

# PI MU EPSILON JOURNAL

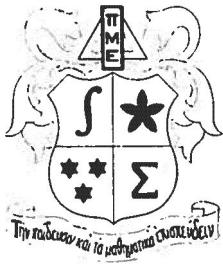
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**PI MU EPSILON JOURNAL**  
**THE OFFICIAL PUBLICATION OF THE**  
**NATIONAL HONORARY MATHEMATICS SOCIETY**

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Problems for solution and solutions to problems should be mailed directly to the PROBLEM EDITOR. Puzzle proposals and puzzle solutions should be mailed to the EDITOR.

The PI MU EPSILON JOURNAL is published at Macalester College twice a year—Fall and Spring. One volume consists of five years (10 issues) beginning with the Fall 19x4 or Fall 19x9 issue, starting in 1949. For rates, see inside back cover.

A MESSAGE FROM THE PRESIDENT  
COMMEMORATING THE SEVENTY-FIFTH ANNIVERSARY  
OF PI MU EPSILON, INC.

The year, 1989, marks the 75th Anniversary of the founding of Pi Mu Epsilon, Inc. and the 40th Anniversary of the establishment of the *Pi Mu Epsilon Journal*. Chartered in the State of New York on May 25, 1914, Pi Mu Epsilon Fraternity, Inc. began as a non-secret organization "to promote scholarship and mathematics" on college and university campuses and at qualified non-academic institutions. The name of this national honorary mathematics society was changed to Pi Mu Epsilon, Inc. in 1988 to reflect the coeducational composition of the organization since it originated.

Like the Panama Canal which officially opened in 1914, Pi Mu Epsilon unlocked the gates to recognizing student talent in the mathematical sciences that same year. Its membership now boasts over 90,000 individuals and 257 chapters across the United States.

Each year since 1952, Pi Mu Epsilon has been holding its National Meeting in conjunction with the Summer Meetings of the American Mathematical Society and the Mathematical Association of America. The Diamond Jubilee celebration will take place from August 7 to August 10, 1989, on the campus of the University of Colorado at Boulder.

The National Honorary Mathematics Society proudly dedicates this commemorative issue of the *Pi Mu Epsilon Journal* to the achievements and aspirations of all the outstanding students of mathematics who have been selected to membership in Pi Mu Epsilon and who will join its ranks in the future, as well as to the loyal Faculty Advisors, Faculty Correspondents, and National Officers who are the backbone of the organization. As the 21st century approaches, Pi Mu Epsilon remains committed to its tradition of serving the nation by recognizing promise in the mathematical sciences and by promoting scholarly activities in mathematics.

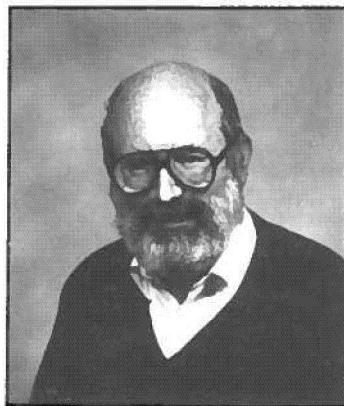
Eileen L. Poiani  
President  
Pi Mu Epsilon, Inc.

**COVERS**

The front and back covers commemorating the 75th Anniversary of the founding of Pi Mu Epsilon were designed and prepared by Professor E. P. Miles, Jr., Florida State University, Tallahassee, Florida, at the FSU Muench Center for Color Graphics, on an INTERCOLOR 2427, DATAVUE, and PRINTACOLOR GP 1024.

Professor Miles presented the J. Sutherland Frame Lecture at the Summer Meeting of Pi Mu Epsilon in Pittsburgh, PA in 1981.

## THE C. C. MACDUFFEE AWARD FOR DISTINGUISHED SERVICE



On August 10, 1988, at the Pi Mu Epsilon National Meeting in Providence, Rhode Island, Dr. Milton D. Cox, was honored by Pi Mu Epsilon with the C. C. MacDuffee Award for Distinguished Service. This prestigious recognition, first bestowed in 1965 upon J. Sutherland Frame, is awarded "often enough to be recognized and seldom enough to be meaningful," according to the Pi Mu Epsilon Governing Council.

As Faculty Advisor of the Ohio Delta chapter at Miami University in Oxford, Ohio, for at least eighteen years, as Councilor of Pi Mu Epsilon from 1975 to 1981 and National President from 1984-87, Dr. Cox has continually contributed to the achievement of Pi Mu Epsilon's goal "to promote scholarship in mathematics." A sort of mathematical "pied piper," he journeyed over the years with many Miami University students - by van, auto, and train, to the annual Pi Mu Epsilon National Meetings at college and university campuses across the United States. His untiring efforts to stimulate student interest in mathematics have been rewarded both by an active Pi Mu Epsilon chapter on his home campus and by a substantial increase in the number of student papers presented at national meetings during his presidency of Pi Mu Epsilon.

Not content to wait for just one summer opportunity for Pi Mu Epsilon students to present papers, Dr. Cox has organized and directed fifteen consecutive Pi Mu Epsilon Student Conferences at Miami University. The sixteenth is scheduled for Fall 1989. These successful conferences continue to serve as models which have been successfully replicated on other campuses.

A graduate of DePauw University, the honoree received his master's and doctoral degrees from Indiana University, specializing in abstract algebra and field theory. At Miami University, he currently divides his time teaching as an Associate Professor of Mathematics and Statistics and serving as the Associate Provost for Teaching Effectiveness Programs. In addition, Dr. Cox coordinates the departmental honors program and chairs the Committee on Student Sections for the Mathematical Association of America's Ohio Section of which he is a past-president.

A member of Phi Beta Kappa, Dr. Cox has lectured on topics ranging from the theory of tiling to the ingredients of an undergraduate mathematics curriculum. He has brought to Pi Mu Epsilon and to its students a contagious enthusiasm for mathematics.

Pi Mu Epsilon is proud to add the name of Dr. Milton D. Cox to the list of recipients of the C. C. MacDuffee Distinguished Service Award.

1964	Dr. J. Sutherland Frame
1966	Dr. Richard V. Andree
1967	Dr. John S. Gold
1970	Dr. Francis Regan
1972	Dr. J. C. Eaves
1975	Dr. Houston Karnes
1980	Dr. Richard Good
1988	Dr. Milton D. Cox

## AMS ESTABLISHES ANNUAL PRIZE

President Eileen Poiani has announced that the American Mathematical Society has established a prize of up to \$1000 per year to promote undergraduate scholarship in mathematics. The prize money will be administered by Pi Mu Epsilon. Officers and councilors of Pi Mu Epsilon will determine how the prize money will be distributed.

Join in PI MU EPSILON'S DIAMOND  
JUBILEE CELEBRATION in Boulder,  
Colorado, from August 7-9, 1989.



*See The Rockies!  
Share Your Ideas!  
Develop New Friendships!*

See your PI MU EPSILON advisor for details".

## PIMU EPSILON CELEBRATES 75 YEARS OF MATHEMATICAL ACTIVITY

by J. Sutherland Frame  
Michigan State University

### 1. Incorporation and Early Days.

The Pi Mu Epsilon Fraternity, incorporated on May 25, 1914, under the laws of the State of New York, is celebrating its 75th anniversary as a national mathematics honor society, with over 250 chapters in 45 states and the District of Columbia. It is a non-secret organization whose purpose is the promotion among students and faculty of scholarly activity in mathematics.

The Syracuse University Mathematics Club, which parented Pi Mu Epsilon, first met on November 30, 1903, at the home of Dr. W. H. Metzler, who was named Director of the club. Dr. Edward Drake Roe, Jr. was elected Vice-Director, Mary B. Quinlan Secretary and Mr. Carpenter Treasurer, as recounted in the archives at Syracuse University. During the next ten years the club carried on an active program with many student papers (preserved in the archives), directed in turn by Metzler, Roe, Warren G. Bullard, Floyd Fiske Decker, Daniel Pratt and Lapine Hall Price. At the tenth anniversary meeting on November 17, 1913, a committee chaired by Professor Roe was established to consider a possible revision of the club's constitution. Its report on December 7 presented four options, included in Professor Decker's historical article about Pi Mu Epsilon (*Pi Mu Epsilon Journal*, Vol. 1, No. 1 (1949), pp. 8-12). The second option "to reorganize the club as a professional fraternity" was approved. Thus Dr. Roe's influential ideas led to the establishment of Pi Mu Epsilon.

On March 2, 1914, a convention was held and a constitution was adopted. On March 23, the choice **E n M** was made among five sets of Greek letters proposed for the fraternity name, subject to changing the order of the letters to **Pi Mu E**. Eight faculty members and 42 students then took the following pledge and signed their names as charter members:

**PLEDGE:** I solemnly promise to give my best efforts to the improvement of my scholarship in all my studies and researches and especially in Mathematics; I will maintain a discreet silence concerning all the aims and obligations of this fraternity; I

will cheerfully accept advice and admonition as long as I am a member of the Fraternity.

Officers elected at the convention on April 27, 1914, were as follows:

Professors E. D. Roe, Jr. and F. F. Decker Director and Vice-Director, undergraduates Helen L. Applebee and Purley J. Bentley Secretary and Treasurer, and Olive Evelyn - Jones Historian. Incorporation followed on May 25, 1914.

Additional chapters were chartered at the Ohio State University in 1919, at the University of Pennsylvania in 1921, and at the Universities of Missouri and Alabama in 1922. The officers of the Alpha Chapter at Syracuse University served as national officers until December 1922, when a national office was established. Additional information about the early days is included in Dr. Decker's article referenced above, and in J. Sutherland Frame's article "Fifty years in the Pi Mu Epsilon Fraternity" (*Pi Mu Epsilon Journal*, Vol. 3, No. 10 (1964), pp. 511-5).

### 2. Pi Mu Epsilon National Reports and Meetings.

Pi Mu Epsilon has always considered its primary aim to be the encouragement of scholarly interest and activity in mathematics among undergraduate and beginning graduate students. This aim was fostered at first mainly by individual chapters, but partly by informal meetings of interested faculty and students, usually held once in three years at meetings of the Mathematical Association of America (MAA). The American Mathematical Monthly (the Monthly) recorded national meetings of Pi Mu Epsilon in 1938 and on January 1, 1942. Meetings without formal talks were held in 1945, 1948, 1953, and 1959. Pi Mu Epsilon activities were reported in the Clubs and Allied Activities section of the Monthly, a section edited from 1931 to June 1935 by F. M. Weida, from December 1935 to July 1938 by F. W. and Helen B. Owens, from August 1938 to August 1942 by E. H. C. Hildebrandt, from January 1942 to December 1946 by J. Sutherland Frame, and from 1947 to 1951 by L. F. Ollmann.

In 1927 a vote of nine chapters on the question "Shall we have a National Publication for **Pi Mu E**?" was 5 to 4 negative. Years later, in 1949, Pi Mu Epsilon launched its own journal, with Ruth Stokes of Syracuse as Editor, Howard C. Bennett as Business Manager, and J. Sutherland Frame, H. T. Kames, N. H. McCoy, and R. J. Walker as Associate Editors. The Journal contains papers written by undergraduates and others deemed to be of interest to them, as well as a puzzle section, a problem department,

chapter reports, invited addresses, and other items of interest to Pi Mu Epsilon members. New member lists were included until 1972.

Pi Mu Epsilon has been especially fortunate in having had a succession of outstanding problem **proposers/solvers** serve as Editor of the Journal's Problem Department. The first so designated was the late Leo Moser, who edited the department beginning with Vol. 1, No. 4. through Vol. 2, No. 7. Together, C. W. Trigg and Leon **Bankoff** edited Vol. 2, No. 8. Murray S. Klamkin assumed the editorship with Vol. 2, No. 9 and continued for ten years. Leon **Bankoff** took over with Vol. 4, No. 9 and sewed through Vol. 7, No. 2. The next three issues were jointly edited by **Bankoff** and Clayton W. Dodge. Beginning with Vol. 7, No. 6, Dodge had been Editor.

Presentation of papers by students at Pi Mu Epsilon national meetings began at the national meetings of the M M and American Mathematical Society (AMS) at Michigan State University in 1952, continued at meetings at the University of Michigan in 1955 and at Pennsylvania State University in 1957, and has continued since 1960 at most subsequent Pi Mu Epsilon national summer meetings, held annually with few exceptions (1962, 1974). Student speakers are supported for their travel expenses, encouraged to submit their papers to the Journal for publication, and lured by prizes for the best papers published by students below or just at the master's level. To facilitate interaction between participants, Pi Mu Epsilon holds an informal get-acquainted reception the evening before student papers are delivered. A complete list of the papers presented appears as Appendix A to this article.

Most Pi Mu Epsilon national meetings have included a one-hour address by an invited speaker, notably that in 1972 at Dartmouth by President John Kemeny, entitled "Mathematical Models and the Computer" (Pi Mu Epsilon Journal, Vol. 5, No. 8 (1973), pp. 373-86). At the 1973 meeting of the Council, Past President J. C. Eaves proposed and the Council approved a motion that this activity be formalized in a series of annual addresses to be known as the J. Sutherland Frame Lectures. Past President Frame was asked to inaugurate the series in 1975, and deeply appreciates the honor. Appendix B is a complete list of speakers and topics.

In 1964 a 50-year history of Pi Mu Epsilon by Frame was printed in the Journal. We look forward to our 75th anniversary meeting at the University of Colorado in Boulder in August 1989.

### 3. Pi Mu Epsilon Financial Perspective.

In 1936 when national Secretary John S. Gold became national **Secretary-Treasurer** for Pi Mu Epsilon his office issued charters for \$10, and registration fees and certificates of membership for \$0.25 each. The latter two charges were combined into a \$1 initiation fee in November 1950. raised to \$2 in 1957. to \$4 in 1971. and to \$10 in 1986, due to increasing expenses. Pi Mu Epsilon has never charged its members annual dues, but does encourage subscriptions to the Journal, which are included for one year in the initiation fee.

In 1937, the L. G. Balfour Company became the official jeweler for Pi Mu Epsilon, and in 1938 began remitting royalties on jewelry which totaled \$3384.42 from 1951 to 1954, and varying amounts in subsequent years. At first, 10-karat gold keys or key-pins were sold for \$3.75, but when the price of gold skyrocketed in the 1970's and the key price rose above \$12, Pi Mu Epsilon began ordering gold-plated pins in quantity from Balfour, and sold them to initiates for \$5 in 1972 and for \$8 beginning in 1979.

The first 4000-copy issue of the Pi Mu Epsilon Journal in 1949 cost \$500. Between 1950 and 1954 the Journal received \$0.75 of each \$1 initiation fee, which paid for a one-year subscription. Printing costs continued to rise. In 1973 the **two-year** subscription price was increased from \$1.50 to \$4 for members, and from \$2 to \$6 for non-members and libraries; it was raised again in 1980 to \$8 and \$12 respectively for two years. In 1975-1976 the Journal income of \$6108.90 included \$4000 from the Pi Mu Epsilon treasury, and expenses were \$5426.26. In 1980-1981 Journal costs of \$10,850.12 were subsidized by \$5000 from the Pi Mu Epsilon treasury. Cash assets of Pi Mu Epsilon on June 30, 1988, totalled \$27,185.79.

In 1966 the Council approved a prize of \$100, to be awarded each year, for the best paper by an undergraduate that was published in the Journal. Beginning in 1971, first, second, and third prizes of \$200, \$100, and \$50 were awarded, and eligibility was extended to graduate students working on or just completing a master's degree. These awards were named the Richard V. Andree Awards by the Council in 1987. Of all the student-written papers, heretofore receiving first prizes, that of Donald John Nicholson on "A Ubiquitous Partition of Sets of  $\mathbb{R}^n$ ," which appeared in Vol. 8, No. 1, pp. 2-7, has been selected as "best" and is reprinted on pages 663-668.

Pi Mu Epsilon also encouraged local chapters by matching prize money up to \$50 per chapter and assisting financially with student participation in sectional meetings. But the largest financial support to students was transportation expense to national meetings. Travel subsidies totaling \$486.21 for the 1952 summer meeting in East Lansing, Michigan, with student speakers, and \$793.24 for the December 1953 meeting in Baltimore for officers and delegates (no student speakers), was first based on full travel expense up to \$60 for speakers, and one-half rail fare plus lower berth up to \$35 for delegates. In later years travel subsidies were based on minimum airfare for a speaker, with half these amounts for a chapter delegate, subject, however, to maximums which rose from \$60 in 1952 to \$150 in 1964, \$300 in 1973, \$400 in 1977, \$500 in 1981, and \$600 in 1985.

In 1957 Pi Mu Epsilon contributed \$300 to help launch the Mu Alpha Theta high school math honorary sponsored by Dick and Josephine Andree. It contributed \$500 in March 1973 to the National Council of Teachers of Mathematics (NCTM) for its building fund, \$500 in 1981 to the NCTM toward a memorial to our past president Houston Karnes, \$3000 in 1978 to the MAA for its building fund, and \$500 in 1987 toward the endowed scholarship in mathematics or computer science at the University of Oklahoma in memory of Richard V. Andree.

#### 4. Pi Mu Epsilon growth in chapters and membership.

When national officers were first elected in 1922, there were five chapters of Pi Mu Epsilon (listed in section 1). When Dr. E. D. Roe, Jr., the first Director General, died in 1929 and was succeeded by Dr. Louis Ingold, there were 18 chapters. There were 31 chapters on April 1, 1936, when a new constitution was adopted and the offices of Secretary, Treasurer, and Librarian were combined in the office of **Secretary-Treasurer**, held by the former Secretary, John S. Gold of Bucknell. Griffith C. Evans succeeded F. W. Owens as Director General, and was followed by W. E. Milne in 1939 and Tomlinson Fort from 1942 to 1948. By fall 1949, when the *Pi Mu Epsilon Journal* was started, and E. H. C. Hildebrandt was Secretary-Treasurer, there were 48 chapters. Membership totals on April 1, 1951, and July 1, 1954, were 18,857 and 22,897. At that time Richard V. Andree replaced J. Sutherland Frame as Secretary-Treasurer and three-year terms for National Officers began July 1 instead of April 1, to facilitate exchanges of files and information.

During C. C. MacDuffee's term as Director-General, 1948-1954, he installed

almost all the new chapters. His successor, S. S. Cairns, had a sabbatical leave in 1954-1955, and asked Vice-Director General Frame to act for him that year. By the end of Frame's three terms as Director General in 1966 there were 120 chapters, of which Frame had installed about 50. Membership in Pi Mu Epsilon increased by ~ 25,198 from 1975 to 1986 when Richard Good was Secretary-Treasurer, and should exceed 90,000 by 1989. On April 1, 1989, there were 257 chapters.

The Constitution and By Laws of 1936 were revised in 1958, 1965, and 1972. A 1958 revision provided that the nominating committee appointed by the Director General should nominate three qualified candidates for the office of Director General, among whom the candidate with the second highest vote of the chapters should become Vice-Director General. Another significant change provided for the chartering of affiliate chapters at Pi Mu Epsilon at non-academic institutions. Only one such chapter was installed, at the urging of George Marks, at the General Electric plant in Evendale, Ohio, in 1958. It became inactive when Marks moved away. Constitutional revisions in 1965 changed the titles of Director General and Vice Director General to President and Vice President, shortened the pledge for initiates, and provided that charter petitions approved by unanimous vote of the Council did not require chapter approval. A 1972 revision changed the office of Vice President to President-Elect, at least one to be nominated, and provided for the automatic succession after a three-year term to a term as President, followed by a term as Past President and Council member.

#### 5. Who were our National Council Members?

Lists of names and dates do not tell the whole story. So what do we know about the men and women who have served as Officers and Councilors of Pi Mu Epsilon? Back numbers of the *Pi Mu Epsilon Journal* provided added information. The reference (1/1/1-9/F49), meaning Vol. 1, No. 1, pp. 1-9, Fall 1949, refers to a photo and biography of the first Director General E. D. Roe, Jr., who was an astronomer as well as a mathematician. Director-General Griffith C. Evans served as Vice President of both the AMS and M M, and was AMS President in 1938-1940. Tomlinson Fort served as AMS Associate Secretary and later as *Bulletin* Editor. C. C. MacDuffee was M M President in 1945 and Frame served two terms (1950-1952 and 1958-1960) on the M M Board of Governors. R. H. Bing was M M President in 1961. Photos and biographies of C. C. MacDuffee, W. M. Whyburn, J. S. Frame, S. S. Cairns and Sophia MacDonald appear in (1/5/191-3/F51). Other portraits and biographies appear in (1/6/229-31/S52).

