

# PI MU EPSILOX JOURNAL

THE OFFICIAL PUBLICATION OF

THE HONORARY MATHEMATICAL **FRATERNITY**

VOLUME 1

NUMBER 10



## CONTENTS

	<b>Page</b>
Mina Spiegel Rees . . . . .	Mildred E. Taylor 395
A Resolution. Adopted by the Council of the American Mathematical Society . . . . .	396
Metric Extension. . . . .	Alan J. Goldman 400
Note by R. C. A. 30 December 1953 . . . . .	407
Corrigenda . . . . .	408
Letter to the Editor . . . . .	Helen E. McAleer 408
Problem Department . . . . .	409
Problems for Solution . . . . .	409
Solutions. . . . .	410
National Meeting of the Pi Mu Epsilon Fraternity. Baltimore. Maryland . . . . .	416
Report of the Chapters. . . . .	421
Medals. Prizes and Scholarships . . . . .	425
News and Notices . . . . .	426
Book Reviews . . . . .	432
Acknowledgments. . . . .	434
Initiates. Academic Year 1951-1952 (Continued) . . . . .	435
Academic Year 1952-1953 (Continued) . . . . .	435
Academic Year 1953-1954 . . . . .	436

---

APRIL

---

1954

PI MU EPSILON JOURNAL  
THE OFFICIAL PUBLICATION  
OF THE HONORARY MATHEMATICAL FRATERNITY

RUTH W. STOKES, Editor

ASSOCIATE EDITORS

J. S. FRAME, Michigan State College, East Lansing, Michigan  
H. T. KARNES, L. S. U., Baton Rouge 3, Louisiana  
N. H. MCCOY, Smith College, Northampton, Massachusetts  
LEO MOSER, University of Alberta, Edmonton, Alberta, Canada  
R. J. WALKER, Cornell University, Ithaca, New York

HOWARD C. BENNETT, Business Manager

GENERAL OFFICERS OF THE FRATERNITY

Director General: C. C. MacDuffee, University of Wisconsin  
Vice-Director General: W. M. Whyburn, University of N. C.  
Secretary-Treasurer General: J. S. Frame, Michigan State College

Councilors General.

Howard C. Bennett, Syracuse University, Syracuse, New York  
S. S. Cairns, University of Illinois, Urbana, Illinois  
Tomlinson Fort, University of Georgia, Athens, Georgia  
Sophia L. McDonald, University of California, Berkeley, Calif.  
Ruth W. Stokes, Syracuse University, Syracuse, New York

All editorial correspondence, including manuscripts, chapter reports, books for review, news items, etc., should be addressed to THE EDITOR OF THE PI MU EPSILON JOURNAL 15 Smith College, Syracuse University, Syracuse 10, New York.

PI MU EPSILON JOURNAL is published semi-annually at Syracuse University.

SUBSCRIPTION PRICE: To Individual Members, \$1.50 for 2 years; to Non-Members and Libraries, \$2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to The Pi Mu Epsilon Journal, 15 Smith College, Syracuse University, Syracuse 10, New York.

# PI MU EPSILOX JOURNAL

THE OFFICIAL PUBLICATION OF  
THE HONORARY MATHEMATICAL FRATERNITY



VOLUME 1

NUMBER 10

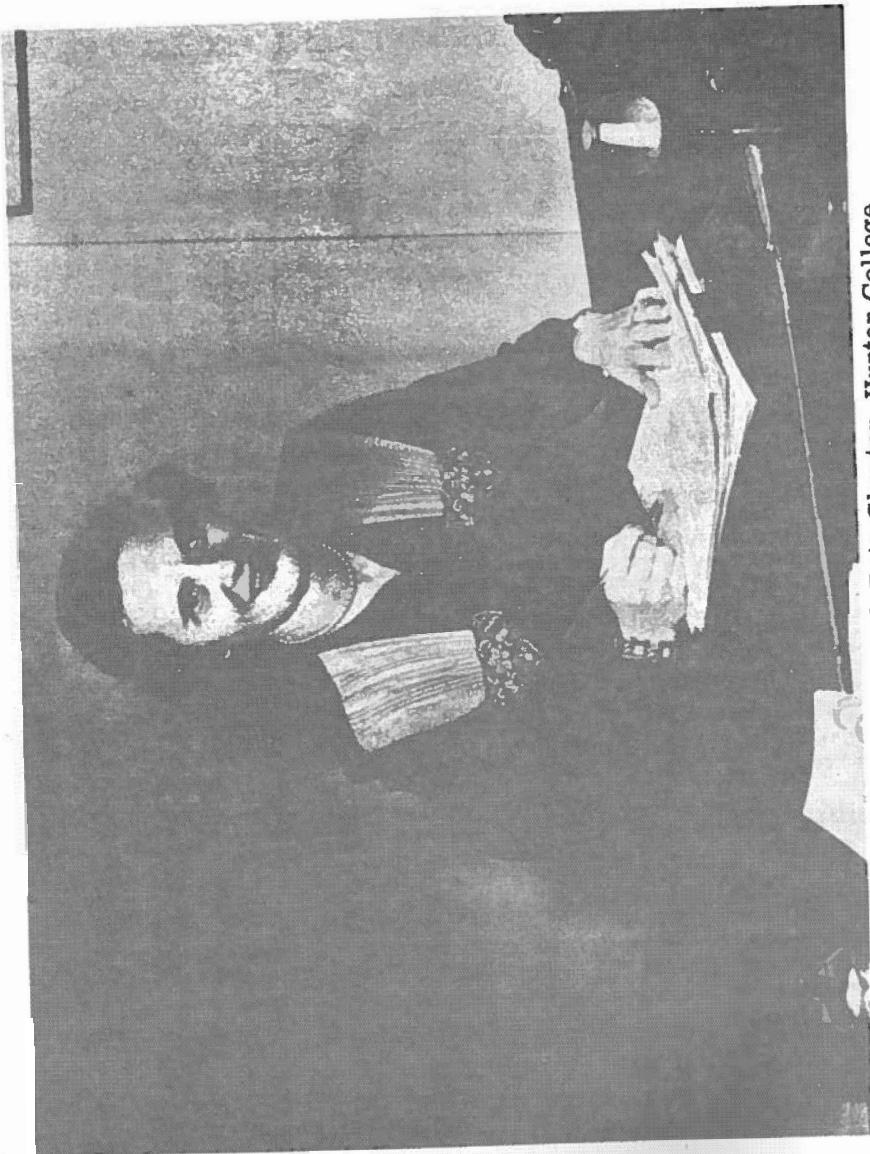
CONTENTS

	Page
Mina Spiegel Rees. . . . .	Mildred E. Taylor 395
A Resolution, Adopted by the Council of the American Mathematical Society . . . . .	396
Metric Extension. . . . .	Alan J. Goldman 400
Note by R. C. A. 30 December 1953 . . . . .	407
Corrigenda. . . . .	408
Letter to the Editor . . . . .	Helen E. McAleer 408
Problem Department. . . . .	409
Problems for Solution . . . . .	409
Solutions.. . . . .	410
National Meeting of the Pi Mu Epsilon Fraternity, Baltimore, Maryland. . . . .	416
Report of the Chapters. . . . .	421
Medals, Prizes and Scholarships . . . . .	425
News and Notices . . . . .	426
Book Reviews . . . . .	432
Acknowledgments. . . . .	434
Initiates, Academic Year 1951-1952 (Continued) . . . . .	435
Academic Year 1952-1953 (Continued) . . . . .	435
Academic Year 1953-1954. . . . .	436

APRIL

1954

Copyright 1954 by Pi Mu Epsilon Fraternity, Inc.



MINA S. REES, New York Beta Chapter, Hunter College  
"OFFICIAL UNITED STATES NAVY PHOTOGRAPH"

## MINA SPIEGEL REES

by

Mildred E. Taylor, Mary Baldwin College

Born in Cleveland, Ohio, a few years prior to World War I, Mina S. Rees, endowed with great mental and spiritual powers, was destined to render invaluable services to her country and to mathematics in particular, during World War II and the years that followed. Within a remarkably short time she had earned for herself both national and international recognition.

And now, crowned with a list of honors and success such as few scholars attain in a whole lifetime, after an absence of ten years from the campus of Hunter College, Mina Rees has returned to her Alma Mater to be Dean of the Faculty. She has more than fulfilled the promise that she showed as an undergraduate. As an undergraduate, she demonstrated exceptional ability and proficiency in mathematics; she easily qualified for membership and was initiated into the Pi Mu Epsilon Fraternity. Besides being an exceptional student she was interested in campus affairs. So, while being editor of the yearbook and president of the student council, she found time to do enough studying to earn a Phi Beta Kappa key and graduate *summa cum laude*. She received an A.M. degree from Columbia University and a Ph.D. degree from The University of Chicago.

Even before receiving her **doctor's** degree she began her career as a highly trained mathematician when in 1926 she joined the faculty of Hunter College where she remained until the second year of the last war when she was granted a leave of absence to go to Washington to serve as Technical Aide and Executive Assistant to the Chief of the Applied Mathematics Panel, National Defense Research Committee, OSRD. In 1946, Dr. Rees was appointed Head of the Mathematics Branch of the Office of Naval Research. In 1949, she became Director of the Mathematical Sciences Division of the Office of Naval Research.

*Science*, honoring **Mina** Rees, published a resolution adopted by the Council of the Institute of Mathematical Statistics, excerpts from which follow:\*

"Under Dr. **Rees'** leadership the Division of Mathematical Sciences of the Office of Naval Research gave whole **hearted** support to basic research, in particular to basic research in mathematical statistics and probability. The whole action was conducted with remarkable foresight and wisdom... .

**The** postwar development of mathematical statistics in the United States owes a great deal to the farsighted policy of the Office of Naval Research ability administered by Dr. Rees. Mathematical statistics owes **Mina** Rees a public "well done," and **extends its best** wishes to her successor at the Office of Naval Research."

In 1953, Dr. Rees accepted an invitation to return to Hunter College as Dean of the Faculty.

Learned societies in which Mina Rees holds membership are: AAAS. (Fellow), A.M.S., M.A.A., I.M.S., Philosophical Society of Washington, N. Y. Academy of Sciences, and **Sigma** Xi.

She has served in an advisory capacity for many of our scientific groups. We list here the chief of these:

Member, Council on Scientific Personnel, 1946

**Member**, UNESCO Panel on Applied Mathematics

Member, Committee on Conference in Applied Mathematics, International Congress of Mathematicians, 1950, Cambridge, Mass.

Member, Mathematics Policy Committee

Member-at-Large, Mathematics Division, National Research Council

Navy Member, Applied Mathematics Advisory Council, National Bureau of Standards, from time of **Us** founding until 1953

---

\*See *SCIENCE*, vol. 118, no. 3065, Sept. 25, 1953, pp. 349-350.

Consultant, Committee to Evaluate the Operation and Functions of the National Bureau of Standards, 1953  
Chairman of Section **A(Mathematics)**, and **Vice-President**, A.A.A.S., 1954.

Member, Committee to Advise Director of National of National Bureau of Standards on mathematics program, 1954-1957.

Everything that Mina Rees has attempted has been done with distinction. As recognition for some of these services she has received the following honors: *King's Medal for Service in the Cause of Freedom*, given by Great Britain; and the *President's Certificate of Merit* from the United States. Last May when she was leaving Washington to return to Hunter College she was given the "OUTSTANDING PERFORMANCE RATING" for her work with **ONR**.\* This citation reads in part:

"From the beginning of her career in the Office of Naval Research, Dr. Rees has foreseen more clearly than anyone else the impending expansion of the use of mathematical modes of thought and analysis in the Navy's theoretical and operational activities, as well as the directions in which this would take place... .

"As Deputy Science Director, Dr. Rees contributed significantly to the development of a corps of **scientifically** trained officers in the Navy by making an Advanced Science Program for Naval Officers, for which ONR is responsible, more attractive to properly qualified officers.

..... "At the same time her contacts with mathematicians, statisticians, and engineers have been such as to enable her to contribute significantly to the vitality of scientific activity in these fields; a fact which has earned her public recognition of the Institute of Mathematical Statistics and the American Mathematical Society."

---

\*Excerpt from JUSTIFICATION FOR OUTSTANDING PERFORMANCE RATING, May, 1953.

Her most recent recognition has been made by the American Mathematical Society, and this is in the form of a resolution printed below, in this journal.

In addition, Dr. Rees is a much traveled young woman. Any list of foreign countries visited by her should include; England, France, Germany, Norway, Sweden, Denmark, Finland, Russia, Central America, Cuba, Italy, Portugal, Switzerland, and The Netherlands.

When any woman has been as busy as Dr. Rees in doing research, we are apt to think of her as being entirely absorbed in her work. She, however, like any normal young person has her hobbies for leisure hours, — painting and tennis. But she has always been interested in the people around her and the things that are happening to them and also to the world in which we live. She is alert to the meaning of events of our time. Dr. Rees has a quick wit, a ready humor and a most attractive smile.

