PHYSICS

1. In propagation of light $\vec{E}$ and $\vec{B}$ oscillate in mutually perpendicular directions.
$\vec{E}\times\vec{B} =$ direction of propagation = +z direction
only option (4) satisfies both conditions of (1) $\vec{E}\times\vec{B}=0$
(2) $\left(\vec{E}\times\vec{B}\right)$ directed
along the z-axis.

2. Half life = 15 hrs. = $\frac{0.693}{\lambda}$

$\lambda = 0.0462 hr^{-1}$ $\qquad$ $N_0 = \frac{1}{24}$ moles of Na

No.of $\beta-$particles (disintegrations) = $N_0 - N_0 e^{-(\lambda\times 7.5)}$

$\frac{1}{24} moles\left(1-e^{-0.35}\right)$
= 0.0122 moles
$\therefore$ no.of $\beta-$particles = 7.4×10^{21}

3. Amplitude in a damped oscillation is given by $A = A_0 e^{-\beta t}$

$energy \propto A^2$

$\therefore \sqrt{E} = \sqrt{E_0}e^{-\beta t}$ where E_0 is initial energy

$\sqrt{15} = \sqrt{45}e^{-\beta 15\sec}$

$3^{\frac{1}{2}} = e^{-15\beta}$

on taking log both sides $-\frac{1}{2}\ln(3) = -15\beta$

$\beta = \frac{\ln 3}{30} = 4$

4. $[e] = IT$

$[m] = M$

$[c] = LT^{-1}$

$[h] = ML^2T^{-1}$

$[\mu_0] = MLI^{-2}T^{-3}$

If $\mu_0 = e^a m^b c^c h^d$

$MLI^{-2}T^{-3} = [IT]^a[M]^b\left[LT^{-1}\right]^c\left[ML^2T^{-1}\right]^d$
by eqating powers, we get
$a=-2, b=0, c=-1, d=1$

$\therefore [\mu_0] = \left[\frac{h}{ce^2}\right]$

5. At 30cm from the magnet on its equitorial plane $\vec{B}_{magnet} = -\overrightarrow{B_M}$ (newtral point)

so by equating their magnitude $\frac{\mu_0}{4\pi}\frac{M}{r^3} = 3.6\times10^{-5}Tesla$

$$\frac{10^{-7}\times M}{(0.3)^3} = 3.6\times10^{-5}Tesla$$

$$M = 3.6\times0.027\times10^2 = 9.7Am^2$$

6. $[e] = IT$

$[m] = M$

$[c] = LT^{-1}$

$[h] = ML^2T^{-1}$

$[\mu_0] = MLI^{-2}T^{-3}$

If $\mu_0 = e^a m^b c^c h^d$

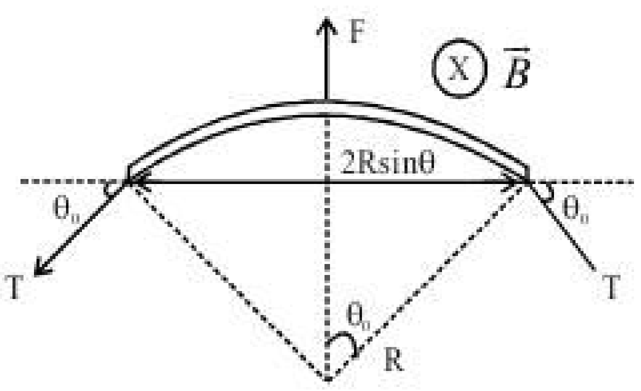
$MLI^{-2}T^{-3} = [IT]^a [M]^b [LT^{-1}]^c [ML^2T^{-1}]^d$

by eqating powers, we get

$a = -2, b = 0, c = -1, d = 1$

$\therefore [\mu_0] = \left[\frac{h}{ce^2}\right]$

7.



For the area to be in equilibrium, $F = 2T\sin\theta$ & $F = I(2R\sin\theta)\times B$

$\therefore 2T\sin\theta = I2R\sin\theta\times B$

$T = IRB$

8. When positive terminal of battery is connected to A, current passes through D1 diode.

$\therefore$ current supplied $= \frac{2V}{5\Omega}$

= 0.4 A

When positive terminal is connected to B current passes through D2.

$\therefore$ current supplied $= \frac{2V}{10\Omega} = 0.2A$

9. As collisions are elastic and masses are equal, velocities of colliding particles get exchanged.

$\Delta\vec{P}$ in each collision with the supports = $2mv$

Time interval between consecutive collisions with one support = $\dfrac{(L-2nr)\times 2}{v}$

$$F_{avg}=\frac{\Delta P}{T}=\frac{2mv}{(L-2nr)2/v}=\frac{mv^2}{L-2nr}$$

10. When the currents are parallel, $I_1\ I_2$ is positive and the force between then is attractive (i.e. negative) similarly when currents are anti parallel $I_1\ I_2$ is negative and the force between than is repulsive (i.e. positive) so option (2) satisfies the conditions.

11. $\therefore \int dV=-\int \vec{E}\cdot d\vec{r}$

$d\vec{r}=dx\,\hat{i}+dy\,\hat{j}$

$\int dV=-\int\left(25\hat{i}+30\hat{j}\right)\cdot\left(dx\hat{i}+dy\,\hat{j}\right)$

$\int_0^V dV=-\int_0^2 25dx+\int_0^2 30dy$

$V-0=-\left[25(x)_0^2+30(y)_0^2\right]$

$V=-\left[25\times 2+30\times 2\right]$

$V=-110\text{ volt}=-110\,J/C$

12. For the given situation

$\overset{V_s}{\vec{S}}\ \underset{\sim\sim}{}f_0$ $\quad\overset{V_0}{\overleftarrow{O}}$ frequency listened by an observer is f.

So, $f=f_0\left[\dfrac{V+V_0}{V-V_s}\right]$

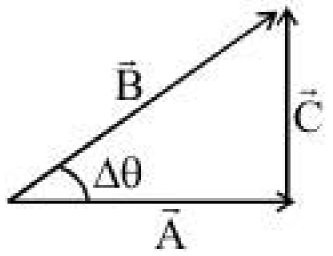
$$f=\frac{f_0V}{V-V_s}+\frac{f_0}{V-V_s}=V_0$$

equating the equation

$y=mx+C$

$$m=\frac{f_0}{V-V_s}$$

So choice is (A).

13. 

