

PI MU EPSILON JOURNAL

THE OFFICIAL PUBLICATION OF
THE HONORARY MATHEMATICAL FRATERNITY



VOLUME 2

NUMBER 9

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PI MU EPSILON JOURNAL is published semi-annually at St. Louis University.

SUBSCRIPTION PRICE: To Individual Members, \$1.50 for 2 years; to Non-Members and Libraries, \$2.00 for 2 years. Subscriptions, orders for back numbers and correspondence concerning subscriptions and advertising should be addressed to the PI MU EPSILON JOURNAL, Department of Mathematics, St. Louis University, 221 North Grand Blvd., St. Louis 3, Mo.

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A MATHEMATICAL INTRODUCTION TO LOGIC CIRCUITS

FRANZ E. HOHN
Illinois, '37

1. Propositional Logics

In the process of communication, we make use of many simple declarative sentences, such as

- "Two plus two is four."
- "It is false that two plus two is not four."
- "The potential at point P is high."
- "The least significant digit of the sum is 0."
- "John is in love with my girl-friend Jane."
- "All jokes are funny."

Sometimes two different sentences, such as the first two above, have the same meaning. This meaning, apart from the actual words used, and apart from any subjective significance the sentence may possibly possess, is called a *proposition*.

Some propositions, like the first above, we know to be true; some, like the last above, we know to be false. Other propositions are true or false, but for lack of knowledge of relevant facts, we may not be able to decide which; for example, "It rained August 1, 1955 in Timbuctou." Finally, there are propositions which are forever undecidable; for example, "*This* proposition is false." One concludes that if the proposition is false, then it is true, and that if it is true, then it is false. In what follows, we restrict our attention exclusively to *decidable propositions*, that is, those which we know are either true or false.

Propositions are often combined in various ways, by what are known as *logical connectives*, in order to form new propositions which may be called *propositional functions* of the original ones. For example, from two propositions P and Q we may form a third, their *conjunction* or *union*, which is denoted by $P \vee Q$ and which means "either P or Q or both". The connective \vee is often called the "inclusive or". Another function of P and Q is their *disjunction* or *product* which is denoted by $P \cdot Q$, or just $P Q$, and which means "both P and Q". Finally, a simple way to obtain a new proposition from a single proposition P is to deny P. This proposition, denoted by \bar{P} is the *complement* or *negative* of P and is true if P is false, false if P is true.

If P is the proposition "Two plus two is four" and if Q is the proposition "Arithmetic is nonsense", then $P \vee Q$ is the proposition "Two plus two is four or arithmetic is nonsense or both". $P Q$ is the proposition "Two plus two is four and arithmetic is nonsense." Q is the proposition "Arithmetic is not nonsense."

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The importance of the three logical connectives just mentioned is suggested by the following definition: Let $\{P\}$ be any non-empty set of decidable propositions such that for all P and Q in $\{P\}$, the propositions $P \vee Q$, $P Q$, and \bar{P} again belong to $\{P\}$. Then $\{P\}$ is called a *propositional logic*. A propositional logic may contain finitely many or infinitely many essentially distinct propositions. Examples of finite propositional logics will be indicated later.

2. Truth Values of Propositions

We now assign a *truth value* p to each proposition P of $\{P\}$. The symbol p has the value "0" when P is false and the value "1" when P is true. The symbols 0 and 1 may be regarded as being simply convenient marks, or as the digits of a binary number system. Two propositions P and Q are then defined to be *equivalent*, written $P = Q$, if and only if their truth values are equal: $p = q$.

The logical connectives *and*, *or*, *not*, and others as well, may be studied conveniently with the aid of what are called *truth tables*. For example, the truth values of P and \bar{P} may be recorded as follows:

P	\bar{P}
0	1
1	0

For $P \vee Q$ and $P Q$ we have the truth table

P	Q	$P \vee Q$	$P Q$
0	0	0	0
0	1	1	0
1	0	1	0
1	1	1	1

which lists the truth values of $P \vee Q$ and $P Q$ for each of the four possible combinations of truth values of P and Q.

From these truth tables it appears that if we define union and logical multiplication of the symbols 0 and 1 by the rules

$$\begin{array}{ll} 0 \vee 0 = 0 & 0 \cdot 0 = 0 \cdot 1 = 1 \cdot 0 = 0 \\ 0 \vee 1 = 1 \vee 0 = 1 \vee 1 = 1 & 1 \cdot 1 = 1 \end{array}$$

then the truth values of the union (product) of two propositions is the union (product) of their truth values. Similarly, if we define complements of 0 and 1 thus:

$$\bar{0} = 1 \quad \bar{1} = 0$$

then it follows that the truth value of the complement of a proposition is the complement of its truth value. From these observations we may conclude by induction that:

The truth value of any function of propositions P, Q, \dots of a propositional logic $\{P\}$, built up by a finite number of applications of the operations $\vee, \cdot, \text{and only}$, is the same function of the truth values p, q, \dots .

We shall call this latter function the *truth function* corresponding to the given propositional function.

3. Truth Tables for Other Connectives

From the definition of equivalence, it follows that two propositional functions of P and Q are equivalent if and only if they have the same truth table. Since the third column of a truth table can be filled out in only $2^4 = 16$ ways (when only two propositions are involved), it appears that there are only 16 essentially distinct propositional functions of two propositions.

Many of these 16 functions have special names. For example, the proposition " P implies Q ", which we write " $P \leq Q$ " and which by *definition* means " Q is true whenever P is", has the following truth table:

P	Q	$P \leq Q$
0	0	1
0	1	1
1	0	0
1	1	1

Note that if P is false, Q is certainly "true whenever P is", so $P \leq Q$ has the truth value 1. The logical connective " \leq " is called *formal implication* to distinguish it from implication in the colloquial sense. This is necessary because formal implication permits a false proposition P to imply formally any proposition Q , true or false.

If we now define 0 and 1 like the familiar inequality " \leq :

$$0 \leq 0, \quad 0 \leq 1, \quad 1 \leq 1$$

it follows that $P \leq Q$ if and only if $p \leq q$.

The proposition $P = Q$, which we have already defined to mean " P and Q have the same truth value", has the truth table:

P	Q	$P = Q$
0	0	1
0	1	0
1	0	0
1	1	1

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The logical connective " $=$ " is what we have called *equivalence*.

From the truth tables

P	Q	$\bar{P} \vee Q$	$P \bar{Q} \vee P \bar{Q}$
0	0	1	1
0	1	1	0
1	0	0	0
1	1	1	1

formed by using the principle announced at the close of the last section, conclude that

$$(P \leq Q) = (\bar{P} \vee Q)$$

and

$$(P = Q) = (P \bar{Q} \vee P \bar{Q}).$$

Another important connective is the *exclusive or* or *sum* also called the *symmetric difference*, denoted by $+$, which is represented by the following table:

P	Q	$P + Q$	$\bar{P} \bar{Q} \vee P \bar{Q}$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	0	0

It follows from the table that $P + Q$ means " P or Q but not both" and that

$$(P + Q) = \bar{P} \bar{Q} \vee P \bar{Q}.$$

It also follows that if we define $+$ for 0 and 1 to be like addition modulo 2:

$$0 + 0 = 0, \quad 0 + 1 = 1 + 0 = 1, \quad 1 + 1 = 0$$

then the truth value of the sum of two propositions is the sum of their truth-values.

Among the 16 possible functions of two propositions are also the propositions T and F defined by the following table:

P	Q	T	F
0	0	1	0
0	1	1	0
1	0	1	0
1	1	1	0

The proposition T is true for all combinations of truth values of P and Q and is known as the *tautology*. The tautology has many equivalent forms:

$$T = P \vee \bar{P}$$

$$T = [(P = Q) = (P \leq Q) (Q \leq P)]$$

and so on. The proposition F is one which is false for all combinations of truth values of P and Q . It is called the *inconsistency* and also has many equivalent forms:

$$F = P \bar{P}$$

$$F = (P \vee Q) \bar{P} \bar{Q}$$

and so on. Since every propositional logic contains some P and hence $P \vee \bar{P}$ and $P \bar{P}$, every propositional logic contains both T and F .

Finally we mention the *Schefferstroke* $P | Q$.

