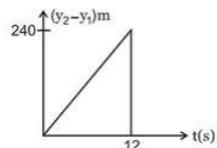
(A)



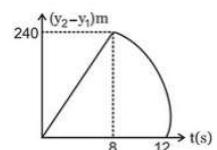
(B)



(C)



(D)



Answer: (D)

$$\text{Solution: } S_1 = 10t - \frac{1}{2}gt^2$$

$$\text{When } S_1 = -240$$

$$\Rightarrow -240 = 10t - 5t^2$$

$$\Rightarrow t = 8s$$

So at $t = 8$ seconds first stone will reach ground

$$S_2 = 20t - \frac{1}{2}gt^2$$

Till $t = 8$ seconds

$$S_2 - S_1 = 30t$$

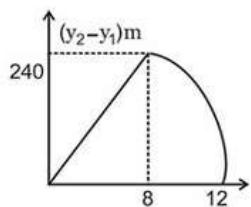
But after 8 second S_1 is constant -240

Relative to stone $t_1 > 8$ seconds displacements of stone 2 $S_2 + 240$

$$\Rightarrow S_2 + 240 = 20t - \frac{1}{2}gt^2$$

And at $t = 12$ seconds stone will reach ground

The corresponding graph of relative position of second stone w.r.t. first is



Topic: Kinematics

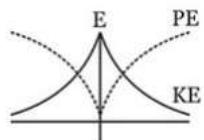
Difficulty: Moderate (Embibe predicted high weightage)

Ideal time: 240

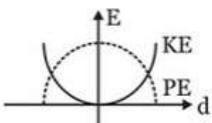
18. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d . which one of the following represents these correctly?

(Graphs are schematic and not drawn to scale)

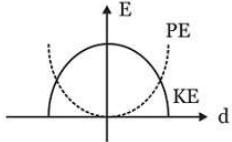
(A)



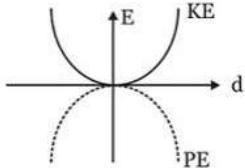
(B)



(C)



(D)



Answer: (C)

Solution:

For simple pendulum performing simple harmonic motion, displacement

$$y = A \sin \omega t$$

$$\begin{aligned} \text{Velocity } \frac{dy}{dt} &= V = \omega A \cos \omega t \\ &= A\omega \sqrt{1 - \sin^2 \omega t} \\ &= A\omega \sqrt{1 - \frac{y^2}{A^2}} \\ &= \omega \sqrt{A^2 - y^2} \end{aligned}$$

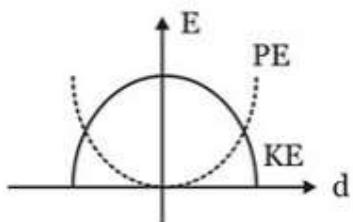
$$\begin{aligned} \text{Kinetic energy} &= \frac{1}{2} mv^2 \\ &= \frac{1}{2} \times m \times \omega^2 (A^2 - y^2) \end{aligned}$$

$$\text{at } y = A \text{ (extreme positions)}$$

$$\text{Kinetic energy} = \frac{1}{2} \omega^2 m (A^2 - A^2) = 0$$

$$\text{Similarly potential energy} = \frac{1}{2} m \omega^2 y^2$$

On plotting graphs of potential energy & Kinetic energy



Topic: Simple Harmonic Motion

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 60

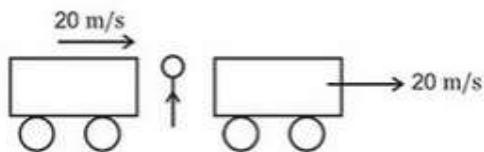
19. A train is moving on a straight track with speed 20ms^{-1} . It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound 320ms^{-1}) close to:

- (A) 24%

- (B) 6%
 (C) 12%
 (D) 18%

Answer: (C)

Solution:



Before $f_0 = 1000 \text{ Hz}$

$$f' = \left(\frac{v}{v - v_s} \right) \times f_0$$

$$= \left(\frac{320}{320 - 20} \right) \times f_0$$

$$= \left(\frac{320}{300} \right) \times f_0$$

$$= \frac{16f_0}{15}$$

$$f'' = \left(\frac{v}{v+v_s} \right) \times f_0$$

$$f'' = \left(\frac{320}{320+20} \right) f_0$$

$$= \left(\frac{320}{340} \right) f_0$$

$$= \left(\frac{16}{17} \right) f_0$$

Change in frequency

$$= \left(\frac{16}{15} - \frac{16}{17} \right) f_0$$

\therefore Percentage change in frequency

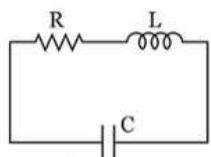
$$= \frac{\left(\frac{16}{15} - \frac{16}{17} \right) f_0}{f_0} \times 100 \approx 12\% \text{ nearly}$$

Topic: Wave & Sound

Difficulty: Moderate (embibe predicted high weightage)

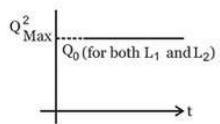
Ideal time: 200

20. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

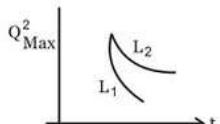


If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (Plots are schematic and not drawn to scale)

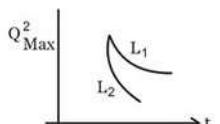
(A)



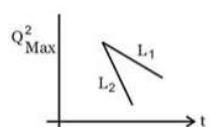
(B)



(C)



(D)



Answer: (C)

Solution:

Comparing to damped pendulum

We write

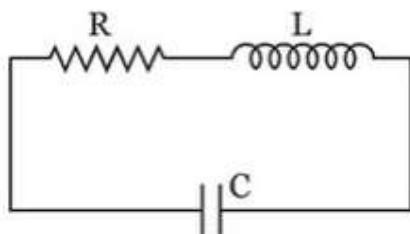
$$m \frac{dv}{dt} = -kx - bv; \text{ bv is resistive force}$$

$$\Rightarrow \text{Amplitude } A = A_0 e^{-\frac{bt}{2m}}$$

Comparing results, we can write

$$\frac{q}{c} = +L \frac{di}{dt} + iR$$

as charge decreasing



$$\frac{q}{c} = L \frac{d^2 q}{dt^2} - \frac{dq}{dt} R$$

$$\Rightarrow A = q; R = b, m = L$$

$$q = q_0 e^{-\frac{Rt}{2L}}$$

$$q^2 = q_0^2 e^{-\frac{Rt}{L}}$$

Exponentially decreasing function more the 'L' losses will $\left(\frac{R}{L}\right)$ and more will be $e^{-\left(\frac{Rt}{L}\right)t}$

L_1 graph has higher values than L_2

Topic: Magnetism

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 150

21. A solid body of constant heat capacity $1J/^{\circ}C$ is being heated by keeping it in contact with reservoirs in two ways:

- (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
- (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat

In both the cases body is brought from initial temperature $100^{\circ}C$ to final temperature $200^{\circ}C$. Entropy change of the body in the two cases respectively is:

- (A) $2\ln 2, 8\ln 2$
- (B) $\ln 2, 4\ln 2$

- (C) $\ln 2, \ln 2$
(D) $\ln 2, 2\ln 2$

Answer: (C)

Solution:

$$\text{Change in entropy } ds = \frac{dQ}{T}$$

$$\Delta Q = \text{heat supplied} = C \Delta T$$

$$dQ = C dT$$

$$ds = \frac{CdT}{T}$$

Integrating both sides

$$\int_{S_i}^{S_f} ds = C \int \frac{dT}{T}$$

$$S_f - S_i = \Delta S = C \ln T \Big|_{100}^{200}$$

$$= C[\ln 200 - \ln 100]$$

$$\Delta S = C \ln 2$$

$$C = 1 \text{ J}/{}^\circ\text{C}$$

$$\Rightarrow \Delta S = \ln 2$$

Entropy change is same for both cases as C is constant, and temperature change (i.e. from 100 to 200) is same.

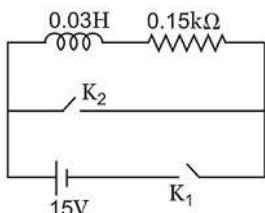
Topic: Heat & Thermodynamics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

22. An inductor ($L = 0.03\text{H}$) and a resistor ($R = 0.15\text{k}\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then

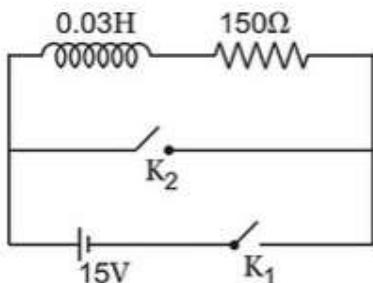
at $t = 0$. K_1 is opened and key K_2 is closed simultaneously. At $t = 1ms$, the current in the circuit will be : ($e^5 \cong 150$)



- (A) 0.67 mA
- (B) 100 mA
- (C) 67 mA
- (D) 6.7 mA

Answer: (A)

Solution:



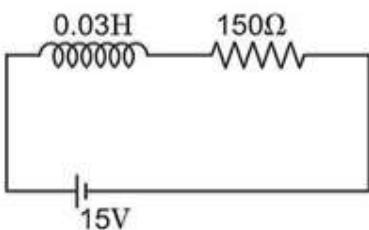
$$L = 0.03H$$

$$R = 0.15k\Omega = 150\Omega$$

$(e^5 \cong 150$ given)

$$E = 15V$$

Case I: K_1 is closed for long time



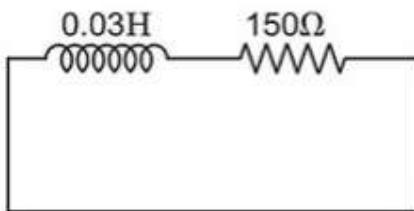
for long time, inductor acts as a conducting wire.

$$\Rightarrow \text{current in the circuit} = \frac{V}{R}$$

$$= \frac{15}{150}$$

$$i_0 = 0.1 \text{ A}$$

Case II: K_1 is open and K_2 is closed



Current in the circuit

$$i = i_0 e^{-\frac{t}{\tau}}; \tau = \frac{L}{R}$$

After $t = 1\text{ms} = 10^{-3}\text{s}$

$$i = i_0 e^{-\left(\frac{10^{-3} \times 150}{3 \times 10^{-2}}\right)}$$

$$= 0.1 e^{-\frac{15}{3}}$$

$$= 0.1 \frac{1}{e^5} = \frac{0.1}{150}$$

$$= 6.67 \times 10^{-4}$$

$$= 0.67 \times 10^{-3} \text{ A}$$

$$= 0.67 \text{ mA}$$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

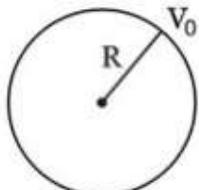
Ideal time: 120

23. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potential $\frac{3V_0}{2}$, $\frac{5V_0}{4}$, $\frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1 , R_2 , R_3 and R_4 respectively. Then

- (A) $2R < R_4$
- (B) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
- (C) $R_1 \neq 0$ and $(R_2 - R_1) > R_4 - R_3$
- (D) $R_1 = 0$ and $R_2 < (R_4 - R_3)$**

Answer: (D)

Solution:



Potential for uniformly charged sphere

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad \text{outside i.e. } r > R$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \quad \text{on the surface}$$

$$V = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right] \quad \text{inside i.e. } r < R$$

Clearly potential is decreasing with r .

$$\Rightarrow \frac{3V_0}{2}, \frac{5V_0}{4} \text{ are inside potentials } [\because r > V_0]$$

$\frac{3v_0}{4}, \frac{v_0}{4}$ are outside potentials [$\because < v_0$]

$$\text{To get } R_1 : \frac{3v_0}{2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \right]$$

$$v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$$

$$\frac{3}{2 \times 4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \right]$$

$$\frac{3}{2} = \frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \Rightarrow R_1 = 0$$

$$\text{To get } R_2 : \frac{5}{4} v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2} \right]$$

$$\frac{5}{4} \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2} \right]$$

$$\frac{5}{4} = \frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2}$$

$$\frac{1}{2} \frac{R_2^2}{R^2} = \frac{1}{4}$$

$$R_2^2 = \frac{R^2}{2}$$

$$R_2 = \frac{R}{\sqrt{2}}$$

$$\text{To get } R_3 : \frac{3v_0}{4} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}$$

$$\frac{3}{4} \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}$$

$$\frac{3}{4R} = \frac{1}{R_3}$$

$$R_3 = \frac{4}{3} R$$

$$\frac{1}{4} \times \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_4}$$

$$R_4 = 4R$$

$$R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} > R_2$$

$$R_1 = 0 \text{ and } R_2 < (R_4 - R_3)$$

Both options are correct.