By triangle rule

$\vec{A}+\vec{C}=\vec{B}$

$\vec{B}-\vec{A}=\vec{C}$

$|\vec{B}-\vec{A}|=|\vec{C}|=|\vec{B}|\sin\Delta\theta$

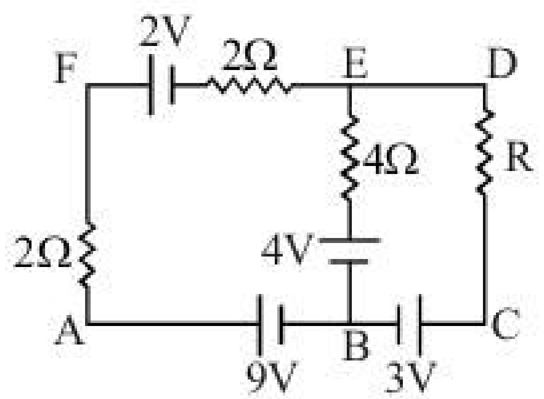
$|\vec{B}-\vec{A}|=|\vec{B}|\Delta\theta$ $\qquad (\because \sin\Delta\theta \simeq \Delta\theta)$

again $|\vec{B}|\cos\Delta\theta=\vec{A}$

$\therefore\ \cos\Delta\theta \simeq 1$

$|\vec{B}|=|\vec{A}|$

so, $|\vec{B}-\vec{A}|=|\vec{B}|\Delta\theta=|\vec{A}|\Delta\theta$

14. 

If current in 4Ω is zero

then $\underset{BCDE}{\varepsilon_v}=0$

$V_{EB}+V_{BC}+V_{CD}+V_{DE}=0$

$-4+3+V_{CD}+0=0$

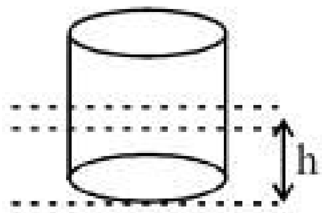
$V_{CD}=1\,\text{volt}$

again $V_A-9+3+1-V_D=0$

$V_A-V_D=5V$

15. Let block is floating with disolve depth h.

Then about equilibrium



$$M_{Block}g = F_{up}$$

$$(AH\rho_B)g = (Ah)\rho_L g \quad \_\_(1)$$

When block depressed by distance x then

$$F_{Net} = F'_{up} - M_{Block}g$$

$$= A(H+x)\rho_L g - AH\rho_B g$$

from equation (1) $F_{Net} = Ax\rho_L g$

$$F_{Net} = -Ax\rho_L g$$

$$\cancel{A}H\rho_{Block} \cdot \frac{d^2x}{dt^2} = -\cancel{A}x\rho_L g$$

$$\frac{d^2x}{dt^2} = -\frac{\rho_L g}{H\rho_{Block}}x$$

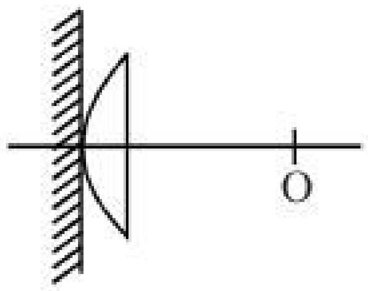
$$\omega^2 = \frac{\rho_L g}{H\rho_B}$$

For simple pendulum

$$\omega^2 = \frac{g}{\ell}$$

Equating $\ell = \dfrac{H\rho_B}{\rho_L}$

$$= \frac{650 \times 54}{900} = 39\,\text{cm}$$

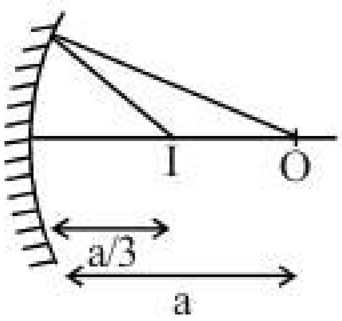
16. 

This combinations will behave like a mirror of power.

$$P_{eq} = 2P_L + P_M$$

$$P_{eq} = 2\frac{1}{f} + 0$$

$$F_{eq} = -\frac{f}{2}$$



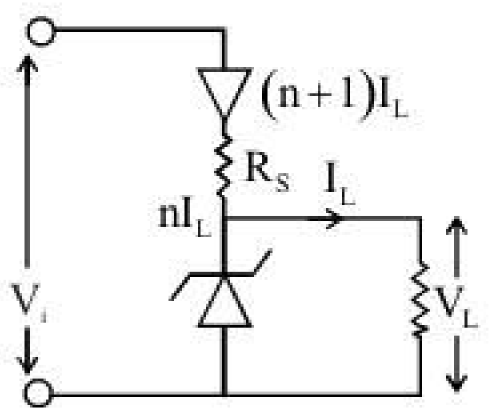
so the behaviour will be like a mirror of focal length $-\frac{f}{2}$

Using mirror equation $\frac{1}{V} + \frac{1}{U} = \frac{1}{f_{eq}}$

$$\frac{1}{-a/3} + \frac{1}{-a} = \frac{-1}{f/2}$$

$$\frac{4}{a} = \frac{-2}{f}$$

$$\boxed{a = 2f}$$

17. 

Voltage drop across zener diode is V_L so voltage drop across K_S

$$V_{R_S} = V_i - V_L = (n+1) I_L R_S$$

$$R_S = \frac{V_i - V_L}{(n+1) I_L}$$

18. Let $\left(\frac{\theta}{A}\right)$ is derived quantity which is derived by three fundamental quantities η, $\left(\frac{S\Delta\theta}{h}\right)$ and $\left(\frac{1}{eg}\right)$

By using property of homogeneity.

$$\left[\frac{\theta}{A}\right]=[\eta]^x\left[\frac{S\Delta\theta}{h}\right]^y\left[\frac{1}{eg}\right]^z$$

$$\left[\frac{\theta}{A}\right]=\left[m^1T^{-3}\right]$$

$$[\eta]=\left[m^1L^{-1}T^{-1}\right]$$

$$\left[\frac{S\Delta\theta}{h}\right]=\left[L^1T^{-2}\right]$$

$$\left[\frac{1}{eg}\right]=\left[m^{-1}L^2T^{+2}\right]$$

$$\left[m^1L^0T^{-3}\right]=\left[m^1L^{-1}T^{-1}\right]^x\left[m^0L^1T^{-2}\right]^y\left[m^{-1}L^2T^{+2}\right]^z$$

$$x+0-z=1,\ -x+y+2z=0\ \&\ -x-2y+2z=-3$$

$$\begin{array}{l}-x+y+2z=0\\ -x-2y+2z=-3\\ \underline{\ \ +\ \ +\ \ -\ \ \ \ \ \ \ \ +\ \ }\\ \ \ \ \ 3y=3\ \Rightarrow y=1,\ \ x=1,\ z=0\end{array}$$

$$\text{so, }\frac{\theta}{A}=\eta\cdot\frac{S\Delta\theta}{h}$$

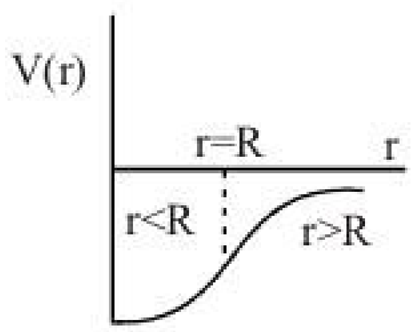
19. Potential V(r) due to large planet of radius R is given by

