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1962

# WHEN LINES ARE POINTS<sup>1</sup>

by Harry Levy

## PI MU EPSILON JOURNAL THE OFFICIAL PUBLICATION OF THE HONORARY MATHEMATICAL FRATERNITY

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The beginning student derives a great deal of pleasure from working in mathematics. In part his sense of satisfaction stems from his realization that mathematical problems have a definiteness that problems in other areas often seem to lack, and that the correctness, or lack of it, of his solutions is not a matter of opinion but a hard mathematical fact. It may therefore be surprising to some of you when I observe that mathematics resembles the fine arts and indeed may be called one of them; it does so because the mathematician like the poet, the sculptor, and the composer, creates something out of nothing using only his talent and his knowledge, the resources of his intellect. This may have been the frame of mind that led the youthful geometer Janos Bolyai to write to his father, in 1823, "I have created a new universe out of nothing." Like all great artists, Bolyai was passionately proud of his creation, and like all great art, Bolyai's work is immortal. It lives on in today's nuclear age to which it has been bound by the work of Lobachevski and Gauss, of Riemann and Ricci, of Einstein, and of many others.

But the artistry of mathematics is only one of its aspects. Mathematics is a science, because mathematicians observe and study existing mathematical structures, and discover and formulate the basic principles that govern them. Perhaps one characteristic that distinguishes mathematics from other areas of human endeavor is that the scientific side of mathematics and its artistic side are united in one coherent whole; I intend this evening to suggest how Riemann, a great German mathematician of the middle nineteenth century, might have been led to the creation of elliptic geometry (one of the two classical non-Euclidean geometries) by a "scientific" study of a particular Euclidean structure.

Before going on to my main objective, I should like to note that when a scientist is confronted with something apparently new, he frequently uses the method of analogy. He compares the new with the familiar, and, drawing on his knowledge of the latter, he is often able to direct his studies in a meaningful way. In using this method in mathematics, proper vocabulary and suitable choice of symbols can serve to illuminate the structure being studied and to clarify the problems whose solution is being sought. The primitive elements of geometry are usually called points, and when a mathematician is investigating any kind of mathematical structure by geometrical methods, he is very likely to call the constituent elements of his structure points, even though, in some other context, he may refer to them as lines, or functions, or matrices, or spheres, or transformations.

The structure that we propose to investigate is a bundle of lines in ordinary, that is, Euclidean (**three-dimensional**) space. For our purposes, a bundle of lines means the set of all lines through a given point. Some

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<sup>1</sup> These remarks were presented to the Undergraduate Mathematics Club, University of Illinois, Urbana, Illinois.

of you may wonder whether such a commonplace structure can possess any exciting properties. Since we wish to examine relationships between lines of the bundle, since our "space", that is, the bundle, is a set of lines, it is clear that we are to be concerned with a geometrical analysis in which the primitive elements are lines. However strange it may seem, we cannot hesitate. From this moment on, whenever we speak of a *point* we shall mean a *line* of the given bundle. We formalize this first step of our analysis thus:

1. There exists a set of objects called points; their totality, together with the relationships between them, is called an elliptic plane.

We shall denote the elliptic plane by  $E_2$ , and the notation  $X \in E_2$  is read " $X$  is an element of  $E_2$ ", that is,  $X$  is a point of the elliptic plane. To prove a theorem in elliptic geometry, it suffices for our purpose to select a suitable theorem about lines of a bundle and then translate the relationships involving lines of the bundle into geometrical language appropriate to the elliptic plane. In the bundle of lines, each two lines determine a real number, their angle, which we can, for convenience, measure in right-angle units. We commented earlier on the importance of vocabulary; what shall we call this number when we are concerned with elliptic geometry? It would seem that the word "distance" might prove useful; using this terminology enables us to put our second observation of elliptic geometry as follows:

2. Each two points of  $E_2$  determine a real number; if  $P, Q \in E_2$ , we call the number they determine the (elliptic) distance between them, and we denote it by  $e(P, Q)$ . We call  $e$  the distance function or the metric of  $E_2$ ; it satisfies the following conditions:

- (a)  $0 \leq e(P, Q) = e(Q, P) \leq 1$ ;
- (b)  $e(P, Q) = 0$  if and only if  $P = Q$ ;
- (c)  $e(P, R) \leq e(P, Q) + e(Q, R)$ .

The proof of (a) and (b) is indeed trivial; (c) can be established by elementary solid analytic geometry if we introduce rectangular coordinates chosen so that the direction cosines of the lines representing  $Q, P, R$  are  $(1, 0, 0), (\cos \vartheta, \sin \vartheta, 0), (\cos \alpha, \cos \beta, \cos \gamma)$ , respectively, where  $\cos \vartheta \geq 0, \cos \alpha \geq 0$ .

The Euclidean distance function satisfies these three conditions with the exception of the second inequality in (a), and indeed these conditions play an important role in defining metric geometries that are more general than either Euclidean or elliptic. The last inequality in (a) tells us that in elliptic geometry, as in spherical geometry, distance is bonded. Condition (c) is called the triangle inequality; it can be used to define the curves that play the role of Euclidean straight lines. But even in elliptic geometry there are difficulties that we need to take care of. Returning to our bundle of lines, we see that  $\angle PQ + \angle QR = \angle PR$ , where  $P, Q, R$  are lines, if and only if  $Q$  is coplanar with  $P$  and  $R$  and lies in the acute angle sector formed by them. But we would like to prolong an elliptic line beyond any one of its points, and we can be satisfied with

the following formulation:

3. If  $P^*, Q$  are distinct points of  $E_2$ , there exists one (and just one) point  $P_Q^*$  such that

$$e(P, Q) + e(Q, P_Q^*) = e(P, P_Q^*) = 1.$$

We define the elliptic line through  $P$  and  $Q$  to consist of all points  $X$  such that

$$e(P, X) + e(X, P_Q^*) = e(P, P_Q^*).$$

Circles arise naturally in any two-dimensional metric geometry.

4. If  $P \in E_2$ , and  $r$  is a positive real number, the (elliptic) circle with center  $P$  and radius  $r$  is the set of points  $X$  distant  $r$  from  $P$ ,

$$e(P, X) = r.$$

Thus an elliptic line corresponds to a pencil in the Euclidean bundle, and an elliptic circle to a circular cone (with one exception)

We now have points, lines, circles, and distance in our emerging geometry. Do these concepts differ significantly from their Euclidean counterparts? If we turn to elementary intersection and incidence relations, we find some results that are ordinary, and others that deviate sharply from the Euclidean pattern.

(A) *Two distinct points in  $E_2$  are incident to (that is, lie on) exactly one line.*

(B) *Two distinct lines in  $E_2$  have exactly one point of intersection.*  
Thus the phenomenon of parallel lines does not exist in  $E_2$ .

(C) *Every line in  $E_2$  is a circle of radius one.*

Here we have a property that suggests spherical geometry, but the parallelism is not complete since every circle in spherical geometry has two centers.

(D) *Concurrent lines in  $E_2$  have collinear centers, and lines with collinear centers are concurrent.*

A somewhat less elementary property of elliptic lines is the following:

(E) *Every line in  $E_2$  is a simple closed curve of length two.*

A Euclidean line is cut when one of its points is deleted, and the two portions form the two rays emanating from the point. Removal of one point of an elliptic line leaves the line connected; to cut the line two points must be removed, and of the two segments then formed, one has length equal to the distance between the deleted points, and the other, two decreased by that distance. But a more interesting situation arises when we examine (E) and (B) together. We are doomed to fail if we attempt to portray these two properties on a blackboard. Have we found a contradiction? No, the validity of (E) and (B) is irrefutable. We may have gone astray in tacitly supposing that in every two-dimensional geometry a simple closed curve determines an exterior and an interior. We need only to consider on a torus either a generating circle or a parallel circle to see the error in such an assumption.

Let us return to our model, the bundle of lines. Let  $p$  be a pencil of lines in the bundle, and let  $P$  and  $Q$  be any two distinct lines not in  $p$ . Then clearly the pencil determined by  $P$  and  $Q$  is separated by  $P$  and  $Q$  into two sectors of lines, one of which has a line in common with  $p$ ,

and the other does not. Thus, in  $E_2$ , any two points not on a given line of  $E_2$  can be joined by a segment which does not intersect the given line. If  $E_2$  were cut along the given line, it would remain connected. We thus have the following:

(F) *An elliptic line is a nonbounding curve.*

There is one farther very fundamental difference between the elliptic plane and the Euclidean plane. The latter is orientable, and it possesses two orientations. A line reflection interchanges the two orientations, whereas rotations and translations preserve both orientations.

Thus an orientation-preserving motion in the Euclidean plane can never be the product of an odd number of line reflections. We cannot here prove that  $E_2$  (like the Möbius band) is nonorientable, but we can prove that the Euclidean distinction between products of an odd and of an even number of line reflections has no significance in  $E_2$ .

A point reflection in Euclidean space leaves fixed each line of the bundle through the point; it therefore acts as the identity transformation when interpreted in  $E_2$ . On the other hand, it is the product of successive reflections in three mutually orthogonal planes. A reflection in a plane when interpreted in  $E_2$  becomes a reflection in a line (the elliptic line corresponding to the pencil in the plane). Thus in  $E_2$  the product of three line reflections can be the identity, so that the product of two line reflections can be a single line reflection.

Those of you who might wish to follow up these preliminary remarks on elliptic geometry could investigate one or more of the following problems:

(a) The problem of coordinatization. One way to establish coordinates in  $E_2$  is to represent a point by direction numbers of the corresponding line of the bundle.

(b) The problem of axioms. It is possible to establish a system of axioms which give  $E_2$  a logical existence independent of Euclidean space.

(c) The development of relationships between an elliptic and a Euclidean plane. Such a development can create for the Euclidean plane a unity which would otherwise be lacking. For example, all "conies" in  $E_2$  are closed curves, and by means of them we can find common properties of conics in the Euclidean plane.

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## INTRODUCTION

In 1953 Professors Peter Scherk and N. D. Lane published a paper (3) which employed unique methods for the study of conformal differential geometry; methods which are rather synthetic in character. If we consider the conformal plane as the one point compactification of the Euclidean plane it is apparent that methods from complex variables might be used in its study. However, certain desired properties are lost when extensions to conformal spaces of higher dimensions are attempted. The methods of Scherk and Lane are directly applicable to higher spaces (2, 5, 7). They are related to methods developed by Scherk in his studies of projective spaces (6).

The results presented in this paper are due primarily to these two men. Some of their work has been modified and new proofs have been offered for particular theorems by both the present writer in his master's report (1) and by F. A. Scherk in his master's thesis (7). The introduction of conformally elementary points and an indication of their use are our goals.

## PRELIMINARIES

The points studied here are considered to be interior points of real arcs embedded in the conformal plane. This restriction to interior points is one of convenience for the theory has also been developed to encompass end points of arcs.

A proper circle  $C$  will decompose the plane into two regions, the interior  $C_*$  and the exterior  $C^*$  of  $C$ . The interior lies to the left of the orientation of  $C$ . The infinite sequence of circles  $C_1, C_2, \dots$  will converge to the circle  $C$  if there exists to every pair  $C^k \subset C$  and  $C^{k+1} \subset C$  an integer  $k = k(C^k, C^{k+1})$  such that  $C^k \subset C_n$  and  $C^{k+1} \subset C_n^*$  for every integer  $n > k$ . Convergence of a sequence of points to a point and convergence of a sequence of circles to a point are defined similarly. It will be convenient to denote by  $C(P, Q, R)$  the circle  $C$  drawn through the three mutually distinct points  $P, Q$  and  $R$ .  $C(\pi P)$  will denote the circle  $C$  of the linear pencil  $\pi$  of circles which passes through the point  $P$ . Only pencils of the second kind with fundamental point  $s$  will be considered (cf. Fig. 1).

Finally, suppose that  $s$  is an interior point of an arc  $A$ . Then  $s$  is called a point of support (intersection) with respect to a given circle  $C$  if there

<sup>1</sup> Received by Editors September 30, 1961. Presented at the National Meeting of Pi Mu Epsilon, Stillwater, Oklahoma, August 29, 1961. This paper was judged best for graduate students. Mr. De Noya won a hundred dollar prize.

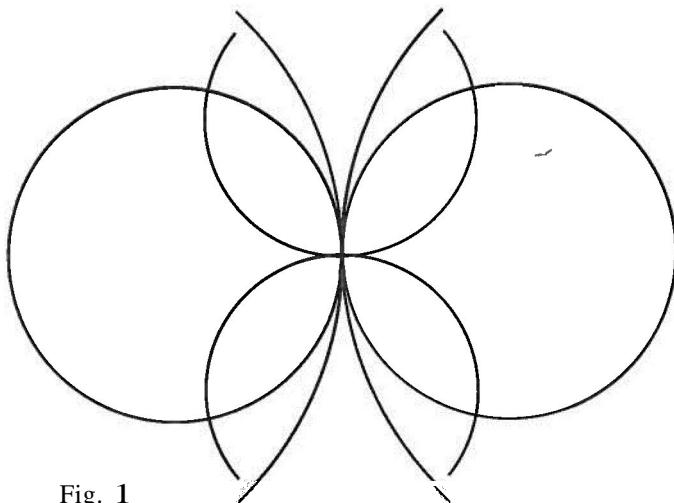


Fig. 1

is some neighborhood of  $\mathbf{s}$  decomposed by  $\mathbf{s}$  into two open neighborhoods which lie in the same (different) regions bounded by  $C$ .  $C$  is termed a supporting (intersecting) circle of  $A$  at  $\mathbf{s}$ . By definition the point circle  $\mathbf{s}$  supports  $A$  at  $\mathbf{s}$ . It may be that for a given arc there are points of the arc which are neither points of support nor of intersection, e.g. the arc

$$t \sin \frac{1}{t} \text{ at } t = 0.$$

#### DIFFERENTIABILITY

We shall say that an arc  $A$  is differentiable at an interior point  $\mathbf{s}$  (or that  $\mathbf{s}$  is a differentiable point) if two conditions are satisfied. The first of which is  $t \sin \frac{1}{t} \text{ at } t = 0$ .

#### Condition I.

Let  $\mathbf{s} \subset A$ . Then for every point  $P \neq \mathbf{s}$  and for every sequence of distinct points  $\{\bar{s}\} - \mathbf{s}$ , such that each  $\bar{s} \subset A$ , there exists a circle  $C_0$  such that  $C(\bar{s}, \mathbf{s}, P) = C_0$ .

The circle  $C_0$  is called the tangent circle of  $A$  at  $\mathbf{s}$  and is clearly independent of how  $\{\bar{s}\}$  converges. However,  $C_0$  does depend continuously upon  $P$ . The point  $s$  is itself the tangent point-circle of  $A$  at  $\mathbf{s}$ .

#### Theorem 1

The set  $\tau = \tau(s)$  of all tangent circles of  $A$  at  $\mathbf{s}$  is a pencil of the second kind with fundamental point  $\mathbf{s}$ .

Proof. Let  $P, Q$  and  $R$  be three mutually distinct points. If the sequence of points  $\{\bar{R}\} - R$ ,  $\bar{R} \neq R$ , then the angle between the circles of the sequence of circles  $C(\bar{R}, R, P)$  and  $C(\bar{R}, R, Q)$  converges to zero. Thus, the circles have (in the limit) the one point  $\bar{R}$  in common and are therefore tangent. Suppose now that  $R = \mathbf{s}$  and  $\bar{R} = \mathbf{s}$ , where  $\mathbf{s} \subset A$  and  $\bar{s} \subset A$ , then

$$\lim_{\substack{s \rightarrow s \\ s \neq s}} [C(\bar{s}, s, P); C(\bar{s}, s, Q)] = 0$$

Since the only restriction placed upon the points  $P$  and  $Q$  was that they be distinct it follows that for every other distinct point  $M$  there is one and only one circle through  $M$  and tangent to  $A$  at  $\mathbf{s}$ . Hence, if two tangent circles have another point in common they must be identical. Moreover, let some circle  $C$  touch the tangent circle  $C_0$  at  $\mathbf{s}$  and let  $P \subset C$ ,  $P \neq s$ . Then  $C_0$  and  $C$  are identical, i.e.  $C \subset \tau$ . Therefore the totality of tangent circles of  $A$  at  $\mathbf{s}$  constitutes a pencil  $\tau = \tau(s)$  of the second kind with fundamental point  $\mathbf{s}$ .

#### Corollary

If  $C(\tau, P)$  and  $C(\tau, Q)$  have another point in common they are identical. Thus, there is one and only one circle of  $T$  through each point  $P \neq s$ .

The above then yields

#### Theorem 2.

Suppose that the point  $\mathbf{s} \subset A$  satisfies Condition I. Let  $\pi$  be a pencil of the second kind with fundamental point  $\mathbf{s}$ ,  $\pi \neq \tau$ . If  $\{\bar{s}\} - s$  ( $\bar{s} \subset A$ ,  $\bar{s} \neq s$ ), then

$$\lim_{\substack{\bar{s} \rightarrow s \\ \bar{s} \neq s}} C(\pi, \bar{s}) = s.$$

#### Theorem 3.

Let  $\mathbf{s}$  be an interior point of an arc  $A$  and assume that  $\mathbf{s}$  satisfies Condition I. Then every non-tangent circle either supports or intersects  $A$  at  $\mathbf{s}$ .

Proof. Suppose that  $C$  neither supports nor intersects  $A$  at  $\mathbf{s}$ . Then the non-support supposition implies that there is a sequence of distinct points  $\{\bar{s}\} - s$  such that  $\bar{s} \subset (A \cap C)$  and  $\bar{s} \neq s$ . Now let  $P \subset C$ ,  $P \neq s$ . Then  $C = C(\bar{s}, s, P)$  for every  $\bar{s} \neq s$ .

But Condition I implies that  $C = C(\tau, P)$  which contradicts the hypothesis  $C$  was a non-tangent circle. Therefore our supposition was false and the theorem is proved.

#### Condition II.

If  $\{\bar{s}\} \rightarrow s$ ,  $\bar{s} \neq s$ , then  $C(\tau, \bar{s})$  converges to some unique circle  $C(s)$ .  $C(s)$  is the osculating circle at  $\mathbf{s}$ .

The arc  $A$  is differentiable at  $\mathbf{s}$  if both conditions I and II are satisfied. It can be shown that the two conditions are independent (1, 3, 7).

The following three theorems deal with non-osculating circles. It will be noted that they, together with the preceding theorem, provide us with a complete classification of distinct types of differentiable points.

#### Theorem 4.

Let  $\mathbf{s}$  be a differentiable interior point of an arc  $A$ . Then every non-osculating circle either supports or intersects  $A$  at  $\mathbf{s}$ .

The proof of this theorem is similar to that of Theorem 3 and will be omitted.

#### Theorem 5.

If the osculating circle  $C(s)$  is different from the point circle  $s$ , then every non-osculating tangent circle supports  $A$  at  $s$ .

Proof. Let  $C$  be any proper non-osculating tangent circle of  $A$  at  $s$ , i.e.  $C \subset \pi$ . Then by Theorem 4  $C$  either supports or intersects  $A$  at  $s$ . Suppose that  $C$  intersects. If a sequence of points  $\{\bar{s}\}$  exists such that  $\bar{s} \subset (A \cap C^*)$ ,  $\bar{s} \neq s$ ,  $\{\bar{s}\} \rightarrow s$ , then each  $C(\tau, \bar{s})$  lies in the closure of  $C^*$ . Hence, by Theorem 1 we have  $C(\tau, \bar{s}) \subset C^* \cup s$  and  $\{\bar{s}\} \rightarrow s$  gives us

$$C(s) \subset (C^* \cup C)$$

Now similar consideration a sequence  $\{\bar{s}_1\} \subset (A \cap C^*)$  with  $\bar{s}_1 \neq s$ ,  $\{\bar{s}_1\} \rightarrow s$  gives us that

$$C(s) \subset (C_* \cup C)$$

By combining  $C(s) \subset (C^* \cup C)$  and  $C(s) \subset (C_* \cup C)$  we have that  $C(s) = C$ , which contradicts the hypothesis. Thus,  $C$  supports  $A$  at  $s$ .

We now consider the final case, when the osculating circle  $C(s)$  is the point circle  $s$ . The proof follows from the above.

#### Theorem 6.

If  $C(s) = s$ , then the non-osculating tangent circles at  $s$  either all support or all intersect  $A$  at  $s$ .

#### CHARACTERISTIC

We now introduce the important concept of the characteristic of a differentiable point. It is not our purpose to exhibit the reasoning behind the definition but to indicate how by using it we may classify differentiable points.

The characteristic  $(a_0, a_1, a_2, i)$  has the following properties

$$i = 1 \text{ or } 2$$

$$a_0 = 1 \text{ or } 2$$

$$a_1 = 1 \text{ or } 2$$

$$a_2 = 1, 2 \text{ or } \infty$$

$i = 1$  if  $C(s) \neq s$ ;  $i = 2$  if  $C(s) = s$ . The number  $a_0$  is even or odd as the non-tangent circles through  $s$  intersect or support;  $a_0 + a_1$  is even or odd as the non-osculating tangent circles support or intersect;  $a_0 + a_1 + a_2$  is even if  $C(s)$  supports, odd if  $C(s)$  intersects, while  $a_2 = \infty$  if  $C(s)$  neither supports nor intersects.

Theorem 5 places a restriction upon the characteristic in that if  $C(2) = s$ ,  $i = 1$  and  $a_0 + a_1$  is even. Moreover, the agreement that the point-circle always supports implies that  $a_0 + a_1 + a_2$  is even. With these restrictions in mind it is seen that there exists ten distinct types of differentiable points in the conformal plane (1, 3, 7).

