

### Solution

#### Mathematics

1. Point of intersection

$$y = kx^2, x = ky^2$$

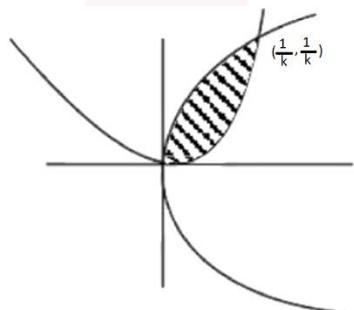
$$x = k(k^2x^4)$$

$$x^3 = \left(\frac{1}{k}\right)^3 \Rightarrow x = \frac{1}{k}$$

$$\text{Point of intersection } \left(\frac{1}{k}, \frac{1}{k}\right)$$

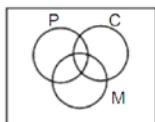
$$\text{Area} = \int_0^{1/k} \left( \sqrt{\frac{x}{k}}, -kx^2 \right) dx = 1 \Rightarrow \left( \frac{1}{\sqrt{k}} \frac{x^{3/2}}{3/2} - \frac{kx^3}{3} \right)_0^{\frac{1}{k}} = 1 \Rightarrow \frac{2}{3k^2} - \frac{1}{3k^2} = 1 \Rightarrow k^2 = \frac{1}{3}$$

$$k = \frac{1}{\sqrt{3}}$$



$$2. P \equiv \left[ \frac{140}{3} \right] = 46$$

$$C \equiv \left[ \frac{140}{5} \right] = 28$$



$$M \equiv \left[ \frac{140}{2} \right] = 70$$

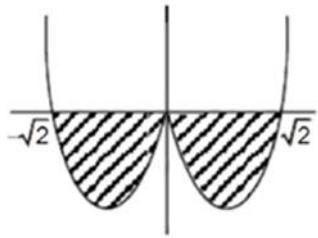
$$n(P \cup C \cup M) = n(P) + n(C) + n(M) - n(P \cap C) - n(C \cap M) - n(M \cap P) + n(P \cap C \cap M)$$

$$= 46 + 28 + 70 - \left[ \frac{140}{15} \right] - \left[ \frac{140}{10} \right] - \left[ \frac{140}{6} \right] + \left[ \frac{140}{30} \right]$$

$$= 144 - 9 - 14 - 23 + 4$$

$$= 102$$

3.  $\int_a^b (x^4 - 2x^2) dx$



From figure minimum area is  $(-\sqrt{2}, \sqrt{2})$

4.  $\frac{dy}{dx} + 3(3 \sec^2 x)y = \sec^2 x$

This is linear differential equation

Integration factor =  $e^{\int 3 \sec^2 x dx} = e^{3 \tan x}$

Hence  $y \cdot e^{3 \tan x} = e^{\int e^{3 \tan x} \cdot \sec^2 x dx}$

$$\Rightarrow ye^{3 \tan x} = \frac{e^{3 \tan x}}{3} + c$$

$$\Rightarrow y = Ce^{-3 \tan x} + \frac{1}{3}$$

Given  $y\left(\frac{\pi}{4}\right) = \frac{4}{3} \Rightarrow \frac{4}{3} = Ce^{-3} + \frac{1}{3}$

$$\Rightarrow C = e^3$$

Hence  $y\left(-\frac{\pi}{4}\right) = e^3 \cdot e^3 + \frac{1}{3} = e^6 + \frac{1}{3}$

5.  $P(7 \text{ or } 8)$

$$= P(H) P(7 \text{ or } 8) + P(T) P(7 \text{ or } 8)$$

$$= \frac{1}{2} \times \frac{11}{36} + \frac{1}{2} \times \frac{2}{9} = \frac{11}{72} + \frac{1}{9} = \frac{19}{72}$$

6.  $\sum_{i=1}^{20} \left( \frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} \right)^3$

Now  $\frac{^{20}C_{i-1}}{^{20}C_i + ^{20}C_{i-1}} = \frac{^{20}C_{i-1}}{^{21}C_i} = \frac{i}{21}$

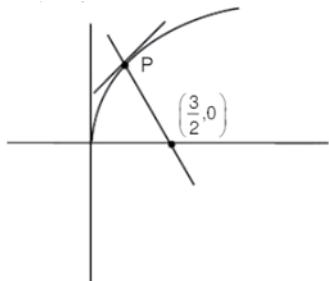
Let given sum be  $s$ , so

$$S = \sum_{i=1}^{20} \frac{(i)^3}{21^3} = \frac{1}{(21)^3} \left( \frac{20 \cdot 21}{2} \right)^2 = \frac{100}{21}$$

Given  $S = \frac{k}{21} \Rightarrow k = 100$

7. If the third term in the expansion of  $(1 + x^{\log_2 x})^5$  is 2560 then the value of  $x$  is
- (A)  $\frac{1}{4}$   
(B)  $2\sqrt{2}$   
(C)  $\frac{1}{8}$   
(D)  $4\sqrt{2}$
8. Let  $5, 5r, 5r^2$  be the sides of a triangle then value of  $r$  cannot be
- (A)  $\frac{7}{4}$   
(B)  $\frac{3}{2}$   
(C)  $\frac{5}{4}$   
(D)  $\frac{3}{4}$
9. Let  $ABC$  is a triangular plane such that  $AB = 7, AC = 6, BC = 5$ , and  $D$  is mid point of  $AC$ . A tower stand at mid point of  $AC$  subtending angle equal to  $30^\circ$  at vertex  $B$ , then height of tower is
- (A)  $\frac{2\sqrt{7}}{3}$   
(B)  $7\sqrt{3}$   
(C)  $2\sqrt{7}$   
(D)  $\frac{3}{2}\sqrt{21}$
10. The equation of tangent to the hyperbola,  $4x^2 - 5y^2 = 20$ , which is parallel to  $x - y = 2$ , is
- (A)  $x - y - 3 = 0$   
(B)  $x - y - 9 = 0$   
(C)  $x - y + 1 = 0$   
(D)  $x - y + 5 = 0$