$$\underset{\text{out}}{V}(r)=-\frac{GM}{r}$$

$r>R$

$$V_s(r)=\frac{-GM}{R}$$

$r=R$

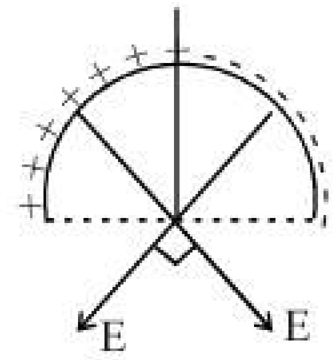


$$V_{in}=-\frac{3}{2}\frac{GM}{R}\left[1-\frac{r^2}{3R^2}\right]$$

$r<R$

20. Due to quarter ring electric field intensity is

$$E = \frac{2k\lambda}{R}\sin\frac{\theta}{2}$$



when $\theta = \pi/2$

So, due to each quarter section, field intensity is

$$E = \frac{2k\lambda}{R} \times \sin\frac{\pi}{4} = \frac{\sqrt{2}k\lambda}{R}$$

so Net $\vec{E}_{Net} = \sqrt{2}\,E$ $\qquad \therefore\ \lambda = \frac{\theta}{\pi R/2}$

$$= \frac{\sqrt{2}\sqrt{2}k\lambda}{R} \qquad \therefore\ \pi R = L$$

$$= \frac{2k\cdot\lambda}{R} = \frac{2k(2\theta)}{\pi R^2} = \frac{4\theta}{4\pi^2\varepsilon_0 R^2}$$

$$\theta = 10^3\varepsilon_0$$

$$\text{so, } E_{Net} = \frac{4\times10^3\varepsilon_0}{4\pi^2\varepsilon_0 R^2} = \frac{4\times10^3}{4\pi^2\left(\frac{L}{\pi}\right)^2}$$

$$= \frac{4\times10^3}{4\cdot L^2} = \frac{4\times10^3}{4\times(0\cdot2)^2} = \frac{4\times10^3}{4\times0.04} = 25\times10^3$$

21. In a potentiometer, the null point will fluctuate due to varying current & voltage.

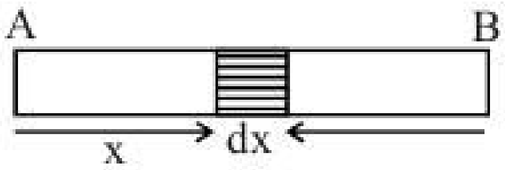
In the moving magnet / coil galvanometer, the dial will be unsteady due to varying current through it.

In hot wire voltmeter, the principle of heat due to current is used to measure the voltage.

$$P_{avg} = \frac{V_{RMS}^2}{R}$$

$$\therefore\ V_{RMs}^2 = R\ P_{avg}$$

$$\therefore\ \boxed{\text{hot wire voltmeter}}$$

22. 

$$x_{COM} = \frac{\int_0^L (\mu dx)\,x}{\int_0^L \mu dx}$$

$$\frac{7}{12}L = \frac{\int_0^L \left(ax + \frac{bx^2}{L}\right)dx}{\int_0^L \left(a + \frac{bx}{L}\right)dx}$$

$$\frac{7}{12}L = \frac{\left(\frac{bL^3}{3L} + \frac{aL^2}{2}\right)}{\left(aL + \frac{bL}{2}\right)}$$

$$\frac{7}{12} = \frac{\frac{a}{2} + \frac{b}{3}}{a + \frac{b}{2}}$$

$$\therefore \boxed{2a = b}$$

23. $$\frac{mV^2}{r} = \alpha r^2$$

$$\therefore \text{K.E.} = \frac{\alpha r^3}{2}$$

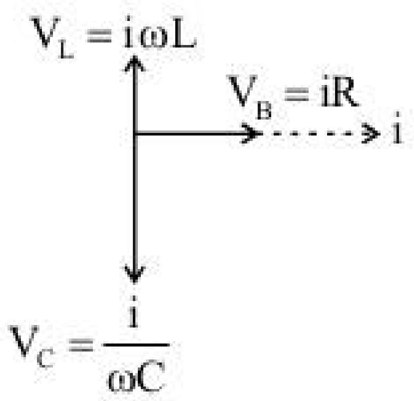
$$\Delta\,\text{P.E} = \int_0^r \alpha r^2 \cdot dr$$

$$\text{P.E} = \frac{\alpha r^3}{3}$$

$$\text{T.E.} = \frac{\alpha r^3}{2} + \frac{\alpha r^3}{3}$$

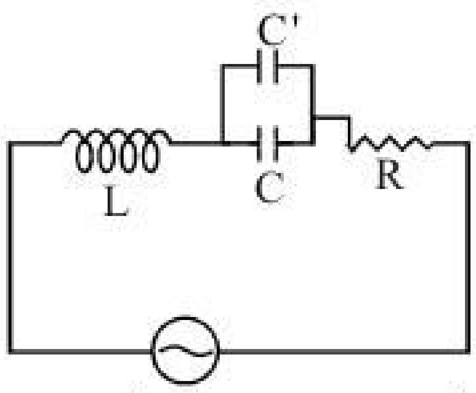
$$\boxed{\text{T.E.} = \frac{5}{6}\alpha r^3}$$

24. As current leads voltage thus



Since power factor has to be made '1'

$\therefore$ Effective capacitance has to be increased thus connecting in parallel.



$\because \cos\phi = 1 \qquad \therefore \phi = 0$

$$i\,\omega L = \frac{i}{\omega(C + C')}$$

$$\therefore C + C' = \frac{1}{\omega^2 L}$$

$$\therefore C' = \frac{1}{\omega^2 L} - C$$

$$\boxed{\therefore C' = \frac{1-\omega^2 LC}{\omega^2 L} \text{ in parallel}}$$

25. $I = 4I_0 \cos^2\left(\phi/2\right)$

$$I_{max} = 4I_0$$

$$\text{Now, } \frac{I_{max}}{2} = 2I_0 = 4I_0\cos^2\left(\phi/2\right)$$

$$\cos\left(\phi/2\right) = \frac{1}{\sqrt{2}}$$

$$\therefore \frac{\phi}{2} = \frac{\pi}{2} \qquad \therefore \phi = \frac{\pi}{2}$$

$$\frac{2\pi}{\lambda}\Delta x = \frac{\pi}{2} \qquad \therefore \Delta x = \frac{\lambda}{4}$$

$$y\frac{d}{D} = \frac{\lambda}{4} \qquad \therefore y = \frac{\lambda D}{4d}$$

$$\therefore \boxed{y = \frac{\beta}{4}}$$

26. For metals, there is no free motion but rather oscillation about mean position.

Thus these have K.E. & P.E., which are almost equal.