#### ORDER

An arc  $A$  in the conformal plane is said to be of finite cyclic order if it has only a finite number of points in common with any circle. If some circle  $C$  meets  $A$   $n$  times and no circle meets  $A$  more than  $n$  times where  $n$  is a specific integer, then  $A$  is said to be of bounded cyclic order and  $n$  is called the (cyclic) order of  $A$ . If  $s$  is any point on  $A$ , the order of  $s$  is the minimum of the orders of all neighborhoods of  $s$  on  $A$ . In this note we shall deal only with arcs of order three. However, the next theorem, which will in part justify our choice for the characteristic, does not specify the order of the arc and yet does concern the order of a point on this arc.

#### Theorem 6.

Let  $s$  be a differentiable interior point of the arc  $A$ . Suppose that  $s$  has the characteristic  $(a_0, a_1, a_2; i)$ . Then the order of  $s$  is not less than  $a_0 + a_1 + a_2$ .

The proof of this theorem is too long to include here. We have listed it because we shall indicate how it can be sharpened when  $s$  is conformally elementary.

Suppose now that we have three mutually distinct points  $u, v, q$  such that  $u$  and  $v$  converge on an arc  $A$  to a point  $s$  and that

$$\lim C(u, v, q) = C$$

Then  $C$  is termed a general tangent circle of  $A$  at  $s$ . Moreover if  $q$  also converges to  $s$ , then  $C$  is a general osculating circle of  $A$  at  $s$ . We say that  $A$  is strongly differentiable at  $s$  if the following two conditions are met.

Condition I'. Let  $R \neq s, q \rightarrow R$ . If two points  $u$  and  $v$  converge on  $A$  to  $s$ , then  $C(u, v, q)$  always converges.

Condition II'.  $C(t, u, v)$  converges if the three mutually distinct points  $t, u, v$  converge on  $A$  to  $s$ .

It can be shown that strong differentiability implies ordinary differentiability and that  $C(s)$  is the one and only osculating circle, for if in I' we let  $Q = R$  and  $u = s$  we have that I' implies I and

$$\lim C(u, v, q) = C(\tau, R)$$

Condition II' and II are similarly related.

Elementary Points. We now have sufficient background for the introduction of conformally elementary points. We say that a point  $s$  of an arc  $A$  is conformally elementary if there exists a neighborhood of  $s$  decomposed by  $s$  into two neighborhoods of order three. It has been shown (4, 6, 7) that their closures are strongly differentiable at  $s$ . Theorem 6 can now be made more exact.

#### Theorem 6'

Let  $s$  be a differentiable conformally elementary point with characteristic  $(a_0, a_1, a_2; i)$ , then the cyclic order of  $s$  is precisely  $a_0 + a_1 + a_2$ .

It is, of course, much more satisfactory to deal with an equality rather than an inequality as in Theorem 6.

We have indicated that strong differentiability implies ordinary differentiability. The converse is not true. However, if the differentiable point is conformally elementary we have

#### Theorem 7

Suppose that  $s$  is a conformally elementary point-on the arc  $A$ .

Then

i)  $s$  satisfies Condition I<sup>1</sup> if and only if it satisfies Condition I and  $a_0 = 1$  (i.e. if the non tangent circles through  $s$  support),

ii)  $A$  is strongly differentiable at  $s$  if and only if it is differentiable at  $s$  and  $a_0 = a_1 = 1$ .

Besides providing a useful link between ordinary and strong differentiability conformally elementary points are used with much success in the study of vertices of closed curves in the conformal plane. Finally, they are instrumental in supplying a conformal proof of the Four Vertex Theorem. The study of conformally elementary points does not belong in an introduction to conformal differential geometry. These points are used to unify certain preliminary results and to give order and meaning to more extensive and more profitable studies. It has been this author's expressed purpose to provide the material necessary for their introduction and to indicate how some of this material is further related.

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Oklahoma State University

## GENERAL SOLUTION

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## TO THE LINEAR DIOPHANTINE EQUATION IN TWO VARIABLES<sup>1</sup>

By T. W. Shook  
Ohio Alpha

#### INTRODUCTION

It is well known that the Diophantine equation

$$ax + by = c \quad (1)$$

has integer solutions if and only if  $(a,b)/c$ , in which case the general solution is given by:

$$x = x_0 + \frac{b}{(a,b)} t$$

$$y = y_0 - \frac{a}{(a,b)} t \quad (t = 0, \pm 1, \pm 2, \dots) \quad (2)$$

where  $x_0$  and  $y_0$  are a solution to equation 1.

Normally, the initial solution is found by an algorithm which is often lengthly and cumbersome. This paper expresses  $x$  and  $y$  directly as a function of  $a$ ,  $b$ , and  $c$ , eliminating the algorithm altogether.

#### THE GENERAL SOLUTION

Suppose  $a$  and  $b$  are two integers with  $0 < a \leq b$ . We define a sequence of remainders recursively as follows:

$$\langle ab \rangle_0 = b$$

$$\langle ab \rangle_1 = a$$

$$\langle ab \rangle_{k+1} = \langle ab \rangle_{k-1} - \langle ab \rangle_k \cdot \left[ \begin{array}{c} \langle a,b \rangle_{k-1} \\ \langle a,b \rangle_k \end{array} \right] \quad (k = 1, \dots, \lfloor \frac{ab}{a} \rfloor - 1) \quad (3)$$

WHERE:

$$\lfloor \frac{ab}{a} \rfloor = \text{MAX } \{ k \mid \langle ab \rangle_k > 0 \} \quad (4)$$

THEN DEFINE:

$$\langle ab \rangle = b \cdot (a,b)^{-1} \sum_{j=0}^{\lfloor \frac{ab}{a} \rfloor - 1} (-1)^j \langle ab \rangle_j \langle ab \rangle_{j+1}$$

$$\langle ab \rangle = -a \cdot (a,b)^{-1} \sum_{j=1}^{\lfloor \frac{ab}{a} \rfloor} (-1)^j \langle ab \rangle_j \langle ab \rangle_{j+1} \quad (5)$$

The following theorem then gives the general solution:

THEOREM — Given the Diophantine Equation

$$ax + by = c \quad (1)$$

<sup>1</sup> Received by Editors October 11, 1961. Presented at the National Meeting of Pi Mu Epsilon, Stillwater, Oklahoma, August 29, 1961. This paper was judged best for undergraduates. Mr. Shook won a hundred dollars prize.

One may assume without loss of generality that  $0 < a \leq b$ . Then (1) is solvable if and only if  $\frac{[a,b]}{(a,b)} / c$ , in which case the general solution is:

$$\begin{aligned} x &= \frac{c}{(a,b)} \left\lfloor \frac{ab}{(a,b)} \right\rfloor + \frac{bt}{(a,b)} \\ y &= \frac{c}{(a,b)} \left\lfloor \frac{ab}{(a,b)} \right\rfloor - \frac{at}{(a,b)} \\ t &= 0, \pm 1, \pm 2, \dots \end{aligned} \quad (6)$$

**PROOF** — We need only show that  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$  and  $\left\lceil \frac{ab}{(a,b)} \right\rceil$  are integers satisfying

$$a \left\lfloor \frac{ab}{(a,b)} \right\rfloor + b \left\lceil \frac{ab}{(a,b)} \right\rceil = 1 \quad (7)$$

If one substitutes the values given in (5) for  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$  and  $\left\lceil \frac{ab}{(a,b)} \right\rceil$ , equation (7) becomes a trivial identity.

We prove that  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$  and  $\left\lceil \frac{ab}{(a,b)} \right\rceil$  are integers by induction on  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$ .

$\alpha$ ) When  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor = 1$ ,  $a \nmid b$  and  $(a,b) = a$ . The Formula (5) yields  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor = 1$  and  $\left\lceil \frac{ab}{(a,b)} \right\rceil = 0$ .

$\beta$ ) Suppose  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor = k > 1$  and let  $r = \left\lfloor \frac{ab}{(a,b)} \right\rfloor_2$ . Then  $\left\lfloor \frac{ra}{(a,b)} \right\rfloor = k-1$  so  $\left\lfloor \frac{ra}{(a,b)} \right\rfloor$  and  $\left\lceil \frac{ra}{(a,b)} \right\rceil$  will be integers by the induction hypothesis.

Also note that  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor_j = \left\lfloor \frac{ra}{(a,b)} \right\rfloor_{j+1}$  and  $(r,a) = (a,b)$ . Then

$$\begin{aligned} \left\lceil \frac{ab}{(a,b)} \right\rceil &= -a \left( \frac{k-1}{(a,b)} \sum_{j=1}^{k-1} \frac{(-1)^j}{\left\lfloor \frac{ab}{(a,b)} \right\rfloor_j \left\lfloor \frac{ab}{(a,b)} \right\rfloor_{j+1}} \right) \\ &= a \left( \frac{k-1}{(r,a)} \sum_{j=1}^{k-1} \frac{(-1)^j}{\left\lfloor \frac{ra}{(a,b)} \right\rfloor_j \left\lfloor \frac{ra}{(a,b)} \right\rfloor_{j+1}} \right) \left\lceil \frac{ra}{(a,b)} \right\rceil, \text{ an integer.} \end{aligned}$$

Solving Equation (7) for  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$ ,

$$\left\lfloor \frac{ab}{(a,b)} \right\rfloor = \frac{(a,b) - b \left\lceil \frac{ab}{(a,b)} \right\rceil}{a} = - \left[ \frac{b}{a} \right] \left\lceil \frac{ab}{(a,b)} \right\rceil + \frac{(a,b) - r \left\lceil \frac{ab}{(a,b)} \right\rceil}{a}$$

The right-hand term of the above expression is

$$\frac{(a,b) - r \left\lceil \frac{ab}{(a,b)} \right\rceil}{a} = \frac{(r,a) - r \left\lceil \frac{ra}{(a,b)} \right\rceil}{a} = \left\lceil \frac{ra}{(a,b)} \right\rceil, \text{ which is an integer.}$$

Hence  $\left\lfloor \frac{ab}{(a,b)} \right\rfloor$  and  $\left\lceil \frac{ab}{(a,b)} \right\rceil$  are integers. This completes the proof.

Ohio State University

## PROBLEM DEPARTMENT

Edited by  
M. S. KLAMKIN  
State University of New York  
at Buffalo

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate, signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Division of Interdisciplinary Studies and Research, State University of New York at Buffalo, Buffalo 14, New York.

### PROBLEMS FOR SOLUTION

144. Proposed by Hüseyin Demir, Kandilli, Eregli, Kdz., Turkey. Find the shape of a curve of length L lying in a vertical plane and having its end points fixed in the plane, such that when it revolves about a fixed vertical line in the plane, generates a volume which when filled with water shall be emptied in a minimum of time through an orifice of given area A at the bottom. (Note: The proposer has only obtained the differential equation of the curve.)
145. Proposed by David L. Silverman, Beverly Hills, California. For what integers a and b ( $0 < a < b$ ) are the roots of  $x^4 + (a+b)x^3 + (a+b+ab)x^2 + (a^2+b^2)x + ab = 0$ , integers?
146. Proposed by C. W. Trigg, Los Angeles City College. Find a set of three-digit numbers, each of which is a permutation of the same three digits, which when divided by the sum of the digits yields two pairs of alternate integers.
147. Proposed by Leo Moser, University of Alberta. Show that the maximum number of terms of different form in a polynomial of degree n in k variables is the same as the maximum number of terms of different form in a polynomial of degree k in n variables.
148. Proposed by M. S. Klamkin, State University of New York at Buffalo. If a convex polygon has three angles of  $60^\circ$ , show that it must be an equilateral triangle.

## SOLUTIONS

124. Proposed by M. S. Klamkin, State University of New York at Buffalo.

Prove the impossibility of constructing the center of a circle with a straightedge only, given a chord and its midpoint. (For the original version of this problem see the spring, 1961 issue, p. 183).

The proof is a slight variation of the one given to prove the impossibility of constructing the center of a given circle with straightedge only (see H. Rademacher and O. Toeplitz, *The Enjoyment of Mathematics*, Princeton University Press, Princeton, 1957, pp. 177–180). It is established there, that there exists a conical projection transforming the given circle into another circle such that the centers do not correspond to each other. Now rotate the given circle about an axis through its center and perpendicular to its plane such that the given chord becomes parallel to the line of intersection formed by the planes of the two circles. It now follows that the midpoint of the chord transforms into the midpoint of the transformed chord (since they are both parallel). The impossibility of the construction now follows by a "Reductio ad Absurdum" argument. We first assume that the center of the given circle with a given chord and its midpoint can be constructed by straightedge alone. The previous construction immediately leads to a contradiction. For, whatever the construction might be, it would consist in drawing a certain number of straight lines and finding their intersection with one another and with the given circle with its given chord and midpoint. Now if the whole figure, consisting of the given circle, chord and midpoint, together with all the points and lines of construction, is conically-projected by the above transformation, the transformed figure will satisfy all the requirements of the construction, but will produce a point other than the center of the transformed circle. Consequently, such a construction is impossible.

While almost all of the previous proof is given in Rademacher and Toeplitz (*ibid*) and also R. Courant and H. Robbins, *What is Mathematics*, Oxford University Press, New York, 1943, P. 152, it was repeated here for the sake of completeness.

132. Proposed by L. Carlitz, Duke University.

Let  $p$  be an odd prime. Find the number of solutions  $(x, y, z)$  of the congruence

$$(1) \quad xyz + a(x+y+z) \equiv 0 \pmod{p},$$

where

$$a \not\equiv 0 \pmod{p}.$$

Solution by the proposer.

The congruence can be written as

$$(2) \quad (xy + a)z \equiv -a(x+y) \pmod{p}.$$

If  $xy + a \not\equiv 0$ , then  $z$  is uniquely determined by (2); thus there are  $p^2-p+1$  solutions satisfying this condition. On the other hand, if  $xy + a \equiv 0$ , it is necessary that  $x + y \equiv 0$ , and  $z$  is arbitrary. Now the system

$$\begin{aligned} xy + a &\equiv 0, \\ x + y &\equiv 0, \end{aligned} \pmod{p},$$

reduces to  $x^2 \equiv a$ , which has at  $1 + (\frac{a}{p})$  solutions, where  $(\frac{a}{p})$  is the Legendre symbol. Hence, the total number of solutions is  $p^2 - p + 1 + p \{ 1 + (\frac{a}{p}) \}$ .

134. Proposed by R. F. Wheeling and R. H. G. Mitchell, Socony Mobil Oil Company.

If  $\phi_1(x)$  and  $\phi_2(x)$  are one-dimensional probability density functions (i.e.,  $\int_{-\infty}^{\infty} \phi_1(x) dx = 1$ ) and if there exists a number  $X_0$  such that

$$\begin{aligned} \phi_2(x) &\geq \phi_1(x) & \text{for } x > X_0, \\ \phi_2(x) &\leq \phi_1(x) & \text{for } x < X_0, \end{aligned}$$

then

$$\int_{-\infty}^{\infty} x \phi_2(x) dx \geq \int_{-\infty}^{\infty} x \phi_1(x) dx,$$

provided both integrals exist.

Solution by F. Zetto, Chicago, Illinois.

Since  $[x-X_0] [\phi_2(x) - \phi_1(x)] \geq 0$ , for  $\infty > x > -\infty$  it follows that

$$\int_{-\infty}^{\infty} [x-X_0] [\phi_2(x) - \phi_1(x)] dx \geq 0,$$

or that

$$\int_{-\infty}^{\infty} x [\phi_2(x) - \phi_1(x)] dx \geq 0.$$

Also solved by Paul Meyers, K. Smith, J. Thomas, M. Wagner and the proposers.

135. Proposed by T. E. Hull, University of British Columbia. Suppose that  $k$  points are placed uniformly around the circumference of a circle with unit radius. Show that the product of the distances from any one point to the others is equal to  $k$ , for any  $k > 1$ .

Solution by David L. Silverman, Beverly Hills, California.

Place the circle in the complex plane with center at  $(0, 0)$  and

one vertex at  $(1, 0)$ . The other vertices will then coincide with the  $k$ th roots of unity. If we denote them by  $\Gamma_1 = 1, \Gamma_2, \Gamma_3, \dots, \Gamma_k$  it follows that

$$z^k - 1 = \prod_{i=1}^k (z - \Gamma_i).$$

Consequently,

$$\prod_{i=1}^k (z - \Gamma_i) = 1 + z + z^2 + \dots + z^{k-1}$$

The desired result is obtained by letting  $z = 1$ . Also solved by John T. Bagwell, Jr., H. Kave, Paul Meyers, Ted Newton, K. Smith, G. Tarns, M. Wagner, F. Zetto and the proposer.

**Editorial Note:** The same problem was proposed by Robert P. Goldberg in the November, 1961, Mathematics Magazine and solved in the May-June, 1962 issue.

#### ERRATA

106. Proposed by M. S. Klamkin, State University of New York at Buffalo.

An equi-angular point of an oval is defined to be a point such that all chords through the point form equal angles with the oval at both points of intersection (on the same side of the chord). It is a known elementary theorem that if all the interior points of an oval are equi-angular, then the oval is a circle.

1. Show that if one boundary point of an oval is equi-angular, the oval is a circle.
2. Determine a class of non-circular ovals containing at least one equi-angular point
3. It is conjectured that a non-circular oval can have, at most, one equi-angular point.

It has been pointed out by Michael Goldberg, Washington, D. C., that there is an error in the proposer's solution to part 2 (Fall, 1961 issue). He also gives a geometrical solution for this part. The corrected version of part 2 should read:

2. Let the origin be the equi-angular point. Then we have to find  $r$  such that

$$r \frac{d\theta}{dr} \Big|_{\theta} + r \frac{d\theta}{dr} \Big|_{\theta+\pi} = 0, \text{ for all } t$$

One obvious solution is .

$$\ln r = \int_0^\infty F(\sin t) dt$$

where  $F$  is odd, i.e.,

$$r = ae^{bc\cos\theta}$$

## BOOK REVIEWS

Edited by

FRANZ E. HOHN, UNIVERSITY OF ILLINOIS

Undergraduate Research in Mathematics, Report of a conference held at Carleton College, Northfield, Minnesota, June 19 to 23, 1961, with support from the National Science Foundation, edited by Kenneth O. May and Seymour Schuster. Carleton Duplicating Service, Northfield, Minnesota, 1961.

Despite the fact that "undergraduate research in mathematics" is not well defined, this report should be of great interest to anyone concerned with the development of mathematical maturity in undergraduate students. Although some of the conferees wished to restrict the term "research" to research leading to original results in mathematics worthy of publication in at least the Monthly, and others wished to extend the word to cover routine term papers of the nature found in undergraduate courses in, say, English, this reviewer feels that despite the semantic disagreement two important contributions are made in this report.

First, there is a discussion of the various techniques used to stimulate undergraduate study. These techniques include, among others, the Moore Method, undergraduate theses, undergraduate seminars, term papers, honors programs and honors sections, competitions, clubs, and local undergraduate publications. There are several detailed reports describing the utilization of these methods.

Second, there is a discussion of the support available for undergraduate research and independent study in mathematics under the NSF's Undergraduate Science Education Program. Here, the discussion ranged from the availability of NSF funds to a summary of the programs now being held under the NSF grants. Mathematicians, it appeared, have not been participating as actively in this program as other scientists.

For the very few hours needed to read this report, the time will be well spent.

University of Illinois

Hiram Paley

Introduction to Probability and Statistics, Second Edition. By H. L. Alder and E. B. Roessler. San Francisco, W. H. Freeman, 1962. xii + 289 pp., \$5.50.

This second edition remains virtually unchanged from the first except for the addition of two new chapters on the **F-distribution** and analysis of variance. As before, the book is intended for use in a one-semester introductory course in statistics where calculus is not a prerequisite.

With the exception of brief discussions on index numbers and time series, the topics covered are the usual ones and, as in other books written on this level, the quality of mathematics suffers. Of course, this is partly due to the severe limitations of writing about statistics on this level. However, at least some of the **definitions** could be improved. For example, the first **definition** given is: "A measurable characteristic is called a variable." Later on, we find: "A measurable characteristic of a population, such as its mean or standard deviation, is called a population parameter or simply a parameter."

A distinguishing positive feature of the book is the authors' easy, fluid style of writing. Also, there are hundreds of exercises requiring only a small amount of computation, and the answers are given in the back of the book.

University of Illinois

Gus Haggstrom

A Second Course in Number Theory. By Harvey Cohn. New York, Wiley, 1962. xiii + 276 pp., \$8.00.

As the title of this book suggests, a reader needs as background a first course in number theory. But titles cannot reveal everything, and so it should be pointed out that a reader also needs some understanding of abstract algebra, particularly a little group theory. The author does give a review of elementary number theory and group theory, but this might not be easy reading without some prior familiarity with these topics.

Following this review material, Professor Cohn launches forth on his main topic, ideal theory in quadratic fields, including discussions of units, the algebra of ideals, unique factorization with ideals, norms, and class numbers. The third and final part of the book comprises the application of this theory of ideals to three topics: a proof of the Dirichlet theorem on the infinitude of primes in an arithmetic progression, quadratic reciprocity, and quadratic forms. There is a good bibliography and several numerical tables.

The writing has a historical orientation which is rare outside of books on the history and culture of mathematics. To illustrate this we cite two examples out of many. In the preface it is remarked that "A student completing this course should acquire an appreciation for the historical origins of linear algebra, for the zeta-function tradition, for ideal class structure, and for genus theory." On page 241 it is observed that "At this juncture number theory was strongly influenced by Riemann's theory of functions. . . .