(1/8/331/S53), (2/1/1,31-4/F54), (2/6/279-81/S57), (5/7/356-8/F72), and (6/3/194-5/F75). Colton MacDuffee was born June 25, 1895, and died August 21, 1961, see (3/5/213/F61). Appendix C is a complete list of Officers and Councilors.

In 1957 Josephine and Richard Andree launched the high school mathematics honorary society Mu Alpha Theta (2/8/394/S58). Henry Alder and later J. C. Eaves (among others) served as Presidents. Cosponsored now by the NCTM and MAA, Mu Alpha Theta now has over 1200 chapters and 40,000 members in 42 states, the District of Columbia, Puerto Rico, Canada, Germany, Japan, and Switzerland.

In 1965 J. C. Eaves proposed the C. C. MacDuffee Awards for Distinguished Service to Pi Mu Epsilon, described in (4/6/229/S67), and followed (4/6/230-3/S67) by portraits of the 1966 honorees J. S. Frame, R. V. Andree and their wives. Subsequent honorees were former Secretary-Treasurer John S. Gold in 1967 (4/8/319-21/S68), Editor Francis Regan in 1970 (5/3/105-6/F70), Presidents J. C. Eaves in 1972 (5/8/371-2/S73), Houston Karnes in 1975 (6/3/123-4/F75), who died in March 1980, Secretary-Treasurer Richard Good in 1980 (7/3/149/F80), and Past-President Milton Cox in 1988 (8/10/634-5/S89).

After 21 years as Secretary-Treasurer, 1954-1975, Richard Andree served as President-Elect and then President, but died May 8, 1987 after a long illness. Richard Good was his able successor as Secretary-Treasurer 1975-1987 and now serves as a Councilor. In 1978 E. Allan Davis followed Houston Karnes as President. The four Councilors of 1975-1978 have all become national officers: E. Maurice Beasley, Milton Cox and Eileen Poiani as Presidents 1981-1984, 1984-1987, and 1987-1990, respectively, and Robert Woodside as Secretary-Treasurer 1987-1990. David Ballew, our Editor 1978-1984, is now President-Elect. Joseph Konhauser followed Ballew as Editor, Milton Cox is now Past-President, and Richard Poss, Robert Eslinger, and J. Douglas Faires are now Councilors. Under Eileen Poiani and her colleagues Pi Mu Epsilon should have a bright future.

## APPENDIX A

### STUDENT PAPERS

1952 -- Michigan State College Meeting

"modification of Infinite Series", **Bevan K. Youse**, University of Georgia  
 "Rapid Square Roots", **Charles D. Parker**, Michigan State University  
 "almost Periodic Functions", **John E. Hoffman**, University of Oklahoma  
 "matrix Inversion", **Verna Lair**, University of Delaware  
 "matrix Extension", **Alan J. Goldman**, University of Kentucky  
 "The G. C. D. Algorithm", **Wilson M. Zaring**, University of Kentucky

There were no meetings in 1953 or 1954.

1955 -- University of Michigan Meeting

"magic Squares", **Walter W. Turner**, Michigan State College  
 "Semigroups", **Thomas Head**, University of Oklahoma  
 "Elliptical Wheels on an Inclined Plane", **James Hinly, Jr.**, University of Georgia  
 "modern Developments in Additive Number Theory", **Roy Lisker**, University of Pennsylvania  
 "Reorderings of Sets", **Ronald Cleary**, Syracuse University  
 "An Equation Concerning the Functional Exponent", **John Stallings**, University of Arkansas  
 "Some Cayley Color Groups of Order Less Than Fifteen", **Chih Han Sah**, University of Illinois  
 "On Extensions of Kasner's Circle", **Manning L. Rose**, University of Kentucky

There was no meeting in 1956.

1957 -- Pennsylvania State University Meeting

"Envelopes of Certain Families of Conics", **Katharine Lippis**, St. Louis University  
 "Mathematics in Turkish High Schools", **Ali Tangoren**, Louisiana State University  
 "Symmetry Groups and molecular Structure", **J. S. Frame**, Michigan State University  
 "On the Convergence of the Fourier Series", **Tom Boehme**, University of Oklahoma  
 "The Perpetual Calendar", **Francis Felix**, Pennsylvania State University

There were no meetings in 1958 or 1959.

1960 -- Michigan State University Meeting

"Konigsberg Bridge Problem", **Fred Howlett**, University of Nebraska  
 "Filters and Ultrafilters", **John Allen**, Oklahoma State University

## 1961 -- Oklahoma State University Meeting

- "Conformally Elementary Points". **Louis DeNoya**, Oklahoma State University  
 "Transcendence of  $e$ ". **John Wells**, Ohio University of Dayton  
 "A Topic in modern Algebra". **Charles Mullins**, Oklahoma State University  
 "General Solutions to the Linear Diophantine Equation in the Two Variables",  
**Thurston Shook**, Ohio State University

The AMS/AMM Summer Meeting was in Canada and there was no Pi Mu Epsilon Meeting in 1962.

## 1963 -- Boulder, Colorado Meeting

- "Raabe's Test", **Charles B. Huelsman**, Miami University  
 "Some Recent number-Theoretic Research Using a High-speed Calculator", **Kenneth E. Kloss**, Carnegie Tech  
 "Definition of Vector Product and Topological Considerations", **Robert Jay Buck**, University of Buffalo  
 "A Characteristic Property of Closed Subgroups of  $E^n$ ". **Stephen E. Crick, Jr.**, Michigan State University  
 "The Approximation of Normal Derivatives and the Numerical Solution of the mixed Boundary Value Problem for Two Dimensional Laplace Equation", **Sergei K. Anto**, Portland State College  
 "Commutative Alternative Rings", **William A. Thedford**, Oklahoma State University  
 "Catenary Rddition -- A method of Sequence Generation", **Thomas William Cusick**, University of Illinois  
 "Tauberian Theorems on the Fractional Function". **M. Alan Feldstein**, UCLA  
 "A Generalization of the Riemann-Lebesgue Lemma", **William E. Heireman**, Georgia Institute of Technology

## 1964 -- University of Massachusetts Meeting

- "Existence Theorems for Hyperbolic Partial Differential Equations", **R. Brewster Knight**, Georgetown University  
 "Theorem Proving on the Computer... the State of the Art", **Carl Douglas Jensen**, Florida State University  
 "Weierstrass Approximation Theorem", **James H. Opolka**, DePaul University  
 "Applications of Contraction mappings", **Richard H. Bouldin**, University of Alabama

## 1965 -- Cornell University Meeting

- "A Study of the Order Properties of Non-Desarguesian Planes", **Beverly Welles**, Denison University  
 "Generation of Content and measure Spaces", **Howard Edward Evans**, Franklin & Marshall College  
 "Some of the Plane Figures Under the Direct motions", **Henry Lee Phillips**, Southern University  
 "Determinants in a Division Ring". **William G. Bridges, Jr.**, University of Connecticut  
 "mappings of Semigroups Associated with Ordered Pairs", **Margaret Vitanza**, SUNY at Buffalo, New York  
 "Sums of Rational numbers", **William A. Webb**, Michigan State University  
 "Subgroups of Rational Numbers", **Alan Schwartz**, University of Wisconsin  
 "Transverse Stability of Vessels", **Joseph C. Zuercher**, Marquette University  
 "Zeros, Identities, and Associativity in Groupoids", **Johnnye Lundgren**, Sacramento State College

## 1966 -- Rutgers - The State University Meeting

- "The Solution of a Simple Differential Equation", **Ralph Edwin Showalter**, North Carolina State at Raleigh  
 "Applications of the Arithmetics of Simple Quadratic Fields to Diophantine Analysis". **Hugh L. Montgomery**, University of Illinois  
 "The Numerical Study of a Boundary Value Problem with One Given Condition at Infinity", **Carol Knudson**, State University of South Dakota  
 "An Introduction to Topological Rings". **David M. Arnold**, Western Washington State College  
 "The Jordan-Schonflies Theorem", **Richard C. Ville, Jr.**, Michigan State University  
 "Solutions of Equations in Finite Groups". **Mark Benin-d**, Tulane University  
 "The Schrödinger Wave Equation". **Glenn Thompson**, University of Toledo  
 "Rddition Program for Turing machine", **James Woepfle**, SUNY at Buffalo, New York  
 "Surveying on the Level: The Schröder-Bernstein Theorem", **Frank R. Bernhart**, University of Oklahoma  
 "Random Variables", **Louis F. Kemp**, Polytechnic Institute of Brooklyn  
 "An Extension of Liouville's Theorem". **Kerb Silverman**, Syracuse University

## 1967 -- Rutgers - The State University Meeting

- "Formal Power Series Over Commutative Ring with Identity", **James William Brewer**, Florida State University  
 "Extensions of mean Value Theorem". **Preston Dinkins**, Southern University  
 "The Fundamental Theorem of Algebra", **Paul J. Campbell**, University of Dayton  
 "Pascal's Triangle Generalized and Polytope Geometry", **Eric Richard Halsey**, Pomona College  
 "Reciprocal Rddition", **James Williams**, Carleton College  
 "The Study of a Recursive Sequence", **James A. Wingert**, John Carroll University  
 "Continued Fractions and the Geometry of moment Spaces", **Thomas McChesney**, Denison University  
 "Construction of a Banach Space Using the Laplace Transform", **Pedro J. Saavedra**, Georgetown University

- "**A. L. P. S.: Algebraic Programming System**", **Ralph Howenstine**, University of Oklahoma  
 "Indices of Primitivity and Imprimitivity of a Cyclically Vertex-Symmetric Graph",  
**Alexander F. Hunter**, Florida Presbyterian College  
 "Further Varieties of Faces of the Hexahexaflexagon". **Robert L. Munson**, Oregon State University
- 1967 -- University of Toronto Meeting**
- "Some Useful Results in Applications of Power Series Transformations to the Solution of Difference Equations", **Ray A. Gaskins**, Virginia Polytechnic Institute  
 "Generalizations of Sequences". **William L. Reynolds**, Florida State University  
 "Survey of Student Activities to Increase Interest in mathematics and mathematical Sciences", **Rout A. Deju**, New Mexico Institute of Mining and Technology  
 "A Preliminary Examination of Round-Robin Tournament Theory", **Charles A. Bryan**, John Carroll University  
 "A Functional Approach to the Singularities of Plane Curves", **Richard J. Bonneau**, College of the Holy Cross
- 1968 -- University of Wisconsin Meeting**
- "The Fibonacci Sequence: An Introduction", **Donald F. Reynolds**, Texas Christian University  
 "A Representation Theorem for Bounded Finite Generation Processes", **James Rosenberg**, Pomona College  
 "Revision of monitor Using Assembly Language", **Arthur P. Stadler**, Denison University  
 "Twin 'n-Primes'", **George F. Grob**, Georgetown University  
 "Series of Polygonal numbers", **Michael C. Kopkas**, John Carroll University
- 1969 -- University of Oregon Meeting**
- "Rational Points on Elliptic Curves", **Bruce Rienzo**, Rutgers University  
 "The Cantor Set", **Jerry Lynn West**, Southern University  
 "A Semi-Number System". **Ann K. Miller**, Saint Louis University  
 "Determination of Polynomial Roots: A Construction", **John Wallis**, Texas Christian University  
 "Simplicial Decompositions of Convex Polytopes", **Allan L. Edmonds**, Oklahoma State University  
 "A Decimal Approximation to 'Pi', Utilizing a Power Series", T. **Golian** and J. **Hanneken**, Ohio University  
 "number Theory, Density of Quadratic [Non-Residues in  $(1, p)$ ]". **Richard W. Johnson**, East Carolina University  
 "Tauberian Theorems", **Addison M. Fischer**, West Virginia University  
 "Topological Rings and Topologies on Rings", **Gary Sargent**, New Mexico Institute of Technology  
 "An Important Counterexample in the Field of Round Robin Tournament Theory", **Anthony J. Kenzie**, John Carroll University

- 1970 -- University of Wyoming Meeting**
- "Linear Programming", **Margaret M. Lynch**, St. Louis University  
 "Program for Finding Endomorphisms and Associated Near Rings of a Finite Group", **Russell Schenck**, University of Southwestern Louisiana  
 "Positive Definite matrices", **Stephen M. Zemyan**, University of Delaware  
 "newton and the Development of Calculus". **Thomas R. Bingham**, State University College of New York  
 "Wronskian Identities". **Martin J. Swiatkowski**, John Carroll University  
 "The Prime Divisors of Second Order Recurrences". **P. A. Catlin**, Carnegie-Mellon University  
 "Phi and Curious Yellow: The Golden Ratio". **Linda Riede**, University of Denver  
 "Game Theory". **Elliot Wiesner**, Polytechnic Institute of Brooklyn  
 "Zero Divisors in an Enveloping Ring", **Robert Ussery**, East Carolina University  
 "On 'Almost Unitary Perfect' numbers", **Sidney West Graham**, University of Oklahoma
- 1971 -- Pennsylvania State University Meeting**
- "Lonesome Points in a Topological Space", **Bobby Beckorn**, East Carolina University  
 "Dieting: An Application of Decision Theory", **Rebecca Jane Klemm**, Miami University  
 "The Legendre Polynomials". **Barbara F. Beier**, Fontbonne College  
 "n-Space Generalization of the Pythagorean Theorem", **Sister Carol Ann Dodd**, Western Washington State  
 "Finite Near Domains", **Russell Schenck**, University of Southern Louisiana  
 "An Example of Non-numerical Analysis", **Frederick C. Drusekis**, University of Arizona  
 "The Unique Equilateral Triangle". **Janet B. McDonald**, State University of New York at Albany
- 1972 -- Dartmouth College Meeting**
- "K-Automorphisms of  $K(t)[[x]]$ ". **Hugh Barnett**, East Carolina University  
 "Integers That Are a Function of Their Digits", **Victor G. Feser**, St. Louis University  
 "A General Test for Divisibility". **Robb T. Koether**, University of Richmond  
 "Eulerian numbers", **Angela Kenison**, Brigham Young University  
 "Characterization of the Differentiable Functions of a Quaternion Variable", **Joseph J. Buff**, New York University  
 "Distributively Generated Near Rings of Groups of Order <15". **Kenneth R. Wadsworth**, Worcester Polytechnic Institute  
 "Happy numbers", **Daniel Wensinger**, John Carroll University  
 "Intuitionism (School of Intuition)". **Beverly Waiters**, Gonzaga University  
 "Goldbach's Conjecture", **Chris Scusset**, Michigan State University  
 "A Program to Determine the Number of Topologies on a Finite Set", **Roger W. Foster**, Miami University

"Computer Generated Random Vectors with Specified Density", **Richard M. White**, Miami University  
 "Complex Analysis", **Ronald Umble**, Temple University

#### 1973 -- University of Montana Meeting

"Some Slick Tricks with Fourier Series", **Joseph Jonathan Buff**, New York University  
 "Niceness of the Socle and a Characterization of Groups of Bounded Order", **San W. Talley**, Western Kentucky University  
 "Applications of Finite Differences to the Summation of Series", **H. Joseph Straight**, State University of New York  
 "Partial Differentiation in metric Spaces", **Roseann Morello**, Seton Hall University  
 "Properties of CT-Topological Spaces", **Daniel Campbell**, Samford University  
 "Book Thickness and Graphs", **David Keys**, Louisiana State University  
 "A Look at Predator-Prey mathematical models", **Raymond J. Marszalowicz**, St. Peter's College  
 "Configurations of the Platonic Solids of Positive Genus", **Kurt J. Schmucker**, Michigan State University  
 "Some Properties of the Hermite Differential Equation", **Shirley M. foreman**, Prairie View A&M College  
 "mathematics and Language", **Vincent Fon**, University of Kansas  
 "The filgebras of Floating Point number Systems", **P a d Vitasek May**, University of Oklahoma  
 "A Formal System for Quantification Theory", **Mitchell Spector**, John Carroll University  
 "The Theory of Queues", **Kristine R. Hueseman** and **Cynthia M. Warren**, St. Louis University  
 "Graphical approach to Game Theory", Evelyn Wallace Jenkins, University of East Carolina  
 "An All-Rule Axiomatization of the Propositional Calculus", **William David Hoak**, Occidental College  
 "Constructing Polyhedron (Regular)", **Karin Ann Stahara**, St. Louis University  
 "Soap Films", **Jack Eckhard**, St. Louis University  
 "metric Topological Invariants and Some Elementary Consequences", **Gerald Hyde Jaimovich**, U.C.L.A.

The AMS/AMM Summer Meeting was not held and thus there was no Pi Mu Epsilon Meeting in 1974.

#### 1975 -- Western Michigan University Meeting

"Sequential Convergence", **Lynn L. Mineo**, Eastern Carolina University  
 "The Philosophy and Proof of Gödel's Incompleteness Theorem", **Charles M. 'Robins**, Brooklyn College  
 "Continuous Non-Differentiable Functions", **Brent Hailpern**, University of Denver  
 "A Proof of the Theorem that Any Two Tape Turing machines may be Simulated by a One Tape Turing Machine", **Zach D. Cox, Jr.**, University of North Carolina at Wilmington

"A Problem in Geometry and Probability", **John Q. Walker, II**, Southern Illinois University at Edwardsville  
 "A Growth model for the Plastics Industry", **Lawrence R. Rogers**, Miami University  
 "The Busy Beaver Problem for Turing machines", **Kathleen Marra**, Manhattan College  
 "Coherent States in mathematical Physics", **Thomas Ronayne**, University of South Florida  
 "mathematical models in nuclear Fuel management", **Michael P. Mahanah**, Michigan State University  
 "Calculus of Variations", **John R. Andreozzi**, Rhode Island College  
 "Distance and Interiority in a Boolean metric Space", **M. Elizabeth Newton**, St. Louis University  
 "numerical Solution of a ion-Linear Electron Conduction Equation with Boundary Values", **James DeLucia**, St. Joseph's College  
 "Snakes on the 7-Dimensional Cube", **Robert Myers**, Hope College

#### 1976 -- University of Toronto Meeting

"Chainable Continuum Not Homeomorphic to an Inverse Limit on [0,1] with Only One Bonding map", **Dorothy Marsh**, University of Houston  
 "An Informal math Lab", **Kevin Bucol**, Creighton University  
 "[0,1] Is not Compact: A Discussion of the Hyper-Reals", **Thomas Sweeney**, St. Louis University  
 "On the Problem of the Lion and the man", **Mark Showers**, Southern Illinois University  
 "Ridge Regression", **Dote Borowiak**, University of Akron  
 "Fugue in Z# major", **William Stone**, University of Utah  
 "Fixed Point Theorems in metric Spaces", **Carol Collins**, East Carolina University  
 "fidditions and Con-actions to Elementary number Theory in Certain Subsets of the Integers I and II", **William Lenhart** and **Karen McConlogue**, St. Joseph's University  
 "Topology on Geometries with Betweenness", **Anees Rozouk**, Andrews University  
 "High School mathematical models", **Ellen Hearn**, St. Peter's College  
 "The Whitney Theory for Maps Between 2-Manifolds", **Jane Hawkins**, College of the Holy Cross  
 "On Difunctional and Circular Relations", **Alma E. Posey**, Hendrix College  
 "Operations Research -- An Approach to the Solution of Problems in the Urbana System", **Elaine flowers**, Alabama State University

#### 1977 -- University of Washington Meeting

"Paradox in the Development of mathematics", **Wayne Heym**, Miami University  
 "The Value of mathematics in Pro-Legal Education", **Bruce Fox**, University of Illinois  
 "Commutative Rings and Fields", **Kathryn Dowdell**, St. Joseph's University  
 "Continuous Convergence of Functions", **Al Robe Childs**, East Carolina University  
 "Continuous Convergence in C(X)", **James Lewis**, East Carolina University  
 "A Characterization of G Sets in metric Spaces", **Jeff Thompson.. Cornell**, University

- "How to Model for Politicians". **Lonita B. Spivey**, College of Charleston  
 "mathematical modeling of a Sewage Plant", **John Gimbel**, Andrews University  
 "The Structure of the Solution Space of 5th Order Linear Differential Equations",  
**Ernest Lowery**, Prairie View A & M University  
 "Sink-Like Structures in Compartmental Analysis", **James Bellinger**, Southern Illinois University  
 "A Computer Application of Linguistics", **Alfredo Garcia**, St. Louis University  
 "Set Simplification Simplified", **Date Watts**, University of Denver  
 "The Effect of Finite Infinities on Rational numbers", **Victor Meyer**, University of Detroit  
 "mathematics Field Day", **Kenneth Pittz**, Creighton University  
 "An Introduction to and an Application of Elliptical Integrals", **Mark Goldsmith**, Miami University  
 "Acceleration of Root Finding Algorithms Through Chebyshev Interpolation", **Richard Daugherty**, Western Kentucky University  
 "Simple Continued Fractions", **David Miyashiro**, Ohio State University  
 "On Distance Attaining Sets", **Robert Goggins**, University of Mississippi  
 "An Introduction to Coding Theory", **Sill Heidler**, Miami University