i.e. $P.E_{avg} = K.E_{avg} = \frac{3}{2}RT$

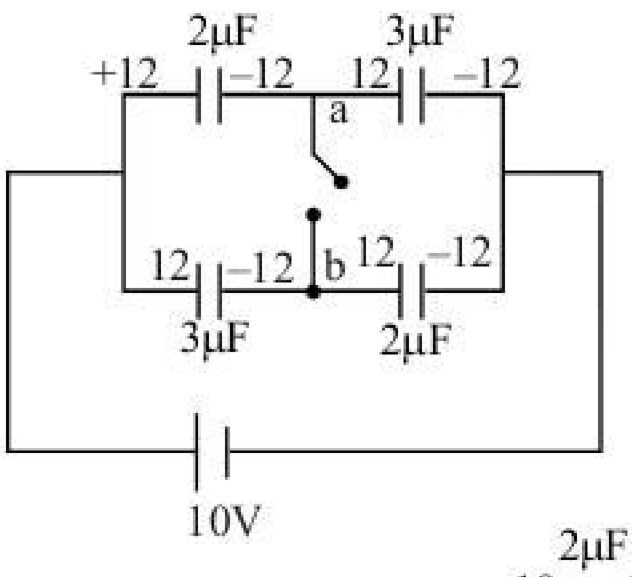
$\therefore$ $T.E. = K.E. + P.E.$

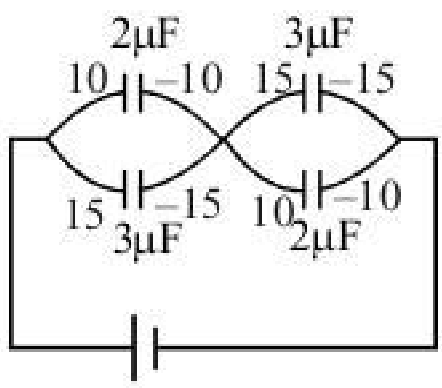
$\therefore$ $T.E. = 3RT$ per mole

$\therefore$ specific heat $C = \frac{3R}{M}$

$$C = \frac{3 \times 8.314}{27 \times 10^{-3}}$$

$$\boxed{C \approx 925 \frac{J}{kg\,K}}$$

27. 



for upper link $C_{eq} = \frac{6}{5}\mu F$

$\therefore Q_{upper} = Q_{lower} = 12\mu C$

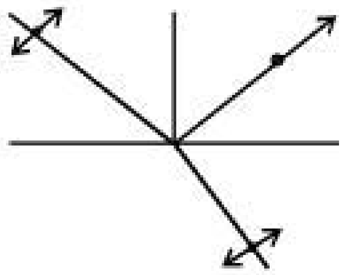
on closing switch charge on $2\mu F$ is $10\mu C$ & that on $3\mu F$ is $15\mu C$

$\therefore q_i = -12 + 12 = 0$

$\therefore q_j = 15 - 10 = 5\mu C$

$\therefore$ $\boxed{\text{charge } 5\mu C \text{ flows from b to a}}$

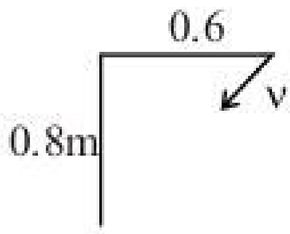
28. At Brewster's angle



$\tan i = \mu$

The reflected light is completely polarized, where as refracted light has both components to electric field. Thus, the reflected ray will have lesser intensity compared to refracted ray.

$\therefore \boxed{I_{reflected} < \frac{I_0}{2}}$

29. 

$v = \omega r = 0.6 \times 12 = 7.2\,m/s$

$\bar{R} = 0.8\hat{k} + 0.6\hat{i}$ m

$\bar{V} = -7.2\hat{j}$ m/s

$\bar{L} = m\bar{R} \times \bar{V}$

$\bar{L} = 2\left(5.76\hat{i} - 4.32\hat{k}\right)$

$\therefore \boxed{|\bar{L}| = 14.4\,kg\,m^2s^{-1}}$

30. $\lambda = \frac{h}{mv}$

also $mvr = \frac{nh}{2\pi}$

$\lambda = \frac{2\pi r}{n}$

$\therefore r \propto n^2$

$\therefore \lambda \propto n$

for $n = 4$, the de Broglie wavelength is four times that of ground state.

CHEMISTRY

1. $A+2B+3C \rightleftharpoons AB_2C_3$

given :

$6.0\,g$ of A, 6.0×10^{23} atoms of B and 0.036 mole of C yields 4.8 gm of compound AB_2C_3.

Atomic mass of $A = 60$ amu

Atomic mass of $C = 80$ amu

Mole of $A = \frac{6}{60} = \frac{1}{10} = 0.1$ mole

Mole of $B = \frac{6.0\times10^{23}}{6.023\times10^{23}} = 1$ mole

Mole of $C = 0.036$

So accroding to reaction $A+2B+3C \rightleftharpoons AB_2C_3$

C is limiting reagent which consumed $= \frac{0.036}{3} \Rightarrow 0.22$ mole

So 0.012 mole of C formed 0.012 mole of AB_2C_3. So

Mole of $AB_2C_3 = \frac{wt}{\text{molecular } wt}$

$0.012 = \frac{4.8}{\text{Molecular wt}}$ of AB_2C_3

So Molecular wt. of $AB_2C_3 = 400$

So atomic mass of $A + 2\times$ Atomic mass of $B + 3$ atomic mass of $C = 400$

$$60 + 2B + 3\times80 = 400$$

So atomic mass of $B = 50$ amu

2. dipole moment $(\mu) = q\times d$

d (distance) $= 1.617\text{Å} = 1.617\times10^{-8}$ cm

$\mu = 0.38D = 0.38\times10^{-18}$ esu$\times$cm

$q = \frac{\mu}{d} = \frac{0.38\times10^{-18}}{1.617\times10^{-8}}$

So fractional charge $= \frac{\text{Particle chagre}}{\text{Total charge}}$

$$= \frac{q}{Q} = \frac{0.38\times10^{-18}}{1.617\times10^{-8}\times4.802\times10^{-10}} = 0.05$$

3. $Cl_2 + H_2O \longrightarrow HCl + HClO$

4. $\left[FeF_6\right]^{3-}$

oxidation state of $Fe = +3$

$Fe^{+3} = \left[Ar\right]3d^5$, F^- is weak field Ligand

↑	↑

eg

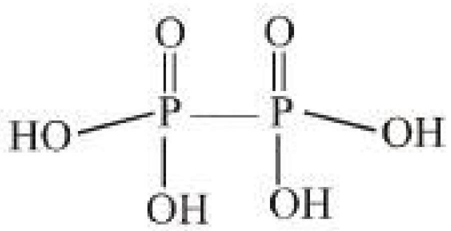
↑	↑	↑

t_2g

5. Lactic acid is formed in muscles during vigorous exercise.
This is due to anaerobic respiration.
Glucose $\rightarrow$ Lactic acid + energy

$C_6H_{12}O_6 \rightarrow 2C_3H_6O_3 + energy$

6. $H_4P_2O_6$



7. Gas deviate the most from its ideal behaviour at high pressure and law temeperature.

8. $BeCl_2$
according to Fajan's rule, covalent nature α small of cation.

