SET A
PART A – PHYSICS

1. The density of a material in the shape of a cube is determined by measuring three sides of the cube and its mass. If the relative errors in measuring the mass and length are respectively 1.5% and 1%, the maximum error in determining the density is:

(A) 2.5%

(B) 3.5%

(C) 4.5%

(D) 6%

Solution: (C)

$e \Rightarrow \frac{M}{L^3}$

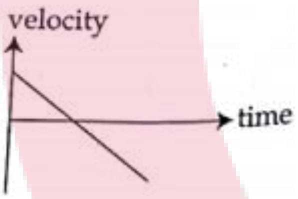
$\frac{\Delta e}{e} \Rightarrow \frac{\Delta M}{M} + \frac{3\Delta L}{L}$

$\frac{\Delta e}{e} \times 100 \Rightarrow 1.5 + 3 \times 1$

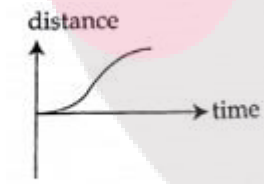
% relative error = 4.5%

2. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up.

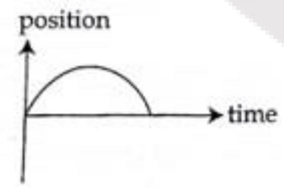
(A)



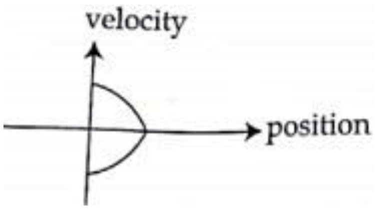
(B)



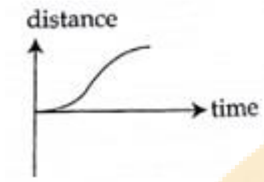
(C)



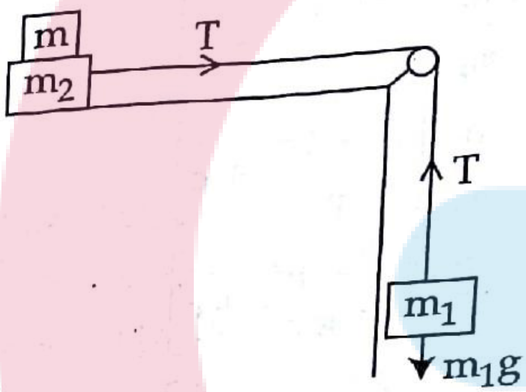
(D)



Solution: (B)



3. Two masses $m_1 = 5\ kg$ and $m_2 = 10\ kg$, connected by an inextensible string over a frictionless pulley, are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m_2 to stop the motion is:



(A)$18.3\ kg$

(B) $27.3\ k$

(C) $43.3\ kg$

(D) $10.3\ kg$

Solution: (Bouns)

Does not match.

In this question coefficient of friction b/w masses m_1 and m_2 is unknown. Without knowing that the problem cannot be solved.

Even is as assume that friction b/w m_1 and m_2 is sufficient to prevent slipping.

Answer does not match.

4. A particle is moving in a circular path of radius a under the action of an attractive potential $U = -\frac{k}{2r^2}$. Its total energy is:

(A) $-\frac{k}{4a^2}$

(B) $\frac{k}{2a^2}$

(C) Zero

(D) $-\frac{3}{2}\frac{k}{a^2}$

Solution: (C)

$V = -\frac{K}{2r^2}$

$F = -\frac{ru}{rr} = \frac{2K}{2r^3} = \frac{K}{r^3}$

$\frac{mv^2}{r} = \frac{K}{r^3}$

$V^2 = \frac{K}{mr^2}$

$\frac{1}{2}mv^2 = \frac{K}{2r^2}$

$E = \frac{K}{2r^2} - \frac{K}{2r^2} = 0$

5. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50% greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is:

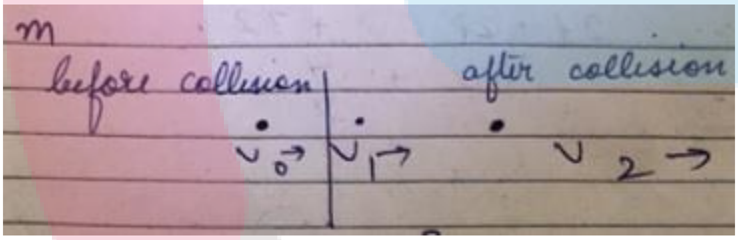
(A) $\frac{v_0}{4}$

(B) $\sqrt{2}\, v_0$

(C) $\frac{v_0}{2}$

(D) $\frac{v_0}{\sqrt{2}}$

Solution: (B)



$KE_2 = \frac{3}{2}mv_0^2$

Conservation of momentum,

$v_1 + v_2 = 2v_0$

Conservation of energy

$\frac{1}{2}mV_1^2 + \frac{1}{2}mV_2^2 = \frac{3}{2}mV_0^2$

$V_1^2 + V_2^2 = 3V_0^2$

$V_1^2 + V_2^2 + 2V_1V_2 = 4V_0$

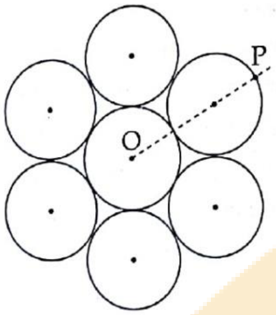
$2V_1V_2 = V_0^2$

$(V_1 - V_2)^2 \Rightarrow V_1^2 + V_2^2 - 2V_1V_2$

$\Rightarrow 3V_0^2 - 2V_0^2$

$\Rightarrow \sqrt{2}V_0$

6. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is:



(A) $\frac{19}{2}MR^2$

(B) $\frac{55}{2}MR^2$

(C) $\frac{73}{2}MR^2$

(D) $\frac{181}{2}MR^2$

Solution: (D)

MI of system about O

$\frac{1}{2}MR^2 + 6 \times \frac{1}{2}MR^2 + 6 \times M \times (2R)^2$

$\Rightarrow \quad I_1 = \frac{7}{2}MR^2 + 24MR^2$

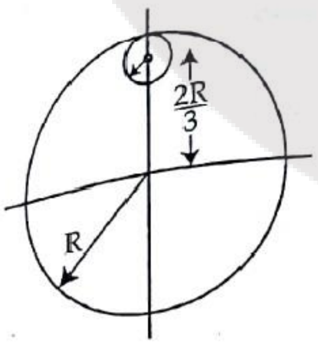
MI of system about P

$\Rightarrow \quad I_1 + 7MD^2$

$\Rightarrow \quad \frac{7}{2}MR^2 + 24MR^2 + 7M \times (3R)^2$

$\Rightarrow \quad \frac{181}{2}MR^2$

7. From a uniform circular disc of radius R and mass 9 M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is:



(A) $4MR^2$

(B) $\frac{40}{9}MR^2$

(C) $10MR^2$

(D) $\frac{37}{9}MR^2$

Solution: (A)

By parallel axis theorem,

MI of smaller disc $= \frac{M\left(\frac{R}{3}\right)^2}{2} + M\left(\frac{3R}{3}\right)^2$

$$MI = \frac{MR^2}{18} + \frac{4MR^2}{9}$$

$$MI = \frac{MR^2}{2}$$

$$MI = MI(Bigger) - MI(Smaller)$$

$$= \frac{9MR^2}{2} - \frac{MR^2}{2}$$

$$= 4MR^2$$

8. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the n^{th} power of R. If the period of rotation of the particle is T, then:

(A) $T \propto R^{3/2}$ for any n

(B) $T \propto R^{\frac{n}{2}+1}$

(C) $T \propto R^{\frac{n+1}{2}}$

(D) $T \propto R^{n/2}$

Solution: (C)

$$F = \frac{K}{r^n}$$

$$\frac{mV^2}{r} = \frac{K}{r^n}$$

$$V^2 \propto \frac{K}{r^{n-1}}$$

$$V \propto \frac{1}{r^{\frac{n-1}{2}}}$$

$$T = \frac{2\pi r}{V} \propto \frac{r}{V}$$

$$T \propto \frac{r}{\frac{1}{r^{\frac{n-1}{2}}}}$$

$$T \propto R^{\frac{n+1}{2}}$$

9. A solid sphere of radius r made of a soft material of bulk modulus K is surrounded by a liquid in a cylindrical container. A massless piston of area a floats on the surface of the liquid, covering entire cross section of cylindrical container. When a mass m is placed on the surface of the piston to compress the liquid, the fractional decrement in the radius of the sphere, $\left(\frac{dr}{r}\right)$, is:

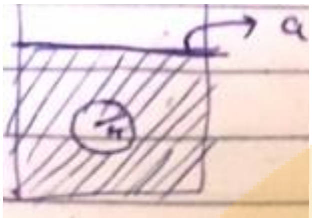
(A) $\frac{Ka}{mg}$

(B) $\frac{Ka}{3mg}$

(C) $\frac{mg}{3kA}$

(D) $\frac{mg}{Ka}$

Solution: (C)



$$P = K\frac{\Delta V}{V}$$

$$P = K.3\left(\frac{\Delta R}{R}\right)$$

$$\frac{mg}{3Ka} = \left(\frac{\Delta R}{R}\right)$$

10. Two moles of an ideal monoatomic gas occupies a volume V at 27^oC. The gas expands adiabatically to a volume 2V. Calculate (a) the final temperature of the gas and (b) change in its internal energy.