It is defined by the table:

P	Q	P	Q	$\bar{P} \vee \bar{Q}$
0	0	1	1	1
0	1	1	1	1
1	0	1	1	1
1	1	0	0	0

From the table it follows that $P | Q$ means "at least one of P and Q is false" and hence

$$P | Q = \bar{P} \vee \bar{Q}.$$

We have listed only the most useful connectives above, but like those listed, *all are readily expressed in terms of \vee , \cdot , and \neg* . From this it follows that the 16 distinct propositional functions of two propositions P and Q are an example of a finite propositional logic. The complete list of 16 functions is as follows:

$$\begin{aligned} T, F, P, \bar{P}, Q, \bar{Q}, \bar{P} \vee \bar{Q}, \bar{P} \vee Q, P \vee \bar{Q}, P \vee Q, \bar{P} \bar{Q}, \bar{P} Q, P \bar{Q}, P Q, \\ P Q \vee \bar{P} \bar{Q}, \bar{P} Q \vee P \bar{Q}. \end{aligned}$$

If $P = T$ and $Q = F$, the propositional logic of 16 functions reduces to a propositional logic of just two.

Recalling the definition of a propositional logic, we may conclude at this point:

Every propositional function—employing a finite number of arbitrary connectives—of propositions P, Q, \dots of a propositional logic $\{P\}$ is again a proposition of $\{P\}$.

For example, we have

$$(P | Q) + (P \leq Q) = (\bar{P} \vee \bar{Q}) (\bar{P} \vee Q) \vee (\bar{P} \vee \bar{Q}) (\bar{P} \vee Q)$$

which involves only \vee , \cdot , and \neg (and which reduces simply to P).

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It should be mentioned that all connectives can also be expressed in terms of \vee and \neg only, in terms of \cdot and \neg only, and also in terms of the Scheffer stroke only. The reader may enjoy establishing these facts.

4. Propositional Logic as Boolean Algebra.

As we have just remarked, a propositional logic *contains*, in addition to simple expressions like $P \vee Q$, $P Q$, and \bar{P} , more complicated expressions built up by use of the various connectives. The truth values of such functions may be tabulated and compared just as in the simpler cases. This process provides a convenient method for establishing the equivalence of functions and, at times, of effecting simplifications. For example, the equivalences

$$\begin{aligned} f(P, Q, R) &= (P \vee Q) R \bar{P} \\ &= \bar{P} Q R \vee P \bar{Q} \bar{R} \vee P \bar{Q} R \vee P Q \bar{R} \\ &= (\bar{P} Q \vee P \bar{Q}) R \vee P \bar{R} \end{aligned}$$

follow from the fact that all three functions have the same truth table:

P	Q	R	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

The correctness of the entries *should* be checked by the principle stated at the close of Section 1.

By this method of comparison, together with appropriate use of definitions already made, we may now establish the following equivalences for arbitrary propositions of a propositional logic $\{P\}$ and for their truth functions. In the right-hand column, the symbol " \Leftrightarrow " denotes "if and only if" and " \rightarrow " is another symbol for "implies".

- | | |
|---|---|
| (1) $P \vee Q = Q \vee P$ | $p \vee q = q \vee p$ |
| $P Q = Q P$ | $p q = q p$ |
| (2) $(P \vee Q) \vee R = P \vee (Q \vee R)$ | $(p \vee q) \vee r = p \vee (q \vee r)$ |
| $(P Q) R = P (Q R)$ | $(p q) r = p (q r)$ |
| (3) $P (Q \vee R) = P Q \vee P R$ | $p (q \vee r) = p q \vee p r$ |
| $P \vee Q R = (P \vee Q)(P \vee R)$ | $p \vee q r = (p \vee q)(p \vee r)$ |

(4)	$P \cdot P = P$	$p \cdot p = p$
	$P \vee P = P$	$p \vee p = p$
(5)	$P \vee F = P$	$p \vee 0 = p$
	$P \cdot F = F$	$p \cdot 0 = 0$
(6)	$P \vee T = T$	$p \vee 1 = 1$
	$P \cdot T = P$	$p \cdot 1 = p$
(7)	$P \bar{P} = F$	$p \bar{p} = 0$
	$P \vee \bar{P} = T$	$p \vee \bar{p} = 1$
(8)	$\overline{P \vee Q} = \bar{P} \bar{Q}$	$\overline{p \vee q} = \bar{p} \bar{q}$
	$\overline{P \cdot Q} = \bar{P} \vee \bar{Q}$	$\overline{p \cdot q} = \bar{p} \vee \bar{q}$
(9)	$\overline{\overline{P}} = P$	$\overline{\bar{p}} = p$
(10)	$P \leq P$	$p \leq p$
(11)	$(P = Q) = (P \leq Q) (Q \leq P)$	$(p = q) = (p \leq q) (q \leq p)$
(12)	$(P \leq Q) (Q \leq R) \leq (P \leq R)$	$(p \leq q) (q \leq r) \rightarrow (p \leq r)$
(13)	$(P \leq Q) = (P \cdot Q = P)$	$p \leq q \Leftrightarrow p \cdot q = p$
(14)	$(P \leq Q) = (P \vee Q = Q)$	$p \leq q \Leftrightarrow p \vee q = q$

Since $\{P\}$ is closed with respect to the operations \vee , \cdot , and $\bar{}$, and since the above 14 laws include the basic postulates of a Boolean algebra, what we have in effect established, once the details are complete, is the following result:

With respect to the connectives \vee , \cdot , $\bar{}$, and with respect to the meanings of T and F assigned above, every propositional logic $\{P\}$ is a Boolean algebra.

Similarly if $\{p\}$ is the set of truth functions of the propositions of $\{P\}$, then $\{p\}$ also is a Boolean algebra.

Actually these 14 rules are not all independent so that the proof that we have in fact Boolean algebras could be made much shorter than the above observations suggest. The above rules are all essential, however, for the application to circuit design.

It is now possible to use the above algebra for the simplification or manipulation of logical functions.

For example:

$$\begin{aligned}(P \leq Q) \cdot (Q \leq P) &= (\bar{P} \vee Q) (\bar{Q} \vee P) \vee (\bar{P} \vee Q) (\bar{Q} \vee P) \\&= P \bar{Q} (\bar{Q} \vee P) \vee (\bar{P} \vee Q) \bar{P} \\&= P \bar{Q} \bar{P} \vee P \bar{Q} \vee \bar{P} Q \vee \bar{P} Q \\&= P \bar{Q} \vee \bar{P} Q = P + Q.\end{aligned}$$

Also: $\overline{P + Q} = \overline{\bar{P} Q \vee P \bar{Q}} = \bar{P} \bar{Q} \cdot \bar{P} \bar{Q} = (P \vee \bar{Q}) (\bar{P} \vee Q) = \bar{P} \bar{Q} \vee P Q.$

Further identities involving the operation t are particularly useful in applications.

5. The Canonical Expansion

One final bit of algebra is necessary before we begin discussing circuits.

Frequently a propositional function $f(P_1, P_2, \dots, P_n)$, where the P 's are given functions, is defined by a truth table rather than being given algebraically. It is always possible, in the following way, to write an algebraic expression for the function by inspection of this table.

Define $P^1 \equiv P$, $P^0 \equiv \bar{P}$.

Then the proposition f defined thus: $f \equiv P_1^{\alpha_1} P_2^{\alpha_2} \dots P_n^{\alpha_n}$ has truth value 1 if and only if each $P_j^{\alpha_j}$ has truth value 1, i.e., if and only if each $P_j^{\alpha_j}$ has truth value 1. Thus the truth table of the proposition f will have a 1 in the row corresponding to the combination $\alpha_1 \alpha_2 \dots \alpha_n$ and a 0 in every other row.

Now, returning to the truth table of f , let us write the union of the products f corresponding in this way to those rows of the truth table for which $f = 1$. Since this union has the value 1 for exactly the same combinations as does f , it is equivalent to f . Thus f has been expressed in algebraic form. This form of f is called its *conjunctive normal form* or its *canonical expansion*.

As an example, consider the table

P	Q	R	f
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

Here $f = \bar{P} \bar{Q} \bar{R} \vee \bar{P} Q \bar{R} \vee P \bar{Q} R \vee P Q \bar{R}$.