9. $Ef = 80$ kj/mole ; Eb = 120 kj/mole

given, $A(g) \rightleftharpoons B(g)$

$\Delta H = -40$ kg/mole

$\frac{Ef}{Eb} = \frac{2}{3}$

we know that

$Ef - Eb = \Delta H$

$Ef - Eb = -40$

$Eb\frac{2}{3} - Eb = -40$

$Eb = 120$ kg/mole

$Ef = 80$ kg/mole

10. C_6H_{14} isomers are

11. Neoprene is polymer of chloroprene,

$$nCH_2=CH-\underset{\substack{|\\ Cl}}{C}=CH_2 \xrightarrow[\text{polymerisation}]{\text{1,4-addition}} \left(-CH_2-CH=\overset{\substack{Cl\\ |}}{C}-CH_2-\right)_n$$

12. Calamine is an ore of zinc.

13. Order of de brog lie wavelength visible photon > Thremal electron> Thermal neutran

14. Higher the reduction potential easier to reduce and oxidise the other species with lower reduction potential.

15. Dihydrogen is infalmmable gas.

16. $Na_2Cr_2O_7$ is more soluble than $K_2Cr_2O_7$

17.

$$CH_3-\overset{\substack{O\\ ||}}{C}-H \xrightarrow{OH^-} \overset{\ominus}{C}H_2-\overset{\substack{O\\ ||}}{C}-H$$

$$H_2O\downarrow\ CH_3-\underset{\substack{||\\ O}}{C}-H$$

$$CH_3-\underset{\substack{|\\ OH}}{CH}-CH_2CHO\,(Aldol)$$

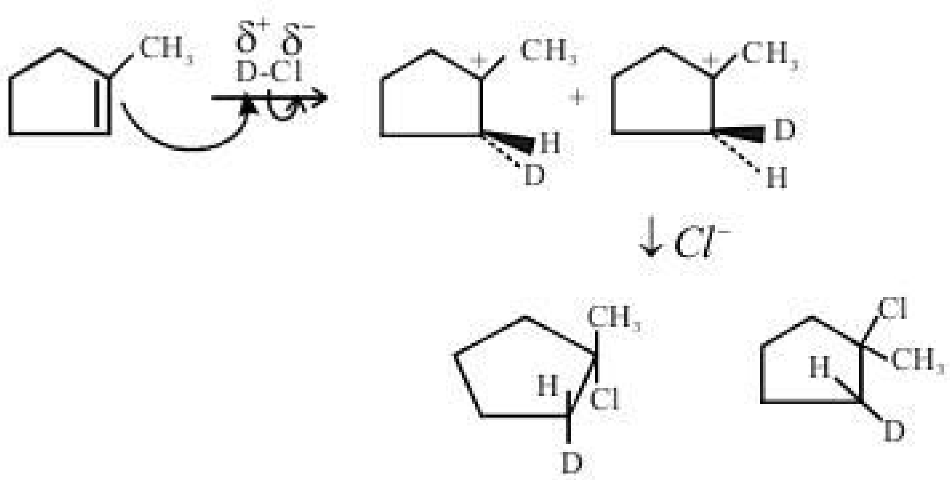
$$-H_2O\downarrow\Delta$$

$$CH_3-CH=CH-CHO$$

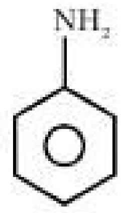
Aldol condensation reaction

18. A pink coloured salt terms blue on heating in the presence of CO^{2+}

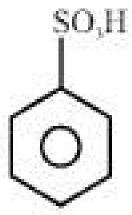
19.



Both 2 & 3 are formed in equal amounts.

20. A)  $\xrightarrow[\Delta]{Na} NaCN \xrightarrow{FeSO_4} Fe_4\left[Fe(CN)_6\right]_3$

Blue ppt

B)  $\xrightarrow{Na} Na_2S \xrightarrow{Na_2[Fe(CN)_5NO]} Na_4\left[Fe(CN)_5NOS\right]$

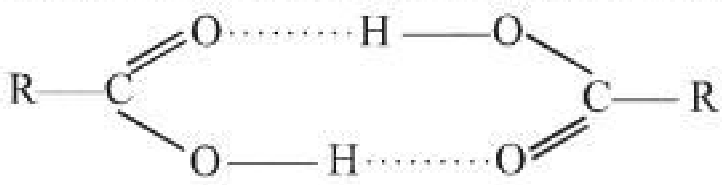
Violet colour

C) $\underset{\text{Thiourea}}{NH_2-\overset{\overset{S}{\|}}{C}-NH_2} \xrightarrow{Na} NaSCN \xrightarrow{FeCl_3} \underset{\text{Red colour}}{Fe_4\left[Fe(SCN)_6\right]_3}$

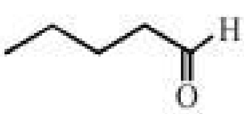
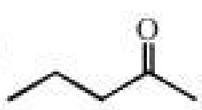
21. Ice $\rightleftharpoons$ water

Volume of Ice is more than compare to water so on increase the pressure reaction shift in the forward direction.

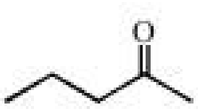
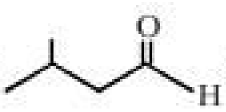
22. Molar mass of acetic acid in benzene using freezing point depreasion is affected by association.

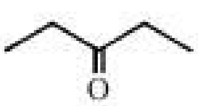
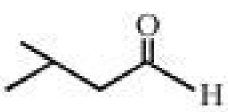


23. Addition of phosphate fertiliser to water bodies caues enhamced growth of algae.

24. 1)  and  are fuctional isomers

2) $CH_3-CH_2-\overset{\overset{O}{\|}}{C}-CH_2-CH_3$ and $CH_3-\overset{\overset{O}{\|}}{C}-CH_2-CH_2-CH_3$ are positional iosmers

3)  and  are fuctional isomers

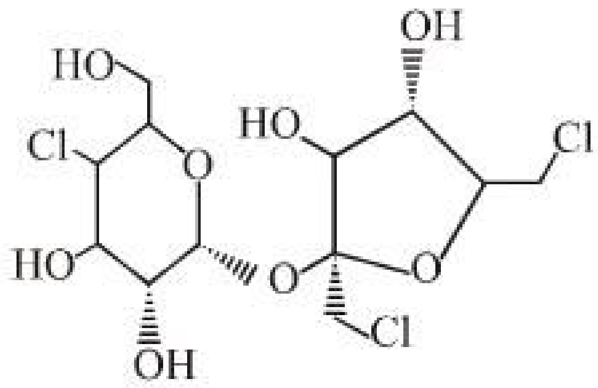
4)  and  are fuctional isomers

25. $[COCl_4]^{2-}$

reaction $\underset{\text{pink}}{\left[CO(H_2O)_6\right]^{2+}} + 4Cl^- \longrightarrow \underset{\text{Blue}}{\left[CO(Cl)_4\right]^{2-}} + 6H_2O$ pink

26. $CH_3\left(CH_2\right)_{15}\overset{\oplus}{N}\left(CH_3\right)_3Br^-$ will form micelles in aqueous solution at lowest molar concentration.