This book fills a gap in the literature very neatly. There is, in fact, no book (certainly not in English) covering these particular topics, although of course there are a few that overlap in minor ways. The author is to be congratulated on opening up a large area of mathematics to a wider class of readers. Graduate students will find that this book is one of those helpful works that serve as bridges between the more elementary books and the articles in the journals.

University of Oregon

Ivan Niven

Discrete Variable Methods in Ordinary Differential Equations. By Peter Henrici. New York, Wiley, 1962. xi + 407 pp., \$11.50.

Professor Henrici has written a book which will be of vital interest to all students of numerical analysis. Although this book is concerned with the numerical solution of ordinary differential equations, the underlying concepts are of importance in all areas of numerical analysis. This text is the first book on the numerical solution of o.d.e. which was written with high speed digital computers in mind.

The book is divided into three parts which are as follows:

- Part 1. One-step methods for initial value problems.
- Part 2. Multistep methods for initial value problems.
- Part 3. Boundary value problems.

In each part, there is a buildup of the theoretical apparatus with precise definitions followed by applications. There is a considerable discussion of roundoff error in terms of random variables, and the numerical examples show the appropriateness of the stochastic model. A good set of problems is given at the end of each chapter.

This book represents a standard of rigor and clarity which all future text books in numerical analysis should aim for.

Stanford University

Gene H. Golub

Theory of Numbers, Second Edition. By G. B. Mathews. New York, Chelsea, 1961. xii + 323 pp., \$3.50 clothbound.

Conic Sections. By G. Salmon. New York, Chelsea, 1962. xv + 399 pp., \$3.50 clothbound, \$1.95 paperbound.

Projective Methods in Plane Analytical Geometry. By C. A. Scott. New York, Chelsea, 1961. xii + 290 pp., \$3.50 clothbound.

The Logic of Chance. By J. Venn. New York, Chelsea, 1962. xxix + 508 pp., \$4.95 clothbound, \$2.25 paperbound.

These four books are reprints of classical works, originally published in the last century, that were outstanding in their day. They are famous for clarity, completeness, and charm and belong in every reference library.

University Calculus with Analytic Geometry. By C. B. Morrey, Jr. Reading, Mass., Addison-Wesley, 1962. xiv + 754 pp., \$12.50.

Like many others of the modern texts published recently by Addison Wesley, this is another excellent book.

It is designed for a sequence of courses, totalling 12 semester hours, for students who are well prepared in algebra and trigonometry but who do not have any background in analytic geometry. The approach and the language of the book are definitely modern. Besides the usual topics discussed in most calculus books, the author devotes a substantial amount of space to analytic geometry and to the theory of vectors in the plane and in space. Theory is emphasized from the beginning, even to the point of requiring the student to prove many of the theorems as exercises.

An early chapter gives an introduction to the fundamental concepts of limit, differentiation, and integration. The discussion is informal at the start, but rigorous proofs are given of all essential theorems. Union, intersection, and difference of two sets and also interior points as well as limit points are defined and made use of in Chapter 8 on the definite integral. The differential is discussed carefully and extensively.

The reviewer recommends this book as a text where a combination geometry-calculus book course is in order.

University of Maryland

Dagmar Henney

Stochastic Service Systems. By John Riordan. New York, Wiley, 1962. + 139 pp., \$6.75.

This book, a member of the SIAM Series in Applied Mathematics, is a well written introduction to the probability theory of queueing and traffic systems, and of certain variants of these which arise especially in telephone problems. The mathematical level is comparable to that of Feller's book, and a student who has had an introduction to probability theory from such a book should find little difficulty in reading Riordan's book. Many interesting topics which are not treated in standard references on queueing theory are contained in the book; for example, nonstandard queueing disciplines such as "last-come, first-served", systems with defections or balking, busy periods, etc. There are also historical remarks and references which are quite up-to-date.

Cornell University

J. Kiefer

Numerical Analysis, with Emphasis on the Application of Numerical Techniques to Problems of Infinitesimal Calculus in Single Variable, Second Edition. By Zdenek Kopal. New York, Wiley, 1962. xvi + 594 pp., \$12.00.

The author has written a highly personal book which strongly reflects his own philosophy and interests. The numerical techniques described are mainly finite difference methods for approximating continuous problems. The first chapter gives a short history of number systems and numerical analysis. Chapters II through VII develop the classical subjects of numerical analysis - polynomial interpolation, numerical differentiation, integration of o.d.e.s, boundary value problems, and mechanical quadrature. Chapter VIII describes methods for solving integral and integro-differential equations and Chapter IX is devoted to operational methods in numerical analysis which include polynomial and rational approximations. The book is concluded by several appendices which contain several useful tables.

Each chapter begins with motivational material. There are extensive bibliographical notes and problems at the conclusion of each chapter. Throughout the text, illustrative numerical examples are given.

Although this is a very readable and interesting book, it will find its greatest use as a reference for the practicing numerical analyst.

Stanford University

Gene H. Golub

Infinite Series. By I. I. Hirschman, Jr. New York, Holt, Rinehart, and Winston, 1962. x + 173 pp., \$4.00.

This book is another welcome addition to the Athena Series of Holt, Rinehart and Winston. As other members of the series, it has an attractive format.

As an indication of the coverage, a listing of the chapter headings and the corresponding lengths in pages is as follows:

- Chapter 1. Tests for Convergence and Divergence (35 pages).
- 2. Taylor Series (20 pages).
- 3. Fourier Series (25 pages).
- 4. Uniform Convergence (21 pages).
- 5. Rearrangements, Double Series, Summability (28 pages).
- 6. Power Series and Real Analytic Functions (18 pages).
- 7. Additional Topics in Fourier Series (14 pages).

#### Appendix

Set and Sequence Operations, Continuous Functions (6 pages)

Although the treatment is rigorous, emphasis has been placed upon the applications of the theory developed rather than upon the theory itself. Consequently, there is more material than usual on problems and applications, which, according to this reviewer, definitely enhances the book. The pace set by the author is rather leisurely, which should make the book attractive to those students studying on their own.

It should be noted that in problem 9, p. 15, it has been tacitly assumed that  $\lambda$  is a constant.

Otherwise,  $\sum_{n=1}^{\infty} n^{-\lambda}$  can diverge even if  $\lambda > 1$  (i.e.  $\lambda = 1 + 1/n$ ).

Similarly for problem 11.  
AVCO Corp.

M. S. Klamkin

Selected Topics in the Classical Theory of Functions of a Complex Variable. By M. H. Heins. New York, Holt, Rinehart and Winston, 1962. xi + 160 pp., \$3.50.

The advent of the present monograph comes as a rare pleasure to the reviewer, as it must to other mathematicians who enjoy classical analysis and its recent developments. For many reasons this is a most interesting and stimulating book.

To begin with, the selection of topics has been done with considerable care. Among the contents are to be found the Cauchy-Goursat theorem, the Mittag-Leffler and Weierstrass theorems, the Riemann mapping theorem, the Bloch and Schottky theorems, the big Picard theorem, the theory of harmonic and subharmonic functions, the Dirichlet problem, the classification of regions (as hyperbolic and parabolic), the theorems of Iverson and Milloux-Schmidt, the Phragmén-Lindöf theorem, Wiman's theorem, Carleman's method, results on the boundary behavior of conformal mappings, and the strong form of the Cauchy integral theorem and formula. In a number of these areas the author has made significant research contributions, and this is not without influence on the development of the text.

A few of the topics discussed may already have been encountered by the student in a first course in function theory. If so, he will find it fruitful to renew his acquaintance with them here; the treatment is almost certain to be different in some respect from that previously seen. This difference is not merely for its own sake, however, since there is always a point to the author's departure from the traditional. For example, his use of Fourier series methods in deriving basic properties of analytic functions is highly illuminating, as is his presentation of the Caratheodory-Koebe proof of the Riemann mapping theorem.

The many topics that are new to the student will greatly enrich his knowledge of function theory and provide a base for attacking the current literature. Indeed, the latter facet of the book is perhaps its strongest. With due acknowledgement of Pólya and Szegő's celebrated *Aufgaben und Lehrsätze*, the author adopts a style in which the problems often form an integral part of the development. The student should therefore work all of them, as well as the informal problems which arise as details in the text itself. This poses a serious challenge, but the student who accepts it and works his way through the book's 155 pages will be amply rewarded in ability to cope with the research literature.

Alternatively, the book lends itself well to use as a source of advanced topics for inclusion toward the end of a first course in complex function theory. An attempt has been made to keep it reasonably self contained, even to the extent of inserting material from real function theory in the appendices, and many of the topics are treated independently of one another.

To carry out a program of the above sort is no mean task. The author has done it skillfully, particularly with regard to the organization of material and the difficult choice of what should be made explicit and what left to the reader. Not the least of the author's skills, however, is his command of a prose style which virtually puts the work in a class by itself. Pleasantly informal, the writing is filled with interesting observations, side remarks, and references, so that one almost has the feeling of being present at a well polished lecture. In all likelihood this book is destined to become a classic in its field.

University of Washington

Maynard G. Arsove

Geometrical Constructions Using Compasses Only. By A. N. **Kostovskii**. New York, **Blaidsell**, 1961. xi + 79 pp., \$0.95. (Translated from the Russian by **Halina Moss**).

The Ruler in Geometrical Constructions. By A. S. Smogorzhevskii. New York, **Blaidsell**, 1961, viii + 86 pp., \$0.95. (Translated from the Russian by **Halina Moss**).

These two booklets, presupposing hardly any more geometry than that to which an American high school student is generally exposed, form a readable introduction to the problems implied by the titles. The authors restrict themselves to the synthetic approach almost exclusively. They develop with a minimum of axiomatic formalization the additional ideas of inversive and projective geometry needed and which are not generally part of the beginning students' background.

These books should make excellent sources of additional material for high school students and will be of interest to anyone who has ever toyed with ruler and compass.

A highlight of the "Ruler" book is a proof of the impossibility of constructing with the ruler alone the centers of two given nonconcentric circles unless they intersect. However, beyond mentioning them, the three classical impossible construction problems are not discussed.

The translation is well done. Only two misprints were detected; these are too minor to point out.

University of Nebraska

Edwin Hafar

Handbook of Automation and Control, Volume 3. Edited by M. **Grabbe, S.** Ramo, and D. E. **Woolridge**. New York, **Wiley**, 1961, xxi + 1153 pp., \$19.75.

Preceding volumes of this Handbook were devoted to the fundamental theories of Logical Design and Information Theory (Vol. 1) and to Programming and Design of Computers (Vol. 2). The latest (and final) volume is called "Systems and Components"; a better title might have been "Process Control". The reviewer feels, as for the first two volumes, that overlap is considerable—compare Ch. 27 of Vol. 3 to Ch. 16 of Vol. 2—and that the depth of thought is highly variable—as witnessed by the complete irrelevance of Ch. 2 ("The Human Component") and the very fine treatment of "Transmission Systems" in Ch. 15.

It seems, however, that in overall quality this new volume is superior to the predecessors: The sections on "Manufacturing Process Control," "Chemical Process Control" and "Industrial Control Systems" (including Nuclear Reactors) are, in general, quite usable as a first introduction to the fields: This contrasts strongly with the approach of Vol. 2 which presupposes often a good deal of preliminary knowledge. Incidentally, Chapter 26 on "Semiconductor Devices" and Chapter 27 on "Transistor Circuits" (both of these are in the Component Section) are examples of outstanding clarity and show handbook writing at its best.

In summary: The reviewer wished that about 50% of the chapters were rewritten to reach the high level of competence of the other half. This criticism, however, can be leveled at almost any compendium. One should congratulate the editors for having achieved an entirely useful encyclopedia of the automation and control field. The density of information on the more than 3,000 pages is definitely high and, this is a very positive point, there seem to be practically no errors.

University of Illinois

W. J. Poppelbaum

Handbook of Statistical Tables. By D. B. Owen. Reading, Mass., **Addison-Wesley**, 1962. xii + 580 pp., \$12.50

This is an unusually complete collection of 113 tables of functions used in statistics, many of them more extensively tabulated than ever before. The volume will be much appreciated by both the advanced student and the practicing statistician.

An Introduction to Probability and Statistics. By Howard G. Tucker. New York, Academic Press, 1962. xii + 228 pp., \$5.75.

Here is a bold new textbook on probability and statistics for undergraduate mathematics majors that differs radically in scope, depth, and presentation from others now on the market. For an introductory text, the author has given us a sound mathematics book which is concise, well written, and relatively free of errors.

In the preface the author draws attention to these features of the book:

- “1. Random variables are treated as measurable functions.
- 2. Sampling is treated in terms of product spaces.
- 3. Distributions are derived by the transformation method.
- 4. Probability is given an axiomatic treatment.
- 5. A chapter on the matrix theory needed is inserted in the middle of the book.
- 6. The Neyman theory of confidence intervals is given a systematic treatment.
- 7. A more natural definition of the multivariate normal distribution is given.
- 8. Expectation is given a unified treatment; the expectation of a random variable X is defined to be

$$\int_{-\infty}^{\infty} P[x > x] dx - \int_{-\infty}^{\infty} P[x \leq x] dx,$$

provided that both of these improper Riemann integrals are finite. Formulas and properties in the discrete and absolutely continuous case are then derived from this definition."

A further indication of the content of the book is that among the theorems proved are a version of the Law of Large Numbers, the Neyman-Pearson Fundamental Lemma, the Cramer-Rao Inequality, and Cochran's Theorem.

Compared to other books aimed at the same level, relatively little space is devoted to motivational material and examples. The exercises are commonly used for filling in steps in proofs and for proving corollaries, but there are still many good exercises on applications. The answers are not given.

It is unfortunate that this excellent text may be at too high a level to be very useful in the classroom. Many teachers will find that a course based on this text will overshoot the maturity of their students, particularly if many of them come directly from calculus. More motivational material and examples would have been helpful without compromising the author's aim to write a sound introductory statistics book.

University of Illinois

Gus Haggstrom

**Stability** by Liapunov's Direct Method, with Applications. By Joseph LaSalle and Solomon Lefschetz. New York, Academic Press, 1961. vii + 134 pp., \$5.50.

This monograph, the fourth in a series edited by Richard Bellman, represents (according to the cover) the first "detailed and elementary account in English of Liapunov's direct (second) method." The book presents no new material but does give a concise account of Liapunov stability theory as it appears in the literature of the past two years. Indeed, in the short space of 134 pages the reader is at least introduced to some of the modern concepts of stability analysis in a rather painless and pleasing way.

In Chapter One, essential material is given for application in later chapters. Ideas are rapidly and lucidly presented in a manner which gives the student an opportunity to learn yet does not insult the more mature reader. By inclusion of this chapter, the book is rendered essentially self contained.

Chapter Two deals with differential equations, immediately and conclusively put in the normal form. The reader is guided quickly through the ideas of trajectories in phase space, critical points and eigenvalues, etc. A great deal of notation is brought out in this chapter and it appears that one entirely unfamiliar with the terminology might easily lose his way among the definitions. However, the main purpose of the chapter is to lay a solid mathematical foundation for the stability theory which is the subject of the book. Having previously given definitions and elementary properties of vector spaces and coordinate transformations, the authors now begin a careful, clear development of stability notions applied to the vector set of differential equations in normal form. Beginning with autonomous systems, Liapunov functions are defined and Liapunov's stability theorems are given with proofs either stated or outlined. Several types of stability are described and examples are given to explain the more subtle aspects of the theory. More general and practically important topics complete this chapter: Stability and the Theorems of Liapunov for Nonautonomous Systems, Converse of the Theorems of Liapunov, the Extent of Asymptotic Stability, Stability under Persistent Disturbances. Chapter Two contains the bulk of the theoretical development of Liapunov's work, condensed into fifty pages. This is, indeed, the main contribution of the book.

Chapter Three is devoted to the study of stability theory applied to control mechanisms. Several typical control problems are worked out in detail and comments are made concerning more desirable ways to approach control problems. It might be considered that this chapter introduces a philosophy of control from the viewpoint of the mathematician. Such a discussion is important to the student learning to apply notions of stability to practical control problems and is welcomed by the systems **analyst** as an analytic approach to the quantitative study of non-linear control systems. The technical content of Chapter Three is not new to the scholar working the control field; however, it is a good concise source of pertinent information for reference.

Chapter Four is a collection of assorted topics in stability and is of interest primarily to the specialist.

In total, the book leaves a good impression. It might be criticized in that there is little discussion of the philosophy underlying the selection of Liapunov functions. Many of the coordinate transformations seem to have insufficient motivation at the outset. The logic behind their selection is apparent only at later stages of the solution. Still, it must be admitted that most people attack stability problems this way and little is really known about the selection of Liapunov functions. This reviewer must emphasize, however, that several significant contributions have been made along these lines since publication of the book. Thus the text does not contain the current advances concerning selection of Liapunov functions. The unconventional

notation used in numbering certain equations was confusing at first to this reviewer. In Chapter Three some simple block diagrams of the systems under discussion would have added to the clarity of the presentation. These minor criticisms are, however, offset by the clear style of discourse and appropriate geometrical diagrams which help give an insight into some of the subtle differences between the several types of stability described.

The book is well suited for use as a reference text in a course on stability or for self study. The authors have written a monograph which should be well received as a clear, concise introduction to an intensely important subject.

Stephen J. Kahne  
University of Illinois.

Lie Algebras. By Nathan Jacobson. New York, Wiley-Interscience, 1962. ix + 331 pp., \$10.50.

This book gives a very concise, closely knit presentation of the theory of Lie algebras. The objective is to provide a thorough background in the most general context in recognition of the growing relevance of the theory in a number of different fields, such as Lie groups, algebraic groups, and free groups. In particular, the restrictions on the base field are minimal and are introduced only when necessary. For example, the classical classification of semi-simple Lie algebras is extended to split semi-simple Lie algebras, namely, those which have a **Cartan** subalgebra H such that for every  $h \in H$ ,  $ad h$  has its characteristic roots in the base field. This is a fairly minor point, but it illustrates the spirit in which the author approaches the subject.

The prerequisites, as stated by the author, are a thorough foundation in linear algebra for the first nine chapters, plus some knowledge of Galois theory and the structure theory of associative algebras for the last chapter. This already puts the book out of reach of the average first year graduate student, to say nothing of that intangible, mathematical maturity, clearly necessary for negotiating such a compact work.

Briefly, the first four chapters are concerned with the structure of Lie algebras and the classification of the semi-simple ones. **Levi's** decomposition theorem and an introduction to the **cohomology** theory of Lie algebras are included, and **Dynkin's** method is followed in the classification. The next chapters are concerned with representations of Lie algebras. The universal enveloping algebra is introduced and used to reduce the problem to that of representations of associative algebras, the theorem of Ado-Iwasawa is proved, **Cartan's** classification of irreducible representations of a semi-simple Lie algebra is given, via modules, and **Weyl's** formula for the character of such a representation is derived. The ninth chapter studies the **automorphism** group of a Lie algebra, while in the last chapter the classification of simple Lie algebras over arbitrary fields of characteristic 0 is investigated. At the end of each chapter there is a section of stimulating, **non-trivial** problems.

Much of this material is obtainable elsewhere, ultimately in the **works** of E. **Cartan**, more readably in the volumes of Chevalley on Lie Groups, the Paris "**Sophus Lie**" seminar notes, **Bourbaki**, and the notes of H. **Freudenthal** on Lie groups. However, the completeness and generality of the present exposition make it a valuable addition to the literature both as a reference and as a text for an advanced graduate-course. Its appearance is sure to be welcomed by many experts in related fields.

Northwestern University

R. J. Crittenden

**The USSR Olympiad Problem Book.** By D. O. Shklarsky, N. M Chentzov, and I. M Yaglom. San Francisco, W. H. Freeman, 1962. xvi + 452 pp., \$9.00. (Revised, edited, and translated by I. Sussman and J. Maykovick.)

**The Contest Problem Book.** By Charles T. Salkind. New York, Random House, 1961. vi + 154 pp., \$1.95.

The first of these two books is a collection of unconventional problems of varying levels of difficulty, many new, some familiar, and a few very well-known. The problems come from arithmetic, elementary number theory, analytical trigonometry, and algebra. Very little knowledge beyond that taught in college algebra and trigonometry is required for their solution. (When unusual knowledge is required, it is provided in the text.) On the other hand, many of them require a high degree of computational skill and insight.

The problems are 320 in number and are arranged according to subject matter. Their statement occupies the first 79 pages of the book. Then follow 343 pages of solutions and, for those who may give up less easily, 30 pages of hints and answers.

These problems are taken from the Olympiad contest examinations given to Russian students from the seventh to tenth grades, which correspond in academic attainment to our ninth to twelfth grades. However, the problems are interesting enough and difficult enough to be useful and challenging to college students in this country.

This volume is an attractive and valuable contribution to the mathematical literature.

The second of these collections is a compilation of the problems from the annual high school contests sponsored by the Mathematical Association of America. Solutions to the problems are given, as is an index which classifies the problems according to subject. The examinations are those for the years 1950 to 1960 inclusive.

These problems are easier than their Russian counterparts. They are also much more varied in nature and scope. (Geometry, for example, is well represented here.) As a result, this book will be more useful in high schools in this country than the first volume will. Indeed, it should be made available to every able mathematics student as a source of diversion, inspiration, and training in original thinking.

University of Illinois

Franz E. Hohn

**Concepts of Tensor Analysis and Differential Geometry.** By Tracy Y. Thomas. New York, Academic Press, 1961. vii + 119 pp., \$5.00.

This is the first volume in a series entitled "Mathematics in Science and Engineering" which is designed to present the theory and application of recent scientific and mathematical developments. The author of this volume gives an introductory account of the subject described in his title. He succeeds in including a great deal of the standard material in his twenty-five chapters. The brevity of his treatment will appeal to those students of applied mathematics who would like to become familiar with the formal aspects of the tensor calculus, and to many others who want only a bird's-eye view of the subject.