Often the canonical expansion of a function may be considerably simplified by the aid of rules (1) - (9) and other rules derived from them and the definitions. In the case of the present example we have

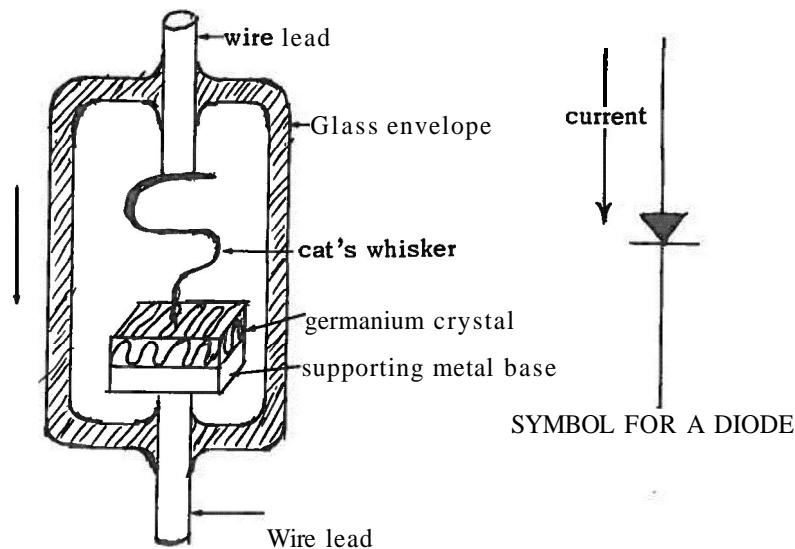
$$f = \bar{P} + Q + R.$$

The canonical expansion of another function was also given in the example at the opening of Section 4.

6. Electronic Realization of Logical Functions

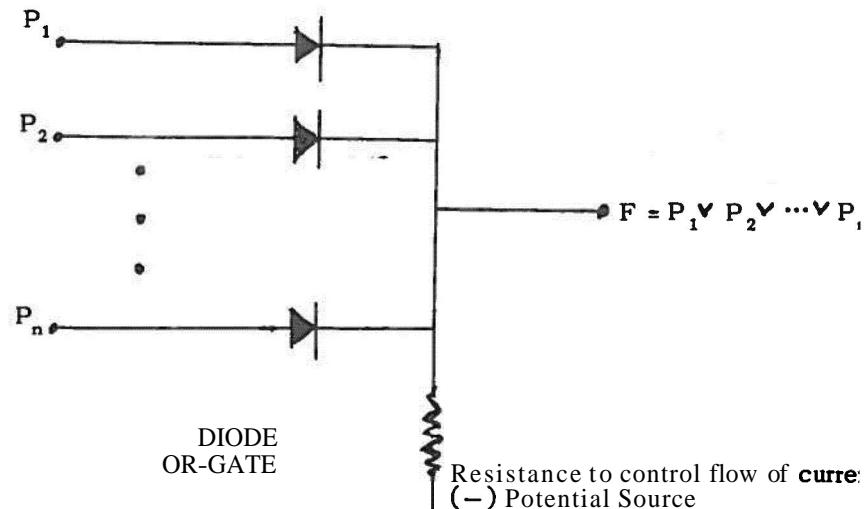
With the above algebra of propositions at our disposal we are in a position to show how logical functions can be realized economically by means of electronic devices and circuits. One way to do this is to let the truth of a proposition P be represented by a suitably high voltage potential (or simply by a voltage pulse) at a certain point or on a certain lead in a circuit, the falsity of P then being represented by a low potential (or no pulse) at the same point in the circuit. Then one uses appropriate electronic devices to exploit this mapping of truth values into voltages for the construction of circuits which realize logical functions.

Among these electronic devices, one of the simplest is the *crystal diode*. It consists of a germanium or silicon crystal in contact with a "cat's whisker":



The characteristic property of the diode is that it conducts current freely in one direction (indicated by the arrow in the diagram) and almost not at all in the other direction. The diode envelope is only about 1/4 inch long and 1/10 in diameter. For symbolic purposes, we assume the diode conducts perfectly in one direction, not at all in the other. In an actual design, the fact that this is not quite true must be recognized and dealt with appropriately if it is of any consequence. This is ordinarily an engineering problem rather than an abstract mathematical problem however.

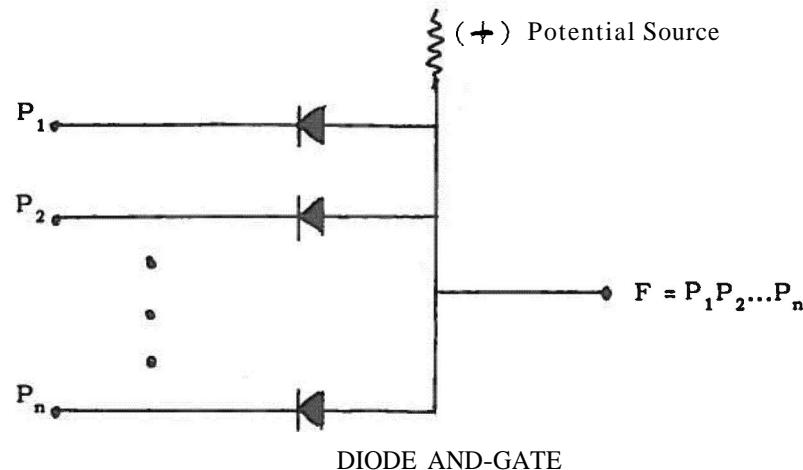
If the truth of a proposition is represented by a positive voltage on a corresponding lead, the "or" connective may be realized by the following diode circuit:



Here if all the leads corresponding to the P_i 's are at a slightly lower potential than the source all the diodes are blocked from right to left and terminal F is at the same low potential as the source. On the other hand, because the diodes are perfect conductors from left to right, if one or more of the P_j 's is high, F is also at this high potential. Thus the potential at F corresponds to the function

$$F = P_1 \vee P_2 \vee \dots \vee P_n.$$

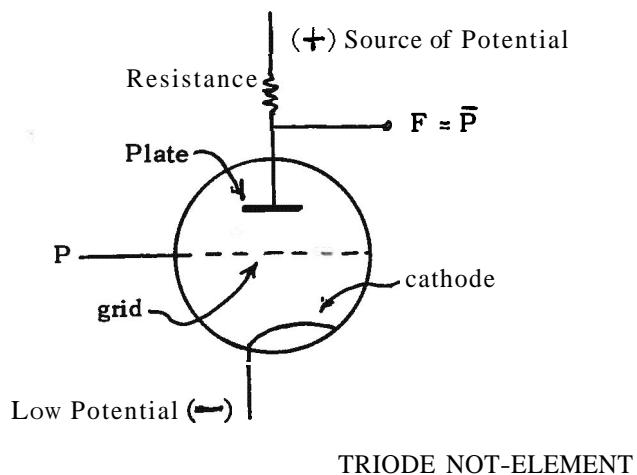
The "and" connective may be realized by the diode circuit



Here if any one of the P_j leads is at a lower voltage than the **source**, the corresponding **diode**, conducting perfectly from right to left, reduces the F lead to the same low potential. On the other hand, if all P_j -leads are at a potential slightly higher than the source, all the diodes are blocked and F is at the same high potential as the source. Thus the potential at F corresponds to the function

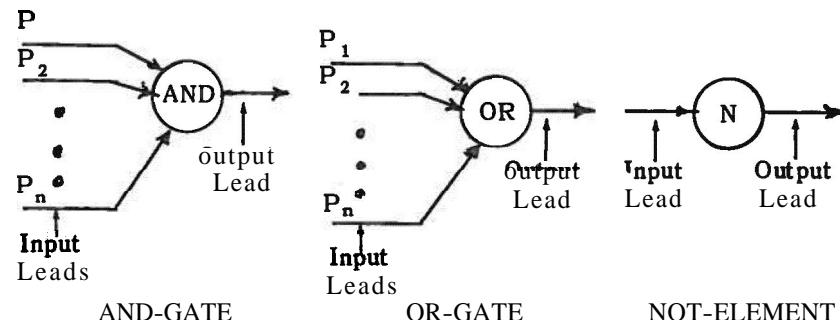
$$F = P_1 P_2 \cdots P_n.$$

The complement of a function may be realized with a vacuum tube. Consider the diagram



Here the tube will not conduct current from plate to cathode unless a sufficiently high voltage is applied to the grid lead at P. Thus, when the potential at P is "low", the potential at F remains "high". However, if P is at a high potential, the tube acts as though it were nearly a short circuit from plate to cathode so that F is at a low potential. Thus the tube realizes the function $F = \bar{P}$.