11. Let  $P$  be the nearest to  $\left(\frac{3}{2}, 0\right)$ , then normal at  $P$  will pass through  $\left(\frac{3}{2}, 0\right)$ . Let Co-ordinates of  $P$  be  $s\left(\frac{t^2}{4}, \frac{t}{2}\right)$



Hence equation of normal is  $y + tx = \frac{1}{2} + \frac{t^3}{4}$

This line passes through  $s\left(\frac{3}{2}, 0\right)$

$$\frac{3t}{2} = \frac{t}{2} + \frac{t^3}{4} \Rightarrow t = 0, 2 \{(-2)\text{ is rejected}\}$$

hence nearest point is  $(1, 1)$

$$\text{distance} = \sqrt{\left(\frac{3}{2} - 1\right)^2 + (1 - 0)^2} = \frac{\sqrt{5}}{2}$$

12.  $\mu = \frac{1+3+8+x+y}{5}$

$$25 = 12 + x + y \Rightarrow x + y = 13 \quad \dots(i)$$

$$\sigma^2 = \frac{\Sigma(x_i - \mu)^2}{N}$$

$$9.2 = \frac{1 + 9 + 64 + x^2 + y^2}{5} - 25$$

$$34.2 \times 5 = 74 + x^2 + y^2$$

$$171 = 74 + x^2 + y^2$$

$$97 = x^2 + y^2 \quad \dots(ii)$$

$$(x + y)^2 = x^2 + y^2 + 2xy$$

$$169 - 97 = 2xy \Rightarrow xy = 36$$

$$T = 4.9$$

So ratio is  $\frac{4}{9}$  or  $\frac{9}{4}$

## 13. Case-I

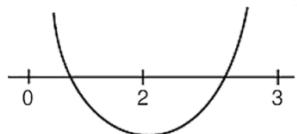
$$c - 5 > 0 \dots \text{(i)}$$

$$f(0) > 0$$

$$c - 4 > 0 \dots \text{(ii)}$$

$$f(2) < 0$$

$$4(c - 5) - 4c + c - 4 < 0$$



$$c < 24 \dots \text{(iii)}$$

$$f(3) > 0$$

$$9(c - 5) - 6c + c - 4 > 0$$

$$4c - 49 > 0 \Rightarrow c > \frac{49}{4} \dots \text{(iv)}$$

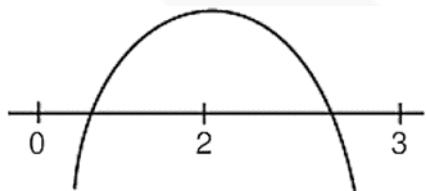
Here (i)  $\cap$  (ii)  $\cap$  (iii)  $\cap$  (iv)

$$c \in \left(\frac{49}{4}, 24\right)$$

## Case-II

$$c - 5 < 0 \dots \text{(i)}$$

$$f(0) < 0$$



$$c < 4 \dots \text{(ii)}$$

$$f(2) > 0 \Rightarrow c > 24 \dots \text{(iii)}$$

$$f(3) < 0 \Rightarrow c < \frac{49}{4} \dots \text{(iv)}$$

(i)  $\cap$  (ii)  $\cap$  (iii)  $\cap$  (iv)  $\Rightarrow c \in \emptyset$

$$\text{Ans. } c \in \left(\frac{49}{4}, 24\right)$$

14.  $x + y + z = 1$ 

$$x + 3y + 5z = \beta$$

$$3x + 4y + az = 9$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 3 & 5 \\ 3 & 4 & \alpha \end{vmatrix} = (3\alpha - 20) - (\alpha - 15) + (4 - 9)$$

$\Rightarrow D = 0$  for  $\alpha = 5$

For this ' $\alpha$ ' equations first and third are

$$3x + 4y + 5z = 9 \quad \dots(i)$$

$$x + y + z = 1 \quad \dots(ii)$$

$$2 \times (1) - 5 \times (2) \Rightarrow x + 3y + 5z = 13$$

While remaining equation is  $x + 3y + 5z = \beta$

Hence for infinite solution  $\alpha = 5, \beta = 13$

15.  $4|z_1|^2 = 9|z_2|^2 \Rightarrow 4z_1 \bar{z}_1 = 9z_2 \bar{z}_2$

$$\Rightarrow \frac{2z_1}{3z_2} = \frac{3\bar{z}_2}{2z_1}$$

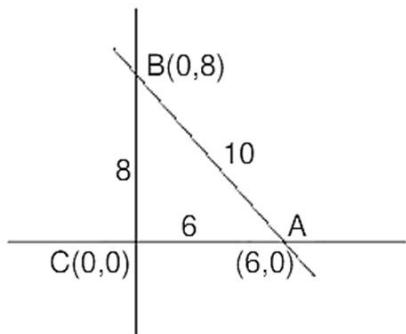
$$\text{Given } Z = \frac{2z_1}{3z_2} + \frac{3z_2}{2z_1} \Rightarrow z = \frac{3\bar{z}_2}{2\bar{z}_1} + \frac{3z_2}{2z_1} = \text{real number}$$