University of Illinois

Harry Levy

**Lectures in Projective Geometry.** By A. Seidenberg. Princeton, Van Nostrand, 1962. x + 230 pp., \$6.50.

This is an extremely well-written introduction to the subject, intended as a text for a two-semester course at the junior or senior level in college. The author introduces the main topics of projective geometry on an intuitive basis "as an extension of high-school geometry", then proceeds axiomatically. The book is completely self-contained, even to the extent of introducing such topics as determinants — the stated prerequisites are two years of high-school mathematics. This greatly increases the accessibility of the material treated: the book could be recommended to the graduate student who has had no time and/or opportunity to take a course in geometry, or equally well to the high-school teacher wishing to enrich his background by independent study. Its value would seem greatest as a course text for the advanced undergraduate; even if one foresees no extensive study of geometry in his future, this is an excellent place to acquire familiarity both with the axiomatic method and with many of the basic concepts of abstract algebra — but a person using the text for the latter purpose should be cautioned that Professor Seidenberg's treatment might well entice him into a full-time study of geometry.

University of Chicago

D. A. Moran

**Russian Reader in Pure and Applied Mathematics.** By P. H. Nidditch. New York, Wiley, 1962. x + 166 pp., \$2.25.

This little book contains one hundred brief readings from many different areas of pure and applied mathematics. The variety is excellent. Interlinear translations of all passages are provided. Extensive notes explaining all grammatical peculiarities are given. The book will be useful to all who are learning to read mathematical Russian. Indeed, it fills a long-felt need for a collection of specifically mathematical readings. Previous collections of scientific passages have only rarely included mathematical material.

University of Illinois

Franz E. Hohn

**The Method of Mathematical Induction.** By I. S. Sominskii. New York, Blaisdell, 1961. vii + 57 pp., \$0.95.

This little booklet begins by outlining the method of mathematical induction and pointing out the equivalence of this principle and the fact that any set of positive integers contains a smallest number. Examples are given in which induction fails because one of two essential components of a proof by induction is omitted or bungled.

There follow 52 theorems appropriate for proof by induction. Some are proved. These are chosen to illustrate a variety of techniques. Others are left as exercises. A brief chapter then gives proofs of some elementary theorems from algebra, and the final chapter gives solutions to the exercises some of which are particularly simple and some of which are challenging. Most of the examples and exercises deal with algebraic identities. Some familiar inequalities are also included. Very few geometrical theorems appear. No determinantal identities are given.

The book is easy to read. Since so little background is presumed, it is suitable for good high school students. They should find in it much that is of value but they may wish the style were a little livelier.

University of Illinois

Franz E. Hohn

Mathematical Statistics. By S. S. Wilks, New York, Wiley, 1962. xvi + 644 pp., \$15.00.

Here is a book that may be fairly said to represent the solid core of modern statistical theory, carefully and systematically presented. The student who has mastered this volume will have a very thorough and balanced grasp of the present state of the subject, and will be in a good position to begin to work at the growing edge of this large area of mathematics.

The task of preparing an up-to-date account of any active field of mathematics is like trying to board a moving train. This book had its beginning in a lithographed text under the same title published by Princeton University Press in 1943. In the ensuing 19 years the subject has advanced so rapidly that one may say that the character of the book has radically changed since the lithographed version. A few topics have receded into the background, or even disappeared. For example, in the original version there was a section devoted to the Pearson system of distribution functions, and another to the chi-square test of goodness of fit; in the present text the first of these topics is not mentioned, and the second appears only in two exercises. Chapter IX of the early version was devoted to analysis of variance, treated from the regression standpoint; in the present version the term "analysis of variance" does not appear in the table of contents, although it is adequately treated in the text.

These changes are, of course, symptomatic of the extensive changes in mathematical statistics over the past several decades. Even more interesting from this standpoint are the topics appearing in Professor Wilks' new book that did not appear in the earlier one. Chapter 1, entitled "Preliminaries", contains a rather detailed account of the necessary set theory (including sequences of sets), Borel fields, probability measures, and probability spaces. These matters, if mentioned at all, received only passing mention in the earlier book. Chapter 16, on statistical decision functions, deals with a topic which, in 1943, was little more than a gleam in the eye of John von Neumann. The chapter presents a rather brief but entirely adequate account of certain central ideas of decision theory. Sequential statistical analysis, dealt with in Chapter 13, has likewise made its debut since 1943; in fact, it may be regarded as a forerunner of decision theory. Another new topic is that of time series, to which Chapter 17 is devoted.

According to the preface, "the purpose of this book is to introduce mathematical statistics to readers with good undergraduate backgrounds in mathematics." Only a very few undergraduates, however, would be sufficiently grounded in mathematics and in statistics to undertake to read this book at the beginning of their graduate work. It presumes a sophistication in set theory and related matters which is likely to be gained only in a rather solid course in real variables. The reader without considerable background in matrix theory would be in difficulties at some points, but this is more easily remedied. Finally, it must be said that this book needs to be supplemented by a good deal of experience with actual data. In practical terms, for almost all students this means that one or two preliminary courses in statistics should precede the study of this book. In this connection, a rather minor criticism is that although the term "experimental design" is used a number of times, it is never defined, as far as the reviewer could determine.

In summary, this is a book which will retain its usefulness for many years, and which is likely to become one of the permanent classics in the field of mathematical statistics.

Earlham College

Howard W. Alexander

Mathematical Programming. By S. Vajda. Reading, Mass., Addison-Wesley, 1961. ix + 310 pp., \$8.50.

Suppose that  $N$  horses run in a race and that a gambler will receive a dollars for each dollar bet if the  $i^{\text{th}}$  horse wins and will lose the amount of the bet otherwise. How should the gambler divide his stake among the horses so that his smallest possible stake after the race is as large as possible?

To see the nature of this problem we may assume that the gambler's stake is unity and that he allocates the amount  $x_i$  to the  $i^{\text{th}}$  horse. Then these variables must satisfy the relations

$$x_1 + x_2 + \dots + x_N = 1$$

$$x_i \geq 0, \quad i = 1, 2, \dots, N.$$

Furthermore, if the  $i^{\text{th}}$  horse wins, the gambler's stake will become

$$a_i x_i + (1-x_i) = (a_i + 1) x_i - 1.$$

If we let

$$\min_i [(a_i + 1) x_i - 1] = v,$$

then our task is to choose the variables  $x_i$  so as to maximize the value of  $v$ . We may restate the relations and problem to read:

Choose the non-negative variables  $x_i$  subject to the conditions

$$(a_i + 1)x_i - 1 \geq v, \quad i = 1, 2, \dots, N,$$

$$\sum_{i=1}^N x_i = 1,$$

so as to permit the largest possible choice of the value  $v$ .

We have just stated a linear programming problem, the solution of which may be found in the book under review. The essential ingredients are that we consider a physical situation demanding an optimization, try to formulate the problem mathematically as the maximization of a linear form whose variables are subject to linear inequality constraints, and then solve the resulting equations computationally. It has been found in recent years that many problems in operations research, game theory, and engineering are of this type. The computational problem is enormous, since we may have to deal with hundreds of inequalities involving thousands of constraints.

The first four chapters of this book are preparatory in nature. Chapter five through eight form a satisfactory introduction to this field and are readily accessible to upper division and graduate students. Interesting sets of problems and answers are available so that the book is suitable for independent study. The remaining chapters, devoted to quadratic programming, stochastic linear programming, and dynamic programming, are not written in the lucid expository style for which the author is well known and may safely be avoided. A useful bibliography is included.

This is a useful introduction to a field that is rapidly growing in importance. R. Kalaba

*Higher Algebra for the Undergraduate, Second Edition.* By M. Weiss. Revised by R. Dubisch. New York, Wiley, 1962. ix + 177 pp., \$4.95.

This book may well be the answer to that difficult problem of changing from "cookbook" problem solving algebra to the axiomatic modern algebra of the present day. Usually, the transition is a very painful one for students and teachers alike. The students simply don't see the need for axioms, and for proving things which seem obviously true. They find modern algebra dull, and very troublesome, and even if they don't give up, they frequently end up with far from a love for the subject. This text should alleviate many of these difficulties.

The original edition was well received, and the present one, retaining the same spirit, will be widely used. Important material on linear algebra has been added, additional exercises included, and the format generally improved.

After introducing the real and complex number systems, the authors discuss the elementary theory of groups, rings, integral domains, and fields, pleasantly guiding the student through these abstract concepts. Next follow the subjects of polynomials, matrices, systems of equations and determinants. The last chapter is devoted to the important topic of homomorphisms. Notice how the best (and most difficult) is saved to the last.

The reviewer feels that a careful study of this text should provide most of the material and much of the maturity needed for a serious graduate course in modern abstract algebra.

San Jose State College

T. J. Cullen

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- \*T. Y. Thomas: *Concepts from Tensor Analysis and Differential Geometry.* New York, Academic Press, 1961. vii + 119 pp., \$5.00.
- T. Y. Thomas: *Plastic Flow and Fracture in Solids.* New York, Academic Press, 1961. ix + 267 pp., \$8.50.
- J. Todd: *A Survey of Numerical Analysis.* New York, McGraw-Hill, 1962. xvi + 589 pp., \$12.50.
- \*H. G. Tucker: *An Introduction to Probability and Mathematical Statistics.* New York, Academic Press, 1962. ix + 228 pp., \$5.75.
- \*J. Venn: *Logic of Chance (Reprint).* New York, Chelsea, 1962. xxix + 508 pp., \$4.95 clothbound, \$2.25 paperbound.
- \*M. Weiss and R. Dubisch: *Higher Algebra for the Undergraduate, Second Edition.* New York, Wiley, 1962. ix + 171 pp., \$4.95.
- H. S. Wilf: *Mathematics for the Physical Sciences.* New York, Wiley, 1962. xii + 284 pp., \$7.95.
- \*S. S. Wilks: *Mathematical Statistics.* New York, Wiley, 1962. xvi + 644 pp., \$15.00.

\*See review, this issue.

NOTE: All correspondence concerning reviews and all books for review should be sent to PROFESSOR FRANZ E. HOHN, 374 ALTGELD HALL, UNIVERSITY OF ILLINOIS, URBANA, ILLINOIS.

#### NOTICE TO INITIATES

On initiation into Pi Mu Epsilon Fraternity, you are entitled to two copies of the Journal. It is your responsibility to keep the business office informed of your correct address, at which delivery will be assured. When you change address, please advise the business office of the Journal.



This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., we are publishing editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

The National Aeronautics and Space Administration was established in 1958 to conduct research into space problems of flight and vehicles, conduct activities for space exploration, and to provide the widest appropriate dissemination of information on these activities. We are fortunate to have an article by Dr. Mattison L. Story from the Educational Services Branch of NASA in this issue. Certainly mathematics in this respect helps make the satellites go round.

The Aeronautical Charting and Information Center is responsible for providing the Air Force with aeronautical charts, flight information, terrain models, maps, intelligence on air facilities, and related cartographic services as well as the research necessary to carry out these objectives. Mr. J. Donald Define, mathematician in the Geophysical Studies Section of the **Geo-Sciences** Branch of ACIC, has participated in this program to increase interest in mathematics, science, and research with a very interesting article, published in this issue.

The following lists contributing corporations with the issue in which their editorials appeared.

Aeronautical Chart and Information Center	Vol. 3, No. 7
Aeronutronics	Vol. 3, No. 2
Army Ballistic Missile Agency	✓ Vol. 2, No. <b>10</b>
AVCO, Research and Advanced Development	Vol. 2, No. <b>10</b>
Bell Telephone Laboratories	Vol. 2, No. <b>10</b>
Bendix Aviation Corporation	Vol. 2, No. 8
David Taylor Model Basin	Vol. 3, No. 5
<b>E. I. du Pont de Nemours</b> and Company	Vol. 3, No. 2
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
Office of Naval Research	Vol. 3, No. 5
Eli Lilly and Company	Vol. 3, No. 2
Mathematics Teachers College, Columbia U.	Vol. 3, No. 3
<b>McDonnell</b> Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
National Aeronautics and Space Administration	Vol. 3, No. 7
National Science Foundation	<b>Vol. 3</b> , No. 3
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7
RAND Corporation	Vol. 3, No. 5
Research Analysis Corporation	Vol. 3, No. 5
Shell Development Company	Vol. 3, No. <b>1</b>
Sperry Rand Corporation	Vol. 3, No. 6
Union Electric Company	Vol. 3, No. 4
Woodrow Wilson Foundation	Vol. 3, No. 3

## NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

### MATHEMATICS AND NATIONAL PURPOSE

**MATTISON L. STORY**

Education Specialist  
National Aeronautics & Space Adm.  
Washington, D. C.



Quantification processes are becoming increasingly vital to every aspect of our national life. Today's complex systems, which are nowhere more evident than in our prodigious space program, daily reflect the endless miracles of modern mathematics. Our so-called "leaping technology" is, in the fullest sense, uniquely dependent upon the amazingly rapid calculations and precise exactitudes now possible in this all-important realm.

The immense challenge which the "organized complexity" of modern life constantly poses could never be met without the systematic formulations and concise organizing principles which mathematics indispensably furnishes. Each Gargantuan accomplishment in modern space technology can be attributed to some specific new implementation of theorem or formula — the actual physical structuring of complex instruments from symbolic predesigns.

It is thus entirely safe to say that mathematics has become a basic and essential tool of national purpose. The vast space enterprise undoubtedly furnishes the single, most dramatic evidence of its strategic new role as a kind of bellwether of national progress. Its increasing essentiality in all areas, stemming again from inevitable conditions of growth and complexity, can be equally demonstrated in the many diverse facets of our daily life as we cope with such commonplaces as gross national product, standard-of-living indices, opinion poll techniques, and population-explosion predictions.

The status of mathematics in relation to other educational disciplines has perhaps undergone a comparable change. While never actually subordinated, it has often been regarded as occupying the position of a prerequisite or intermediate discipline in the curricular scale. Today it is additionally claiming a highly independent stature as a kind of interface between man's scientific concepts and the vehicles and artifacts which enable him to bring them to realization.

Evidence of the dramatically changed status of mathematics can be

**demonstrated** in a variety of ways. A highly pragmatic aspect of its enhanced recognition lies in the simple fact that mathematical know-how is now worth a great deal of money. Happily, the professional skills of the trained mathematician are not readily marketable but continue to be in the highest possible demand.

Far more significant, as a factor of change, **is the** growth of a new attitude toward the mathematician. He has, in a real sense, become the new heroic image of our society. Nothing is of greater social significance, perhaps, than our changing ethic of heroism which promises to supplant the warrior with the thinker as a symbol of greatness. The archetypal hero, traditionally celebrated in epic and fable, has invariably been the man of action, the warrior-statesman whose deeds of physical prowess marked him out for leadership and for worshipful adulation of the masses. The emergence of such intellectual giants as Albert Einstein to positions of widespread public recognition and heroic esteem augurs well for a future in which a new and civilizing recognition of intellectual greatness will perhaps become the keystone of our value system.

The gratifying progress now being made in the improvement of mathematics education is perhaps the most felicitous trend of all. Not only in our advanced curricula for specialized mathematics students but in the common core of education for all citizens, mathematics programs are being **strengthened** and refurbished in school systems throughout the land. This is our surest guarantee of national prestige. It is the bedrock of our future space program. Nothing is more heartening than this recognition that a widespread study of mathematics is, in effect, the best means of strengthening and undergirding our national potential for future greatness.

## AERONAUTICAL CHART AND INFORMATION CENTER

### ACIC AND ITS AEROSPACE MISSION

By  
**J. Donald Define**  
 Geophysical Studies Section  
 Geo-Sciences Branch of ACIC



Mathematics, the Queen of the Sciences, is inherent in many fields, often where least expected. At the Aeronautical Chart and Information Center (ACIC), you might expect to find a large cartographic operation producing many navigational charts and aids for normal use as well as special support activities such as the Mercury Orbit Chart which has been and will be used by the Astronauts. Some glimmer of the Aerospace Mission might have come to your attention when reading of the Center's work on Lunar Charts and the production of a Lunar Atlas as part of the preparation for a manned lunar landing. At this point however, you might consider this the sum total of **ACIC's** work and, as a mathematician, consider that the cartographic field, even when allied with astronomy, is not a field worthy of your attention.

However, this impression, as with many impressions, only partially reveals the situation. Specifically, there are a number of fields with which the Centet is concerned: Cartography, certainly, but also photogrammetry, geodesy, and astronomy along with the requisite computer support.

Cartography, although basically concerned with accurate geographic portrayal, must preserve certain physical relationships of the earth's surface as precisely as possible. One of the more challenging aspects of cartography is the separate field of map projections where the principal concern is the representation of a three dimensional space. Here a good mathematical background is essential with emphasis on such areas of study as Theory of Functions of a Complex Variable, Differential Geometry of Curves and Surfaces and Advanced Calculus.

Photogrammetry is involved principally with obtaining reliable measurements through photography. At ACIC, aerial photographs are used to provide improved cartographic coverage. What sort of background is needed for a photogrammetrist? Error Theory, Matrix

Algebra, Vector Analysis and Least Squares Adjustments are certainly essential.

Geodesy - the science involving study of the exact size and shape of the earth and its internal structure - is a branch of applied mathematics. In the future, geodesy will surely embrace a similar study of the moon (selenodesy) and most probably the planets - the term planetodesy has already been proposed. A variety of subjects are required background for this science. Geodesists are keenly interested in positioning points on a non-mathematical surface, the geoid, which is an equipotential mean sea level surface; and the transfer of the points to a mathematical surface, the ellipsoid, for distance and azimuth computations. Some essential mathematical tools include: Numerical Analysis (including the Numerical Solution of Differential Equations, Numerical Integration, Fourier Analysis, **Legendre** Polynomials and Least Squares), Vector Analysis, Matrix Algebra, Projective and Differential Geometry, Bessel Functions, Spherical Harmonics, Theory of the Potential, Error Theory and Probability and a smattering of Matrix and Tensor Calculus.

Astronomy may be considered as another branch of applied mathematics. Positional astronomy is used to determine point locations on the earth's surface and, together with geodesy, contributes to determination of the earth's size and shape. Solar eclipses, star occultations, precise observations of the moon and of the motion of artificial earth satellites are of vital interest to ACIC astronomers. Background mathematics should include: Bessel Functions, **Legendre** Polynomials, Least Squares Adjustments, Calculus of Finite Differences, Differential Correction Methods, Celestial Mechanics, Potential Theory, and Numerical Integration of Orbits.

The academic background of an applied mathematician at ACIC must be varied and, at the same time, complete. To provide some of the necessary knowledge, ACIC has offered individuals with appropriate mathematical foundation the opportunity of attending one year of advanced training in geodesy and photogrammetry at the Ohio State University's Institute of Cartography, **Photogrammetry**, and Geodesy. Similarly, a number of qualified ACIC employees have received graduate instruction in astronomy at the University of Cincinnati and the Yale University Observatory. Advanced mathematics and computer courses are included in both programs. The Center will no doubt continue to provide advanced training related to current and future projects.

Mathematicians are an integral part of the technical and scientific work force at ACIC. A variety of opportunity exists for the application of mathematical knowledge to the cartographic, photogrammetric, geodetic, and astronomic projects currently in work at the Center or anticipated for the future to support the **USAF** aerospace effort.

## NEWS AND NOTICES

### Edited By

**Mary L. Cummings, University of Missouri**

Army Major Robert J. Weeks, a 1949 graduate of the University of Illinois, was one of a class of 665 U. S. officers and 85 officers from 48 allied nations, who completed the regular course at the U. S. Command and General Staff College, Fort Leavenworth, Kansas, June, 1962.

Army Second Lieutenant **Eldon J. Nosari** recently completed the eight week officer orientation course at the Artillery and Missile Center, Fort Sill, Oklahoma. He received bachelor's and master's degrees from the University of Arkansas, and was formerly an instructor at the University.

Army Second Lieutenant Bruce C. Tyson, Jr. has been assigned to the Chemical Research and Development Laboratories at the U. S. Army Chemical Center, Maryland. He is a 1958 graduate of Duke University and a 1960 graduate of Princeton University.

Army Second Lieutenant Alfred E. Bruns, recently completed the eight week field artillery officer orientation course at the Artillery and Missile School, Fort Sill, Oklahoma. The school emphasizes leadership as well as the practical application of artillery tactics and techniques. Lt. Bruns is a graduate of the University of Missouri.

Paul Henley, a 1962 graduate of the College of Engineering, University of Missouri, has received a Chemical and Engineering News merit award as one of the country's outstanding scholars. He is one of twelve students from American universities to receive the 1962 award, which is given for scholarship and outstanding achievement in campus extra-curricular activities. In March, 1961, **Henley** was presented the American Institute of Chemical Engineers Scholarship Award as the member of the student chapter of the Institute at the University with the highest scholastic rating during his freshman and sophomore years.

We note with interest that **Henley** was the 1961 captain of the University of Missouri football team. As left guard, he made the All Conference first team during his junior year, while during his senior year he was named to the second team in both the Academic All American and All Conference selections.

A lecture by the famous French mathematician, Professor Jean Dieudonne, was the high point of a series of lectures presented by Pi Mu Epsilon at the University of Maryland. Seventy-five midshipmen of the Naval Academy were present at the lecture in addition to many others. Dr. Dieudonne outlined the aims and methods of that famous body of French mathematicians (the **Bourbaki** group) that tries to put order into large chunks of mathematics. Though they try to classify mathematics, they do not attempt to write an encyclopedia. The books published by the society are painstakingly prepared, the completion of a single book taking perhaps ten to twenty years. One person is elected to write a first draft of the book, which is read during a general session and discussed at great length. A second man writes a second draft, and it is read during the following year, with further examination and rewriting. Finally, after ten, twelve, fourteen years, if agreement has been reached, the work is turned over to the editor.