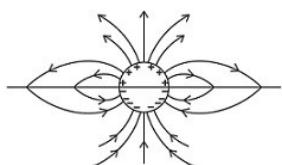
Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

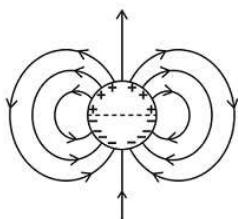
Ideal time: 210

24. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:
 (Figures are schematic and not drawn to scale)

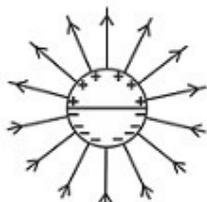
(A)



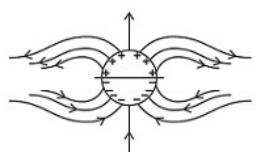
(B)



(C)

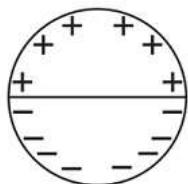


(D)

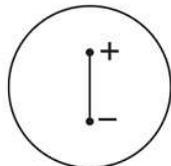


Answer: (B)

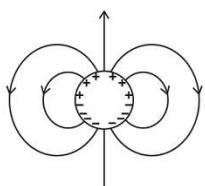
Solution:



Consider cross section of cylinders which is circle the half part of circle which has positive charge can be assume that total positive charge is at centre of mass of semicircle. In the same way we can assume that negative charge is at centre of mass of that semicircle.



Now it acts as a dipole now by the properties of dipole and laws of electric field line where two lines should not intersect the graph would be



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

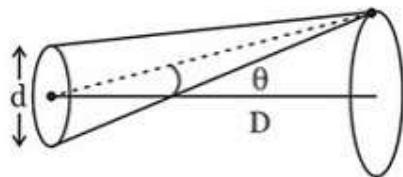
Ideal time: 90

25. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

- (A) $300\mu m$
- (B) $1\mu m$
- (C) $30\mu m$**
- (D) $100\mu m$

Answer: (C)

By fraunhofer diffraction through a circular aperture $\theta = \frac{1.22\lambda}{d}$



$$D = \text{diameter of pupil} = 2 \times 0.25 = 0.5 \text{ cm}$$

$$\lambda = 500 \text{ nm}$$

First dark ring is formed by the light diffracted from the hole at an angle θ with the axis

Viewing distance $D = 25 \text{ cm}$

\therefore minimum separation between

$$2 \text{ objects} = D\theta$$

$$= \frac{25 \times 10^{-2} \times 1.22 \times 500 \times 10^{-9}}{5 \times 10^{-1}}$$

$$= 30 \times 10^{-6} \text{ m}$$

$$= 30 \mu m$$

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

26. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequency of the resultant signal is/are:

- (A) 2000 kHz and 1995 kHz
- (B) 2 MHz only
- (C) 2005 kHz, and 1995 kHz
- (D) 2005 kHz, 2000 kHz and 1995 kHz**

Answer: (D)

Solution:

$$\text{Frequency of single wave} = 5\text{kHz} = f$$

$$\text{Carrier wave frequency} = 2\text{MHz}$$

$$= 2000 \text{ kHz} = f_c$$

$$\text{Resultant signal maximum frequency}$$

$$= f + f_c$$

$$= 5 + 2000 \text{ kHz}$$

$$= 2005 \text{ kHz}$$

$$\text{Resultant signal minimum frequency}$$

$$= f_c = f$$

$$= 2000 - 5\text{kHz}$$

$$= 1995 \text{ kHz}$$

Topic: Communication Systems

Difficulty: Easy (embibe predicted high weightage)

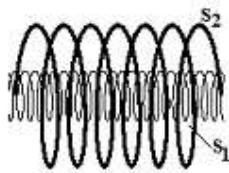
Ideal time: 60

27. Two coaxial solenoids of different radii carry current I in the same direction. Let \vec{F}_1 be the magnetic force on the inner solenoid due to the outer one and \vec{F}_2 be the magnetic force on the outer solenoid due to the inner one. Then:

- (A) \vec{F}_1 is radially outwards and $\vec{F}_2 = 0$
 (B) $\vec{F}_1 = \vec{F}_2 = 0$
 (C) \vec{F}_1 is radially inwards and \vec{F}_2 is radially outwards
 (D) \vec{F}_1 is radially inwards and $\vec{F}_2 = 0$

Answer: (B)

Solution:



S_2 is solenoid with more radius than S_1 field because of S_1 on S_2 is 0

\therefore force on S_2 by $S_1 = 0$

In the uniform field of S_2 S_1 behaves as a magnetic dipole

\therefore force on S_1 by S_2 is zero because force on both poles are equal in magnitude and opposite in direction.

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

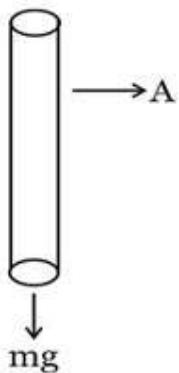
Ideal time: 120

28. A pendulum made of a uniform wire of cross sectional area A has time period T. When an additional mass M is added to its bob, the time period changes to T_M . If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
 $(g = \text{gravitational acceleration})$

- (A) $\left[1 - \left(\frac{T}{T_M}\right)^2\right] \frac{A}{Mg}$
 (B) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$
 (C) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{Mg}{A}$
 (D) $\left[1 - \left(\frac{T_M}{T}\right)^2\right] \frac{A}{Mg}$

Answer: (B)

Solution:



Initial length = ℓ

$$\text{Time period } T = 2\pi \sqrt{\frac{\ell}{g}} \quad \dots(i)$$

After suspending mass M,

$$\text{Young's modulus } Y = \frac{\text{Stress}}{\text{Strain}}$$

$$= \frac{F}{A\ell} = \frac{F\ell}{\Delta\ell A}$$

$$\text{Change in length of wire } \Delta\ell = \frac{F\ell}{Ay}$$

$$\text{Now Time period } T_M = 2\pi \sqrt{\frac{\ell + \Delta\ell}{g}} \quad \dots(ii)$$

$$\frac{T}{T_M} = \frac{2\pi \sqrt{\frac{\ell}{g}}}{2\pi \sqrt{\frac{(\ell + \Delta\ell)}{g}}} \quad \left[\because \frac{i}{ii} \right]$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell + \Delta\ell}$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell + \frac{F\ell}{Ay}} \quad [\text{putting } \Delta\ell \text{ value}]$$

$$\left(\frac{T}{T_M}\right)^2 = \frac{1}{1 + \frac{F}{Ay}}$$

$$1 + \frac{F}{Ay} = \left(\frac{T_M}{T}\right)^2$$

$$\frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{F}$$

$$F = mg$$

$$\Rightarrow \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$$

Topic: Simple Harmonic Motion

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

29. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

(A) $\frac{4MR^2}{3\sqrt{3}\pi}$

(B) $\frac{MR^2}{32\sqrt{2}\pi}$

(C) $\frac{MR^2}{16\sqrt{2}\pi}$

(D) $\frac{4MR^2}{9\sqrt{3}\pi}$

Answer: (D)

Solution: Let a be length of cube for cube with maximum possible volume diagonal length = 2R

$$\Rightarrow \sqrt{3}a = 2R \Rightarrow a = \frac{2R}{\sqrt{3}}$$

As densities of sphere and cube are equal. Let M' be mass of cube

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M'}{a^3}$$

$$M' = \frac{3Ma^3}{4\pi R^3}$$

Moment of inertial of cube about an axis passing through centre

$$\frac{M'(2a^2)}{12}$$

$$\frac{3Ma^3}{4\pi R^3} \times \frac{2a^2}{12}$$

$$\frac{Ma^5}{8\pi R^3}$$

$$a = \frac{2}{\sqrt{3}}R$$

$$\frac{M \times 32R^5}{8\pi \times 9\sqrt{3}R^3}$$

$$\frac{4MR^2}{9\sqrt{3}\pi}$$

Topic: Rotational Mechanics

Difficulty: Moderate (embibe predicted Low Weightage)

Ideal time: 300

30. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5 \times 10^{-4} \text{ ms}^{-1}$. If the electron density in the wire is $8 \times 10^{28} \text{ m}^{-3}$, the resistivity of the material is close to:

- (A) $1.6 \times 10^{-5} \Omega \text{m}$
- (B) $1.6 \times 10^{-8} \Omega \text{m}$
- (C) $1.6 \times 10^{-7} \Omega \text{m}$
- (D) $1.6 \times 10^{-6} \Omega \text{m}$

Answer: (A)

Solution:

Potential difference = 5V

length = 0.1m = ℓ

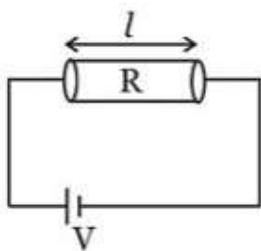
Electron speed = drift velocity $v_d = 2.5 \times 10^{-4} \text{ m/s}$

electron density (n) = $8 \times 10^{28} \text{ m}^{-3}$

charge on each electron(e) = $1.6 \times 10^{-19} \text{ C}$

We know $i = n A e v_d$... (i)

And $v = iR$... (ii)



Resistance R is also equal $\frac{\rho \ell}{A}$

$$R = \frac{\rho \ell}{A}$$

$$\rho = \frac{AR}{\ell} \quad [\rho = \text{Resistivity}]$$

$$= \frac{A}{\ell} \times \frac{v}{i} \quad [\text{from (ii)}]$$

$$= \frac{Av}{\ell \times nA e v_d} \quad [\text{from (i)}]$$

$$= \frac{v}{\ell \times n \times e \times v_d}$$

$$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$$

$$= 0.16 \times 10^{-4} \Omega \text{m} = 1.6 \times 10^{-5} \Omega \text{m}$$

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)



JEE MAIN 2015

Detailed Solution - Offline 4th April



Ideal time: 120

Chemistry

1. The vapour pressure of acetone at $20^\circ C$ is 185 torr. When 1.2 g of a non-volatile substance was dissolved in 100 g of acetone at $20^\circ C$, its vapour pressure was 183 torr. The molar mass ($gmol^{-1}$) of the substance is :
- (A) 488
 - (B) 32
 - (C) 64
 - (D) 128

Solution: (C) $\Delta P = 185 - 183 = 2 \text{ torr}$

$$\frac{\Delta P}{P^o} = \frac{2}{185} = X_B = \frac{\frac{1.2}{M}}{\left(\frac{1.2}{M}\right) + \frac{100}{58}}$$

$$M_{(CH_3)_2CO} = 15 \times 2 + 16 + 12 = 58 \text{ g/mole}$$

$$\begin{aligned} \frac{1.2}{M} &\ll \frac{100}{58} \\ \Rightarrow \frac{2}{185} &= \frac{1.2}{M} \times \frac{58}{100} \\ M &= \frac{58 \times 1.2}{100} \times \frac{185}{2} \\ 64.38 &\approx 64 \text{ g/mole} \end{aligned}$$

2. 3 g of activated charcoal was added to 50 mL of acetic acid solution (0.06N) in a flask. After an hour it was filtered and the strength of the filtrate was found to be 0.042 N. The amount of acetic acid adsorbed (per gram of charcoal) is:

- (A) 54 mg
- (B) 18 mg
- (C) 36 mg
- (D) 42 mg

Solution: (B) Meqs of CH_3COOH (initial) $50 \times 0.06 = 3 \text{ Meqs}$

Meqs CH_3COOH (final) $50 \times 0.042 = 2.1 \text{ Meqs}$

$$CH_3COOH \text{ adsorbed} \quad 3 - 2.1 = 0.9 \text{ Meqs}$$

$$9 \times 10^{-1} \times 60 \text{ g/Eq} \times 10^{-3} \text{ g}$$

$$540 \times 10^{-4} = 0.054g$$

54mg

$$\text{Per gram} = \frac{54}{3} = 18 \text{ mg/g of Charcoal}$$

3. Which of the following is the energy of a possible excited state of hydrogen?

- (A) +6.8 eV
- (B) +13.6 eV
- (C) -6.8 eV
- (D) -3.4 eV

$$\text{Solution: (D)} E_n = \frac{-13.6}{n^2} \text{ eV}$$

$$\text{Where } n = 2 \Rightarrow E_2 = -3.40 \text{ eV}$$

4. Which among the following is the most reactive?

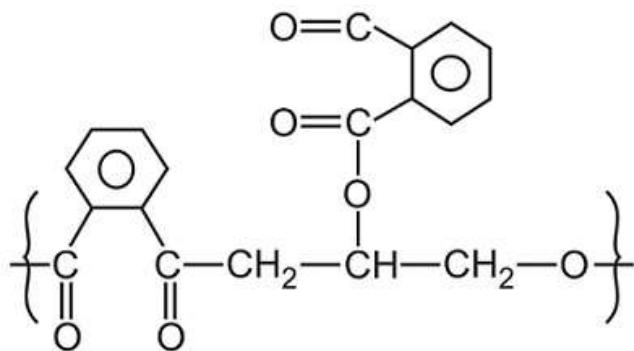
- (A) ICl
- (B) Cl_2
- (C) Br_2
- (D) I_2

Solution: (A) $I - Cl$ bond strength is weaker than I_2 , Br_2 and Cl_2 (Homonuclear covalent).