Because Dr. Rees has done so much for mathematics and mathematicians. Pi Mu Epsilon is very proud to have her as a member and to pay this small tribute to her.

#### DEAN MINA S. REES

Honored by the American Mathematical Society

At a meeting of the Council of the American Mathematical Society, held in Levering Hall of The Johns Hopkins University, in Baltimore, Maryland, on December 29, 1953, the following resolution was adopted. It was read before the Society, by Secretary E. G. Begle, at the Business Meeting in Remsen Hall the next afternoon.

"The Council of the American Mathematical Society here takes cognizance of the resignation this past September of Dr. Mina Rees as Head of the Mathematics Section of the Office of Naval Research. She has accepted a position as Dean of the Faculty at Hunter College. We congratulate Hunter College on this wise selection and can only say that our heavy loss as mathematicians is the gain of Hunter College.

"The very striking and brilliant contributions made by pure (non-military, non-applied) science not least of these by mathematics, to the winning of World War II is well known. It was clearly seen by the government and those responsible for the armed services that a large scale fostering by the U. S. government of fundamental research, the basis of all research, was unavoidable. Only thus could we hope to hold our own in years to come, and incidentally build up a suitable reserve of talented men for emergencies. This was actually acted upon by the Navy who thus took the lead by some years with the creation of the Office of Naval Research. Needless to say as the purest of all sciences, mathematical research might well have lagged behind in such an undertaking. That nothing of the sort happened is beyond any doubt traceable to one person — Mina Rees. Under her guidance, basic research in general, and especially in mathematics, received the most intelligent and whole hearted support. No greater wisdom and foresight could have been displayed and the whole postwar development of mathematical research in the United States owes an immeasurable debt to the pioneer work of the Office of Naval Research and to the alert, vigorous and farsighted policy conducted by Miss Rees. The influence of these policies has been such that it vitally affected later developments: The activities of Air Force and Ordnance research, the National Science Foundation itself. It is well known that in these more recent organizations Mina Rees was constantly appealed to for counsel and guidance.

"As Miss Rees leaves her task, the Council of the American Mathematical Society desires to express to her in the name of the whole mathematical community its warmest feelings of appreciation for her past performance and extends to her its best wishes for the future."

Her most recent recognition has been made by the American Mathematical Society, and this is in the form of a resolution printed below, in this journal.

In addition, Dr. Rees is a much traveled young woman. Any list of foreign countries visited by her should include; England, France, Germany, Norway, Sweden, Denmark, Finland, Russia, Central America, Cuba, Italy, Portugal, Switzerland, and The Netherlands.

When any woman has been as busy as Dr. Rees in doing research, we are apt to think of her as being entirely absorbed in her work. She, however, like any normal young person has her hobbies for leisure hours, — painting and tennis. But she has always been interested in the people around her and the things that are happening to them and also to the world in which we live. She is alert to the meaning of events of our time. Dr. Rees has a quick wit, a ready humor and a most attractive smile.

Because Dr. Rees has done so much for mathematics and mathematicians. Pi Mu Epsilon is very proud to have her as a member and to pay this small tribute to her.

#### DEAN MINA S. REES

Honored by the American Mathematical Society

At a meeting of the Council of the American Mathematical Society, held in Levering Hall of The Johns Hopkins University, in Baltimore, Maryland, on December 29, 1953, the following resolution was adopted. It was read before the Society, by Secretary E. G. Begle, at the Business Meeting in Remsen Hall the next afternoon.

"The Council of the American Mathematical Society here takes cognizance of the resignation this past September of Dr. Mina Rees as Head of the Mathematics Section of the Office of Naval Research. She has accepted a position as Dean of the Faculty at Hunter College. We congratulate Hunter College on this wise selection and can only say that our heavy loss as mathematicians is the gain of Hunter College.

"The very striking and brilliant contributions made by pure (non-military, non-applied) science not least of these by mathematics, to the winning of World War II is well known. It was clearly seen by the government and those responsible for the armed services that a large scale fostering by the U. S. government of fundamental research, the basis of all research, was unavoidable. Only thus could we hope to hold our own in years to come, and incidentally build up a suitable reserve of talented men for emergencies. This was actually acted upon by the Navy who thus took the lead by some years with the creation of the Office of Naval Research. Needless to say as the purest of all sciences, mathematical research might well have lagged behind in such an undertaking. That nothing of the sort happened is beyond any doubt traceable to one person — Mina Rees. Under her guidance, basic research in general, and especially in mathematics, received the most intelligent and whole hearted support. No greater wisdom and foresight could have been displayed and the whole postwar development of mathematical research in the United States owes an immeasurable debt to the pioneer work of the Office of Naval Research and to the alert, vigorous and farsighted policy conducted by Miss Rees. The influence of these policies has been such that it vitally affected later developments: The activities of Air Force and Ordnance research, the National Science Foundation itself. It is well known that in these more recent organizations Mina Rees was constantly appealed to for counsel and guidance.

"As Miss Rees leaves her task, the Council of the American Mathematical Society desires to express to her in the name of the whole mathematical community its warmest feelings of appreciation for her past performance and extends to her its best wishes for the future."

## METRIC EXTENSION\*

Alan J. Goldmann, \*\*

Gamma of New York, Brooklyn College

The problem we wish to discuss in this paper can be stated in general terms as follows: If a space  $K$ , in which a notion of "distance" has been defined, lies in a larger space  $S$ , then how can we extend the distance-concept of  $K$  to all of  $S$ ? We shall show that under a special hypothesis concerning the relationship of  $K$  and  $S$ , the extension can be completed in one and only one way after it has been carried out "halfway," and that the method involved is a "natural" one.

First we make precise the way in which "distance" is to be defined. By a metric space is meant a set of elements called points, for all ordered pairs of which there is defined a real-valued metric or distance-function  $F$  such that:

- M 1.  $F(x,y) \geq 0$  for all choices of points  $x,y$  in the space.
- M 2.  $F(x,x) = 0$ .
- M 3.  $F(z,x) \leq F(x,y) + F(y,z)$ .

M1 and M2 are reasonable properties to demand of any candidate for the name of "distance"; M3 is called the "triangle" inequality, and reduces to the famous inequality of that name when  $F$  is the usual distance in the Euclidean plane.

A useful concept in a metric space is that of *convergence*; if  $p$  is a point and  $p_1, p_2, \dots = \{p_n\}$  a sequence of points in the space, we write  $\{p_n\} \rightarrow p$  or  $\lim p_n = p$  if

\*A paper presented before the Pi Mu Epsilon at the scientific session, Monday evening during the National Meeting held at Michigan State College, East Lansing, Michigan, on December 28, 1952.

\*\*The Author of this paper was present at the National Meeting.

$\lim F(p_n, p) = 0$ ; this reduces the idea of convergence in a metric space to the (known) one of convergence in the system of real numbers.

It should be noted that the same space can be assigned many different metrics; for instance, in the Cartesian "r-s" plane, the distance from point  $P(r_1, s_1)$  to point  $Q(r_2, s_2)$  might be given by the familiar  $\sqrt{(r_2-r_1)^2 + (s_2-s_1)^2}$  or by  $|r_2-r_1| + |s_2-s_1|$ . In either case M1, M2, M3 are satisfied; geometrically, use of the second metric instead of the first involves replacing the length of the hypotenuse of a right triangle by the sum of the lengths of its legs. These considerations show that the distance-function is not implicit in the mathematical space, but is an additional construct.

Both distance-functions given above for the Cartesian plane satisfy an additional condition:

$$M\ 4.\ F(x,y) = 0 \text{ only if } x = y.$$

It is customary to impose this condition on a metric, in addition to M1, M2, M3; such a restriction is extremely desirable, in that without it, for instance, a sequence of points might converge to more than one limit point. M4 cannot be usefully introduced here, however, and at the end of the paper we shall explain why.

Lemma: If  $F$  satisfied M2 and M3, then  $F(x,z) = F(z,x)$ .

Proof. In M3, take  $y = x$  and apply M2. The result is  $F(z,x) \leq F(x,z)$ . Then in M3 first interchange  $x$  and  $z$ , getting  $F(x,z) \leq F(z,y) + F(y,x)$ , and then take  $y = z$ , obtaining  $F(x,z) \leq F(z,x)$ . The two inequalities imply the desired equality.

An obvious consequence is that in a metric space, the distance between two points does not depend on the order in which the points are considered. Any respectable "distance" should certainly have this property, and it will be used without explicit mention in the proofs of the theorems to come. This comment concludes the generalities on "distance."

The considerations of this paper were suggested by a rather specific situation which we proceed to describe.

Suppose we wish to measure the distance between points P and Q lying on the edge of a "ruler." If the ruler bears a metric scale with subdivisions of all orders of fineness, then we might well adopt a procedure described sufficiently by the following particular case: Suppose the unit-cm. point which is the closest to P of those preceding P on the scale is the **1-cm.-point**, while the corresponding point for Q is the **3-cm.-point**. Then as a first approximation,  $\overline{PQ} \approx (3-1) = 2\text{cm}$ . Working with the **10<sup>-1</sup>-cm.-points**, we might find **1.3cm.** nearest P and **3.6cm.** nearest Q, so that our next approximation would be  $\overline{PQ} \approx (3.6-1.3) = 2.3\text{cm}$ . Continuing in similar manner, we might obtain as a sequence of points of scale-division (to be called scale-points) for P, {1, 1.3, 1.33, 1.333, etc.}, and perhaps the sequence {3, 3.6, 3.66, 3.666, etc.] for Q. Then the sequence of approximations for PQ is (2, 2.3, 2.33, 2.333, etc.) and we should conclude that  $\overline{PQ} = 2\frac{1}{3}\text{ cm}$ .

We now translate certain salient features of this situation into the language of our general problem. The set of all scale-points constitutes a space K on which a **distance-measuring metric**  $F(x,y)$  is already defined by the numbering of the scale, and this K is embedded in the space S consisting of all points on the ruler's edge. We assumed that we could approximate any given point P (or Q) of S by a sequence of points of K,  $\{p_n\} \rightarrow P$ ; this state of affairs is described by saying that **K is dense** in S. But to say  $\{p_n\} \rightarrow P$  is to say that the distance between  $p_n$  and P tends to zero, and so we must be able to measure such distances; although we cannot yet measure the distance between any two points of S, we must suppose that we can measure the distance between a point of K and a point of S. Such distances define an extension  $G(x,y)$  of  $F(x,y)$ .

Before proceeding to the actual construction, we state the general problem in its final form. A metric space K, of distance-function  $F(x,y)$ , lies in a larger space S.  $F(x,y)$  has already been extended somehow to a  $G(x,y)$  defined whenever one of  $x,y$  is in K;  $G(x,y)$  is nearly a metric for S, in that it satisfied M1, M2, **M3**, but fails to be one since it is not defined for all point-pairs of S. K is dense in S, in terms of G, and we wish to extend  $G(x,y)$ , and thus  $F(x,y)$ , to a metric  $H(x,y)$  for S.

The essential ideas of the method are indicated by the specific situation considered. Given points P and Q of S, we "approximate" each of them by a sequence of points in K, and consider the limit of the resulting approximate distances." We show that this limit exists (Theorem 1), is independent of the choice of the approximating sequences (Theorem 2), and is really a metric (Theorem 3) which extends  $G(x,y)$  (Theorem 4). It is even the only extension of  $G(x,y)$  to S(Theorem 5).

We begin with 2 trivial extension of  $G(x,y)$ , by defining  $G(x,x) = 0$  even when x is not in K. Now our Lemma applies to  $G(x,y)$ . Note that G and F are equal when x and y are in K.

**Theorem 1.** Given P and Q in S, and sequences of points of K:  $\{p_n\} \rightarrow P$  and  $\{q_n\} \rightarrow Q$ . Then  $\lim F(p_n, q_n)$  exists.

Proof:

$$\begin{aligned} F(p_n, q_n) &= G(p_n, q_n) \leq G(q_n, q_m) + G(q_m, p_n) \\ &\leq G(q_n, q_m) + G(p_m, p_n) + G(p_m, q_m), \end{aligned}$$

$$\begin{aligned} F(p_m, q_m) &= G(p_m, q_m) \leq G(q_m, q_n) + G(q_n, p_m) \\ &\leq G(q_m, q_n) + G(p_m, p_n) + G(p_n, q_n). \end{aligned}$$

Then

$$G(p_n, q_n) - G(p_m, q_m) \leq G(q_n, q_m) + G(p_m, p_n),$$

and

$$G(p_m, q_m) - G(p_n, q_n) \leq G(q_m, q_n) + G(p_m, p_n).$$

The left hand sides of the proceeding two inequalities are numerically the same and one is positive.

$$\begin{aligned} \text{So } |F(p_n, q_n) - F(p_m, q_m)| &\leq G(q_m, q_n) + G(p_m, p_n) \\ &\leq G(q_m, Q) + G(Q, q_n) + \\ &\quad G(p_m, P) + G(P, p_n). \end{aligned}$$

Since  $\{p_n\} \rightarrow P$  and  $\{q_n\} \rightarrow Q$ , the last inequality implies that the sequence of approximate distances  $F(p_n, q_n)$  Satisfied the Cauchy Convergence Criterion.