Professor Dieudonne treated his audience to some astounding remarks, namely, that trigonometry and analytic geometry are silly subjects taught in a silly way, and that lattice theory and non-associative algebra are weeds in the rapidly growing theory of mathematics. What do **you think**?

At the University of Kentucky, the Pi Mu Epsilon Distinguished Mathematician Award for scholastic achievement went to Thomas **Steadman Bagby**. He received mathematics books valued at ten dollars and ten more dollars to be spent on books of his choice.

The Pi Mu Epsilon Key Award for the best graduate seminar paper-content

and presentation was presented to Jim Caveny. The award consists of a Pi Mu Epsilon key and a book award equivalent to that of Mr. **Bagby**.

The Pi Mu Epsilon chapter at the University of Buffalo awarded Richard M. Meyer and Albert D. **Polimeni** memberships in the Mathematical Association of America, the former for having achieved the highest average in mathematics, and the latter for having obtained the highest grade on the comprehensive examinations.

We wish to point out that **Carleton** College, which **has one** of our newest chapters, publishes a journal of undergraduate mathematics called *Delta Epsilon*. It is published by students and faculty of the college and serves as a "medium for immediate publication for private circulation and discussion of undergraduate work." Donald C. **Olivier** is student editor, while Professor Kenneth O. May is faculty editor. The paper first appeared in 1960. A four year grant from the National Science Foundation enables the staff to continue with the publication.

Students and faculty at Carleton, **alumni** of the college, mathematicians at other institutions interested in undergraduate research, and libraries, may receive *Delta Epsilon* by requesting it.

Two members of Pi Mu Epsilon at the University of Missouri, Wilson E. **Brumley**, and David Norman Martin, were graduated with honors in **mathematics**. Both **Brumley** and Martin held Continental Oil Company scholarships.

Other members at the University of Missouri receiving awards and scholarships were:

David O. Lambeth, a Henry N. Ess Scholarship for scholastic excellence.

Patrick D. Harris, Bendix Aviation Corporation Honors Scholarship in Science and Engineering. Harris also received the Weinbach Prize in Electrical Engineering for being the highest ranking member of the graduating class in electrical engineering.

Kathryn Lee Boehm, the Sigma Delta Epsilon Award of \$25 for excellence in science.

Billy G. Kay, Hamilton Watch Award to the person who has most successfully combined a proficiency in his major field of science with achievements in the social sciences and the humanities. Kay also received the Missouri Engineers of Chicago Scholarship.

Herbert Black, the Frederick O. Norton Memorial Prize for high scholastic standing and promise of leadership in social service appropriate to the engineering profession.

Edward K. Bower, first prize in the annual Calculus Competition.

**Mathukumalli** Vidyasagar, second prize in the Calculus Competition.

#### DIGITAL COMPUTING FOR PROMISING UNDERGRADUATES

Beginning June 17, 1963, the Digital Computer Laboratory of the University of Illinois will conduct an eight-week undergraduate training and working program, concerned with the use and construction of computers, for a limited number of advanced undergraduates.

Students in residence at any college or university in the United States or Canada who will be juniors or seniors in the fall of 1963 and who are interested in learning about and working with stored-program digital computers are invited to apply for admission.

Successful applicants will receive a stipend of \$400.00 and travel expenses to and from Urbana, Illinois.

No academic credit will be given to students engaged in this program.

Application forms may be obtained by writing to Professor C. W. Gear, Digital Computer Laboratory, University of Illinois, Urbana, Illinois.

The closing date for receiving applications is February 15, 1963.

#### A STATEMENT ON THE PEACE CORPS

FROM

R. SARGENT SHRIVER, JR., DIRECTOR

The United States is sending some of its most outstanding young men and women as Peace Corps Volunteers to the developing nations. As teachers, engineers, nurses, coaches and surveyors, and in community development work, these Volunteers are providing leadership and knowledge to people throughout the world.

Fraternities and sororities have prided themselves on their ability to attract and develop leadership. Responsibility, too, has come with this leadership.

Let me suggest that an even greater responsibility and challenge awaits you now. The chance to serve overseas, and thus to continue the work of more than 4,000 Peace Corps Volunteers now in the field, offers a rare fulfillment and experience. Inform yourself about the Peace Corps and how you may become a part of it after college. Contact the Peace Corps Liaison Officer on your campus, or write directly to PEACE CORPS, College and University Division, Washington 25, D.C.

## NEW CHAPTERS INSTALLED

On April 19, 1962, Professor J. Sutherland Frame installed Utah Beta Chapter (the eighty-seventh) of Pi Mu Epsilon at Utah State University in Logan, Utah.

The charter members were thirteen initiates, namely, **Antone Bringhurst**, Lavelle Day, Norman Eggert, Julia Frandsen, Danny Goodrich, Robert Hammond, Ivan Keller, Mary Nelson, Janet Olsen, Bruce **Orcutt**, Paul Peterson, Wendell Pope, Wayne Rich, and four former members of the fraternity: Lawrence Cannon, Neville **Hunsaker**, Konrad Suprunowicz, E. E. Underwood.

Before the installation, Professor Frame gave a lecture on "Continued Fractions"; at the banquet he gave a talk about the history of Pi Mu Epsilon and showed some of the historical documents.

The eighty-eighth chapter, Rhode Island Alpha, was installed on April 26, at the University of Rhode Island in Kingston. Professor Frame was again the installing officer. The installation ceremony was at 4:30 p.m., followed by a banquet at 5:30 both events taking place at the Larchmont Inn at nearby Wakefield. Later, Professor Frame gave a talk on "A Bridge to Relativity." Charter members were Harry Bender, Mary Cummings, Albert Bennett, Carl Garabedian, Jerald Greenberg, John James, Clifford **Leitaco**, E. M. J. Pease, Nancy Randall, Jacob Stauffer, **Rammath** Suryanarayanan. The following additional nine members were initiated after the presentation of the charter: Richard Bender, Joel Cohen, Diana Drew, James Foster, **Domenick Lombardi**, Mersima Moskos, Maureen **Russo**, Robert Salhany, Ronald **Tourgee**.

# CHAPTER ACTIVITIES

Edited By

Houston T. Karnes, Louisiana State University

**EDITOR'S NOTE:** According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the *Pi Mu Epsilon Journal*. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. Karnes, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

## REPORTS OF THE CHAPTERS

### ALPHA OF MARYLAND, University of Maryland

The Maryland Alpha Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

"**Bourbaki**," by Professor Jean Dieudonne, France.

"Adele Algebra," by Professor Andre Well.

"The Four Vertex Theorem," by Professor Stanley Jackson.

"Why Use 29?" by Professor Richard A. Good.

"Solving Linear Algebraic Systems on a Computer," by L. Kenton Meals, Head of Engineering Application Branch at the David Taylor Model Basin.

"Determining Pi to 100,000 Digits," by Dr. Daniel Shanks, DTMB.

"Foundations of Probability and Statistics," by Dr. Syski.

Professor Jean Dieudonne gave the initiation dinner address. The title of his paper was, "A Panorama of Mathematics." Sixteen new members were initiated. Professor Leon Cohen, Chairman of the Mathematics Department, awarded the Abramowitz prize in Mathematics to an undergraduate student, Mr. Hyman. A scroll was presented to the faculty advisor, Mrs. Dagmar Henny, as a token of appreciation for time and effort given to the Chapter.

Miss Evelyn Wooley was the winner of a puzzle-solving contest sponsored by the Chapter.

Officers for 1961-62 were: Director, Howard Wilson; Vice-Director, David Sprecher; Treasurer, Den-ill Bordelon; Secretary, Alan G. Henney; Faculty Advisor, Mrs. Dagmar R. Henney.

### GAMMA OF MISSOURI, St. Louis University

The Missouri Gamma Chapter held four meetings during the academic year 1961-62. The following papers were presented:

"Symmetric and Self-Distributive Systems," by Mr. Patrick Cassens.

"Group Multiplication by Computers," by Mr. Grattan P. Murphy.

"Plane Polygons," by Mr. Gerald Harshany.

"What is Topology?" by Dr. Saunders MacLane, University of Chicago.

One hundred seventy-two new members were initiated into the Chapter this year.

At the annual banquet Dr. Waldo Vezeau presented the following awards: Pi Mu Epsilon Contest - Senior Division, Mr. Al Ciplickas; Junior Division, Mr. Nick Eissen. Freshman Achievement Award - Miss Mary Middleton. James W. Garneau Mathematics Award for the Outstanding Senior - Mr. Suthep Chantrasomsak.

At the first meeting of the year Mr. William Reddy was elected Vice-Director to succeed Mr. Richard Doyle and Miss Barbara Resnik was elected Secretary to succeed Miss Marceline C. Gratiaa. At the final meeting, Mr. S. Patrick Cassens was elected Director for 1962-63. Dr. Francis Regan has

# CHAPTER ACTIVITIES

resigned from the position of Faculty Advisor and permanent **Secretary-Treasurer** of the Chapter and Dr. Edwin Eigel, Jr. will succeed him.

### BETA OF FLORIDA, Florida State University

The Florida Beta Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

"Fibonacci Sequence," by Mr. Donald Vander Jagt.

"Contributions of Geometry to Mathematics," by Dr. Raymond Wilder, University of Michigan.

"A Conjecture of Borsuk's - A Problem in n-Dimensional Space," by Dr. R. W. Jollensten, National Security Agency.

"Geometric Construction with Compass Alone," by Mr. Peter Rice.

"Ancient Number Systems," by Mr. Fredrick Zerla.

"Inter-Product Spaces," by Dr. Ralph McWilliams.

During the year, twenty-eight new members were initiated into the Chapter. Officers for 1962-63 are: Director, Peter Rice; Vice-Director, Chris Schaufele; Secretary-Treasurer, Sharon Moses; and Faculty Advisor, Dr. H. C. Griffith.

### ALPHA OF GEORGIA, University of Georgia

The Georgia Alpha Chapter held twelve meetings during the academic year 1961-62. The following papers were presented:

"Why Statistics?" by Dr. C. H. Kapadia.

"Osgood's Curve," by Dr. John Jewett.

"Programming the 1960 IBM Computer," by Mr. J. C. Fortson.

"Join Systems," by Mr. Gene Worth.

"Dimension Raising Functions," by Mr. Mike Donahue.

"Computer Programming," by Mr. L. M. Quattelbaum.

"Functions Everywhere Continuous and Nowhere Differentiable," by Mr. Julio Bastida.

"Implicit Function Theorem," by Mr. Britt Williams.

"Semigroups," by Mr. Chun Jai Rhee.

"Why the Number Pi is Transcendental," by Dr. L. W. Anderson.

"Mathematics of Antiquity," by Dr. R. P. Hunter.

"Odds at Dice," by Dr. J. H. Henkel.

Officers for the spring quarter, 1962 through the winter quarter, 1963 are: Director, Charles Christmas; Vice-Director, Earl Lavendar; Secretary, Saralyn Souter; and Treasurer, John Boyd.

### GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"Theory of Groups," (a series of three lectures) by Robert Kaufman, Nathaniel Riesenber, and Michael Tinkler.

"Cardinal Numbers," by Nathaniel Riesenber.

"Creativity in Mathematics," by Professor James Singer, Chairman, Department of Mathematics.

Honorable Mention in the National Science Fellowship competition was given to Larry Smith, Nathaniel Riesenber, Charles Rose, and Harvey Abramson.

Seven of the Chapter's graduates will be entering graduate schools throughout the country to continue their studies.

Officers for 1961-62 were: Director, Nathaniel Riesenber; Vice-Director, Larry Smith; Treasurer, Robert Kaufman; and Secretary, Harvey D. Abramson.

Officers for 1962-63 are: Director, Shaul Stahl; Vice-Director, Robert Blumenthal; Secretary, William Messing; and Treasurer, Edward Ross.

### ALPHA OF LOUISIANA, Louisiana State University

The Louisiana Alpha Chapter held four meetings during the academic year 1961-62. The following papers were presented:

"The Existence of Fixed Points and Applications," by Dr. James Keisler.

"Mathematical Induction," by Dr. Haskell Cohen.  
 "Pathological Function," by Dr. T. Henry **Hildebrandt**.  
 "The Structure of the Real Number System," by Dr. Ronald C. **Bzoch**.  
 Louisiana Alpha held three initiations, one in the fall, spring, and summer. A total of forty students were initiated into the Chapter.

The Annual Honors Examination Awards were presented, at the University's Annual Honors Day Convocation, to the following: **Freshmen** Award to Daniel Elven Jones, and senior award to Charles Sparks Rees.

Officers for 1961-62 were: Director, Kenneth Glen Freeman; Vice-Director, DeWitt Lee Summers; Secretary, Claire Fasullo; Treasurer, Jeffrey B. **Fariss**; Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston T. Karnes.

Officers for 1962-63 are: Director, **DeWitt L.** Summers; Vice-Director, Donald Roy Cowsar; Secretary, Kathleen Dolese; Treasurer, Harold **Reiter**; Faculty Advisor, Dr. Haskell Cohen; and Corresponding Secretary, Dr. Houston T. Karnes.

#### GAMMA OF LOUISIANA, Tulane University

The Louisiana Gamma Chapter was installed on November 27, 1961, and held five meetings during the year. The Chapter was installed by the Director General, Dr. J. Sutherland Frame. His installation paper was "Continued Fractions." At the installation dinner Dr. Frame spoke on "The History of Pi Mu Epsilon."

During the balance of the year four meetings were held with the following papers being presented:

- "The Axiom of Choice," by Dr. G. S. Young.
- "Packings of the Plane," by Dr. A. C. Woods.
- "The Bieberbach Conjecture," by Dr. A. A. **Armendariz**.
- "A Set Similar to the Cantor Set," by Dr. Bruce **Treybig**.

Seven new members were initiated into the Chapter at the spring initiation. Officers for 1961-62 were: Director, George W. Tiller, Vice-Director, Robert **E. Bonini**; Secretary, Sylvia A. **Ibele**; and Treasurer, Charles R. Blackburn, II.

Officers for 1962-63 are: Director, Henry W. **Frantz**; Vice-Director and Treasurer, Donald E. **Ramirez**; and Secretary, Patricia **Merkel**.

#### ALPHA OF NORTH CAROLINA, Duke University

The North Carolina Alpha Chapter held two meetings during the 1961-62 academic year. One a business meeting and the other the annual initiation. The initiation address, "Continuous Functions," was given by Professor **F. G. Dressel**. Five students were initiated.

Officers for 1962-63 are: Director, James M. White; Vice-Director, Robert Chapman **Newman**; Secretary, Robert E. Smith; and Treasurer, Dabney W. Townsend, Jr.

#### EPSILON OF OHIO, Kent State University

The Ohio Epsilon Chapter held three meetings during the academic year 1961-62. At the annual initiation banquet on April 11, 1962, Professor Earle Bush, Head of the Department of Mathematics at Kent State University spoke on "What Is Mathematics?" Twenty-two members were initiated.

Films shown at the two business meetings were: *The Earliest Numbers* and *Minute-Men, Missles, and Mission*.

Miss Vera Melinda Chapman received the Pi Mu Epsilon award at the annual Honors Day assembly. She was awarded a plaque and a check for \$40.

Officers for 1962-63 are: Director, Lois Wilson; Vice-Director, James Weaver; Secretary, Barbara Grills; and Treasurer, Richard Schooley.

#### BETA OF NORTH CAROLINA, University of North Carolina

The North Carolina Beta Chapter held five meetings during the academic year 1961-62. The following papers were presented:

- "Use of the Mathematics-Physics Library," by Dr. A. T. Brauer.

#### CHAPTER ACTIVITIES

"Some Topics in Integration," by Dr. W. M. **Whyburn**.

"Infinite Processes," by Dr. F. B. Jones.

Nineteen new members were initiated into the Chapter during the year. Officers for 1961-62 were: Director, Albert Deal; Vice-Director, Clifton **Whyburn**; Secretary, Sandra Ness; Treasurer, Warren Boeh.

#### ALPHA OF NEBRASKA, University of Nebraska

The Nebraska Alpha Chapter held seven meetings during the academic year 1961-62. The following talks were given:

- 'Can Mathematicians Know Anything; If So, What?' by Professor Dewey.
- "Tunnel **Diode**," by David Gustavson.
- "Telemetering of Physiological Responses of Athletes," by Dr. Rose.

Seventeen new members were initiated into the Chapter at the fall initiation and thirty-five were initiated at the spring initiation.

Officers for 1961-62 were: Director, Larry **Dornhoff**; Vice-Director, David Bliss; Secretary, William T. White; Treasurer, Richard Altrock; Faculty Advisor, Dr. Hubert **Hunzeber**.

Officers for 1962-63 are: Director, Jon **Froemke**; Vice-Director, Stephen Lange; Secretary, Robert **Ladd**; Treasurer, Ken Chatfield; Faculty Advisor, Dr. John **Kimber**, Jr.

#### BETA OF VIRGINIA, Virginia Polytechnic Institute

The Virginia Beta Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

- "Continued Fractions," by Dr. J. Sutherland Frame, Director-General of Pi Mu Epsilon.

"**Riemann** Functions and Their Applications to the Solution of Initial Value Problems in Fluid Dynamics," by Dr. Tom Chang.

"Using a Digital Computer to Obtain Bounds for the Solution of Certain Problems," by Dr. Walter S. Snyder.

"Matrix and Metrics," by Dr. L. B. **Rall**.

Officers for 1961-62 were: Director, Roger Flora; Vice-Director, Robert **Hanson**; Secretary, Janet **Yates**; Treasurer, Fred Patterson.

#### BETA OF OREGON, Oregon State University

The Oregon Beta Chapter held three meetings during the academic year 1961-62. No papers were presented at these meetings since the colloquium meetings of the mathematics department as a whole are available to the members of this chapter.

Social activities included a picnic during the fall term and the Initiation Banquet on May 22 at which Mr. David Johnson spoke on the subject of network flow.

A freshman-sophomore competition was held on March 26. The first prize of \$50 was awarded to Stephen P. Ogard and the second prize of \$30 to Gary J. Ford. The four students who received honorable mention and a book were Thomas L. Mundres, Vasilios Papakonstantinou, John **E. Ferguson**, and Janet Allison. Approximately 35 students took the examination.

Officers for 1961-62 were: Director, John J. **Kohfeld**; Vice-Director, Botond G. Eross; Secretary, Betty Kvarda; Corresponding Secretary-Treasurer, Dr. A. R. Poole.

Director for 1962-63 will be Botond G. Eross. Other officers will be elected during the fall term.

#### BETA OF OKLAHOMA, Oklahoma State University

The Oklahoma Beta Chapter held fourteen meetings during the **1961-62** academic year. The following papers were presented:

"Experiences in Europe," by Professor R. B. Deal.

"Matrix of a Magic Square," by Gene **Sturm**.

"**Applications** of Boolean Algebra to Logic," by Professor R. W. Gibson.

"An Astronomer Looks at Space Travel," by Professor H. S. Mendenhall.

"Orientation," by Louis De **Noya**.

'Finite Differences,' by Professor Robert **Hultquist**.  
 "A Topic from Physics," by Professor **H. P. Hotz**.  
 "Fixed Point Theorems," by Professor **O. H. Hamilton**.  
 "Inverse Ratio System of Counting **Ballots**," by Professor Robert **W. Gibson**

"An Extension of Pascal's Triangle," by Jimmie **Lakin**.  
 "Perfect Numbers" by Mary Johnson.

"A Concept of Time for Scientists," by Mr. Savoe Lottenville.  
 At the annual banquet on May 4, 1962, Professor L. Wayne Johnson presented the Freshman Mathematics Award to Lynn Carpenter. The banquet address was given by Professor R. B. Deal whose topic was, "A La **Sorbonne**."

Officers for 1961-62 were: Director, Gene **Sturm**; Vice-Director, Stuart Reeves; Secretaries, Mary Lou **Thurman** and Linda Priest West; Treasurer, Dale **Kearns**; and Faculty Advisor, Professor John E. Hoffman.

Officers for 1962-63 are: Director, Jimmie **Lakin**; Vice-Director, Joseph S. Greene; Secretary, Mary Ann Smith; Treasurer, Bruce Edgar; and Faculty Advisor, Professor John E. Hoffman.

#### **ALPHA OF ALABAMA**, University of Alabama

The Alabama Alpha Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"The Seven Bridges of **Königsberg**," by Dr. Charles Neville Maxwell.  
 "Non-Interacting Control Systems," by Dr. O. R. **Ainsworth**.  
 "Numerical Solutions for Differential Equations," by Dr. Walter L. Wilson.  
 "Philosophy of Mathematics," by Mr. George W. Roberts.  
 Dr. Clark, Assistant Dean of the College of Arts and Sciences, gave the annual banquet address; and the annual spring picnic was held in **Moundville**, Alabama.

Two initiations were held, in the fall and spring, and a total of eighty-six students were initiated into the Chapter.

Officers for 1962-63 are: Director, Dieter **Pukatzki**; Vice-Director, **J. Z. Higgs**; Secretary, Patricia Lucas; Treasurer, Nancy Charles Jones; Social Co-Chairmen, **Adonice** Hereford and James **Dixon**; and Faculty Advisor, Dr. Henry Miller.

#### **ZETA OF OHIO**, University of Dayton

The Ohio Zeta Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

"Derivatives of Composite Functions," by Mr. Thomas Grilliot.  
 "Iterations of the Exponential Function," by Professor **McIntyre**.  
 "Research and Mathematics," by Professor Buck.  
 "Structure of Mathematics and Convex Sets," by Professor **Pettis**.  
 "Applications of Differential Geometry to Hydrodynamics," by Professor **Mishra**.