#### 1978 -- Providence Meeting

- "Why I like Graph Theory", **Nancy L. Burger**, State University of New York at Potsdam  
 "Calculation of the Period of the Lotka-Volterra Predator Prey Model", **Kathy Stuewe**, University of Tennessee  
 "The Unique Number 15", **J. B. Zipperer, Jr.**, Armstrong State College  
 "A Paradox in Quantum Theory", **Linda Whaleyman**, Sam Houston State University  
 "Is It Possible to Lose the Ol' Magic?", **Douglas W. Boone**, Miami University  
 "The Double Ferris Wheel Problem", **Albert E. Parish**, College of Charleston  
 "The Use of Fractional Calculus in Solving Certain Difference Equations", **David Challener**, Iowa State University  
 "Causes of math Anxiety at the University", **Kathleen V. Walker**, Southern Illinois University at Edwardsville  
 "Non-Linear Additive Functions", **Julie D. Anderson**, Hendrix College  
 "The Converse of Lagrange's Theorem and Finite Nilpotent Groups", **Marcia James**, East Carolina University  
 "matrix models in Biology", **Michael Young**, Portland State University  
 "multivariate Discriminant Analysis and the Prediction of Loan Defaults", **Nick Bellotti**, University of Northern Florida  
 "Scaling in mammals", **Timothy O'Shea**, St. Peter's College  
 "Concerning Irreducible Compact Continua", **W. Dwayne Collins**, University of Houston  
 "Exam Scheduling: An Example of math modeling", **Carole H. Cook**, Miami University  
 "Generalized Lipschitz Criteria for First Order Differential Equations", **Mark L. Burton**, Hendrix College  
 "A method of Finding the Complement of a Sequence", **Susan McClintock**, East Carolina University

- "A Pulse-Time model for mathematics Class Enrollments", **Julie Montgomery**, Sam Houston State University  
 "Division Algorithms for Prime Factorization", **John Anderson**, Rose-Hulman Institute of Technology  
 "Problems: Stimulation to Research and Application", **Steven From**, Rose-Hulman Institute of Technology  
 "numerical Treatment of meteorological Data", **Gregory Battle**, Washington University  
 "Magic Card Squares, Cubes, and Hypercubes", **Bernard P. Smith**, St. Louis University

#### 1979 -- Duluth Meeting

- "On Continued Fraction Representations of Liouville numbers", **Michael Filaseta**, University of Arizona  
 "Behavior of the Permanent of a Special Class of Doubly Stochastic Matrices", **Phuong Anh Vu**, University of Houston  
 "Physical models for Applications of Rational numbers in Reduces Form", **JoAnn Fiene**, Southern Illinois University-Carbondale  
 "Topological Properties of the Generalized Long Line", **Ali Erayat**, Iowa State University  
 "Enzyme Kinetics of Phenylketonuria", **C. P. Kuchinad**, Marquette University  
 "A Hard (But Interesting) Problem", **Michael L. Call**, Rose-Hulman Institute of Technology  
 "Time as a Dynamical Variable in Quantum Physics", **Stephen L. Ken**, University of Tennessee-Knoxville  
 "Love Sonnet for a mathematician", **Mark Walker**, South Dakota School of Mines and Technology  
 "Infinitesimals: Where They Come From and What They Can Do", **Professor H. Jerome Keisler** and **J. Sutherland Frame**, University of Wisconsin-Madison  
 "The Axiom of Choice", **John B. Vaughn**, St. Louis University  
 "The Kernel of the Laplace Transformation", **David C. Sutherland**, Hendrix College  
 "The Knight's Tour Problem", **Gary Ricard**, South Dakota School of Mines and Technology  
 "A Simple Proof of a Theorem by H Scheffe", **Peter Westfall**, University of California-Davis  
 "mathematics and the Boiling Points of Alkanes", **Michael J Schell**, University of North Carolina at Charlotte  
 "Two Problems in number Theory", **Janet Reid**, University of North Florida  
 "Uniform Algebras and Scattered Spaces", **Robert C. Smith**, University of Arkansas  
 "A Case Against Computer Crime", **Lonnie Eward**, Northwest Missouri State University  
 "measure Construction Using Cauchy Sequences", **Stephen W. Semmes**, Armstrong State College  
 "Cyclic numbers", **Richard O. Griffin**, University of Lowell

"The **Consequences of Cauchy's** Integral Theorem", **Alfred Earl Byrum**, East Carolina University  
 "An Infinite number of magic Squares". **Stephen J. Hubert**, Miami University

**1980** -- University of Michigan Meeting

"**Some** Sums of Sums", **Ann Zabinski**, College of St. Benedict  
 "Simulating Transformations in the Plans **Symmetry** Patterns Using the **Tektronix 4051**", **James McElheny**, Hope College  
 "A New numeric algorithm To Solve Convolution Kernel **Volterra** Integral Equations", **Aly Graham**, University of Arizona  
 "Characterization of Primes". **Kathlyn Nolan**, University of Arkansas at Pine Bluff  
 "Resolution of Russell's Paradox", **Christopher J. Roesmer**, University of Dayton  
 "Equivalent Statements", **Wood-Wai Lee**, Lamar University  
 "Applications of the Programmable Calculator in mathematics and Statistics", **Daniel Pollak**, Miami University  
 "mean Value Theorem Revisited", **Larry N. Stroud**, East Carolina University  
 "numerical Recovery of **Noisy** Exponential Sums Using Generalized Differential flpproximation", **Nersi Nazari**, Southern Illinois University  
 "The **Area** of a Triangle Formed by Three Lines". **Michael L. Orrick**, Macalester College  
 "Singular Functions", **Sandra Cousins**, Hendrix College  
 "Unpacking the Knapsack **Algorithm**", **Mark Walker**, South Dakota School of Mines and Technology  
 "Properties of the Infinite **Symmetric** Group". **Stephen Semmes**, Washington University  
 "Self-Taught Calculus", **Marie Spetseris**, College of Charleston  
 "English Soccer Predictions". **Kevin Saylors**, Pomona College  
 "Correctness Proofs For Flow Chart Programs". **Beverly J. Skeans**, Marshall University  
 "Discrete Versus Continuous: Or, What is the Calculus of Finite Differences?", **Michael L. Calk**, Rose-Hulman Institute of Technology  
 "The Centralizer of a Linear Transformation", **Douglas Eugene Jewett**, Texas A&M University  
 "Finite **State Machines**", **Martha Blackwelder**, Appalachian State University  
 "nested Interval Theorem With Two Applications". **Kurt L. Wiese**, University of Arkansas  
 "Relativity in **Perspectivity**". **William Terkeurst**, Hope College  
 "Problems With **Infinity**", **Michael K. May**, S.J., St. Louis University

**1981** -- Pittsburgh Meeting

"Introduction to Box-Jenkins Time Series", **Beth Snyder**, Miami University  
 "Stokes' Theorem for QuaternionIntegral Operators", **Dan Mock**, South Dakota School of Mines and Technology  
 "A Look at Formal **Theory**", **Dean Shea**, St. John's University  
 "An flpproximation to the normal Distribution", **Edward D. Cowry**, Western Washington University

"The Gamma Function and Extensions". **Bro. Longinus Anyanwu**, Morgan State University  
 "Dirichlet Integrals and Their flpplications", **Ravi Salgia**, Loyola University  
 "Transformation of Computer Programs into Functions", **Brian Sumner**, University of Denver  
 "A Complex Parabola in Four Dimensions", **Robert Kear**, West Carolina University  
 "Using the method of maximum Likelihood Estimation in Genetics", **Margaret It-Wallace**, Miami University  
 "Rubik's magic Cubes", **Kevin Saylors**, Pomona College  
 "Descartes: Philosopher or mathematician?", **James F. Goeke, S.J.**, St. Louis University  
 "A Non-Parametric multiple Comparison Test for Differences in Variances". **Dean Follmann**, Northern Illinois University  
 "The Relations of Differentiable Functions and the Power Series". **Brian Bunsness**, South Dakota School of Mines and Technology  
 "mazes and Their Passage". **Donna 1. Ford**, Miami University  
 "mathematical **Analysis** of Inflation", **Elias Kosmas**, Oklahoma State University

**1982** -- University of Toronto Meeting

"Forward and Backward **Eigenvalues**", **Kriss Schueler**, Youngstown State University  
 "application of the Golden number to Fibonacci **Algebra**", **Duane A. Cooper**, Atlanta University Center  
 "The **Influence** of the math **Meeting** on **Recruiting**". **Mario Spetseris**, College of Charleston  
 "Volume of an n-Dimensional Unit Sphere", **Ravi Salgia**, Loyola University  
 "A mathematical model for Paired-flssociate Learning". **Dore Ann Celentano**, St. Peter's College  
 "An **Analysis** of **Monopoly** Strategies", **Thomas Chenier** and **Cathy Lynn Vandiford**, East Carolina University  
 "Population Competition and Crop Yield", **Donna 1. Ford**, Miami University  
 "Approximating Partial Sums of the Harmonic Series", **Karim K. Carter**, University of Arkansas  
 "Application of Stochastic Control to Battlefield Replacements", **Andrew Stump\$\$**, Washington University  
 "Effects of **Sir** Pollution on Pulmonary Functions of Children". **Deborah Pennell**, University of Montana  
 "A Geometric **Analysis** on the Relative Sizes of Classical Orthogonal Polynomials", **Arthur W. Mifflin**, Southern Illinois University  
 "The Stone-Cech Compactification in the Structure Space of a Distributive Lattice", **Christopher M. Brislawn**, Pomona College  
 "A mathematical **Approach** to Improving Your Backgammon Skills", **Joan Hart**, Miami University  
 "Estimating the Inclusion Radius of Polynomial Zeros", **William R. Somsky**, Iowa State University  
 "Creating Kaleidoscopes", **Mary Anne Bromelmeier**, Miami University  
 "A Derivation of a Fifth Order Predictor-Corrector method". **Eric D. Stutz**, Rutgers University

"Computer Graphics in numerical **Analysis**", **Daniel John Pierce**, Northern Illinois University  
 "Peppermint Patty's Dilemma -- **Math Anxiety**", **Cathie Spino**, Miami University  
 "Love and **Psycho-Mathematics**", **Richard R. Porter**, Douglass College  
 "Investigations of **Maxfield's Theorem**", **Laura Southard**, University of Oklahoma  
 "Alternative Tic-Tac-Toes", **Kevin Saylor**, Miami University

**1983 -- Albany Meeting**

"**A BASIC** Program for the **Schredering** Equation", **Mary Anne Bromelmeier**, Miami University  
 "Subset Selection", **David Van Brackle**, University of Central Florida  
 "Two **Ancient** Greek Construction Problems in Euclidean and Hyperbolic **Geometry**",  
**Jack M. Rau**, Oklahoma State University  
 "What Difference Does it **Make?**", **Dariush Saghaei**, John Carroll University  
 "The Gamma Function and Log-Concavity", **Karen Anderson**, Hendrix College  
 "**A** Ubiquitous Partition of Subsets of  $\mathbb{R}^n$ ", **Donald John Nicholson**, Iowa State University  
 "Leo **Moser's** Theorem", **Susan McDonald Britt**, East Carolina University  
 "**Exploratory** Data **Analysis** Using **Microcomputers**", **Thomas TenHoeve III**, Hope College  
 "**Cryptography** .. The Science of Secret Writing", **Denise Vining**, Miami University

**1984 -- University of Oregon Meeting**

"Complements -- mathematically Speaking", **Leslie Youngdahl**, Miami University  
 "Finding the Center of Complaints", **Michael H. Cox**, Marshall University  
 "Solving [Ion-Linear Systems of Equations]", **Deborah Witfield**, Youngstown State University  
 "**Money** and **Math**: An Investigation of Linear Economic models", **Renee L. Larson**, South Dakota State University  
 "Dealings in n-Dimensional Geometry", **Karin Remington**, College of St. Benedict  
 "Wedging Those Vector Integral Theorems", **Calvin Johnson**, University of California-Davis  
 "**A** mathematical **Model** of Voter Participation", **Mary Beth Dever**, Northern Illinois University  
 "number **Line**", **Patrick Tamer**, Appalachian State University  
 "Graph measure in Euclidean n-Space", **Jody Trout**, The College of Charleston  
 "Finite **Laplace Transforms**", **David W. Barnette**, East Carolina University  
 "**A** Way to Generate **Arbitrarily** High-Order Root-Finding methods", **Jeffrey Michael Kubina**, Youngstown State University  
 "Samuelson's Interaction Between the Accelerator and the multiplier", **Suguna Pappu**, Miami University  
 "Commutativity and **Distributivity**: Different Perspectives", **Benjamin L. Marshall**, Hendrix College

**1985 -- University of Wyoming Meeting**

"Box-Jenkins **Autoregressive** Integrated moving **Average** Forecasting of Common Stock Prices", **Milam W. Aiken**, University of Oklahoma  
 "An Historical Look at Some **Interesting** Functions", **George M. Alexander**, St. John's University  
 "**A** Two-Dimensional lion-Linear Population **Model**", **Kenneth Mark Alo**, University of Houston-Downtown  
 "**A** Shortcut to Solving **Polynomial** and Rational **Inequalities**", **Christa Blackwell**, University of Oklahoma  
 "Create Your Own **Geometry**", **David Cameron**, Miami University  
 "**Metagame Theory** and Political Behavior", **Andrew Chin**, University of Texas Which Came First, the Explicit Formula for the Determinant or its Properties?",  
**Kevin T. Christian**, University of California-Davis  
 "Historic Geometric Problems Involving Compass and Straightedge Construction".  
**Anthony Clacko**, Youngstown State University  
 "**lion**-negative Integer Solutions of  $\sum_{i=1}^n x_i = k$ ", **Marie Coffin**, South Dakota State University  
 "Some Applications of the **Gray Code**", **Henry L. Culver**, Youngstown State University  
 "Using Extrapolation to Obtain Approximations to 'Pi'", **Raymond E. Flannery, Jr.**, Youngstown State University  
 "**A** Report on the Precision of Floating Point **Math** Functions on Selected Computers".  
**John C. Flaspoehler**, Georgia Institute of Technology  
 "**A** Recursive Descent Scanner in LOGO", **Mark E. Frydenberg**, University of Hartford  
 "monte **Carlo** Studies in Statistical Research", **Kathleen A. Steigelm**, Northern Illinois University  
 "Fooling **Around** with mother nature .. An Application in Genetics", **John Greskovich**, Miami University  
 "Mysior's Example of a **Regular** Topological Space That is **Not** Completely **Regular**",  
**Elizabeth Stratton**, Miami University  
 "Some **Results** in the **Theory** of Filmicable **Numbers**", **James B. Hart**, Hendrix College  
 "The Shifted **QR** Algorithm in Real matrix Computations", **Karalee Howell**, Occidental College  
 "Sabermetrics", **James D. Johnston**, St. Bonaventure University  
 "Randomness and Complexity", **Jorge O. Ochoa-Lions**, University of Arizona  
 "Expanding a System of Propositional Logic", **Mitchell Pollack**, Bucknell University  
 "Valuations on **Monoids**", **Terry Reilly**, University of Montana  
 "mathematical **Models** in International Relations", **Sandra R. Rogers**, Auburn University  
 "Hidden Lines, The Unseen Graphic", **Henry Rosche III**, Southeastern Louisiana University  
 "**A** microcomputer Application of Karmarkar's Polynomial-Time **Algorithm** for Linear Programming", **Annuar Mohd Saffar**, University of Missouri-Columbia  
 "From Lead **Pipes** to Telesat .. History and Applications of Fourier **Analysis**", **James J. Shea**, Worcester Polytechnic Institute

- 'network modeling in a Transportation Environment", **Cynthia M. Stuber**, St. Norbert College
- "A General Solution Procedure for the Steady-State Probability of State-Homogeneous Production Line models". **Karim Tofang-Sazi** and **Sencer Yeralan**, University of Missouri-Columbia
- "A Phase-Plane Analysis of Coulomb Damping", **John Tokar**, Rose-Hulman Institute of Technology
- "Dale's Cone of Experience and Improved Math Teaching", **Rob Walling**, Miami University
- "Lattices of Periodic Functions". **Kirk Wetter**, Hope College
- "Boolean Idempotent matrices and Applications", **Sherry Wheeler**, Alabama State University
- "Figureate numbers". **Terry Wiencek**, Youngstown State University
- "From Pac-Man to the Dihedral Group,  $D_4$ : An Application of Group Theory to Video Games", **Erik L. Wynters**, University of New Hampshire

#### 1986 -- University of California-Berkeley Meeting

- "Generation of One million Prime numbers", **William O. Haas**, Southeastern Louisiana University
- "A Proof of Primality Utilizing Fermat's Theorem", **Sara S. Fagan**, Marquette University
- "Solving Linear Diophantine Equations Using Euclid's Algorithms", **Kathy Kowell**, Miami University
- "Lame's Theorem and the Euclidean Algorithm", **Frederick Taverner**, University of California-Davis
- "A New Proof of a Lemma to the Quadratic Reciprocity Law", **David Claussen**, Miami University
- "A Representation of Squares in Generalized Fibonacci Sequences", **Russ Shuttleworth**, Wichita State University
- "Class numbers of Cyclotomic Fields", **John E. Fischer, Jr.**, University of Pennsylvania
- "Starting with Pascal's Triangle", **Erstan Wheeler II**, Virginia Polytechnic Institute
- "\$1000 Howard: Sam Loyd's 14-15 Puzzle". **Judith Borciszewicz-Symalla**, St. John's University
- "mathematical models in Population Genetics", **Ridwan Tabbaa**, University of Houston-Downtown
- "Entropy of the M/G/1 Queueing System". **Milam W. Aiken**, University of Oklahoma
- "Estimating Age-Specific Fecundity of Soft Shell Clams", **Anne Kochendorfer**, Fairfield University
- "An Objective Analysis of Rainfall Data". **Thomas A. Kreitzberg**, Drexel University
- "The Absorbing Markov Process as Applied to a Random Behavior Model", **Bridget Moore**, Miami University
- "Linear-Time Three-Dimensional Graphics with Hidden Line Elimination", **Jeffrey S. Bonwick**, University of Delaware
- "A Look at the DoD's Trusted Computer System Evaluation Criteria", **John Nasopofler**, Georgia Institute of Technology

- "The mathematical World of Cryptology", **Barry Schoch**, Fairfield University
- "Symbolic Computation". **Emit J Volcheck**, University of Delaware
- "Does  $\sum_{k=1}^{\infty} \frac{1}{k^k}$  Converge or Diverge?", **Dawn Alisha Lott**, Bucknell University
- "Infinitesimals", **Hunter Marshall**, Texas A&M University
- "A Method of Defining Infinitesimals and Extending Functions", **Karen Sue Billings**, Hendrix College
- "Beauty from Boredom -- A Visa of Fractal Geometry", **fun Shea**, Worcester Polytechnic Institute
- "Fractal Curves", **Michael J. Cullen**, Marquette University
- "The Dynamics of  $F(z) = z^2 - 1$ ", **Connie Lou Overzet**, Boston University
- "The Equilic Quadrilateral", **Rob Walling**, Miami University
- "Counting Rectangles in a Multirectangular Region", **Steven D. Van Lieshout**, St. Norbert College
- "Graphs Uniquely Hamiltonian-Connected from a Vertex". **Carolyn R. Thomas**, St. Lawrence University
- "Approximating the Solution to Ordinary Differential Equations Using Taylor Polynomial Expansions", **Anthony Clacko**, Youngstown State University
- "Stability on a Finite Interval of Time-Average Differential Equations", **Michael P. Perrone**, Worcester Polytechnic Institute
- "Pressure Analysis in a Biomedical Device", **Paula A. Michaels**, Miami University
- "The Iraq-Iran War", **Conchita Minor**, Alabama State University
- "Locating Emergency Facilities in Order to minimize Response Time", **Craig J Cole**, Miami University
- "The Effect of Einstein's Theory of Relativity on Interstellar navigation", **James G. Kirklin**, Mount Union College
- "modeling a magnetic Oscillator", **Bradley O. Pad**, Miami University
- "Applications of Helly's Theorem to the Approximation of Functions by Polynomials", **Donna Vigeant**, University of Lowell
- "Using Residues to Evaluate Certain Infinite Series". **Mark Hassell Smith**, East Carolina University
- "The Development of Outstanding Secondary mathematics Students", **Brian A. Twitchell**, University of Maine