Suppose we wish to measure the distance between points P and Q lying on the edge of a "ruler." If the ruler bears a metric scale with subdivisions of all orders of fineness, then we might well adopt a procedure described sufficiently by the following particular case: Suppose the unit-cm. point which is the closest to P of those preceding P on the scale is the **1-cm.-point**, while the corresponding point for Q is the **3-cm.-point**. Then as a first approximation,  $\overline{PQ} = 2\text{cm}$ . Working with the **10<sup>-1</sup>-cm.-points**, we might find **1.3cm.** nearest P and **3.6cm.** nearest Q, so that our next approximation would be  $\overline{PQ} \approx (3.6 - 1.3) = 2.3\text{cm}$ . Continuing in similar manner, we might obtain as a sequence of points of scale-division (to be called scale-points) for P,  $1, 1.3, 1.33, 1.333, \dots$ , and perhaps the sequence  $\{3, 3.6, 3.66, 3.666, \dots\}$  for Q. Then the sequence of approximations for  $\overline{PQ}$  is  $\{2, 2.3, 2.33, 2.333, \dots\}$  and we should conclude that  $\overline{PQ} = 2\frac{1}{3}\text{ cm}$ .

We now translate certain salient features of this situation into the language of our general problem. The set of all scale-points constitutes a space K on which a **distance-measuring metric**  $F(x,y)$  is already defined by the numbering of the scale, and this K is embedded in the space S consisting of all points on the ruler's edge. We assumed that we could approximate any given point P (or Q) of S by a sequence of points of K,  $\{p_n\} \rightarrow P$ ; this state of affairs is described by saying that K is **dense** in S. But to say  $\{p_n\} \rightarrow P$  is to say that the distance between  $p_n$  and P tends to zero, and so we must be able to measure such distances; although we cannot yet measure the distance between any two points of S, we must suppose that we can measure the distance between a point of K and a point of S. Such distances define an extension  $G(x,y)$  of  $F(x,y)$ .

Before proceeding to the actual construction, we state the general problem in its final form. A metric space K, of distance-function  $F(x,y)$ , lies in a larger space S.  $F(x,y)$  has already been extended somehow to a  $G(x,y)$  defined whenever one of x,y is in K;  $G(x,y)$  is nearly a metric for S, in that it satisfied M1, M2, **M3**, but fails to be one since it is not defined for all point-pairs of S. K is dense in S, in terms of G, and we wish to extend  $G(x,y)$ , and thus  $F(x,y)$ , to a metric H(x,y) for S.

The essential ideas of the method are indicated by the specific situation considered. Given points P and Q of S, we "approximate" each of them by a sequence of points in K, and consider the limit of the resulting "approximate distances." We show that this limit exists (Theorem 1), is independent of the choice of the approximating sequences (Theorem 2), and is really a metric (Theorem 3) which extends  $G(x,y)$  (Theorem 4). It is even the only extension of  $G(x,y)$  to S (Theorem 5).

We begin with 2 trivial extension of  $G(x,y)$ , by defining  $G(x,x) = 0$  even when x is **not** in K. Now our Lemma applies to  $G(x,y)$ . Note that G and F are equal when x and y are in K.

**Theorem 1.** Given P and Q in S, and sequences of points of K:  $\{p_n\} \rightarrow P$  and  $\{q_n\} \rightarrow Q$ . Then  $\lim F(p_n, q_n)$  exists.

Proof:

$$\begin{aligned} F(p_n, q_n) &= G(p_n, q_n) \leq G(q_n, q_m) + G(q_m, p_n) \\ &\leq G(q_n, q_m) + G(p_m, p_n) + G(p_m, q_m), \end{aligned}$$

$$\begin{aligned} F(p_m, q_m) &= G(p_m, q_m) \leq G(q_m, q_n) + G(q_n, p_m) \\ &\leq G(q_m, q_n) + G(p_m, p_n) + G(p_n, q_m). \end{aligned}$$

Then

$$G(p_n, q_n) - G(p_m, q_m) \leq G(q_n, q_m) + G(p_m, p_n),$$

and

$$G(p_m, q_m) - G(p_n, q_n) \leq G(q_m, q_n) + G(p_n, p_m).$$

The left hand sides of the preceding two inequalities are numerically the same and one is positive.

$$\begin{aligned} \text{So } |F(p_n, q_n) - F(p_m, q_m)| &\leq G(q_m, q_n) + G(p_m, p_n) \\ &\quad G(q_m, Q) + G(Q, q_n) + \\ &\quad G(p_m, P) + G(P, p_n). \end{aligned}$$

Since  $\{p_n\} \rightarrow P$  and  $\{q_n\} \rightarrow Q$ , the last inequality implies that the sequence of approximate distances  $F(p_n, q_n)$  Satisfied the Cauchy Convergence Criterion.

Theorem 2. Given  $P$  and  $Q$  in  $S$ , and sequences of points of  $K$ :  $\{p_n\} \rightarrow P$ ,  $\{\bar{p}_n\} \rightarrow P$ ,  $\{q_n\} \rightarrow Q$ ,  $\{\bar{q}_n\} \rightarrow Q$ . Then  $\lim F(p_n, q_n) = \lim F(\bar{p}_n, \bar{q}_n)$ .

*Proof:* The two limits exist by Theorem 1.

$$\begin{aligned} F(p_n, q_n) &= G(p_n, q_n) \leq G(q_n, \bar{p}_n) + G(\bar{p}_n, p_n) \\ &\leq G(q_n, \bar{q}_n) + G(\bar{q}_n, \bar{p}_n) + \\ &\quad G(\bar{p}_n, p_n) \\ &\leq G(\bar{p}_n, \bar{q}_n) + G(q_n, Q) + \\ &\quad G(Q, \bar{q}_n) + G(p_n, P) + \\ &\quad G(P, \bar{p}_n), \end{aligned}$$

$$\begin{aligned} F(\bar{p}_n, \bar{q}_n) &= G(\bar{p}_n, \bar{q}_n) \leq G(\bar{q}_n, p_n) + G(p_n, \bar{p}_n) \\ &\leq G(p_n, q_n) + G(\bar{q}_n, q_n) + \\ &\quad G(p_n, \bar{p}_n) \\ &\leq G(p_n, q_n) + G(q_n, Q) + \\ &\quad G(Q, \bar{q}_n) + G(\bar{p}_n, P) + \\ &\quad G(P, p_n), \end{aligned}$$

so

$$|F(p_n, q_n) - F(\bar{p}_n, \bar{q}_n)| \leq G(p_n, P) + G(\bar{p}_n, P) + \\ G(q_n, Q) + G(\bar{q}_n, Q) \rightarrow 0.$$

Thus  $\lim F(p_n, q_n) = \lim F(\bar{p}_n, \bar{q}_n)$ .

**Definition:** Let  $P, Q$  be any points of  $S$ . Since  $K$  is dense in  $S$ , there exist sequences of points of  $K$ :  $\{p_n\} \rightarrow P$  and  $\{q_n\} \rightarrow Q$ . Theorems 1 and 2 permit us to define our new **distance-function** by:

$$H(P, Q) = \lim F(p_n, q_n).$$

**Theorem 3.**  $H(P, Q)$  is a metric.

**Proof:** M1:  $F(p_n, q_n) \geq 0$ , so  $H(P, Q) = \lim F(p_n, q_n) \geq 0$ .

M2: Take  $p_n = q_n$ . Then  $H(P, P) = \lim F(p_n, p_n) = 0$ .

M3: Since  $F(r_n, p_n) \leq F(p_n, q_n) + F(q_n, r_n)$ , we have  $\lim F(r_n, p_n) \leq \lim F(p_n, q_n) + \lim F(q_n, r_n)$ , or  $H(R, P) \leq H(P, Q) + H(Q, R)$ .

**Theorem 4.**  $H(P, Q)$  is an extension of  $G$ .

**Proof:**

Take any  $p$  in  $K$  and  $q_n$  in  $K$ , and any  $Q$  in  $S$ .

Take  $p_n = p$ , and  $\{q_n\} \rightarrow Q$ . Then

$G(p, Q) \leq G(Q, q_n) + G(q_n, p)$  and

$G(p, q_n) \leq G(q_n, Q) + G(Q, p)$ , so

$0 \leq |G(p, Q) - G(p, q_n)| \leq G(q_n, Q) \rightarrow 0$ .

Thus  $G(p, Q) = \lim G(p, q_n) = \lim F(p, q_n) = H(p, Q)$ .

**Theorem 5.**  $H(P, Q)$  is the only possible extension of  $G$  to a metric for  $S$ .

**Proof:** Let  $J$  be another such extension of  $G$ . Then in particular it is an extension of  $F$ . Given any  $P, Q$  in  $S$ , we choose approximating sequences as usual. Then

$$J(P, Q) \leq J(P, p_n) + J(p_n, Q) \leq J(P, p_n) + J(p_n, q_n) + J(q_n, Q),$$

$$J(p_n, q_n) \leq J(p_n, P) + J(P, q_n) \leq J(P, p_n) + J(P, Q) + J(q_n, Q);$$

And

$$|J(P, Q) - J(p_n, q_n)| \equiv |J(P, Q) - F(p_n, q_n)| \leq J(P, p_n) + J(q_n, Q) \rightarrow 0.$$

Thus  $J(P, Q) = \lim F(p_n, q_n) = H(P, Q)$ , and  $J$  coincides with  $H$ .

In order to justify our failure to restrict  $F$  by M4, we conclude by proving that such a restriction would be unprofitable, in that  $H$  might not obey M4 even if  $F$  did. To show this, the following example suffices: Let  $p$  be a fixed positive prime, and for any positive integer  $N$  define  $V_p(N)$  as the exponent of the highest power of  $p$  dividing  $N$ . In the space  $S$  composed of the positive rationals  $m/n$  in lowest terms we choose as metric

$$H\left(\frac{m_1}{n_1}, \frac{m_2}{n_2}\right) = |V_p(m_1 n_1) - V_p(m_2 n_2)|$$

clearly satisfying M1, M2, M3. Let K be the set of integers 1, p,  $p^2$ ,  $p^3$ , etc. Define F and G over their proper domains

as equal to H. For any  $m/n$ , we have  $G(m/n, p^{V_p(mn)}) = O$ ,

so that the sequence obtained by iterating  $p^{V_p(mn)}$  is a sequence of points of K converging to  $m/n$ . Thus K is dense in S, and our previous discussion applies. Furthermore, F obeys M4, since for s, t  $\geq O$ , we have  $O = F(p^s, p^t) = H(p^s, p^t) = |s-t|$  only if  $s = t$  and thus only if  $p^s = p^t$ . M4 fails for H, however, since  $H(p, p^{-1}) = |1-1| = O$ . Thus in spite of all our efforts, S may turn out to be a rather queer metric space in which two distinct points can be zero-distant.

#### NOTE by R. C. A., 30 December 1953

It is a most extraordinary fact that during the past six years three thirteenth century manuscripts of Omar Khayyām's Rubāiyāt became available for scholars. Apparently they were originally in the library of a learned mullah, who died some years ago in Persia, and came to light when sold at auction. The oldest of these manuscripts, dated 1207, was acquired by the University of Cambridge in 1950. Its 252 quatrains are translated into 252 double quatrains by Professor A. J. Arberry in his *Omar Khayyām. A New Version Based upon Recent Discoveries* (1952).

The New York manuscript, dated 1216, was discussed in some detail by Professor Arberry in Studies Presented to Vladimir Minorsky, London University, School of Oriental and South African Studies, *Bulletin*, v. 14, part 3, 1952, "Omar again," p. 413-419. In this manuscript are 247 quatrains. Still further items of interest with regard to the manuscript are in the illustrated auctioneer's catalogue of Parke-Bernet Galleries, New York, December 15, 1953, 16p.: *The Second Earliest Known Manuscript of the Rubaiyat of Omar Khayyam. Sold by Order of the Present Owner*. The manuscript was sold to a Philadelphia collector for \$4,500.

The third manuscript, in the possession of the "noble and discriminating patron of oriental art and letters," Alfred Chester Beatty of Dublin, was published in a sumptuous volume (London, 1949) with the following title: *THE RUBĀ'İYĀT OF OMAR KHAYYĀM, edited from a newly discovered manuscript dated 658 (1259 - 60) in the possession of A. Chester Beatty Esq., by A. J. Arberry, Sir Thorne-as Adams's Professor of Arabic in the University of Cambridge, with Comparative English Versions by Edward FitzGerald, E. H. Whinfield, and the Editor*. This manuscript contains 172 quatrains. The New York manuscript contains all of these quatrains, and 245 of the 252 quatrains in the Cambridge manuscript.

