12-th Canadian Mathematical Olympiad 1980

1. Determine digits a, b ($a \neq 0$) so that the decimal number $\overline{a679b}$ is divisible by 72.

2. Fifty cards numbered 1 to 50 are shuffled and then laid out face up in 5 rows of 10 cards each. The cards in each row are rearranged in the increasing order from left to right. The cards in each column are then rearranged in the increasing order from top to bottom. Do the cards in the final arrangement still increase from left to right?

3. Among all triangles with a fixed angle A and the incircle of fixed radius, determine which triangle has the least perimeter.

4. A gambling student tosses a fair coin and scores one point for each head and two points for each tail. Prove that the probability of the student scoring exactly n points is $\dfrac{2+(-2)^{-n}}{3}$.

5. A parallelepiped has the property that all cross sections which are parallel to any fixed face F have the same perimeter as F. Determine whether or not any other polyhedron has this property.

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