(A) (a) $189\ K$ (b) $2.7\ kJ$

(B) (a) $195\ K$ (b) $-2.7\ kJ$

(C) (a) $189\ K$ (b) $-2.7\ kJ$

(D) (a) $195\ K$ (b) $2.7\ kJ$

Solution: (C)

$$TV^{\gamma-1} = Constant$$

$$\gamma = \frac{5}{3}$$

$$300(V)^{\frac{2}{3}} = T(2V)^{\frac{2}{3}}$$

$$T = 189K$$

$$\Delta u = ncVaT$$

$$= 2 \times \frac{3}{2} \times 8.314 \times 11$$

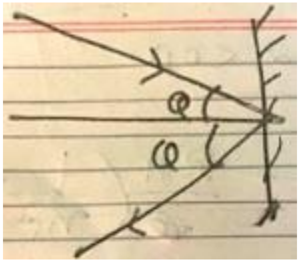
$$= 2.7\ kJ(-ve)$$

11. The mass of a hydrogen molecule is $3.32 \times 10^{-27} kg$. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area $2\ cm^2$ at an angle of 45^o to the normal, and rebound elastically with a speed of $10^3\ m/s$, then the pressure on the wall is nearly:

(A) $2.35 \times 10^3\ N/m^2$ (B) $4.70 \times 10^3\ N/m^2$

(C) $2.35 \times 10^2\ N/m^2$ (D) $4.70 \times 10^2\ N/m^2$

Solution: (A)



$F = 2mnV cos\theta$

$P = \frac{2mnV cos\theta}{A}$

$P = \frac{2 \times 3.32 \times 10^{-27} \times 10^{23} \times 10^{3}}{\sqrt{2} \times 2 \times 10^{-4}}$

$P = 2.35 \times 10^{3} \ N/m^{2}$

12. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of $10^{12}/sec$. What is the force constant of the bonds connecting one atom with the other? (Mole wt. of silver = 108 and Avagadro number = $6.02 \times 10^{23} gm\ mole^{-1}$)

(A) $6.4 \ N/m$

(B) $7.1 \ N/m$

(C) $2.2 \ N/m$

(D) $5.5 \ N/m$

Solution: (B)

$v = 10^{12} s^{-1}$

$w = \sqrt{\frac{k}{m}}$

$T = 2\pi \sqrt{\frac{k}{m}}$

$v = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$

$v = \frac{1}{\pi} \sqrt{\frac{k}{m}}$

$v = \frac{1}{2\pi} \sqrt{\frac{K \times 6.02 \times 10^{23}}{108 \times 10^{-3}}}$

$K = \frac{4\pi^{2} \times 10^{24} \times 108 \times 10^{-3}}{6.02 \times 10^{23}}$

$\approx 7.1 \ N/m$

13. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is $2.7 \times 10^{3} \ kg/m^{3}$ and its Young's

modulus is $9.27 \times 10^{10}\ Pa$. What will be the fundamental frequency of the longitudinal vibrations?

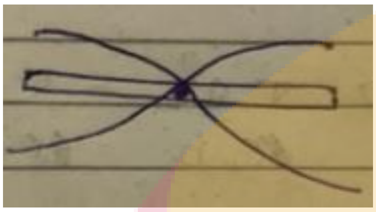
(A) 5 kHz

(B) 2.5 kHz

(C) 10 kHz

(D) 7.5 kHz

Solution: (A)



$$L = \frac{\lambda}{2}$$

$$v = \sqrt{\frac{T}{e}}$$

$$v = 2L\sqrt{\frac{T}{e}}$$

$$v = \frac{1}{2 \times 0.6}\sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^{3}}} \approx 4.88 \times 10^{3} HZ$$

14. Three concentric metal shells A, B and C of respective radii a, b and c (a < b <c) have surface charge densities $+\sigma, -\sigma$ and $+\sigma$ respectively. The potential of shell B is:

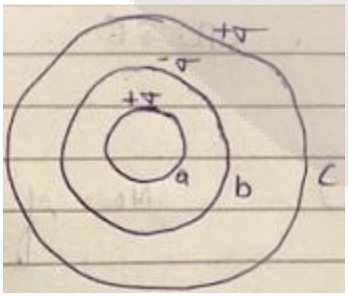
(A) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{a} + c\right]$

(B) $\frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b} + c\right]$

(C) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{b} + a\right]$

(D) $\frac{\sigma}{\varepsilon_0}\left[\frac{b^2-c^2}{c} + a\right]$

Solution: (B)



Potential $\frac{1}{4\pi\varepsilon_0} \frac{\sigma 4\pi R^2}{d}$

$$= \frac{\sigma R^2}{\varepsilon_0 d}$$

Required potential of b

$= \frac{\sigma a^2}{\varepsilon_0 b} - \frac{\sigma b^2}{\varepsilon_0 b} + \frac{\sigma c^2}{\varepsilon_0 c}$

$= \frac{\sigma}{\varepsilon_0}\left(\frac{a^2 - -b^2}{b} + c\right)$

15. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be:

(A) 1.2 nC

(B) 0.3 nC

(C) 2.4 nC

(D) 0.9 nC

Solution: (A)

$Q_1 = CV_1$Q

$Q = KCV$

$Q = \frac{5}{3} \times 90 \times 10^{-12} \times 20$

$5 \times 30 \times 20 = 0.3\ nC$

16. In an a.c circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30t$

$r = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattles current are, respectively:

(A) 50,10

(B) $\frac{1000}{\sqrt{2}}, 10$

(C) $\frac{50}{\sqrt{2}}, 10$

(D) 50,0

Solution: (B)

$e = 100 \sin 30t$

$r = 20 \sin\left(30t - \frac{\pi}{4}\right)$

$Average\ power = V_{rms}\ I_{rms} cos\phi$

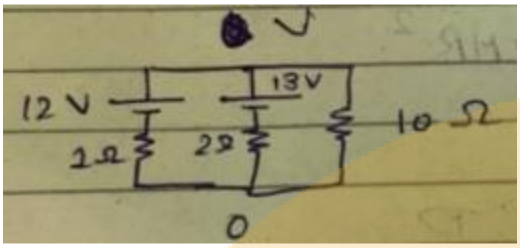
$= \frac{100}{\sqrt{2}} \times \frac{20}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= \frac{1000}{\sqrt{2}}$

17. Two batteries with e.m.f. 12 V and 13 V are connected in parallel across a load resistor of 10Ω. The internal resistances of two batteries are 1Ω and 2Ω respectively. The voltage across the load lies between:

(A) 11.6 V and 11.7 V

(B) 11.5 V and 11.6 V
(C) 11.4 and 11.5 V
(D) 11.7 V and 11.8 V
Solution: (B)



$$\frac{V-12}{1}+\frac{V-13}{2}+\frac{V}{10}=0$$

$$V+\frac{V}{2}+\frac{V}{10}\Rightarrow 12+\frac{13}{2}$$

$$V\Rightarrow\frac{12+\frac{13}{2}}{1+\frac{1}{2}+\frac{1}{10}}=11.562$$

18. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:

(A) $r_e > r_P = r_\alpha$
(B) $r_e < r_p = r_\alpha$
(C) $r_e = r_p < r_\alpha$
(D) $r_e < r_\alpha < r_p$

Solution: (B)

$$r=\frac{mV}{qB}=\frac{\sqrt{2mK}}{qB}$$

$$r\propto\frac{\sqrt{m}}{q}$$

$$r_e=\frac{\sqrt{m_e}}{q}$$

$$r_a=\frac{\sqrt{u_m}}{2q}$$

$$r_p=\frac{\sqrt{m}}{q}$$

$$=\frac{2\sqrt{m}}{2q}$$

$$r_e=r_a<r_p$$

19. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2 . The ration $\frac{B_1}{B_2}$ is:

(A) 2
(B) $\sqrt{3}$
(C) $\sqrt{2}$

(D) $\frac{1}{\sqrt{2}}$

Solution: (C)
Dipole moment = IA
For $Loop_1$
$m = I\pi R^2$
$B_1 = \frac{\mu_0 i}{2r}$
For $Loop_2$
If current is constant m is doubled.
$r = \frac{r}{\sqrt{2}}$
$B_2 = \frac{\mu_0 i}{\frac{r}{\sqrt{2}}}$
$\frac{B_1}{B_2} = \sqrt{2}$

20. For an RLC circuit driven with voltage of amplitude v_m and frequency $\omega_0 = \frac{1}{\sqrt{LC}}$ the current exhibits resonance. The quality factor, Q is given by
(A) $\frac{\omega_0 L}{R}$
(B) $\frac{\omega_0 R}{L}$
(C) $\frac{R}{(\omega_0 C)}$
(D) $\frac{CR}{\omega_0}$
Solution: (A)
Quality factor $= \frac{1}{R}\sqrt{\frac{L}{C}}$
But, $\omega_0 = \frac{1}{\sqrt{LC}}$
So, $\frac{\omega_0 L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$

21. An EM wave from air enters a medium. The electric fields are $\vec{E}_1 = E_{01}\hat{x}\cos\left[2\pi v\left(\frac{z}{c} - t\right)\right]$ in air and $\vec{E}_2 = E_{02}\hat{x}\cos[k(2z - ct)]$ in medium, where the wave number k and frequency v refer to their values in air. The medium is non-magnetic. If ϵ_{r_1} and ϵ_{r_2} refer to relative permittivities of air and medium respectively, which of the following, option is correct?
(A) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 4$
(B) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = 2$
(C) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

(D) $\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{2}$

Solution: (C)

$\frac{\epsilon_{r_1}}{\epsilon_{r_2}} = \frac{1}{4}$

22. Unpolarized light of intensity I passes through an ideal polarizer A. Another identical polarizer B is placed behind A. The intensity of light beyond B is found to be $\frac{1}{2}$. Now another identical polarizer C is placed between A and B. The intensity beyond B is now found to be $\frac{1}{8}$. The angle between polarizer A and C is

(A) 0^o

(B) 30^o

(C) 45^o

(D) 60^o

Solution: (C)

When unpolarized light passes through a polarizer.

Intensity becomes $\frac{I}{2}$

$\frac{I}{2}\cos^2 a = \frac{I}{8}$

$\cos^2 a = \frac{1}{4}$

$a = \frac{\pi}{3}$

23. The angular width of the central maximum in a single slit diffraction pattern is 60^o. The width of the slit is $1\mu m$. The slit is illuminated by monochromatic plane waves. If another slit of same width is made near it, Young's fringes can be observed on a screen placed at a distance 50 cm from the slits. If the observed fringe width is 1 cm, what id slit separation distance?

(i.e. distance between the centres of each slit.)

(A) $25\ \mu m$

(B) $50\ \mu m$

(C) $75\ \mu m$

(D) $100\ \mu m$

Solution: (A)

$25\ \mu m$

24. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let $\wedge_n$ be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants)

(A) $\wedge_n \approx A + \frac{B}{\lambda_n^2}$ (B) $\wedge_n \approx A + B\lambda_n$

(C) $\wedge_n^2 \approx A + B\lambda_n^2$ (D) $\wedge_n^2 \approx \lambda$

Solution: (A)

$\wedge_n \approx A + \frac{B}{\lambda_n^2}$

25. If the series limits frequency of the Lyman series is $v_L{}'$ then the series limits frequency of the P fund series is:

(A) $25v_L$

(B) $16v_L$

(C) $\frac{v_L}{16}$

(D) $\frac{v_L}{25}$

Solution: (D)

$\frac{1}{\lambda} = RHZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$v = f\lambda$

$v = CRHZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$

$v_1 = \frac{Lyman}{CRHZ^2}\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$

P-fund

$v_2 = CRHZ^2\left(\frac{1}{S^2} - \frac{1}{\infty^2}\right)$

$V_2 = \frac{V_1}{25}$

It is found that if a neutron suffers an elastic collinear with deuterium at rest, fractional loss of its energy is P_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is P_c. The values of P_d and P_c are respectively.