5. Which polymer is used in the manufacture of paints and lacquers?

- (A) Poly vinyl chloride
- (B) Bakelite
- (C) Glyptal
- (D) Polypropene

Solution: (C) Glyptal is polymer of glycerol and phthalic anhydride.



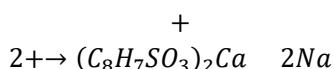
6. The molecular formula of a commercial resin used for exchanging ions in water softening is $C_8H_7SO_3Na$ (Mol. wt. 206). What would be the maximum uptake of Ca^{2+} ions by the resin when expressed in mole per gram resin?

(A) $\frac{1}{412}$

(B) $\frac{1}{103}$

(C) $\frac{1}{206}$

(D) $\frac{2}{309}$



Solution: (A) $+ + Ca_{(aq)}$

$-Na$

$2C_8H_7SO_3$

$2 + ions$

$$2 \text{ moles(resin)} = 412 \text{ g} \equiv 40 \text{ g Ca}$$

$2 + ions$

1 mole of Ca

$$Ca^{2+}/g \text{ of resin} = \frac{1}{412}$$

Moles of

7. In Carius method of estimation of halogens, 250 mg of an organic compound gave 141 mg of AgBr. The percentage of bromine in the compound is : (Atomic mass : Ag = 108, Br = 80)

(A) 60

(B) 24

(C) 36

(D) 48

Solution: (B) $R \xrightarrow{-Br} Cariusmetho \quad AgBr$

250 mg organic compound is RBr

$$141 \text{ mg AgBr} \Rightarrow 141 \times \frac{80}{188} \text{ mg Br}$$

Br in organic compound

$$141 \times \frac{80}{188} \times \frac{1}{250} \times 100 = 24$$

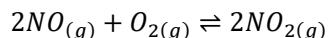
8. Assertion: Nitrogen and Oxygen are the main components in the atmosphere but these do not react to form oxides of nitrogen.

Reason: The reaction between nitrogen and oxygen requires high temperature.

- (A) Both the Assertion and Reason are incorrect.
- (B) Both Assertion and Reason are correct and the Reason is the correct explanation for the Assertion.
- (C) Both Assertion and Reason are correct, but the Reason is not the correct explanation for the Assertion.
- (D) The Assertion is incorrect but the Reason is correct.

Solution: (B) $N_{2(g)}$ & $O_{2(g)}$ react under electric arc at $2000^{\circ}C$ to form $NO_{(g)}$. Both assertion and reason are correct and reason is correct explanation.

9. The following reaction is performed at 298 K.



The standard free energy of formation of $NO_{(g)}$ is 86.6 kJ/mol at 298 K. What is the standard free energy of formation of $NO_{2(g)}$ at 298 K? ($K_P = 1.6 \times 10^{12}$)

- (A) $2 \times 86,600 - R(298)\ln\frac{0.5}{(1.6 \times 10^{12})}$
- (B) $R(298)\ln(1.6 \times 10^{12}) = 86,600$
- (C) $86,600 + R(298)\ln(1.6 \times 10^{12})$
- (D) $86,600 - \frac{\ln(1.6 \times 10^{12})}{R(298)}$

Solution: (A) $\Delta G_{reac}^o = -2.303RT\log K_P$

$$-RT\ln K_P$$

$$-R(298)\ln(1.6 \times 10^{12})$$

$$\Delta G_{reac}^o = 2\Delta G_{f(NO_2)}^o - 2\Delta G_{f(NO)}^o$$

$$2\Delta G_{f(NO_2)}^o - 2 \times 86.6 \times 10^3$$

$$2\Delta G_{f(NO_2)}^o = -R(298)\ln(1.6 \times 10^{12}) + 2 \times 86,600$$

$$\Delta G_{f(NO_2)}^o = 86,600 - \frac{R(298)}{2}\ln(1.6 \times 10^{12})$$

$$0.5[2 \times 86,600 - R(298)\ln(1.6 \times 10^{12})]$$

10. Which one of the following alkaline earth metal sulphates has its hydration enthalpy greater than its lattice enthalpy?

- (A) $SrSO_4$
- (B) $CaSO_4$
- (C) $BeSO_4$
- (D) $BaSO_4$

Solution: (C) $\Delta H_{Hydration} > \Delta H_{Lattice}$

Salt is soluble. $BeSO_4$ is soluble due to high hydration energy of small Be^{2+} ion. K_{sp} for $BeSO_4$ is very high.

11. The number of geometric isomers that can exist for square planar

$[Pt(Cl)(py)(NH_3)(NH_2OH)]^+$ is (py = pyridine):

- (A) 6
- (B) 2
- (C) 3
- (D) 4

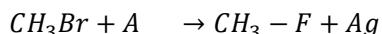
Solution: (C) Complexes with general formula $[Mabcd]^+$ square planar complex can have three isomers.

12. The synthesis of alkyl fluorides is best accomplished by:

- (A) Swarts reaction
- (B) Free radical fluorination
- (C) Sandmeyer's reaction
- (D) Finkelstein reaction

Solution: (A)

Alkyl fluoroiodes is best accomplished by swarts reaction i.e. heating an alkyl chloride/bromide in the presence of metallic fluoride such as AgF , Hg_2F_2 , CoF_2 , SbF_3 .



The reaction of chlorinated hydrocarbons with metallic fluorides to form chlorofluoro hydrocarbons, such as CCl_2F_2 is known as swarts reaction.

13. The intermolecular interaction that is dependent on the inverse cube of distance between the molecules is:

- (A) Hydrogen bond
- (B) Ion-ion interaction
- (C) Ion-dipole interaction
- (D) London force

Solution: (A) $Ion - ion interaction \propto \frac{1}{r^2}$

$$Ion - dipole interaction \propto \frac{1}{r^4}$$

$$London forces \propto \frac{1}{r^6}$$

And $Hydrogen bond \propto \frac{1}{r^3}$

14. In the context of the Hall-Heroult process for the extraction of Al, which of the following statements is false?

- (A) Na_2AlF_6 serves as the electrolyte.
- (B) CO and CO_2 are produced in this process.
- (C) Al_2O_3 is mixed with Ca which lowers the melting point of the mixture and brings conductivity.
- (D) $\overset{3+}{Al}$ is reduced at the cathode to form Al.

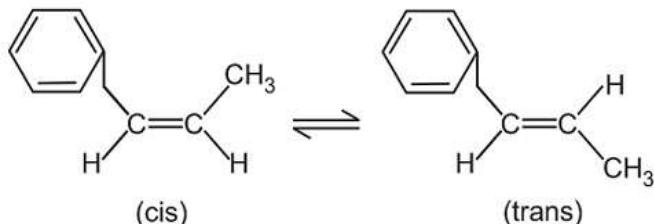
Solution: (A) Hall-Heroult process for extraction of Al. Al_2O_3 is electrolyte Na_3AlF_6 reduces the fusion temperature and provides good conductivity.

15. Which of the following compounds will exhibit geometrical isomerism?

- (A) 1, 1 - Diphenyl - 1 - propane
- (B) 1 - Phenyl - 2 - butene
- (C) 3 - Phenyl - 1 - butane
- (D) 2 - Phenyl - 1 - butane

Solution: (B)

1 - Phenyl - 2 - butene:



16. The ionic radii (\AA) of $3-$, O and F^- are respectively:
 N

- (A) 1.71, 1.36 and 1.40
- (B) 1.36, 1.40 and 1.71
- (C) 1.36, 1.71 and 1.40
- (D) 1.71, 1.40 and 1.36

Solution: (D) As $\frac{Z}{e} \uparrow$ ionic radius decreases for isoelectronic species.

$$3 - \left(\frac{Z}{e} \right) = \frac{7}{10}$$

N

$$2 - \left(\frac{Z}{e} \right) = \frac{8}{10}$$

O

$$- \left(\frac{Z}{e} \right) = \frac{9}{10}$$

F

—
 2 → F
 3 → O
 N

17. From the following statements regarding H_2O_2 , choose the incorrect statement:

- (A) It has to be kept away from dust
- (B) It can act only as an oxidizing agent
- (C) It decomposes on exposure to light
- (D) It has to be stored in plastic or wax lined glass bottles in dark.

Solution: (B) It can act both as oxidizing agent and reducing agent.

18. Higher order (3) reactions are rare due to:

- (A) Loss of active species on collision
- (B) Low probability of simultaneous collision of all the reacting species
- (C) Increase in entropy and activation energy as more molecules are involved
- (D) Shifting of equilibrium towards reactants due to elastic collisions.

Solution: (B) Probability of an event involving more than three molecules in a collision are remote.

19. Match the catalysts to the correct process:

	Catalyst		Process
A.	$TiCl_3$	i.	Wacker process
B.	$PdCl_2$	ii.	Ziegler – Natta polymerization
C.	$CuCl_2$	iii.	Contact process`
D.	V_2O_5	iv.	Deacon's process

- (A) A → iii, B → i, C → ii, D → iv
- (B) A → iii, B → ii, C → iv, D → i
- (C) A → ii, B → i, C → iv, D → iii
- (D) A → ii, B → iii, C → iv, D → i

Solution: (C) The Wacker process originally referred to the oxidation of ethylene to acetaldehyde by oxygen in water in the presence of tetrachloropalladate (II) as the catalyst.

In contact process, Platinum used to be the catalyst for this reaction, however as it is susceptible to reacting with arsenic impurities in the sulphur feedstock, vanadium (V) oxide (V_2O_5) is now preferred.

In Deacon's process, The reaction takes place at about 400 to $450^\circ C$ in the presence of a variety of catalysts, including copper chloride($CuCl_2$).

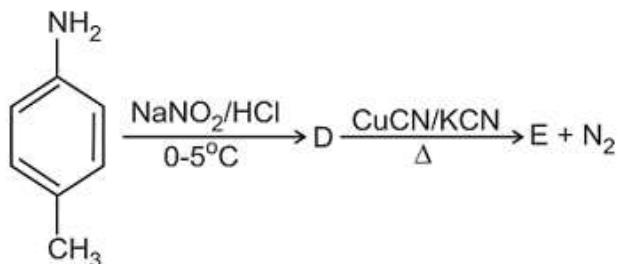
In Ziegler-Natta catalyst catalyst, Homogenous catalysts usually based on complexes of Ti, Zr or Hf used. They are usually used in combination with different organ aluminium co-catalyst.