#### 1987 -- University of Utah Meeting

- "Deterministic and Probabilistic Fire Models", **Warren E. Maisel**, Worcester Polytechnic Institute
- "The Epidemiology of the RIDS Virus", **Aaron Klebanoff**, University of California-Davis
- "The Strangely Attracted Bouncing Ball", **Stephanie Ruth Land**, University of Texas
- "applications of Signal Processing", **Debra Shure**, University of Lowell
- "Self-Calibration of Complex Visibility Data from a Very Large Array of Antennas", **Ali Sofaei-nili**, Iowa State University
- "Representations and Characters of Groups", **Ken Chick**, Miami University
- "An Algebraic Construction of a Projective Geometry", **Stephanie Dumoski**, Occidental College
- "The RSA Public Key Cryptosystem: An Application for Modern Algebra", **Stephen Fiete**, West Virginia University

"**16/64 = 1/4 and Other n-Digit Cancellations**", **David A. Messineo**, University of Hartford  
 "An filgebraist's View of Competitive Games", **Erlan Wheeler II**, Virginia Polytechnic Institute  
 "Perpendicular Least Square Entimators", **Brian Anderson**, Western Kentucky University  
 "The Isoperimetric Inequality", **Jeff Diller**, University of Dayton  
 "Inverting a Pin in  $\mathbb{R}^2$ ", **Russell Godwin**, University of Arkansas  
 "Fourier Series and the 'Best' Mean Square flpproximation", Margaret M. Lineberger, East Carolina University  
 "Dynamic Programming Applied to Computer Voice Recognition", Thomas Eugene Gibbons, St. John's University  
 "A Formal Sum method to the Traveling Salesman Problem", **Jeffrey Horn**, Marquette University  
 "A Bit of Checking and Correcting", **Summer Quimby**, St. Norbert College  
 "A Graphical Illustration of the Convergence of Karmarkar's Linear Programming flgorithm", **Josef S. Crepeau**, University of Montana  
 The Existence of Eulerian and Hamiltonian Circuits in Graphs and their Line Graphs", **Carol Parker**, Hendrix College  
 "Circuit Spaces and Cut-Spaces of a Connected Graph", **Michael Tackett**, Miami University  
 "Yet Another Discussion of Graceful Graphs", **Andrew P. Ferreira**, Miami University  
 "Sub-Families of Venn Diagrams", **Philip Beymer**, University of Oregon  
 "Soap Films as minimal Surfaces", **George Matter**, St. John's University  
 "Theology, mathematics and meaning", **Mary Ehle**, St. Norbert College  
 "Curves Length Minimizing Modula v in  $\mathbb{R}^n$ ", **Jeff Abramson**, Massachusetts Institute of Technology  
 "NP-Completeness and the Traveling Salesman Problem", **Melanie X. Breaker**, Northeast Missouri State University  
 "The Classical Problems of flntiquity in the Hyperbolic Plane", Robert Curtis, University of California-Santa Cruz  
 "A Physical Derivation of the Well-Tempered musical Scale", **Timothy Koponen**, Aquinas College  
 "Games of Timing with Two or Three Players", Timothy P. Ronana, Moravian College  
 "A Glimpse at the Theory of Restricted Choice", Scott Krutsch, Rose-Hulman Institute of Technology

#### 1988 -- Providence Meeting

"maximal Polygons for Convex, Periodic Tilings", **Annette M. Matthews**, Portland State University  
 "An Introduction to Equitansitive Tilings", **Gerry Wuchter**, Miami University  
 "Finding a Generator of a Finitely Generated Rbelian Group", David L. fakes, Miami University  
 "The Cross Product in n-Space", **Joel Atkins**, Rose-Hulman Institute of Technology  
 "A Continued Fraction flpproach for Factoring Large numbers", Robert Coury, University of Washington

'Perfect numbers, Abundant numbers, and Deficient numbers', Sarah Taylor, East Carolina University  
 'The Problem of the Traveling Salesman - A study in Computational Complexity and Heuristic Algorithms", **Douglas Galarus**, The University of Montana  
 "Using Portable Intermediate Code in Compiler Construction", **Paul Stodgill**, Dickinson College  
 'numerical Solutions for the Three-Dimensional Heat Conduction Equation", **Charles Jabbour**, University of Houston-Downtown  
 "The Effect of Intra-Cluster Correlations on the Regression Estimation in the Finite Population Inference", **Dewi T. Saleh**, Wichita State University  
 "QR Revisited -- A Parallel flpproach to the QR - Decomposition", **Lara Aist**, University of Maryland-Baltimore County  
 "A Brute Force flpproach to Solving a Putnam Problem", **Summer Quimby**, St. Norbert College  
 The Sturm-Liouville mathematical System", **Lorie Ceremuga**, Youngstown State University  
 "The Annihilation Operator", **Marc Ahrendt**, Elmhurst College  
 "Mathematical measures in the Analysis of Image", Clifford D. Krumwieda, Texas A&M University  
 "Squaring a Square", Terry Henderschott, Miami University  
 "The Area of Wasted Space in a Rosette Design", **George Anderson**, Miami University  
 "Some Results for Chromatic and flssociated Polynomials of Graphs", Laura Cuthbertson, Hendrix College  
 "The Greedy Spy", **Janelle Becker**, St. John's University/College of St. Benedict  
 "Heuristic Arguments in mathematics", David Petry, University of Oregon  
 "Removing the flbsolute from Relativity", **Katie Coener**, St. Norbert College  
 "Dynamics of PDP models", **George Ashline**, St. Lawrence University  
 "A Venture into Chaos", **Erlan Wheeler**, Virginia Tech  
 "Fractals -- mathematical Chaos", **Michael F. Marcon**, McNeese State University

Editor's Note: The author and the Editor are indebted to David Ballew for compiling Appendices A and C.

## APPENDIX B

## J. SUTHERLAND FRAME LECTURES

The J. Sutherland Frame Lecture is named in honor of Pi Mu Epsilon's ninth president who served from 1957-1966. In 1952, Sud Frame initiated the concept and made possible the Pi Mu Epsilon student participation and student paper sessions at the Joint Summer Meetings of the AMS and MAA. Since then he has offered insights and inspiration to our student mathematicians at the summer meetings.

<b>1988</b>	<b>Boris W. Schottschneider</b> "You, Too, Can Tile the <b>Conway</b> Wag"
<b>1987</b>	<b>Clyton W. Dodge</b> "Reflections of a Problems Editor"
<b>1986</b>	<b>Poul Halmos</b> "Problems I Cannot Solve"
<b>1985</b>	<b>Ernst Snapper</b> "The Philosophy of mathematics"
<b>1984</b>	<b>John L. Kelley</b> "The Concept of Plane <b>Area</b> "
<b>1983</b>	<b>Henry Alder</b> "How to Discover and Prove Theorems: A Demonstration with Partitions"
<b>1982</b>	<b>Israel Halperin</b> "The Changing Face of mathematics"
<b>1981</b>	<b>E. P. MOBS, Jr.</b> "The <b>Beauties</b> of mathematics Revealed in Color Block Graphs"
<b>1980</b>	<b>Richard A. Askey</b> "Ramanujan and Some Extensions of the Gamma and Beta Functions"
<b>1979</b>	<b>H. Jerome Keisler</b> "Infinitesimale: Where They Come From and What They Can Do"
<b>1978</b>	<b>Herbert E. Robbins</b> "The Statistics of Incidents and Accidents"
<b>1977</b>	<b>Ivan Niven</b> "Techniques of Solving <b>Extremal</b> Problems"
<b>1976</b>	<b>H. s. MacDonald Coxeter</b> "The <b>Pappus</b> Configuration and Its Groups"
<b>1975</b>	<b>J. Sutherland frame</b> "matrix Functions: A Powerful Tool"

## APPENDIX C

## NATIONAL OFFICERS FOR PI MU EPSILON

	DIRECTOR-GENERAL	VICE-DIRECTOR-GENERAL	SECRETARY-	TREASURER-GENERAL	LIBRARIAN
<b>1914-1915</b>	<b>E. D. Roe, Jr.</b>	<b>F. F. Decker</b>	Helen Applebee	P. J. Bentley	Olive Jones

Local Officers of the Syracuse Chapter served as General Officers until 1922.

<b>1922-1927</b>	E. D. Roe, Jr.	W. Houston	W. G. Bullard	Louisa Lotz	Mabel Kessler
<b>1927-1928</b>	<b>E. D. Roe, Jr.</b>	<b>T. Fort</b>	H. S. Everett	Louisa Lotz	Mabel Kessler
<b>1928-1929</b>	<b>E. D. Roe, Jr.</b>	<b>T. Fort</b>	John S. Gold	Louisa Lotz	Mabel Kessler
<b>1929-1933</b>	Louis Ingold	T. Fort	John S. Gold	Louisa Lotz	Mabel Kessler
<b>1933-1936</b>	<b>F. W. Owens</b>	<b>T. M. Putnam</b>	John S. Gold	E. R. Smith	Mabel Kessler

The Offices of Secretary and Treasurer were combined and the Office of Librarian was discontinued.

	SECRETARY-TREASURER		
<b>1936-1939</b>	<b>G. C. Evans</b>	W. E. Milne	John S. Gold
<b>1939-1942</b>	W. E. Milne	Lincoln LaPaz	John S. Gold
<b>1942-1945</b>	Tomlinson Fort	H. H. Downing	John S. Gold
<b>1945-1948</b>	Tomlinson Fort	E. H. C. Hildebrandt	John S. Gold
<b>1948-1951</b>	<b>C. C. MacDuffee</b>	<b>D. R. Holt</b>	E. H. C. Hildebrandt

The PI Mu Epsilon Journal was first published in 1949 with Ruth Stokes editing the first issues.

	EDITOR OF THE <b>PI MU EPSILON JOURNAL</b>		
<b>1951-1954</b>	<b>C. C. MacDuffee</b>	W. M. Whyburn	J. S. Frame
<b>1954-1957</b>	S. S. Calms	J. S. Frame	R. V. Andree
<b>1957-1960</b>	J. S. Frame	Orrin Frink	R. V. Andree
<b>1960-1963</b>	J. S. Frame	R. H. Bing	R. V. Andree
<b>1963-1966</b>	J. S. Frame	H. T. Kernes	R. V. Andree
<b>1966-1969</b>	J. C. Eaves	H. T. Kernes	R. V. Andree
<b>1969-1972</b>	J. C. Eaves	H. T. Kernes	R. V. Andree
<b>1972-1975</b>	H. T. Kernes	E. Allan Davis	R. V. Andree
<b>1975-1978</b>	E. Allan Davis	R. V. Andree	Richard Good
<b>1978-1981</b>	R. V. Andree	E. Maurice Beasley	Richard Good
<b>1981-1984</b>	E. Maurice Beasley	Milton Cox	Richard Good
<b>1984-1987</b>	Milton Cox	Eileen Polani	Richard Good
<b>1987-1990</b>	Eileen Polani	David Ballew	Robert Woodside

Ruth Stokes  
Franz Hohn  
Francis Regan  
Francis Regan  
Seymour Schuster (& R.  
V. Andree in 1966-67)  
Kenneth Loewen  
Kenneth Loewen  
David Kay  
David Kay  
David Ballew  
David Ballew  
Joseph D. E. Konhauser  
Joseph D. E. Konhauser

## COUNCILORS OF Pi MU EPSILON

1914-1922	Florence A. Lane	Helen Mary <b>Barnard</b>	Edward Cottrell	Adolph <b>Sussman</b>
1922-1926	E. R. Hedrick	W. H. Roever	S. E. Rasor	
1926-1929	R. D. Carmichael			
1929-1933	E. D. Roe, Jr.	R. D. Carmichael	E. R. Hedrick	U. G. Mitchell
1933-1936	E. R. Hedrick	Tornlinson Fort	C. S. Latmin	Louis <b>Ingold</b>
1936-1939	W. C. Brenke	Alan Campbell	D. Lehmer	F. W. Owens
1939-1942	H. H. Downing	W. W. <b>Elliott</b>	G. C. Evans	R. A. Johnson
1942-1945	W. C. Brenke	P. J. Daus	E. H. C. <b>Hildebrandt</b>	W. P. On
1945-1948	George Williams	C. A. Hutchinson	C. H. Richardson	E. R. <b>Smith</b>
1948-1951	S. S. Calms	Tornlinson Fort	J. S. Gold	A. J. Kempner
1951-1954	<b>S. S. Calms</b>	Tornlinson Fort	S. L. McDonald	Ruth Stokes
1954-1957	H. C. <b>Bennett</b>			Ruth Stokes
1957-1960	Wealthy Babcock	R. F. Graesser	S. L. McDonald	
1960-1963	H. S. Thurston	H. W. Farnham		
1963-1966	E. H. C. <b>Hildebrandt</b>	Harriet Griffin	R. L. <b>San Soucie</b>	R. F. Graesser
1966-1969	J. C. Eaves	Marion Fort, Jr.	Ivan Niven	Angus E. Taylor
1969-1972	Josephine Chanter	Roy Dubisch	Kenneth O. May	Francis <b>Regan</b>
1972-1975	L. E. Bush	Roy Dubisch	<b>Irving Reiner</b>	R. H. Sorgenfrey
1975-1978	L. E. Bush	E. Maurice <b>Beasley</b>	Irving Reiner	William <b>Harkness</b>
1978-1981	Gloria Hewitt	E. Maurice <b>Beasley</b>	Dale W. Lick	Eileen <b>Polani</b>
1981-1984	Milton Cox	E. Maurice <b>Beasley</b>	Robert Woodslde	Eileen <b>Polani</b>
1984-1987	Milton Cox	A. D. Stewart	Fredric <b>Zerla</b>	Eileen <b>Polani</b>
1987-1990	David Barnette	A. D. Stewart	Robert Eslinger	Robert Woodslde
	Richard Good	Virginia <b>Taylor</b>	Robert Eslinger	J. Douglas <b>Faires</b>
		Richard Poss	Robert Eslinger	

## FROM THE CONSTITUTION OF Pi MU EPSILON

ARTICLE VII.  
FORMS AND CEREMONIES

Section 1. Motto. The Pi Mu Epsilon motto is the following:



"To Promote Scholarship and Mathematics"

Section 2. Insignia. The insignia of the fraternity are the following:

Colors—Violet, gold, and lavender.

Seal—Round, design of pin raised in center. The words, "Pi Mu Epsilon, Incorporated" around the edge.



Flower—Violet.  
Shield—Divided into four parts. In the upper right hand corner, the design of a conventionalized violet; in the lower right, a summation sign; in the upper left, an in-



tegral sign; and in the lower left, three stars representing fraternity, morality and scholarship. Just above the shield proper the pin is represented, and below, on a streamer, is the Greek motto.

*Paper judged to be the "best"  
since Student Paper Competition  
began in 1968.*

A UBIQUITOUS PARTITION OF SUBSETS OF  $R^n$ 

by Donald John Nicholson  
Iowa State University

We are about to embark on an adventure into the beauty of mathematical inquiry. The journey will begin with a simple yet wonderful property of the real numbers, and in the end we will arrive at an elegant result: a ubiquitous partition of subsets of  $R^n$ , i.e., a partition in which every element of the partition has a non-empty intersection with each neighborhood (abbreviated nbd) in the space.

The path will lead only through Euclidean spaces, so each space must be understood to be a subspace of  $R^n$ . When nbds are mentioned, they refer to nonempty intersections of open subsets of  $R^n$  with the space. We will use the symbol  $\bar{X}$  to represent the closure in  $R^n$  of a set  $X$ ,  $X'$  to denote the set of limit points in  $R^n$  of  $X$ , and then define  $X'' = X' - X$ . Finally,  $U_r(x_0)$  is the set of all points whose distance from  $x_0$  is less than  $r$ , and  $Br(x_0)$  is its closure. Now let us begin.

Let  $Q$  and  $P$  be the rational and irrational numbers respectively, and let  $U$  be a nonempty interval in  $R$ . Both  $U \cap Q$  and  $U \cap P$  are infinite no matter how small  $U$  is. Consequently, each point of  $U \cap Q$  is a limit point of the set, but there must always be irrational limit points as well. It is this property which we shall generalize in the following definition.

**Definition 1.** A space  $X$  is a Grunowix space\* iff  $Vnbd U$  in  $X$   $\bar{U} - X \neq \emptyset$ .

**Proposition 1.** If  $X$  is a Grunowix space, then  $X$  is dense in itself

**Proof.** Let  $x_0$  be an isolated point in  $X$ . Then  $\{x_0\}$  is a nbd of  $x_0$  in  $X$ . But  $\{x_0\} - X = \emptyset$ , thus  $X$  is not a Grunowix space.

\* — see excerpt from a letter by the author at end of the paper

If  $X$  is a Grunowix space, then  $X \subset Y'$ , but  $X \# Y'$ . This raises a question: what kind of space is  $X''$ ? Before answering, let us define a new concept.

**Definition 2.** Two spaces  $X$  and  $Y$  are mutually dense iff  $\overline{X} = \overline{Y}$ . An arbitrary collection of spaces is mutually dense iff they are mutually dense in pairs.

**Lemma 1.** Let  $X$  and  $Y$  be disjoint spaces. Then the following are equivalent: (a)  $\overline{X} = \overline{Y}$ , (b)  $X \subset Y''$  and  $Y \subset X''$ , and (c)  $X \subset Y'$  and  $Y \subset X'$ .

**Proof.** Suppose  $\overline{X} = \overline{Y}$ . Then  $X \subset \overline{Y}$  and  $Y \subset \overline{X}$ , and since  $X$  and  $Y$  are disjoint,  $X \subset Y''$  and  $Y \subset X''$ . Note that  $X'' \subset Y'$  and  $Y'' \subset X'$ , hence  $X \subset Y'$  and  $Y \subset X'$ . But this implies that  $X \subset \overline{Y}$  and  $Y \subset \overline{X}$ , so  $\overline{X} \subset \overline{Y}$  and  $\overline{Y} \subset \overline{X}$ . This returns us to our original supposition that  $\overline{X} = \overline{Y}$ .

**Lemma 2.** If  $X$  is a Grunowix space, then  $\overline{X} = \overline{X''}$ .

**Proof.** By the definition of  $X''$ ,  $X$  and  $X''$  are disjoint, and  $X'' \subset X'$ ; therefore, to satisfy Lemma 1 we need only show that  $X \subset (X'')'$ . Let  $x_0 \in X$  and let  $U$  be a nbd of  $x_0$  in  $\mathbb{R}^n$ . Then  $\exists r > 0 \ni U_r(x_0) \subset U$ . Now choose  $r' \ni 0 < r' < r$ ; this will give us  $U_{r'}(x_0) \subset B_{r'}(x_0) \subset U_r(x_0) \subset U$ . Observe that  $\overline{U_{r'}(x_0)} \cap X \subset B_{r'}(x_0)$ , and  $U_{r'}(x_0) \cap X$  is a nbd of  $x_0$  in  $X$ , so that, since  $X$  is a Grunowix space,  $\overline{U_{r'}(x_0)} \cap X = X \# \emptyset$ . This implies that  $B_{r'}(x_0)$  and thus  $U$  contain points in  $X''$ ; it follows that  $x_0$  is a limit point of  $X''$ . Since  $x$  is arbitrary, we infer that  $X \subset (X'')'$ .

**Theorem 1.** Let  $X$  and  $Y$  be mutually dense, disjoint spaces. Then  $X$  and  $Y$  are Grunowix spaces.

**Proof.** Let  $x_0 \in X$  and let  $U$  be a nbd of  $x_0$  in  $\mathbb{R}^n$ . By Lemma 1  $X \subset Y'$  and  $Y \subset X'$ , thus  $\exists y_0 \in U \cap Y$   $\exists y_0$  is a limit point of  $U \cap X$ . This implies that  $y_0 \in \overline{U \cap X} = X$ ; since  $U \cap X$  is an arbitrary nbd of  $x$  in  $X$ ,  $X$  is a Grunowix space. By a similar argument  $Y$  is a Grunowix space.

Lemma 2 and Theorem 1 answer our question: if  $X$  is a Grunowix space, then so is  $X''$ ; e.g.,  $Q$  and  $P$  are Grunowix spaces. But Theorem 1 has a much more interesting consequence: every collection of spaces which is mutually dense and pairwise disjoint is a collection of Grunowix spaces, and we shall see that such a collection is a

ubiquitous partition of its union.