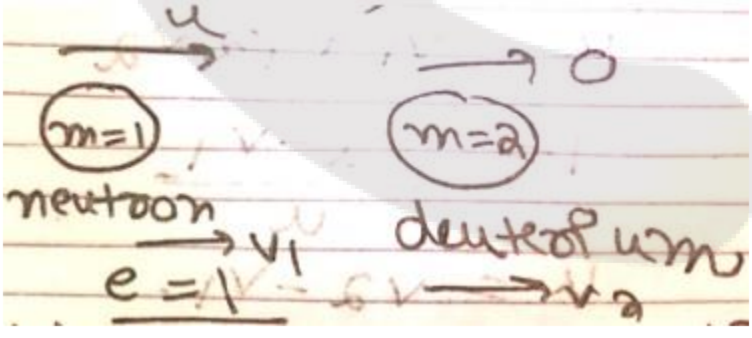
(A) (.89, .28)

(B) (.28, .89)

(C) (0 0)

(D) (0, 1)

Solution: (A)



By momentum conservation

$u = V_1 + 2V_2$

$e = 1 = \frac{V_2 - V_1}{u}$

$u = V_2 - V_1$

$V_2 = \frac{2u}{3}\,; V_1 = -\frac{u}{3}$

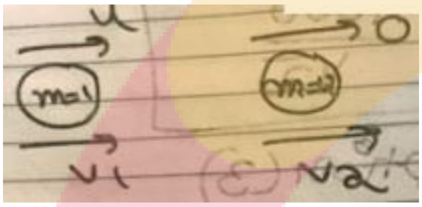
Initial energy $= \frac{1}{2}u^2$

Final energy $= \frac{1}{2} \times 1 \times \frac{u^2}{9}$

$= \frac{u^2}{18}$

Fractional change $= \frac{\frac{u^2}{2} - \frac{u^2}{18}}{\frac{u^2}{2}}$

$= 0.88$



$u = V_1 + 12V_2$

$1 = \frac{V_2 - V_1}{u}$

$u = V_2 - V_1$

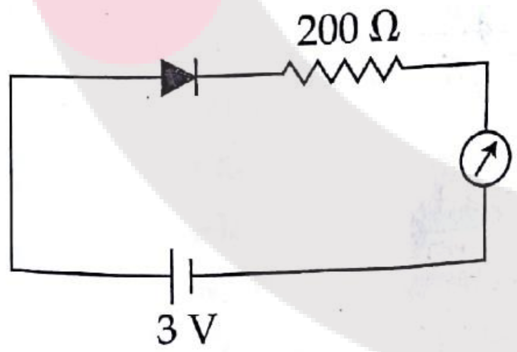
$V_2 = \frac{2u}{13}$

$V_1 = -\frac{11u}{13}$

Change in energy $= u^2 - \left(\frac{11u}{13}\right)^2 \times 100$

$= 0.294$

27. The reading of the ammeter for a silicon diode in the given circuit is:



(A) 0
(B) 15 mA
(C) 11.5 mA
(D) 13.5 mA

Solution: (B)

The diode is in forward bias.

So its resistance = 0

Therefore, $i = \frac{V}{R}$

$i = \frac{3}{200} = 15\,mA$

28. A telephonic communication service is working at carrier frequency of 10 GHz. Only 10% of it is utilized for transmission. How many telephonic channels can be transmitted simultaneously if each channel requires a bandwidth of 5 kHz?

(A) 2×10^3

(B) 2×10^4

(C) 2×10^5

(D) 2×10^6

Solution: (C)

No. of channel $\Rightarrow \frac{10^6}{5 \times 10^3} = 2 \times 10^5$

29. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell.

(A) 1Ω

(B) 1.5Ω

(C) 2Ω

(D) 2.5Ω

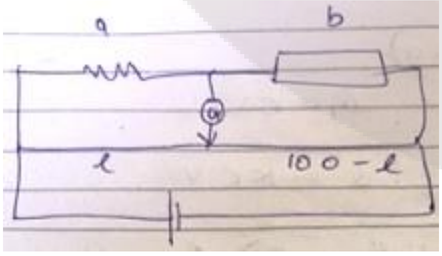
Solution: (B)

2.5Ω

30. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10cm. The resistance of their series combination is $1k\Omega$. How much was the resistance on the left slot before interchanging the resistances?

(A) 990 Ω (B) 505 Ω (C) 550 Ω (D) 910 Ω

Solution: (C)



$a + b = 1000$

$\frac{a}{b} = \frac{l}{100 - l}$

$\frac{b}{a} = \frac{l - 10}{110 - l}$

$$1 = \frac{;(l-10)}{(100-l)(110-l)}$$

PART B – CHEMISTRY

31. The ratio of mass percent of C and H of an organic compound $(C_X H_Y O_Z)$ is 6 : 1. If one molecule of the above compound $(C_X H_Y O_Z)$ contains half as much oxygen as required to burn one molecule of compound $C_X H_Y$ completely to CO_2 and H_2O. The empirical formula of compound $C_X H_Y O_Z$ is:

(A) $C_3H_6O_3$ (B) C_2H_4O (C) $C_3H_4O_2$ (D) $C_2H_4O_3$

Solution: (D)

$$\frac{mass\ of\ C}{mass\ of\ H} = \frac{6}{1}$$

$12x = 6y$

$x : y = 1 : 2$

$$C_y H_y + \left(\frac{\frac{x+y}{2}}{2}\right) O_2 \rightarrow xCO_2 + \frac{y}{2}\, H_2O$$

$$\left(\frac{x+\frac{y}{2}}{2}\right) \times \frac{1}{2} = Z$$

Empirical formula = $C_2H_4O_3$

32. Which type of 'defect' has the presence of cations in the interstitial sites?

(A) Schottky defect

(B) Vacancy defect

(C) Frenkel defect

(D) Metal deficiency defect

Solution: (C)

Frenkel defect

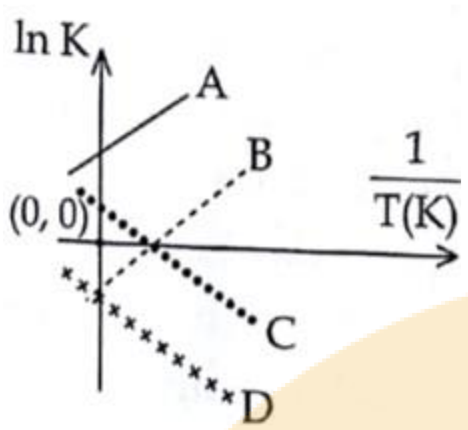
33. According to molecular orbital theory,©ich of the following will not be a viable molecule?

(A) He_2^{2+} (B) He_2^{+} (C) H_2^{-} (D) H_2^{2-}

Solution: (D)

In H_2^{2-} bond order is zero

34. Which of the following lines correctly show the temperature dependence of equilibrium constant, K, for an exothermic reaction?



(A) A and B
(B) B and C
© C and D
(D) A and D
Solution: (A)
In K $= -\frac{\Delta H}{RT} + C$
$K = Ae^{\frac{-\Delta H}{RT}}$

35. The combustion of benzene (l) gives $CO_2(g)$ and $H_2O(l)$. Given that heat of combustion of benzene at constant volume is $-3263.9\ kJ\ mol^{-1}$ at 25^oC; heat of combustion $(in\ kJ\ mol^{-1})$ of benzene at constant pressure will be:
$(R = 8.314\ JK^{-1}\ mol^{-1})$

(A) 4152.6 (B) −452.46 (C) 3260 (D) −3267.6

Solution: (D)
$C_6H_6 + O_2 \rightarrow CO_2 + H_2O$
Use, $\Delta H_p = \Delta H_c + \Delta n_g RT$
$-3267.6\ kJ\ mol^{-1}$

36. For 1 molal aqueous solution of the following compounds, which one will show the highest freezing point?
(A) $[Co(H_2O)_6]Cl_3$
(B) $[Co(H_2O)_5Cl]Cl_2.H_2O$
© $[Co(H_2O)_4Cl_2]Cl.2H_2O$
(D) $[Co(H_2O)_3Cl_3].3H_2O$
Solution: (D)
Melting point is lower for more solute concentration $[Co(H_2O)_3Cl_3].3H_2O$ doesn't dissociate into ions and has least solute conc. Is the answer.

37. An aqueous solution contains 0.10 $M\ H_2S$ and 0.20 M HCl. If the equilibrium constants for the formation of HS^- from H_2S is 1.0×10^{-7} and that of S^{2-} from HS^- ions is 1.2×10^{-13} then the concentration of S^{2-} ions in aqueous solution is:
(A) 5×10^{-8}

(B) 3×10^{-20}

© 6×10^{-21}

(D) 5×10^{-19}

Solution: (B)

$$\begin{array}{lll} H_2S & \rightleftharpoons & HS^- + HH^7 \\ 0.1 - x & & x \ \ (x+0.2) \end{array}$$

$$\frac{x(x+0.2)}{0.1-x} = 10^{-7}$$

$$x = \frac{1}{2} \times 10^{-7}$$

$$\begin{array}{lll} HS^- & \rightleftharpoons & S^2 + H^7 \\ x - y & & y \quad x + y + 0.2 \end{array}$$

$$\frac{y(x+y+0.2)}{(x+y)} = 1.2 \times 10^{-13}$$

$y = 3 \times 10^{-20}$

38. An aqueous solution contains an unknown concentration of Ba^{2+}. When 50 Ml of a 1 M solution of Na_2SO_4 is added, $BaSO_4$ just begins to precipitate. The final volume is 500 Ml. The solubility product of $BaSO_4$ is 1×10^{-10}. What is the original concentration of Ba^{2+}?