16.  $\vec{n} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 2 \\ 1 & 2 & 3 \end{vmatrix} = -7\hat{i} - 7\hat{j} + 7\hat{k}$

$$\text{Equation of plane is } -7(x - 4) - 7(y + 1) + 7(z - 3) = 0$$

$$\Rightarrow -7x - 7y + 7z = 0 \Rightarrow x + y = z$$

17.  $I \equiv \left( \frac{ax_1 + bx_2 + cx_3}{a+b+c}, \frac{ay_1 + by_2 + cy_3}{a+b+c} \right)$



$$= \left( \frac{8(6) + 6(0) + 10(0)}{24}, \frac{10(0) + 6(8) + 8(0)}{24} \right) = (2, 2)$$

18.  $\sin^2 2\theta + \cos^4 2\theta = \frac{3}{4}$

$$\Rightarrow 1 - \cos^2 2\theta + \cos^4 2\theta = \frac{3}{4}$$

$$\text{Let } \cos^2 2\theta = t \Rightarrow t^2 - t + \frac{1}{4} = 0 \Rightarrow \left(1 - \frac{1}{2}\right)^2 = 0$$

$$\Rightarrow t = \frac{1}{2} \Rightarrow \cos^2 2\theta = \frac{1}{2}$$

$$\Rightarrow 2 \cos^2 2\theta - 1 = 0 \Rightarrow \cos 4\theta = 0 \Rightarrow 4\theta = (2n+1)\frac{\pi}{2}$$

$$\Rightarrow \theta = (2n+1)\frac{\pi}{8} \Rightarrow \theta = \frac{\pi}{8}, \frac{3\pi}{8} \in \left[0, \frac{\pi}{2}\right]$$

Sum of values of  $\theta$  is  $\frac{\pi}{2}$

19.  $\int \frac{(\sin^n \theta - \sin \theta)^{\frac{1}{n}} \cos \theta}{\sin^{n+1} \theta} d\theta$

$$= \int \frac{(t^n - t)^{\frac{1}{n}} dt}{t^{n+1}} \quad (\text{Put } \sin \theta = t)$$

$$= \int \frac{t \left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^{n+1}} dt$$

$$= \int \frac{\left(1 - \frac{1}{t^{n-1}}\right)^{\frac{1}{n}}}{t^n} dt$$

$$\text{Put } 1 - \frac{1}{t^{n-1}} = z \Rightarrow \frac{(n-1)}{t^n} dt = dz$$

$$\Rightarrow I = \frac{1}{n-1} \int z^{\frac{1}{n}} dz = \frac{z^{\frac{1}{n}+1}}{\left(\frac{1}{n}+1\right)(n-1)} + c = \frac{n(1-t^{1-n})^{\frac{1}{n}+1}}{n^2-1} + c$$

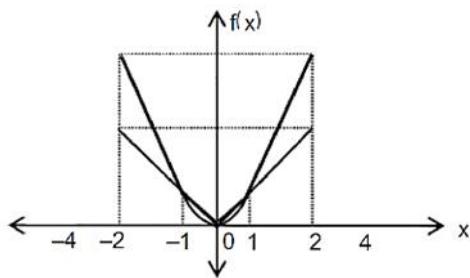
$$= \frac{n}{n^2-1} \left(1 - \frac{1}{\sin^{n-1} \theta}\right)^{\frac{n+1}{n}} + c$$

20.  $\lim_{x \rightarrow 1^+} \frac{(1-|x|+\sin|1-x|) \sin([1-x]\frac{\pi}{2})}{|1-x|[1-x]}$

$$= \lim_{x \rightarrow 1^+} \frac{(1-x+\sin(x-1)) \sin\left(-\frac{\pi}{2}\right)}{(x-1)(-1)}$$

$$= \lim_{x \rightarrow 1^+} \frac{-(x-1)+\sin(x-1)}{(x-1)} = -1+1=0$$

21.



From the graph we can easily conclude that  $f(x)$  is non-derivable at  $a = -2, -1, 0, 1, 2$

22. Normal to there 2 curves are

$$y = m(x - c) - 2bm - bm^3$$

$$y = mx - 4am - 2am^3$$

If they have a common normal

$$(c + 2b)m + bm^3 = 4am + 2am^3$$

$$\Rightarrow c + 2b - 4a = (2a - b)m^2 \quad (m = 0 \text{ corresponds to axis})$$

$$\Rightarrow m^2 = \frac{c}{2a - b} - 2 > 0$$

23. Tangent at  $(1, -1)$  is

$$x(1) + y(-1) + 2(x + 1) - 3(y - 1) - 12 = 0$$

$$\Rightarrow 3x - 4y = 7$$

Required circle is

$$(x - 1)^2 + (y + 1)^2 + \lambda(3x - 4y - 7) = 0$$

It passes through  $(4, 0)$

$$\Rightarrow 9 + 1 + \lambda(12 - 7) = 0 \Rightarrow \lambda = -2 \Rightarrow x^2 + y^2 - 8x + 10y + 16 = 0$$

$$\text{radius } \sqrt{16 + 25 - 16} = 5$$

24.  $P(m) = m^2 - m + 41$ 

$$P(3) = 9 - 3 + 41 = 47$$

$$P(5) = 25 - 5 + 41 = 61$$

Hence  $P(3)$  and  $P(5)$  are both prime

25. Two digit numbers of the form  $7\lambda + 2$  are 16, 23, ..., 93

Two digit numbers of the form  $7\lambda + 5$  are 12, 19, ..., 96

Sum of all the above numbers equals to  $\frac{12}{2}(16 + 93) + \frac{13}{2}(12 + 96) = 654 + 702 = 1356$