20. Which one has the highest boiling point?

- (A) *Xe*
- (B) *He*
- (C) *Ne*
- (D) *Kr*

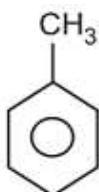
Solution: (A) Due to higher Vander Waal's forces. *Xe* has the highest boiling point.

21. In the reaction,



The product E is:

- (A)



- (B)



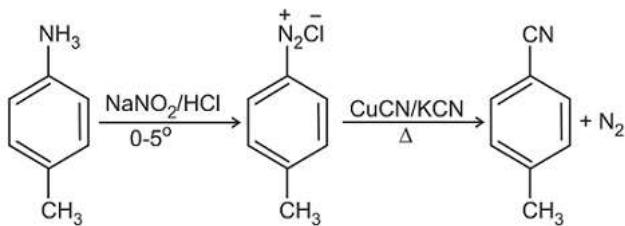
- (C)



- (D)



Solution: (D)

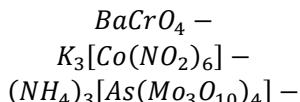


22. Which of the following compounds is not colored yellow?

- (A) $BaCrO_4$
- (B) $Zn_2[Fe(CN)_6]$
- (C) $K_3[Co(NO_2)_6]$
- (D) $(NH_4)_3[As(Mo_3O_{10})_4]$

Solution: (B)

Cyanides not yellow.



23. Sodium metal crystallizes in a body centred cubic lattice with a unit cell edge of 4.29\AA . The radius of sodium atom is approximately:

- (A) 0.93\AA
- (B) 1.86\AA
- (C) 3.22\AA
- (D) 5.72\AA

Solution: (B)

For B.C.C

$$4r = \sqrt{3}a$$

$$r = \frac{\sqrt{3}}{4}a = \frac{1.732}{4} \times 4.29$$

$$1.86\text{\AA}$$

24. The standard Gibbs energy change at 300 K for the reaction $2A \rightleftharpoons B + C$ is 2494.2 J. At a given time, the composition of the reaction mixture is $[A] = \frac{1}{2}$, $[B] = 2$ and $[C] = \frac{1}{2}$. The $K - mol, e = 2.718$
 reaction proceeds in the: $R = 8.314J/$

- (A) Reverse direction because $Q < K_C$
- (B) Forward direction because $Q > K_C$
- (C) Reverse direction because $Q > K_C$
- (D) Forward direction because $Q < K_C$

Solution: (C) $\Delta G^\circ = -RT \in K_C$

$$2494.2 = -8.314 \times 300 \in K_C$$

$$2494.2 = -8.314 \times 300 \times 2.303 \log K_C$$

$$\frac{-2494.2}{2.303 \times 300 \times .314} = -0.44 = \log K_C$$

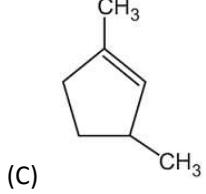
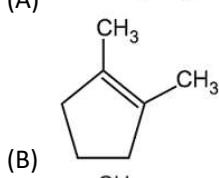
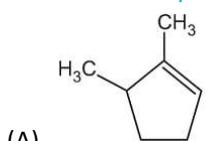
$$\log K_C = -0.44 = 1.56$$

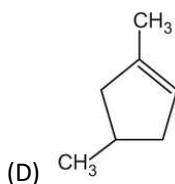
$$K_C = 0.36$$

$$Q_C = \frac{2 \times \frac{1}{2}}{\left(\frac{1}{2}\right)^2} = 4$$

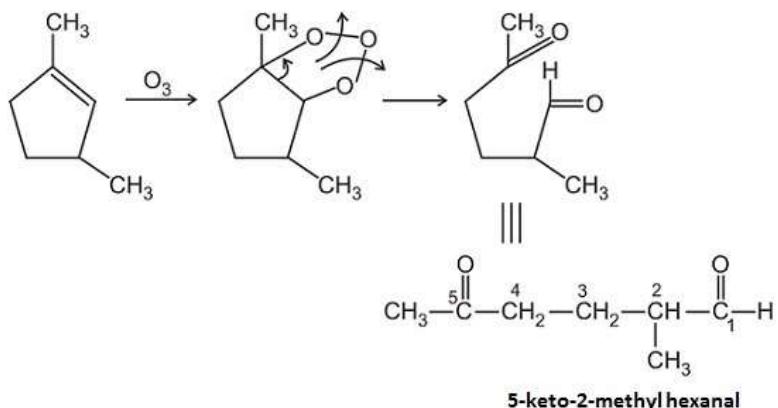
$Q_C > K_C$ reverse direction

25. Which compound would give 5-keto-2-methyl hexanal upon ozonolysis:





Solution: (C)

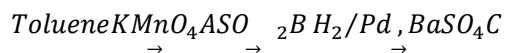


26. Which of the following compounds is not an antacid?

- (A) Rantidine
- (B) Aluminium hydroxide
- (C) Cimetidine
- (D) Phenelzine

Solution: (D) Ranitidine, Cimetidine and metal hydroxides i.e. Aluminum hydroxide can be used as antacid but not phenelzine. Phenelzine is not an antacid. It is an antidepressant.
Antacids are a type of medication that can control the acid levels in stomach. Working of antacids: Antacids counteract (neutralize) the acid in stomach that's used to aid digestion. This helps reduce the symptoms of heartburn and relieves pain.

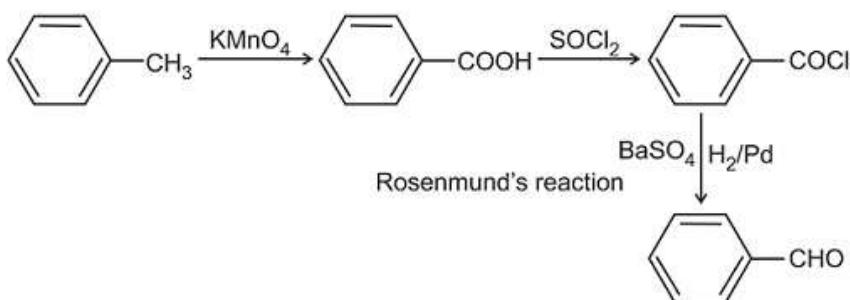
27. In the following sequence of reactions:



The Product C is:

- (A) C_6H_5CHO
- (B) C_6H_5COOH
- (C) $C_6H_5CH_3$
- (D) $C_6H_5CH_2OH$

Solution: (A)



28. Which of the vitamins given below is water soluble?

- (A) Vitamin K
- (B) Vitamin C
- (C) Vitamin D
- (D) Vitamin E

Solution: (B) B complex vitamins and vitamin C are water soluble vitamins that are not stored in the body and must be replaced each day.

29. The color of KMnO_4 is due to:

- (A) $\sigma - \sigma$ transition
- (B) $M \rightarrow L$ charge transfer transition
- (C) $d - d$ transition
- (D) $L \rightarrow M$ charge transfer transition

Solution: (D) Charge transfer from $\frac{2}{O}^-$ to empty d-orbitals of metal ion (Mn^{+7})

30. Two Faraday of electricity is passed through a solution of CuSO_4 . The mass of copper deposited at the cathode is: (Atomic mass of Cu = 63.5 amu)

- (A) 127 g
- (B) 0 g
- (C) 63.5 g
- (D) 2 g

Solution: (C) $2F \equiv 2\text{EqsofCu}$

$$2 \times \frac{63.5}{2} = 63.5 \text{ g}$$

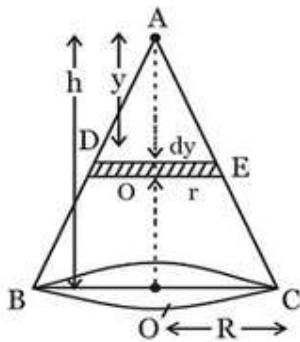
Mathematics

1. Distance of the centre of mass of a solid uniform cone from its vertex is z_0 . If the radius of its base is R and its height is h then z_0 is equal to:

- (A) $\frac{5h}{8}$
- (B) $\frac{3h^2}{8R}$
- (C) $\frac{h^2}{4R}$
- (D) $\frac{3h}{4}$

Answer: (D)

Solution:



Consider an elementary disc of radius r and thickness dy .

If total mass of cone = M and density = ρ

Then mass of elementary disc is $dm = \rho dv = \rho \times \pi r^2 dy$ (1)

In similar Δ 's AOE and $AO'C$

$$\frac{y}{h} = \frac{r}{R} \Rightarrow r = \frac{y}{h} R \quad \dots\dots (2)$$

Put (2) in (1)

$$dm = \rho(\pi) \left(\frac{y}{h} R \right)^2 dy$$

$$dm = \rho \times \frac{\pi R^2}{h^2} y^2 dy$$

\therefore The centre of mass of cone lying on the line AO' at a distance y_{cm} from A can be calculated as

$$\begin{aligned} y_{cm} &= \frac{\int(dm)y}{\int dm} = \frac{\int \rho \pi R^2}{h^2 M} \cdot \frac{y^3 dy}{\int dm} \\ &= \frac{\rho \pi R^2}{h^2 M} \int_0^h y^3 dy \end{aligned}$$

$$\therefore M = \rho \times \frac{1}{3} \pi R^2 h$$

$$\Rightarrow y_{cm} = \frac{\rho \pi R^2}{h^2 \rho \times \pi R^2 h} \times \frac{h^4}{4} = \frac{3h}{4}$$

Topic: Centre of Mass

Difficulty: Easy (embibe predicted Low Weightage)

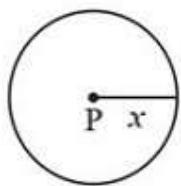
Ideal time: 30

2. A red LED emits light at 0.1 watt uniformly around it. The amplitude of the electric field of the light at a distance of 1 m from the diode is:

- (A) 5.48 V/m
- (B) 7.75 V/m
- (C) 1.73 V/m
- (D) 2.45 V/m

Answer: (D)

Solution:



For a point source of power = P , then intensity at a point at a separation x from the source is

$$I = \frac{\text{Power}}{\text{Area}} = \frac{P}{4\pi x^2}$$

∴ Average intensity of EM wave is given by

$$I = \frac{1}{2} C \epsilon_0 E_0^2$$

$$\Rightarrow E_0 = \sqrt{\frac{2P}{4\pi \epsilon_0 C x^2}}$$

$$\therefore \frac{1}{4\pi \epsilon_0 C} = 9 \times 10^9, P = 0.1 \text{ W}, x = 1 \text{ m}$$

$$C = \text{Speed of light} = 3 \times 10^8 \text{ m/s}$$

$$\Rightarrow E_0 = \sqrt{\frac{2 \times 0.1 \times 9 \times 10^9}{3 \times 10^8 \times 1^2}} = \sqrt{6} = 2.45 \text{ V/m}$$

Topic: Optics

Difficulty: Easy (embibe predicted high weightage)

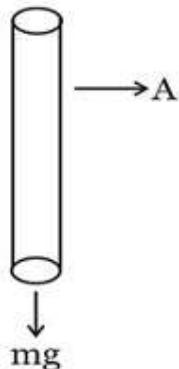
Ideal time: 120

3. A pendulum made of a uniform wire of cross sectional area A has time period T . When additional mass M is added to its bob, the time period changes to T_M . If the Young's modulus of the material of the wire is Y then $\frac{1}{Y}$ is equal to:
 (g = gravitational acceleration)

- (A) $\left[1 - \left(\frac{T_M}{T}\right)^2\right] \frac{A}{Mg}$
 (B) $\left[1 - \left(\frac{T}{T_M}\right)^2\right] \frac{A}{Mg}$
 (C) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$
 (D) $\left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{Mg}{A}$

Answer: (C)

Solution:



Initial length = ℓ

$$\text{Time period } T = 2\pi \sqrt{\frac{\ell}{g}} \quad \dots(i)$$

After suspending mass M,

Young's modulus Y = $\frac{\text{Stress}}{\text{Strain}}$

$$= \frac{F}{A\ell} = \frac{F\ell}{\Delta\ell A}$$

$$\text{Change in length of wire } \Delta\ell = \frac{F\ell}{Ay}$$

$$\text{Now Time period } T_M = 2\pi \sqrt{\frac{\ell + \Delta\ell}{g}} \quad \dots(ii)$$

$$\frac{T}{T_M} = \frac{2\pi \sqrt{\frac{\ell}{g}}}{2\pi \sqrt{\frac{\ell + \Delta\ell}{g}}} \quad \left[\because \frac{i}{ii} \right]$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell + \Delta\ell}$$

$$\frac{T^2}{T_M^2} = \frac{\ell}{\ell + \frac{F\ell}{Ay}} \quad [\text{putting } \Delta\ell \text{ value}]$$

$$\left(\frac{T}{T_M}\right)^2 = \frac{1}{1 + \frac{F}{Ay}}$$

$$1 + \frac{F}{Ay} = \left(\frac{T_M}{T}\right)^2$$

$$\frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{F}$$

$$F = mg$$

$$\Rightarrow \frac{1}{y} = \left[\left(\frac{T_M}{T}\right)^2 - 1\right] \frac{A}{Mg}$$

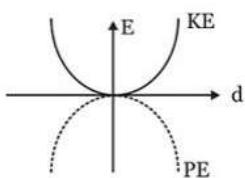
Topic: Simple Harmonic Motion

Difficulty: Difficult (embibe predicted high weightage)

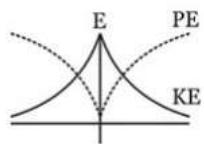
Ideal time: 120

4. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d . which one of the following represents these correctly?
 (Graphs are schematic and not drawn to scale)

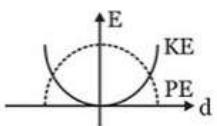
(A)



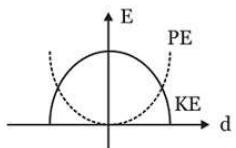
(B)



(C)



(D)



Answer: (D)

Solution:

For simple pendulum performing simple harmonic motion, displacement

$$y = A \sin \omega t$$

$$\text{Velocity } \frac{dy}{dt} = V = \omega A \cos \omega t$$

$$= A\omega \sqrt{1 - \sin^2 \omega t}$$

$$= A\omega \sqrt{1 - \frac{y^2}{A^2}}$$

$$= \omega \sqrt{A^2 - y^2}$$

$$\text{Kinetic energy} = \frac{1}{2} mv^2$$

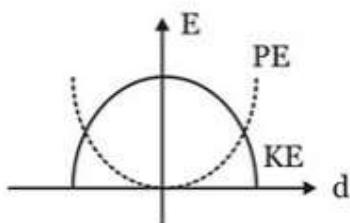
$$= \frac{1}{2} \times m \times \omega^2 (A^2 - y^2)$$

$$\text{at } y = A \text{ (extreme positions)}$$

$$\text{Kinetic energy} = \frac{1}{2} \omega^2 m (A^2 - A^2) = 0$$

$$\text{Similarly potential energy} = \frac{1}{2} m \omega^2 y^2$$

On plotting graphs of potential energy & Kinetic energy



Topic: Simple Harmonic Motion

Difficulty: Easy (embibe predicted high weightage)

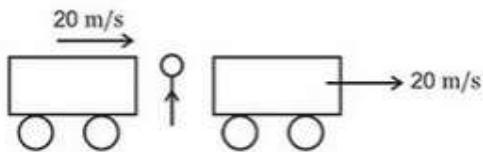
Ideal time: 60

5. A train is moving on a straight track with speed 20ms^{-1} . It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound 320ms^{-1}) close to:

- (A) 18%
- (B) 24%
- (C) 6%
- (D) 12%

Answer: (D)

Solution:



Before $f_0 = 1000 \text{ Hz}$

$$f' = \left(\frac{v}{v - v_s} \right) \times f_0$$

$$= \left(\frac{320}{320 - 20} \right) \times f_0$$

$$= \left(\frac{320}{300} \right) \times f_0$$

$$= \frac{16f_0}{15}$$

$$f'' = \left(\frac{v}{v + v_s} \right) \times f_0$$

$$f'' = \left(\frac{320}{320 + 20} \right) f_0$$

$$= \left(\frac{320}{340} \right) f_0$$

$$= \left(\frac{16}{17} \right) f_0$$

Change in frequency

$$= \left(\frac{16}{15} - \frac{16}{17} \right) f_0$$

\therefore Percentage change in frequency

$$= \frac{\left(\frac{16}{15} - \frac{16}{17} \right) f_0}{f_0} \times 100 \approx 12\% \text{ nearly}$$

Topic: Wave & Sound

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 200



JEE MAIN 2015

Detailed Solution - Offline 4th April



6. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is $2.5 \times 10^{-4} \text{ ms}^{-1}$. If the electron density in the wire is $8 \times 10^{28} \text{ m}^{-3}$, the resistivity of the material is close to:

- (A) $1.6 \times 10^{-6} \Omega \text{m}$
(B) $1.6 \times 10^{-5} \Omega \text{m}$
(C) $1.6 \times 10^{-8} \Omega \text{m}$
(D) $1.6 \times 10^{-7} \Omega \text{m}$

Answer: (B)

Solution:

$$\text{Potential difference} = 5\text{V}$$

$$\text{length} = 0.1\text{m} = \ell$$

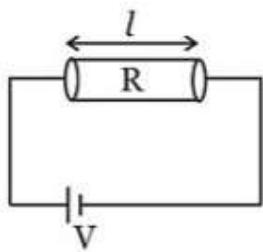
$$\text{Electron speed} = \text{drift velocity } v_d = 2.5 \times 10^{-4} \text{ m/s}$$

$$\text{electron density (n)} = 8 \times 10^{28} \text{ m}^{-3}$$

$$\text{charge on each electron (e)} = 1.6 \times 10^{-19} \text{ C}$$

$$\text{We know } i = n A e v_d \quad \dots \text{(i)}$$

$$\text{And } v = iR \quad \dots \text{(ii)}$$



Resistance R is also equal $\frac{\rho l}{A}$

$$R = \frac{\rho l}{A}$$

$$\rho = \frac{AR}{l} \quad [\rho = \text{Resistivity}]$$

$$= \frac{A}{l} \times \frac{v}{i} \quad [\text{from (ii)}]$$

$$= \frac{Av}{\ell \times nA \cdot ev_d} \quad [\text{from (i)}]$$

$$= \frac{v}{\ell \times n \times e \times v_d}$$

$$= \frac{5}{0.1 \times 8 \times 10^{28} \times 1.6 \times 10^{-19} \times 2.5 \times 10^{-4}}$$

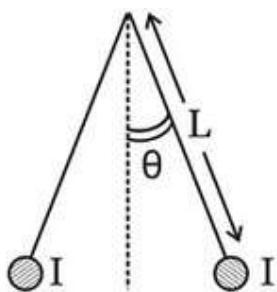
$$= 0.16 \times 10^{-4} \Omega \text{m} = 1.6 \times 10^{-5} \Omega \text{m}$$

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 120

7.

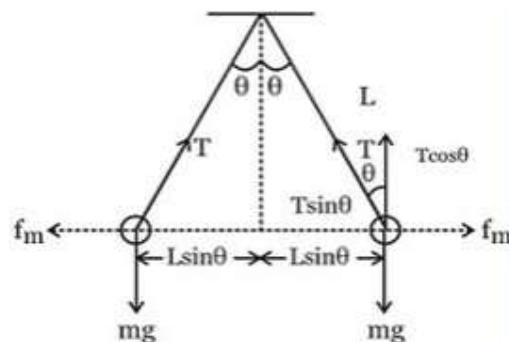


Two long current carrying thin wires, both with current I , are held by insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle ' θ ' with the vertical. If wires have mass λ per unit length then the value of I is:
(g = gravitational acceleration)

- (A) $2\sqrt{\frac{\pi g L}{\mu_0} \tan \theta}$
 (B) $\sqrt{\frac{\pi \lambda g L}{\mu_0} \tan \theta}$
 (C) $\sin \theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos \theta}}$
 (D) $2 \sin \theta \sqrt{\frac{\pi \lambda g L}{\mu_0 \cos \theta}}$

Answer: (D)

Solution:



Two wires will repel each other due to current in the same direction and due to magnetic force.

\therefore magnetic force per unit length is

$$\frac{df}{dl} = \frac{\mu_0 I^2}{2\pi(2L \sin \theta)} = \frac{\mu_0 I^2}{4\pi L \sin \theta}$$

and mass per unit length of each mix = $\frac{dm}{dl} = \lambda$

So, magnetic force on total length ℓ' of the mix is $f_m = \frac{\mu_0 I^2 \ell'}{4\pi L \sin \theta}$

and weight = $\lambda \ell' g$

By equilibrium of mix,

$$T \sin\theta = f_m \text{ and } T \cos\theta = mg$$

$$\Rightarrow \frac{T \sin\theta}{T \cos\theta} = \frac{f_m}{mg} \Rightarrow f_m = mg \tan \theta$$

$$\Rightarrow \frac{\mu_0 I^2}{4\pi L \sin\theta} \ell' = mg \frac{\sin\theta}{\cos\theta} = \lambda \ell' g \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow \frac{\mu_0 I^2}{4\pi L \sin\theta} \ell' = \lambda \ell' g \frac{\sin\theta}{\cos\theta}$$

$$\Rightarrow I^2 = \frac{\lambda g \pi L}{\mu_0 \cos\theta} 4 \sin^2 \theta$$

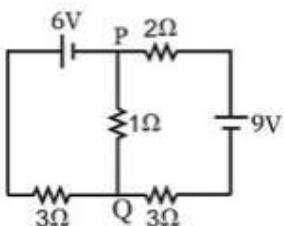
$$I = 2 \sin\theta \sqrt{\frac{\lambda \pi g L}{\mu_0 \cos\theta}}$$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

8.

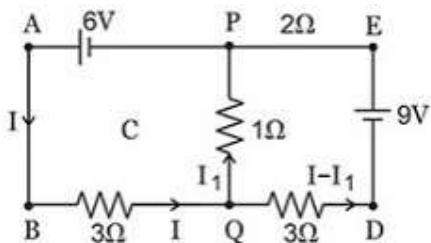


In the circuit shown, the current in the 1Ω resistor is:

- (A) 0.13 A, from Q to P
- (B) 0.13 A, from P to Q
- (C) 1.3 A, from P to Q
- (D) 0 A

Answer: (A)

Solution:



The distribution of current according to Kirchhoff's first law is as shown in the circuit.
By Kirchoff's second law (voltage rule)

In loop APQBA using sign curve line

$$6 - 3I - I_1 = 0$$

$$\Rightarrow 3I + I_1 = 6 \quad \dots(i)$$

In loop QD & PQ

$$\Rightarrow -3(I - I_1) + 9 - 2(I - I_1) + 1 \times I_1 = 0$$

$$\Rightarrow 9 - 5(I - I_1) + I_1 = 0$$

$$\Rightarrow 9 + 6I_1 - 5I = 0$$

$$\Rightarrow 5I - 6I_1 = 9 \quad \dots(ii)$$

(Multiplying (i) by 5) - (Multiplying (ii) by 3)

$$\Rightarrow 15I + 5I_1 = 30$$

$$15I - 18I_1 = 27$$

$$- + -$$

$$23I_1 = 3 \Rightarrow I_1 = \frac{3}{23} \text{ A} = 0.13 \text{ A}$$

+ve sign of I_1 shows that current 0.13 A flows from Q to P.