(A) $5 \times 10^{-9} M$

(B) $2 \times 10^{-9} M$

© $1.1 \times 10^{-9} M$

(D) $1.0 \times 10^{-10} M$

Solution: ©

$BaSO_4 \rightleftharpoons Ba^{2+} + SO_4^{2-}$

$x \qquad \underbrace{x + 0.05}_{\text{from } Na_2SO_4}$

$$\frac{(x)}{0.5}\frac{(x+0.05)}{0.5} = 10^{-10}$$

X is small

$x = 5 \times 10^{-10}$

$x + 0.05 \approx 0.05$

Required answer $= 5 \times 10^{-10} \div 0.95 = 1.1 \times 10^{-9} M$

39. At 518^oC, the rate of decomposition of a sample of gaseous acetaldehyde, initially at a pressure of 363 Torr, was $1.00\ Torr\ s^{-1}$ when 5% had reacted and $0.50\ Torr\ s^{-1}$ when 33% had reacted. The order of the reaction is:

(A) 2

(B) 3

(C) 1

(D) 0

Solution: (A)

Rate $\alpha(cone)^n$

Let initial concentration be x

$0.95x \rightarrow 1\ torr\ s^{-1}$

$0.67x \rightarrow 0.5\ torr\ s^{-1}$

$\left(\frac{0.95}{0.67}\right)^n = \frac{1}{0.5}$

$\Rightarrow\ \ n = 2$

40. How long (approximate) should water be electrolyzed by passing through 100 amperes current so that the oxygen released can completely burn 27.66 g of diborane? (Atomic weight of B = 10.8 u)

(A) 6.4 hours

(B) 0.8 hours

© 3.2 hours

(D) 1.6 hours

Solution: ©

$B_2H_6 + 3O_2 \rightarrow B_2O_3 + 3H_2O$

$2H_2O \rightarrow 2H_2 + O_2$

$4e^-\ (0.5)$

$\frac{It}{F}$ =number of moles of electron

$\frac{100t}{69500} = 3 \times 4$

$t = 3.2$ hours

41. The recommended concentration of fluoride ion in drinking water is up to 1 ppm as fluoride ion is required to make teeth enamel harder by converting $[3Ca_3(PO_4)_2\,.Ca(OH)_2]$ to:

(A) $[CaF_2]$

(B) $[3(CaF_2).Ca(OH)_2]$

© $[3Ca_3(PO_4)_2.CaF_2]$

(D) $[3\{Ca(OH)_2\}.CaF_2]$

Solution: ©

$[3Ca_3(PO_4)_2.CaF_2]$

42. Which of the following compounds contain(s) no covalent bond(s)?

$KCl, PH_3, O_2, B_2H_6, H_2SO_4$

(A) KCl, B_2H_6, PH_3

(B) KCl, H_2SO_4

© KCl

(D) KCl, B_2H_6

Solution: ©

KCl

43. Which of the following are Lewis acids?

(A) PH_3 and BCl_3 (B) $AlCl_3$ and $SiCl_4$
© PH_3 and $SiCl_4$ (D) BCl_3 and $AlCl_3$

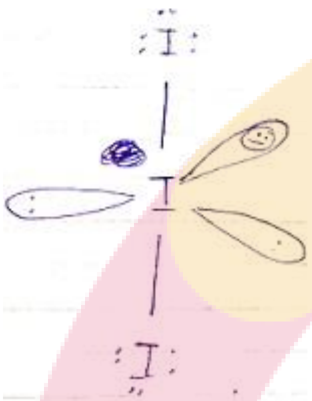
Solution: (B,D)

Has sextet configured

44. Total number of lone pair of electrons in I_3^- ions is:

(A) 3 (B) 6 (C) 9 (D) 12

Solution: ©



45. Which of the following salts is the most basic in aqueous solution?

(A) $Al(CN)_3$
(B) CH_3COOK (C) $FeCl_3$ (D) $Pb(CH_3COO)_2$

Solution: (B)

CH_3COOK

46. Hydrogen peroxide oxidises $[Fe(CN)_6]^{4-}$to $[Fe(CN)_6]^{3-}$ in acidic medium but reduces $[Fe(CN)_6]^{3-}$to$[Fe(CN)_6]^{4-}$ in alkaline medium. The other products formed are, respectively:

(A) $(H_2O + O_2)$ and H_2O
(B) $(H_2O + O_2)$ and $(H_2O + OH^-)$
© H_2O and $(H_2O + O_2)$
(D) H_2O and $(H_2O + OH^-)$

Solution: ©

H_2O and $(H_2O + O_2)$

47. The oxidation states of Cr in $[Cr(H_2O)_6]Cl_3$, $[Cr(C_6H_6)_2]$, and $K_2[Cr(CN)_2(O)_2(O)_2(NH_3)]$ respectively are:

(A) $+3, +4$ and $+6$
(B) $+3, +2$ and $+4$
© $+3, 0$ and $+6$
(D) $+3, 0$ and $+4$

Solution: ©

$+3, 0$ and $+6$

48. The compound that does not produce nitrogen gas by thermal decomposition is:

(A) $Ba(N_3)_2$ (B) $(NH_4)_2Cr_2O_7$
© NH_4NO_2 (D) $(NH_4)_2SO_4$

Solution: (D)

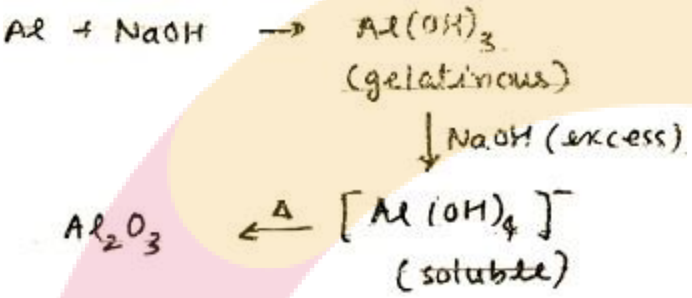
$(NH_4)_2SO_4$

49. When metal 'M' is treated with NaOH, a white gelatinous precipitate 'X' is obtained, which is soluble in excess of NaOH. Compound 'X' when heated strongly gives an oxide which is used in chromatography as an adsorbent. The metal 'M' is:

(A) Zn (B) Ca (C) Al (D) Fe

Solution: ©

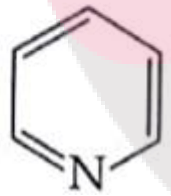
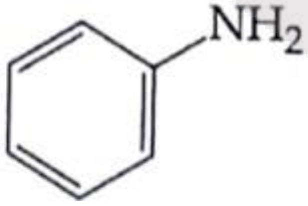
Alumina (Al_2O_3) is used in chromatography

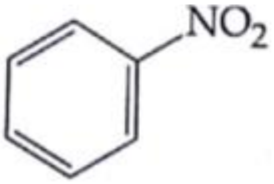


50. Consider the following reaction and statement:

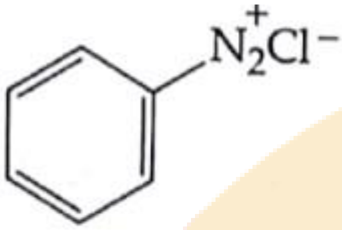
$[Co(NH_3)_4Br_2]^+ + Br^- \rightarrow [Co(NH_3)_3Br_3] + NH_3$

(D) Two isomers are produced if the reactant complex ion is a cis – isomer.

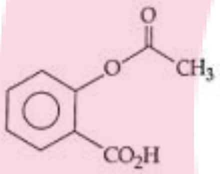
(ii) Two isomers are produced if the reactant complex ion is a trans – isomer.
(iii) Only one isomer is produced if the reactant complex ion is a trans – isomer.
(iv) Only one is produced if the reactant complex ion is a cis – isomer.
The correct statements are:
(A) (i) and (ii)
(B) (i) and (iii)
(C) (iii) and (iv)
(D) (ii) and (iv)
Solution: (B)
Trans → only meridonial
Cis → fac and mer
51. Glucose on prolonged heating with HI gives:
(A) n – Hexane
(B) 1 – Hexene
(C) Hexanoic acid
(D) 6 – iodohexanal
Solution: (A)
$C_6H_{12}O_6 + HI + \Delta \rightarrow C_6H_{14}$
52. The trans-alkenes are formed by the reduction of alkynes with:
(A) $H_2 - Pd/C, BaSO_4$
(B) $NaBH_4$ (C) $Na/liq.NH_3$ (D) $Sn - HCl$
Solution: ©
$Na/liq.NH_3$
53. Which of the following compounds will be suitable for Kjeldahl's method for nitrogen estimation?
(A)

(B)

©



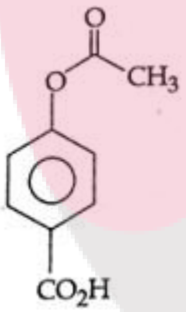
(D)



Solution: (B)
Kjedlahl's method can't be used for testing nitrogen in azo nitro and nitrogen in aromatic ring.
54. Phenol on treatment with CO_2 in the presence of NaOH followed by acidification produces compound X as the major product. X on treatment with $(CH_3CO)_2$ in the presence of catalytic amount of H_2SO_4 produces:
(A)



(B)



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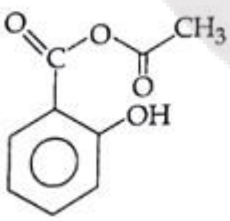
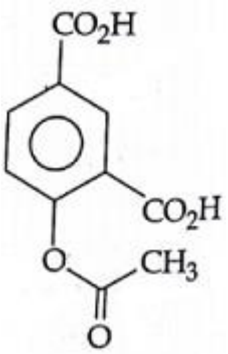
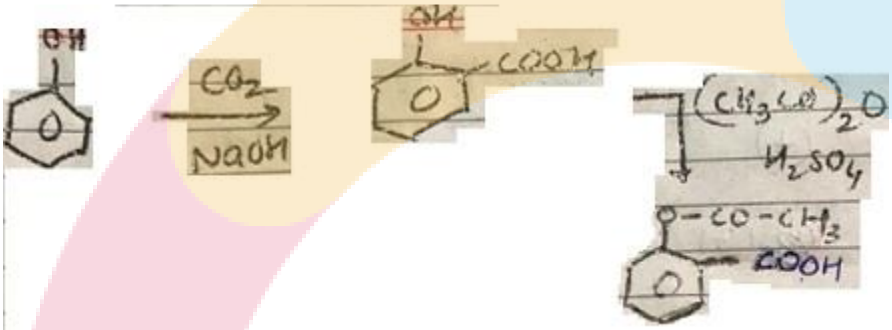


(D)



Solution: (A)



55. An alkali is titrated against an acid with methyl orange as indiactor, which of the following is a correct combination?