27. Sucralose contains chlorine as it is trichloroderivative of sucrose.

Sucralose $\rightarrow$ 

28. $A+2B\rightarrow C$

$\left(R_1\right)$ Rate $=K[A][B]$ .....(1)

According to condition

$\left(R_2\right)$ Rate $=K[A][2B]$ ...(2)

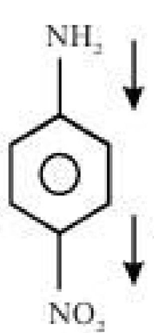
equation eq $2\div$ eq 1

$\frac{R_2}{R_1}=2$

$\boxed{R_2=2R_1}$

29. Incorrect formula is X_2Cl_3

30. Dipolemoment $\propto$ e^- transfer (or) e^- delocalisation.

In  e^- transfer is maximum.

MATHEMATICS

1. $\tan 60° = \left|\dfrac{m-\left(-\sqrt{3}\right)}{1+m\left(-\sqrt{3}\right)}\right|$

$\Rightarrow \quad \left(m+\sqrt{3}\right)^2 = 3\left(1-m\sqrt{3}\right)^2$

$\Rightarrow \quad m = 0 \text{ or } m = \sqrt{3}$

$\therefore$ equation of required line is $y+2=\sqrt{3}\left(x-3\right)$

i.e., $y-\sqrt{3}x+2+3\sqrt{3}=0$

2. $f\left(x\right)=\dfrac{\left(1+x\right)^{3/5}}{1+x^{3/5}}$

$f'\left(x\right)=0 \Rightarrow x=1$

$\therefore \quad f\left(0\right)=1, \quad f\left(1\right)=\dfrac{2^{0.6}}{2}$

$= 2^{-0.4}$

$\therefore \; f\left(x\right)\in\left(2^{-0.4},1\right)$

3. $\displaystyle\int \frac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}\,dt=\frac{1}{2}\left(g\left(t\right)\right)^2+C$

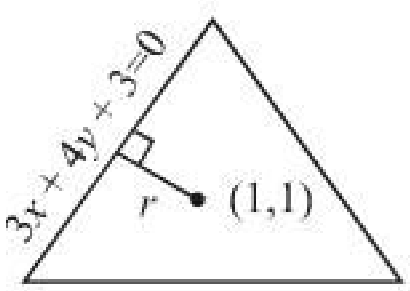
Differentiating both sides

$\dfrac{\log\left(t+\sqrt{1+t^2}\right)}{\sqrt{1+t^2}}=g\left(t\right)g'\left(t\right)$

$\Rightarrow g\left(t\right)=\log\left(t+\sqrt{1+t^2}\right)$

$\therefore \; g\left(2\right)=\log\left(2+\sqrt{5}\right)$

4. In an equilateral triangle incentre & curcumcentre all same & $R=2r$



Now, $r=\dfrac{\left|3+4+3\right|}{\sqrt{9+16}}$

$=2$

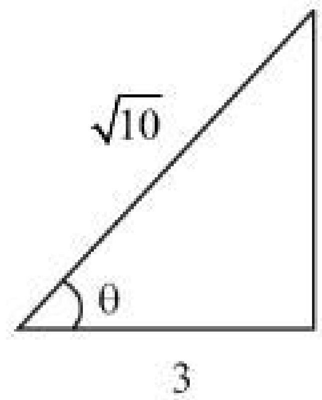
$\Rightarrow \quad R=4$

$\therefore$ equation of circumcircle is $\left(x-1\right)^2+\left(y-1\right)^2=16$

$\Rightarrow \; x^2+y^2-2x-2y-14=0$

5. $$2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{3}{2} \quad ...(1)$$

$$2\sin\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)=\frac{1}{2} \quad ...(2)$$



Dividing (2) by (1), $\tan\left(\frac{\alpha+\beta}{2}\right)=\frac{1}{3}$

$$\Rightarrow \quad \tan\theta=\frac{1}{3} \quad \left(\because\ \theta=\frac{\alpha+\beta}{2}\ \text{given}\right)$$

$$\Rightarrow \quad \sin\theta=\frac{1}{\sqrt{10}}$$

$$\& \quad \cos\theta=\frac{3}{\sqrt{10}}$$

$$\sin 2\theta+\cos 2\theta=2\sin\theta\cos\theta+2\cos^2\theta-1$$

$$=2\times\frac{1}{\sqrt{10}}\times\frac{3}{\sqrt{10}}+2\left(\frac{9}{10}\right)-1$$

$$=\frac{6}{10}+\frac{18}{10}-1$$

$$=\frac{7}{5}$$

6. symbolic form

$$(P\Lambda\sim R)\longleftrightarrow Q$$

$$\therefore \sim\left[(P\Lambda\sim R)\longleftrightarrow Q\right]$$

$$\boxed{\sim Q\longleftrightarrow(P\wedge\sim R)}$$

Using Demerojans law

7. $$|5\ adj\ A|=5$$

$$\Rightarrow \quad 5^3|adj\ A|=5$$

$$\Rightarrow \quad |adj\ A|=\frac{1}{5^2}$$

$$\Rightarrow \quad |A|^{3-1}=\frac{1}{5^2}$$

$$\Rightarrow \quad |A|=\pm\frac{1}{5}$$

8. $a+b+2c=0$

$2a+3b+4c=0$

$\Rightarrow \quad \dfrac{a}{-2}=\dfrac{b}{0}=\dfrac{c}{1}$

For a point on pout $z=0$

$\Rightarrow \quad x+y=3$

$2x+3y=4$

By solving we get $x=5,\ y=-2,\ z=0$

$\therefore$ Point is $(5,-2,0)$

equation line is $\dfrac{x-5}{-2}=\dfrac{y+2}{0}=\dfrac{z}{1}$

Shortest distance $=\left|\dfrac{(\bar{a}_2-\bar{a}_1)\times\bar{b}}{|\bar{b}|}\right|=2$

9. $\lim\limits_{x\to 0} f(x)=f(0)$

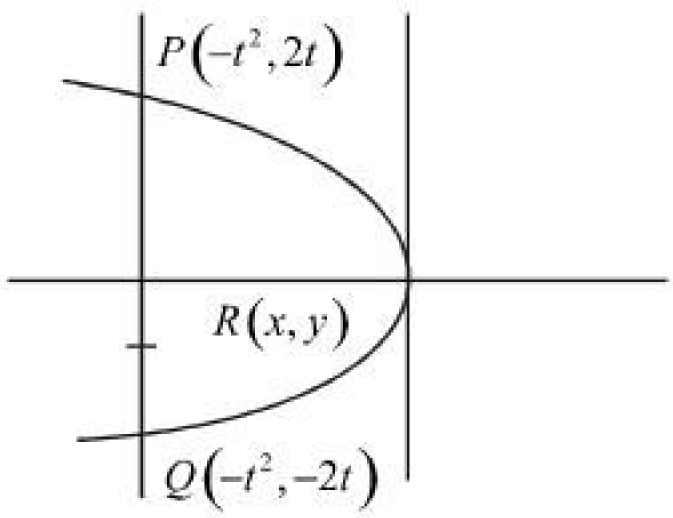
$$\Rightarrow \lim_{x\to 0}\frac{\dfrac{(e^x-1)^2}{x^2}}{\dfrac{\sin\left(x/k\right)}{k\ \ x/k}\ \dfrac{\log\left(1+x/4\right)}{4\ \ x/4}}=12$$