For Swinburne's reaction, upon acquisition of his penny Copy of FitzGerald's 1859 masterpiece, see Lionel Stevenson, *The Ordeal of George Meredith*, New York, 1953, p. 113-114.

## CORRIGENDA, VOLUME 1, NUMBER 9

- R. C. Archibald, *Notes on Omar Khayyām* (1050-1122) and *Recent Discoveries*.
- p. 352, line 20, for **quatrians**, read quatrains  
lines - 2-3, far 101 quotations, read 101 quatrains
- p. 354, for  $x^3 + cx^2 + bx = a$ , read  $x^3 + cx^2 + bx = a$   
for  $x^3 + bx = cxta$ , read  $x^3 + bx = cx^2 + a$   
footnote 7, lines 2 and 7, for **Woepcke**, read Woepcke  
and in line -3 for Lucky, read Luckey
- p. 357, line -13, for Lucky, read Luckey  
line -2, for **Beitrage**, read **Beiträge**  
and for **Chajjam**, read **Chajjām**  
line -1, for **Chajjam**, read **Chajjām**  
for **ältesten Auszüge**, read **ältesten Auszüge**  
and for **Chajjams**, read **Chajjāms**
- p. 358, line I, for **Ruba'ijjat**, read **Rubā'ijjāt**  
line 10, for Khayyam, read **Khayyām**  
line -1, for **nam**, read name  
for Merciful, read Merciful  
Plate, for **Khayyam**, read **Khayyām**

\* \* \*

## LETTER TO THE EDITOR

Dear Miss Stokes:

Thank you for remembering to send to me the interesting Pi Mu Epsilon Journal of November 1953, which contained the article by R. C. Archibald, "Notes on Omar Khayyām" (1050 - 1122) and Recent Discoveries."

I am very much interested in any Omar material and am collecting quite a shelf of the subject. I was with my uncle, Dr. David Eugene Smith, when he was doing research for his book of **Rubāiyāt** of Omar Khayyām. He endeavored to get an Iranian student interested in doing research in Iran when we were there in 1933.

Cortland, New York  
February 7, 1954.

Helen E. McAleer

## PROBLEM DEPARTMENT

Edited by

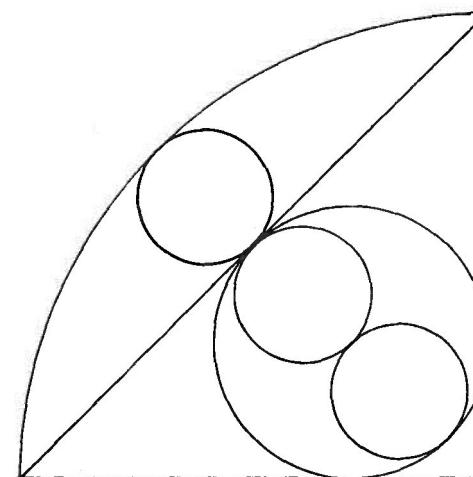
Leo Moser, University of Alberta

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's degree. Solutions of these problems should be submitted on separate, signed sheets within five months after publication. Address all communications concerning problems to Leo Moser, Mathematics Department, University of Alberta, Edmonton, Alberta, Canada.

## PROBLEMS FOR SOLUTION

63. *Proposed by Leon Bankoff, Los Angeles, California*

State and solve the problem suggested by the following diagram.



64. *Proposed by Fred Gross, Brooklyn College*

Prove that

$$x^9 - 6x^7 + 9x^5 - 4x^3$$

is divisible by 27 for all positive integral values of x.

65. *Proposed by Martin Schechter, Brooklyn, N. Y.*

Prove that every simple polygon which is not a triangle has at least one of its diagonals lying entirely inside of it.

66. *Proposed by C. W. Trigg, Los Angeles City College*

If three circles with radii a, b, c, are externally tangent, there are two circles with radii  $r$ , R which touch the three circles. Show that

$$\frac{1}{r} - \frac{1}{R} = 2\left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

and that

$$\frac{1}{r} + \frac{1}{R} = 4\sqrt{\frac{a+b+c}{abc}}$$

67. *Proposed by Pedro Piza, San Juan, Puerto Rico*

Find four numbers in arithmetical progression and three in geometrical progression such that the sum of the squares of the four is equal to the sum of the squares of the three.

## SOLUTIONS

49. *Proposed by C. S. Venkataraman, Trichur, India*

If  $s = (a + b + c + d)/2$  and  $S = abcd$ , prove that

$$s^4 + (s - b - c)^4 + (s - c - d)^4 + (s - d - b)^4 - (s - a)^4 \\ - (s - b)^4 - (s - c)^4 - (s - d)^4 = 12S.$$

*Solution by the Proposer*

Replacing s by its value we find that the right hand side is homogeneous of the fourth degree and vanishes when  $a = 0, b = 0, c = 0, d = 0$ . Hence it is equal to  $KS$  and  $K$  can readily be found to be 12 by taking  $a = b = c = d = 1$ .

Also solved by Louisa S. Grinstein, Buffalo, New York and C. W. Trigg, Los Angeles City College.

52. *Proposed by R. T. Sharp, McGill University, Montreal*

A set of n smooth dominoes  $1'' \times 2'' \times \frac{1}{4}''$  is piled on a table, one horizontally placed domino in each layer. Find the largest distance that the top domino can be made to overhang the bottom one.

*Solution by C. W. Trigg, Los Angeles City College*

If a pile of  $(K - 1)$  rectangular parallelepipeds, each of length  $2x$ , is placed on another horizontal parallelepiped and slid along until its center of gravity lies above the leading edge of the Kth parallelepiped, the center of gravity of the entire configuration lies a horizontal distance  $x/K$  from the leading edge of the Kth parallelepiped.

It follows that a columnar pile of the n dominoes, with  $x = 1''$ , may be slid between two parallel vertical planes  $1''$  apart so that the successive overhangs from the top down constitute the series,

$$1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n-1},$$

thus forming a half-arch in equilibrium. Since the harmonic series is divergent, the sum of this series will increase with n and the overhang of the top domino over the bottom one may be made any desired amount.

In order that the *entire* top domino may overhang the bottom one, n must be at least 5, in which case the total overhang of the leading edge of the top domino is  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}$  or approximately 2.0833''. For n = 12, the total overhang is 3.0199''.

If the pile be **slid** so that the diagonals of the dominoes lie in the same **vertical** plane and the centers of gravity of the various **sub-piles** from the top down lie over a corner of the leading **edge of** the supporting domino, then the total overhand will be  $\sqrt{1^2 + (\frac{1}{2})^2}$  or  $\frac{1}{2}\sqrt{5}$  times that of the first mentioned **arrangement**.

Also solved by A. E. Livingston, University of Washington, and the proposer.

**53. Proposed by Leon Bankoff, Los Angeles, California**

A rectangular slab of width  $w$  is moved horizontally from one **corridor of width**  $a$  into another at right angles to it, of width  $b$ . What is the maximum value of the length  $\ell$ , of the slab, that will permit passage?

*Solution by the Proposer*

Let  $\theta$  denote the angle between the side  $\ell$  and either wall of the **corridor** of width  $a$ . Referring to the diagram, it is evident that

$$\ell = a \csc \theta + b \sec \theta - w \tan \theta - w \cot \theta \quad (1)$$

Then  $\frac{d\ell}{d\theta} = -a \csc \theta \cot \theta + b \sec \theta \tan \theta - w \sec^2 \theta + w \csc^2 \theta$ .

The minimum value of  $\ell$ , found by equating the first derivative to zero, represents the maximum value of  $\ell$  that will permit passage of the slab from one corridor to the other. Thus

$$b \sec \theta \tan \theta - a \csc \theta \cot \theta + w(\csc^2 \theta - \sec^2 \theta) = 0. \quad (2)$$

Now, if we let  $\sin \theta = x$ , we get

$$\tan \theta = \frac{x}{\sqrt{1-x^2}}; \cot \theta = \frac{\sqrt{1-x^2}}{x}; \sec \theta = \frac{1}{\sqrt{1-x^2}}; \csc \theta = \frac{1}{x}.$$

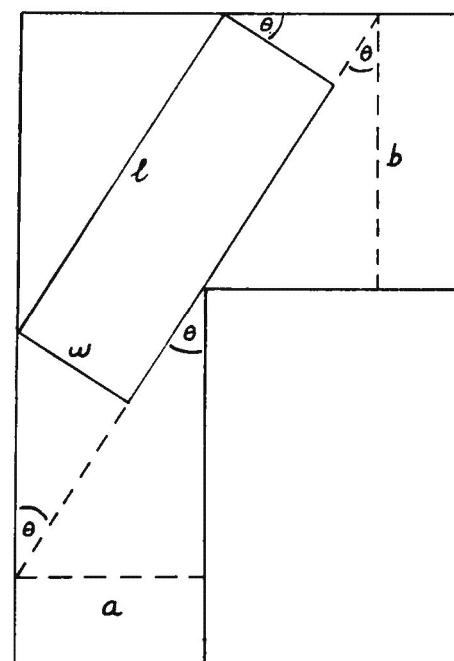
Substituting for  $\theta$  in (2), we get

$$\frac{bx}{1-x^2} - \frac{a\sqrt{1-x^2}}{x} + w\left(\frac{1}{x^2} - \frac{1}{1-x^2}\right) = 0.$$

Simplifying and squaring, we obtain

$$x^6(a^2 + b^2) - 4wbx^5 + x^4(4w^2 + 3a^2) + 2wbx^3 - x^2(3a^2 - 4w^2) + w^2 - a^2 = 0. \quad (3)$$

Since all the coefficients of the powers of  $x$  are given the **sextic** may be solved for  $x$ , yielding the value of  $\sin \theta$  for which  $\ell$  is extreme. Substituting corresponding values for function of  $\theta$  in (1), we obtain the desired value of  $\ell$ .



**58. Proposed by C. W. Trigg, Los Angeles City College**

Unscramble each of the numbered words and phrases to obtain the names of twenty-nine mathematicians. It will be observed that the initials of the names, in order, read PI MU EPSILON JOURNAL, MATHEMATICS.

- (1) REAP COIN; (2) AIM HIS CLUB; (3) MEN SNEER;
- (4) SPUN KEYS; (5) NINE TIES; (6) A PLOT; (7) NO

**ALMS;** (8) **IN ZEBRA;** (9) **TALL HOPI;** (10) **THE COURD;** (11) **SHUN A COMIC;** (12) **I ADJOURN;** (13) **OO!! DOGS;** (14) **WONDER DUO;** (15) **RED CORE;** (16) **PANIER;** (17) **BAR A CHILD;** (18) **MEN IN LAND;** (19) **MORE IN CASH;** (20) **SAUL IN POOL;** (21) **FETCH, BY HEC;** (22) **CROP HIS PATE;** (23) **OX DUE US;** (24) **GNOME;** (25) **BALE;** (26) **CHERISH SUN TAN;** (27) **FLED IN;** (28) **CARTON;** (29) **SLIT RING.**

*Solution by Nathaniel Grossman, Aurora, Ill.*

(1) POINCARE; (2) IAMBlichus; (3) MERSENNE; (4) USPENSKY; (5) EINSTEIN; (6) PLATO; (7) SALMON; (8) IBN EZRA; (9) L'HOPITAL; (10) OUGHTRED; (11) NICOMACHUS; (12) JOURDAIN; (13) OSGOOD; (14) UNDERWOOD; (15) RECORDE; (16) NAPIER; (17) ARCHIBALD; (18) LINDEMANN; (19) MASCHERONI; (20) APOLLONIUS; (21) TCHEBYCHEF; (22) HIPPOCRATES; (23) EUDOXUS; (24) MONGE; (25) ABEL; (26) TSCHIRNHAUSEN; (27) INFELD; (28) CANTOR OR CARNOT; (29) STIRLING.

Also solved by Leon Bankoff, Ruth Stokes and the proposer.

**Bankoff** points out that omitted from the list is the name of the editor of the problem section of the Pi Mu Epsilon Journal, which **may** be written: SORE MOLE; MORE SOLE; LOSE MORE; EEL ROOMS; OLE MORSE; ORE MOLES; etc.

### 59. Proposed by P. Piza, San Juan, Puerto Rico

Let

$$a = 1 + 2 + 3 + 4 + \dots + n,$$

$$b = 1^3 + 2^3 + 3^3 + 4^3 + \dots + n^3,$$

$$c = 1^5 + 2^5 + 3^5 + 4^5 + \dots + n^5,$$

$$d = 1^7 + 2^7 + 3^7 + 4^7 + \dots + n^7.$$

And let  $t_n = n(n+1)/2$  be the  $n$  th triangular number.