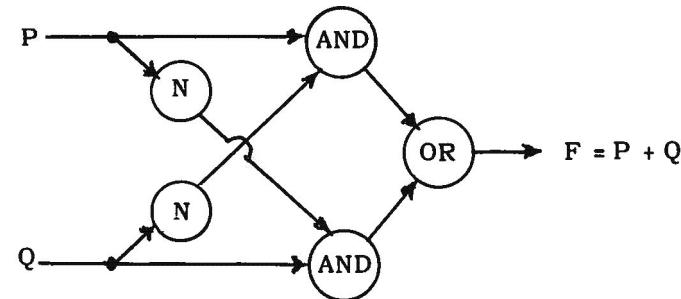
In a similar fashion, one can realize other connectives and combinations thereof electronically. There are also many other ways of realizing the above three connectives. Such realizations are commonly called "decision elements". The three decision elements we have shown are also called or-gates, and-gates, and not-elements or inverters, respectively. The actual devices used to realize these decision elements need not concern us further, so we symbolize them respectively as follows:



Using these symbols, we can now draw a logical diagram as it is called, for a circuit that realizes any given propositional function. Once the logical diagram is obtained and the actual devices to be used are decided on, the construction of the circuit is a routine matter. For example, the function

$$F = \bar{P} Q \vee P \bar{Q} = P + Q$$

has the realization



Here if the voltages at P and Q are both high or both low, each and-gate has one low input and is therefore closed so that the voltage at F is low. On the other hand, if P is high and Q is low or P is low and Q is high, then one of the two and-gates has two high leads and hence transmits a high potential through the or-gates to F. Thus the voltage at F represents the truth value of the function $F = P + Q$.

If we let the truth value 0 correspond to a low voltage and the truth value 1 correspond to a high voltage, then in the above diagrams we can replace the capital letters representing propositions by the corresponding small letters representing their truth values. Then the circuits realize the corresponding functions of truth values. This is simpler and more useful, so we adopt this procedure from now on.

7. The Simplification Problem

Since a logical function has many different but equivalent algebraic **forms**, we must always consider the possibility that we have chosen a form which renders the corresponding circuit unnecessarily complex. For example, in the last section we have used the fact that

$$P + Q = \bar{P}Q \vee P\bar{Q}$$

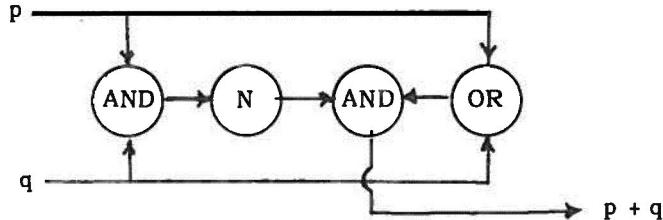
and have drawn the corresponding logical diagram. On the other hand, we also have

$$P + Q = (\bar{P} \vee \bar{Q}) (P \vee Q) = \bar{P}\bar{Q} (P \vee Q)$$

or, for truth values,

$$p + q = p\bar{q} (p \vee q).$$

The logical diagram for the latter form of the function is:

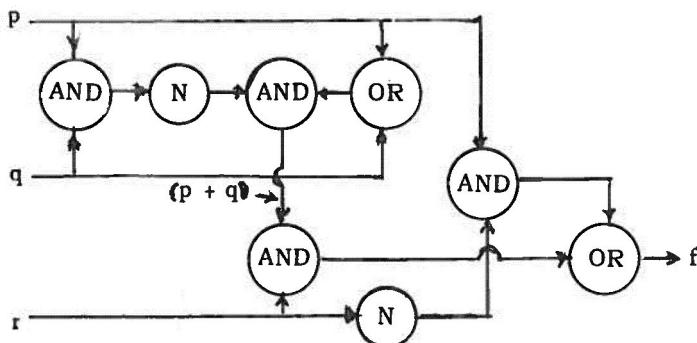


which requires one less not-element than does the previous one.

Again, the function

$$f = (\bar{p} \vee q \vee p \bar{q}) r \vee p \bar{r} = (p + q) r \vee p \bar{r}$$

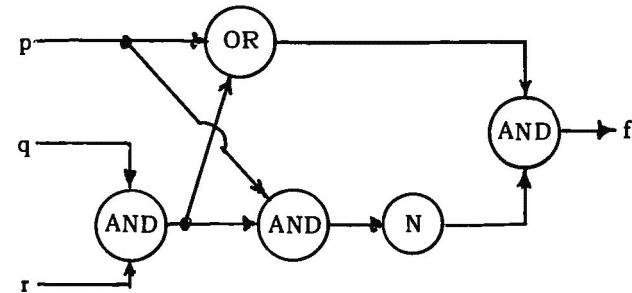
has one realization requiring six gates and two not-elements:



We have however seen that we may also write

$$f = (p \vee q \bar{r}) \bar{p} \bar{q} \bar{r}$$

Here the product $q \bar{r}$ appears twice but need be generated only once in the circuit:



This diagram requires only four gates and one not-element.

The problem of writing a given truth function in its simplest form for the purposes of logical design has not been completely solved.

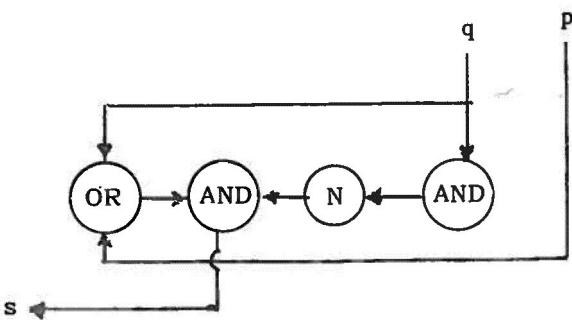
8. Logic Circuits and computation

In industrial control systems and telephone systems the propositions whose truth values are mapped by logic circuits may be in effect statements that certain processes are complete in which case the outputs of these circuits may be used as signals to certain equipment to start the next process of the chain being controlled. A more familiar situation is that in which computation is being effected with numbers expressed in the binary system. This computation can be reduced to a set of propositions which say that certain digits are 0 or 1. If we let a high potential denote the digit 1 and a low potential denote the digit 0, then propositions in arithmetic may be mapped in a 1-1 fashion into propositions about the potential on certain leads of a circuit. For example, suppose we wish to add two binary digits, modulo 2. Then the function we wish to realize has the table

p	q	s
0	0	0
0	1	1
1	0	1
1	1	0

that is, it is simply the sum $p + q$.

Thus the circuit



is a modulo 2 adder, or *half-adder* as it is commonly called. It is the same circuit we have drawn before except that the orientation has been changed to correspond to the direction in which addition proceeds.

Since the addition of two numbers often involves a carry operation we next design a circuit which combines an addend digit p , an augend digit q and a carry digit c_o to yield a sum digit s and a carry digit c according to the following table:

p	q	c_o	s	c
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

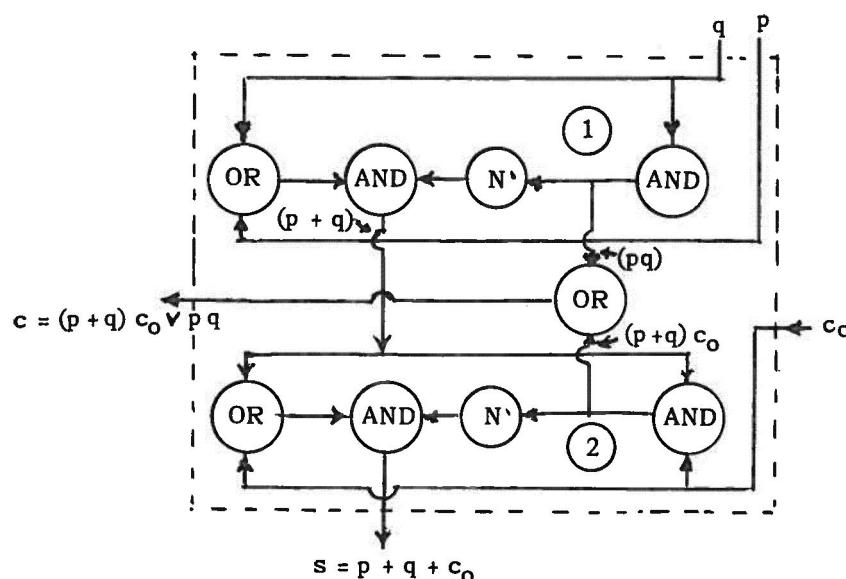
Here s is the mod 2 sum of p , q , and c_o so that we have at once

$$s = p + q + c_o.$$

We have also, from the table and from the algebraic rules (1) - (14):

$$\begin{aligned} c &= \bar{p} q c_o \vee p \bar{q} c_o \vee p q \bar{c}_o \vee p q c_o \\ &= (\bar{p} q \vee p \bar{q}) c_o \vee p q (\bar{c}_o \vee c_o) \\ &= (p + q) c_o \vee p q \end{aligned}$$