"Calculus of Variations," by Professor **Cesari**.  
 "Problems in General Geometrical Dynamics," by Professor **Hlavaty**.  
 "Observations on Recent Trends in School and College Mathematics," by Professor Reingold.

Mr. David Schweickart, a recent initiate, was named as the sophomore who had excelled most in mathematics and was awarded the two volumes of "Differential and Integral Calculus" by Courant.

Officers for 1962-63 are: Director, Mr. Thomas Grilliot; Vice-Director, Mr. David Van **Hausen**; Secretary-Treasurer, Mr. Max **Gruendl**.

#### **GAMMA OF NORTH CAROLINA**, North Carolina State College

The North Carolina Gamma Chapter held fourteen meetings during the 1961-62 academic year. The following papers were presented:

"The Law of Growth and Decay," by Dr. John W. Cell, Head, Department of Mathematics.  
 "Applied Mathematicians and What They Do," by Dr. Horace Trent,

#### CHAPTER ACTIVITIES

#### Ballistic Research Laboratories, Washington

"The Changing Undergraduate Curriculum," by Dr. R. Creighton Buck, Chairman, Committee on the Undergraduate Program in Mathematics, University of Wisconsin.

"The Rocket Research Project at North Carolina State College," by Dr. R. C. Bullock.

"A Method of Solving a System of Differential Equations," by Mr. Jafar **Hoomani**.

Officers for 1961-62 were: Director, Walter B. **Cummings**; Vice-Director, Betty G. Harris; Secretary-Treasurer, Philip Nanzetta; and Faculty Adviser, Dr. James B. Wilson.

Officers for 1962-63 are: Director, Thomas H. Banks; Vice-Director, William S. **Guion**; Secretary-Treasurer, Richard Shachtman; and Faculty Adviser, Dr. James B. Wilson

#### **GAMMA OF WASHINGTON**, Seattle University

The Washington Gamma Chapter held four meetings during the academic year 1961-62. The following papers were presented:

"Orthogonal Polynomials," by Dr. Theodore Chihara.

"Conformal Mappings," by Dr. William **Woolf**, University of Washington

"Famous Problems, Solved and Unsolved," by Dr. R. H. Bing, University of Wisconsin.

"Mathematical Curiosities," by Dr. R. H. Bing.

Eleven students were initiated into the Chapter in two initiations held during the year.

Officers for 1961-62 were: Director, Gary Haggard; Secretary-Treasurer, Mary Ann Hoare; and Faculty Advisor, Dr. Theodore Chihara.

#### **MU OF NEW YORK**, Yeshiva University

The New York Mu Chapter held several meetings during the academic year 1961-62. At the final meeting, the following were elected officers for 1962-63: Director, Benjamin **Volk**; Vice-Director, Martin **Braun**; and Secretary-Treasurer, Robert **Reinerman**.

#### **ALPHA OF OKLAHOMA**, University of Oklahoma

The Oklahoma Alpha Chapter held eight meetings during the academic year 1961-62. A paper was presented at each meeting. Dr. N. A. Court gave the annual banquet address. His subject was: "What is Mathematics?" Thirteen students were initiated during the year.

Officers for 1962-63 are: Director, Earl **LaFon**; Vice-Director, Leo Pratte; Secretary-Treasurer, Maureen Pratte; Faculty Sponsor, Dr. Allen Davis; and Faculty Correspondent, Dr. Dora **McFarland**.

#### **BETA OF KANSAS**, Kansas State University

The Kansas Beta Chapter held five meetings during the academic year 1961-62. The following papers were presented:

"Simulation of Fractional Derivative and Integral Operators on an **Analog-Computer**," by Mr. Gordon E. **Carlson**.

"Bernoulli Numbers and Bernoulli Polynomials," by Professor Paul J. **McCarthy**, University of Kansas.

"The Elastic Bar Variational Problem," by Professor Raville. Head, Department of Applied Mechanics.

"Rational Function Approximation," by Dr. Henry Thacher, **Argonne** National Laboratory.

"Recent Developments at Kansas State University," by Dean Paul Young. Thirty-seven members were initiated into the Chapter at the spring initiation.

Officers for 1962-63 are: Director, Neal Poland; Vice-Director, Robert Crank, Secretary, Michael Miller; and Treasurer, William Stamey.

#### **ALPHA OF KENTUCKY**, University of Kentucky

The Kentucky Alpha Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

"Mathematics, Computers, and Business," by Dr. J. B. Cornelison.  
 "Mathematicians and the Computer Revolution," by Dr. Silvio **Navarro**.  
 An annual banquet was held with Dr. Wendell De Marcus of the Department of Physics giving the address. Dr. De Marcus' topic for the banquet address was, "The Peeled Earth."

Winner of the Pi Mu Epsilon Distinguished Mathematician Book Award was Thomas **Steadman Bagby**. Jim Caveny received the Pi Mu Epsilon Key Award for the best mathematical content seminar paper with the best presentation.

Five members of the chapter have received fellowships to aid in their continued studies.

Officers for 1961-62 were: Director, John Pfaltzgraff; Vice-Director, Clifford **Swauger**; Treasurer, **Lael Kinch**; Secretary, Evelyn Rupard; Librarian, Jim Caveny; and Faculty Advisor, Dr. T. J. **Pignani**.

Officers for 1962-63 are: Director, Jackson Lackey; Vice-Director, Evelyn Rupard; Treasurer, Adelbert Roark; Secretary, Mary E. Logan; Librarian, James Miller; and Faculty Advisor, Dr. T. J. **Pignani**.

#### KAPPA OF NEW YORK, Rensselaer Polytechnic Institute

The New York Kappa Chapter held four meetings during the academic year 1961-62. At the annual banquet Dr. Richard C. **Diprima** spoke on "The Mathematician in Industry and in the Academic World."

Twelve members were initiated into the Chapter at the fall and spring initiations.

A student-faculty lecture series was established by the Chapter and two lectures were presented in this program.

Officers for 1962-63 are: Director, Richard Mateosian; Vice-Director, Larry Levine; and Secretary-Treasurer, Howard Kushner.

#### ALPHA OF NEW YORK, Syracuse University

The New York Alpha Chapter held six meetings during the academic year 1961-62. The following papers were presented:

- "Proofs of Irrationality," by Dr. Albert Edrei.
- "Topology and the Imagination," by Dr. Mark Mahowald.
- "Opportunities in Mathematics," by Dr. Erik **Hemming sen**.

In April the annual banquet was held. Dr. Clyde **Hardin** of the Department of Philosophy gave the banquet address. His topic was, "Can Machines Think?" At the banquet, twenty-one students were initiated into the Chapter.

In March the third annual mathematics contest for high school seniors of the Central New York area was sponsored and prepared by the Chapter. Syracuse University donated a full tuition scholarship for one year for the winner and the Chapter purchased other prizes.

Officers for 1961-62 were: Director, Donald V. Davies; Vice-Director, Bradley Fullager; Secretaries, Linda Sullivan and Marilyn **Meinhardt**; and Treasurer, Donald A. Lutz.

Officers for 1962-63 are: Director, Lee Bryant; Vice-Director, David **Kahn**; Secretaries, Leonora **Zebkiw** and Linda **Serednick**; Treasurer, Adrian Warntz; Faculty Advisor, Professor Mark Mahowald; and Faculty Correspondent, Professor Nancy Cole.

#### ALPHA OF ARIZONA, University of Arizona

The Arizona Alpha Chapter held eight meetings during the academic year 1961-62. The following papers were presented:

- "The Indian Mathematician **Ramanujan**," by Dr. L. J. **Mordell**.

- "Fibonacci Sequences," by Mr. Charles **Haskell**.

- "The Area Theorem," by Dr. L. M. Milne-Thompson.

- "Eigenvalues of the Sturm-Liouville System," by Mr. Joseph Broder.

Membership requirements were raised during the year and the practice of holding two initiations instead of one was begun. The fall semester's initiation was held at a banquet on January 4, 1962. Dr. John Webb gave the banquet address, "The Mathematician in Society".

Officers for 1962-63 are: Director, Bob **DeVore** and Secretary-Treasurer, John Bunch.

#### ALPHA OF MONTANA, Montana State University

The Montana Alpha Chapter held twelve meetings during the academic year 1961-62. The following papers were presented:

- "What to do About  $J[\sqrt{-5}]$ ?" by Dr. William Ballard.

- "Riemann Surfaces," by Mrs. Hilda **Haltz**.

- "Areza's Generalized Theorem," by Mr. Richard **NanKervis**.

- "Uncertainty and Probability," by Mr. George Trickey.

- "Number Theoretic Functions," by Mr. Allen Luedecke.

- "Waring's Problem," by Mr. Fred **DeMarinis**.

- "Buffon's Needle Problem," by Mr. Richard Konesky.

- "Orthonormal Sets," by Mr. Harry Bauer.

- "Dimensional Analysis," by Mr. Edward **Kopitzke**.

- "Rain on the Roof," by Mr. George **McRae**.

- "Two Parameter Eigenvalue Problems," by Dr. Felix **Arscott**.

- "Topological Surfaces," by Dr. R. H. **Bing**.

Montana Alpha presented awards in the form of books to Mr. Barry Davis - outstanding student in undergraduate mathematics and Mr. John **Ulvia** - outstanding student in undergraduate physics. The Chapter also contributed \$50 for prize money for the 1962 Montana Science Fair. Winners, and the schools they represented, were each awarded books.

Officers for 1961-62 were: Director, Allen Luedecke; Vice-Director, George Trickey; and Secretary-Treasurer, Robert **Svehla**.

#### ALPHA OF NEW MEXICO, New Mexico State University

The New Mexico Alpha Chapter held nine meetings during the academic year 1961-62. The following papers were presented:

- "Pontrijagin Duality for Semigroups," by Mr. Charles Austin, University of Washington.

- "A Topic from Statistics," by Dr. Rudolph Borges, University of Cologne.

- "Summability," by Mr. J. P. **Braunen**, University of Texas.

- "Modem Modem Algebra," by Professor S. Eilenberg, Columbia University.

- "The Riemann Hypothesis and Space Communication," by E. C. **Posner**, Jet Propulsion Laboratory.

- "Generalized Information Theory," by N. **Scarratt**, Purdue University.

- "Statistical Metric Spaces," by Professor B. **Schweizer**, University of Arizona.

- "Sheaves, Germs, and Lattices," by Professor John D. Thomas.

- "Dedekind Rings and Linear Algebra," by Professor Robert J. Wisner, Michigan State University, Oakland.

- "An Application of Finite Induction to Number Theory," by Professor Robert J. Wisner,

Officers for 1961-62 were: Director, Carol L. Peercy; Vice-Director, Edmund J. Peake, Jr.; Secretary-Treasurer, William L. Caudle. Professor Seymour Goldbert, chapter advisor and permanent faculty correspondent, is on leave of absence during the year 1962-63. Consequently, Professor John D. Thomas will assume these duties.

#### LAMBDA OF NEW YORK, Manhattan College

The New York Lambda Chapter held thirteen meetings during the 1961-62 academic year. The following papers were presented:

- "The Mathematics of Computer Programming," by Mr. John **Morrissey**, IBM.

- "The Hierarchy of Geometry," by Mr. Nicholas De **Lillo**, Fordham University.

- "The Actuarial Profession," by Mr. Robert Johansen, F. S. A.

- "Equivalence Relations," by Mr. Donald **McCarthy**, New York University.

- "Orthogonal Function," by Charles Weber.

- "Generating Functions," by Jeremiah Kelly.

- "Introduction to Infinite Products," by Stanley S. Leory.

- "Theory of Numbers," by Whitney S. Harris.

Twelve members were initiated into the Chapter at the initiation held in the spring.

Three members of the chapter, Charles Weber, Frank J. Gordon, and John **Stout**, represented Manhattan College in the Putnam Mathematical Contest and ranked 86th out of the 165 teams entered into the contest.

Five members have received awards which will aid in their continuing studies.

Officers for 1961-62 were: Director, Jeremiah Kelly; Vice-Director, Stanley S. Leroy; and Secretary-Treasurer, Harry **J. Neylan**.

Officers for 1962-63 are: Director, Thomas G. Walsh; Vice-Director, Gerald Silva; and Secretary-Treasurer, John Stout.

#### ALPHA OF VIRGINIA, University of Richmond

The Virginia Alpha Chapter held six meetings during the academic year 1961-62. The following papers were presented:

"On Decimal Fractions," by Professor Harley Stafford.

"Programming for Digital Computers," by Professor Frederick Schmidt, Medical College of Virginia.

"A Model for the Real Number System," by Barbara Sue Oglesby.

"The Axiom of Choice," by Kent Wright.

The following awards were issued by the Chapter:

Freshman Mathematics Examination - First Prize (\$10), John Bsiley; Second Prize (\$5), Thomas Hicks. Sophomore Mathematics Examination - First Prize (\$7.50 each), a tie between Van **Bowen** and Michael Kusheba.

The James D. Crump Prize, a University award for excellence in mathematics, was won by Charlotte Adams.

Officers for 1961-62 were: Director, Paul Cohen; Vice-Director, Alice Hall; Secretary, Charlotte Adams; and Treasurer, Harold Smith.

Officers for 1962-63 are: Director, Kay **Koontz**; Vice-Director, William **R. Tolbert**; Secretary, **Cecelia** Stiff; Treasurer, Hans Carter; Corresponding Secretary, E. Sherman **Grable**; and Faculty Advisor, H. Pearce **Atkins**.

#### ALPHA OF FLORIDA, University of Miami

The Florida Alpha Chapter held eleven meetings during the academic year 1961-62. The following papers were presented:

"Mathematics, Science of Game?" by Professor L Rosenbaum.

"The Revolution in Geometry," by Professor F. **Borges**.

"Introduction to Some Recent Concepts in Topology," by Dr. Morton L. Curtis.

"Application of Modular Arithmetic to Computing Machines," by Dr. **H. Aiken**.

"Complex Numbers and Quaternions," by Dr. Herman Meyer.

"How to Talk to a Computer," by J. B. Kaplan.

At the Initiation Banquet Dr. Cesare **Emiliani** of the University of Miami Institute of Marine Sciences gave the address. His topic was, "The Possibility of **Extraterrestrial** Life as Evidenced by Certain Types of Meteorites." Thirty-two new members were initiated this year.

A scroll was presented to Mrs. Georgia Del **Franco**, Faculty Correspondent, who is a charter member of the Florida Alpha Chapter. In addition, winners of the Pi Mu Epsilon Mathematics Contest were: first prize, James **W. Rose** and second prize, Paul Brown.

Officers for 1961-62 were: Director, Lawrence **Hawkins**; Vice-Director, William **Fiennin**; Secretary-Treasurer, Gloria **Cashin**; and Faculty Advisor, Miss Patricia Elliott.

Officers for 1962-63 are: Director, Gloria **Cashin**; Vice-Director, Stephen Love; Secretary-Treasurer, Sofia Pappatheodorou; and Faculty Advisor, J. B. Kaplan.

## INITIATES

#### ALABAMA ALPHA, University of Alabama (Spring, 1962)

Martha F. Alexander	William L. Gamble
Robert D. Alien	Carolyn A. <b>Gilchrist</b>
Hollis P. <b>Behannon</b>	Laura Gonzales
Bettye A. Blake	James A. <b>Guin</b>
James A. Brasher	Robert W. <b>Gunderson</b>
Patricia A. <b>Bruening</b>	Adonice C. Hereford
James L. <b>Chesster</b>	John T. <b>Hubbard, Jr.</b>
Harold L. Daniel	Eddi T. Hunt
William A. Dark	Jane S. Jones
Bernard <b>Digiorio</b>	Jetty V. Lindsey
Eleanor J. Dudley	William T. <b>Mauldin</b>
Judith K. Dunn	James A. Maxwell
Ann D. <b>Fite</b>	David N. <b>McNelis</b>
Olney R. <b>Fortier</b>	

Charles G. Montgomery	Charles G. Montgomery
<b>Johonna Nichol</b>	Gregory T. <b>Ogden</b>
Jimmy Porter	Charles Rampacek
Lynn H. Rice	James E. Simmons
<b>Kathie</b> Simon	Thomas C. <b>Smitherman</b>
Donald Thompson	John D. <b>Warmbrod</b>
James W. <b>Wiggins</b>	Clifton T. <b>Windham</b>

#### ALABAMA BETA, Auburn University (May 10, 1962)

Hubert R. <b>Adkins</b>	Ann <b>Grogan</b>
Robert S. Ballard	Phillip R. Henderson
Robert F. Baits	<b>Raymon A. Henton</b>
Johney E. <b>Burkhalter</b>	Mary E. <b>Hinton</b>
William H. Butler	Stewart V. Horn
Don <b>Chambless</b>	<b>Azalia A. Osborn</b>
Floyd L. <b>Currie</b>	James R. <b>Pitts</b>
Harry L. <b>Defferbach</b>	Gerald A. Pounds
Kay <b>Finney</b>	<b>Athanasiros Prakouras</b>
Sandra Gray	Naomi Robbins

<b>Franke A. Rusche</b>	Edwin W. Smith
Thomas H. Springfield	Thomas H. Springfield
Lawrence H. Stone	Lawrence H. Stone
Jetty F. Thomas	Martha S. Thomas
Francis E. Watson	John P. <b>Weidner</b>
John P. <b>Weidner</b>	Jerry F. <b>Williams</b>
Jerry F. <b>Williams</b>	Joe E. Young

#### ARIZONA ALPHA, University of Arizona (June 4, 1962)

John Louis Bunch	Raphael <b>Finkelstein</b>	Bernard A. <b>Fischer</b>
	Helen Gin	

#### ARKANSAS ALPHA, University of Arkansas (Match 19, 1962)

Donald R. Allen	John K. <b>Harris</b>	Kim L. Mitchell
Robert Arlington	Terry J. <b>Henley</b>	Jetty K. Ott
<b>Mary K. Beavers</b>	Ronald E. Hill	George E. Rouse
Nancy W. Brown	Robert L. Hudson	Randall C. Stephens
William D. <b>Bushmaier</b>	James T. <b>Karam</b>	Jack B. Swift
Webster T. Gotten	Marwin K. Kemp	Phillip A. Terry
Bryce E. <b>Curtisinger</b>	Joe B. <b>Locke</b>	George K. Wallace
Ronald E. Eddy	Judy M. Loux	Robert J. <b>Welborn</b>
Robert O. Fisher	Rex A. <b>Martin</b>	Leonard A. <b>Wiggins</b>
Melvin V. Foster	Phyllis G. <b>McFarland</b>	Edward G. Woods
Rebecca A. <b>Frazier</b>		

#### CALIFORNIA ALPHA, University of California, Los Angeles (February 4, 1962)

Edwin S. <b>Beckenbach</b>	Motley A. <b>Feldstein</b>	John S. Lane
<b>Sorell Berman</b>	Allesandro <b>Figa-Talamanca</b>	Joseph Motkin
James R. Bock	Arthur P. <b>Gittleman</b>	Howard Ritea
<b>Fook Eng</b>	Robert Gold	Bernard Russo
Robert J. <b>Engert</b>	Theodore Gold	Howard B. Vein

#### CALIFORNIA GAMMA, Sacramento State College (Fall, 1961)

James <b>Chrislock</b>	Charles <b>Hagopian</b>	Evelyn <b>Larsen</b>
Ann Cornelius	Byron <b>Hendrix</b>	Ray <b>McAfee</b>
Eric <b>Frerking</b>	Barthel W. Huff	Cynthia Speed

**DISTRICT of COLUMBIA ALPHA**, Howard University (May 19, 1962)

Anjean B. Carter  
 Basil Livingston Cleare  
 Willie E. Cook, Jr.

Oscar H. Criner  
 Eugene M. DeLoath  
 Robert Gamble  
 Ray C. Titus

William Gee  
 Sandra Ann Gittens  
 James Edward Joseph, Jr.