Prove the following relation true for arbitrary  $n$ :

$$a + b + c + d = 2(t_n^4 + t_{t_n}).$$

*Solution by Louisa S. Grinstein, Buffalo, New York*

It is well known that

$$a = n(n+1)/2, \quad b = n^2(n+1)^2/4$$

$$c = n^2(n+1)^2(2n^2 + 2n - 1)/12$$

$$d = n^2(n+1)^2(3n^4 + 6n^3 - n^2 - 4n + 2)/24$$

Adding and regrouping we find

$$a + b + c + d = n^4(n+1)^4/8 + [n(n+1)/2]$$

$$[n(n+1)/2 + 1] = 2(t_n^4 + t_{t_n}).$$

Also solved by Fred Gross, Brooklyn College, Martin Schechter, Brooklyn, N. Y., C. W. Trigg, Los Angeles City College, and the Proposer. The solutions of Trigg and Gross were almost identical with the one given above while **Piza** used induction over  $n$ .

NATIONAL MEETING  
OF THE PI MU EPSILON FRATERNITY  
BALTIMORE, MARYLAND

Minutes of the Meeting, December 28, 1953

A national meeting of the Pi Mu Epsilon Fraternity was held in the south lounge of Levering Hall on the Homewood Campus of Johns Hopkins University, Baltimore, Maryland, on December 28, 1953.

An informal gathering of delegates was held from 10:00 a.m. to 12:00 during the meeting of the National Council. Luncheon at noon was followed by a business meeting which adjourned at 2:30 p.m. Then after a brief intermission about half of the group stayed on until 4:00 p.m. to hear something of the history of Pi Mu Epsilon and to participate in further exchange of ideas.

Twenty-two chapters were represented at the business meeting by the following delegates and members.

California Alpha	Sophia McDonald (Councilor General)
Delaware Alpha	L. Vernon Lewis
D. C. Alpha	G. C. Webber
Georgia Alpha	Arthur N. Thorpe
Illinois Alpha	T. Fort (Councilor General)
Iowa Alpha	Peter G. Braunfeld
Kansas Alpha	S. S. Cairns (Councilor General)
Kentucky Alpha	Franz Hohn
Michigan Alpha	Gary H. Meisters
Missouri Alpha	Elliott Ward Cheney, Jr.
Missouri Gamma	Joseph B. Cornelison
	J. A. Ward
	Delia Koo
	J. S. Frame (Secretary-Treas. General)
	John D. Bentley
	Gerald T. Dobrindt

1954

## NATIONAL MEETING

417

New York Alpha

William Kopka  
Ruth Stokes  
(P.M.E. Journal Editor & C.G.)

New York Beta

June Zimany  
Ellen Praus  
Sandra Biderman  
J. H. Bushey  
Frances Goldman

North Carolina Beta

W. M. Whyburn  
(Vice-Director General)

Ohio Alpha  
Ohio Delta  
Oklahoma Alpha  
Oklahoma Beta  
Pennsylvania Beta

Phyllis Rubin  
Frederick G. Werner  
John E. Hoffman  
Yaroslaw Stachiw  
Ronald H. Broadhurst  
Bruce Cloud  
Lawrence J. Wallen  
G. L. Raynor  
H. P. Pettit  
S. Ashby Foote  
C. C. MacDuffee  
(Director General)

Pennsylvania Gamma

Wisconsin Alpha  
Wisconsin Beta

The minutes of the September 1952 meeting at East Lansing, Michigan, were read and approved.

Director General MacDuffee reported on the installation of the Alabama Beta chapter at Alabama Polytechnic Institute on April 21, 1953, and of the New York Theta chapter at Cornell on May 22, 1953. The pending petition of Rutgers University has 17 favorable votes. If it is approved, New Jersey Alpha will be the 56th chapter.

Secretary General Frame reported an increase in members of 1048 in the year 1953, bringing the total membership to 22,062. Then Treasurer General Frame gave a financial report indicating a balance in the treasury of \$4901.13, from which travel expenses to the meeting are still to be paid.

Miss Ruth Stokes, in her report as Editor of the Pi Mu Epsilon Journal, indicated that volume 1 would close with number 10, and recommended that in the future each volume

should cover the period of service of one editor. She commended the work of the Associate Editors, Walker, McCoy, Frame, **Moser**, and **Karnes**. She presented the financial report and notes that the Journal has **297** paid subscribers in addition to initiates. She urged students to gain experience by writing expository articles for publication in the Journal, and to contribute by sending in problems and solutions.

The report of the nominating committee was presented, as given on the enclosed ballot (a copy of the ballot follows these minutes, p. **419**). Since no candidate for business manager of the **Journal** had been found, the name of Paul Gilbert was suggested by Miss Stokes, but he has subsequently declined the nomination. The rest of the slate was approved without additions from the floor.

Two amendments were presented by the National Council and were approved by the meeting after discussion.

**1.** Moved by G. C. Webber of Delaware Alpha, seconded by Delia Koo of Michigan Alpha, and unanimously approved for referral to the chapters: To amend Article V, section 3 of the Constitution and By Laws of the Pi Mu Epsilon Fraternity by changing the date "April 1" to "July 1."

As explained in the agenda for the meeting, the months of April and **May** are the busiest months of the year for the preparation of certificates by the Secretary General; and not the easiest time for a new man to learn his job, and for transferring the files. It would also be difficult for a new Journal Editor to take over on April 1, just before the Spring issue is off the press.

**2.** Moved by **H. P. Pettit** of Wisconsin Alpha, seconded by **Ashby** Foote of Wisconsin Beta and approved for referral to the chapters: To amend Article **IX** to include the sentence "Each chapter of Pi Mu Epsilon shall select a permanent corresponding secretary (to whom official correspondence for the chapter shall be sent) who shall be a member of the mathematics department with the rank of instructor or higher at the institution where the chapter is located."

The purpose of this amendment is to keep a continuing and up-to-date mailing list of chapters for sending **commu-**

nications from the Secretary General, the Journal Editor, and other officers. Not infrequently when student officers leave their institutions they fail to report the names of their successors.

It was moved by June Zimany, New York Beta, and seconded by Ronald **Broadhurst**, Pennsylvania Beta, and supported by the convention that the following policy on travel reimbursement (similar to that of the Mathematical Association) be approved, subject to budgeting limitations: That the Director General, Secretary-Treasurer General, and Journal Editor be paid a sum up to first class rail fare plus **pullman** for one meeting a year, less travel expense available from other sources; that the Vice-Director General and other General Officers be paid travel expense on the basis of one-third the above rate, provided that the total reimbursement from all sources does not exceed first class fare; and that reimbursement for student delegates be decided each year on recommendation of the National Council.

The place and time of the next meeting was discussed, and it was suggested that the location be west of the Mississippi, but no action was taken.

Mr. John Hoffman described and distributed copies of the mathematical newsletter printed by the Oklahoma Alpha chapter.

There being no further business, the meeting was adjourned.

Respectfully submitted,

J. Sutherland Frame  
Secretary-Treasurer General

Just as the manuscript for the **April** issue of the Journal was going to press the editor received a copy of the ballot showing not only nominations for General Officers of Pi Mu

Epsilon who will take office on July 1, 1954, for a **three-year** term, but also showing the election results. So, instead of the ballot as originally printed, we give here the results of the election.

There were 31 ballots received by March 15, and the election was extremely close.

#### General Officers for Three-Year Term 1954-1957

Director General: S. S. Cairns, University of Illinois  
 Vice-Director General: J.S. Frame, Michigan State College  
 Secretary-Treasurer General: R. V. Andree, University of Oklahoma

#### Councilors General:

Wealthy Babcock, University of Kansas

R. F. Graesser, University of Arizona

Sophia L. McDonald, University of California, Berkeley

H. S. Thurston, University of Alabama

Editor of the Official Journal: Ruth W. Stokes, Syracuse University

Business Manager of the Official Journal: (Office not yet filled; so, by the constitution (p. 11), the present Business Manager Howard C. Bennett will continue to hold office until his successor has been elected and duly qualified. Mr. Bennett asked to be relieved of his duties.)

The chapters voted approval of the two amendments to the constitution (See statements of the amendments, as given in the minutes of the Baltimore meeting, above.)

#### REPORTS OF THE CHAPTERS

Edited by

Houston T. Karnes, Louisiana State University

(Send reports to Ruth W. Stokes, 15 Smith College, Syracuse University, Syracuse 10, New York.)

**EDITOR'S NOTE.** According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary-General and to the **Director-General**, an annual report of the chapter activities including programs of meetings, results of elections, etc." The **Secretary-General** now suggests that an additional copy of the annual report of each chapter be sent to the editor of the Pi Mu Epsilon Journal. Besides the information listed above we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships. These annual reports will be published in the chronological order in which they are received.

#### Eta of New York, University of Buffalo

The following papers were presented at regular meetings of the New York Eta chapter during the 1952-1953 year:

"Pretty Patterns of Polygons or Regular Tesselations in Curved Spaces" by George W. Walker

"Celestial Mechanics<sup>a</sup>" by Dr. William H Davis of the Physics Department

"Fibonacci Numbers" by Rita Mochan

"An Area Expressed as a Limit" by Mary Harrington

"Semi-Complex Graphs" by Dr. Louis Kramer

"Mathematical Developments of Some Vibration Problems" by Robert C. Kroeger

"Measure<sup>p</sup>" by Dr. Daniel Orloff of the Cornell Aeronautical Laboratory.

The meeting on December 16, 1952, was a dinner meeting. Mathematical games and recreation were enjoyed by the members.

The following were elected to serve as officers for the year 1953-54: Director, Richard Barnes; Vice-Director, Leonard

Darbee; Secretary, Joseph Siciliano; Treasurer, Thomas Harris; Faculty Advisor, Howard W. Baeumler.

#### Alpha of Alabama, University of Alabama

During the academic year 1952-1953, the Alabama Alpha chapter held five meetings. In addition, the chapter had a Christmas banquet and the annual spring picnic. The first meeting was held on October 16, when the director, Miss Betty Ellis,\* presented a report on the Pi Mu Epsilon National Convention held during the summer. At subsequent meetings the following papers were presented:

**"Photoelasticity"** by Dr. William D Jordan, Department of Engineering Mechanics

"A Problem in Potential Theory<sup>D</sup>" by Dr. J. H. Hornback, Mathematics Department

"The Logical Design for an Automatic Digital Computer<sup>D</sup>" by Dr. A S. Householder of the Oak Ridge Laboratories

"Inverting a Matrix by the Monte **Carlo** Method<sup>D</sup>" by Dr. Paul Hummel of the School of Commerce.

Eighteen new members were initiated during the year.

Officers elected for the year 1953-1954 were: Director, A. W. Yonda; Secretary, Betty Ellis; Treasurer, Roger Wicks; Faculty Advisor, Dr. Carl **Seebeck**; Social Chairman, Lou Ann Ray; Scholarship Chairman, Dr. J. H. Hornback.

#### Epsilon of New York, The St. Lawrence University

At the first meeting of the New York Epsilon chapter for the academic year of 1952-53, the following officers were elected: President, Heidi Genhart; Secretary, Hilda Budelman; Treasurer, George Van Wyck; Director, Dr. O. Kenneth Bates; Permanent Secretary, Dr. Ruth Peters.

Meetings were held monthly with the presentation of the following papers:

"Non-Euclidean Geometry<sup>D</sup>" by Hilda **Budelman**

"Wartime Radar Equipment<sup>D</sup>" by George Van Wyck

"The Actuarial Field<sup>D</sup>" by Bernard Silkowitz

"The Poisson Distribution Curve<sup>D</sup>" by Stuart Collins

" $e^{ix} = \cos x + i \sin x$ <sup>D</sup>" by Heidi Genhart

"The Steel Square<sup>D</sup>" by William **Dixon**.

---

\*Student delegate to the National Convention at Michigan State College, 1952.

On May 12 the **annual** picnic was held.

Officers for 1953-1954 are: President, Hilda Budelman; Secretary, Ellen Keenholts; Treasurer, Jack Kinney; Director, Dr. O. Kenneth Bates; Permanent Secretary, Dr. Ruth Peters.

#### Alpha of New York, Syracuse University

The New York Alpha chapter held six meetings during the year 1952-1953. The following papers were presented:

"Analogue Methods<sup>\*</sup>" by Dr. **Otway** Pardee

"Surfaces<sup>\*</sup>" by Dr. Erik Hemmingsen

"IBM Machines and the Computation Field<sup>D</sup>" by Dr. Walter Murdock of the IBM Corporation

"Looking Between the Stars<sup>D</sup>" by Dr. Kurt Sitte of the Physics Department

"Boolean Algebra<sup>D</sup>" by George Mulfinger and "Duplex Numbers" by Frank Raymond

"Curves of Constant Width" by Dr. Walter Baum.

The annual initiation banquet, welcoming thirty-three new members, was held December 17. The guest speaker was Dr. W. A. **Hurwitz** of the **Cornell** University Mathematics Department. He spoke on "Trigonometry Without Geometry."