(A)

Base	Acid	End point
Weak	Srtong	Colourless to pink

(B)

Base	Acid	End point
Strong	strong	Pinkish red to yellow

©

Base	Acid	End point
Weak	Strong	Yellow to pinkish red

(D)

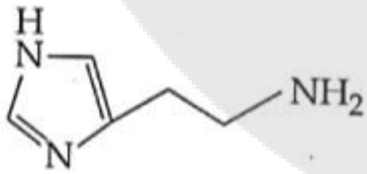
Base	Acid	End point
Strong	Strong	Pink to colourless

Solution: ©

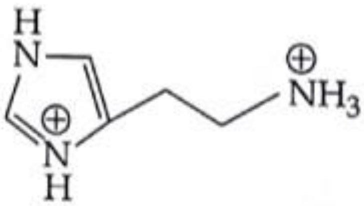
Methyl orange is yellow in basic and reddish id acid

56. The predominant form of histamine presenr in human blood is $(pK_a, \text{Histidine} = 60.)$

(A)



(B)



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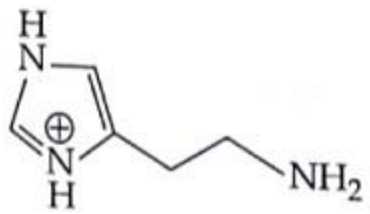
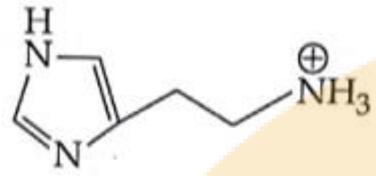


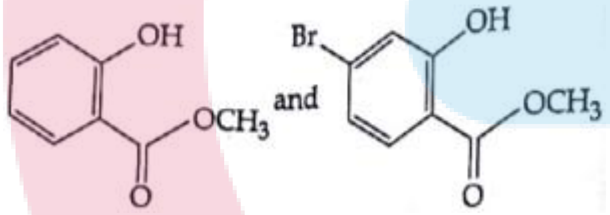
(D)



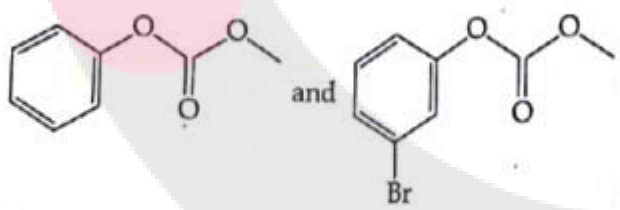
Solution: (D)

57. Phenol reacts with methyl chloroformate in the presence of $NaOH$ to form product A. A reacts with Br_2 to form product B. A and Bare respectively:

(A)



(B)



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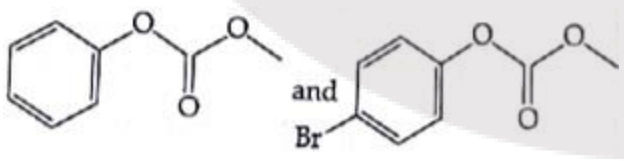
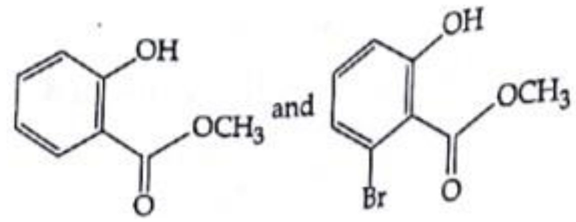
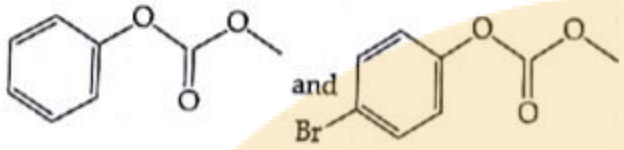


(D)



Solution: ©

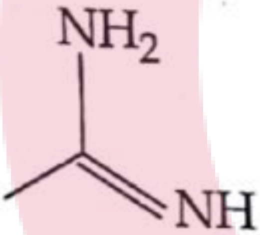


58. The increasing order of basicity of the following compounds is:

(i)

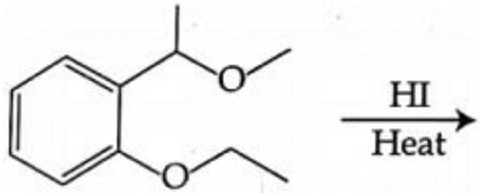
(ii)

(iii)

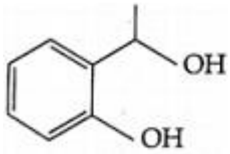


(iv)

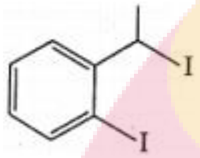
(A) (i) < (ii) < (iii) < (iv)
(B) (ii) < (i) < (iii) < (iv)
© (ii) < (i) < (iv) < (i)
(D) (iv) < (ii) < (i) <(iii)
Solution: ©
(iii) Is stabilized by resonance when lone pair is donated
(ii) Is most unstable because position charge in double bond is unstable
(i) Is more stable than (d)
59. The major product formed in the following reaction is:



(A)



(B)



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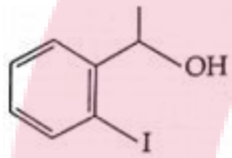
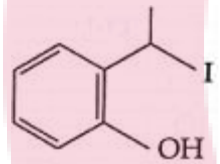
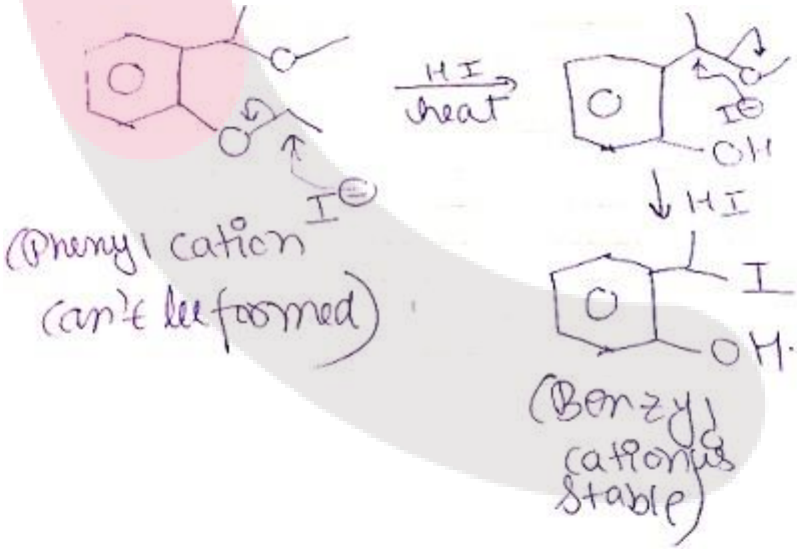


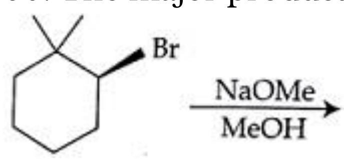
(D)



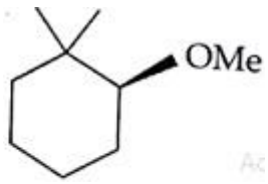
Solution: (A)



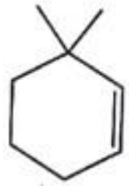
60. The major product of the following reaction is:



(A)



(B)



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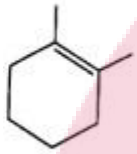
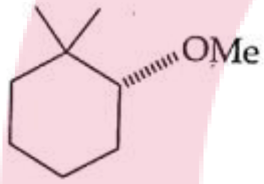
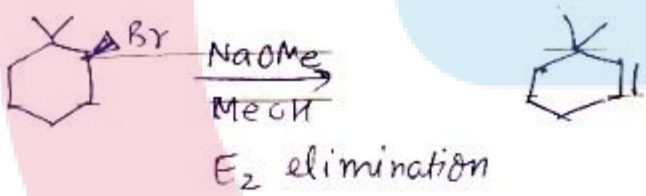


(D)



Solution: (B)



PART C – MATHEMATICS

61. Two sets A and B are as under:

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \leq 36\}$. Then:

(A) $B \subset A$

(B) $A \subset B$

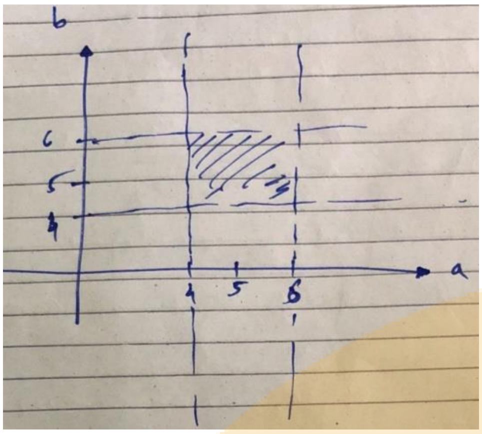
(C) $A \cap B = \phi$ (an empty set)

(D) Neither $A \subset B$ nor $B \subset A$

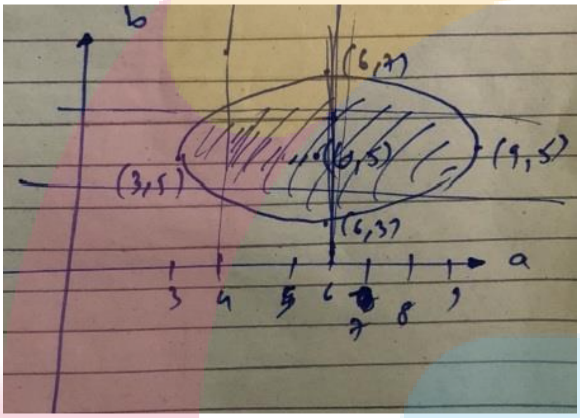
Solution: (B)

$A = \{(a, b) \in R \times R : |a - 5| < 1 \text{ and } |b - 5| < 1\}$;

$$\frac{(a-6)^2}{9} + \frac{(b-5)^2}{4} \leq 1$$



$\frac{4}{9}+\frac{1}{4}\leq 1$ (4, 4) lies in B



(4,6) lies in B

(6, 4) lies in B (6,3) lies in A

62. Let $S=\{x\in R: x\geq 0 \text{ and } 2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S:

(A) Is an empty set

(B) Contains exactly one element

(C) Contains exactly two elements

(D) Contains exactly four elements

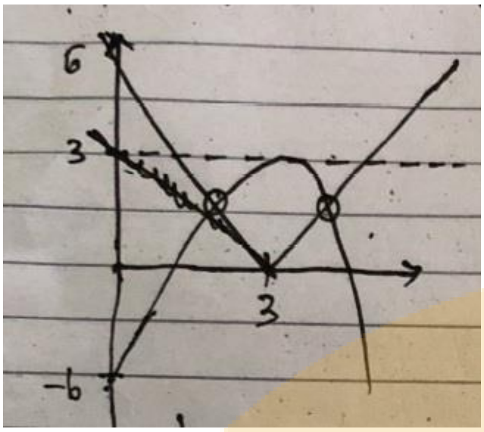
Solution: (C)