26.  $f(x) = x^3 + ax^2 + bx + c$

$$f'(x) = 3x^2 + 2ax + b$$

$$f''(x) = 6x + 2a$$

$$f'''(x) = 6$$

$$a = f'(1) = 3 + 2a + b \Rightarrow a + b = -3$$

$$b = f'' = 12 + 2a \Rightarrow 2a - b = -12$$

$$c = f'''(3) \Rightarrow c = 6 \text{ and } a = -5, b = 2$$

$$\Rightarrow f(x) = x^3 - 5x^2 + 2x + 6$$

$$\Rightarrow f(2) = 8 - 20 + 4 + 6 = -2$$

## Physics

1.  $\rho = \frac{M}{L^3}$

$$n_1 u_1 = n_2 u_2$$

$$\therefore 128[M_1 L_1^{-3}] = n_2 [M_2 L_2^{-3}]$$

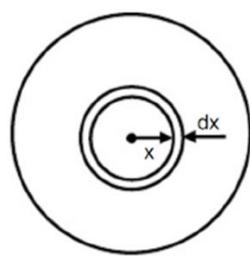
$$\therefore n_2 = 128 \times \left[ \frac{M_1}{M_2} \right] \left[ \frac{L_2}{L_1} \right]^3$$

$$= 128 \times \left[ \frac{1000}{50} \right] \times \left[ \frac{25}{100} \right]^3$$

$$= 128 \times 20 \times \frac{1}{64}$$

$$= 40$$

2.



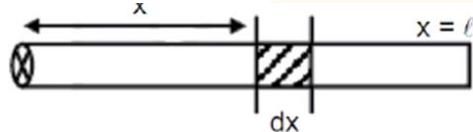
Net torque =  $\sum Torque \text{ on ring}$

$$\int d\tau = \int_0^R \mu \times \frac{F}{\pi R^2} \times 2\pi x \cdot x dx$$

$$\tau = 2\mu F R$$

3.  $\frac{Q}{A} = K \times \frac{1000 - 100}{1} = 90 \text{ W/m}^2$

4.  $\omega = 2\pi n \text{ rad/s}$



$$dQ = \rho \cdot dx$$

$$= \frac{\rho_0}{l} x dx$$

$$dI = \frac{dQ \cdot \omega}{2\pi}$$

$$dM = dI \times A$$

$$\int dM = \int_0^l \frac{\omega}{2\pi} \cdot \frac{\rho_0}{l} \times \pi x^2 dx$$

$$M = \frac{\pi n \rho_0 l^3}{4}$$

5.  $f_1 = \left( \frac{340}{340-34} \right) f$

$$f_2 = \left( \frac{340}{340-17} \right) f$$

$$\frac{f_1}{f_2} = \left( \frac{340-17}{340-34} \right) = \frac{19}{18}$$

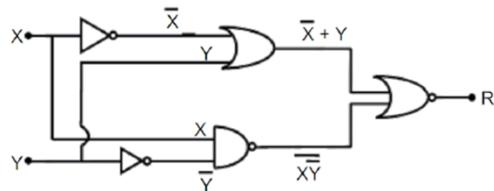
6.  $x = \frac{\varepsilon}{13r} \times \frac{12r}{l} = \frac{12\varepsilon}{13l}$

$$\frac{\varepsilon}{2} = xl'$$

$$\frac{\varepsilon}{2} = \frac{12\varepsilon}{13l} l'$$

$$l' = \frac{13l}{24}$$

7.



$$R = \overline{(\bar{X} + Y)} + \overline{X\bar{Y}} = \overline{(\bar{X} + Y)} \cdot \overline{X\bar{Y}} = (\bar{\bar{X}}\bar{Y}) \cdot X\bar{Y} = (X\bar{Y})(X\bar{Y})$$

8.  $D = \sqrt{2h_T R} + \sqrt{2h_R R}$

$$D = \sqrt{2 \times 140 \times 64 \times 10^5} + \sqrt{2 \times 40 \times 64 \times 10^5}$$

$$D = 8 \times 10^3 [\sqrt{28} + \sqrt{8}]$$

$$D = 8 \times 10^3 [2\sqrt{7} + 2\sqrt{2}]$$

$$D = 16 \times 10^3 [2\sqrt{7} + 2\sqrt{2}]$$

$$D = 16 \times 10^3 [2.6 + 1.4]$$

$$= 64000m = 64km \approx 65km$$

9.  $\frac{1}{f_1} = (\mu_1 - 1) \left( \frac{1}{R} - \frac{1}{\infty} \right)$

$$\frac{1}{f_2} = (\mu_2 - 1) \left( \frac{1}{R} - \frac{1}{\infty} \right)$$

$$\Rightarrow \frac{f_2}{f_1} = \frac{\mu_1 - 1}{\mu_2 - 1} = \frac{1}{2}$$

$$\Rightarrow 2\mu_1 - 2 - \mu_2 - 1$$

$$\Rightarrow 2\mu_1 - \mu_2 = 1$$

10.  $\lambda \begin{pmatrix} 0 \\ A \end{pmatrix} = \sqrt{\frac{150}{V}} \Rightarrow 7.5 \times 10^{-2} = \sqrt{\frac{150}{V}}$