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

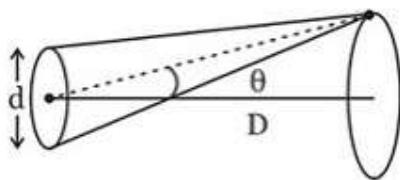
Ideal time: 90

9. Assuming human pupil to have a radius of 0.25 cm and a comfortable viewing distance of 25 cm, the minimum separation between two objects that human eye can resolve at 500 nm wavelength is:

- (A) $100\mu m$
- (B) $300\mu m$
- (C) $1\mu m$
- (D) $30\mu m$

Answer: (D)

By Fraunhofer diffraction through a circular aperture $\theta = \frac{1.22\lambda}{d}$



$$D = \text{diameter of pupil} = 2 \times 0.25 = 0.5 \text{ cm}$$

$$\lambda = 500 \text{ nm}$$

First dark ring is formed by the light diffracted from the hole at an angle θ with the axis

Viewing distance $D = 25 \text{ cm}$

\therefore minimum separation between

$$2 \text{ objects} = D\theta$$

$$= \frac{25 \times 10^{-2} \times 1.22 \times 500 \times 10^{-9}}{5 \times 10^{-1}}$$

$$= 30 \times 10^{-6} \text{ m}$$

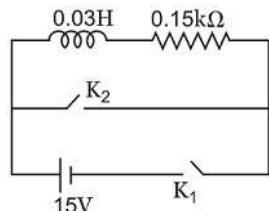
$$= 30 \mu \text{m}$$

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

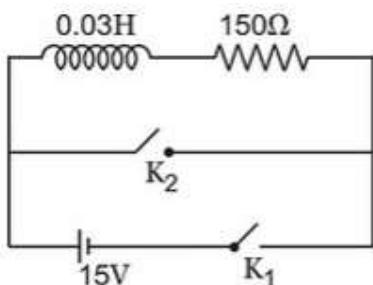
10. An inductor ($L = 0.03H$) and a resistor ($R = 0.15k\Omega$) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at $t = 0$, K_1 is opened and key K_2 is closed simultaneously. At $t = 1ms$, the current in the circuit will be : ($e^5 \approx 150$)



- (A) 6.7 mA
- (B) 0.67 mA**
- (C) 100 mA
- (D) 67 mA

Answer: (B)

Solution:



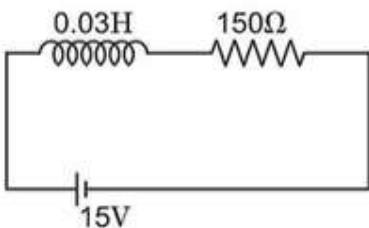
$$L = 0.03H$$

$$R = 0.15k\Omega = 150\Omega$$

$$(e^5 \approx 150 \text{ given})$$

$$E = 15V$$

Case I: K_1 is closed for long time



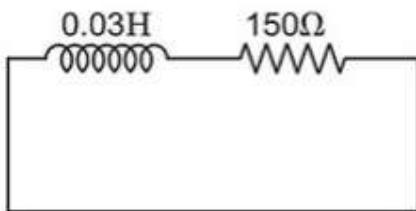
for long time, inductor acts as a conducting wire.

$$\Rightarrow \text{current in the circuit} = \frac{V}{R}$$

$$= \frac{15}{150}$$

$$i_0 = 0.1 \text{ A}$$

Case II: K₁ is open and K₂ is closed



Current in the circuit

$$i = i_0 e^{-\frac{t}{\tau}}; \tau = \frac{L}{R}$$

After t = 1ms = 10⁻³s

$$i = i_0 e^{-\left(\frac{10^{-3} \times 150}{3 \times 10^{-2}}\right)}$$

$$= 0.1e^{-\frac{15}{3}}$$

$$= 0.1 \cdot \frac{1}{e^5} = \frac{0.1}{150}$$

$$= 6.67 \times 10^{-4}$$

$$= 0.67 \times 10^{-3} \text{ A}$$

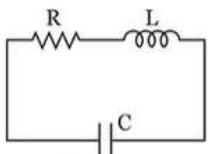
$$= 0.67 \text{ mA}$$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

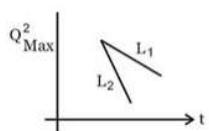
Ideal time: 120

11. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below:

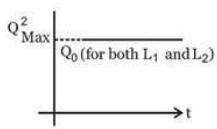


If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

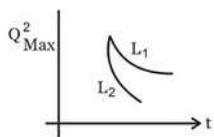
(A)



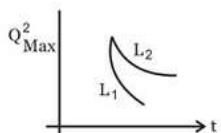
(B)



(C)



(D)



Answer: (C)

Solution:

Comparing to damped pendulum

We write

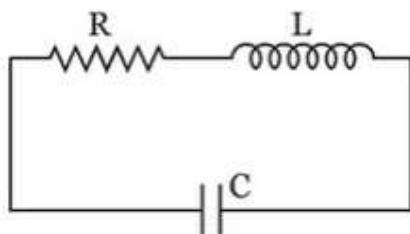
$$m \frac{dv}{dt} = -kx - bv; bv \text{ is resistive force}$$

$$\Rightarrow \text{Amplitude } A = A_0 e^{-\frac{bt}{2m}}$$

Comparing results, we can write

$$\frac{q}{c} = +L \frac{di}{dt} + iR$$

as charge decreasing



$$\frac{q}{c} = L \frac{d^2 q}{dt^2} - \frac{dq}{dt} R$$

$$\Rightarrow A = q; R = b, m = L$$

$$q = q_0 e^{-\frac{Rt}{2L}}$$

$$q^2 = q_0^2 e^{-\frac{Rt}{L}}$$

$$q^2 = q_0^2 e^{-\frac{Rt}{L}}$$

Exponentially decreasing function more the 'L' losses will $\left(\frac{R}{L}\right)$ and more will be $e^{-(\frac{Rt}{L})t}$

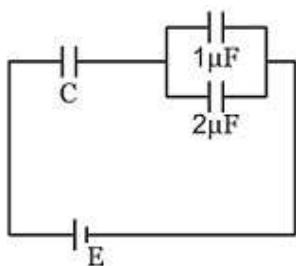
L_1 graph has higher values than L_2

Topic: Magnetism

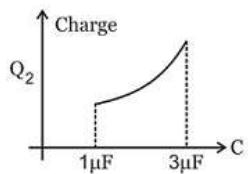
Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 150

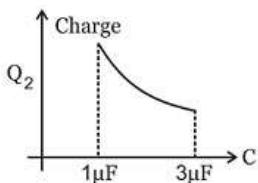
12. In the given circuit, charge Q_2 on the $2\mu F$ capacitor changes as C is varied from $1\mu F$ to $3\mu F$. Q_2 as a function of ' C ' is given properly by: (figure are drawn schematically and are not to scale)



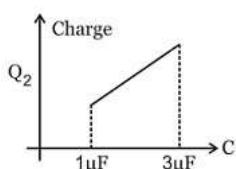
(A)



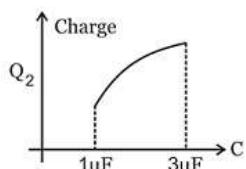
(B)



(C)

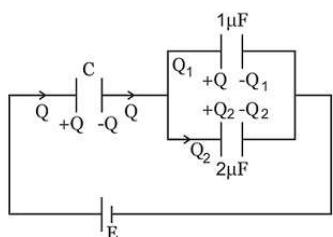


(D)

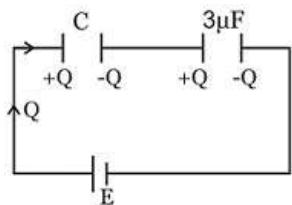


Answer: (D)

Solution:



$\because 1 \wedge 2 \mu F$ are in parallel.



∴ Equivalent capacitance of the series combination is

$$C_{eq} \text{ is } \frac{3c}{c+3}$$

$$\text{So total charge supplied by battery is } Q = C_{eq} = \frac{3CE}{c+3}$$

∴ Potential difference across parallel combination of $1\mu F$ ad $2\mu F$ is

$$\Delta V = \frac{Q}{3} = \frac{CE}{c+3}$$

So charge on $2\mu F$ capacitor is

$$Q_2 = C_2 \Delta V = \frac{2CE}{c+3}$$

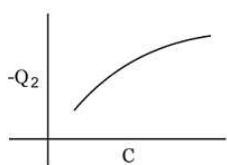
$$\Rightarrow \frac{Q_2}{2E} = \frac{C}{c+3} \Rightarrow \frac{Q_2}{2E} = \frac{c+3-3}{c+3}$$

$$\Rightarrow \frac{Q_2}{2E} = 1 - \frac{3}{c+3} \Rightarrow \left(\frac{Q_2}{2E} - 1 \right) = \frac{-3}{c+3}$$

$$\Rightarrow (Q_2 - 2E)(C + 3) = -6E$$

Which is of the form $(y - \alpha)(x + \beta) < 0$

So the graph in hyperbola. With down ward curve line. i.e



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

13. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:

- (A) $\frac{4MR^2}{9\sqrt{3}\pi}$
- (B) $\frac{4MR^2}{3\sqrt{3}\pi}$
- (C) $\frac{MR^2}{32\sqrt{2}\pi}$
- (D) $\frac{MR^2}{16\sqrt{2}\pi}$

Answer: (A)

Solution: Let a be length of cube for cube with maximum possible volume diagonal length = $2R$

$$\Rightarrow \sqrt{3}a = 2R \Rightarrow a = \frac{2R}{\sqrt{3}}$$

As densities of sphere and cube are equal. Let M' be mass of cube

$$\frac{\frac{M}{4\pi R^3}}{\frac{1}{3}} = \frac{M'}{a^3}$$

$$M' = \frac{3Ma^3}{4\pi R^3}$$

Moment of inertial of cube about an axis passing through centre

$$\frac{M'(2a^2)}{12}$$

$$\frac{3Ma^3}{4\pi R^3} \times \frac{2a^2}{12}$$

$$\frac{Ma^5}{8\pi R^3}$$

$$a = \frac{2}{\sqrt{3}}R$$

$$\frac{M \times 32R^5}{8\pi \times 9\sqrt{3}R^3}$$

$$\frac{4MR^2}{9\sqrt{3}\pi}$$

Topic: Rotational Mechanics

Difficulty: Moderate (embibe predicted Low Weightage)

Idal time: 300

14. The period of oscillation of a simple pendulum is $T = 2\pi \sqrt{\frac{L}{g}}$. Measured value of L is 20.0 cm known to 1 mm accuracy and time for 100 oscillations of the pendulum is found to be 90s using a wrist watch of 1s resolution. The accuracy in the determination of g is:

- (A) 1%
- (B) 5%
- (C) 2%
- (D) 3%**

Answer: (D)

$$\text{Solution: } \therefore T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow g = 4\pi^2 \frac{L}{T^2}$$

\therefore Error in g can be calculated as

$$\frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta T}{T}$$

\therefore Total time for n oscillation is $t = nT$ where T= time for oscillation.

$$\begin{aligned} & \Rightarrow \frac{\Delta t}{t} = \frac{\Delta T}{T} \\ & \Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + \frac{2\Delta t}{t} \end{aligned}$$

Given that $\Delta L = 1\text{mm} = 10^{-3}\text{m}$, $L = 20 \times 10^{-2}\text{m}$

$$\Delta t = 1\text{s}, t = 90\text{s}$$

error $\in g$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t} \right) \times 100$$

$$\frac{\Delta g}{g} \times 100 = \left(\frac{\Delta L}{L} + \frac{2\Delta t}{t} \right) \times 100$$

$$\left(\frac{10^{-3}}{20 \times 10^{-2}} + \frac{2 \times 1}{90} \right) \times 100$$

$$\frac{1}{2} + \frac{20}{9}$$

$$0.5 + 2.22$$

$\cong 3$

Topic: Unit & Dimensions

Difficulty: Easy (embibe predicted easy to score)

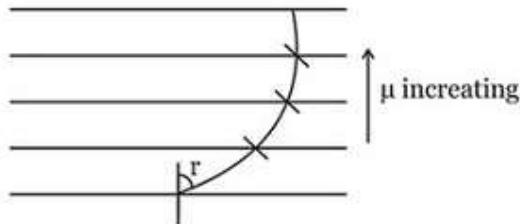
Ideal time: 90

15. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam:

- (A) Bends downwards
- (B) Bends upwards**
- (C) Becomes narrower
- (D) Goes horizontally without any deflection

Answer: (B)

Solution: Consider air layers with increasing refractive index.



At critical angle it will bend upwards at interface. This process continues at each layer, and light ray bends upwards continuously.