$\Rightarrow \quad 4k=12$

$k=3$

10. End points of bouble ordinate can be taken as $(-t^2,2t)$ & $(-t^2,-2t)$ according to given condition.

$x=\dfrac{-2t^2-t^2}{3}$ & $y=\dfrac{-4t+2t}{3}$



$\Rightarrow \quad 3x=-3t^2$ & $3y=-2t$

i.e., $\quad x=-t^2$ & $t=-3/2\,y$

eliminatry t

$x=-\left(-\dfrac{3y}{2}\right)^2$

i.e., $x=-\dfrac{9}{4}y^2$

i.e., $9y^2=-4x$

11. $2ae = \frac{1}{2}\left(\frac{2b^2}{a}\right)$

$\Rightarrow\ 2ae = \frac{b^2}{a}$

$\Rightarrow\ 2e = \frac{b^2}{a^2}$

also $e^2 = 1 - \frac{b^2}{a^2} \Rightarrow e^2 = 1 - 2e \Rightarrow e = \sqrt{2} - 1$

12. $np = 2 \quad npq = 1$

$\Rightarrow\ q = \frac{1}{2},\ \ p = \frac{1}{2}, n = 4$

$p(x \geq 1) = 1 - p(x < 1)$

$= 1 - p(x = 0)$

$= 1 - {}^4C_0\left(\frac{1}{2}\right)^0\left(\frac{1}{2}\right)^4$

$= 1 - \frac{1}{16}$

$= \frac{15}{16}$

13. Put $x = 1$ both side

$\Rightarrow \begin{vmatrix} 2 & 2 & -1 \\ 4 & 3 & 0 \\ 6 & 1 & 1 \end{vmatrix} = a - 12$

$\Rightarrow a = 24$

14. Required probility $= \frac{1}{27}$

NOTE : Don't consider equilateral triangle

Consider only 21 cases of isosceles triangle, each case occuring thrice

$(2,2,1)(2,2,3)\ (3,3,1)\ (3,3,2)\ (3,3,4)\ (3,3,5)\ (4,4,1)$

$(4,4,2)\ (4,4,3)\ (4,4,5)\ (4,4,6)\ (5,5,1)\ (5,5,2)\ (5,5,3)$

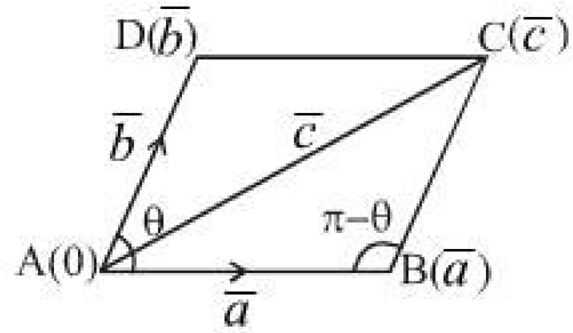
$(5,5,4)\ (5,5,6)\ (6,6,1)\ (6,6,2)(6,6,3)\ (6,6,4)\ (6,6,5)$

out of which $(6,6,5)$ has maximum area. Hence requaired probility is $\frac{3}{63} = \frac{1}{21}$

NOTE : If we consider equilateral triangle, there are $21 \times 3 = 63$ occurrences of non equilateral isoscales triangles and 6 occurences of equilateral triangle out of which $(6,6,6)$ has maximum area. so the required probability would have been $\frac{1}{69}$ and not $\frac{1}{27}$.

15. $\bar{a}\cdot\bar{b}=|a||b|\cos\theta$ ...(1)

$\bar{a}+\bar{b}=\bar{c}$



$\Rightarrow \quad [a+b]^2=|c|^2$

$\Rightarrow \quad a^2+b^2+2a\cdot b=c^2$

$\Rightarrow \quad a\cdot b=\dfrac{c^2-a^2-b^2}{2}$ ....(2)

Now

$\overrightarrow{DB}\cdot\overrightarrow{AB}$

$=(\vec{a}-\vec{b})\cdot\vec{a}=a^2-a.b$

$=a^2-\dfrac{c^2-a^2-b^2}{2}=\dfrac{1}{2}(3a^2+b^2-c^2)$

Hence none of the answers is correct.

16. $\int_0^{\sin x} f(x)\,dt=\dfrac{\sqrt{3}}{2}x$

Differentiating both sides w.r.t.x

$f(\sin x)\cos x=\dfrac{\sqrt{3}}{2}$

$x=\dfrac{\pi}{3}$

$\Rightarrow y\left(\dfrac{\sqrt{3}}{2}\right)\times\dfrac{1}{2}=\dfrac{\sqrt{3}}{2}$

$\Rightarrow f\left(\dfrac{\sqrt{3}}{2}\right)=\sqrt{3}$

17. $v^2-u^2=2gh$

$0-(48)^2=2(-32)h$

$\Rightarrow h=\dfrac{2304}{64}$

$=36$

$\therefore$ The greatest height $=64+36=100$ meters

18. $(a-1)(x^2+x+1)(x^2-x+1)+(a+1)(x^2+x+1)^2=0$

$\Rightarrow \; x^2+x+1 \;$ or $\; (a-1)(x^2-x+1)+(a+1)(x^2+x+1)=0$

$\Rightarrow \; ax^2+x+a=0$

For leal & unequal roots $D>0$

$\Rightarrow \quad 1-4a^2>0$

$\Rightarrow \quad a\in\left(-\frac{1}{2},\frac{1}{2}\right)-\{0\} \qquad \because \; a\neq 0$

19. The general term is second bracket is ${}^8C_r\left(2x^2\right)^{8-r}\left(-\frac{1}{x}\right)^r$

Total exponent of x is $16-3r$

Term indenpendent of $x=1x\;\exp.of\;x^0+(-1)+\exp.of\;x+3\times\exp.of\cdot\frac{1}{x^5}$

$$=0-{}^8C_5\;2^3(-1)+3\left({}^8C_7 2(-1)\right)$$

$$=-400$$

20. $x=0 \Rightarrow y=0$

Differentiating we have

$$\cos y\frac{dy}{dx}=x\cos\left(\frac{\pi}{3}+y\right)\frac{dy}{dx}+\sin\left(\frac{\pi}{3}+y\right)$$