$$V = \frac{150}{7.5 \times 7.5 \times 10^4} = \frac{80}{3} kV$$

Nearby value is 25 keV

11.  $e = Bvl = 0.1(2 \times 10^{-2}) \times 6 = 12 \times 10^{-3} = 12mV$

12.  $\frac{A_1}{A_2} = \frac{\pi(R_{\max})_1^2}{\pi(R_{\max})_2^2} = \frac{u_1^4}{u_2^4} = \frac{1}{16}$

$$\left[ \because \text{max. Range} = \frac{u^2}{g} \right]$$

13.  $\frac{q_1}{a^2} = \frac{q_2}{b^2} = \frac{q_3}{c^2} = \text{constant}$

$$q_1 + q_2 + q_3 = Q$$

$$q_1 = \frac{a^2(Q)}{a^2+b^2+c^2}$$

$$q_2 = \frac{b^2(Q)}{a^2+b^2+c^2}$$

$$q_3 = \frac{c^2(Q)}{a^2+b^2+c^2}$$

$$V = \frac{Kq_1}{a} + \frac{Kq_2}{b} + \frac{Kq_3}{c}$$

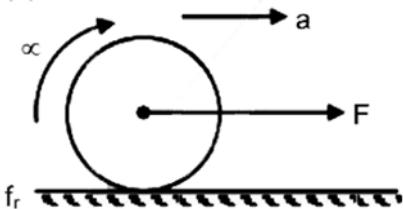
$$V = \frac{1}{4\pi\epsilon_0} [Q] \left( \frac{a+b+c}{a^2+b^2+c^2} \right)$$

14.  $E = \phi KE$

$$KE = E - Q$$

$$hV = \phi + KE$$

15.



$$F - f_r = ma \dots (i)$$

$$f_r R = I\alpha = \frac{mR^2}{2}\alpha \dots (ii)$$

For pure rolling

$$a = \alpha R \dots (iii)$$

From (i)(ii) and (iii)

$$F - \frac{mR\alpha}{2} = m\alpha R$$

$$F = \frac{3}{2}mR\alpha$$

$$\alpha = \frac{2F}{3mR}$$

16.  $V = f\lambda$

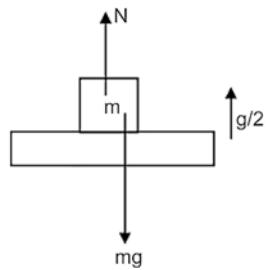
$$\sqrt{\frac{T}{\mu}} = f\lambda$$

$$\sqrt{\frac{8}{5 \times 10^{-3}}} = 100\lambda$$

$$\lambda = 40 \text{ cm}$$

$$\frac{\lambda}{2} = 20 \text{ cm}$$

17.



$$N - mg = \frac{mg}{2}$$

$$N = \frac{3mg}{2}$$

$$s = ut + \frac{1}{2}at^2$$

$$= 0 + \frac{1}{2}\frac{g}{2}(1)^2$$

$$= \frac{g}{4}$$

$$W_N = \frac{3mg}{2} \times \frac{g}{4}$$

$$= \frac{3mg^2}{8} = \frac{3mg^2}{8}$$

18.  $C = \frac{E_0 A}{d}$

$$C_1 = \frac{K_1 E_0 A}{3d} = \frac{K_1 C}{3}$$

$$C_2 = \frac{K_2 C}{3}$$

$$C_3 = \frac{K_3 C}{3}$$

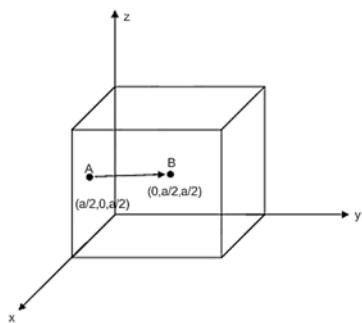
$$C_{eq} = K_{eq} = \frac{C}{3}(K_1 + K_2 + K_3)$$

$$K_{eq} = \frac{K_1 + K_2 + K_3}{3} = \frac{36}{3} = 12$$

19.  $V_e = \sqrt{2}v$

$$K.E = \frac{1}{2}m(\sqrt{2}v)^2 = mV^2$$

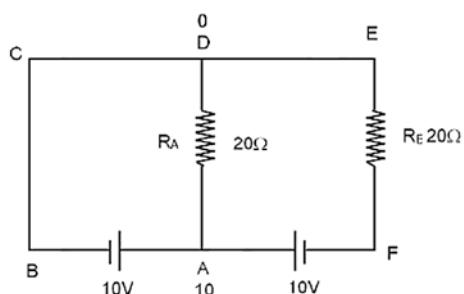
20.



$$\bar{r} = \bar{r}_B - \bar{r}_A$$

$$= -\frac{a}{2}\hat{i} + \frac{a}{2}\hat{j}$$

21.