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 60

16. A signal of 5 kHz frequency is amplitude modulated on a carrier wave of frequency 2 MHz. The frequency of the resultant signal is/are:

- (A) 2005 kHz, 2000 kHz and 1995 kHz**
- (B) 2000 kHz and 1995 kHz
- (C) 2 MHz only
- (D) 2005 kHz, and 1995 kHz

Answer: (A)

Solution:

$$\text{Frequency of single wave} = 5\text{kHz} = f$$

$$\text{Carrier wave frequency} = 2\text{MHz}$$

$$= 2000 \text{ kHz} = f_c$$

Resultant signal maximum frequency

$$= f + f_c$$

$$= 5 + 2000 \text{ kHz}$$

$$= 2005 \text{ kHz}$$

Resultant signal minimum frequency

$$= f_c = f$$

$$= 2000 - 5\text{kHz}$$

$$= 1995 \text{ kHz}$$

Topic: Communication Systems

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 60

17. A solid body of constant heat capacity $1J/\text{ }^\circ\text{C}$ is being heated by keeping it in contact with reservoirs in two ways:

- (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
- (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat

In both the cases body is brought from initial temperature $100\text{ }^\circ\text{C}$ to final temperature $200\text{ }^\circ\text{C}$. Entropy change of the body in the two cases respectively is:



JEE MAIN 2015

Detailed Solution - Offline 4th April



- (A) $\ln 2, 2\ln 2$
- (B) $2\ln 2, 8\ln 2$
- (C) $\ln 2, 4\ln 2$
- (D) $\ln 2, \ln 2$

Answer: (D)

Solution:

$$\text{Change in entropy } ds = \frac{dQ}{T}$$

$$\Delta Q = \text{heat supplied} = C \Delta T$$

$$dQ = CdT$$

$$ds = \frac{CdT}{T}$$

Integrating both sides

$$\int_{S_i}^{S_f} ds = C \int_{T_i}^{T_f} \frac{dT}{T}$$

$$S_f - S_i = \Delta S = C \cdot \ln T \Big|_{100}^{200}$$

$$= C[\ln 200 - \ln 100]$$

$$\Delta S = C \ln 2$$

$$C = 1 \text{ J}/^{\circ}\text{C}$$

$$\Rightarrow \Delta S = \ln 2$$

Entropy change is same for both cases as C is constant, and temperature change (i.e. from 100 to 200) is same.

Topic: Heat & Thermodynamics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

18. Consider a spherical shell of radius R at temperature T. the black body radiation inside it can be considered as an ideal gas of photons with internal energy per unit volume

$u = \frac{U}{V} \propto T^4$ and pressure $p = \frac{1}{3} \left(\frac{U}{V} \right)$. If the shell now undergoes an adiabatic expansion the relation between T and R is:

- (A) $T \propto \frac{1}{R}$
- (B) $T \propto \frac{1}{R^3}$
- (C) $T \propto e^{-R}$
- (D) $T \propto e^{-3R}$

Answer: (A)

Solution: \because in an adiabatic process.

$$dQ = 0$$

So by first law of thermodynamics

$$\begin{aligned} dQ &= dU + d \\ \Rightarrow 0 &= dU + d \\ \Rightarrow dW &= -dU \\ \therefore dW &= PdV \end{aligned}$$

$$\Rightarrow PdV \quad \dots \text{(i)}$$

$$\text{Given that } \frac{U}{V} \propto T^4 \Rightarrow U = kVT^4$$

$$\Rightarrow dU = k(VT^4) = K(T^4dV + 4T^3VdT)$$

$$\text{Also, } P = \frac{1}{3} \frac{U}{V} = \frac{1}{3} \frac{kVT^4}{V} = \frac{KT^4}{3}$$

Putting these values in equation

$$\begin{aligned} \Rightarrow \frac{KT^4}{3} dV &= -k(T^4dV + 4T^3VdT) \\ \Rightarrow \frac{TdV}{3} &= -TdV - 4VdT \\ \Rightarrow \frac{4T}{3} dV &= -4VdT \\ \Rightarrow \frac{\frac{1}{3} dV}{V} &= \frac{-dT}{T} \end{aligned}$$

$$\Rightarrow \frac{1}{3} \ln V = -\ln T \Rightarrow \ln V = l^{-3}$$

$$\Rightarrow VT^3 = \text{constant}$$

$$\frac{4}{3}\pi R^3 T^3 = \text{constant}$$

$$RT = \text{constant}$$

$$\Rightarrow T \propto \frac{1}{R}$$

Topic: Heat & Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

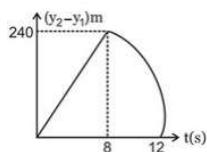
Ideal time: 120

19. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first?

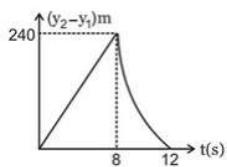
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10 \text{ ms}^{-2}$)

(the figure are schematic and not drawn to scale)

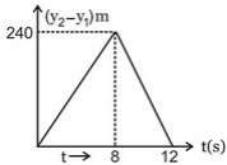
(A)



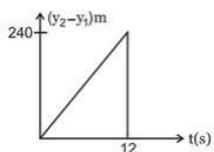
(B)



(C)



(D)



Answer: (A)

$$\text{Solution: } S_1 = 10t - \frac{1}{2}gt^2$$

$$\text{When } S_1 = -240$$

$$\Rightarrow -240 = 10t - 5t^2$$

$$\Rightarrow t = 8s$$

So at $t = 8$ seconds first stone will reach ground

$$S_2 = 20t - \frac{1}{2}gt^2$$

Till $t = 8$ seconds

$$S_2 - S_1 = 30t$$

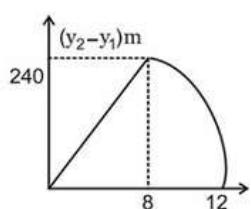
But after 8 second S_1 is constant -240

Relative to stone $t_1 > 8$ seconds displacements of stone 2 $S_2 + 240$

$$\Rightarrow S_2 + 240 = 20t - \frac{1}{2}gt^2$$

And at $t = 12$ seconds stone will reach ground

The corresponding graph of relative position of second stone w.r.t. first is



Topic: Kinematics

Difficulty: Moderate (Embibe predicted high weightage)

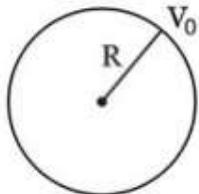
Ideal time: 240

20. A uniformly charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potential $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then

- (A) $R_1 = 0$ and $R_2 < (R_4 - R_3)$
- (B) $2R < R_4$
- (C) $R_1 = 0$ and $R_2 > (R_4 - R_3)$
- (D) $R_1 \neq 0$ and $(R_2 - R_1) > R_4 - R_3$

Answer: (A)

Solution:



Potential for uniformly charged solid sphere

$$v = \frac{1}{4\pi\epsilon_0} \frac{Q}{r} \quad \text{outside i.e } r > R$$

$$v = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \quad \text{on the surface}$$

$$v = \frac{1}{4\pi\epsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{r^2}{R^2} \right] \quad \text{inside i.e. } r < R$$

Clearly potential is decreasing with r .

$$\Rightarrow \frac{3v_0}{2}, \frac{5v_0}{4} \text{ are inside potentials } [\because r > v_0]$$

$\frac{3v_0}{4}, \frac{v_0}{4}$ are outside potentials [$\because < v_0$]

$$\text{To get } R_1 : \frac{3v_0}{2} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \right]$$

$$v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R}$$

$$\frac{3}{2 \times 4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \right]$$

$$\frac{3}{2} = \frac{3}{2} - \frac{1}{2} \frac{R_1^2}{R^2} \Rightarrow R_1 = 0$$

$$\text{To get } R_2 : \frac{5}{4} v_0 = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2} \right]$$

$$\frac{5}{4} \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} \left[\frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2} \right]$$

$$\frac{5}{4} = \frac{3}{2} - \frac{1}{2} \frac{R_2^2}{R^2}$$

$$\frac{1}{2} \frac{R_2^2}{R^2} = \frac{1}{4}$$

$$R_2^2 = \frac{R^2}{2}$$

$$R_2 = \frac{R}{\sqrt{2}}$$

$$\text{To get } R_3 : \frac{3v_0}{4} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}$$

$$\frac{3}{4} \frac{1}{4\pi\varepsilon_0} \frac{Q}{R} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{R_3}$$

$$\frac{3}{4R} = \frac{1}{R_3}$$

$$R_3 = \frac{4}{3} R$$

$$\frac{1}{4} \times \frac{1}{4\pi\epsilon_0} \frac{Q}{R} = \frac{1}{4\pi\epsilon_0} \frac{Q}{R_4}$$

$$R_4 = 4R$$

$$R_4 - R_3 = 4R - \frac{4R}{3} = \frac{8R}{3} > R_2$$

$$R_1 = 0 \text{ and } R_2 < (R_4 - R_3)$$

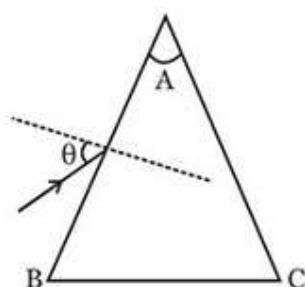
Both options are correct.

Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 210

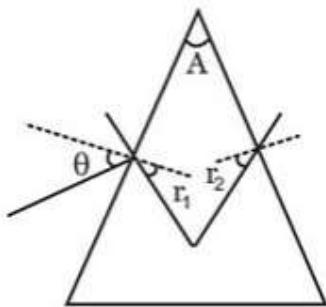
21. Monochromatic light is incident on a glass prism of angle A. If the refractive index of the material of the prism is μ , a ray, incident at an angle θ , on the face AB would get transmitted through the face AC of the prism provided:



- (A) $\theta > \cos^{-1} \left[\mu \sin \left(A + \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (B) $\theta < \cos^{-1} \left[\mu \sin \left(A + \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (C) $\theta > \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$
- (D) $\theta < \sin^{-1} \left[\mu \sin \left(A - \sin^{-1} \left(\frac{1}{\mu} \right) \right) \right]$

Answer: (C)

Solution:



For emergence $r_2 <$ critical angle

$$\Rightarrow r_2 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$A = r_1 + r_2$$

$$\Rightarrow A - r_1 = r_2$$

$$\Rightarrow A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow A - r_1 < \sin^{-1}\left(\frac{1}{\mu}\right)$$

$$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < r_1$$

\therefore By shells law

$$\sin \theta = \mu \sin r_1$$

$$\Rightarrow r_1 = \sin^{-1}\left(\frac{\sin \theta}{\mu}\right)$$

$$\Rightarrow A - \sin^{-1}\left(\frac{1}{\mu}\right) < \sin^{-1}\left(\frac{\sin \theta}{\mu}\right)$$

$$\Rightarrow \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \frac{\sin \theta}{\mu}$$

$$\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right) < \sin \theta$$

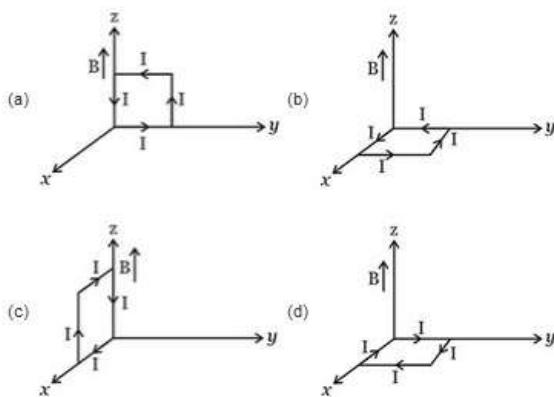
$$\Rightarrow \theta > \sin^{-1}\left(\mu \sin\left(A - \sin^{-1}\left(\frac{1}{\mu}\right)\right)\right)$$

Topic: Optics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 240

22. A rectangular loop of sides 10 cm and 5cm carrying a current I of 12 A is placed in different orientations as shown in the figure below:



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium?