Officers elected for 1953-54 were: Director, Norbert Bischof; Vice-Director, Richard H. Brans; Corresponding Secretary, Joan **Farber** Johnson; Recording Secretary, Patricia Hansell; Treasurer, George Finkbeiner.

#### Alpha of Florida, University of Miami

During the academic year of 1952-1953, the Florida Alpha chapter held six meetings. The following papers were presented:

"Hypercomplex Numbers<sup>D</sup>" by Dr. Harry S. Robertson

"Riemann Geometry and Relativity<sup>D</sup>" by Dr. **Mayme** I. Logsdon

"Sub-tropical Topology<sup>D</sup>" by Dr. **Wayman** L. Strother

"Directed Sets<sup>D</sup>" by Dr. Herman Meyer

"Relation Theory and Multivalued Functions<sup>D</sup>" by Dr. David **Foulis**

"Selected Mathematical Problems" by Dr. Elton J. **Moulton**.

The annual Christmas party was postponed until January 5. Entertainment was provided by members of the chapter after which refreshments were served.

Officers for 1952-1953 were: Director, Dr. **Mayme** I. Logsdon; Vice-Director, John Maecher; Secretary, Robert W. **FitzGerald**; Treasurer, David **Foulis**.

Officers for 1953-1954 are: Director, William **Gaylor**; Vice-Director, Walter **Roop**; Secretary, Edward Berger; Treasurer, Zwni Abboushi.

#### Beta of Illinois, Northwestern University

(Report of the activities of the chapter, Fall and Winter 1953-1954)

During the first two quarters of the academic year 1953-54, six meetings were held. At these meetings, six papers were presented, three by members of the faculty and three by student members of Pi Mu Epsilon. The papers were:

- "Decision Problems" by Dr. James Dekker, chapter advisor
- "Critical points" by Dr. George Springer
- "The Mathematics of an Armament Race" by Dr. Ralph Boas
- "Some Aspects of Set Theory" by Robert Rieger
- "Simple Mechanical Computers" by Robert Fultyn
- "The Random Walk Problem" by Jerome Green.

On March 4, the Illinois Beta chapter of Pi Mu Epsilon will have a mathematics demonstration and lecture, as part of the annual Open House held by the Northwestern Technological Institute.

On January 12, election of officers for the following year took place, these officers to hold office until the Winter quarter of the academic year 1954-55. The following officers were elected: President, Robert E. Briney; Vice-president & Treasurer, Robert Rieger; Secretary, Raymond Ettinger; and Corresponding Secretary, Dr. James Dekker.

Officers for the previous year, 1953, were: President, Miles V. **Klein**; Vice-president, Jerome Green; Treasurer, Robert Fultyn; and Secretary, Mary Lou Stanfield.

At the initiation banquet held in May, 1953, Dr. William **Jenner** spoke on "Prime Numbers." The next initiation will be held in May, 1954.

#### MEDALS, PRIZES AND SCHOLARSHIPS

On April 25, 1953, the New York Epsilon chapter sponsored a high school mathematics contest, now an annual event. The individual winner was Earl **Dunn**, a junior from Tupper Lake High School, Tupper Lake, N. Y. The highest scoring senior, William **Bartlett** from **Potsdam** High School, **Potsdam**, N. Y., was offered a \$1,000 scholarship to St. Lawrence University, according to the rules of the contest.

The latter part of April, 1954, the Illinois Beta Chapter, Northwestern University, will hold its annual mathematics contest, a campus-wide competition with cash prizes.

The Eta of New York chapter, University of Buffalo, makes an annual award of dues for one year's membership in the Mathematical Association of America paid by the chapter for the winner. The award for the academic year 1952-1953 went to Mrs. Miriam **Hayman** Brown for outstanding achievement in mathematics.

At the November 12, 1952, meeting of the Kansas Gamma chapter, University of Wichita, announcement was made of two additions to the Pi Mu Epsilon Scholarship Fund: \$200 by Mr. H. K. Sears in memory of Mrs. Sears; \$100 by Mrs. E. B. **Wedel** in memory of Professor **Wedel** (former professor of mathematics at the University of Wichita).



## NEWS AND NOTICES

### PI MU EPSILON CELEBRATES ITS FORTIETH ANNIVERSARY

The first meeting of the Mathematical Fraternity, at Syracuse University, which was soon to become a national fraternity was held in the Hall of Languages, April 27, 1914. At that meeting Dr. E. D. Roe, Jr., was unanimously elected Director of the Fraternity. In less than one month, that is, on May 20, 1914, the Pi Mu Epsilon Fraternity, as it was then called, was granted a charter by the State of New York. The fraternity was incorporated under the name: "Pi Mu Epsilon Fraternity, Inc." The Certificate of Incorporation was signed by P. C. J. De Angelis, Justice of Supreme Court, and on May 25, 1914, the certificate was placed on file in the Office of the Secretary of State, in Albany, New York. The fee for recording was \$11.00.

As Number 10, Volume 1, of *The Pi Mu Epsilon Journal* goes to press, the editor has before her all of these precious documents, pertaining to the founding of the fraternity, kindly loaned by the present Secretary General.

We mention these matters at this time because May 20, 1954, will mark the fortieth anniversary of Pi Mu Epsilon, and we think it is fitting to pause and give some thought to the beginnings of our great national honorary mathematical fraternity.

"A chapter of Pi Mu Epsilon may be chartered only in an academic institution whose standards are excellent in all liberal arts departments and particularly so in mathematics," so says our present constitution. The fraternity has experienced steady growth both in strength of membership and number of chapters. By 1923, there were six chapters, and presiding over these six chapters was Dr. E. D. Roe, first Director General. These chapters are listed here in order of installation: at Syracuse University, Ohio State University, University of Pennsylvania, University of Missouri, University of Alabama and Iowa State College of Agriculture and Mechanical Arts.

In forty years the number of chapters has reached fifty-six.

\* \* \*

#### A New Chapter

We are happy to announce that the chapters voted approval of the petition from the Mathematics Club of Rutgers University for

1954

NEWS AND NOTICES

427

a chapter of Pi Mu *Epsilon*. So it looks as if New Jersey Alpha will soon be our **fifty-sixth** chapter. The installation ceremony probably will be on April 22, 1954, with Director General C. C. MacDuffee officiating.

\* \* \* \*

#### News Items Gleaned from Chapter Reports

In April, 1953, the Alpha of Alabama sent three delegates, Director Betty Ellis and two other members, to represent the chapter at the installation of the Beta of Alabama chapter at Alabama Polytechnic Institute.

On March 4, 1954, the Illinois Beta chapter of Pi Mu Epsilon featured a mathematics demonstration and lecture, as part of the annual Open House held by the Northwestern Technological Institute.

The New Hampshire Alpha chapter sponsors a program called the "Math-Aid" which functions every Monday evening and members assist other students having difficulty with mathematics in courses ranging from algebra to differential equations.

\* \* \* \*

#### News of Alumni Members

Army Home Town News Center,  
Kansas City, Missouri  
January 28, 1954



David L. Weeks of Oklahoma City, Okla., recently was promoted to first lieutenant in Korea, where he is wire officer with the signal section at I Corps headquarters. Before arriving overseas last October, Lieutenant Weeks was stationed at Camp Gordon, Ga. A graduate of Oklahoma A & M College where he was a member of Lambda Chi Alpha, Phi Kappa Phi, PI MU EPSILON and Phi Eta Sigma, the Oklahoma City officer entered the Army in July 1952. His parents, Mr. and Mrs. Leo E. Weeks, live at 1230 St. Charles Street.

(U.S. Army Photo)

Mr. Vincent Naramore (New York Alpha) has been appointed head of the Mathematics Department at St. Michael's College, Winooski, Vermont.

Dr. John W. Brace (New York Theta) took his **Ph.D.** degree at Cornell University, June 1953, and is now a member of the mathematics staff at the University of Maryland.

\* \* \* \*

### Deaths

Professor E. B. **Wedel**, member of Pi Mu Epsilon (University of Oklahoma) and professor of mathematics at the University of Wichita, died in July, 1952.

Among the initiates for the Missouri Alpha chapter, University of Missouri, May 1, 1953, there appeared the name of George F. Schwaebe (See vol. 1, no. 9, p. 390). Last Fall our Business Manager, Howard C. Bennett, received a letter from the parents bearing the news that their son had been **killed** in a Navy plane crash, July 17 (1953). We are sure the other members of Pi Mu Epsilon will be saddened by this news and will wish to join the Journal Staff in an expression of sympathy to the parents. Details of the plane crash are given in Mr. and Mrs. **Schwaebe's** letter to Professor Bennett, excerpts of which are printed below.

"We received a copy of the Pi Mu Epsilon Journal addressed to our son, George F. Schwaebe in which was enclosed the attached form to renew his subscription.

"It is with great sorrow that I write to tell you that George was killed in a Navy plane crash at Milton, Florida, July 17, with 39 other midshipmen while on their summer training tour.

"We do want you to know that George took great pride and joy in his association with Pi Mu **Epsilon**, and Mrs. Schwaebe, with the permission of the local chapter at Missouri University, in Columbia, has his Pi Mu Epsilon pin on a gold chain around her neck as her most valued memento of our only son. We hope that you also will approve of this.

"With our best wishes for your continued success in encouraging all young college people in attaining high proficiency in mathematics, we are...

(Signed) Mr. and Mrs. Geo. E. Schwaebe."

\* \* \* \*

### *Pi Mu Epsilon Doctorates*

Among the persons receiving doctorates in mathematics in 1952 in the United States (and listed in the *Bulletin* of the American Mathematical Society, vol. 59, no. 3 [May, 1953], p. 286-296) the following sixty members of Pi Mu Epsilon received **Ph.D.'s** at the institution where they were initiated into Pi Mu Epsilon. They are listed below by chapter and initiation date.

California Alpha	<b>J. J. Brandstatter</b>	(June 7, 1946)
	R. M. Hayes	(Dec. 20, 1946)
	J. R. Jackson	(Jan. 6, 1950)
	E. E. Osborne	(May 29, 1947)
California Beta	T. A. Jeeves	(Sept. 25, 1943)
	W. L. Parker	(April 27, 1947)
	R. G. Stoneham	(Dec. 3, 1947)
	R. F. Tate	(Dec. 19, 1942)
	F. B. Thompson	(Nov. 22, 1949)
	R. A. Willoughby	(Dec. 11, 1946)
Georgia Alpha	R. K. Butz	(Jan. 19, 1949)
	B. F. Hadnot	(Jan. 19, 1949)
	A. T. Hind, Jr.	(Nov. 8, 1950)
	G. N. Wallon	(Feb. 1, 1950)
Illinois Alpha	B. V. Dean	(May 16, 1951)
	C. B. Hanneken	(May 9, 1950)
	J. H. Hornback	(May 9, 1950)
	G. R. Livesay	(May 19, 1944)
	E. R. Mullins, Jr.	(May 9, 1950)
Illinois Beta	D. J. Thompson	(Nov. 16, 1948)
Iowa Alpha	V. W. Bolie	(Apr. 19, 1949)
	Fred Chong	(May 17, 1951)
	A. M. Feyerherm	(May 23, 1950)
	L. D. Gates, Jr.	(Apr. 19, 1949)
	C. H. Lindahl	(Apr. 20, 1948)
	M. R. Mickey, Jr.	(Apr. 20, 1948)
Kansas Alpha	H. D. Mills	(Dec. 4, 1947)
	R. C. Fisher	(Jan. 5, 1949)
	W. C. Foreman	(Apr. 25, 1941)
	J. R. Larkin	(Jan. 5, 1949)
	J. D. Riley	(May 13, 1947)
Kentucky Alpha	W. C. Royster	(Feb. 26, 1947)
	R. E. Wheeler	(Apr., 1948)
Missouri Alpha	W. L. Stamey, Jr.	(May, 1949)

New York Alpha	H. G. Rice Samuel Schechter	(Dec. 15, 1948) (Dec. 11, 1947)
New York Delta	Bernard Fleishman W. M. Hirsch	(1948) (Dec. 13, 1946)
North Carolina Beta	M. D. Kruskal Meyer Dwass	(Dec. 13, 1946) (Nov. 16, 1951)
Ohio Alpha	Emilie V. Haynsworth <b>E. R. Fadell</b>	(Dec. 6, 1949) (June 2, 1949)
Oregon Alpha	E. W. Martin, Jr. R. V. Mendenhall	(May 14, 1948) (May 14, 1947)
Pennsylvania Alpha	H. E. Tinnappel I. J. Christopher A. E. Livingston C. K. Tsao	(June 6, 1939) (May 25, 1950) (Apr. 29, 1949) (Apr. 29, 1949)
Pennsylvania Epsilon	Jean M. Calloway Hyman Kamel David Rosen Marvin Tomber	(May 18, 1948) (Dec. 15, 1939) (May 22, 1950) (May 18, 1948)
Wisconsin Beta	G. J. Habetler C. D. Harris Phillips Whidden R. J. Buehler <b>G. P. Dineen</b> G. F. Rose W. N. Smith Arnold Wendt	(May 26, 1950) (May 30, 1948) (May 30, 1948) (May 21, 1948) (May 21, 1948) (May 21, 1948) (May 21, 1948) (March 24, 1949)

Since we have no master list of membership, we are very grateful to Secretary General J. Sutherland Frame for his work in preparing the above partial list of our members receiving their doctorates in mathematics in the year 1952. It is a partial list because he could only select from the 207 doctorates listed in the Bulletin (**AMS**, May, 1953) those who received degrees at institutions having chapters of Pi Mu Epsilon. It was a laborious task which involved many hours of searching through the Fraternity's files for each of these institutions, giving lists of initiates each year since the chapter was installed. Though exercising the greatest care, because of pressure of time and other matters, he is not certain that he did not miss some names. Nevertheless sixty **Ph.D.'s** earned in one year by members of Pi Mu Epsilon reflect great credit on the Fraternity! We realize, however, that many of our members, after being initiated, transfer to other universities to work on their doctorates, in some cases to a university within the U.S. where there is no chapter of Pi Mu **Epsi-**

lon, and some few go to countries beyond our borders for their doctorates. In such cases there was no way for the Secretary to check membership in the Fraternity.