Assume  $V_F = as'0'$

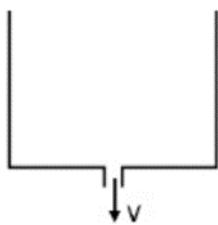
as  $V_E = V_F = 0$

$\Rightarrow$  Current in  $R_B$  is zero

$$V_A - V_D = 10V$$

$$\Rightarrow I = \frac{10}{20} = \frac{1}{2}A \text{ from } A \text{ to } D$$

22.



$$\sqrt{2gh} \times A = V$$

$$h = \frac{(10^{-4})^2}{2 \times 9.8 \times (10^{-4})^2} = 5.1 \text{ cm}$$

$$23. \quad \eta_1 = 1 - \frac{T_2}{T_1}$$

$$\eta_2 = 1 - \frac{T_3}{T_2}$$

$$\eta_3 = 1 - \frac{T_4}{T_3}$$

Given  $\eta_1 = \eta_2 = \eta_3$

$$\frac{T_2}{T_1} = \frac{T_3}{T_2} = \frac{T_4}{T_3}$$

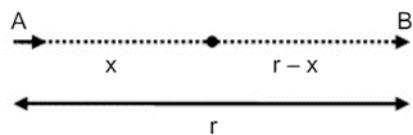
$$T_2^2 = T_1 T_3 \dots (1)$$

$$T_2^2 = T_1 T_3 \dots (1)$$

(1) and (2)

$$T_3 = (T_1 T_4^2)^{1/3}, T_2 (T_1^2 T_4)^{1/3}$$

24.



$$V_1 + V_2 = 0$$

$$\frac{K4Qa}{x^2} - \frac{K2Qa}{(x-r)^2} = 0$$

$$x^2 = 2(x-r)^2$$

$$x = \left( \frac{\sqrt{2}r}{\sqrt{2}-1} \right)$$

25.  $\text{Req} = \frac{R_1 R_2}{R_1 + R_2} = \frac{6 \times 12}{6 + 12} = 4\Omega$

26.  $\frac{E}{B} = C \Rightarrow E = B.C. = 10^{-4} \times 3 \times 10^8 V/m = 3 \times 10^4 V/m$

27.  $d \sin\theta = n\lambda$

$$\therefore \lambda = \frac{d \sin\theta}{n} = 10^{-4} \times \frac{1}{40} \times \frac{1}{n}$$

$$\therefore \lambda = \frac{2.5 \times 10^{-6}}{n} = \frac{2500 \times 10^{-9}}{n} = \frac{2500}{n} (\text{nm})$$

Putting  $n = 1, \lambda_1 = 2500 \text{ nm}$

$n = 2, \lambda_2 = 1250 \text{ nm}$

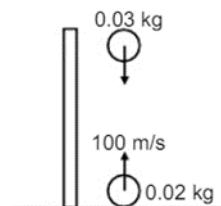
$n = 3, \lambda_3 = 833 \text{ nm}$

$n = 4, \lambda_4 = 625 \text{ nm}$

$n = 5, \lambda_5 = 500 \text{ nm}$

$n = 6, \lambda_6 = 357 \text{ nm}$

28.



Using relative velocity



$$a_{\text{rel}} = 0$$

$$v_{\text{rel}} = 100$$

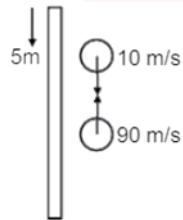
$$100 = v_{\text{rel}} \times t$$

$$t = \frac{100}{100} = 1 \text{ sec}$$

$$v_{\text{bullet}} = 100 - 1 \times 10 = 90 \text{ m/s}$$

$$v_{\text{particle}} = 10 \times 1 = 10 \text{ m/s}$$

$$S = 100 \times 1 - \frac{1}{2} \times 10 \times 1 = 95 \text{ m}$$



$$P_i = P_f$$

$$\Rightarrow 90 \times 0.02 - 10 \times 0.03 = 0.05V$$

$$V = 30 \text{ m/s} \Rightarrow h = \frac{v^2}{2g} = \frac{900}{20} = 45 \text{ m}$$

So from top of building

$$45 - 5 = 40 \text{ m}$$

29. Green  $\rightarrow$  5, Black  $\rightarrow$  0 Red  $\rightarrow$   $10^2$  Brown  $\rightarrow$  10% tolerance

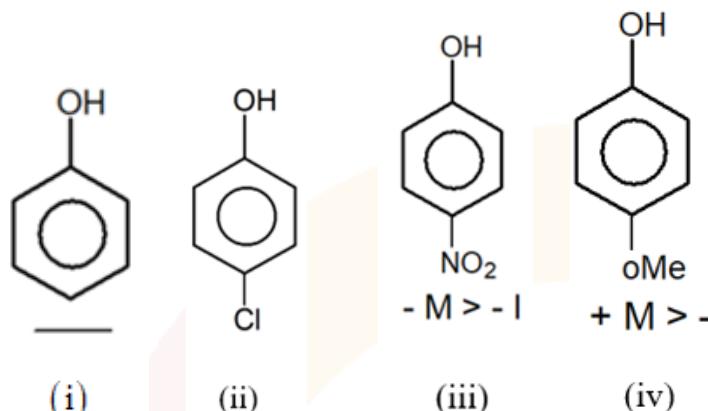
$$I^2R = P$$

$$I^2 = \frac{2}{50 \times 10^2} = 4 \times 10^{-4} A$$

$$I = 2 \times 10^{-2} A = 20 \text{ mA}$$

## Chemistry

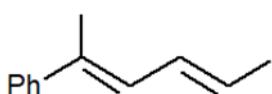
1.