- (A) (b) and (d), respectively
- (B) (b) and (c), respectively
- (C) (a) and (b), respectively
- (D) (a) and (c), respectively

Answer: (A)

Solution: For a magnetic dipole placed in a uniform magnetic field the torque is given by $\vec{\tau} = \vec{M} \times \vec{B}$ and potential energy U is given as

$$U = -\vec{M} \cdot \vec{B} = -MB\cos\theta$$

When \vec{M} is in the same direction as \vec{B} then $\vec{\tau} = 0$ and U is min = - MB as $\theta = 0^\circ$

\Rightarrow Stable equilibrium is (b). and when \vec{M} then $\vec{\tau} = 0$ and U is max = + MB

As $\theta = 180^\circ$

Unstable equilibrium in (d).

Topic: Electrostatics

Difficulty: Easy (embibe predicted high weightage)

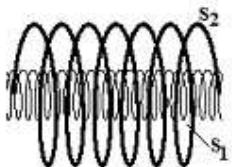
Ideal time: 30

23. Two coaxial solenoids of different radii carry current I in the same direction. Let \vec{F}_1 be the magnetic force on the inner solenoid due to the outer one and \vec{F}_2 be the magnetic force on the outer solenoid due to the inner one. Then:

- (A) \vec{F}_1 is radially inwards and $\vec{F}_2 = 0$
- (B) \vec{F}_1 is radially outwards and $\vec{F}_2 = 0$
- (C) $\vec{F}_1 = \vec{F}_2 = 0$**
- (D) \vec{F}_1 is radially inwards and \vec{F}_2 is radially outwards

Answer: (C)

Solution:



S_2 is solenoid with more radius than S_1 field because of S_1 on S_2 is 0

\therefore force on S_2 by $S_1 = 0$

In the uniform field of S_2 S_1 behaves as a magnetic dipole

\therefore force on S_1 by S_2 is zero because force on both poles are equal in magnitude and opposite in direction.

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 120

24. A particle of mass m moving in the x direction with speed $2v$ is hit by another particle of mass $2m$ moving in the y direction with speed v . If the collision is perfectly inelastic, the percentage loss in the energy during the collision is close to:

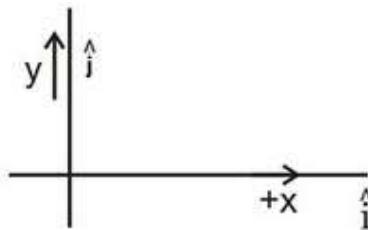
- (A) 56%
- (B) 62%
- (C) 44%
- (D) 50%

Answer: (A)

Solution:

The initial momentum of system is $\vec{P}_i = m(2V)\hat{i} + (2m)v\hat{j}$

According to question as



On perfectly inelastic collision the particles stick to each other so.

$$\vec{P}_f = 3m\vec{V}_f$$

By conservation of linear momentum principle

$$\vec{P}_f = \vec{P}_i \Rightarrow 3m\vec{V}_f = m2V\hat{i} + 2mV\hat{j}$$

$$\Rightarrow \vec{V}_f = \frac{2V}{3}(\hat{i} + \hat{j}) \Rightarrow V_f = \frac{2\sqrt{2}}{3}V$$

\therefore loss in KE of system $K_{initial} - K_{final}$

$$\frac{1}{2}m(2V)^2 + \frac{1}{2}(2m)V^2 - \frac{1}{2}(3m)\left(\frac{2\sqrt{2}V}{3}\right)^2$$

$$2mV^2 + mV^2 - \frac{4}{3}mV^2 = 3mV^2 - \frac{4mV^2}{3}$$

$$\frac{5}{3}mV^2$$

$$\% \text{ change in KE } 100 \times \frac{\Delta K}{K_i} = \frac{\frac{5}{3}mV^2}{3mV^2} = \frac{5}{9} \times 100$$



JEE MAIN 2015

Detailed Solution - Offline 4th April



$$\frac{500}{9} = 56$$

Topic: Magnetism

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

25. Consider an ideal gas confined in an isolated closed chamber. As the gas undergoes an adiabatic expansion, the average time of collision between molecules increase as V^q , where V is the volume of the gas. The value of q is:

$$\left(\gamma = \frac{C_p}{C_v} \right)$$

- (A) $\frac{\gamma+1}{2}$
- (B) $\frac{\gamma-1}{2}$
- (C) $\frac{3\gamma+5}{6}$
- (D) $\frac{3\gamma-5}{6}$

Answer: (A)

Average time of collision

$$t = \frac{\text{mean free path}(\lambda)}{\text{average speed } (v)}$$

$$t \propto \frac{\lambda}{v}$$

$$\therefore \lambda \propto \frac{1}{\text{no. of molecules per unit volume}}$$

$$\lambda \propto \frac{1}{\left(\frac{N}{V}\right)}$$

$$\Rightarrow \lambda \propto V$$

$$\text{And } \bar{v} \propto \sqrt{T}$$

$$\Rightarrow \bar{v} \propto \sqrt{PV}$$

$$\therefore P \propto V^{-\gamma}$$

for adiabatic process where γ = adiabatic coefficient

$$\Rightarrow \bar{v} \propto \sqrt{V^{-\gamma} V}$$

$$\Rightarrow \bar{v} \propto V^{\frac{1-\gamma}{2}}$$

So average time

$$\therefore t_{avg} \propto \frac{V}{V^{\frac{1-\gamma}{2}}}$$

$$t_{avg} \propto V^{1 - \left(1 - \frac{\gamma}{2}\right)}$$

$$t_{avg} \propto V^{\frac{1+\gamma}{2}}$$

$$\therefore q = \frac{1+\gamma}{2}$$

Topic: Heat & Thermodynamics

Difficulty: Difficult (embibe predicted high weightage)

Ideal time: 120

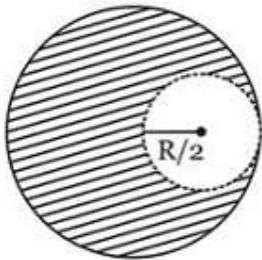
26. From a solid sphere of mass M and radius R , a spherical portion of radius $\frac{R}{2}$ is removed, as shown in the figure. Taking gravitational potential $V = 0$ and $r = \infty$, the potential at the center of the cavity thus formed is:
 $(G = \text{gravitational constant})$



- (A) $\frac{-2GM}{3R}$
- (B) $\frac{-2GM}{R}$
- (C) $\frac{-G}{2R}$
- (D) $\frac{-GM}{R}$

Answer: (D)

Solution:



Potential due to whole sphere if cavity is not there at distance $\frac{R}{2}$ from centre

$$= \frac{-GM}{R^3} \left(\frac{3}{2} R^2 - 0.5 r^2 \right)_{r=\left(\frac{R}{2}\right)}$$

$$= \frac{-GM}{R^3} \left(\frac{3}{2} R^2 - \frac{R^2}{8} \right)$$

$$= \frac{-GM}{R^3} \left(\frac{12 R^2 - R^2}{8} \right)$$

$$= \frac{-11GM}{8R} \quad \text{--- (1)}$$

Potential due to sphere of radius $\frac{R}{2}$ at its centre let M' be mass of this sphere (equating densities)

$$\frac{M}{\frac{4}{3}\pi R^3} = \frac{M'}{\frac{4}{3}\pi\left(\frac{R}{2}\right)^3}$$

$$M' = \frac{M}{8}$$

Potential due to the sphere of $\frac{R}{2}$ radius at its centre is

$$= \frac{3}{2} \frac{GM'}{\frac{R}{2}}$$

$$= \frac{3}{2} \frac{GM \times 2}{8R}$$

$$= \frac{-3}{8} \frac{GM}{R} \quad \text{--- (2)}$$

\therefore Potential at $r = \frac{R}{2}$ is $= (1) - (2)$

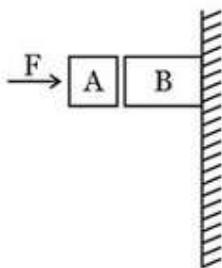
$$= \frac{-11}{8} \frac{GM}{R} + \frac{3}{8} \frac{GM}{R} = \frac{-GM}{R}$$

Topic: Gravitation

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 360

27.



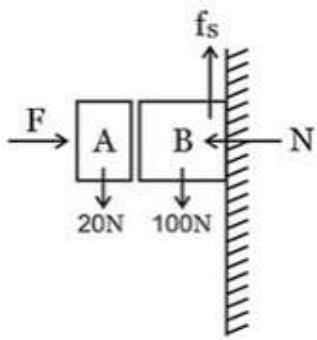
Given in the figure are two blocks A and B of weight 20N and 100N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is:

- (A) 120 N
 (B) 150 N

- (C) 100 N
 (D) 80 N

Answer: (A)

Solution:



For complete state equilibrium of the system. The static friction on the block B by wall will balance the total weight 120 N of the system.

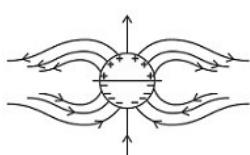
Topic: Laws of Motion

Difficulty: Moderate (embibe predicted Low Weightage)

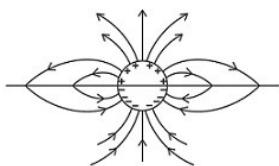
Ideal time: 60

28. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge $-\sigma$ in the lower half. The electric field lines around the cylinder will look like figure given in:
 (Figures are schematic and not drawn to scale)

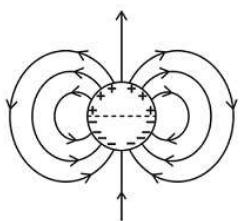
(A)



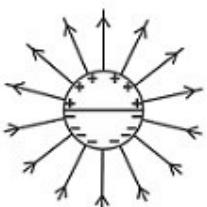
(B)



(C)

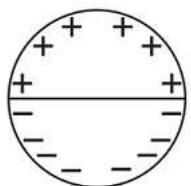


(D)

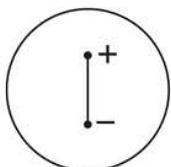


Answer: (C)

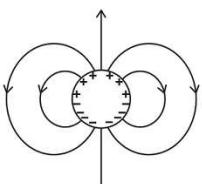
Solution:



Consider cross section of cylinders which is circle the half part of circle which has positive charge can be assume that total positive charge is at centre of mass of semicircle. In the same way we can assume that negative charge is at centre of mass of that semicircle.



Now it acts as a dipole now by the properties of dipole and laws of electric field line where two lines should not intersect the graph would be



Topic: Electrostatics

Difficulty: Moderate (embibe predicted high weightage)

Ideal time: 90

29. As an electron makes transition from an excited state to the ground state of a hydrogen like atom/ion:

- (A) Kinetic energy decreases, potential energy increases but total energy remains same
- (B) Kinetic energy and total energy decreases but potential energy increases
- (C) Its kinetic energy increases but potential energy and total energy decrease
- (D) Kinetic energy, potential energy and total energy decrease

Answer: (C)

$$\text{Solution: } U = \frac{-e^2}{4\pi\epsilon_0 r}$$

U = potential energy

$$k = \frac{e^2}{8\pi\epsilon_0} r$$

K = kinetic energy

$$E = U + k = \frac{-e^2}{8\pi\epsilon_0 r}$$

E = Total energy

∴ as electron de-excites from excited state to ground state k increases, U & E decreases

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30

30. Match list-I (Fundamental Experiment) with List-II (its conclusion) and select the correct option from the choices given below the list:

	List-I		List-II
A	Franck-Hertz Experiment	(i)	Particle nature of light
B	Photo-electric experiment	(ii)	Discrete energy levels of atom
C	Davison-Germer Experiment	(iii)	Wave nature of electron
D		(iv)	Structure of atom

- (A) A – (ii); B – (i); C – (iii)
(B) A – (iv); B – (iii); C – (ii)
(C) A – (i); B – (iv); C – (iii)
(D) A – (ii); B – (iv); C – (iii)

Answer: (A)

Solution: Frank-Hertz experiment demonstrated the existence of excited states in mercury atoms helping to confirm the quantum theory which predicted that electrons occupied only discrete quantized energy states.

Phot-electric experiment = Demonstrate that photon is the field particle of light which can transfer momentum and energy due to collision.

Davisson-Germer experiment = this experiment shows the wave nature of electron.

Topic: Modern Physics

Difficulty: Easy (embibe predicted high weightage)

Ideal time: 30