$x=0 \quad y=0$

$\Rightarrow \quad \frac{dy}{dx}=\frac{\sqrt{3}}{2}$

$\Rightarrow \quad \frac{-dx}{dy}=\frac{-2}{\sqrt{3}}$

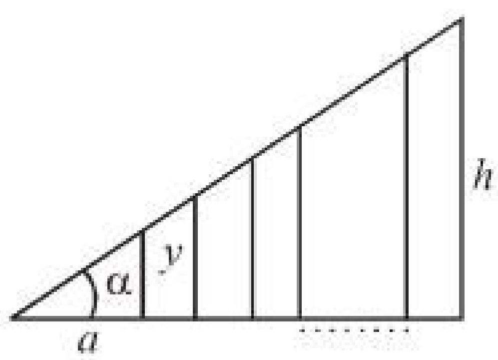
$\therefore$ equation of normal is $y=\frac{-2}{\sqrt{3}}x$

i.e., $\qquad \sqrt{3}\;y=-2x$

$\qquad 2x+\sqrt{3}y=0$

21. $\frac{h}{a+9x}=\frac{y}{a}$

$y=a\tan\alpha$



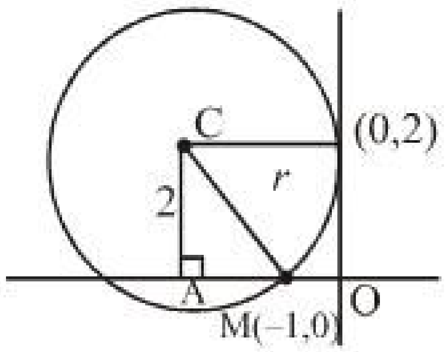
$\Rightarrow \quad \frac{h}{a+9x}=\frac{a\tan\alpha}{a}$

$\Rightarrow \quad a+9x=\frac{h}{\tan\alpha}$

$\Rightarrow x=\frac{h-a\tan\alpha}{9}$

$$=\frac{(h\cos\alpha-a\sin\alpha)}{9\cos\alpha}$$

22.



$AM = r - 1$

$\therefore$ Using pythogoras theorem is ΔCAM

$$2^2 + (r-1)^2 = r^2$$

$$\Rightarrow \quad 4 + r^2 - 2r + 1 = r^2$$

$$\Rightarrow \quad 4 + r^2 - 2r + 1 = r^2$$

$$\Rightarrow \quad r = 5/2$$

23. $$y - (x + 2y^2)\frac{dy}{dx} = 0$$

$$\Rightarrow \quad y = (x + 2y^2)\frac{dy}{dx}$$

$$\Rightarrow \quad y\frac{dy}{dx} = x + 2y^2$$

$$\Rightarrow \quad \frac{dx}{dy} + \left(-\frac{1}{y}\right)x = 2y$$

$$\text{I.F.} = e^{\int -\frac{1}{y}dy} = e^{-\ell ny} = y^{-1} = 1/y$$

$\therefore$ solution is $x\left(\frac{1}{y}\right) = \int (2y) \times \frac{1}{y}\, dy + c$

$$\Rightarrow \quad \frac{x}{y} = 2y + c$$

$x = 1, \quad y = -1 \qquad \Rightarrow c = 1$

$$\frac{x}{y} = 2y + 1$$

put $y = 1$

$x = 2 + 1$

$= 3$

24. $${}^{n}C_2 - n = 54$$

$\Rightarrow$ $$\frac{n(n-1)}{2} - n = 54$$

$\Rightarrow$ $$n^2 - 3n - 108 = 0$$

$\Rightarrow$ $$n = 12$$

25. $$\sum_{n=1}^{5} \frac{1}{n(n+1)(n+2)(n+3)} = \frac{k}{3}$$

$\Rightarrow$ $$\frac{1}{1.2.3.4} + \frac{1}{2.3.4.5} + \ldots\ldots\ldots\ldots + \frac{1}{5.6.7.8} = \frac{k}{3}$$

$\Rightarrow$ $$\frac{1}{3}\left[\frac{1}{1.2.3} - \frac{1}{6.7.8}\right] = \frac{k}{3}$$

$\Rightarrow$ $$\frac{1}{3}\left[\frac{1}{6} - \frac{1}{336}\right] = \frac{k}{3}$$

$\Rightarrow$ $$k = \frac{55}{336}$$

26. $f(2-x) = f(2+x) \Rightarrow$ function is symmetrical about $x = 2$

& $f(4-x) = f(4+x) \Rightarrow$ function is symmetrical about $x = 4$

$\Rightarrow$ $f(x)$ is periodic with period .2

$\Rightarrow$ $$\int_{10}^{50} f(x)dx = \int_{2(5)}^{2(25)} f(x)dx = (25-5)\int_{0}^{2} f(x)dx = 20 \times 5 = 100$$

27. $A(3,2,0)$ & $B(1,2,3)$ all in the plane

$\Rightarrow$ $\overline{AB} = 2\hat{i} + 0\hat{j} + (-3)\hat{k}$ is in the plane

$\therefore$ Vector normal of plane $= \left(2\hat{i} - 3\hat{k}\right) \times \left(\hat{i} + 5\hat{j} + 4\hat{k}\right)$

$$= 15\hat{i} - 11\hat{j} + 10\hat{k}$$

$\therefore$ equation of plane is

$$\left(\overline{r} - \left(3\hat{i} + 2\hat{j} + 0\hat{k}\right)\right) \cdot \left(15\hat{i} - 11\hat{j} + 10\hat{k}\right) = 0$$

$\Rightarrow$ $15x - 11y + 10z - 23 = 0$

28. $ar^2 + ar^3 = 60$ & $a \times ar \times ar^2 = 1000$

$\Rightarrow$ $ar\left(r + r^2\right) = 60$ $\Rightarrow a^3r^3 = 1000$

$\Rightarrow ar = 10$

$\Rightarrow$ $r + r^2 = 6$

$\Rightarrow$ $r = -3, 2$

$\Rightarrow$ $r = 2$

$\Rightarrow$ $a = 5$

$T_7 = ar^6$

$= 5 \times 2^6$

$= 320$

29. Let $Z = r\left(\cos\theta + i\sin\theta\right)$

$\Rightarrow$ $25 - r5\left(\cos 5\theta + i\sin 5\theta\right)$

$$\Rightarrow \quad \frac{\operatorname{Im} Z^5}{\left(\operatorname{Im} Z\right)^5} = \frac{\sin 5\theta}{\sin^5\theta}$$

Let $Z = \dfrac{\sin 5\theta}{\sin^5\theta}$

$$\frac{dz}{d\theta} = \frac{\sin^5\theta 5\cos 5\theta - \sin 5\theta\, 5\sin^4\theta\cos\theta}{\left(\sin^5\theta\right)^2}$$

$\Rightarrow$ $5\sin^4\theta\left(\sin\theta\cos 5\theta - \cos\theta\sin 5\theta\right) = 0$

$\Rightarrow$ $\sin\theta = 0$ or $\sin\left(-4\theta\right) = 0$

$\Rightarrow$ $\theta = n\pi$ or $\theta = \dfrac{n\pi}{4}$

$\theta = -\dfrac{\pi}{4}$ $\Rightarrow$ $Z_{\min} = -4$

30. Selection of three element in A such that $f\left(x\right) = y_2 = {}^7C_3$

Now for remaining 4 elements in A we have 2 elements in B

$\therefore$ Total number of onto function $= {}^7C_3 \times \left(2^4 - {}^2C_1\left(2-1\right)^4\right) = {}^7C_3 \times 14$