Acidic strength of phenol is dependent on the substituents attached to it. Presence of electron withdrawing groups increase the tendency to attract electrons (Acidity), thereby decreasing the pKa value.  $\text{NO}_2$  is a strong withdrawing group, Cl is a weak electron withdrawing group as it has  $+M$  effects whereas  $\text{OCH}_3$  is an electron donating group. Hence, the correct order of pKa values is:

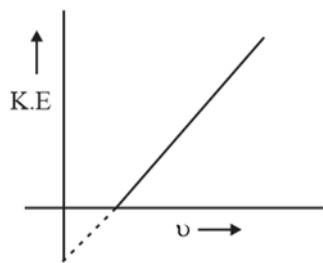
order of pKa values:  $III < II < I < IV$

2. A water supply with a BOD level of 3-5 ppm is considered moderately clean. In water with a BOD level of 6-9 ppm, the water is considered somewhat polluted because there is usually organic matter present and bacteria are decomposing this waste. Hence, B with a higher value of BOD level is more polluted.
3. Addition of alc.  $KOH$  to haloalkanes follows the Saytzeff rule which points towards greater stability. Saytzeff's rule implies that elimination reactions favour formation of alkene with greater number of substituents.



Not only that, but it also gives the molecule an extended conjugate system which further enhances its stability.

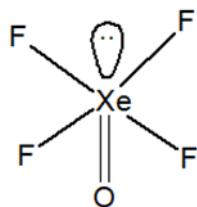
4.  $\because KE_{\max} = bv - hv_0 \dots\dots\dots$  Photoelectric effect.



Intercept must be  $-hv_0$

5. The most inorganized system has  $a \neq b \neq c$  and  $\alpha \neq \beta \neq \gamma = 90^\circ$  is called the triclinic system.

- 6.



Steric no. = 6, hybridisation =  $sp^3d^2$

Geometry = octahedral

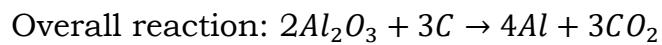
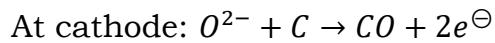
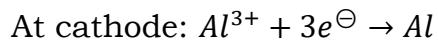
No. of lone pairs on Xe = 1

- 7.

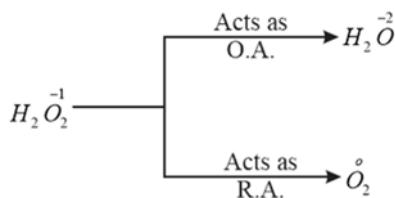
Possible Structural isomers	No of geometrical isomer by each
$[MCl(NH_3)(NO_2)(SCN)]$	3
$[MCl(NH_3)(ONO)(SCN)]$	3
$[MCl(NH_3)(NO_2)(NCS)]$	3
$[MCl(NH_3)(ONO)(NCS)]$	3

Total number of isomers will be 12.

8. Hall Heroult's process:



9.



$H_2O_2$  acts as an oxidizing agent, as well as a reducing agent in both acidic as well as basic mediums.

10. According to MOT; electronic configuration of  $N_2^+$  is:

$$\sigma 1S^2 \sigma^* 1S^2 \sigma 2S^2 \sigma^* 2S^2 \pi 2p_x^2 \pi 2p_y^2 \sigma 2p_z^1$$

$$B.O. = \frac{1}{2}(9 - 4) = 2.5$$

i.e., 2 $\pi$  bonds and 0.5  $\sigma$  bonds.

11. Chlorotris(triphenylphosphine)rhodium(I), is known as Wilkinson's catalyst. It is used as a homogeneous hydrogenation catalyst. It is a square planar 16-electron complex. The oxidation state of Rhodium in it is +1.

12.  $\because K_P = K_C(RT)^{\Delta ng}$

$$\frac{K_P}{K_C} = (RT)^{\Delta ng}$$

$$\text{Equations 1: } \Delta ng = 0 \therefore \frac{K_P}{K_C} = 1$$

$$\text{Equations 2: } \Delta ng = 1 \therefore \frac{K_P}{K_C} = RT$$

$$\text{Equations 3: } \Delta ng = -2 \therefore \frac{K_P}{K_C} = (RT)^{-2}$$

13. Since,  $P_A^0 = 7 \times 10^3 Pa$ ;  $P_B^0 = 12 \times 10^3 Pa$  and  $X_A = 0.4$

According to Raoult's Law:  $P_T = P_A^0 x_A + P_B^0 x_B$

$$P_T = (7 \times 10^3)(0.4) + (12 \times 10^3)(0.6)$$

$$P_T = 10 \times 10^3 Pa$$

Method 1

$$\text{Now, } P_A^0 x_A = P_T y_A$$

$$y_A = \frac{P_A^0 x_A}{P_T} = \frac{7 \times 10^3 \times 0.4}{10^4} \Rightarrow 0.28$$

$$y_B = 1 - y_A \Rightarrow 1 - 0.28 \Rightarrow 0.72$$

### Method 2

We have the total pressure and the individual partial vapour pressures of the two liquids,  $P_T = 10 \times 10^3 \text{ Pa}$ ,  $P_A = 28 \times 10^2 \text{ Pa}$  and  $P_B = 72 \times 10^2 \text{ Pa}$ . Each gas exerts a partial pressure proportional to its mole fraction. Hence, mole fractions of A and B in vapour phases are 0.28 and 0.72 respectively.

14. (A) Since Combustion of Coal is a fast and spontaneous process, no catalyst is required.

(B) Catalyst used is  $\text{Ni}(s)$

(C) Catalyst used is  $\text{Fe}_2\text{O}_3(s)$ ,  $\text{K}_2\text{O}_{(s)}$  and  $\text{Al}_2\text{O}_3(s)$ .