Let us construct such a collection. Let  $r, s \in (0, 1)$  and define  $r \sim s$  iff  $\log_r s \in Q$ . It should be a simple exercise for the reader to show that  $\sim$  is an equivalence relation and hence partitions  $(0, 1)$ . Let  $[r]$  and  $[s]$  be distinct equivalence classes in the partition of  $(0, 1)$ . Then  $[r]$  and  $[s]$  are disjoint. Furthermore,  $\log_r s \in P$ ; i.e.,  $\exists p \in P \ni s = r^p$ . Since  $p \in P$ ,  $\exists q_i \ni i=1 \text{ to } \infty \ni q_i \rightarrow p$ , thus  $r^{q_i} \rightarrow r^p = s$ . Each  $r^{q_i} \in [r]$ , thus  $s$  is a limit point of  $[r]$  as is every element of  $[s]$ . Similarly, every element of  $[r]$  is a limit point of  $[s]$ , thus  $[r] = [s]$  by Lemma 1, and  $[r]$  and  $[s]$  are Grunowix spaces by Theorem 1. Note that each equivalence class is countable, whereas  $(0, 1)$  is uncountable, thus  $\{|[r]:r \in (0, 1)\}$  is uncountable.

The above partition is interesting in that the elements are mutually dense; they are like chemically inert gases in a closed container at thermal equilibrium. Just as the molecules of each gas distribute themselves throughout the container, every nbd in  $(0, 1)$  contains points from every equivalence class; i.e., the partition is ubiquitous.

Let us define a Grunowix partition as a partition consisting of mutually dense spaces. By Lemma 1 and the definition of a limit point, this is equivalent to the definition of a ubiquitous partition. Thus every Grunowix partition is ubiquitous, but no other partition is; this is the beauty of the Grunowix space.

DO NOT BLINK! We are about to show how to construct Grunowix partition's of an infinite number of spaces by using only three theorems and our partition of  $(0, 1)$ !

**Theorem 2.** Let  $\{X_\alpha : \alpha \in A\}$  where  $A$  is an indexing set be a Grunowix partition of  $X$ , and let  $f: X \rightarrow Y$  be a continuous bijection. Then  $\{f(X_\alpha) : \alpha \in A\}$  is a Grunowix partition of  $Y$ .

**Proof.** The bijectivity condition insures that  $\{f(X_\alpha) : \alpha \in A\}$  will be a partition of  $Y$ . If  $x_0 \in X_a$  and  $a \neq b$ ,  $\exists \{x_i\}_{i=1}^\infty \subset X_b \ni x_i \rightarrow x_0$ . Since  $f$  is continuous,  $f(x_i) \rightarrow f(x_0)$ , thus  $f(X_a)$  and  $f(X_b)$  are mutually dense. It follows that  $\{f(X_\alpha) : \alpha \in A\}$  is a Grunowix partition of  $Y$ .

**Theorem 3.** Let  $\{X_i^\alpha : \alpha \in A\}$  be a Grunowix partition of the  $i^{\text{th}}$  factor space of  $X = X_1 \times X_2 \times \dots \times X_n$ . Then  $\{X_1 \times \dots \times X_i^\alpha \times \dots \times X_n : \alpha \in A\}$  is a Grunowix

partition of  $X$ .

**Proof.** Let  $a, b \in A$  and  $a \neq b$ . Then  $\overline{x_i^a} = \overline{x_i^b}$ . Now  $\overline{x_1 x_2 \dots x_i^a x_{i+1} \dots x_n} = \overline{x_1 x_2 \dots x_i^b x_{i+1} \dots x_n} = \overline{x_1 x_2 \dots x_n}$ , thus  $\{x_1 x_2 \dots x_i^a x_{i+1} \dots x_n : a \in A\}$  is a mutually dense collection of spaces. Since they are disjoint and their union is  $X$ , they form a Grunowix partition of  $X$  (we will call this a Grunowix partition of  $X$  in the  $i$ 'th factor space or  $i$ 'th coordinate).

**Theorem 4.** Let  $\{X : a \in A\}$  be a Grunowix partition of  $X$ , and  $B$  an open subset of  $X$ . Then  $\{X \cap B : a \in A\}$  and  $\{X_a \cap \overline{B} : a \in A\}$  are Grunowix partitions of  $B$  and  $\overline{B}$  respectively.

**Proof.** Here we will use the equivalence of the definitions of Grunowix and ubiquitous partitions. Now  $\forall x \in X$ ,  $\forall$  nbd  $U$  of  $x$  in  $X$ , and  $\forall a \in A$ ,  $U \cap X_a \neq \emptyset$ . Since  $B$  is open in  $X$ , each nbd in  $B$  is a nbd in  $X$ , thus  $\forall x_0 \in B$ ,  $\forall$  nbd  $U$  of  $x_0$  in  $B$ , and  $\forall a \in A$ ,  $U \cap X_a \neq \emptyset$ . But  $U \cap X_a = U \cap X_a \cap B$ , thus  $\{X_a \cap B : a \in A\}$  is a Grunowix partition of  $B$ .

Now let  $x \in B''$  and  $U$  a nbd of  $x$  in  $X$ . Then  $U \cap B \neq \emptyset$ , and since  $B$  is open in  $X$ ,  $U \cap B$  is open in  $X$ , so that  $U \cap B \cap X_a \neq \emptyset \quad \forall a \in A$ . Since

$U \cap B \cap X_a \subset U \cap \overline{B} \cap X_a$ ,  $\{X_a \cap \overline{B} : a \in A\}$  is a Grunowix partition of  $\overline{B}$ .

These three theorems are our tools for constructing Grunowix partitions. We may construct a Grunowix partition of any open subset of  $R^n$  or its closure by collecting the intersections of the set with each element of a Grunowix partition of  $R^n$ . A Grunowix partition of  $R^n$  may be easily constructed by mapping  $(0,1)$  homeomorphically onto one or more factor spaces of  $R^n$  and using Theorem 3. Look how many spaces we can ubiquitously partition with a simple equivalence relation on  $(0,1)$ !

We would like to make one final observation. Constructing a Grunowix partition of  $R^n$  by partitioning one or more of its factor spaces imposes a variety of geometric structures on the connected components of the elements of the partition. Since the elements of a Grunowix partition of  $R$  are totally disconnected, a Grunowix partition of  $R^3$  in one coordinate will consist of elements whose components are parallel planes, in two coordinates the components will be parallel lines, and in three coordinates the components will be points. In general the components of the elements of a Grunowix partition of  $R^n$

in  $m$  of its coordinates will be  $n-m$  dimensional hyperplanes.

There is more. We may use any curvilinear coordinate system in  $R^n$  to construct a Grunowix partition so long as each point has unique coordinates. By using spherical or cylindrical coordinates we may have a Grunowix partition of  $R^3$  whose elements consist of concentric spheres or coaxial cones or cylinders. The mysteries of the Grunowix space know no bounds!

The purpose of Figure 1 is to aid in visualizing the Grunowix partition  $\{Q^3, R^3 - Q^3\}$  of  $R^3$ . The element  $Q^3$  is totally disconnected, and since it is countable,  $R^3 - Q^3$  is connected. A very rough description of the partition is a countable collection of infinitesimal "boxes" enclosing points. How exquisite! And it all began with the property of the real numbers that every interval contains infinitely many rationals and irrationals.

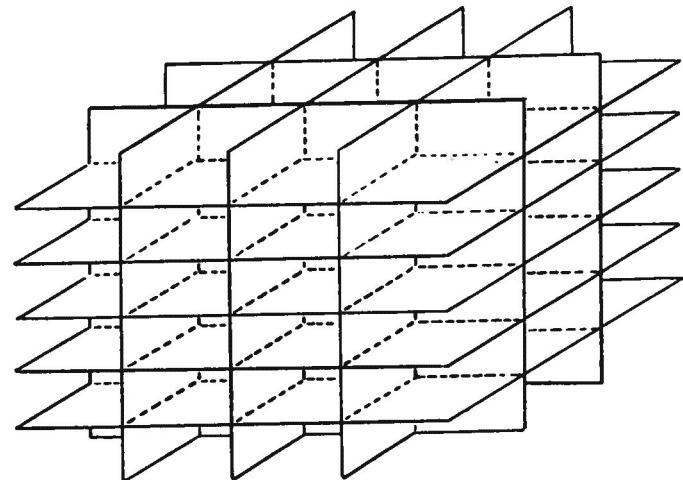


FIGURE 1  
Part of  $R^3 - Q^3$

## REFERENCES

- Levine, Norman, *On-Spaces Nowhere Locally Compact*, Kyungpook Mathematics Journal, 22, No. 2 (December 1982), 167-173.
- Munkres, James R., *Topology: A First Course*, Prentice-Hall, Inc., 1975.
- Naber, Gregory L., *Topological Methods In Euclidean Spaces*, Cambridge University Press, 1980.

## EXCERPT FROM A LETTER

... The idea of a Grunowix space arose from an undergraduate topology homework problem in reference 3. The problem is Exercise 1-28 on page 25: prove that the set of rationals as a subspace of the reals is not locally compact.

When I finally decided what property of the set of rationals prevented it from being locally compact, it struck me as being very fascinating. I asked the instructor if this property had a name. He knew of none, so I generalized the property and tried to come up with a name myself. My wife suggested inventing a word, so we tried constructing words with a scrabble board and letter tiles, and 'Grunowix' was the first construction we could pronounce.

My original definition for the Grunowix space was formulated different &, but it was equivalent to the present one. If my paper is devoid of references, it is only because none of the math professors I was acquainted with could steer me toward any sources that helped me with this interesting Apace, nor did any searching on my part uncover any useful references. I was forced to devetop the definitions and propositions on my own; it required walking many months and wandering down home btind alleys, but I am pleased with the results and what I learned from the experience. It has made me love mathematics.

I would like to thank Dr. Donald Sanderson of Iowa State University for the multitude of occasions on which he listened to me sort out and try to clarify my ideas and for being kind and patient enough to read and constructively criticize five versions of this paper ...

Sincerely,  
Donald John Nicholson

**Editor's note:** This paper was prepared while the author was a senior undergraduate majoring in mathematics and physics at Iowa State University. Donald presented the paper at the National Meeting of Pi Mu Epsilon in Albany, NY in August 1983 and has entered the paper in the Journal's National Paper Competition. The competition is open to students who have not received their master's degree at the time of submission. Papers may be submitted to the Editor at any time.

## X-MATRICES: GENERALIZED COMPLEX NUMBERS

by Samuel Councilman  
California State University, Long Beach

In introductory Abstract and Linear Algebra courses one encounters the familiar isomorphism between complex numbers and  $2 \times 2$  matrices.

$$a_1 + a_2 i \leftrightarrow \begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix}$$

Thus all matrices of the form

$$\begin{pmatrix} a_1 & -a_2 \\ a_2 & a_1 \end{pmatrix} = a_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + a_2 \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

yield a commutative subalgebra of  $M_{2,2}(R)$ , the algebra of  $2 \times 2$  real matrices. A natural question is "can this be generalized to higher dimensions so as to obtain commutative subalgebras of  $M_n(R)$  for  $n > 2$ ?"

In even dimensions  $n = 2k$  the matrix

$$J = \begin{pmatrix} 0 & & & & & & & -1 \\ & \ddots & & & & & & \\ & & \ddots & & & & & \\ & & & -1 & & & & \\ & & & & \ddots & & & \\ & & & & & 1 & & \\ & & & & & & \ddots & \\ & & & & & & & 0 \end{pmatrix}$$

behaves just like an "imaginary unit."  $J^2 = -I$ . Thus one may be inclined to consider matrices of the form  $a_1 I + a_2 J$ . But you can do better! The scalars  $a_1$  and  $a_2$  can be replaced by *bisymmetric diagonal matrices* that is, diagonal matrices which are symmetric with respect to the diagonal from the lower left to the upper right corner.

Definition.  $D = (d_{ij})$  is a *bisymmetric diagonal* matrix if and only if  $d_{ij} = 0$  for  $i \neq j$  and  $d_{ii} = d_{(n+1-i)(n+1-i)}$ .

In  $M_{2k,2k}(R)$ , consider the set of matrices of the form

$$D_1 I + D_2 J = D_1 + D_2 J$$

where  $D_1$  and  $D_2$  are bisymmetric diagonal matrices and  $J$  is the imaginary unit matrix defined above. Because of the pattern formed by the (possibly) non-zero elements, these  $D_1 + D_2 J$  matrices may be called "*X-matrices*."

The set of *X-matrices* is obviously closed under scalar multiplication, and the commutative and distributive properties of matrix addition show that X-matrices add just like complex numbers:

$$(D_1 + D_2 J) + (D_3 + D_4 J) = (D_1 + D_3) + (D_2 + D_4) J.$$

To verify that X-matrices *multiply* like complex numbers, a preliminary observation is required: since premultiplication by a diagonal matrix multiplies rows and postmultiplication multiplies columns, it is easily seen that

$$DJ = JD$$

for any bisymmetric diagonal matrix  $D$ . With this result, a straightforward application of commutativity of matrix addition and the distributive law shows that X-matrices multiply exactly like complex numbers:

$$\begin{aligned} (D_1 + D_2 J)(D_3 + D_4 J) &= D_1 D_3 + D_1 D_4 J + D_2 J D_3 + D_2 J D_4 J \\ &= D_1 D_3 + D_1 D_4 J + D_2 D_3 J + D_2 D_4 J J \\ &= (D_1 D_3 - D_2 D_4) + (D_1 D_4 + D_2 D_3) J. \end{aligned}$$

Since multiplication of diagonal matrices is commutative, this multiplication of X-matrices is also commutative, and in even dimensions  $2k$  the set of X-matrices  $D_1 + D_2 J$  is a commutative subalgebra of  $M_{2k,2k}(R)$ .

To complete the analogy with complex numbers, one should note that the X-matrix  $D_1 + D_2 J$  will have inverse  $(D_1^2 + D_2^2)^{-1}(D_1 - D_2 J)$  just in case the bisymmetric diagonal matrix  $D_1^2 + D_2^2$  is nonsingular (has no 0 elements on its main diagonal).

Since the imaginary unit matrix  $J$  fails to exist in odd dimensions, one may wonder whether analogous commutative algebras exist in  $M_{n,n}(R)$

$$E = \begin{pmatrix} 0 & & & & & 1 \\ & \ddots & & & & \\ & & 1 & & & \\ & & & \ddots & & \\ & & & & 1 & \\ & & & & & 0 \end{pmatrix}$$

instead of  $J$  (here  $E^2 = I$ ), and considering X-matrices of the form

$$D_1 + D_2 E$$

with  $D_1, D_2$  bisymmetric diagonal, one obtains a subalgebra with commutative multiplication

$$(D_1 + D_2 E)(D_3 + D_4 E) = (D_1 D_3 + D_2 D_4) + (D_1 D_4 + D_2 D_3)E.$$

These X-matrices  $D_1 + D_2 E$  exist in all dimensions, behave much like complex numbers and are symmetric with respect to both diagonals.

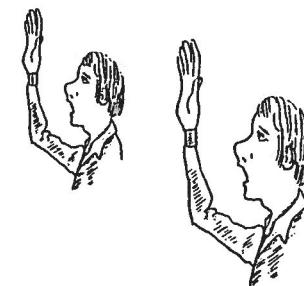


Struggling through an  $\epsilon$ - $\delta$  proof of the continuity of a simple function, the instructor stopped, faced the students, and asked "Are there any questions?"

One student asked "How much do we have to make to get an A?"

The instructor answered "Somewhere in the neighborhood of 85."

Another student, who seemed to have been asleep, woke up and asked "How large is  $\epsilon$ ?"



A.R. Amm-Moy

## NOTE ON A WELL-KNOWN LIMIT

by W. Vance Underhill  
East Texas State University

In the Spring 1986 issue of this journal Norman Schaumberger obtained the limit

$$\lim_{n \rightarrow \infty} \frac{n\sqrt[n]{n!}}{n} = \frac{1}{e}$$

by use of the Mean Value Theorem. In that derivation, however, it was necessary to first establish an intermediate double inequality. Here is a shorter and quite different elementary approach to the same limit.

The radius of convergence  $R$  of a power series  $\sum_{n=0}^{\infty} a_n x^n$  is given by

by either of the formulas

$$(1) \quad \frac{1}{R} = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$(2) \quad \frac{1}{R} = \lim_{n \rightarrow \infty} \sqrt[n]{a_n}$$

provided the limit in (1) exists. Consider the series  $\sum_{n=0}^{\infty} \frac{n!}{n^n} x^n$ .

Using (1), a routine calculation shows that

$$\frac{1}{R} = \frac{1}{e}.$$

Using (2), we find that

$$\frac{1}{R} = \lim_{n \rightarrow \infty} \sqrt[n]{\frac{n!}{n^n}} = \lim_{n \rightarrow \infty} \frac{n\sqrt[n]{n!}}{n}.$$

Therefore,

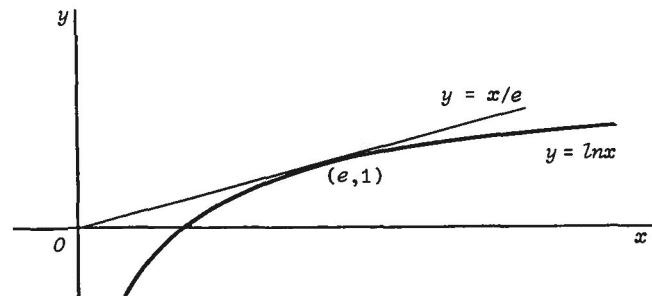
$$\lim_{n \rightarrow \infty} \frac{n\sqrt[n]{n!}}{n} = \frac{1}{e}.$$

 $p^q > q^p$  VIA COORDINATE GEOMETRY

by Norman Schaumberger  
Bronx Community College, Bronx, NY

In this note we offer a geometric proof of the familiar theorem that states that if (a)  $q > p \geq e$ , or if (b)  $0 < q < p \leq e$ , then  $p^q > q^p$ .

To prove case (a), we observe that the graph of  $y = \ln x$  is concave downward and that the tangent line,  $y = x/e$ , at  $(e, 1)$  lies above the curve. Hence  $x/e \geq \ln x$  for all positive  $x$ , with equality only in case  $x = e$ .



Putting  $x = eq/p$  in this inequality gives  $q/p > \ln(eq/p)$ , or

$$(1) \quad q - p > \ln(q/p)^p$$

with strict inequality because  $q/p \neq 1$ .

Since  $p/e \geq 1$  and  $q - p > 0$ , it follows that  $(q - p)\ln(p/e) \geq 0$ , or

$$(2) \quad \ln(p)^{q-p} \geq q - p.$$

Combining (1) and (2) gives the desired result.

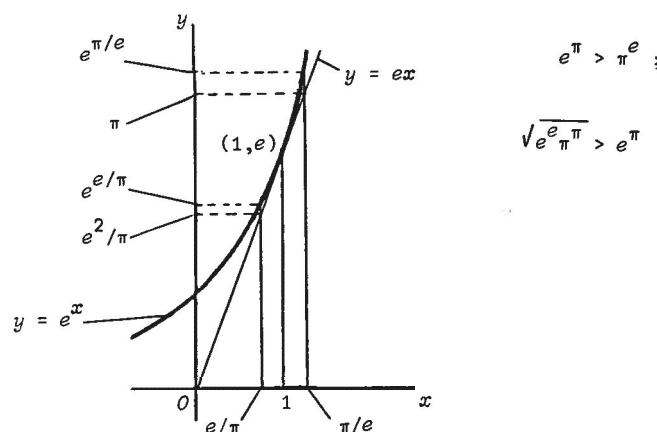
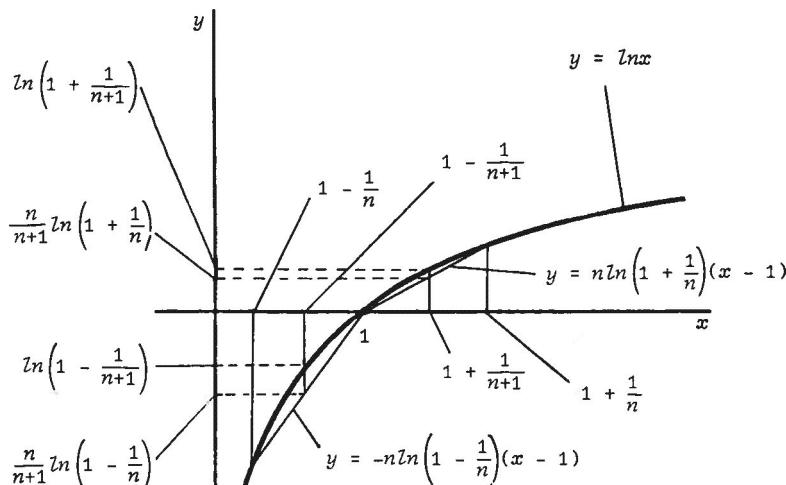
A special case of (a) is  $e^m > m^e$ .