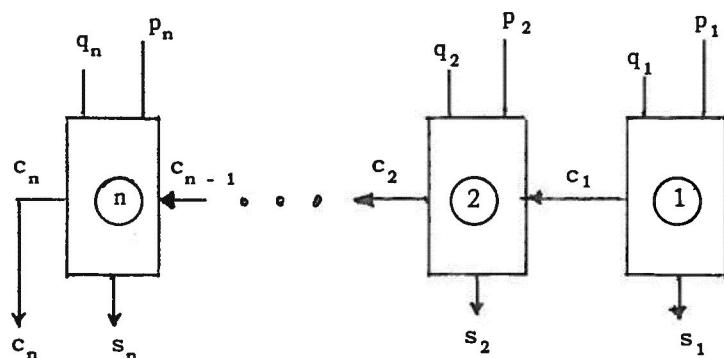
Hence the adder we desire can be realized by combining into one **circuit** a circuit for s and a circuit for c . We draw the logical diagram for s by forming first $p + q$, then $(p + q) + c_o$, in each case proceeding in the same manner as was used in the previous example:



Then we take a $p q$ lead from point 1 and one giving $(p + q) c_o$ from point 2 and feed these into an OR-gate the output of which is therefore c . This circuit is called a **full adder** or simply an adder.

A block diagram for one type of **n-position** binary adder may now be given. It is a circuit which performs the binary **addition**

$$\begin{array}{r} p_n \dots p_3 p_2 p_1 \\ q_n \dots q_3 q_2 q_1 \\ \hline c_n s_n \dots s_3 s_2 s_1 \end{array}$$



This type of adder is called a **serial adder** because the unit performing the j^{th} step must wait for the unit performing the $(j-1)^{\text{th}}$ step to finish its work before going ahead.

9. Further Issues in Circuit Design

From the preceding section it appears that we are well on the way toward designing a computer. There **are**, however, some problems remaining. For example, one must design translators for translating decimal digits into binary digits and back **again**, circuits to remember numbers which are to be used again in later computations, etc. For these **tasks**, the essential logical machinery has been given above.

There remain however more serious problems. We have **tacitly** assumed that all logical operations are performed instantaneously by the decision elements, which is not in fact the case. For some purposes the small but finite delay in each decision element is of little or no consequence; for others such delays are of major importance. In the latter event one must insert **delay elements** in appropriate leads in order to guarantee that signals arrive at the proper point at the proper time. The logical design of such circuits is facilitated by the introduction of time as an operator into the Boolean algebra already developed, but that is the subject of another discussion than this.

University of Illinois

MATHEMATICS IN TURKISH HIGH SCHOOLS¹

by ALI IHSAN TANGOREN
Louisiana ALPHA, '55

The land that is today Turkey was formerly the core of the sprawling Ottoman Empire, which at its peak embraced vast portions of **Europe, Asia**, and Africa. The Empire began in the 13th century and endured until the **20th**.

The Ottoman Empire came to the end of its historic road after World War I. A Turkish army commander named Mustafa **Kemal**, later called Ataturk-Father of the Turks, united a force of nationalists and launched the Turkish war of independence. They achieved a world startling victory in the struggle which culminated in 1923 with the founding of the modern Republic of Turkey. Ataturk became the first president. Along with other sweeping reforms the Republic completely revised the system of education in the country.

The Turkish educational system is highly centralized, headed by a Minister of Education and an Undersecretary. The minister in Ankara affects every school in the country. Policies are made in the capitol and through a series of persons, responsible to those above them they are gradually shifted downward and outward until each teacher is informed by a directive. In such a centralized educational hierarchy the development of well-defined system is expected. The Turkish republic, through the philosophy of its great revolutionary hero Ataturk, and through an early school survey by the philosopher John **Dewey**, has a strong educational system which is based on democratic ideas.

Its objective is the conduction of an energetic and continuous campaign against illiteracy. With this objective in view the Turkish Government is making every endeavor to increase the number of schools, to improve **them**, and to **insure that** every child is taught along modern scientific lines.

In Turkey an essential factor for success in life is to be educated. The new generation must be trained in the best way in the mathematical sciences to build a more powerful nation. Pupils start to study mathematics in the first grade and continue until they are ready to go to a university.

The primary education lasts five years. There is an oral examination at the end of this period. A student may go to a secondary school after graduation from elementary school if he is between the ages of 11 and 18. The secondary training is a six year course split into two periods; the **first**, lasting **3 years**, corresponds roughly to the American Junior High; and the remaining **3**, at a **lycee**, are the

¹Presented at the national meeting at Pennsylvania State University, August 26, 1957; received by the editors, October 18, 1957.

counterpart of Senior High School. There is an oral examination at the end of the third year. Those who pass the examination are allowed to take a matriculation examination which qualifies them to enter into the high school.

The curriculum in the secondary school consists of Turkish, history* geography, civics* mathematics* physics, chemistry, a foreign language, art, music, physical education, home economics, handicraft (electives for girls) or agriculture (electives for boys). At the senior high, philosophy and military science are added. The curriculum is fixed and every student has to follow it. If the student pursues the classical curriculum, at the eleventh grade mathematics loses its emphasis.

The language and science courses, especially mathematics* are the most highly stressed subjects, and for this reason not every student who started secondary education can finish **it**, and some go to other schools, (like trade or commercial schools), where they can succeed. Even in those schools mathematics is stressed very much, and I think most of the mathematics teachers are agreed that the curriculum is heavy for some of the students, and, therefore, try to help students in every way. The teacher in conducting his class lectures, works some sample problems, assigns home work problems, gives class problems, oral examinations and also written examinations. In every school at least one teacher gives free lectures to those who need help in mathematics. Especially before the final and matriculation examinations, most of the students study in groups and teachers sometimes meet the classes out of school hours. Matriculation examinations are given at the end of the 8th and 11th grades and are prepared by the Department of National Education. They examine the matter covered in four different subjects. One of these subjects is mathematics.

To indicate how many hours of mathematics is required of the student I would point out that the 9th grade has 5 hours of mathematics per week. the 10th grade 4 and the third year in the classical section 2, and the scientific section has 8 hours of mathematics per week. Mathematics in scientific classes covers: Theoretical Arithmetic, 1 hour a week; Astronomy, 1 hour a week; Trigonometry, 1 hour a week; Algebra, 2 hours a week; Geometry, 2 hours a week; and Mechanics, 1 hour a week.

At this moment I would like to point out that there is an oral examination at the end of the fifth grade, an oral and matriculation examination at the end of the 8th and again an oral and a matriculation examination at the end of the 11th grade. An average student passes the tests either in June or in September, but at the 11th grade not only those in scientific classes but also those in the classical classes sometimes wait one year just to pass the matriculation examination* especially in mathematics. I think you will agree with the wisdom of such a delay after reading the sample questions taken from the 1956 11th grade matriculations examinations listed at the end of

this paper. In 2 sections of scientific classes I remember one year when only one student passed the oral and the matriculation examination in June. Usually half of the classes finish their examination in September. The students in classical sections graduate more easily and with less effort than in mathematics. I myself graduated from classical section* because I did not want to wait one year for matriculations examination in mathematics. Still I failed in mathematics in June and passed it in September.