**FLORIDA ALPHA**, University of Miami (May 12, 1962)

Irving Abel  
 Richard H. Ault  
 John M. Blackstock  
 Gerald A. Bottorff  
 Everett H. Boyd  
 Alton T. Butson  
 John W. Cooper  
 William L. Davis, Jr.  
 James Philip Dean  
 John E. Dennis, Jr.  
 Raymond F. Dickman

Edwin Duda  
 Sue Edwards  
 Delvis A. Fernandez  
 Jerrold Gans  
 Robert J. German  
 Beryle A. Greenberg  
 Harvey J. Greenberg  
 Giacomo Grippo  
 Barbara A. Hopf  
 Stephen F. Love  
 William S. Moore, III

Sofia A. Pappatheodorou  
 James G. Pardew  
 Fred Perlwein  
 Leonard R. Rubin  
 Eugene Sadowski  
 Victor Sung  
 Marinel Thompson  
 Manuel Villar  
 Thomas J. Walend  
 Edward R. Young

**FLORIDA BETA**, Florida State University (May 15, 1962)

Lee H. Armstrong  
 Cavin O. Cotev  
 David P. Hayes  
 Martin N. Heinezer

Wade T. Macey  
 Beverly A. Martin  
 Stanley E. Payne  
 Chris B. Schaufele

Harriet H. Sibley  
 Kurt A. Snover  
 Janace A. Speckman  
 Daniel S. Yates

**GEORGE ALPHA**, University of Georgia (March 7, 1962)

Alan Atwood  
 Tommy R. Dell  
 John Goode

Robert B. Hafer  
 Albert G. Horrocks  
 Larry K. Johnson

Hillary L. Kitchens  
 Linda J. Sutton  
 Susan E. Tilghman

**GEORGIA BETA**, Georgia Institute of Technology (May 20, 1962)

Hagood Bellinger  
 Gary D. Bent  
 Donald G. Bodnar  
 David L. Brown

Milton E. Cram  
 K. Bruce Erickson  
 James R. Gard  
 William C. Lineberger

Edward B. Saff  
 Sanford M. Wiener  
 John D. Wright  
 Mehdi S. Zarghamee

**ILLINOIS ALPHA**, University of Illinois (May 2, 1962)

Carol S. Abe  
 Muhammad S. AbuSalih  
 M. Edward Borasky  
 Milo F. Byrn  
 Thomas A. Bums  
 Shu-Park Chan  
 Herbert Y. Chang  
 Kenneth R. Conklin  
 Charles G. Denlinger  
 Donald J. DeSmet  
 John H. Esbin, Jr.  
 Kenneth E. Evans, Jr.  
 Robert E. Gaines  
 Paul M. Grabarkewitz

Frank D. Grosshans  
 Donald U. Gubser  
 Arthur W. Hammar  
 Robert M. Jones  
 Robert J. Kansky  
 Jose Katz  
 James J. Kelleher  
 Robert G. Lange  
 Thomas M. Latta  
 Ralph A. Liguori  
 Lawson Lobb  
 Bernadette G. Londak  
 Joseph B. Miles  
 Lois E. Minning

David D. Morrison  
 Billy R. Nail  
 Andrew R. Neureuther  
 Carol M. Perersen  
 Edward F. Rittenhouse  
 Alexander P. Stone  
 Stanley L. Urwiller  
 Zalman Usiskin  
 Henry Waldman  
 William Weitzman  
 Carroll O. Wilde  
 Bing Kuen Wong  
 Henderson C. H. Yang

**INITIATES****ILLINOIS BETA**, Northwestern University (November, 1962)

Vernon L. Baily  
 Larry W. Broberg  
 Roger L. Cooke  
 Lee A. Eggerman  
 Stephen P. Fox  
 Donald R. Frederick  
 Alyce L. Gagopian  
 Robert L. Getlach  
 Roland W. Gubisch  
 William J. Hankley  
 Charles F. Hepner

Melvyn E. Huff  
 Harvey R. Huttas  
 Patricia A. Johnson  
 Robert J. Kilian  
 Sara E. King  
 David E. Kullman  
 John B. McCollum  
 Dennis J. Mueller  
 Carol Muse  
 Richard Nielsen

Robert L. Obenchain  
 Deane M. Peterson  
 Robert Rice  
 Jeffrey R. Sampson  
 John L. Schofill, Jr.  
 Sherman M. Shand  
 Richard A. Soderberg  
 William F. Stasior  
 Robert Streitmatter  
 Robert L. Ward

**ILLINOIS DELTA**, Southern Illinois University (May 22, 1962)

Allen R. Campbell  
 Wilbur H. Clark  
 Ralph Czerwinski  
 Lenard A. Defend  
 Richard Fulkerson

Marie A. Hughes  
 Carolyn J. Jarick  
 David W. Kammer  
 Theodore Kramme  
 Barbara K. McMillan

David C. Mueth  
 Stanley B. Poe  
 Mary C. Scott  
 Charles T. Wright, Jr.

**IOWA ALPHA**, Iowa State University (Spring, 1962)

Charlotte Louise Ashbaugh  
 Geoffrey S. Boehm  
 Susan Chamberlin  
 Lloyd Craig Davis  
 Henri Feiner  
 David Leroy Hench  
 John H. Hoper  
 Richard Ewers Horron  
 David Wayne Jachson

Howard Jackson  
 Gene A. Kemper  
 David Kennison  
 Mads Ledet  
 Allan Theodore Leffler, II  
 Ronald Wilson Moses, Jr.  
 Larry L. Orr  
 Robert Dennis Pedersen

R. Dean Riess  
 Steven R. Ritland  
 Richard E. Robinson  
 Marilyn Russell  
 Donald E. Schmidt  
 Dennis Paul Swain  
 Curran S. Swift  
 James R. Veale  
 James Brian Wahrenbck

**KANSAS ALPHA**, University of Kansas (April 27, 1962)

Harold W. Breedlove  
 Edwin W. Buchert  
 Emanuel G. Calys  
 Charles R. Combs  
 Marvin E. Donaldson  
 Roger Eggerling  
 David L. Erickson  
 Robert H. Feitz  
 Donald L. Foster  
 John B. Johnston

James P. Kirk  
 Kenneth G. Klenke  
 David G. Lash  
 Floyd D. Lee  
 Thomas W. Loewen  
 Michael C. Mackey  
 Charles D. Marshall, II  
 Philip N. Merritt  
 Richard A. Moore  
 Jean M. O'Dell

Kenny L. Peterson  
 Frederick Pilcher  
 Dieter A. Reetz  
 Harold Schick  
 David C. Scott  
 Jeanne L. Sebaugh  
 Sara Y. Simcoe  
 Lee M. Sonneborn  
 Urs P. Wild  
 Fred L. Wilson

**KANSAS BETA**, Kansas State University (May 22, 1962)

Frederic C. Appl  
 Billy H. Bailey  
 Gary A. Bogar  
 Elmer E. Branstetter  
 James D. Callen  
 Kenneth H. Carpenter  
 Curt H. Chadwick  
 Richard D. Chelikowsky  
 Kent Crawford  
 William Davis  
 Judith A. Dreiling  
 Mark J. Dreiling  
 Gerald W. Ebker

Margaret F. Flynt  
 Mohan L. Garg  
 Everett E. Haft  
 John C. Hassler  
 Thomas M. Keegan  
 Harry D. Knostman  
 Dale D. Koelling  
 Samuel L. Lesseig  
 Bill Livesay  
 Francis E. Masat  
 Ulrich Mathis  
 Judith A. Mawdsley  
 Charles H. Murish

John H. Nichols  
 Larry D. Noble  
 Chia-ven Pao  
 Jerry W. Pence  
 David Poon  
 Gary S. Spencer  
 William W. St. Cyr  
 John E. Tuecke  
 Hugh S. Walker  
 Cha Ho Wang  
 Wayne L. Woodworth  
 Jerald J. Wray  
 Leroy J. Yock

**KANSAS GAMMA**, University of Wichita (May 4, 1962)

Robert L. <b>Blumenshine</b>	James O. <b>Matous</b>	Charles Thomas Snyder
Norbert W. <b>Deneke</b>	Michael R. Mendenhall	Donald H. <b>Wetmore</b>
Ronald E. Dutton	Paul J. Schwindt	Frank L. Wright
Elmer A. Hoyer		

**KENTUCKY ALPHA**, University of Kentucky (May 10, 1962)

Francis P. Clarke	James E. Miller	Saeed <b>Salehi</b>
Bradley B. Cox	Betty B. Robinson	William C. Setzer

**LOUISIANA ALPHA**, Louisiana State University (May 16, 1962)

Charles F. Blank	Donald R. Cowsar	Waker H. Grant
<b>Maunsell</b> W. Brousseau III	Kathleen <b>Dolese</b>	Edward J. Pisa
Edward E. <b>Counce</b> , Jr.	Kenneth W. <b>Eiswirth</b>	Richard S. <b>Thomason</b>

**LOUISIANA ALPHA**, Louisiana State University (August 3, 1962)

Sr. Mary Sylvester <b>DeConge</b>	Georgiana <b>Mary Landry</b>	Charles S. <b>Van Arsdall</b>
Charlotte <b>Eloquin</b>	Inez <b>Chappell</b>	Virginia Louise Williams
Margaret Austine Hunter		Joyce Lester Wilson

**LOUISIANA BETA**, Southern University (April 25, 1962)

James Anderson	Kenneth Joseph	Stella Roberson
<b>Montrust</b> <b>Burrell</b>	<b>Norma J.</b> Lewis	Herbert Rolans
Leon <b>Daughtry</b>	Kenneth Maloney	<b>Merdis J.</b> Taylor
Ulyssis Days	<b>Doris Matthews</b>	Henry Thurman
Chester Givens	Ivan <b>McGowan</b>	Doris A. Williams
Archie Harris	Betty J. Neal	Vernon Williams
Colonel Johnson, Jr.	Willis Porter	Austin <b>Woolfolk</b>

**LOUISIANA GAMMA**, Tulane University (April 5, 1962)

Henry W. Frantz III	Donald E. <b>Ramirez</b>	Call S. <b>Weisman</b>
Lawrence M. <b>Hanafy</b>	Fred L. Smith, Jr.	

**MARYLAND ALPHA**, University of Maryland (April 29, 1962)

Harry N. Bragg	Barty Kaminsky	Eleanor Schwartz
Charles M. <b>Eckert</b>	Nancy Loweth	Sadegh Siahatgar
Paul A. Gerhard	<b>Leo J. Mueller, Jr.</b>	Nolan Wallach
<b>Ulrich</b> <b>Gerlach</b>	Ronald D. <b>Pirtle</b>	William J. Wickless, Jr.
Sallie A. Hartwood	Dickson Preston	Wayne L. <b>Milnot</b>
Daniel M. Hyman		

**MICHIGAN ALPHA**, Michigan State University (March 1, 1962)

William Brewer	Kenneth Malich	Morteza Rahimi
Jerry Chateau	Steve <b>Onderwyzer</b>	Batty Smith
Michael Cooper	Carla Oviatt	James Stewart
William Graham	Gerald Pacholke	Douglas Strickland
<b>Jin-Sheng</b> <b>Huang</b>	Paul Pennock	Tom Tucker
Richard Johnson	<b>Allan Petersen</b>	Sandra <b>Wilcox</b>
James MacKenzie		

**INITIATES****MINNESOTA ALPHA**, Carleton College (June, 1962)

Duane Edwin Anderson	Richard Earl Hammer
Brace David	Douglas Ralph Jones
George Stanley Eugster	James <b>Edling</b> Johnson
Alan Dale <b>Fiala</b>	Garry Robert Kampen
William Tibbets Ford	John M. Karon
William Russell Gage	Stanley Lewis <b>Klein</b>
	George Koehler

Frederick Wilbur <b>Lott, III</b>
Felicia Giare <b>Oldfather</b>
Jonathan Beach Skinner
Anthony Smith
William <b>Sudman</b>
Katherine Anne Wier

**MISSOURI ALPHA**, University of Missouri (May 1, 1962)

Conrad W. Bowers	Kathryn L. <b>Kordes</b>
Edward K. Bower	Lynn Kulova
William A. Brock, III	Charles A. Leech, III
Ernest Brockelmeyer	Fook Wah Leong
Charles R. Coe	Steve Ligh
John P. Creason	Mary A. Logan
Ronald L. Cole	Doris R. Long
Richard L. Francis	Carl L. Ludwig
Theodore J. Gibson	David O. Lambeth
Jerome M. <b>Ginden</b>	Charles B. <b>McLane</b>
Thomas G. <b>Hallam</b>	Martin J. Megeff
William Herder	John L. Meyer
Ronald L. Hollrah	Kathy <b>Mussman</b>
William H. Jennings	Pamela Nett
Verlin Koper	Nancy L. Nitz

Robert F. Randall
Robert Ritenour
Robert M. <b>Sandford</b>
Norbert Schmidt
Norma Jo <b>Smith</b>
Randall W. Stone
Floyd G. Teaney
Richard D. <b>Teaney</b>
Ester L. <b>Tichenor</b>
David E. Tomlin
Mathukumalli Vidyasagar
Marvin <b>Wolfmeyer</b>
Farroll T. Wright
Robert E. Yorke
Sergio <b>Lerda-Olberg</b>

**MISSOURI GAMMA**, St. Louis University (April 11, 1962)

Noel J. Abkemeier	Kosta M. <b>Dedo</b>
Raymond M. <b>Albers</b>	Earl E. Dee
Patrick Argos	Julia R. Deimund
Virginia A. <b>Anoldy</b>	William M. Denny
Joseph H. Austin, Jr.	John M. Deslogue
Robert Babione	Donald F. Deutsch
Victor L. <b>Badillo, S.J.</b>	Neil C. <b>DeVries</b>
Charles J. Balsarotti	Andrew J. Dufner, S.J.
Helen M. Barren	Patricia J. Durse
Paul T. Bauer	Donald M. <b>Eisel</b>
S. Louise <b>Beasley</b>	Robert F. Emmett
John A. <b>Beuchman</b>	Robert G. Erwin
Claire M. Birkbeck	ST. M. <b>Marcelline Falk, C.P.S.</b>
Gerald Charles Blausey	Fadlullah M. <b>Farouq</b>
Jean M. Bordeaux	ST. M. <b>Geraldine Farrant, S.S.J.</b>
Charles J. Brez	Paul F. <b>Feldker</b>
Kathleen P. Brady	Thomas D. <b>Fiorino</b>
Patricia A. Brown	ST. M. <b>Carlita Fitzpatrick, RSM.</b>
John P. Bufo, Jr.	Gerald W. Folk
Benjamin R. Bullock	John F. Fox
ST. M. L. <b>Busk, O.P.</b>	Joan F. Franz
St. M. J. Carbonneau, R.S.M.	Edward E. Funke
Charles P. Casey	Darell Gage
Mary J. <b>Chirpich</b>	Dorothy Clark
Dorothy Clark	Elizabeth A. <b>Coerver</b>
Elizabeth A. <b>Coerver</b>	Thomas P. Comer
Thomas P. Comer	Ronald R. <b>Connors</b>
Ronald R. <b>Connors</b>	ST. M. <b>Coleman Conroy, O.S.F.</b>
ST. M. <b>Coleman Conroy, O.S.F.</b>	Major Ralph P. <b>Corbell</b>
Major Ralph P. <b>Corbell</b>	John <b>Cortina, S.J.</b>
John <b>Cortina, S.J.</b>	Jack Cronin
Jack Cronin	Ann <b>Corrigan</b>
Ann <b>Corrigan</b>	Patrick Henry <b>Crowe</b>
Patrick Henry <b>Crowe</b>	Ruth A. David

St. M. <b>deLourdes</b> Hardesty, O.S. F.
Vincent M. Harmon
Anthony D. Harris
William H. Heidger
James A. Helwig
Thomas E. Henderson
Karen Ann <b>Herbst</b>
Fiances Hewitt
Gertrude T. <b>Heyl</b>
John T. Hogan
Edward J. Hoffman
Richard L. Holdener
Jetty L. <b>Hollingsworth</b>
Judy A. <b>Hoog</b>
John J. <b>Horenkamp</b>
St. Elizabeth M. <b>Huber</b> , B.B.M.
Jeanne H. Huesemann
Wayne L. Humphrey
Marion A. Joncich, S.J.
St. Martina Kalinosky
James L. Kappel
St. James M. Keezer, S.U.S.C.
Francis X. Ken
Joan E. Kessling
St. Dorothy Kiel, C.D.P.
ST. M. <b>Fintan Killian</b> , RSM.
Kenneth J. <b>Kintz</b>
Carol Koeppen
Orest Koropecky
Philip M. Krause
Joan L. Kristof

Annette **J.** Krygiel  
 Barbara **M.** Kuehne  
 Sr. Lois **M.** Lapeyre, D.C.  
 Frank A. Latuda  
 Sr. **M.** Basil **LeDuc**,  
 O.S.D.  
 Vincent K. Lee, S.J.  
 Joan L. **Leiper**  
 Joan M. Leysaght  
 John H. **Liebe**  
 John Chian Wun Linn  
 Ronald D. Luczak  
 Hubert **J.** Ludwig  
 Nancy L. **M.** Mallonee  
 John E. Mansfield, S.J.  
 Donald **J.** Manson, S.J.  
 Madge T. Mao  
 Margo T. Mao  
**St. DePaul** Massoni, D.C.  
 John J. Matejcic  
 Matthew N. McKay  
 Mark D. McKenzie, S.J.  
**Michaela K.** McKittrick  
 John G. Weber  
 James G. Miller

**MONTANA ALPHA**, Montana State University (November 14, 1961)

Kenneth V. Bakke  
 David L. Brownman  
 Denny D. Culbertson  
 Barry P. Davis

Mary D. Montie  
 Mary E. Murphy  
 Sara K. Olivier  
**ST. M.** Patrick O'Tolle,  
 S.S.N. D.  
**Joseph M.** Paikeday  
 John M. Piet  
 Mary K. Pisatek  
 John D. Pope  
 Thomas J. Quinn, S.J.  
 Ronald M. Reap  
 Thomas C. Rochow  
 Roy W. Robinson  
 Judy E. Ross  
 Earl J. Scalet  
 David G. Schmidt  
 Raymond J. Schmitt  
**R. Gary** Schouborg, S.J.  
 George W. Schroeder  
 James C. Schuff  
 Juan Sobrino, S.J.  
 Laurence J. Sowash  
 Jerome E. Steinke  
 Dennis E. Starbuck  
**St. M.** Barbara Stastny, OSF  
 Joyce Stratman

Charles D. Stubbs  
**Peter M.** Szucs  
 Frederick M. Tasch  
 Margaret Thiel  
 Mary E. Thompson  
**John T.** Thompson, S.J.  
 Anthony P. Tieber  
**Joseph A.** Tikvatt  
 Mother M. Patricia  
 Thro, RSCJ  
**Dilawar** F. Uthman  
**Guillermo** Van Hoorde  
 M. Virginia Vanice  
 Michael F. Vezeau  
 Wynn A. Volkett  
 Francis J. Willy, S.J.  
 Margaret L. Wolf  
 Donald A. Wuebbels  
 Zuhair Aziz Yacu  
**Anthony J.** Zappanti  
 Raymond W. Zavisla k  
 Eugene A. Zerega  
**Gaylord** Zimmerman, Jr.  
 John H. Zupez, S.J.

**MONTANA BETA**, Montana State College (May 14, 1962)

Mir Kursheed Ali  
 Bernice J. Benson  
 John Bircher  
 David Booth  
 Roswitha Bullinga

Brace Furness  
 Gene Leslie Gallagher  
**Maurine** Hager  
 Subodhenee die Mehta  
**Hartwig** Melbye

James D. Mildenberger  
 Robert L. Svehla  
 Elliot A. Welsh  
 William E. Whitelaw

**Kjell** Nielson  
**Celia** Smith  
 David Stabio  
 Charles Tolliver  
 Robert Eugene Tracy

**NEBRASKA ALPHA**, University of Nebraska (May 6, 1962)

Jeanne A. Baird  
**Fredric** L. Bauman  
**Dwain** E. Blum  
**Jerre** E. Bratt  
 Larry G. Bryant  
 Edward L. Calvin  
 Hugh S. Carroll  
 Marvin E. Criswell  
 James A. Davis  
 Patty A. Edmiston  
 Stanley B. Eliason  
 Loren E. Fairbanks

Anthony E. Hoffman  
 Alan C. Hurd  
**James E.** Huwaldt  
 Kenneth W. Kaufman  
 William G. Kaufman  
 Stephen G. Kellison  
 Patrick H. Kelly  
 Jin Bai Kim  
 Gary D. Klussman  
 Thomas O. Kotouc  
 Robert J. Kvali  
 Phillip M. Leopold

James W. Lindsay  
**Roger J.** Mattson  
 Robert C. Miley, Jr.  
 John I. Molinder  
**Menna M.** Prettyman  
 Enid L. Reeder  
 Ronald L. Rogowski  
 Roger H. Schwabauer  
**Mark L.** Teply  
 James R. Wall  
**Khostow** Hady Youssef

**NEVADA ALPHA**, University of Nevada (April 4, 1962)

Stanley E. Bush  
**Gail M.** Chadwell  
 Robert V. Garcia  
 Charles E. Gervie  
 Michael C. Hislop  
 Robert D. Horn

James M. Hoyt  
 Reginald Meeker  
 Kathleen I. Miller  
 Ann L. Rafetto  
 Jeanne M. Sadler

Gary M. Tachoiries  
 Alan S. Thomas  
 Calvin E. Thompson  
 James D. Whitlock  
 Charles W. Wilmore, Jr.

**NEW HAMPSHIRE ALPHA**, University of New Hampshire (May 24, 1962)

William A. Cooney  
 Ronald E. Cote  
 Judith A. Flagg  
 John A. Hinchee  
 William E. Lunt

Christine M. Malkowski  
 Robert J. Moore  
 Elizabeth A. Nichols  
 John L. Olesniewicz  
 Marilyn J. Staples

Tmng Ngde Quy  
 Janet P. Ray  
 Charles M. Sawyer  
 Marilyn J. Staples

**NEW JERSEY BETA**, Douglas College (May 14, 1962)

Marie L. Crawford  
 Mary Ann Ferrarese  
 Anne L. Grandin  
 Elizabeth M. Grieder

Marianne Haulenbeck  
 Susan R. Kolba  
 Judith A. Moraller

Nancy F. Peters  
 Linda E. Peterson  
 Elaine R. Scheines

**NEW MEXICO ALPHA**, New Mexico State University (May 14, 1962)

Ralph W. Ball  
 Richard K. Fergin  
 Hugh R. Gardner  
 Conrad G. Keyes  
 Maurice Lipton

Alfred R. Mitchell  
 Roger W. Mitchell  
 George P. Mulholland  
 Charles R. Parsons

Peter J. Ruch  
 Harold G. Rutherford, II  
 Hal L. Wallace  
 Benjamin Ward, Jr.

