Crux

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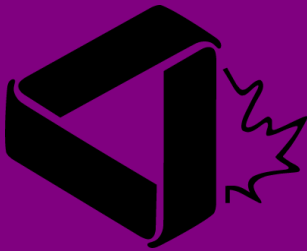
The Back Files

The CMS is pleased to offer free access to its back file of all issues of Crux as a service for the greater mathematical community in Canada and beyond.

Journal title history:

➢ The first 32 issues, from Vol. 1, No. 1 (March 1975) to Vol. 4, No.2 (February 1978) were published under the name *EUREKA*.

➢ Issues from Vol. 4, No. 3 (March 1978) to Vol. 22, No. 8 (December 1996) were published under the name *Crux Mathematicorum*.

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➢ Issues since Vol. 38, No. 1 (January 2012) are published under the name *Crux Mathematicorum*.

Mathematicorum

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MATHEMATICS, ANYONE?

Have you come across an interesting problem lately? Do you have a comment
to make about matters mathematical? Let your friends and colleagues know about
it, through this magazine.

The following members of the Carleton - Ottawa Mathematics Association
(COMA),

> R. Duff Butterill, Ottawa Board of Education,
> H.G. Dworschak, Algonquin College,
> Viktors Linis, University of Ottawa,
> F.G.B. Maskell, Algonquin College,
> Léo Sauvé, Collège Algonquin,
> Richard J. Semple, Carleton University,

acting in their private capacity, recently met and decided to launch this mag-
azine to provide a forum for the exchange of mathematical information, especially
interesting problems and solutions, among the members of the mathematical com-
munity in the Ottawa region, students and teachers alike.

The undersigned member of the above-mentioned group (this sounds like a
mini-max problem) has agreed to serve as editor for the time being. All com-
munications should be sent to him. The magazine will be published monthly,
as long as enough support is received from its readers.

How can you show your support? Here's how:

1. First, and most important, *send your name, address, and educational
affiliation to the editor.* Do this right away, since forthcoming issues will
only be sent to those who have expressed interest by getting their name on
the mailing list. There is no subscription fee, for the present at any rate.

2. Propose problems for which readers will send in solutions. The editor cannot, of course, guarantee to publish all problems submitted; but he will surely publish, more or less in the order received, all those that are interesting in formulation, or for which some ingenuity is required in the solution. They can be anything from elementary to advanced, from easy to difficult, and the editor will try to see that a varied diet is served in every issue of the magazine.

Original problems are particularly sought. But not many of us can create six original problems every morning before breakfast. So please submit any interesting problem you come across, especially those from books that are not easily accessible. Many problems in current textbooks have been floating around for a long time anyway, and they are often included without acknowledgment of any sort. However if you know who *originated* the problem you submit, it would be laudable to give credit where credit is known to be due.

In the preface to his two-volume work, *Analysis* (Blaisdell, 1964), the well-known American mathematician Einar Hille quotes the following lines from Rudyard Kipling:

> When 'Omer smote 'is bloomin' lyre,
> He'd 'eard men sing by land an' sea;
> An' what he thought 'e might require,
> 'E went an' took—the same as me!

What is good enough for Einar Hille.....

3. Send in solutions to proposed problems, including problems you propose yourself. Preferred solutions will be short and elegant rather than long and laborious, and they will be typewritten or very neatly handwritten on signed, separate sheets, in a form suitable for publication. All correct solutions submitted will be acknowledged in print, and the best ones will be published in full.

4. Send in comments on mathematics or the teaching of mathematics, announcements, expository articles, historical vignettes, quotations, yea, even jokes, provided they have some relevance, however far-fetched, to mathematics.

Cette revue sera bilingue dans ce sens qu'aucun des problèmes, solutions, commentaires, etc, qui y paraîtront ne sera traduit; ils seront tous publiés dans leur langue originale, que ce soit l'anglais ou le français.

Pour ouvrir le bal, nous publions plus bas une première liste de dix problèmes, à peu près dans un ordre croissant de difficulté, pour lesquels nous sollicitons l'envoi de vos solutions, en français ou en anglais.

<div align="right">

Léo Sauvé
Collège Algonquin

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The first EUREKA dinner will be held at 7 p.m. on April 1, 1975, at the Rideau Campus of Algonquin College, 200 Lees Ave., following the COMA Council meeting. This is your invitation to attend. For additional information, contact Fred Maskell at 237-3120.