Why is mathematics so emphasized and the students have to study so hard in order to pass it? I think because it is the most positive science* and our educators believe that every student needs it. Why do they need it? Let me explain this by recalling some of what I heard from my teachers. I am still influenced by my most beloved mathematics and philosophy teacher. From time to time my mathematics teacher left the subject of the text book to tell us something about the world's great mathematicians, what they did, how they studied* the importance of mathematics* and its position in the scientific world. Newton* Descartes, and others were the heros of the stories. I also learned that it is the most positive science; it gives the logical deductions of consequences from the general premises of all reasoning. Then the philosophy teacher comes and says, "Nothing can be positive without using some mathematics in it and experimenting the facts. Yes, nothing but mathematics. It is the only one giving others exactness." In another lecture I heard that certain types of illness, especially some of the mental illnesses, can be cured by mathematics. I think today our curriculum is prepared for genius students* and lets the others take their chance to catch up with them. I think it is really working well by close cooperation of students, teachers* and the parents. Many times my teachers explained the fact that to be successful in mathematics one needs some gifts from Almighty God. But still they convinced us that if we wanted we could do it. By working more sample problems, I could learn it more quickly and solve the problems more rapidly. Sometimes the teacher assigned the new chapter to one of the students who had difficulties in mathematics. The student gives the lecture on the new chapter, works the sample problems on the board* and the instructor helps him to explain it more clearly by asking questions and repeating the important points. Sometimes the teacher assigned the students who made good grades as tutors to those who had difficulties with the subject* or seated them next to each other in the classroom. The teachers try from time to time to convince the student that every one can learn **it**, the only difference being in the speed of learning. They also try to help the student love this course. They explain from time to time the importance of it. Everything one can see has some mathematics in **it**, in one way or another. Most of the high schools have their weekly newspaper* which is prepared only by students. These papers and the newspapers of our towns have mathematical puzzles, and most people try to solve these

puzzles while they are drinking the famous Turkish coffee in our cafes. These reflections **will, I hope**, help the reader understand a bit of the attitude in Turkey towards mathematics.

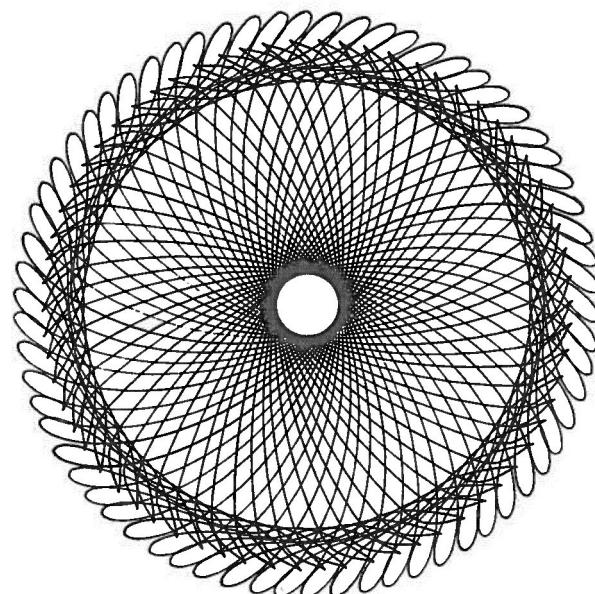
Perhaps the most significant feature of Turkish education since the founding of the Republic in 1923 has been its astonishing growth. It has graduated thousands upon thousands of young men and women from its schools who are not only well trained in their own field, but in mathematics as well. I should point out that our country is still undeveloped, and though we are proud of having one of the finest armies in the world **today**, we do not have enough technicians, and this is certainly one of the reasons that the Department of Education stresses the great importance of training in the scientific fields.

The sample questions taken from the 1956 mathematics matriculation examinations are:

- 1) Prove that any line with slope (m) through the point of inflection cuts the curve $y = x^3 + ax^2 + bx + c$ in 2 points that are symmetric with respect to the point of inflection.
- 2) Find the area enclosed by the curve $y = x^3 + ax^2 + bx + c$, and the x-axis.

3) Discuss and draw the graph of $y = 1 - 4 \cos^2 x$.

4) Draw any circle which passes through a point m on a parabola and through its focus f . The parabola's tangent at m cuts the circle at p . Prove that the tangent drawn to parabola from p is also tangent to the circle.



PROBLEM DEPARTMENT

Edited by

M. S. Klamkin, AVCO Research and Advanced Development Division

This department welcomes problems believed to be new and, as a rule, demanding no greater ability in problem solving than that of the average member of the Fraternity, but occasionally we shall publish problems that should challenge the ability of the advanced undergraduate and/or candidate for the Master's Degree. Solutions of these problems should be submitted on separate signed sheets within four months after publication. Address all communications concerning problems to M. S. Klamkin, Avco Research and Advanced Development Division] R-6, Wilmington, Massachusetts.

Problems for Solution

102. Proposed by Leo Moser, University of Alberta

Give a complete proof that two equilateral triangles of edge 1 cannot be placed, without overlap, in the interior of a square of edge 1.

103. Proposed by Lawrence Shepp, Princeton University

$$\text{If } \frac{F(x) - F(y)}{x - y} \leq F'(\frac{x+y}{2})$$

for all x and y in a bounded interval, then $F(x) = ax^2 + bx + c$.

104. Proposed by D. J. Newman, Massachusetts Institute of Technology

$$\text{If } x_{n+1} = a_n x_n + b_n x_{n-1},$$

$$\text{where } a_n + b_n = 1,$$

$$a_n, b_n \geq 0,$$

find a necessary and sufficient condition on the a_n, b_n such that $\{x_n\}$ converges for all initial conditions.

105. Proposed by C. D. Olds, San Jose State College
Show that

$$\int_1^{\sqrt{2}} \frac{(x^4 - 2x^2 + 2)^n}{x^{2n+1}} dx = \int_1^{\sqrt{2}} \frac{(x^2 - 2x + 2)^n}{x^n} dx.$$

106. Proposed by M. S. Klamkin, Avco RADD

An equi-angular point of an oval is defined to be a point such that all intersecting chords through the point form equal angles with the oval at both points of intersection (on the same side of the chord). It is a known elementary theorem that if all the interior points of an oval are equi-angular then the oval is a circle.

1. Show that if one boundary point of an oval is equi-angular, the oval is a circle.

2. Determine a class of non-circular ovals containing at least one equi-angular point.

3. It is conjectured that a non-circular oval can have, at most, one equi-angular point.

Solutions

97. Proposed by Alan J. Goldman, Princeton University

Prove that a triangle of area $1/\pi$ has a perimeter greater than 2.

I. Solution by Jack R. Porter, Oklahoma University

A corollary (which is easily proven) in plane geometry states, "Of all triangles of equal areas, the equilateral triangle has the least perimeter." The stated inequality follows immediately.

II. Solution by the proposer.

The desired inequality follows from $A = r p/2$ and $\pi r^2 < A$, where $A = \text{area}$, $p = \text{perimeter}$, $r = \text{radius of the inscribed circle}$.

Also solved by Patrick G. Carr, James J. Dodd, Richard K. Fry, Fred Gross, Arthur F. Kaupe, Jr., Mitchell A. Krasny and Robert A. Alig, George Peters, T. A. Porschning, Guy Torchinelli, Jack Winter, and Jack Silver.

98. Proposed by C. W. Trigg, Los Angeles City College

Each of the letters in (AH) (ME) = EEE represents a distinct digit. Decode the equation.

Solution by Arthur F. Kaupe, Jr.

$$\begin{aligned} (\text{AH}) (\text{ME}) &= \text{E}(111) = (3\text{E})(37); & \text{whence } \text{E} = 7, \text{A} = 2, \text{H} = 1, \text{M} = 3. \\ &= (37)(3\text{E}); & \text{whence } \text{E} = 5, \text{A} = 3, \text{H} = 7, \text{M} = 1. \\ &= (74)(3\text{E}/2); & \text{whence no solution.} \\ &= (3\text{E}/2)(74); & \text{whence no solution.} \end{aligned}$$

Consequently, the only solutions are $(21)(37) = 777$,

$$(37)(15) = 555.$$

Also solved by Patrick G. Carr, Jack R. Porter, Wolfe Snow, J. J. Dodd, Jack Winter, Jack Silver, John D. Steben and the proposer. One solution by Ralph Beals, Richard K. Fry, Fred Gross, Mitchell A. Krasny and Robert A. Alig, and Dimitri Mihalas.

Note: If we allow a zero first digit, Snow obtains a third solution

$$(06)(74) = 444.$$

99. Proposed by D. C. B. Marsh, Texas Technological College

A student, asked to write the equation of $79x^2 - 2xy + 79y^2 = 1$ after the xy -term has been removed by rotating the axes, uses the facts that $(B^2 - 4AC)$ and $(A + C)$ are **invariants**, whence $A' + C' = 158$ and $-4A'C' = 2^2 - 4(79)^2$, or $A' C' = (78)(80)$. The transformed equation must be $78x'^2 + 80y'^2 = 1$ or $80x'^2 + 78y'^2 = 1$, but which is it? How can he tell?

Solution by the proposer.

If the $Ax^2 + Bxy + Cy^2 + \dots + F = 0$ is transformed by a rotation of less than 90° (conventional) into $A'x'^2 + B'x'y' + C'y'^2 + \dots + F' = 0$, then the signs of B and $A' + C'$ are the same since $B(A' + C') = B(A \cos 2\theta + B \sin 2\theta) = \{(A - C)^2 + B^2\} \sin 2\theta$. The first solution is therefore the correct one.