In case (b),  $p - q > 0$  and  $e/p \geq 1$ . Hence  $(p - q)\ln(e/p) \geq 0$  which gives the inequality in (2).

## PROOFS WITHOUT WORDS

by Norman Schaumberger  
Bronx Community College, Bronx, NY

$$\left(1 + \frac{1}{n+1}\right)^{n+1} > \left(1 + \frac{1}{n}\right)^n ; \quad \left(1 + \frac{1}{n}\right)^{n+1} < \left(1 + \frac{1}{n-1}\right)^n , \quad n > 1$$



## COMMENT ON "COUNTING BIT STRINGS WITH A SINGLE OCCURRENCE OF 00" \*\*

by J. Sutherland Frame  
Michigan State University

Dear Professor Moore:

I enjoyed reading your *Pi Mu Epsilon Journal* article (Vol. 8, No. 9, pp. 572-5) on "Counting Bit Strings with a Single Occurrence of 00," and was especially interested in the role played by the Fibonacci numbers.

Perhaps I overlooked it, or perhaps you did not notice that the sum  $a_n$  in (\*), p. 573, can be expressed as follows for  $n > 0$ .

$$a_n = ((n+1)F_{n-1} + (n-1)F_{n+1})/5$$

Proof: Let  $r$  and  $r'$  denote the positive and negative roots of the equation  $r^2 = r + 1$ . Then for  $n > 0$  we have

$$\begin{aligned} r^n - r'^n + 5^{1/2}F_n, & \quad r^n + r'^n = F_{n-1} + F_{n+1} \\ \text{Thus } a_n = \sum_{k=1}^{n-1} F_k F_{n-k} & = (115) \sum_{k=1}^{n-1} (r^k - r'^k)(r^{n-k} - r'^{n-k}) \\ 5a_n & = (n-1)(F_{n-1} + F_{n+1}) - 2 \sum_{k=1}^{n-1} r^{n-k} r'^k \\ & = (n-1)(F_{n-1} + F_{n+1}) - 2rr'(r^{n-1} - r'^{n-1})/(r - r') \\ & = (n+1)F_{n-1} + (n-1)F_{n+1} \end{aligned}$$

Verification:

n	1	2	3	4	5	6	7	8	9	10	11
F <sub>n</sub>	1	1	2	3	5	8	13	21	34	55	89
(n+1)F <sub>n-1</sub>	0	3	4	10	18	35	64	117	210	374	
(n-1)F <sub>n+1</sub>	0	2	6	15	32	65	126	238	440	801	
a <sub>n</sub>	0	1	2	5	10	20	38	71	130	235	

I thought that this might interest you.

Sincerely yours,  
J. Sutherland Frame

\*\* ... printed with permission

## IN PRAISE OF CALCULUS AND A CALCULUS LAMENT - POETRY

by Hubert Walczak

College of St. Thomas

## In Praise of Calculus

## Oh Calculus!

You free me from the false and inexact  
 You elevate my mind to thoughts abstract

## Oh Calculus!

My fountain of delight  
 You please me - when my answers come out right!

## A Calculus Lament

## Oh Calculus!

You fill me with frustrations  
 Plagues of limits, weird manipulations  
 Derivatives, those ghosts of vanishing fractions  
 Defy my most determined actions  
 Related rates, a mystery profound  
 Who cares how shadows move around?  
 Square roots, sines and secants, sources of my sorrow  
 I hope they all clear up - before that test tomorrow!

## PUZZLE SECTION

*Edited by Joseph D. E. Konhauser  
 Macalester College*

The PUZZLE SECTION is for the enjoyment of those readers who are addicted to working doublecrostics or who find an occasional mathematical puzzle or word puzzle attractive. We consider mathematical puzzles to be problems whose solutions consist of answers immediately recognizable as correct by simple observation and requiring little formal proof. Material submitted and not used here will be sent to the Problem Editor if deemed suitable for the PROBLEM DEPARTMENT.

Address all proposed puzzles and puzzle solutions to Professor Joseph D. E. Konhauser, Mathematics and Computer Science Department, Macalester College, St. Paul, MN 55105. Deadlines for puzzles appearing in the Fall Issue will be the next February 15, and for the puzzles in the Spring Issue will be the next September 15.

## PUZZLES FOR SOLUTION

1. *Imposed By Man. Wayne, Holiday, Florida.*

Restore the digits:

\*\*\*\*\*! - \*\*\*\*\*

2. *'Proposed by Alan Wayne, Holiday, Florida.*

Devise a rule of formulation for the square array

9	3	2
4	8	1
5	6	7

3. *'Proposed by the 'Editor.*

Characterize the members of the set {1, 3, 4, 9, 10, 12, 13, 27, 28, 30, 31, 36, 37, 39, 40, 81, 82.84, 85, 90, 91, 93, 94, 108, 109, 111, 112, 117, ... }.

4. *Imposed by the 'Editor*

A laddergram (shades of 1928). By changing just one letter at each step and not changing the order of the letters, one can go from SOUP to NUTS in just four (the minimum possible number) steps:

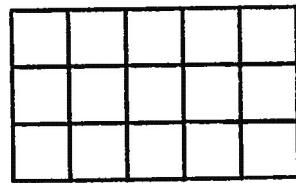
SOUP - SOUS - SOTS - NOTS - NUTS

In how few steps can you go from ONE to TWO?

5. *Proposed by the Editor.*

In a test of short-term memory, four participants { A, B, C, D } were shown cards numbered 1 through 4. Each card bore the picture of one of these common objects - a key, a comb, a book and a pen - each appearing on just one card. After two minutes, the participants were asked to pair card numbers and objects. The results are tabulated below on the left

	1	2	3	4
A	comb	pen	book	key
B	comb	book	key	pen
C	key	book	comb	pen
D	pen	comb	key	book



Three of the participants got exactly two correct matches and the fourth did not get any correct. Match the numbers and the objects.

6. *Contributed.*

Place the integers 1 through 15 in the 3x5 array (above right) so that the five column sums are equal and so that the three row sums are equal.

7. *contributed.*

The "transform" of an isosceles triangle with base angles **B** is an isosceles triangle with vertex angle **B**. A "chain" is a succession of transforms. For example, the transform of the 80-80-20 triangle is the 50-50-80 triangle, and the transform of the latter is the 65-65-50 triangle. If the first triangle is not equilateral, what is the length of the longest chain that can be formed in which all the triangles have an integral number of degrees?

COMMENTS ON PUZZLES 1 • 7, FALL 1988

ALAN WAYNE submitted a program in BASIC for the solution of Puzzle # 1. He determined "the only base ten prime (less than 100) which has the same period length in both base ten and base two and then also in bases four, eight, and sixteen is 71 (length 35)." WAYNE continued "In base thirty-two, the period length of 71 is 7 so the pattern does not continue." In a further remark, he says "I mentioned this problem on a card to Samuel Yates, Delray, FL (author of *Prime Period Lengths*, 1975; and *Reunits and Repetends*, 1982) and he instantly wrote back ... I went directly to pages 98-99 of Beiler's book [Albert H. Beiler, *Recreations in the Theory of Numbers*, Second Edition, Dover Publications, Inc., New York, 1966] and looked to see which primes p had the same exponent to which 2, 4, 8, 10, and 16 belong (mod p) .... Of course, this gives the instantaneous answer,  $p = 71$ ."

RICHARD I. HESS provided the additional computer-generated solutions 191, 311, 359, 479, 599, 719 and 839, with corresponding n values 95, 155, 179, 239, 299, 359 and 419. Other responses to Puzzle # 1 were received from BILL BOULGER, WILLIAM CHAU and EMIL SLOWINSKI.

Puzzle # 2 (see Figure 1) drew twelve responses. A composite of the solutions follows. "From the area condition,  $(AB)^2 = 2(AM)^2$ . From the perimeter condition,  $2(AM) = 2(AB - AM) + BC$ . It follows that  $BC = (2\sqrt{2} - 2)AB$ . The angle opposite side BC has measure  $2\sin^{-1}[(BC)/2(AB)] = 2\sin^{-1}(\sqrt{2} - 1) \approx 48.94^\circ$ ." Solvers were ALI AMIR-MOEZ, JEANETTE BICKLEY, BILL BOULGER, WILLIAM CHAU, RICHARD DUNLAP, GEORGE P. EVANOVICH, MARK EVANS, RICHARD I. HESS, DONALD B. ONNEN, DON PFAFF, EMIL SLOWINSKI and ALAN WAYNE.

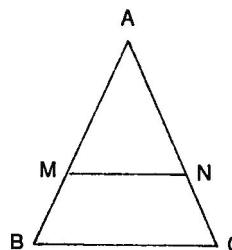


Figure 1

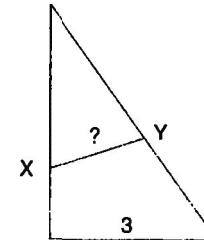


Figure 2

Eleven readers submitted solutions to Puzzle # 3 (see Figure 2). Only WILLIAM CHAU, RICHARD DUNLAP and RICHARD I. HESS avoided differentiation. CHAU's argument is given next. "Let  $x$ ,  $y$  and  $z$  denote the lengths of AX, AY and XY, respectively, where A is the vertex opposite the side of length 3. Since the two portions separated by the line segment XY have equal area, it is clear that  $2xysinA = (3)(4)$ . But  $\sin A = 315$ , so  $xy = 10$ . By the law of cosines,  $z^2 = x^2 + y^2 - 2xycosA$ . But  $xy = 10$  and  $\cos A = 415$ , so  $z^2 = x^2 + y^2 - 16 = x^2 + y^2 - 20 + 4 = (x - y)^2 + 4$ . The minimum of  $z^2$ , or  $z$ , attains clearly at  $x = y$ . Therefore, the line segment XY is shortest when  $AX = AY = 10$ ." THOMAS MITCHELL solved the same problem for XY connecting the other two pairs of sides of the original 3-4-5 triangle. The lengths of minimizing segments turned out to be 46 and 243. MITCHELL commented "... the XY connecting the sides of lengths 4 and 5 is the shortest of the three. In each case, the minimum-length line segment XY is the base of an isosceles triangle. This is likely an intuitive clue which some would employ to solve the puzzle." To this point, here is RICHARD I. HESS' argument. "It must happen that  $AX = AY$  to minimize XY. This can be seen by imagining otherwise and pivoting XY about its midpoint in a small increment. Areas won't change but length will change unless  $AX = AY$ ." Comments, anyone? ALI AMIR-MOEZ generalized the problem by considering an arbitrary right triangle. Other contributors of solutions to Puzzle # 3 were JEANETTE BICKLEY, BILL BOULGER, GEORGE P. EVANOVICH, MARK EVANS, EMIL SLOWINSKI and ALAN WAYNE.

Puzzle # 4 drew only two responses and one of these was incorrect. RICHARD I. HESS submitted the colorings in the two arrays on the left, where a "1" denotes one color and a "2" denotes the other. The coloring on the right is from Martin Gardner's *New Mathematical Diversions from Scientific American*, Simon and Schuster, New York, 1966. According to Gardner, other solutions are possible with different combinations of 1's and 2's in the positions which are underlined in the array on the right.

1	2	2	2	1	2
2	2	1	1	1	
1	2	2	1	2	1
1	2	1	2	2	1
1	1	1	1	2	2
2	1	2	2	2	1

1	2	2	2	1	2
2	2	1	1	1	
1	2	2	1	2	2
1	2	1	2	2	1
1	1	1	1	2	2
2	1	2	1	2	1

HESS made the following assertions:

1. For a 3x3 square there are 20 ways.
2. For a 4x4 square there are 221 ways.
3. For a 5x5 square there are 338 ways.
4. For squares larger than 6x6 there are no ways.

He then asks "How about results for three colors or k colors or mxn rectangles?" HESS' results came "from a program written in BASIC on a PC."

Puzzle # 5 drew responses from twelve readers. The nine respondents with the correct answer 8/21 were CHARLES ASHBACHER, BILL BOULGER, WILLIAM CHAU, RICHARD DUNLAP, GEORGE P. EVANOVICH, MARK EVANS, RICHARD I. HESS, JOHN M. HOWELL and DONALD B. ONNEN. CHAD's argument was 'There are eight ways to locate three O's in the same row, column or diagonal. For each, we need six ways to locate the remaining five X's and the one O. These total 48 ways. But overall we have '9 choose 4' or 126 arrangements of the X's and O's. So the required probability is 48/126 = 8/21.'

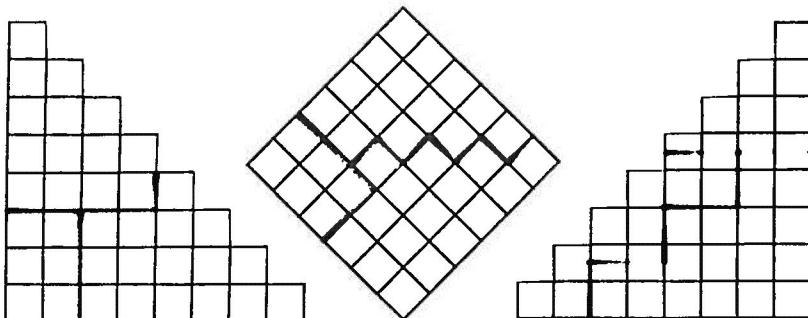
RICHARD I. HESS and JOHN M. HOWELL added that the probability of at least one column, row or diagonal with three X's is 7/9. HESS further stated that the probability of both is 2/7 and of neither is 8/63.

Puzzle # 6 asked for a rule of formulation for the 3x3 array with 1, 6 and 2 in the first row, 5, 4 and 9 in the second, and 8, 7 and 3 in the third. JEANETTE BICKLEY observed "The numbers in row 1 (2, 3) each have three (four, five) letters and are arranged in alphabetical order." The same observation was made by DONALD B. ONNEN (who credited MIKE SANKNER with the solution), BOB PRIELIPP and ALAN WAYNE. In a second solution, WAYNE showed that

$$a_{ij} = -6i^2j^2 + 21i^2j + 51ij^2/2 - 31i^2/2 - 183ij/2 - 24j^2 + 143i/2 + 89j - 69,$$

where  $a_{ij}$  is the jth entry in the ith row.

VALERIE ALBANO, JEANETTE BICKLEY, WILLIAM CHAU, RICHARD DUNLAP and RICHARD I. HESS all provided essentially the same dissection of the staircase-like piece in Puzzle # 7 into three pieces which can be reassembled to form a square. Below, their dissection is shown at the left and their reassembly in the middle. A different dissection found by the Editor is shown at the right. The reassembly is left as an exercise.



The staircase-like piece in Puzzle # 7 has eight steps and can be dissected into three pieces which can be reassembled to form a square. Given a staircase-like piece with nine (or some number other than eight) steps, are you able to dissect the piece into three pieces which can be reassembled to form a square?

**Editor's Note:** Apologies are due RICHARD I. HESS for overlooking his correct solution to Puzzle # 7 in the Spring 1988 issue. In the FALL 1988 issue the Editor said that no correct solutions had been submitted when, in fact, HESS had supplied an infinity of them. In the Editor's solution one need only replace the number 15 by any positive number less than 30, say a, and the number 150 by 180 - 2a, then one has a set of nine points such that each is one unit away from exactly four others.

Solution to Mathacrostic No. 27 (See FALL 1988 issue.)

#### WORDS:

A	endless cycles	J	iota	S	nephrolith
B	limitrophe	K	Newton's fluxions	T	Desargues
C	implicate	L	functor	U	Bernoulli
D	moiety	M	immies	V	etiolate
E	affright	N	napiform	W	ylem
F	oomph	O	isothetic	X	orphic
G	red shift	P	tetragrammaton	Y	Nernst
H	thwart	Q	yttrium	Z	disphenoid
I	optimal shape	R	acuminate		

#### AUTHOR ANDTITLE: ELI MAOR TO INFINITY AND BEYOND

QUOTATION: The common theme of all Islamic art is geometric regularity, spatial rhythm, periodic repetition. Islam, with its central creed of an omnipotent God to all humans must humbly defer, found in the infinite pattern a supreme artistic expression of its philosophy.

SOLVERS: JEANETTE BICKLEY, Webster Groves High School, MO; CHARLES R. DIMINNIE, St. Bonaventure University, NY; ROBERT FORSBERG, Lexington, MA; META HARRSEN, Neptune High School, NJ; DICK AND JOAN JORDAN, Indianapolis, IN; DR. THEODOR KAUFMAN, Brooklyn, NY; CHARLOTTE MAINES, Rochester, NY; DON PFAFF, University of Nevada-Reno, NV; STEPHANIE SLOYAN, Georgian Court College, Lakewood, NJ; MICHAEL J. TAYLOR, Indianapolis Power and Light Co., Indianapolis, IN; and ALAN WAYNE, Holiday, FL.

**Editor's Note:** From the Random House Dictionary of the English Language, 2nd edition, Unabridged

immie • n. Informal. agate. [imitation] marble + - IE].

For clue, see page 97 of the April, 1988, Smithsonian Magazine.

#### Mathacrostic No. 28

*Proposed by Joseph D. E. Konhauser*

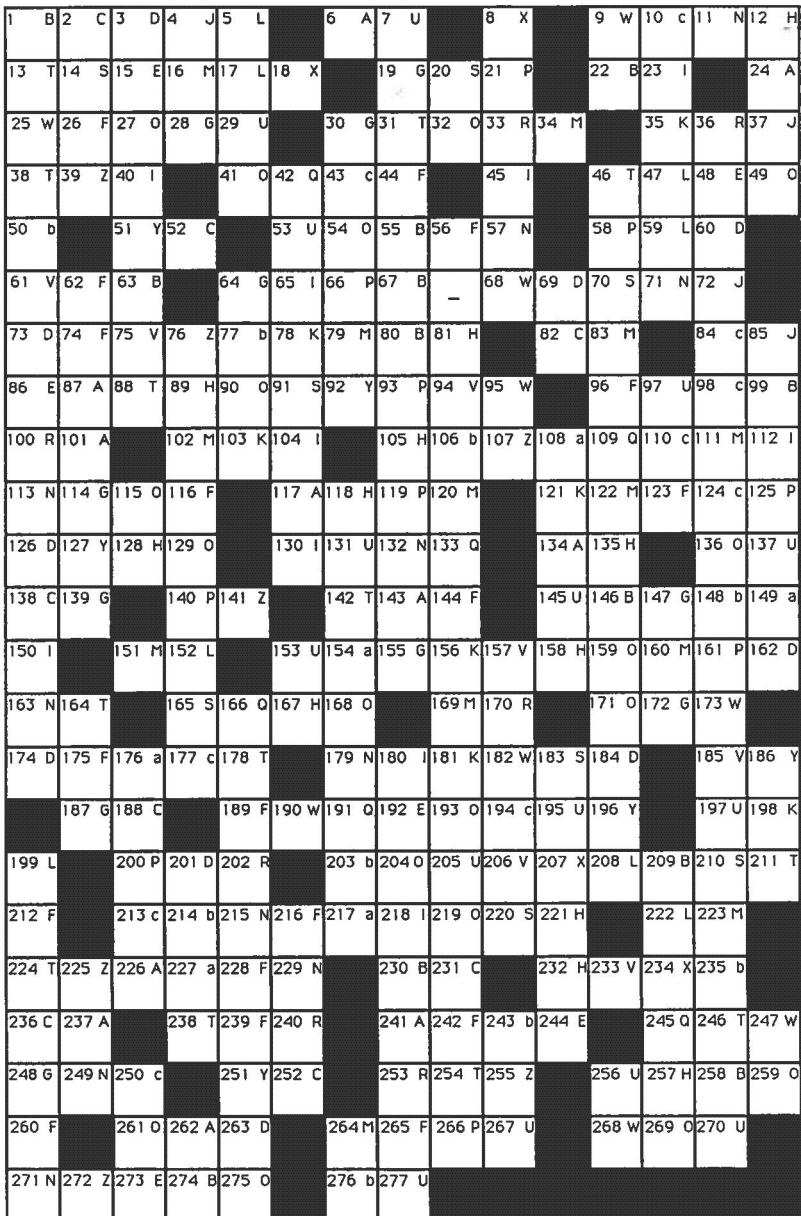
The 277 letters to be entered in the numbered spaces in the grid will be identical to those in the 29 keyed Words at the matching numbers. The key numbers have been entered in the diagram to assist in constructing the solution. When completed, the initial letters of the Words will give the name(s) of the author(s) and the title of a book; the completed grid will be a quotation from that book.