$2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0$

Let $\sqrt{x}=t>0$

$2|t-3|+t(t-6)+6=0$

$=2|t-3|=-(t^2-6t+6)=-(t^2-6t+9)+3$



Has 2 solutions for t = 2 solutions for x

63. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to:

(A) -1 (B) 0 (C) 1 (D) 2

Solution: (C)

$x^2 - x + 1 = 0$

Roots are $-w, -w^2$

$\alpha^{101} + \beta^{107} = (-w)^{101} + (-w^2)^{107}$

$= -w^{101} - w^{214}$

$= -w^2 - w$

$= 1$

As $(1 + w + w^2 = 0)$

64. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$, then the ordered pair (A, B) is equal to:

(A) $(-4, -5)$ (B) $(-4, 3)$ (C) $(-4, 5)$ (D) $(4,5)$

Solution: (C)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A + Bx)(x - A)^2$$

Put $x = 0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow 4$

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx - 4)(x + 4)^2$$

$$\begin{vmatrix} 1-\frac{4}{x} & 2 & 2 \\ 2 & 1-\frac{4}{x} & 2 \\ 2 & 2 & 1-\frac{4}{x} \end{vmatrix} = \left(B - \frac{4}{x}\right)\left(1 + \frac{4}{x}\right)^2$$

Put $x \to \infty \Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B = 5$

Ordered pair (A, B) is (-4, 5)

65. If the system of linear equations

$x+ky+3z=0$

$3x+ky-2z=0$

$2x+4y-3z=0$

has a non-zero solution (x,y,z), then xz/y^2 is equal to:

(A) -10 (B) 10 (C) -30 (D) 30

Solution: (B)

$\begin{vmatrix} 1 & k & 3 \\ 3 & k & -2 \\ 2 & 4 & -3 \end{vmatrix} = 0$

$(-3k+8)-3(-3k-12)+2(-5k)=0$

$-4k+44=0 \qquad k=11$

$x+11y+3z=0$ .... (i)

$3x+11y-2z=0$ ..... (ii)

$2x+4y-3z=0$ ... (iii)

Equation (ii) and (i)

$2x-5z=0$

$x=\frac{5z}{2}=-5y$

Put it in

$2z+4y=0$

$z=-2y$

$\frac{xz}{y^2}=\frac{(-2y)\times(-5y)}{y^2}=10$

66. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is:

(A) At least 1000

(B) Less than 500

(C) At least 500 but less than 750

(D) At least 750 but less than 1000

Solution: (A)

N_1, N_2, D, N_3, N_4

N → Novels

D→ Dictionary

D can be chosen in 3 ways

$N_1 \to 6$

$N_2 \to 5$

$N_3 \to 4$

$N_4 \rightarrow 3$

Total ways $= 3 \times 6 \times 5 \times 4 \times 3 = 1080$

67. The sum of the co-efficient of all odd degree terms in the expansion of $\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5, (x > 1)$ is:

(A) -1 (B) 0 (C) 1 (D) 2

Solution: (D)

$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5 \; x > 1$

$f(x) = 2[x^5 + {}^5C_3 x^3 (x^3 - 1) + {}^5C_3 x (x^3 - 1)^2]$

For sum of add power coefficient it

Put $\frac{f(1) - f(-1)}{2}$

$f(1) = 2[10]$

$f(-1) = 2[-1 + {}^5C_3 \times 20 - {}^5C_3 \times 4]$

$f(-1) = 2[-1 + 20 - 20] = -2$

$$\frac{f(1) - f(-1)}{2} = \frac{2 - (-2)}{2} = 2$$

68. Let $a_1, a_2, a_3, \dots \dots, a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \dots + a_{17}^2 = 140m$, then m is equal to:

(A) 66 (B) 68 (C) 34 (D) 33

Solution: (C)

Let $a_1 = 1$ and d = common difference.

$a_n = 1 + (n - 1)d$

Given, $\sum_{k=0}^{12} a_{4k+1} = 416$

$a_1 + a_5 + a_9 + \cdots a_{49} = 416$

$a + a + 4d + d + 8d + \cdots + d + 48d = 416$

$$13a + \frac{4 \times 23 \times 13d}{2} = 416$$

$a + 24d = 32$ ... (i)

$a_9 + a_{43} = 66$

$a + 8d + a + 42d = 66$

$2a + 50d = 66$

$a + 25d = 33$ ... (ii)

On solving we get,

$d = 1, a = 8$

Now, $a_1^2 + a_2^2 + \cdots a_{17}^2 = 140m$

$a^2 (a + d)^2 + \cdots (a + 16d)^2$

$\Rightarrow a^2 + a^2 + d^2 + 2d + a^2 + 4d^2 + 4d + \cdots a^2 + 256d^2 + 32d$

$\Rightarrow 17a^2 + d^2(1 + 4 + 9 + \cdots 256) + 2d(1 + 2 + \cdots 16)$

$$\Rightarrow 17a^2 + d^2 \times \frac{16 \times 17 \times 33}{6} + 2d\left(\frac{16 \times 17}{2}\right)$$

$\Rightarrow 17 \times 64 + 1496 + 272 \times 8$

$\Rightarrow 1088 + 1496 + 272 \times 8$
$= 4760$

69. Let A be the sum of the first 20 terms and B be the sum of the first 40 terms of the series
$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \cdots$.
If $B - 2A = 100\lambda$, then λ is equal to:
(A) 232 (B) 248 (C) 464 (D) 496
Solution: (C)
$S = 1 + 2.2^2 + 3^2 + 3 + 2.4^2 + 5^2 + 2.6^2 + \cdots$
$S = 1 + 2^2 + 3^2 + 4^2 + \cdots$
$S_1 = \frac{20\times21\times41}{6} + \frac{4\times10\times11\times21}{6}$
$S - 1\ A = 4410$
$B = \frac{40 \times 41 \times 81}{6} + \frac{4 \times 20 \times 21 \times 41}{6}$
$= 33620$
$B - 2A = 24800$
$= 100\lambda = 7\lambda = 248$

70. For each $\in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim\limits_{x\to0+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots + \left[\frac{15}{x}\right]\right)$
(A) Is equal to 0
(B) Is equal to 15
(C) IS equal to 120
(D) Does not exist (in R)

Solution: (C)
$\lim\limits_{x\to0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \cdots \left[\frac{15}{x}\right]\right)$
$\lim\limits_{x\to0^+} x\left(\frac{1}{x} + \frac{2}{x} + \cdots \frac{15}{x}\right)$
$\left[\frac{1}{x}\right] - \left[\frac{2}{x}\right] \ldots. \left[\frac{15}{x}\right] \to$ these terms will be neglected as $\lim\limits_{x\to0^+} x\left\{\left[\frac{1}{x}\right] - \left[\frac{2}{x}\right] \ldots. \left[\frac{15}{x}\right]\right\}$ lying between 0 and 1
$\lim\limits_{x\to0^+} x\left(\frac{1 + 2 + \cdots 15}{x}\right)$
$1 + 2 + \cdots 15 = \frac{15 \times 16}{2}$
$= 120$

71. Let $S = \{t \in R : f(x) = |x - \pi| \cdot (e^{|x|} - 1)\sin|x|$ is not differentiable at $t\}$. Then the set S is equal to:
(A) ϕ (an empty set) (B) $\{0\}$
(C) $\{\pi\}$ (D) $\{0, \pi\}$
Solution: (A)

$f(x) = (x - \pi).\left(e^{|x|} - 1\right).sin|x|$

$f'(\pi + h) = \lim_{x \to h+\pi} \frac{f(\pi + h) - f(\pi)}{h} = \frac{|h| \sin(\pi + h)\,(e^{\pi} - 1)}{h} = 0$

$f'(\pi + h) = \lim_{x \to h+\pi} \frac{f(\pi) - f(\pi - h)}{h} = \frac{|h| \sin(\pi + h)\,(e^{\pi} - 1)}{h} = 0$

Differentiable at π

$f'(0 + h) = \lim_{L \to 0} \frac{f(h) - f(0)}{h} = |\pi| \times \frac{|h|sin|h|}{h} = 0$

$f'(0 - h) = \lim_{L \to 0} \frac{f(0) - f(h)}{h} = |\pi| \times \frac{|h|sin|h|}{h} = 0$

Differentiable at 0

F(x) is d9fferentiable everywhere

$S = \phi$

72. If the curves $y^2 - 6x, 9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is:

(A) 6 (B) $\frac{7}{2}$ (C) 4 (D) $\frac{9}{2}$

Solution: (A)

$y^2 = 6x$

$9x^2 + by^2 = 16$

$2y_1 \frac{dy}{dx} = 6$

$\Rightarrow \frac{dy}{dx} = \frac{3}{y_1}$

$18x_1 + 2by_1 \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} = \frac{-9x_1}{by_1}$

$\left(\frac{3}{y_1}\right) \times \left(\frac{-9x_1}{by_1}\right) = -1$

$27x_1 = by_1^2$

$y_1^2 = 6x_1$

$27x_1 = b(6x_1)$

$x_1 \neq 0$

$b = \frac{27}{6} = \frac{9}{2}$

73. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}, x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$is:

(where C is a constant of integration)

(A) 3 (B) -3 (C) $-2\sqrt{2}$ (D) $2\sqrt{2}$

Solution: (D)

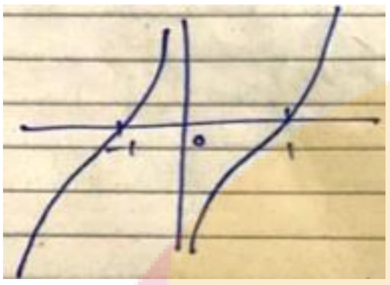
$f(x) = x^2 + \frac{1}{x^2} = \left(x - \frac{1}{x}\right)^2 + 2$

$g(x) = x - \frac{1}{x}$

$x \in R - \{-1.0, 1\}$

Let $x - \frac{1}{x} = t$

$h(x) = \frac{f(x)}{g(x)} = \frac{t^2+2}{t} = t + \frac{2}{t}$



$p(x) = x - \frac{1}{x}$

$p'(x) = 1 + \frac{1}{x^2}$

$t = x - \frac{1}{x}$

Since $x \neq (-1, 0, 1)$

$t \in R - \{0\}$

$h(x) = t + \frac{2}{t}$

$AM \div \geq GM$

Min of $h(x) \Rightarrow \frac{t+\frac{2}{t}}{2} \geq \sqrt{t \times \frac{2}{t}}$