**NEW YORK ALPHA**, Syracuse University (April 24, 1962)

Lee T. Bryant  
 Anthony P. Carpenter  
 John J. Doyle, Jr.  
 James E. Dunn  
**Linda M.** Dykeman  
 Jack T. Ferguson  
 Lois Fero

Marcia G. Jackson  
 David F. Johnson  
 David J. Kahn  
 Alan Kaplan  
 Carolyn R. Kurgan  
 Vicki Newman  
 Jon B. Pangborn

Barbara O. Rauch  
 Richard Reed  
 Gary F. Rose  
 Linda E. Serednick  
 James P. Walsh  
 Adrian A. Warntz  
 Leonora A. Zobkiw

**NEW YORK BETA**, Hunter College (April 4, 1962)

George S. Alland  
**Ahuva** Barnett  
 Sally A. Burgeson  
 Susan M. Dotz  
 Lenore L. Feigelson

Ilene J. Goldstein  
 Harriet E. Hayes  
 Susan A. Kahn  
 Helene S. Lessinger

Philomena Russo  
 Susan B. Sommerfeld  
**Albert J.** Weibel  
 Pauline Winkelmann

**NEW YORK GAMMA**, Brooklyn College (May 21, 1962)

Harvey Braverman  
 Michael Brozinsky  
 Bernard Dickman

Kenneth Klein  
 William Messing  
 Larry Padwa

Charles Rose  
 Edward Ross  
 Jack Shapiro

**NEW YORK DELTA**, New York University (February 13, 1962)

Edward H. Bersoff

Michael P. Swirnoff

John T. Wadsworth

**NEW YORK EPSILON**, St. Lawrence University (March 14, 1962)

John Victor Bauer

Lucretia Anne Cognetta

**NEW YORK ETA**, University of Buffalo (March 14, 1962)

Judith S. Ackerman  
 Arthur P. Altman  
 David V. Bateman  
**Robert J.** Buck  
 Ralph L. Disney  
 Call F. Evans

Mary E. Graves  
 Marilyn A. Kanczak  
 Michael D. Kerwan  
 Anthony T. Lauria  
 Gordon L. Liles  
 Penelope A. Miller

William E. Price  
 John Rogozinski  
**Karl J.** Schmeder  
 Ellen R. Schwartz  
 Irene L. Sharpe  
 Marilynn A. Tober

**NEW YORK IOTA**, Polytechnic Institute of Brooklyn (June 12, 1962)

Paul Feder                    Stewart Nagler                    Anthony J. Naro

**NEW YORK KAPPA**, Rensselaer Polytechnic Institute (February 11, 1962)

Alan W. Adler               Irwin Hirsch                Ralph E. Morganstern  
 Phil J. Best                Frederick A. Lehrer        Ralph B. Prescott  
 Stephen H. Davis          S. Richard Mateosian        William P. Rogers

**NEW YORK KAPPA**, Rensselaer Polytechnic Institute (June 1, 1962)

George Svetlichny           Lawrence Elliott Levine    Robert Frank Anastasi  
 Michael John Arcidiacono                                      Howard Butt Kushner

**NEW YORK LAMBDA**, Manhattan College (Spring, 1962)

Kevin C. Anderson           Whitney S. Harris            Raymond F. Scanga  
 Joseph L. Bertorelli       John R. Mager                Gerald S. Silva  
 Jack Bova                   Bernard J. McCabe         John Stout  
 Joseph P. Genovese        Ronald R. Ragonese        Thomas G. Walsh

**NEW YORK MU**, Yeshiva College (May 1, 1962)

Elliot Beller                Steven E. Grossman        Lawrence Schulman  
 Martin Braun               David Jacobson                Bert Sirote  
 Robert Feinennan           Ben M. Schreiber            Benjamin Volk  
 Joel Grossman

**NORTH CAROLINA ALPHA**; Duke University (Spring, 1962)

Kenneth G. Brown           Jan M. Hollis                Hugh H. Mills, Jr.  
 Joan S. Dimpf               Edwin E. Messikomer

**NORTH CAROLINA BETA**, University of North Carolina (November 20, 1961)

Anne E. Britton            Richard D. Hofler            Sue F. Ross  
 Peter J. Brown            Robert J. Hursey, Jr.       Charles W. Shearin  
 Judith E. Bush            Knox H. Jones                Jance C. Shearin  
 Bobby F. Caviness        Elizabeth A. McLeod        John N. Tunstall  
 Thomas G. Eason           Martha A. Myers            James R. J. Watkins  
 Charles E. Grigsby        Oscar R. Ogg

**NORTH CAROLINA BETA**, University of North Carolina (February 26, 1962)

Robert Linn Bernhardt, III                                    Phillip Morris Kannan

**NORTH CAROLINA GAMMA**, North Carolina State University (April 26, 1962)

Robert R. Allran           Thomas C. Harris                Ralph E. Showalter  
 Anne L. Fakler            James L. Klingennan         David C. Shreve  
 Henry L. Fisher, Jr.     Bobby E. Phillips            Jason L. Sox  
 John D. Fulton           Gary D. Richardson        William J. Tanner  
 Claude D. Greeson       Robert T. Rood

**INITIATES****OHIO ALPHA**, Ohio State University (May 25, 1962)

Robert D. Baer            James T. Hanlon  
 James K. Brooks           William H. Hosken  
 Stephen D. Comer        Kenneth R. Kimble  
 Ralph T. Compton, Jr.    Gary G. Koch  
 David L. Deever           Frederick S. Koehl  
 St. M. Noel Dreska       James Teh-zen Koo  
 Dorothy S. Guy           Harry G. Miller

Charles E. Ryan, Jr.  
 Thurston W. Shook  
**Allan D. Silver**  
 Alan L. Tyree  
**Jacob M. Weinberg**  
 Thomas C. Wesselkamper

**OHIO BETA**, Ohio Wesleyan University (March 21, 1962)

Linda B. Connolly        Donald S. Hetzel  
 Frederick M. Haney      Eric S. Johnson  
 Wilfred James Hansen     John D. Kessler

Betsy L. Rittenhouse  
 Stanley D. Shawhan

**OHIO ETA**, Fenn College (June 15, 1962)

James Bevevino            George Herron  
 Ray Brezic                Kenneth Kasper  
 Richard Cerny            Robert Kime

James Logan  
 Norman Moore  
 Robert Weisensed

**OHIO ETA**, Fenn College (July 25, 1962)

F. Norman Lutz

Joseph Petsche

**OHIO GAMMA**, University of Toledo (May 6, 1962)

Robert S. Brundage       Thomas J. Mazuchowski  
 Ellen C. Brunt            Susan Nichols  
 Peggy Loo                Keith C. Richards  
 Dennis F. Marvin

**Carmen Vittoria**  
 Stephen M. Wegian, Jr.  
 Carole A. Wright

**OHIO DELTA**, Miami University (May 1, 1962)

Kenneth R. Adams        George E. Fredericks  
 Nancy W. Boswell        Carol V. Medlar

Lois J. Snyder

**OHIO EPSILON**, Kent State University (April 11, 1962)

Joyce L. Burrell           Susan K. Hill  
 James H. Buxton           Donald L. Hunston  
 Douglas A. Cope           Sandra L. Jackson  
 Dianne E. Coyne           Neil M. Kettewell  
 Clifford H. Curtis        Carl D. Lytle  
 Donald A. Furey           Glenice A. Nocjar  
 Robert L. Furey           Mary J. Orovany  
 Barbara A. Grills        Kenneth A. Pew

Barbara L. Pletcher  
 Robert Schappelle  
 Richard G. Schooley  
 Donald B. Siano  
 James R. Weaver  
 Richard M. Williams  
 Lois I. Wilson

**OHIO ZETA**, University of Dayton (Spring, 1962)

Michael Barnoski        Laurens W. Houttuin  
 Joseph P. Brzozowski    James A. Kass  
 Thomas S. Clifford, SM.   John Edward Kauflin  
 Max G. Gruendel           Joseph D. Kolesar  
 Darrell J. Horwath       Frank T. Kozuh, SM.

Marcella A. Sakalas  
 Thomas E. Scheper, S.M.  
 Charles D. Schweickart  
 Alan G. Stevens  
 James D. VanHausen

**OHIO ETA**, Fenn College (February 23, 1962)

John E. Conroy           Bruce Johansen  
 Jack E. Crow              C. Robert Klotz  
 David Herlacher           Frank Lozier  
 Wayne Hudson            Henry A. Putze

Kenneth Schwartzkopf  
**David Sealey**  
 John J. Whately  
 Edgar D. Young

**OKLAHOMA ALPHA**, University of Oklahoma (February 13, 1962)

John R. Lesem	Monte D. Miller	Gerald L. Smith
Charles E. Maudlin, Jr.	Helen M. Murphy	Robert W. Vaughan

**OKLAHOMA ALPHA**, University of Oklahoma (May 4, 1962)

Lynne F. Capehart	Stephen DeCanio	Leo J. Pratte
Charles H. Cook	Gary F. Esch	G. Joseph Wimbish, Jr.
	Forrest R. Miller, Jr.	

**OKLAHOMA BETA**, Oklahoma State University (January 4, 1962)

A. Paul Brokaw	Ann Hemker	Billy G. Monks
David R. Bryan	Parshall L. Howe	Karen Owen
E. Ann Campbell	Jerry Johnson	John R. Perkins
Sue Coates	Mary Johnson	Linda L. Priest
Ralph Ferguson	Jimmie G. Lakin	Robert Sasaki
Darrell Gimlin	James C. Loggins	Margaret Sauls
Joseph S. Greene, Sr.	Lyman L. McDonald	Rose Stuntz

**OKLAHOMA BETA**, Oklahoma State University (April 19, 1962)

Anthony R. Abernethy	Brace C. Edgar	Barton S. Pertinne, Jr.
George W. Brewer	Ronald Gardner	Willa E. Pickering
Jerry W. Brown	Vadis G. Godbey	George F. Reese
Norman Bryant	Glenn Hightower	Jack B. Reid
George M. Butler	William Ralph Lee	Song-yeong Ruu
Burton K. Chambers	Terral L. McKellips	Many A. Smith
Clifford C. Chrisman	George W. Miller	Wayne R. Stoddard
Boyd A. Christensen, Jr.	Nancy Page	Homer H. Tang
Fred E. Collings	Chin-loh Pan	

**OREGON ALPHA**, University of Oregon (May 29, 1962)

Roger Backen	Jimmy Hinkhouse	Leroy Peterson
Virginia Emilie Brooke	Paul William Janus	Roger Price
Donald Leroy Bmw	Robert Carl Juola	Darrell Ray Quarles
David Welland Chapman	Karen Kreuder Hoff	Karl Reitz
Jean Therese Collins	Douglas Kelker	Chaika Kim Rhee
Gary Lee Corkiss	Susan Jean Krutsch	Pamela Robinson
Mary Katherine Davidson	David Leeming	Stewart Sawyer
Kathleen Donaldson	Patricia Jo McCorkle	Robert Siegenthaler
John Edlund	James Wells McCoy	Harry Dean Smith
Barbara Louise Elerath	Allen McDaniels	Delbert Wayne Snyder
Robert Charles Ghent	Gary Hunter Newton	Carol Jean Somekawa
Sharlyn Lee Gillis	Phyllis Nicholson	Larry Virgin
Peggy Lee Huston	Patricia Novak	John Williams

**INITIATES****OREGON BETA**, Oregon State University (May 22, 1962)

Robert J. Albright	Phillip M. Gregg	Sandra Schlinkman
Janet E. Allison	Jerry R. Grunwald	Richard G. Schluter
James J. Auburn	Keith R. Hacker	Arnold Leon Schmeder, Jr.
Rodney Allan Badger	John A. Hocken, Jr.	William R. Scofield
Gerald L. Belt	James H. Husband	George Edward Seymore
Jo Ann Berryman	Ronald E. Johnson	Robin H. Shaylor
Robert Russel Beutler	Kenneth Boyd Knechtel	Richard E. Siemens
Leslie H. Birdsall	Sid G. Knox	Gerald Silke
Chester O. Bishop, Jr.	Lawrence John Koth	Keith B. Snyder
John Amos Boles	David H. Y. Lin	Raj Pal Soni
Jimmie H. Boone	Sharon H. S. Long	J. R. Spoel
Gary L. Brown	Michael H. Malmos	Robert Lee Steele
Manuelita A. Brown	Kaye Kent Manchester	Frederick J. Sterk
William G. Brown	Michael Franklin McCoy	Donald P. Stockard
Thomas Danforth Burnett	Robert W. McCoy	Alan R. Stoebig
Gilbert Wayne Butler	James Lee McElroy	Michael J. Stomp
Don L. Campbell	Roger L. McNair	James Akiyoshi Sunamoto
Theodore W. Cannon	Penelope K. Miller	Patricia Sweet
Jerry Joel Congdon	John L. Morack	Michael T. Swotakowski
Charles J. Conlee	David R. Moser	Michael Allen Talvola
Larry R. Cooper	Mahmoud Abd-elfattah	
Elizabeth Louis Simpson	Moustafa	
Curl	Thomas Munkres	Alice A. Thompson
William W. Curtis	Junior A. Nagaki	Jay R. Turner
Bernard E. Davis	Mark I. Nelsen	Sommai Vongsure
Arthur F. Deardorff	George Papageorge	James D. Wagner
Jack D. Dennon	Bill Papakonstantinou	Darryl William Walker
Sandra D. Dunagan	David Harrison Paup	John P. Ward
Stuart Ames Eide	Dean Edward Perry	Michael S. Waterman
John Alan Evans	John W. Powell, Jr.	Ronald N. Webb
John E. Ferguson, Jr.	Merwyn C. Powell	Floyd R. Wendberger
Lloyd A. Fraser	Arthur Douglas Ritchie	Jane M. Whitcomb
Raymond F. Fry	William Earl Gilbert	Jean M. Wick
David E. Gooding	David E. Gooding	John L. Wilkerson
George Joseph Grammens	George Joseph Grammens	Frank O. Williams
		James B. Wuori
		James W. York

**PENNSYLVANIA ALPHA**, University of Pennsylvania (Spring, 1962)

Robert Averell	Paul Cotton	Betty Rarig
Elliott M. Badder	Jay E. Israel	Bruce Schoenberg
Melanie Costa	Elliott D. Kieff	Michael Tinkleman
	Eugene H. Poppel	

**PENNSYLVANIA GAMMA**, Lehigh University (November 9, 1961)

Thomas E. Bachman	Charles W. Hofer	Walter H. Nichols
James A. Begley	John P. Janowski	Norman L. Owsley
R. Barry Bischoff	Gary K. Kohler	John R. Pivnichny
Philip H. Brandt	Donald Lookingbill	Thomas A. Reilly
Charles A. Falcone	Thomas G. Ludwig	Thomas A. Slivinski
Leo B. Freeman, Jr.	Michael J. Mendelsohn	Philip H. Swain
Robert J. Galgon	Frank R. Moore	John K. Wagner
Ronald J. Hartranft	Bernard E. Musch, Jr.	R. Dennis Wayson
G. Raymond Hodil, Jr.	J. Calvin Nafziger	John R. Webb

**PENNSYLVANIA ZETA**, Temple University (March 26, 1962)

Mary Becker	Anne Glass	Herb Silverman
Sue Berwind	Joseph Mandelbaum	Robert Stout
Arthur Bobrove	Marlene Rosenzweig	Joseph Yeager
Sheldon Eisenberg	Arnold Rubin	

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**PENNSYLVANIA DELTA**, Pennsylvania State University (May 15, 1962)

Richard A. <b>Alo</b>	Douglas H. Frank
Mary E. Angstadt	Stanislaw Mrowka
James B. <b>Barto</b>	Anthony J. Felice, Jr.
William S. Bickel	Walter L. <b>Ghering</b>
Dorothy M. <b>Blankeslee</b>	Carole L. Gibson
David L. Briggs	Charlotte R. Gilson
John J. Broderick	Barry S. <b>Goluboff</b>
Aviva R. Brown	John A. Gustavson
Joan B. <b>Byers</b>	Beverly A. <b>Heckman</b>
Judith Q. <b>Chouinard</b>	Charles E. Houck
Richard L. Clark	William M. Houck
Edward E. Collins	Robert L. Hunt
James L. <b>Cooly</b>	Leland T. <b>Mahood</b>
John C. Davis	Donald J. <b>McMahon</b>
John W. Diercks	George E. Mitchell
Lynne E. <b>Eisenstaedt</b>	Robert C. Moore
	Linda Moritz

**RHODE ISLAND ALPHA**, University of Rhode Island (April 26, 1962)

Harry A. Bender	Carl A. Garabedian
Richard Bender	<b>Jerald</b> H. Greenberg
Albert A. Bennett	John T. James, Jr.
Joel Cohen	Clifford D. Leitao
Mary H. Cummins	<b>Domenick</b> Lombard!
Stephen DeMetrick	Mersina Moskos
Diana Drew	Edward M. Pease
James Foster	Nancy E. Randall
	Maureen Russo
	Robert J. Salhany
	Jacob R. Stauffer
	William Strawderman
	E. <b>Ramnath</b> Suryanarayanan
	Ronald A. <b>Tourgee</b>

**SOUTH CAROLINA ALPHA**, University of South Carolina (May 1, 1962)

Robert F. Bradley	David B. <b>Heape</b>
John G. <b>Breland</b> , Jr.	Thomas E. Hogarth
Sheang P. Chan	Carolyn Honeycurt
Carolyn P. <b>Crawley</b>	Barbara <b>Loewe</b>
Peggy A. <b>Crawley</b>	Susan <b>Loewe</b>
Rodney F. Epting	Roy Lucas, Jr.
Dianne Fredy	Edwina <b>Rudisill</b>
George T. <b>Hawkins</b>	Ann Sanders
	Wei-Mei Shyu
	<b>Pelham</b> Thomas
	W. Joseph Van Dyke
	James Wallace
	Lindley H. Webb
	Ronald A. Young

**SOUTH CAROLINA ALPHA**, University of South Carolina (August, 1962)

Nathan E. <b>Hardwick</b>	Marshall Pace
	Edward C. Thomas

**SOUTH DAKOTA ALPHA**, University of South Dakota (May 2, 1962)

Elwood D. Baas	Joy Hamrin
William D. Carda	<b>Shapleigh</b> W. Howell
<b>Cleland</b> V. Cook	Eugene S. Jacobs
Ervin M. <b>Eltze</b>	Helmut B. Nunn
Charles I. <b>Ernst</b>	Harley M. Oien
James C. <b>Fargo</b>	Joyce E. Palmer
Eugene M. Frohling	John E. Powell
Allison J. Haeder	Roger F. <b>Reinking</b>
	Arthur L. <b>Robertson</b>
	Jacqueline M. Rose
	David E. Rumelhart
	George D. Schmieg
	Byron K. Schulke
	James W. Shouse
	Robert H. <b>Steling</b>
	John D. Snyder

**TEXAS ALPHA**, Texas Christian University (Spring, 1962)

**Braynard** R. Traweek

## INITIATES

**VIRGINIA BETA**, Virginia Polytechnic Institute (May 15, 1962)

Jean M. Carr	Margaret A. Hockensmith
James F. Chew	<b>Merrill</b> W. <b>Hume</b>
Bryant Chow	Michael H. Kutner
Dan Chun	Jack M. <b>Mendel</b>
Bennie A. <b>Clemmer</b>	William C. Nelson
Selwyn L. Flounoy	James E. <b>Norman, Jr.</b>
John R. Hanson	Janet Yates

**WASHINGTON BETA**, University of Washington (March 8, 1962)

Brandt R. Allen	Thomas D. <b>Hayward</b>
Naydene Chadson	James P. Jones
<b>Gordon</b> P. <b>Chinn</b>	Robert G. Krause
Peter W. <b>Cornue</b>	Elizabeth M. Kutter
Donald E. <b>Ekman</b>	Karen M. <b>Lenzie</b>
M. Owen Englehart	George F. Mason
William L. Ferris	Jean M. <b>Murakami</b>
Victor Hasselblad	Emiko J. Nakamura

**WASHINGTON BETA**, University of Washington (May 22, 1962)

Dave Barnette	Duane Alfred King
John L. Bergaren	Larry E. Knop
John R. Cherevnik	Frank <b>Kukla</b>
Janet <b>Crist</b>	Ronald Lim
John <b>Engstrom</b>	John <b>Lippert</b>
Victor I. Francisco	Diana Lubash
Louise B. Haack	Gary Miller
	Pamela Johnson

**WASHINGTON GAMMA**, Seattle University (Spring, 1962)

John E. <b>Meany</b>	David L. Predeck
Michael D. Moran	Leon M. Puzon

**WISCONSIN ALPHA**, Marquette University (April 7, 1962)

Kenneth J. <b>Bukowski</b>	Susan <b>Forrestal</b>
Karen A. Case	Edward J. <b>LaClare</b>
Paul S. Cheng	Robert Stanley <b>Matulis</b>
Mary E. <b>DeRosso</b>	Charles J. <b>Mazza</b>
Michael H. Eggle	Robert M. <b>Nehs</b>
Richard E. <b>Fedler</b>	

**WISCONSIN BETA**, University of Wisconsin (February 8, 1962)

David F. <b>Appleyard</b>	James E. Hall
E. H. <b>Battistella</b>	Beverly Henderson
<b>Bruce</b> C. Berndt	David Henderson
Mary C. Carroll	Kenneth M. <b>Kapp</b>
Franklin D. Cheek	Arthur <b>Knowlton</b>
Nahida H. Gordon	

**WISCONSIN BETA**, University of Wisconsin (May 22, 1962)

Barbara D. Antolovich	Dennis W. Kuba
Charles K. <b>Chui</b>	Elizabeth G. Lee
William B. <b>Dykema</b>	Lee C. Marquardt
Walter J. <b>Froehlich</b>	Caryl A. Milkowski
Martin I. Goldstein	Toby J. Mitchell
	Claire L. Mount
	Eunice G. Pollack
	Stephen M. Robinson
	Lynn Van <b>Vleet</b>

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