Following the recently voted amendment to our constitution concerning the corresponding secretaries of each chapter, the editor will be able to work more closely with each chapter in a study like that given above. Additional data on the successful candidates, like where presently employed, may be obtained.

---

A 1954 Summer Conference for Mathematics teachers will be held at the University of Oklahoma in the air conditioned Memorial Union building on June 9-10.

## BOOK REVIEWS

**Handbook of Probability and Statistics with Tables.** By Richard S. **Burlington** and Donald C. May, Jr., Sandusky, Ohio, Handbook Publishers, Inc., 1953. 340 pages.

This handbook attempts to present the basic principles of probability and statistics in a compact, pocket size form which can be used by people in various applied fields. It is quite comprehensive in its coverage of **basic** theory and definitions used in statistical analysis. The phenomenal growth of statistical methodology is evidenced by the author's devotion of three-quarters of the handbook to theory and explanation and only one-quarter to statistical tables. This might be contrasted with **Burlington's** "Handbook of Mathematical Tables and Formulas" where only one-third of the book is devoted to definitions and formulas and two-thirds to tables. The new handbook fills a long-felt need in providing a companion handbook to the original and should become an integral part of every applied statistician's library.

The authors have covered the basic ideas in what I believe to be a wise choice of notation. This is an important point as there are almost as many notations as there are textbooks in this field. After the usual discussion of basic statistical measures, combination, permutations and probability, the authors present the general idea of a distribution function followed by a discussion of specific distributions such as the Binomial, Poisson, Normal, etc. Material is included on regression theory, time series, correlation, a short discussion of multiple and partial correlation, statistical inference, sampling distributions and a short section on analysis of variance. Finite differences and sequential analysis are also discussed and a 'once over lightly' on sampling inspection and quality control.

The tables are the ones a statistician likes to have handy including the binomial function ( $n = 1$  to **20**), the Poisson, normal curve, F, t, z,  $x^2$  and a few mathematical tables useful in statistics.

It is certainly difficult to cover the amount of material these authors attempt in a way that will satisfy all readers. In the later chapters, the book ceases to be a handbook and becomes a mathematical discussion. For example, in the discussion of analysis of variance, the essential principles are there but not the **tech-**

1954

BOOK REVIEWS

433

**niques** actually **used** in solving problems such as the problem on page 208. I believe **the** choice of heading in table 17.43.2 on page 236 is most unfortunate as the American Society for Quality Control has adopted symbols for these values which are widely accepted in quality control circles. It would seem worthwhile to publish an addendum showing that the author's  $M'/R_0$  is **A**,  $b(n)$  is  $c_2$ ,  $k_n$  is  $d_2$ , etc. It is true, however, that the symbols used are more descriptive of the actual statistical process involved but would be very confusing to the quality control man.

In general, I believe that this handbook fulfills its basic purpose and presents a wealth of material and tables in a handy, compact form.

Charles R. Hicks

**Asst.** Professor of Mathematics and Research Associate  
in the Statistical Laboratory, Purdue University

**Theory of Equations.** By C. C. **MacDuffee**. New York, John Wiley and Sons, Inc., 1954. vii + 120 pages. \$3.75.

Professor **MacDuffee's** text provides an excellent introduction to the theory of equations for the general science student. It is interesting and novel that such a book should introduce, and apply to concrete problems, such concepts as "equivalence relation," "ring," "field," "euclidean ring" and others. In spite of the introduction to abstract algebra the text leans towards the practical side of the subject. The many excellent exercises allow the student to test his skill and relate the theory to practical problems.

The table of contents reads: -1. Linear Systems. -2. Rational Solutions. -3. Polynomials. -4. Real Roots. -5. Complex Roots. -6. Relations among the Roots. -7. Systems of Higher Degree. Answers to Exercises. Index.

The publishers are to be congratulated on the format of the book. The reviewer found no misprints.

## ACKNOWLEDGMENTS

Besides acknowledging with thanks the assistance of the associate editors and business manager, the editor wishes to express appreciation to other persons who have given so freely of their services and have thus made possible the publishing of this issue of the Journal. We are grateful for:

Technical and literary contributions from: R. C. Archibald, E. G. Begle (for the AMS), J. Sutherland Frame, A. J. Goldman, Helen J. McAleer and Mildred Taylor.

Refereeing papers: Abe Gelbart, Harry W. Reddick and Warren V. Shepard.

Book Reviews: Walter R. Baum, Charles R. Hicks and George Leger.

Problems and Solutions: We wish especially to thank Editor Leo Moser and his collaborators for their continued and loyal support of the Problem Department.

Services rendered in typing manuscript: Barbara Hughbanks, Joan Seha and Rets Spaulding.

Final composition of the manuscript for the camera: The careful work of June Hegendorfer and her very able staff.

Financial support from our advertisers: L. G. Balfour Company; Cushing-Malloy, Inc.; Handbook Publishers, Inc.; H. M. Gehman, Secretary-Treasurer of MAA; John Wiley and Sons, Inc.

Finally, we wish to acknowledge receipt of complimentary copies of new publications:

O. U. MATHEMATICS LETTER, Vol. III, Nos. 1 and 2, Sept. and Dec., 1953.

PREPARING A TECHNICAL MANUSCRIPT, McGraw-Hill Book Company, Inc.

PROFESSIONAL OPPORTUNITIES IN MATHEMATICS, 2nd. ed., Jan. 1954, from H. M. Gehman, Secretary-Treasurer, MAA.

THE MATHEMATICS STUDENT JOURNAL (Sponsored by the NCTM), Washington, Vol. 1, No. 1, Feb. 1954.

THEORY OF EQUATIONS, C. C. MacDuffee, 1954, John Wiley and Sons, Inc.

THE TRANSIT OF CHI EPSILON, Fall 1953, Vol. 25, No. 2.

INITIATES, ACADEMIC YEAR 1951 - 1952  
(Continued from Vol. 1, No. 2)

### KENTUCKY ALPHA, University of Kentucky (Spring, 1952)

Carl Berger  
John R. Davis

George Farney  
Virginia Hanly

Robert D. Haun, Jr.  
Richard Sprague

INITIATES, ACADEMIC YEAR 1952 - 1953  
(Continued from Vol. 1, So. 5)

### ALABAMA BETA, Alabama Polytechnic Institute

Charter members who were already members of the Fraternity at the time the Chapter was installed, April 21, 1953:

J. C. Eaves  
Ernest Ikenberry

Nathaniel Macon  
E. P. Miles, Jr.

B. E. Mitchell  
W. V. Parker

Other students elected to the Auburn Mathematics Club since the time of the petition and initiated into Pi Mu Epsilon, 1953:

Ann Alvord	Henry Hartsfield
Warren Andrews	John Herron
H. B. Armitage	M. L. Hicks
R. M. Arthur	Marjorie Higgins
Claire Aucoin	Johnny Johnson
Clayton Aucoin	H. W. Kelley
Frank Barker	Darwin Kilbourn
Howard Barringer	Dale Koehler
Clare Ann Campbell	Parker Lamb
Carolyn Cosby	John C. Lampkin
J. C. Dowdle	Doris Jeanne Lindsey
M. M. Duncan	June Long
Paul Gallier	Alfred Matthews

R. B. McMillan
Irene Pace
John S. Parke
Emily Ann Parker
George Sarinoperilos
Nettie Maye Staggers
Alex Taylor
Carl David Todd
Arvid Wahlquist
Jim Watson
Porter Webster
W. H. Whitaker
Paul Williams

CALIFORNIA ALPHA, University of California, Los Angeles  
(January 9, 1953)

Afton H. Cayford  
John W. Lindsay  
Masako Oba

Robert R. Phelps  
Dana E. Quade

Wilbur K. Sepetoski  
Roger Haines Skinner  
Jess Richard Westlake

## NEW YORK ALPHA, Syracuse University

(December 1952)

Walter R. Baum  
 Norbert Bischof  
 Richard H. Bruns  
 John D. Chase  
 Helene Cooper  
**Robert** E. Downing  
 Virginia **Feldmann**  
 George **Finkbeiner**  
 Robert I. Gray  
 Patricia R. **Hansell**  
 Fritz J. Hemmer

Charles A. Johnson  
 Sally A. Keller  
 Paul A. **Kenline**  
 Warren A. Lombard  
 Robert A. Mack  
 Alfred U. **MacRae**  
 Thomas E. Manwarren  
 George L. **Mulfinger**  
 Ira **Nemeroff**  
 Victor J. **Pietrafesa**  
 Frank A. Raymond

Joseph E. **Rizzo, Jr.**  
 Herbert M. Schoen  
 Charles W. Serby  
 Joyce E. **Shorin**  
 Harold **Siegel**  
**Emmanuel**
**William** F. Terrell  
 Rolf L. Thorkildsen  
 John H. Van De **Walker**  
 Clarence Vanselow  
 Ralph S. Wiegand, Jr.

## NEW YORK THETA, Cornell University

(May 22, 1953)

Norman N. **Axelrod**  
 Douglas P. **Baird**  
 Robert M. Blumenthal  
 John W. Brace  
 Chao K. Chow  
 Miodren **Cohen**  
 Leonard Evens  
 Janet Fowler  
 Mary Anne **Friederich**  
 Marcia L. **Goldberg**  
 Tema Hasnas

Raquel Heller  
 David Hertzig  
 Daniel J. **Kleitman**  
 David **Lubell**  
 Elliot Mendelson  
 Jean-Pierre Meyer  
 Norman Morse  
 Steven Orey  
 Jack **Peretz**  
 Joel D. Pincus  
 Filomena **Reyes**  
 Jerome Sacks

Harvey M. Schein  
 David E. **Schroer**  
 Stanley Siegleman  
 Charles **Standish**  
 Norman B. Stein  
 Gilbert A. Stengle  
 Roberta Torrance  
**R. J.** Walker  
 Steven **Weinberg**  
 Berhard Weisblum  
 Jacob Wolfowitz

## NORTH CAROLINA ALPHA, Duke University

(December 10, 1952)

Joan Corgett

Francis P. Knowles

(We regret that the above two names were received by the Editor after Number Nine of the Journal went to press.)