$t + \frac{2}{t} \geq 2\sqrt{2}$

Local minimum $= 2\sqrt{2}$

74. The integral $\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$ is equal to:

(A) $\frac{1}{3(1+\tan^3 x)} + C$ (B) $\frac{-1}{3(1+\tan^3 x)} + C$

(C) $\frac{1}{1+\cot^3 x} + C$ (D) $\frac{-1}{1+\cot^3 x} + C$

Solution: (B)

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x)^2} dx$$

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^2 x\,(\sin^2 x + \cos^2 x) + \cos^3 x\,(\cos^2 x + \sin^2 x))^2} dx$$

$$\int \frac{\sin^2 x \cos^2 x}{(\sin^3 x + \cos^3 x)^2} dx$$

$$\int \frac{\sin^2 x \cos^2 x}{(\cos^3 x)^2 (\tan^3 x + 1)^2} dx$$

$$\int \frac{\tan^2 x \sec^2 x}{(\tan^3 x + 1)^2} dx$$

Put $\tan^3 x + 1 = t$

$$dt = 3\tan^2 x \sec^2 x \, dx$$

$$\Rightarrow \int \frac{dt}{3(t)^2}$$

$$\Rightarrow \frac{-1}{3t} + C$$

$$\Rightarrow \frac{-1}{3(\tan^3 x + 1)} + C$$

75. The values of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is:

(A) $\frac{\pi}{8}$ (B) $\frac{\pi}{2}$ (C) 4π (D) $\frac{\pi}{4}$

Solution: (D)

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ ......(i)

$\int_a^b f(x) \, dx$

$$\int f(x) = \int (a + b - x)$$

$$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2(-x)}{1 + 2^{-x}} dx$$

$I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{2^x \sin^2 x}{1+2^x} dx$ .....(ii)

Equations (i) and (ii)

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x \, (1 + 2^x)}{(1 + 2^x)} \, dx$$

$$2I = 2 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x \, dx$$

$$I = \int_0^{\frac{\pi}{2}} \cos^2 x \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} (\sin^2 x + \cos^2 x) \, dx$$

$$2I = \int_0^{\frac{\pi}{2}} 1 \, dx$$

$I = \frac{\pi}{2} \times \frac{1}{2} = \frac{\pi}{4}$

76. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and $\alpha, \beta (\alpha < \beta)$ be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. units) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha, x = \beta$ and $y = 0$, is:

(A) $\frac{1}{2}(\sqrt{3} - 1)$ (B) $\frac{1}{2}(\sqrt{3} + 1)$

(C) $\frac{1}{2}(\sqrt{3} - \sqrt{2})$ (D) $\frac{1}{2}(\sqrt{2} - 1)$

Solution: (A)

Given $g(x) = \cos x^2, f(x) = \sqrt{x}$

$18x^2 - 9\pi x + \pi^2 = 0$

The roots : $x = \frac{9\pi \pm \sqrt{81\pi^2 - 72\pi^2}}{2 \times 18}$

$x = \frac{9\pi \pm \sqrt{9\pi^2}}{36}$

$x = \frac{(9 \pm 3)\pi}{36}$

$x = \frac{\pi}{3}, \frac{\pi}{6}$

$y = goflus$

As $\cos(\sqrt{x})^2 = \cos x$

Area $= \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \; dx$

$= [\sin x]_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$\Rightarrow \frac{\sqrt{3}}{2} - \frac{1}{2}$

77. Let $y = y(x)$ be the solution of the differential equation $\sin x \frac{dy}{dx} + y \cos = 4x, x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to:

(A) $\frac{4}{9\sqrt{3}}\pi^2$ (B) $\frac{-8}{9\sqrt{3}}\pi^2$ (C) $-\frac{8}{9}\pi^2$ (D) $-\frac{4}{9}\pi^2$

Solution: (C)

$\frac{dy}{dx} + y\cot x = \frac{x}{\sin x} \quad x \in (0, \pi)$

$I.F = e^{\int \cot x} = e^{\ln \sin x} = \sin x$

$\therefore y \sin x = \int \frac{x}{\sin x} . \sin x \, dx$

$y\sin x = 2x^2 + x$

$y\left(\frac{\pi}{2}\right) = 0$

$0 \times 1 = 3 \times \left(\frac{\pi}{2}\right)^2 + c$

$= c = -\frac{\pi^2}{2}$

$\left(\frac{\pi}{6}\right) = \frac{2\times\left(\frac{\pi}{6}\right)^2 - \left(\frac{\pi}{2}\right)^2}{\frac{1}{2}} = \left(\frac{\pi^2}{18}\right) - \left(\frac{\pi^2}{2}\right) \times 2$

$= \frac{\pi^2}{9} - \pi^2$

$= -\frac{8\pi^2}{9}$

78. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is:

(A) $3x + 2y = 6$ (B) $2x + 3y = xy$

(C) $3x + 2y = xy$ (D) $3x + 2y = 6xy$

Solution: (C)

Let the equation of line be

$\frac{x}{p} + \frac{y}{q} = 1$

(p, 0), (O,q) are the points & Q representing passes through (2, 3)

$\Rightarrow \frac{2}{p} + \frac{3}{q} = 1$

Q locus of R is (p, q)

$p = x, q = y$

$\frac{4}{2x} + \frac{6}{2y} = 1$

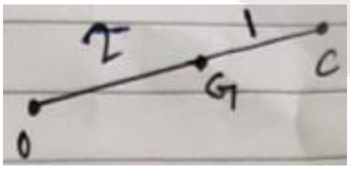
$y + 6x = 2xy$

$= 2y + x = xy$

79. Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is:

(A) $\sqrt{10}$ (B) $2\sqrt{10}$ (C) $3\sqrt{\frac{5}{2}}$ (D) $\frac{3\sqrt{5}}{2}$

Solution: (C)



Centroid divides the line joining O and C in the ratio 2 : 1

A = O, B = G

$OG = \sqrt{(3+3)^2 + (5-3)^2} = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$

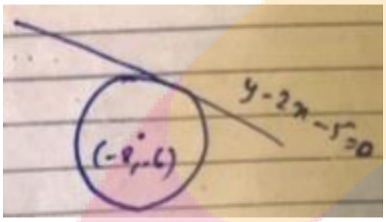
$OC = \frac{3}{2} \times 2\sqrt{10} = 3\sqrt{10}$

Radius $= \frac{OC}{2} = 3\sqrt{\frac{10}{4}} = 3\sqrt{\frac{5}{2}}$

80. If the tangent at $(1,7)$ to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is:

(A) 195 (B) 185 (C) 85 (D) 95

Solution: (D)



$$2x = \frac{dy}{dx}$$

$$\left.\frac{dy}{dx}\right|_{(1,7)} = 2 \times 1 = 2$$

$$\frac{y-7}{x-1} = 2$$

$$y = 2x + 5$$

Distance from centre =

$$\left[\frac{-6 - 2(-8) - 5}{\sqrt{1^2 + 2^2}}\right]$$

$$\left|\frac{5}{\sqrt{5}}\right| = \sqrt{5}$$

Radius of circle $= \sqrt{8^2 + 6^2 - c}$

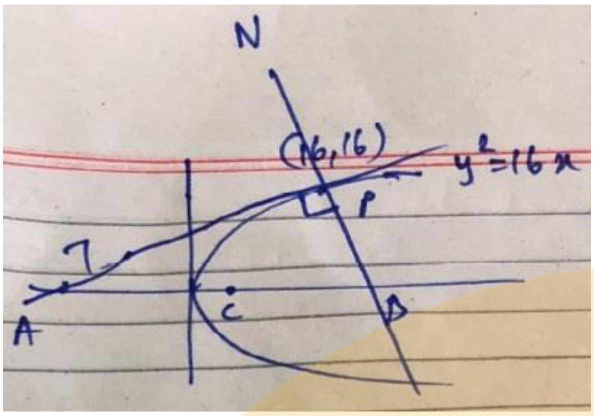
$$\sqrt{100 - c} = \sqrt{5}$$

$$c = 95$$

81. Tangent and normal are drawn at $P(16,16)$ on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is:

(A) $\frac{1}{2}$ (B) 2 (C) 3 (D) $\frac{4}{3}$

Solution: (**B**)



Equation of tangent $= ty = x + at^2$

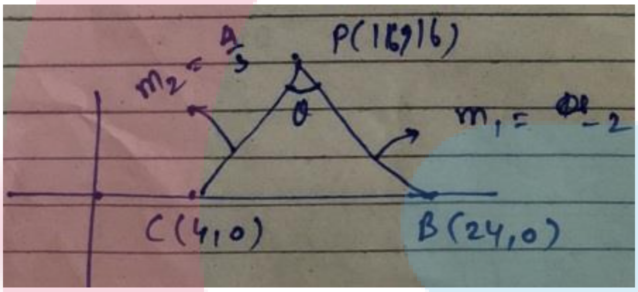
Equation of normal $= y + xt = 2at + at^3$

Tangent $= 2y = x + 16\ (-16,0)$

Normal $= y + 2x = 48\ (24,0)$

Centre - $\left(\frac{24-16}{2}, 0\right)$

Centre $= (4,0)$



$$\tan\theta = \left|\frac{\frac{4}{3}+2}{1-2\times\frac{4}{3}}\right| = \left|\frac{10}{-5}\right| = 2$$

82. Tangents are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point $T(0,3)$ then the area (in sq. units) of ΔPTQ is:

(A) $45\sqrt{5}$ (B) $54\sqrt{3}$ (C) $60\sqrt{3}$ (D) $36\sqrt{5}$

Solution: (A)

$4x^2 - y^2 = 36$

$$\frac{x^2}{9} - \frac{y^2}{36} = 1$$

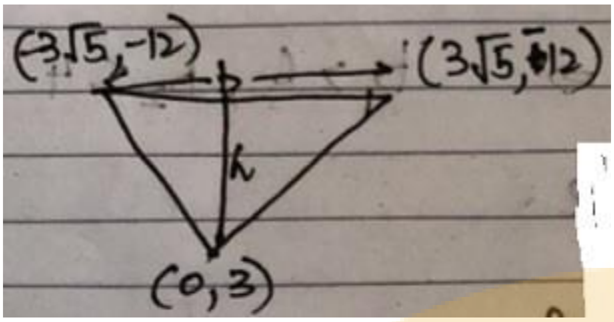
Equation of tangent: $\frac{xx_1}{9} - \frac{yy_1}{36} = 1$