## Definitions

- A. a counter **take-away** game played in China under the name **tsyan-shidzi** (picking stones) (2 wds.)  
117 101 241 143 134 87 237 24 262 6 226
- B. dubbed "darts" and "**kites**" these quadrilaterals, properly arranged, provide non-periodic tessellations of the plane (2 wds.)  
146 63 274 55 99 230 80 1 22 67 258 209
- C. derived branch  
236 52 252 138 2 82 231 188
- D. inelegant  
162 69 73 174 60 126 184 201 3 263
- E. **Greek** word for a construction technique using a marked straightedge  
86 244 48 192 273 15
- F. **Mark Kac's** expression for a sterile generalization pursued for its own sake (2 Ms.)  
56 26 239 116 212 242 265 96 144 260 44 228 216  
189 62 74 123 175
- G. a coupling device **that** permits relative rotational motion of the connected parts (2 wds.)  
139 64 147 248 172 30 19 114 167 28 155
- H. temporal reflection (2 wds.)  
221 257 232 128 135 105 12 118 89 8L 167 158
- I. a commonly used cipher **uncrackable** wen In principle (2 wds., I comp.)  
112 23 40 150 180 130 65 218 45 104
- J. an authorized rule or standard  
72 85 4 37
- K. a zone of **intergradation** between ecological communities  
181 121 198 35 78 103 156
- L. residue  
59 208 152 222 47 17 5 199
- M. first stated by **Zermelo** in 1904, its acceptance or rejection reflects underlying philosophical conceptions about the nature of mathematics and mathematical existence (3 Ms.)  
102 79 169 223 111 122 83 160 264 151 16 34 120
- N. why the early pages of a table of logarithms become dogeared first (2 wds.)  
11 229 215 179 249 132 271 57 113 163 71
- O. to the physicist what a connection on a fibre bundle Is to a topologist (3 Ms., 1 corn~)  
168 261 204 115 90 193 4L 136 49 275 54 269 32  
159 171 27 219 259 129
- P. an **Escher** lithograph featuring three totally **diff.** **ferent** worlds built together into a unified whole  
119 125 66 58 200 93 266 140 161 2L
- Q. "An intellectual is someone whose mind watches \_\_\_\_." Albert **Camus**  
42 109 191 133 166 245
- R. axon  
170 240 36 100 33 253 202
- S. name for an encoding function that can be deserted exactly while keeping its inverse secret  
210 220 9L 165 183 70 20 14
- T. an imaginary **roominghouse** with o rooms (2 wds.)  
46 88 246 38 13 178 238 224 254 3L 142 211 164
- U. Its utilization enables the Voyager spacecraft to continue its journey through the solar system (2 wds.)  
145 195 153 270 29 7 97 131 277 267 197 256 137
- V. a statistical observation that is not homogeneous with others of a sample  
94 233 6L 157 185 206 75  
53 205
- W. William Gospels space-fillingcurve based on an arrangement of seven congruent regular hexagons  
268 182 190 173 9 95 247 68 25
- X. a track or channel in which somethingrolls or slides  
207 8 234 18
- Y. physicist for whom the annual Hungarian mathematical competition, open to high school graduates of that year, is named  
196 5L 92 127 251 186  
---
- Z. wind-blown  
76 255 272 39 107 225 141

## Words

- a. name given by T. H. O'Birne to the only rep-4 pentagon  
108 227 176 149 154 217
- b. term coined in the 19th century, for **curves** employed in Pseudo-Platonic geometric constructions  
214 106 235 203 243 148 276 77 50
- c. visual illusion, **devised** in 1832 by a Swiss naturalist, which spontaneously reverses In depth (2 wds.)  
213 98 84 43 177 250 124 10 194 110



Editor's Note: The grid for Mathacrostic No. 28 was prepared on a Macintosh II by Peter Bergstrom.

## PROBLEM DEPARTMENT

Edited by Clayton W. Dodge  
University of Maine

This department welcomes problems believed to be new and at a level appropriate for the readers of this journal. Old problems displaying novel and elegant methods of solution are also invited. Proposals should be accompanied by solutions if available and by any information that will assist the editor. An asterisk (\*) preceding a problem number indicates that the proposer did not submit a solution.

All communications should be addressed to C. W. Dodge, Math. Dept., University of Maine, Orono, ME 04469. Please submit each proposal and solution preferably typed or clearly written on a separate sheet (one side only) properly identified with name and address. Solutions to problems in this issue should be mailed by December 15, 1989.

## Problems for Solution

691. Proposed by Charles W. Trigg, San Diego, California.

Find the smallest possible FACE on the largest possible CUBE of this addition alphametic:

$$\begin{array}{r} \text{SIX} \\ + \text{FACE} \\ \hline \text{CUBE} \end{array}$$

692. Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.

Solve the equation

$$3(30^x) - 6(15^x) - 3(6^x) + 6(3^x) + 2(5^x) - 10^x + 2^x - 2 = 0.$$

693. Proposed by Barry Brunson, Western Kentucky University, Bowling Green, Kentucky.

Solve the equation

$$3 \cdot 2^{\log_x(3x-2)} + 2 \cdot 3^{\log_x(3x-2)} = 5 \cdot 6^{\log_y(3x-2)},$$

where  $y = x^2$  (from Problem Book in High School Mathematics, A. Prilepko, ed., Mir Publishers, Moscow, 1982, p. 43, partial solution on p. 184).

694. Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.

An ellipse has its foci at the vertices of a hyperbola and its vertices at the hyperbola's foci. Under what conditions, if any, will the ellipse and the hyperbola be orthogonal at their points of intersection.

695. Proposed by Jack Garfunkel, flushing, New York.  
If ABC is a triangle, prove that

$$\sqrt{\sin A} + \sqrt{\sin B} + \sqrt{\sin C} \geq 6 \sqrt{\sqrt{3} \sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2}}.$$

696. Proposed by Robert C. Gebhardt, Hopatcong, New Jersey.

Consider  $\int \int_{\text{region}} xy \sqrt{x^2 + y^2} dA$ , where the integral is taken over the region

bounded by the y-axis and the semicircle  $x = \sqrt{1 - y^2}$  that lies in the first and fourth quadrants.

In rectangular coordinates we have

$$\begin{aligned} \int_{-1}^1 \int_0^{\sqrt{1-y^2}} xy \sqrt{x^2 + y^2} dx dy &= \int_{-1}^1 \frac{y}{3} (x^2 + y^2)^{3/2} dy \\ &= \frac{1}{3} \int_{-1}^1 y(1-y^3) dy = \frac{-2}{15}. \end{aligned}$$

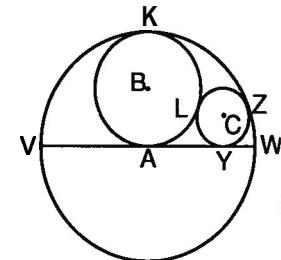
In polar coordinates, however, we have

$$\int_{-\pi/2}^{\pi/2} \int_0^1 r^4 \cos \theta \sin \theta dr d\theta = \frac{1}{5} \int_{-\pi/2}^{\pi/2} \cos \theta \sin \theta d\theta = 0.$$

What is the correct answer? Explain the discrepancy.

697. Proposed by Keith Goggin, Joseph Puthoff and John Ruebusch, St. Xavier High School, Cincinnati, Ohio.

Circle (B) is internally tangent to circle (A) at K and to diameter VAW at center A. Circle (C) is internally tangent to circle (A) at Z, externally tangent to circle (B) at L, and tangent to segment AW at Y, as shown in the figure at right. Find the ratios of the areas of the three circles to one another. (This problem was adapted from an MAA test.)



698. [Proposed by John M. Howell, Littlerock, California.]

Find all solutions in integers  
a) to the equation

$$2^n = n^x.$$

\*b) to the simultaneous inequalities

$$2^n > n^x \text{ and } 2^{n-1} < (n-1)^x.$$

699. [Proposed by Peter A. Lindstrom, North Lake College, Irving, Texas.]

Let  $A(k)$  be the unpaid principal on a loan of \$A after the  $k$ th payment out of  $n$  equal payments has been made, so that  $A(0) = \$A$  and  $A(n) = \$0$ . Let  $i$  be the interest rate per payment period and  $I$  be the total interest paid over the life of the loan. Show that

$$\sum_{k=0}^{n-1} A(k) = \frac{1}{i}.$$

700. [Proposed by R. S. Luthar, University Of Wisconsin Center, Janesville, Wisconsin.]

Let  $a, b, c, p, q, r$  be any positive numbers satisfying the equation  $\frac{1}{p} + \frac{1}{q} + \frac{1}{r} = 1$ . Prove that

$$\frac{a^p}{p} + \frac{b^q}{q} + \frac{c^r}{r} \geq abc.$$

701. [Proposed by Dmitry P. Marvlo, Moscow, U.S.S.R.]

Let  $L$  and  $B$  be nonnegative numbers such that  $L\sqrt{3} + 9B = 9\sqrt{3}$ . Prove that in any triangle ABC

$$\frac{\tan \frac{A}{2} + \tan \frac{B}{2} + \tan \frac{C}{2}}{\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2}} \geq 4 \left( \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \right)^2 + B \left( \tan \frac{A}{2} \tan \frac{B}{2} \tan \frac{C}{2} \right)$$

with equality if and only if the triangle is equilateral.

702. [Proposed by Dmitry P. Marvlo, Moscow, U.S.S.R.]

In right triangle ABC with right angle at C the altitude CD and the median CE are drawn. Determine the ratio of the sides containing the right angle if  $AB = 3 \cdot DE$  (from the SYMP-86 Entrance Exam).

703. [Proposed by Christopher Stuart, New Mexico State University, Mesilla Park, New Mexico.]

If  $f(x) \neq 0$  and differentiable, find  $\lim_{h \rightarrow 0} \left[ \frac{f(x+h)}{f(x)} \right]^{1/h}$ .

## SOLUTIONS

665. [Spring 1988] [Proposed by John M. Howell, Littlerock, California.]

An international committee translated the EINS-ZWEI-DREI problem, Problem 626 [Fall 1986, Fall 1987]. Even though the languages are all mixed up, there is a base ten solution in which O, D, and T, the initial letters of ONE, DOS, and TRE, are in increasing arithmetic progression. Find that solution.

$$\begin{array}{r} \text{O N E} \\ \text{D O S} \\ + \text{T R E} \\ \hline \text{S E X} \end{array}$$

*Solution by Thomas Mitchell, Southern Illinois University at Carbondale, Carbondale, Illinois.*

The only possibilities for the ordered triple (O,D,T) are (1,2,3), (2,3,4), and (1,3,5). In any case, E  $\neq$  0 and E  $\neq$  5. Let (O,D,T) = (1,2,3) and suppose S = 6. Clearly X is even. If X = 4, then E = 9, so N + R = 6, which is not possible. If X = 8, then E = 1 or 6, another impossibility. If X = 0, then E = 7 and N + R = 4, again unobtainable. Thus we cannot have S = 6.

If (O,D,T,S) = (1,2,3,7), then X is odd. If X = 5, then either E = 4 and N + R = 12 or E = 9 and N + R = 16, neither of which is possible. If X = 9, the only other possibility, then E = 6 and N + R = 14, another hopeless equation.

If (O,D,T,S) = (1,2,3,8), then 2 must be carried from the ten's column, requiring that N + R be at least 17, which is not possible. Thus (O,D,T) cannot be (1,2,3).

If (O,D,T) = (2,3,4), then S = 9 and X is odd. If X = 1, then E = 6 and N + R = 2, which is not possible. If X = 5, then E = 8 and N + R = 4, again not solvable. Finally, if X = 7, then E = 4 or E = 9, neither case being admissible.

Hence (O,D,T,S) = (1,3,5,9) and X is odd. Hence X = 7 and E = 4, which leads to N + R = 2. Then (N,R) = (2,0) or (0,2), and the problem has the two solutions:

$$\begin{array}{r} 104 \\ 319 \\ 524 \\ \hline 947 \end{array} \quad \text{and} \quad \begin{array}{r} 124 \\ 319 \\ 504 \\ \hline 947 \end{array}$$

*Also solved by STEVE ASHER, McNeil Pharmaceutical, Spring House, PA, CHARLES ASHBACHER, Mount Mercy College, Cedar Rapids, IA, FRANK P. BATTLES, Massachusetts Maritime academy, Buzzards Bay, ROY BENTON, Columbia Union College, Takoma Park, MD, WILLIAM BOULGER, St. Paul Academy, MN, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, MARK EVANS, Louisville, KY, VICTOR G. FESER, University of May, Bismarck, ND, RICHARD I. HESS, Rancho Solos Denies, CA, WADE H. SHERARD, Furman University, Greenville, SC, AMITABHA TRIPATHI, SUNY at Buffalo, L. J. UPTON, Mississauga, Ontario, Canada, H. J. MICHEL WIJERS, Helmond, The Netherlands, and the PROPOSER.*

666. [Spring 1988] [Proposed by John M. Howell, Littlerock, California.]

Five dice are rolled to form a "poker hand." Find the probabilities of the hands: no matches, one pair, two pairs, three of a kind, four of a kind, five of a kind, full house, and straight.

*I. Solution by Henry S. Lieberman, Waban, Massachusetts.*

We count the number of ways each "poker hand" can be accomplished, then divide by  $6^5 = 7776$ , the total number of permutations of the five dice, to get the desired probability.

Straight: There are 2 straights and  $5!$  permutations of the numbers for  $2 \cdot 5! = 240$  ways of obtaining a straight.

No matches and no straight: There are  $C(6,5) = 6$  subsets of five elements taken from the set  $\{1,2,3,4,5,6\}$  and  $5!$  permutations of the five subset elements. Since 240 of these are straights, then there are  $6 \cdot 5! - 240 = 480$  "bust" hands.

One pair: There are  $C(6,1) = 6$  choices for the pair candidate and then  $C(5,3) = 10$  choices for the rest of the hand. Each hand has  $5!/2! = 60$  permutations, so one pair occurs in  $6 \cdot 10 \cdot 60 = 3600$  ways.

Two pair: There are  $C(6,2) = 15$  choices for the 'pair' numbers and then  $C(4,1) = 4$  possibilities for the odd card. Since each hand has  $5!/2! \cdot 2! = 30$  permutations, there are  $15 \cdot 4 \cdot 30 = 1800$  two-pair hands.

Three of a kind: Similarly there are  $C(6,1) = 6$  and  $C(5,2) = 10$  choices for the triplet number and the remaining numbers and the hand has  $5!/3! = 20$  permutations, yielding 1200 hands.

Four of a kind: Here the numbers are  $C(6,1) = 6$ ,  $C(5,1) = 5$ , and  $5!/4! = 5$ , producing 150 hands.

Five of a kind: There are  $C(6,1) = 6$  choices for this quintet.

Full house: There are  $C(6,1) = 6$  choices for the triple and then  $C(5,1) = 5$  for the pair. Each hand has  $5!/2! \cdot 3! = 10$  permutations, so the full house occurs in 300 ways.

We summarize these results in the following table.

HAND	OCCURRENCES	PROBABILITY
No match - no straight	480	0.0617
One pair	3600	0.4630
Two pair	1800	0.2315
Three of a kind	1200	0.1543
Four of a kind	150	0.0193
Five of a kind	6	0.0008
Full house	300	0.0386
Straight	240	0.0309

*II. Comment by James E. Campbell, Indiana University at Bloomington, Bloomington, Indiana.*

A challenging problem, but I would expect #666 to be a bit more, um, ... devilish!

*Also solved by CHARLES ASHBACHER, Mount Mercy College, Cedar Rapids, IA, JAMES E. CAMPBELL, Indiana University at Bloomington, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, RICHARD DUNLAP, Georgia Tech, Atlanta, MARK EVANS, Louisville, KY, RICHARD I. HESS, Rancho Palos Verdes, CA, JEREMY PARTIN, Hendrix College, Conway, AR, and the proposer.*

**667. [Spring 1988] Proposed by John M. Howell, Little Rock, California.**

Each special die has one face with 1 spot, two faces with 2 spots each, and three faces with 3 spots each. Find the probability of tossing a sum of 8 with four special dice.

*Composite of solutions submitted by Murray S. Klamkin, University of Alberta, Edmonton, Alberta, Canada, and Michael V. Williams, Wake Forest University, Winston-Salem, North Carolina.*

More generally, the probability  $Pr(s,n)$  of a sum  $s$  with  $n$  special dice ( $s \geq n$ ) is the coefficient of  $x^s$  in the expression

$$\left(\frac{x}{6} + \frac{2x^2}{6} + \frac{3x^3}{6}\right)^n = \left(\frac{x}{6}\right)^n (1 + 2x + 3x^2)^n.$$

$$\text{In particular, } Pr(8,4) = \frac{214}{1296} = 0.1651.$$

*Also solved by CHARLES ASHBACHER, Mount Mercy College, Cedar Rapids, IA, STEVE ASHER, McNeil Pharmaceutical, Spring House, PA, WILLIAM BOULGER, St. Paul Academy, MN, JAMES E. CAMPBELL, Indiana University at Bloomington, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, RICHARD DUNLAP, Georgia Tech, Atlanta, DAVID EHREN, University of Wisconsin, Milwaukee, MARK EVANS, Louisville, KY, ROBERT C. GEBHARDT, Hopatcong, NJ, RICHARD I. HESS, Rancho Palos Verdes, CA, PATRICK P. T. LEONG, John Carroll University, Cleveland, OH, HENRY S. LIEBERMAN, Waban, MA, HENRY SCHAEFER, St. Bonaventure University, NY, and the proposer.*

Asher, Dudley, Evans, and Hess each showed that the numbers of ways of obtaining the sums 4 through 12 with four special dice were 1, 8, 36, 104, 214, 312, 324, 216, and 81, respectively, out of the total of 1296 possibilities.

**668. [Spring 1988] Proposed by R. S. Luthar, University of Wisconsin Center, Janesville, Wisconsin.**

Evaluate the integral

$$\int_0^{\pi/2} \frac{2 \cos^3 x}{2 - \sin 2x} dx$$

*I. Solution by Murray S. Klamkin, University of Alberta, Edmonton, Alberta, Canada.*  
By making the substitution  $x = \pi/2 - t$ , we have that

$$J = \int_0^{\pi/2} \frac{2 \cos^3 x}{2 - \sin 2x} dx = \int_0^{\pi/2} \frac{\cos^3 x dx}{1 - \sin x \cos x} = \int_0^{\pi/2} \frac{\sin^3 t dt}{1 - \sin t \cos t}$$

Thus we have

$$2J = \int_0^{\pi/2} \frac{3}{1 - \sin x \cos x} dx = \int_0^{\pi/2} (\cos x + \sin x) dx = 2,$$

and  $J = 1$ .

More generally, if  $G(a - x) = G(x)$ , then the substitution  $t = a - x$  verifies that

$$\text{if } J = \int_0^a \frac{F(x)}{G(x)} dx, \text{ then } J = \int_0^a \frac{F(a-x)}{G(x)} dx$$

so that

$$J = \frac{1}{2} \int_0^a \frac{F(x) + F(a-x)}{G(x)} dx.$$

For example, since

$$\sin^5 x + \cos^5 x = (\sin x + \cos x) \left(1 - \frac{\sin 2x}{2} - \frac{\sin^2 2x}{4}\right),$$

$$\int_0^{\pi/2} \frac{4 \cos^5 x}{4 - 2 \sin 2x - \sin^2 2x} dx = 1 \text{ also.}$$

*II. Solution By Amitabha Tripathi, SUNY at Buffalo, Buffalo, New York.*

We observe that

$$\begin{aligned} & \frac{2 \cos^3 x}{2 - \sin 2x} - \frac{\cos^3 x}{1 - \sin x \cos x} = \\ & \frac{\cos^3 x (1 + \sin x \cos x)}{1 - \sin^2 x \cos^2 x} = \frac{\cos x (1 - \sin^2 x)}{1 - \sin^2 x \cos^2 x} + \frac{\cos^4 x \sin x}{1 - \sin^2 x \cos^2 x} \end{aligned}$$

so that we have

$$\begin{aligned} & \int_0^{\pi/2} \frac{2 \cos^3 x}{2 - \sin 2x} dx \\ &= \int_0^{\pi/2} \frac{\cos x (1 - \sin^2 x)}{1 - \sin^2 x \cos^2 x} dx + \int_0^{\pi/2} \frac{\cos^4 x \sin x}{1 - \sin^2 x \cos^2 x} dx \\ &= \int_0^1 \frac{1 - u^2}{1 - u^2 (1 - u)} du + \int_0^1 \frac{u^4}{1 - u^2 (1 - u^2)} du \\ &= \int_0^1 \frac{1 - u^2 + u^4}{1 - u^2 + u^4} du = 1 \end{aligned}$$

*III. Solution by Richard I. Hess, Rancho Palos Verdes, California.*

We have that

$$\begin{aligned} & \frac{2 \cos^3 x}{2 - \sin 2x} - \frac{\cos^3 x}{1 - \sin x \cos x} \\ &= \sin x + \frac{\cos^3 x - \sin x (1 - \sin x \cos x)}{1 - \sin x \cos x} \\ &= \sin x + \frac{\cos^3 x - \sin x + \cos x - \cos^3 x}{1 - \sin x \cos x} \\ &= \sin x + \frac{\cos x - \sin x}{1 - \sin x \cos x} \end{aligned}$$

By letting  $u = \pi/2 - x$  in the following integral  $J$ , we find that

$$J = \int_0^{\pi/2} \frac{\cos x - \sin x}{\sin x \cos x} dx = \int_0^{\pi/2} \frac{\sin u - \cos u}{1 - \sin u \cos u} du = -J$$

so  $J = 0$ . Now we have that

$$\int_0^{\pi/2} \frac{2 \cos^3 x}{2 - \sin 2x} dx = \int_0^{\pi/2} \sin x dx + J = 1 + 0 = 1.$$

*Also solved by FRANK P. BATTLES, Massachusetts Maritime Academy, Buzzards Bay, ROY BENTON, Columbia Union College, Takoma Park, MD, DAVID G. CARABALLO, Hillside, NJ, CHARLES R. DIMINNIE, St. Bonaventure University, NY, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, RICHARD DUNLAP, Georgia Tech, Atlanta, ROBERT I. EGBERT, The Wichita State University, KS, EDWIN M. KLEIN, University of Wisconsin-Whitewater, HENRY S. LIEBERMAN, Waban, MA, BOB PRIELIPP, University of Wisconsin-Oshkosh, G. MAVRIGIAN, Youngstown State University, OH, WADE H. SHERARD, Furman University, Greenville, SC, SAHIB SINGH, Clarion University of Pennsylvania, and the PROPOSER. One incorrect solution was received.*

**669. [Spring 1988]** Proposed by Mohammad K. Azarian, University of Evansville, Evansville, Indiana.

Let  $n$  be any positive integer. Show that  $\prod_{k=n}^{2n} (k!+1)$  has a factor greater than  $\prod_{k=n}^{2n} k$ .