(D) Catalyst used is  $\text{Pt}_{(s)}$

- 15.



$$0.1 \text{ mol} \quad \frac{1}{142} \text{ moles} \Rightarrow 0.007$$

E. R.                            L.R.

$$(0.1 - 0.007) \simeq 0.1 \quad 0 \quad 0.007 \text{ moles} \quad 0.007 \text{ moles}$$

$$W_{\text{CaSO}_4} = 0.007 \times M_{\text{CaSO}_4}$$

$$= 0.007 \times (136)$$

$$= 0.952 \text{ g}$$

$$[\text{OH}^-] = ?$$

Remaining is  $0.01 \sim 0.007 \simeq 0.1 \text{ moles}$

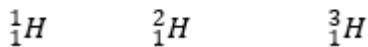
The solution will have  $\text{OH}^-$  ions coming from both  $\text{Ca(OH)}_2$  and  $\text{NaOH}$ . The concentration of  $\text{NaOH}$  is 0.007, which is negligible as compared to 0.1 moles of  $\text{Ca(OH)}_2$ .

$$\therefore [\text{OH}^-] = \frac{2 \times 0.1}{100} \times 1000$$

$$\Rightarrow 2M$$

**16. Isotopes of Hydrogen:**

Protium, Deuterium & Tritium



Among all three isotopes only Tritium is radioactive.

**17.** Dichloromethane and water are immiscible in nature. Also, Dichloromethane has higher density than water so forms bottom layer (Layer *II*) in the separating funnel.

**18.** Higher is the reduction potential, higher will be its tendency to get oxidized and thus higher will be its reducing property. Consequently, its reduction ability will be low.

**19.** EN of *Al* = 1.61

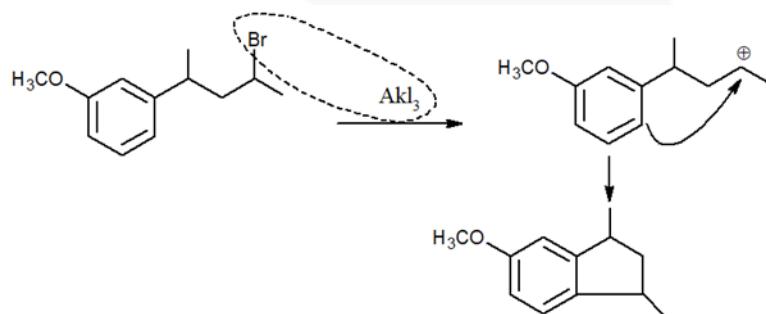
EN of *Be* = 1.57

EN of *B* = 2.04

EN of *C* = 2.55

Due to diagonal relationship between *Be* and *Al*, they exhibit similar properties and thus, have similar electronegativity values.

**20.**

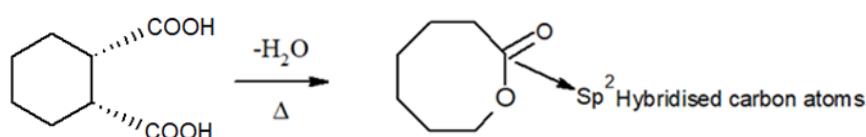


**21.** → *NaBH*<sub>4</sub> doesn't reduce esters and alkenes.

**22.** The speed of hydrolysis will depend on how likely a molecule is to accepting OH<sup>-</sup> ions. More acidic a molecule, higher will be the speed of reaction. NO<sub>2</sub> is a strong EWG (Electron withdrawing group), Cl is a weak EWG and OCH<sub>3</sub> is an EDG.

Hence, the increasing order of rate of hydrolysis will be: 3 < 2 < 4 < 1

23.

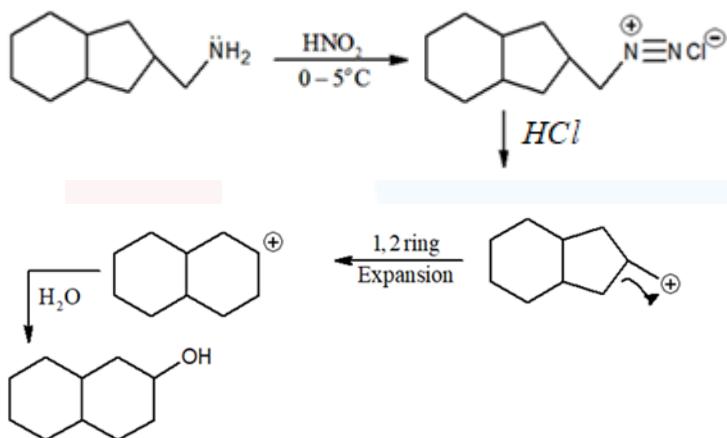


Cyclic anhydrides in 7 membered ring are less stable.

24. Allylic radical is more stable due to resonance.

Rate of reaction  $\propto$  Stability of free radical

25.



26. Arrhenius Solution:  $K = Ae^{-Ea/RT}$

$K$  decreases exponentially with  $B_a$

The second graph should be an asymptotic curve tangential to positive x axis.

1<sup>st</sup> graph is correct and second is incorrect.

27. Beryllium is used in making windows in X-ray tubes.

28.  $\Delta G = \Delta H - T\Delta S$  ..... (Gibbs Helmholtz Equation)

For a reaction to be spontaneous,  $\Delta G < 0$

So,  $\Delta H - T\Delta S < 0$

$\Delta H < T\Delta S$

$$\frac{\Delta H}{\Delta S} < T$$

i.e. Minimum temp. should be  $\frac{\Delta H}{\Delta S} \Rightarrow \frac{200}{40} = 5K$