$x = 0, y = 3$

$$\Rightarrow \frac{x_1 \times 0}{9} - \frac{y_1 \times 3}{36} = 1$$

$\Rightarrow y_1 = -12$

$$x_1 = \sqrt{\frac{36 + (144)^2}{4}} = \pm 3\sqrt{5}$$



$h = 3 + 12 = 15$

$b = 6\sqrt{5}$

Area $= \frac{1}{2} \times 6\sqrt{5} + 15 = 45\sqrt{5}$

83. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0, x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0, 3x - y + 2z - 1 = 0$, then the distance of the origin from the plane, containing the lines L_1 and L_2 is:

(A) $\frac{1}{4\sqrt{2}}$ (B) $\frac{1}{3\sqrt{2}}$ (C) $\frac{1}{2\sqrt{2}}$ (D) $\frac{1}{\sqrt{2}}$

Solution: (B)

Dr'S of L_1

$$\begin{vmatrix} i & j & k \\ 2 & -2 & 3 \\ 1 & -1 & 1 \end{vmatrix}$$

$i(1) - j\,(-1) + k(0)$

$(1, 1, 0)$

But $y = 0$

$\left.\begin{matrix} 2x + 3z = 2 \\ x + z = -1 \end{matrix}\right] \rightarrow$ (-5, 0, 4) point

Equation of L_1

$$\frac{x+5}{1} = \frac{y}{1} = \frac{z-4}{0} = \alpha$$

Similarly for L_2

$$\frac{x - \frac{7}{5}}{3} = \frac{y}{-5} = \frac{z + \frac{8}{5}}{-7} = \beta$$

Point of intersection

(-1, 4, 4)

$$\hat{n}\text{ (Plane)} \begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 3 & -5 & -7 \end{vmatrix} = (-7, 7, -2)$$

Equation of plane

$-7x + 7y - 8z = C.$

(-1, 4, 4)

$c = 3$

Equation of plane

$7x - 7y + 2\,z + 3 = 0$

Distance from origin

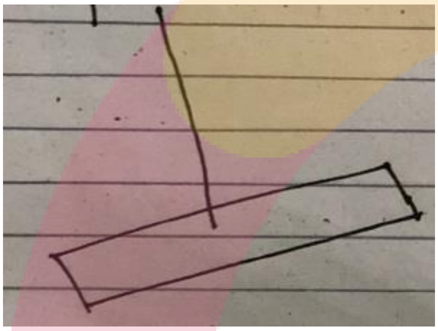
$= \frac{3}{\sqrt{162}} = \frac{3}{9\sqrt{2}}$

$= \frac{1}{3\sqrt{2}}$

84. The length of the projection of the line segment joining the points (5, -1, 4) and (4, -1, 3) on the plane, $x + y + z = 7$ is:

(A) $\frac{2}{\sqrt{3}}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) $\sqrt{\frac{2}{3}}$

Solution: (D)



$A(5, -1, 4)\,;\, b(4, -1, 3)$

Vector joining A, B

$AB = 1\hat{\imath} + 0\hat{\jmath} + 1\hat{k}$

Normal vector $\hat{\imath} + \hat{\jmath} + \hat{k}$

$|AB| = \sqrt{2}$

$|AB_1| = \frac{1+1}{\sqrt{3}} = \frac{2}{\sqrt{3}}$

$AB_{11} = \sqrt{2 - \frac{4}{3}} = \sqrt{\frac{2}{3}}$

85. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{\imath} + 3\hat{\jmath} - \hat{k}$ and $\vec{b} = \hat{\jmath} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u} \cdot \vec{b} = 24$, then $|\vec{u}|^2$ is equal to:

(A) 336 (B) 315 (C) 256 (D) 84

Solution: (A)

$\vec{u} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$

$\begin{vmatrix} x & y & z \\ 2 & 3 & -1 \\ 0 & 1 & 1 \end{vmatrix} = 0$

$4x - 2y + 2z = 0$

$4y - 2z = 0$

$2x - y + z = 0$

$y = 27$

$\vec{u} \, . \, \vec{a} = 0 \;\; 2x + 3y - z = 0$

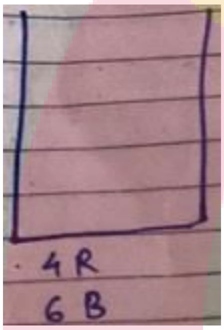
$y + z = 24$

$z = 8$
$y = 16$
$x = 4$
$\therefore\ 4^2 + 8^2 + 16^2$
$= 336$

86. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returned to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is:

(A) $\frac{3}{10}$ (B) $\frac{2}{5}$ (C) $\frac{1}{5}$ (D) $\frac{3}{4}$

Solution: (B)



P(red, red) + p(black, red)

$$\frac{4}{(4+6)} \times \frac{6}{(4+6+2)} + \frac{6}{6+4} \times \frac{4}{6+4+2}$$

$$= \frac{24+24}{10 \times 12} = \frac{48}{10 \times 12} = \frac{2}{5}$$

87. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, \ldots, x_9$ is:

(A) 9 (B) 4 (C) 2 (D) 3

Solution: (C)

$$\sum_{i=1}^{9}(x_i - 5) = 9$$

Mean $= \frac{9}{9} + 5 = 6$

$\overline{x} = 6$

$\sum_{i=1}^{9}(x_i - 5)^2 = 459$

$$\sum_{i=1}^{9} x^2 - 10\sum_{i=1}^{9} x\,;+25 \times 9 = 45$$

$$\sum_{i=1}^{9} x_1^2 = 45 - 25 \times 9 + 10 \times (9 + 5 \times 9)$$

$= \ 360$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{9}(x_i - 6)^2}{9}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^{9} x^2 - 12\sum_{i=1}^{9} x_i + 36 + 9}{9}}$$

$$\sigma = \sqrt{\frac{360 - 12 \times 54 + 36 + 9}{9}}$$

$= \ 2$

88. If sum of all the solutions of the equation $8\cos x . \left(\cos\left(\frac{\pi}{6} + x\right) . \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) = 1$ in $[0, \pi]$ is $k\pi$, then k is equal to:

(A) $\frac{2}{3}$ (B) $\frac{13}{9}$ (C) $\frac{8}{9}$ (D) $\frac{20}{9}$

Solution: (C)

$$8\cos x . \left(\cos\left(\frac{\pi}{6} + x\right) . \cos\left(\frac{\pi}{6} - x\right) - \frac{1}{2}\right) = 1$$

We would use the formula

$\cos(A + B) . \cos(A - B) = \cos^2 A - \sin^2 B$

Now, $\cos\left(\frac{\pi}{6} + x\right) . \cos\left(\frac{\pi}{6} - x\right) = \cos^2\frac{\pi}{6} - \sin^2 x$

$$= \frac{3}{4} - \sin^2 x$$

$$8\cos x \left(\frac{3}{4} - \sin^2 x - \frac{1}{2}\right) = 1$$

$$8\cos x \left(\frac{1}{4} - \sin^2 x\right) = 1$$

$$8\cos x \left(\frac{1}{4} - (1 - \cos^2 x)\right) = 1$$

$$8\cos x \left(\cos^2 x - \frac{3}{4}\right) = 1$$

$$8\cos^3 x - 6\cos x = 1$$

$$2(4\cos^3 x - 3\cos x) = 1$$

$$2\cos 3x = 1$$

$$\cos 3x = \frac{1}{2}$$

$$3x = 2n\pi \pm \frac{\pi}{3}$$

$$x = \frac{2n\pi}{3} \pm \frac{\pi}{9}$$

$$x \in [0, \pi]$$

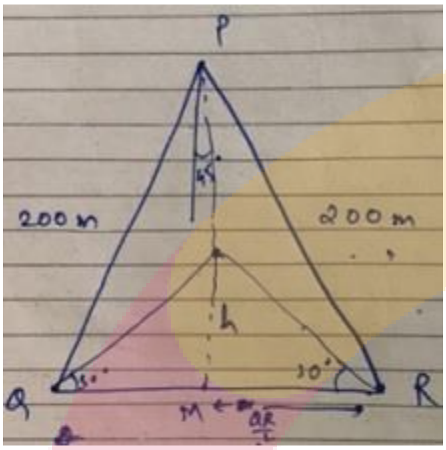
$$\therefore \ x = \frac{\pi}{9}, \frac{7\pi}{9}$$

Sum $= \frac{8\pi}{9}$ $k = \frac{8}{9}$

89. PQR is a triangular park with PQ = PR = 200m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively $45^o, 30^o$ and 30^o, then the height of the tower (in m) is:

(A) 100 (B) 50 (C) $100\sqrt{3}$ (D) $50\sqrt{2}$

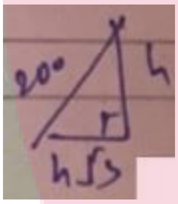
Solution: (A)



$\tan 30 = \frac{2R}{QR}$

$QR = \frac{2h}{\left(\frac{1}{\sqrt{3}}\right)} = 2h\sqrt{3}$

$\tan 45^o = \frac{h}{PM}$ $PM = h$



So, $4h^2 = 200^2$

$h^2 + 3h^2 = 200^2$

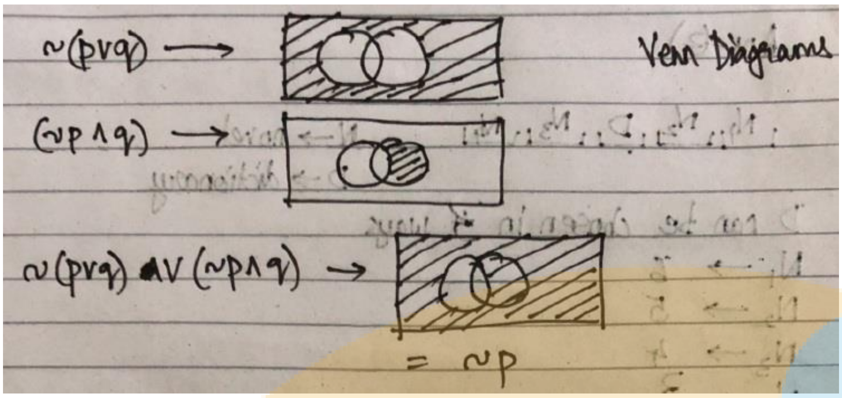
$2h = 200$

$h = 100$

90. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to:

(A) $\sim p$ (B) p (C) q (D) $\sim q$

Solution: (A)



$\sim(p \vee q) = (\sim P \wedge \sim q) \vee (\sim p \wedge q)$

$= \sim p \wedge (\sim q \vee q)$

$= \sim p \wedge q = \sim p$