PROBLEMS -- PROBLÈMES

Problem proposals, preferably accompanied by a solution, should be sent to the editor, whose address appears on p. 1.

For the problems given below, solutions, if available, will appear in EUREKA No. 3, to be published around May 15, 1975. To facilitate their consideration, your solutions, typewritten or neatly handwritten on signed, separate sheets, should be mailed to the editor no later than May 1, 1975.

1. *Proposed by Léo Sauvé, Algonquin College.*
75 cows have in 12 days grazed all the grass in a 60-acre pasture, and 81 cows have in 15 days grazed all the grass in a 72-acre pasture. How many cows can in 18 days graze all the grass in a 96-acre pasture? (Newton)

2. *Proposed by Léo Sauvé, Algonquin College.*
A rectangular array of m rows and n columns contains mn distinct real numbers. For $i = 1, 2, \ldots, m$, let s_i denote the smallest number of the ith row; and for $j = 1, 2, \ldots, n$. let l_j denote the largest number of the jth column. Let $A = \max \{s_i\}$ and $B = \min \{l_j\}$. Compare A and B.

3. *Proposed by H.G. Dworschak, Algonquin College.*
Prove that in any set of ten different two-digit numbers one can select two disjoint subsets such that the sum of numbers in each of the subsets is the same.

4. *Proposed by Léo Sauvé, Algonquin College.*
It is easy to verify that $2\sqrt{3} + i$ is a cube root of $18\sqrt{3} + 35i$. What are the other two cube roots?

5. *Proposed by F.G.B. Maskell, Algonquin College.*
Prove that, if (a,b,c) and (a',b',c') are primitive Pythagorean triples, with $a > b > c$ and $a' > b' > c'$, then either

$$aa' \pm (bc' - cb') \text{ or } aa' \pm (bb' - cc')$$

are perfect squares.

6. *Proposed by Léo Sauvé, Algonquin College.*
(a) If n is a given nonnegative integer, how many distinct nonnegative integer solutions are there for each of the following equations?

$$x + y = n, \qquad x + y + z = n, \qquad x + y + z + t = n.$$

(b) Use (a) to conjecture and then prove a formula for the number of distinct nonnegative integer solutions of the equation

$$x_1 + x_2 + \ldots + x_r = n.$$

7. *Proposed by H.G. Dworschak, Algonquin College.*
Find a fifth degree polynomial $P(x)$ such that $P(x) + 1$ is divisible
by $(x - 1)^3$ and $P(x) - 1$ is divisible by $(x + 1)^3$.

8. *Proposé par Jacques Marion, Université d'Ottawa.*
Etudier la convergence de la suite $\{a_n\}$ définie par

$$a_n = \sqrt{1 + \sqrt{1 + \sqrt{1 + \ldots + \sqrt{1}}}}, \qquad (n \text{ radicaux})$$

et déterminer $\lim_{n \to \infty} a_n$ si elle existe.

9. *Proposé par Jacques Marion, Université d'Ottawa.*
Etudier la convergence de la suite $\{b_n\}$ définie par

$$b_n = \sqrt{1 + \sqrt{2 + \sqrt{3 + \ldots + \sqrt{n}}}}.$$

10. *Proposé par Jacques Marion, Université d'Ottawa.*
On sait que l'équation $e^x = x$ n'a pas de racine réelle; mais l'équation
$e^z = z$ a-t-elle des racines *complexes*?

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He was in logic a great critic,
Profoundly skilled in analytic;
He could distinguish, and divide
A hair 'twixt south and southwest side;
On either which he would dispute,
Confute, change hands, and still confute.
He'd undertake to prove, by force
Of argument, a man's no horse;
He'd prove a buzzard is no fowl,
And that a lord may be an owl;
A calf an alderman, a goose a justice,
And rooks committee-men and trustees.
He'd run in debt by disputation,
And pay with ratiocination.
All this by syllogism, true
In mood and figure he would do...
...In mathematics he was greater
Than Tycho Brahe or Erra Pater;
For he, by geometric scale,
Could take the size of pots of ale;
Resolve by sines and tangents, straight
If bread or butter wanted weight;
And wisely tell what hour o' th' day
The clock does strike, by algebra.

SAMUEL BUTLER, in *Hudibras* (1662)