Also solved by Patrick G. Carr, Mitchell A. Krasny and Robert A. Alig, Jack R. Porter, J. J. Dodd, Guy Torchinelli, and Jack Silver.

100. Proposed by Leon Bankoff, Los Angeles, California

A right triangle ABC ($AC > CB$) is inscribed in a semicircle (O) whose diameter is AB. The radius OS, perpendicular to AB, cuts AC in R, and CD is the altitude upon AB. Find the ratio SO/RO for which triangles ODC and CDB are both Pythagorean.

Solution by the proposer, also solved by Guy Torchinelli.

If we let $\frac{SO}{RO} = k$, then

$$\frac{CD}{DB} = \frac{AD}{CD} = \frac{AO}{RO} = \frac{SO}{RO} = k. \quad (1)$$

Also,

$$\frac{OC}{OD} = \frac{AO}{OD} = \frac{AR}{RC} = \frac{\overline{AR}^2}{\overline{AR} \cdot \overline{RC}} = \frac{\overline{AO}^2 + \overline{RO}^2}{SR(\overline{RO} + \overline{SO})} =$$

$$\frac{\overline{SO}^2 + \overline{RO}^2}{SR(\overline{RO} + \overline{SO})} = \frac{k^2 + 1}{k^2 - 1}. \quad (2)$$

By (1) and (2),  CDB and ODC are Pythagorean only if $\frac{SO}{RO} = k = \frac{m^2 - n^2}{2mn}$ where m and n are integers such that $k > 1$. The same result can be shown to hold for the case $AC < CB$. Also solved by Arthur F. Kaupe, Jr.

101. Proposed by Norman Anning, Alhambra, California

In Dantzig's "Bequest of the Greeks" (Scribner, 1955) p. 38, the statement is made, "Eudoxus...discovered the sections of this surface (a Torus) by planes parallel to the axis of revolution, quartic curves which today are called Cassinian ovals". Show that these section are, or are not, Cassinian ovals.

Solution by Jack R. Porter, Oklahoma University

The equation of the torus generated by revolving the circle

$(x-h)^2 + z^2 = r^2$ about the z-axis is $(x^2 + y^2 + z^2 + h^2 - r^2)^2 =$

$4h^2(x^2 + y^2)$. The equation of a Cassinian oval in the $y-z$ plane is $(y^2 + z^2 + a^2)^2 = c^4 + 4a^2z^2$. In order for the intersection of the plane $x = \text{constant}$ with the torus be a Cassinian oval, we must have

$x^2 + h^2 - r^2 = h^2$ or that $x = \pm r$. Consequently, the cross-sections by planes parallel to the axis of revolution are only Cassinian if and only if the plane is a distance r from the axis.

Also solved by Arthur F. Kaupe, Jr.



DO YOU KNOW

Esperanto: **unu**, du, tri, kvar, kvin,
ses, sep, ok, nau, dek.

Russian: odjn, dva, tri, chetyre, pyat',
shest, **sem'**, **vosem'**, devyat', desyat'.

German: eins, zwei, **drei**, vier, **fünf**,
sechs, sieben, acht, neun, zehn.

French: un, **deux**, trois, quatre, cinq,
six, sept, huit, **neuf**, dix.

BOOK REVIEWS

Edited by
Franz E. Hohn, University of Illinois

Selections from Modern Abstract Algebra. By R. V. Andree. Henry Holt & Co., New York, 1958. xii + 212 pp., \$6.50.

Intended for undergraduates at the sophomore or junior level, this book contains a wide selection of topics from number theory, Boolean algebra, groups, matrices, linear systems, fields and rings.

The best aspect of the book is the way abstract ideas are carefully and slowly introduced, so that the author has a valid claim that his method will help develop the "mathematical maturity" which other authors postulate. Perhaps the weakest aspect is the inclusion of several sections of advanced material too sketchily done to be useful and involving undefined terms.

On the whole the text, examples, and exercises are well suited to provide a transition from the traditional high school and college algebra toward the upper level abstract algebra courses. Surely the ideas of postulational thinking will rub off, met here again and again at various levels of difficulty. The excitement of interesting applications (as in electrical circuits that illustrate Boolean algebra) should be contagious. And the feeling that mathematics is a living, active science will certainly be generated by the scores of well chosen reading suggestions in recent periodicals and texts.

For the student or teacher examining the book critically we append a few notes suggested by a first reading.

p. 58: $x + y + z = x + y + z$, the second t is omitted.

p. 101: The notion of coset-multiplication is never explicitly defined.

p. 102: It seems a bit high flown to introduce the Jordan-Hölder theorem, albeit merely as an example of an advanced theorem.

p. 115: The discussion of elementary matrices E suffers from the omission, both in text and problems, of any description of the inverses E^{-1} .

p. 119: The reference to Section 4-1 should be to 5-2.

pp. 124, 162, et al: It seems puzzling (in a book whose typography is otherwise outstandingly good) that a few matrices are printed in bold-face.

p. 134: The reference to problem 1 should be to problem 11.

pp. 184-185: As printed here the charts classifying various algebraic systems have to be read from the bottom to the top.

p. 189: The coset-operations for a ring have not been explicitly defined. The "discussion" can be used to show the operations well-defined, after they are defined.

p. 191: The use of the words "finite field" for "field of finite order over a **subfield** of the real field" is confusion confounded, ignoring the existence of **Galois** fields (those normally called finite fields) as well as extensions of finite order of other fields (say p-adic fields) which are not subsets of the real field.

p. 196: Reference is made to theorems about solutions of a homogeneous system, but by some oversight these theorems seem not to be explicitly stated in the text or problems.

Michigan State University

B. M. Stewart

Vector Analysis. By Louis Brand. John Wiley & Sons, Inc., New York, 1957. ix + 282 pp., \$6.00.

The book under review is a more elementary treatment of some of the material given in the author's Vector and Tensor Analysis. The scope of the text can be surmised, somewhat, by the chapter headings which are in order: Vector Algebra, Line Integrals, Vector Functions of One Variable, Differential Invariants, Integral Theorems, Dynamics, Fluid Mechanics, Electrodynamics and Vector Spaces. Confining himself for the most part to three dimensions, the author nevertheless indicates a need for a more general concept of a vector. In the last chapter on Vector Spaces there is presented a short but excellent introduction to this important topic.

Clearly written, with an abundance of illustrated examples as well as problems to be solved by the student, Vector Analysis provides the reader with the fundamental theory necessary to an understanding of certain aspects of differential geometry, mechanics, electrodynamics and analysis. Moreover, because of the variety of topics treated, a mastery of the contents will give the student the necessary knowledge to employ vectors as a tool in the physical sciences.

University of Illinois

E. J. Scott

Elements of Pure and Applied Mathematics. By Harry Lass. New York, McGraw Hill Book Company, Inc., 1957. xi + 491 pp., \$7.50.

This text is one in the International Series in Pure and Applied Mathematics. One can gain an idea of its scope from the opinion of the author that "this book can be used to teach almost any undergraduate mathematics course from the junior level onward, and that it can be used as an applied mathematics text in a variety of ways which depend on the subjects to be stressed by the instructor."

While this ambitious claim is certainly somewhat unjustified, there is no denying that a large number of mathematical subjects are discussed, many of them in greater detail and with greater clarity than is usual in a book principally designed to cover those topics in classical mathematics of most interest to the engineer, physicist, or applied mathematician. The text is written in such a way that each chapter is independent of the others, although there are many cross references which help to unify the material. If this were not the case, it would be somewhat surprising to find, for example, a chapter on complex variable theory preceding one on real variable theory.

As one might surmise, it is a difficult matter to motivate the study of a large number of somewhat unrelated topics of usefulness in applied mathematics, and this book spends but little time doing so. The reviewer would have welcomed a stronger attempt in this direction. However this is a common criticism of texts in this area and perhaps a certain "hodge-podge" effect is inevitable, especially in a book which might well serve as a source of reference.

This text should find its principal use in a junior or senior level course in advanced engineering mathematics, possibly having a course in elementary differential equations as a prerequisite. The instructor would have ample material available to develop an adequate course, by choosing from such assorted topics as determinants and matrices, vector and tensor analysis, complex variable theory, ordinary differential equations, orthogonal polynomials, Fourier series and integrals, the Stieltjes integral, the Laplace transform and the calculus of variations.