*I. Solution by the Proposer.*

For each positive integer  $k$  there is a prime  $p_k$  such that  $p_k | (k! + 1)$  and  $p_k > k$ .

Hence

$$\prod_{k=n}^{2n} p_k \mid \prod_{k=n}^{2n} (k^2 + 1) \quad \text{and} \quad \prod_{k=n}^{2n} p_k > \prod_{k=n}^{2n} k.$$

*II. Solution* By Underwood Dudley, DePauw University, Greencastle, Indiana.

One answer would be to say that the given product has itself as a factor; another answer is that the last factor  $(2n)^2 + 1$  of the given product also satisfies the stated conditions. Both answers are unsatisfying and lead one to suspect that the problem is not stated correctly. However, the first emendation that springs to mind -- "has a prime factor greater than" -- will not do, since the statement is then false for  $n = 2$ . The problem thus remains a puzzle.

*Also solved by* FRANK P. BATTLES, Massachusetts Maritime Academy, Buzzards Bay, WILLIAM BOULGER, St. Paul Academy, MN, DAVID G. CARABALLO, Hillside, NJ, RICHARD DUNLAP, Georgia Tech, Atlanta, RUSSELL EULER, Northwest Missouri State University, Maryville, MARK EVANS, Louisville, KY, VICTOR G. FESER, University of Mary, Bismarck, ND, RICHARD I. HESS, Rancho Palos Verdes, CA, PETER A. LINDSTROM, North Lake College, Irving, TX, HENRY S. LIEBERMAN, Waban, MA, DON PFAFF, University of Nevada, Reno, SAHIB SINGH, Clarion University of Pennsylvania, TOM, KIM, CAMMIE and CHRISTINE, Massachusetts Gamma, Bridgewater State College, and AMITABHA TRIPATHI, SUNY at Buffalo. Most solvers gave the solution  $(2n)^2 + 1$ .

Clearly, the statement of the problem is inadequate and the editor should have caught the difficulty before it got into print. One rewording would be "has a factor that is a product of not more than  $n + 1$  primes and is greater than."

**670.** [Spring 1988] *Proposed by* Teter A. Lindstrom, North Lake College, Irving, Texas.

If  $F_n = 2^{2^n} + 1$  is the  $n$ th Fermat number, find all values of  $n$  so that  $F_n$  and  $F_{n-2}$  are twin primes.

*I. Solution* By Mark Evans, Louisville, Kentucky.

Clearly one integer in any three consecutive integers is divisible by 3. Since  $F_n - 1$  obviously is not, then either  $F_n$  or  $F_n - 2$  must be divisible by 3. Hence only  $F_1 - 2 = 3$  and  $F_1 = 5$  are twin primes.

*II. Solution* by Sahib Singh, Clarion University of Pennsylvania, Clarion, Pennsylvania.

The only twin primes of the family are 3 and 5 for  $n = 1$ . The number  $F_n - 2$  is never prime when  $n > 1$  because  $F_m$  divides  $F_n - 2$  whenever  $n > m \geq 0$ . See Burton, *Elementary Number Theory* (1980), page 237.

*Also solved by* WILLIAM BOULGER, St. Paul Academy, MN, DAVID G. CARABALLO, Hillside, NJ, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, RICHARD DUNLAP, Georgia Tech, Atlanta, DAVID EHREN, University of Wisconsin, Milwaukee, RUSSELL EULER, Northwest Missouri State University, Maryville, VICTOR G. FESER, University of Mary, Bismarck, ND, RICHARD I. HESS, Rancho Palos Verdes, CA, FRANK HUBENY, University of Maine, Orono, HENRY S. LIEBERMAN, Waban, MA, DON PFAFF, University of Nevada, Reno, BOB PRIELIPP, University of Wisconsin-Oshkosh, SAHIB SINGH, Clarion University of Pennsylvania, AMITABHA TRIPATHI, SUNY at Buffalo, and the PROPOSER.

**S. LIEBERMAN**, Waban, MA, THOMAS E. MOORE, Bridgewater State College, MA, DON PFAFF, University of Nevada, Reno, BOB PRIELIPP, University of Wisconsin-Oshkosh, HENRY SCHAEFER, St. Bonaventure University, NY, JOHN A. SCHUMAKER, Rockford College, IL, WADE H. SHERARD, Furman University, Greenville, SC, TOM, KIM, CAMMIE and CHRISTINE (2 solutions), Massachusetts Gamma, Bridgewater State College, AMITABHA TRIPATHI, SUNY at Buffalo, MICHAEL D. WILLIAMS, Wake Forest University, Winston-Salem, NC, and the PROPOSER.

**671.** [Spring 1988] *Proposed by* J. S. frame, Michigan State University, East Lansing, Michigan.

Find all sequences of  $2k + 1$  consecutive integers  $a, a + 1, \dots, a + 2k$  such that the sum of the squares of the first  $k + 1$  of these integers is equal to the sum of the squares of the last  $k$ . That is, find a formula for  $a = a_k$  as a function of  $k$ . For example,  $a_1 = 3$  since  $3^2 + 4^2 = 5^2$ .

*Solution* By Mark Evans, Louisville, Kentucky.

Letting all the summations run from  $i = 1$  to  $k$ , we wish to have

$$\begin{aligned} a^2 + \sum_{i=1}^k (i+1)^2 &= \sum_{i=1}^k (i+k)^2 = \sum_{i=1}^k (i+1)^2 + 2\sum_{i=1}^k (i+k)i + \sum_{i=1}^k k^2 \\ \text{so that } a^2 &= 2\sum_{i=1}^k ak + 2\sum_{i=1}^k ik + \sum_{i=1}^k k^2 = 2ak + k^2(k+1) + k^3, \\ 0 &= a^2 - 2k^2a - 2k^3 - k^2 = (a+k)(a-2k^2-k), \end{aligned}$$

and  $a = 2k^2 + k$  (the  $2k$ th triangular number) or  $a = -k$ .

*Also solved by* STEVE ASHER, McNeil Pharmaceutical, Spring House, PA, FRANK P. BATTLES, Massachusetts Maritime Academy, Buzzards Bay, WILLIAM BOULGER, St. Paul Academy, MN, JAMES E. CAMPBELL, Indiana University at Bloomington, CHARLES R. DIMINNIE, St. Bonaventure University, NY, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, DAVID EHREN, University of Wisconsin, Milwaukee, RUSSELL EULER, Northwest Missouri State University, Maryville, VICTOR G. FESER, University of Mary, Bismarck, ND, RICHARD I. HESS, Rancho Palos Verdes, CA, FRANK HUBENY, University of Maine, Orono, HENRY S. LIEBERMAN, Waban, MA, DON PFAFF, University of Nevada, Reno, BOB PRIELIPP, University of Wisconsin-Oshkosh, SAHIB SINGH, Clarion University of Pennsylvania, AMITABHA TRIPATHI, SUNY at Buffalo, and the PROPOSER.

**672.** [Spring 1988] *Proposed by* Barry Brunson, Western Kentucky University, Bowling Green, Kentucky.

Find a series representation for

$$\int_0^1 x^x dx$$

*Solution* by Murray S. Klamkin, University of Alberta, Edmonton, Alberta, Canada. More generally,

**673.** [Spring 1988] *Proposed by Stanley Rabinowitz, Alliant Computer Systems, Littleton, Massachusetts.*

Let AB be an edge of a regular tesseract (a four-dimensional cube) and let C be the tesseract's vertex that is furthest from A. Find the measure of angle ACB.

*Solution by Richard I. Mess, Rancho Palos Verdes, California.*

We take the n-dimensional simplex whose vertices are the points  $(x, x, x, \dots, x)$  where each x is either a 0 or a 1. We take  $A = (0, 0, 0, \dots, 0)$ , a nearest neighbor  $B = (1, 0, 0, \dots, 0)$ , and the farthest point  $C = (1, 1, 1, \dots, 1)$ . Then we have

$$\cos ACB = \frac{(C - B) \cdot (C - A)}{|C - B| |C - A|} = \frac{(0, 1, 1, \dots, 1) \cdot (1, 1, 1, \dots, 1)}{\sqrt{n-1} \sqrt{n}} = \sqrt{\frac{n-1}{n}}.$$

For  $n = 4$  we have  $\cos ACB = \sqrt{3}/2$ , so  $ACB = 30^\circ$ . Angles for other values of n are now readily computed. For example,  $ACB = 45^\circ$  for the square ( $n = 2$ ) and  $ACB = 35.264\dots^\circ$  for the cube ( $n = 3$ ).

*Also solved by JAMES E. CAMPBELL, Indiana University at Bloomington, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, RICHARD DUNLAP, Georgia Tech, Atlanta, HENRY S. LIEBERMAN, Waban, MA, and the PROPOSER.*

**674.** [Spring 1988] *Proposed by Russell Euler, Northwest Missouri State University, Maryville, Missouri.*

Find necessary and sufficient conditions for the arithmetic mean of the roots of the polynomial equation

$$a_0 x^n + a_1 x^{n-1} + \dots + a_{n-1} x + a_n = 0$$

to be equal to the geometric mean of the roots.

*Solution by ÅEOPRIELIPP, University of Wisconsin-Oshkosh, Oshkosh, Wisconsin.*

It is known that the sum of the roots of the given polynomial is equal to  $-a_1/a_0$  and their product is  $(-1)^n a_n/a_0$ . Hence the arithmetic mean of the roots equals their geometric mean if and only if

$$(-a_1/a_0)/n^n = ((-1) a_n/a_0)^{1/n},$$

which can be rewritten as

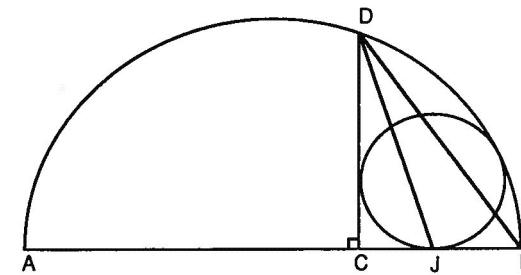
$$a_1^n = n^n a_0^{n-1} a_n.$$

*Also solved by SEUNG-JIN BANG, Seoul, Korea, DAVID G. CARABALLO, Hillside, NJ, JAMES E. CAMPBELL, Indiana University at Bloomington, DAVID DELSESTO, North Scituate, RI, CHARLES R. DIMINNIE, St. Bonaventure University, NY, UNDERWOOD DUDLEY, DePauw University, Greencastle, IN, DAVID EHREN, University Of Wisconsin, Milwaukee, ROBERT C. GEBHARDT, Hopatcong, NJ, RICHARD I. HESS, Rancho Palos Verdes, CA, AMITABHA TRIPATHI, SUNY at Buffalo, and the PROPOSER.*

Although this is a relatively simple problem, one must be careful to examine signs when taking roots. Thus, for example,  $((-1)^n)^{1/n} = (-1)^n$  and not  $= -1$ . Now, if all the roots are positive, then the stated condition implies that the roots must all be equal. If some of the roots are negative, of course their geometric mean must be a real number. Thus, if n is even, an even number of roots must be negative or a root must be zero. A nontrivial example is the quartic polynomial whose zeros are  $3 - 3\sqrt{10}$ ,  $3 - \sqrt{10}$ ,  $3 + \sqrt{10}$ , and  $3 + 3\sqrt{10}$ .

**675.** [Spring 1988] *Proposed by John H. Scott, Macalester College, Saint Paul, Minnesota.*

Erect a semicircle on segment AB as diameter. From point D on the semicircle drop a perpendicular to point C on AB. Draw a circle tangent to CB at J and tangent to the semicircle and to segment CD. Prove that angles CDJ and JDB have equal measures. See the figure below.



*Solution by Henry S. Lieberman, Waban, Massachusetts.*

On pages 84 and 85 of *Problems and Solutions in Euclidean Geometry* by Aref and Wernick (Dover, 1968), it is proven that  $AJ = AD$ . From this it follows that  $\angle ZAD = \angle AJD = \angle CJD$ . But

$\angle ZAD = \angle ZAD + \angle ZCDJ$  and  $\angle CJD = \angle ZCB + \angle ZJD$ . Since  $\angle ZAD$  and  $\angle ZCB$  are both the complement of  $\angle BAD$ , it follows that they are equal. Therefore  $\angle ZCDJ = \angle ZJD$ .

*Also solved by JACK GARFUNKEL, Flushing, NY, RICHARD I. HESS, Rancho Palos Verdes, CA, KING W. JAMISON, Middle Tennessee State University, Murfreesboro, TN, MAVRIGIAN, Youngstown State University, OH, WADE H. SHERARD, Furman University, Greenville, SC, and the PROPOSER.*

Sherard, whose solution closely parallels the featured solution, located the theorem that  $AJ = AD$  as problem 494 in this *Journal* [Spring 1982], proposed by Zelda Katz and solved by Henry S. Lieberman. Jack Garfunkel was one of the also-solvers. Also, this is not the first time that the editor of this Department has missed (related) problems that have appeared here previously.

**676.** [Spring 1988] *Proposed by John M. Howell, Littlerock, California.*  
Show that, for  $k > 0$ ,

$$\sum_{i=1}^n \left[ \prod_{j=0}^k (i+j) \right]^{-1} = \frac{1}{k \cdot k!} - \frac{1}{k} \left[ \prod_{j=1}^k (n+j) \right]^{-1}$$

*Solution by Sahib Singh, Clarion University of Pennsylvania, Clarion, Pennsylvania.*  
The left side of the given equation can be written as

$$S = \sum_{r=0}^{n-1} \frac{r!}{(k+r+1)!} = \frac{1}{k} \sum_{r=0}^{n-1} \left( \frac{r!}{(k+r)!} - \frac{(r+1)!}{(k+r+1)!} \right),$$

which is a collapsing series that reduces to

$$S = \frac{1}{k} \left( \frac{1}{k!} - \frac{n!}{(k+n)!} \right),$$

a form equal to the right side of the given equation.

*Also solved by UNDERWOOD DUDLEY, DePauw University, Greencastle, IN; RICHARD DUNLAP, Georgia Tech, Atlanta, MARK EVANS, Louisville, KY; RICHARD I. HESS, Rancho Palos Verdes, CA; MURRAY S. KLAMKIN, University of Alberta, Canada; HENRY S. LIEBERMAN, Waban, MA, and the PROPOSER. Klamkin pointed out that this result is well known.*

677. [Spring 1988] *Proposed by Jack Garfunkel, Flushing, New York.*  
If A, B, and C are the angles of a triangle, then show that

$$\frac{\cos A \cos B \cos C}{\cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2}} \leq \frac{\sqrt{3}}{9}.$$

*Solution by Murray S. Klamkin, University of Alberta, Edmonton, Alberta, Canada.*

The inequality is obviously true if any angle is greater than or equal to  $90^\circ$ , so we assume that the triangle is acute. Multiplying the numerator and denominator of the left side by  $8 \sin(A/2) \sin(B/2) \sin(C/2)$ , we get

$$\frac{[8 \sin(A/2) \sin(B/2) \sin(C/2)] \cos A \cos B \cos C}{\sin A \sin B \sin C} \leq \frac{\sqrt{3}}{9}.$$

This result follows from items 2.12 and 2.40 in Bottema et al, *Geometric Inequalities*, Noordhoff, Groningen, 1968:

$$\sin \frac{A}{2} \sin \frac{B}{2} \sin \frac{C}{2} = \frac{r}{4R} \leq \frac{1}{8} \quad \text{and} \quad \cot A \cot B \cot C \leq \frac{\sqrt{3}}{9},$$

the latter inequality valid only for acute triangles.

*Also solved by MARK EVANS, Louisville, KY; J. S. FRAME, Michigan State University, Lansing, RICHARD I. HESS, Rancho Palos Verdes, CA; EDWIN M. KLEIN, University of Wisconsin, Whitewater, YOSHINOBU MURAYOSHI, Portland, OR; BOB PRIELIPP, University of Wisconsin-Oshkosh, AMITABHA TRIPATHI, SUNY at Buffalo, and the PROPOSER. Klein, Murayoshi, and Tripathi each pointed out that equality holds if and only if the triangle is equilateral.*

$$S = \int_0^1 x^{ax^r} dx = \int_0^1 e^{ax^r \ln x} dx = \sum_{k=0}^{\infty} \frac{a^k}{k!} \int_0^1 x^{rk} \ln^k x dx.$$

It follows by integration by parts that

$$\int_0^1 x^m \ln^n x dx = \frac{(-1)^n n!}{(m+1)^{n+1}}.$$

Thus

$$S = \sum_{k=0}^{\infty} \frac{(-1)^k a^k}{(rk+1)^{k+1}}.$$

The given problem corresponds to the special case  $a = r = 1$ .

*Also solved by SEUNG-JIN BANG, Seoul, Korea; FRANK P. BATTLES, Massachusetts Maritime Academy, Buzzards Bay; DAVID G. CARABALLO, Hillside, NJ; UNDERWOOD DUDLEY, DePauw University, Greencastle, IN; RUSSELL EULER, Northwest Missouri State University, Maryville, J. S. FRAME, Michigan State University, Lowing, ROBERT C. GEBHARDT, Hopatcong, NJ; RICHARD I. HESS, Rancho Palos Verdes, CA, and the PROPOSER.*

R. P. Boas, Seattle, WA, referred to Polya and Szegö, *Problems and Theorems in Analysis*, vol. 1, 1972 (English translation), p. 36, problem 160, which deals with

$\int_0^1 x^{-x} dx$ , which is essentially equivalent to our problem. Barry Brunson, Western Kentucky University, Bowling Green, quoted from "A Report from the Selberg Symposium," held in Oslo in June 1987, in the *Mathematical Intelligencer* 10 #3, Summer 1988, 46-47:

"The birthday cake served ... was decorated with a formula for which Selberg published a proof in *Norsk Matematisk Tidsskrift* when he was 15 years old. ..."

$$\sum_{n=1}^{\infty} \frac{1}{n} = \int_0^1 \frac{dx}{x},$$

Edwin M. Klein, University of Wisconsin, Whitewater, and Bob Priellipp, University of Wisconsin-Oshkosh, each pointed out that the stated problem was problem A-4 in the 1969 William Lowell Putnam Mathematical Competition (see the August-September 1970 issue of *The American Mathematical Monthly*, pages 723 and 725). Priellipp also noted that on page 676 of the June-July 1971 issue of the *Monthly* C. D. Olds gives a detailed solution to this problem, including its numerical value 0.78343051....

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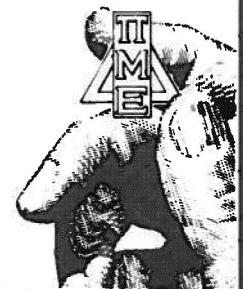
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