## INITIATES, ACADEMIC YEAR 1953 • 1954

## ALABAMA BETA, Alabama Polytechnic Institute

(January 26, 1954)

**Denis** Harold Alcock  
 Eleanor Allison  
 Newton Steele Andrews  
 Elizabeth R. **Barnhart**  
 Robert Clothier

Gerald Cunningham  
 Anna Jane Gritz  
 Nolan **Hardin**  
 James **Hinton**  
 Miriam Horton

Ho **Kang** Liu  
 Robert Lowder  
 James **Lowry**  
 John E. Rives  
 George **Sarinopoulos**

## 1954 INITIATES, ACADEMIC YEAR 1953-1954

## 437

## COLORADO BETA, University of Denver

(November 18, 1953)

Marvin F. Anderson

Sarah **Gorelick**  
 Bill Yuen Lee

**Thayer Masoner**

## DISTRICT OF COLUMBIA ALPHA, Howard University

(November 30, 1953)

Harriett Elizabeth Gregg

William Harrell **Smith**  
 James Sunnie **Winbush**

## ILLINOIS ALPHA, University of Illinois

(December 10, 1953)

Donald Ray Arnold  
**Algirdas A. Avizionis**  
 Donald Lester Bitzer  
 Ernest William **Carperos**  
 Phil Ramon **DeProtino**  
 Luella **Seyer Dickhaut**  
 Allen Dana Harper  
 Russell Harold Hopps  
 Eugene Lyle **Hubbard**

Richard Jay **Kenyon**  
 Raymond F. **Kramer, Jr.**  
 Herbert W. Kuchne  
 Edward **Kung**  
 Sager Daryl Larson  
 William **Lichtenberger**  
 Clifford **Allan Long**  
 Charles Leo **McDonough**

Benjamin H. **McLemore**  
 James Blaney Rice  
 Kenneth Rose  
 Lily Hannah Seshu  
**Sundaram** Seshu  
 John H. **Strathman**  
 Tommy Stanley **Ulom**  
 Porter J. Womeldorf  
 Keith A. **Yarborough**

## MICHIGAN ALPHA, Michigan State College

(October 20, 1953)

William M. **Crampton**  
 Julian A. **Crawford**  
 John R. Davis  
 Arden D. Day  
 Cecil O. Etter  
 M. Paul Hagelberg

Douglas J. Harvey  
 Clyde M. Hyde  
 Gerald W. Isaacs  
**Leo Jedynak**  
 Robert M. Jones

George I. **Kingsley**  
 E. Louis **LeBay**  
 John J. **Lenosky**  
 John S. Perry  
 Marjorie A. Premo  
 Richard D. **Verheul**

## MISSOURI ALPHA, University of Missouri

(December 9, 1953)

Rex Miller Barnes  
 John Dwight **Bentley**  
 Kenneth Wilbur **Campen**  
 Thomas Marshall Carter  
 Franklin Voris Engle

Clayton Wayne **Freear**  
 Virginia Ann **Froerer**  
 Eugene Kent Johnson  
 Jimmy Gordon **Karo**  
 Marvin B. **Kaufman**  
 Michael F. McDonald

**Merril** Eugene Neal  
 David Lee Ohsiek  
 James Deane Patterson  
 Charles B. **Philibert**  
 Frank J. Pike

## MONTANA ALPHA, Montana State University

(Spring, 1954)

Ralph Bingham  
 John Blackwood  
**John** **Frankino**  
 Albert **Gilman**

Charles **Grunn**  
 Noel H. Johnson  
 William **Lien**  
 Edward **Overturf**  
 John **Maruin**

Frank **Mentala**  
 Robert **McRae**  
 Sheldon Rio  
 Gertrude Stene

NEW HAMPSHIRE ALPHA, University of New Hampshire  
(December 7, 1953)

Laurence <b>Baldi</b>	Constance M. Foley	Paul R. Josephson
Edward Ralph Brooks	Robert R. Garipay	Chris D. <b>Kehas</b>
Patrick Thomas Cahill	Gunnar <b>Heskstad</b>	Donald Shaw Kelly
Arthur R. Calawa	Wayne <b>Olin Jackson</b>	Wayne <b>Overman</b>
Charles Edwin Campbell	Wendell <b>Jesseman</b>	Charles H. Snow
John C. Chadbourne	Nicolas Johnson	Ralph <b>Stajdohar</b>
Robert F. Church	Evelyn Jones	Donald <b>Penn</b> Varney

NEW YORK ALPHA, Syracuse University  
(February 12, 1954)

Everett E. Aldridge	Walter W. Jones	Stanley A. <b>Mosier</b>
Donald Austin	Henry J. Juda	Albert A. <b>Mullin</b>
Andrew A. Benvenuto	John F. Jureller	Florence <b>Oglebay</b>
Edward J. Ballard	Ronald H. <b>Kanner</b>	John W. Schaefer
Albert Dewey, Jr.	James A. Kennedy	Richard <b>Seikaly</b>
Olga J. Guarini	Robert A. <b>Kurtz</b>	Rou F. Shortt
Simon Hellerstein	Wylan A. Liddle	Kelvin C. Smith
James E. <b>Hennessy</b>	George L. Luther	Vonda J. <b>Stratton</b>
Richard I. Hirshberg	Donald R. <b>MacRae</b>	Reinhold E. Tomek
Robert J. Holbrook	Myron L. Moffitt	Robert J. Wallis
Leo F. Johnson		Charles J. <b>Zablocki</b>

NEW YORK BETA, Hunter College  
(October 27, 1953)

Sandra <b>Biderman</b>	Frances <b>Goldman</b>	Danuta Milford
Lucille Chang	Blanche <b>Kaye</b>	Rivkah Molotin
Rhoda Davis	Jean <b>Lenz</b>	Lorraine Nayer

NEW YORK GAMMA, Brooklyn College  
(Fall, 1953)

Edmund <b>Eisenberg</b>	Martin Greenlinger	Gary Sales
Stanley Feinrock	Bernard <b>Kalmanowicz</b>	Arthur <b>Schlissel</b>
Philip Feuer	Stephen <b>Krulik</b>	Edward Sinreich

NEW YORK DELTA, New York University  
(December 15, 1953)

Abraham <b>Goldberg</b>	Renée <b>Greenwald</b>	Edward J. <b>McDermott</b>
-------------------------	------------------------	----------------------------

## 1954 INITIATES. ACADEMIC YEAR 1953-1954

## 439

NEW YORK EPSILON, St. Lawrence University  
(Fall, 1953)

Frank M. <b>Catalano</b>	Robert J. Moll	Ellis E. Pierce
Raymond G. Fryer		Stephanie M. Wilson

NEW YORK ETA, University of Buffalo  
(December 2, 1953)

William D. Comstock	Michael S. <b>Kistel</b> , Jr.	Robert W. <b>McClure</b>
John Alfred D'Angelo	Dennis P. <b>Malone</b>	Joseph Nazon
W. Paul <b>Ganley</b>		Robert Vincent Nolan

NORTH CAROLINA ALPHA, Duke University  
(December, 1953)

William A. Baxley	Barbara L. Corbeels	Dorothy U. <b>McCaleb</b>
Peter C. Burkholder		Elizabeth A. Wright

NORTH CAROLINA BETA, University of North Carolina  
(December 3, 1953)

Charles Ward Barnes	Luther B. Clapp	Virginia R. <b>Swann</b>
Barry Wayne Bryant	Philip W. Dibben	Thomas H. Sumner
Louis Child	Edward S. Johnson	Robert L. <b>Hubbard</b>
Julius Grady Cox	Earl Mason Page	Robin A. Mickle
	Nathan R. Roberson, Jr.	

(January 7, 1954)

Bonnie Marie Baker	Mary C. Myers	E. Elizabeth Stewart
Anthony John Houghton	Allan K. <b>Norberg</b>	Tiny Marguerite Morrow

OHIO DELTA, Miami University  
(March 25, 1954)

Mary Elizabeth Baker	William J. Hunter	William <b>Barnitz Oglesby</b>
	Stanton L. Jones	

OKLAHOMA ALPHA, University of Oklahoma  
(November 3, 1953)

Mrs. Ingeborg H. Davis	Dr. John B. Giever	Sandra Jean Swan
Robert T. Dooley	John J. Hole	Jack W. Warhurst
Larry B. Flood	Robert F. Jones	Cleon W. <b>Winslow</b>
	Margaret Shoemaker	

OKLAHOMA BETA, Oklahoma A. & M. College  
(December 13, 1953)

William J. Boston	Kerry S. <b>Havner</b>	A. L. Rose
Larry G. <b>Burdick</b>	Jean Kivett	Malcolm S. Soule
William L. Fash		Carol Thompson

## PENNSYLVANIA BETA, Bucknell University

November 18, 1953)

Leo F. Black  
 Ronald H. Broadhurst  
 Bruce M. Cloud  
 Lawrence O. Comfort  
 Jerome Felcher

Jerald Finkel	Arlene M. Small
Glen R. Grove	William P. Walsh
Arthur D. Harrison, Jr.	Saul H. Wishnofi
Carole L. Itjen	Carle C. Zimmerman, Jr.
Edward J. Mizma	Richard M. Zucker
Marianne Shutack	

## VIRGINIA ALPHA, University of Richmond

(Fall, 1953)

L. Henry Anderson  
 Thomas F. Armstrong

John William Guy III	Jean Olin Ruddle
Jacquelyn Ayre Mack	John F. Swanson
Ann Lindsay Pettit	

## WASHINGTON BETA, University of Washington

(November 18, 1953)

Chu Chin Chang  
 Richard L. Goen

Dorothy Atsuko Iwasaki	George Lomax
	Ellen Jane Shea

## WISCONSIN BETA, University of Wisconsin

(November 12, 1953)

Robert E. Barks  
 Joyce Bauer  
 John W. Blattner  
 Louis R. Bragg  
 John A. Brown  
 Jame Bruce  
 Warren D. Davidson  
 Harry M. Friedman

Mary Ann Garbrecht	Thomas W. Osterburg
Bruce A. Jensen	Carol J. Plautz
Ruth L. Krentz	James M. Van Buskirk
Frederick W. Leysieffer	Walter B. Watson
Robert K. Meany	Richard K. Wendt
Gordon D. Mock	Taro Yamane
Alistair Neill	Etta A. Zuber
	Martin S. Zucker

Meets the needs of both majors and non-majors

## THEORY

## OF

## EQUATIONS

By CYRUS COLTON MACDUFFEE

Professor of Mathematics, University of Wisconsin  
 Director General, Pi Mu Epsilon

This multi-purpose text is designed to meet the needs of both majors and non-majors in the standard junior-senior course, 'Theory of Equations.' It meets these divergent requirements in a subtle, effective manner by covering the standard material in a fairly conservative way and at the same time introducing the important concepts of modern algebra which enable the majoring student to go on to abstract algebra without dislocation. These concepts — including such topics as field, ring, and Euclidean ring — are introduced so gradually and naturally that they in no way disturb the nature of the course itself.

Theory of Equations is the first elementary text in English to give a complete treatment of systems of linear equations in terms of the modern methods which avoid the use of determinants. Of particular value to the student who expects to continue with mathematics is the very full coverage of the theory of polynomials by modern methods. The computational aspects of the subject are not neglected, and there is an unusually thorough discussion of systems of equations of higher degree, and a concise introduction to the theory of numbers.

1954. 120 pages. \$3.75.

Send today for an on-approval copy

JOHN WILEY &amp; SONS, Inc., 440-4th Avenue, New York 16, N. Y.

*A review of this book appears in this issue  
of Pi Mu Epsilon Journal.*

**JUST OFF THE PRESS**

A Book For Those Who Need Probability  
& Statistical Information

HANDBOOK OF PROBABILITY AND STATISTICS  
with  
TABLES  
by

Richard S. Burington and Donald C. May

This book is a convenient summary of theory, working rules and tabular material useful in practical problems in probability and statistics. It brings together information which is not otherwise readily available in simple form except by reference to numerous journals, tables, and treatises on the subject.

It meets the needs of students and workers in this subject. Readers without detailed statistical training will find this volume a sufficient guide for the more commonly met statistical aspects of their studies. Those with statistical training will find it a convenient summary of the material most often needed.

The book is divided into two main portions. The first part includes a summary of the more important formulas and definitions of elementary statistics and probability theory.

— this part of the book makes it always valuable for review purposes.

The second part consists of tables of distributions and other quantities of frequent use in statistical work.

— this part of the book makes it always valuable for quick reference, computations and calculations.

340 pages

5 1/4 x 7 3/4

50 figures

SINGLE COPY PRICE,

**\$4.50**

Published by

**HANDBOOK PUBLISHERS, INC.**

SANDUSKY, OHIO

Now available:

**PROFESSIONAL OPPORTUNITIES IN MATHEMATICS**

Second Edition: January 1954

A Publication of the Arnold **Buffum** Chace Fund

The report of a Committee of the Mathematical Association of America, consisting of H. W. **Brinkmann**, Z. I. Mosesson, S. A. **Schelkunoff**, S. S. Wilks, and Mina Rees, Chairman.

24 pages, paper covers

25¢ for single copies;

10¢ each for orders of ten or more

Send order to: Harry M. Gehman, Secretary-Treasurer  
Mathematical Association of America  
University of Buffalo  
Buffalo 14, New York

**ORDER YOUR KEY NOW**

Your Key is a beautiful symbol of your fraternity association.  
Wear it with pride.

Price List

Standard keys, 1 piece:

10K gold . . . . . \$3.75

14K gold . . . . . 4.50

Standard key, 3 piece with applied ends:

10K gold . . . . . 4.75

14K gold . . . . . 5.50

Standard badge or pin:

10K gold . . . . . 3.25

14K gold . . . . . 4.00

Pin joint and catch 75¢ additional

Keys & pins must be ordered on triplicate order blank forms, one to be sent to us from your chapter, one to the Central **Office** and one kept by chapter.

10% Federal Tax and any State Tax in Addition.

Official Jeweler to Pi Mu Epsilon

**L. G. BALFOUR Company**

Massachusetts

Attleboro,

In Canada... contact your nearest **BIRK'S STORE**

# **QUALITY LITHOPRINTING**

**including**

**Typewriter and Letterpress Composition**

---

**CUSHING - MALLOY, Inc.**

**1350 North Main Street**

**Ann Arbor, Michigan**



**The most economical process  
for limited editions and out-of-print material.**