To extend the possible audience to others seeking an introduction to methods of applied mathematics, the author has also inserted material on group theory, algebraic equations, probability theory, and real variable theory. The chapter on real variable theory includes many of the topics usually discussed in an advanced Calculus course, including in the last page and a half an introduction to Lebesgue integration.

The usefulness of the book is enhanced by a total of 242 illustrative exercises embodied in the text and solved in considerable detail. In addition there are frequent problem lists; however, very few answers are given. Reference lists occur at the end of each chapter. The format is pleasing, and no typographical errors were observed.

Michigan State University

E. A. Nordhaus

Games and Decisions. By R. D. Luce and H. Raiffa. New York, John Wiley & Sons, 1957. xix + 509 pp., \$8.75.

Within the last ten years, we have seen a variety of books dealing with various aspects of game theory and its applications. In writing a textbook, one has the tendency to discuss those aspects of a subject which have reached a more or less final form, and in which the mathematical theory has achieved a certain degree of elegance. In particular, a textbook on game theory is apt to center on the two-person zero-sum game, treated against a background of the study of convex sets in linear spaces.

In subtitling the present book "a critical survey", the authors have taken an entirely different point of view. Although the reader will find here a lucid account of most of the basic facts in two-person theory, the chief concern of the authors has been to present an account of the investigations of the past decade into the general n-person game situation, and its sociological implications. As such, it is an extraordinarily valuable contribution. It should be required reading for every college or university teacher of mathematics, for I cannot think of a better introduction to the point of view that mathematics is becoming a valuable tool in the behavioral sciences.

The first four chapters, together with portions of Chapter 7 and appendices 1 through 7, provide an introduction to 2-person zero-sum games; the emphasis is on exposition and explanation rather than proofs and illustrations. There are no exercises, so that a reader would benefit by coming to the book with the elementary knowledge contained in "Finite Mathematics" or "The Compleat Strategist". The remaining ten chapters deal with multiperson games, and the general problem of decision making under uncertainty. Sample topics: cooperative and non-cooperative games, formalized arbitration schemes, axiom systems for "solutions", equilibrium solutions, the Von Neumann characteristic function approach, games "against" Nature, utility theory, voting schemes, stochastic games, bankruptcy games.

The authors have thus presented, with critical comments, a wide selection of current opinions and approaches to the study of general conflict situations; one would perhaps like to see more devoted to the experimental results obtained by groups at RAND and elsewhere. However, no one could read this book without emerging with the feeling that here is a controversial area of research which offers exciting prospects for future exploration.

University of Wisconsin

R. C. Buck

Analytic Geometry. By E. J. Purcell. New York, Appleton-Century-Crofts, 1958. x + 289 pp., \$4.50.

This book gives unusually careful attention to precise definitions and statements of the conditions under which conclusions hold. It begins with the formal statement of certain principles which are ordinarily tacitly assumed. Then the basic tools of analytic geometry are developed in clear and detailed fashion. In this development, some particularly useful terminology is introduced. For example, the phrase "symmetric partner of a point" simplifies many statements, and it is useful to have the concept of an "essentially unchanged" equation rigorously defined.

The conics are treated in a unified, compact, and simple fashion by means of the focus-directrix definition in a manner which provides excellent training in mathematical thinking. The treatment here and throughout the

book places minimum emphasis on memorization of formulas. The emphasis is rather on learning to obtain needed results from a limited number of basic definitions, theorems, and formulas. The student who masters this text should therefore be well trained in the application of analytical tools.

Tangents to conics are presented in an optional chapter and the general equation of the second degree is treated in a brief but comprehensive way. Curve tracing and polar coordinates are discussed with special care and some particularly interesting examples are included.

The chapters on three-dimensional analytic geometry concern themselves with rectangular coordinates exclusively. Again the treatment is notably precise and **clear, and** is rather more detailed than average. Cylindrical and spherical coordinates are apparently left to the calculus.

The main purpose of teaching mathematics rather than formulas and rules is exceptionally well attained by this book which is clearly the record of a careful, competent, and inspiring teacher.

University of Illinois

Franz E. Hohn

College Algebra, Fourth Edition. By J. B. Rosenbach, E. A. **Whitman**, B. E. **Meserve**, and P. M. **Whitman**. Boston, Ginn and Co., 1958. xiv + 579 + xlvi pp., \$5.25.

This edition of one of the leading college algebra texts leaves the basic spirit of earlier editions unaltered. Like its predecessors, the fourth edition is notable for ample, clear precise explanations, unusually many illustrative examples of what to do and what not to do, and many interesting and valuable exercises of various levels of difficulty.

The exercises are, in fact, largely new. In particular, the statement problems are obviously relevant to modern affairs and thus provide a welcome contrast to the uninspired problems of many other texts.

There are many topics newly treated in this edition. For example, the treatments of the logarithmic charts and of basic matrix concepts are much to the point, but the treatment of finite differences is too brief to be of real value. A substantial chapter on descriptive statistics has also been introduced. Although this is in accord with current trends, the reviewer doubts the value of treating descriptive statistics independently of the corresponding tests of significance. The result of these additions, in consequence of which the book contains all things for all people, is a **larger-than-average** volume. Many teachers will no doubt prefer the shorter edition, "Essentials of College Algebra", (\$4.50) which does not contain the infrequently taught chapters.

There remain, as in every book, certain improvements to be made in the next edition. The subscript notation for the terms of a progression would get away from the often misleading fact that the letter "l" is the first letter of the word "last." The notions of the matrix as an **array** and the determinant as a number (or function) associated with a square array are not carefully distinguished. Thus, one does not interchange columns of a determinant; one interchanges columns of a matrix and examines the effect on the determinant, etc.

Such objections are, however, relatively minor and do not alter the fact that this is one of the clearest, most flexible, and most teachable college algebra texts available. It will be widely appreciated by teacher and student alike.

University of Illinois

Franz E. Hohn

A Comprehensive Bibliography on Operations Research. By the OR-Group, Case Institute. New York, John Wiley, 1958. xi + 188 pp., \$6.50

This bibliography, which is No. 4 in the series "Publications in Operations Research" includes "listings of all material published in operations research through December 1957." The listings are cross-indexed and each

is provided with a code number which serves to classify it in so detailed a fashion that its potential usefulness for a given purpose is readily estimated. Apart from its obvious value for the expert, this volume, in conjunction with the "Introduction to Operations Research" by Churchman, Ackoff, and Arnoff (reviewed last fall in these pages) serves to make this important and expanding field widely available to newcomers to the area.

University of Illinois

Franz E. Hohn

Mathematics and Wave Mechanics. By R. H. **Atkin**. New York. John Wiley, 1957. xv + 348 pp., \$6.00

The topic of wave mechanics is based upon a framework which is a synthesis of many diverse aspects of mathematics. The author has attempted successfully to present those topics pertinent to wave mechanics in a unified whole. The book is divided into essentially two sections: a discussion of the mathematics to be used (and some related topics) and a brief introduction to various topics in quantum mechanics, including chemical applications, relativity, and radiation. These sections are bridged by a brief treatment of classical mechanics.

The treatment of the topics is concise and will require a beginning student to refer to the many excellent references if this volume is to be more than a summary. For a student versed in mathematical analysis the book provides an excellent introduction to the techniques of wave mechanics.

Rigorous proofs are occasionally sacrificed for brevity but adequate references are given to the more complete treatments. The book is relatively free from typographical errors and has a good format. It is recommended to students who wish to become acquainted with the applications of mathematics and to those interested in obtaining a firm knowledge of the mathematical basis of wave mechanics.

Southern Illinois University

Boris Musulin

Models of Man, Social and Rational. By Herbert A. Simon. John Wiley and Sons, Inc., New York, 1957. xiv + 287 pp., \$5.00.

Mathematical model building in economics has a comparatively long history, but its extension to other areas of social science is recent. The uninitiated might even imagine that political science is one area into which mathematics is unlikely to penetrate. The work of Professor Simon is convincing proof to the contrary. A political scientist by formal training, now Professor of Administration and Head of the Department of Industrial Management at the Carnegie Institute of Technology, he is one of the newer generation of social scientists with little respect for traditional boundaries of specialization and with the habit of boldly applying all tools in their possession, including both classical and modern mathematics, wherever they seem appropriate.

The volume under review contains reprints of sixteen articles published during the previous ten year period. They are divided into four groups: Causation and Influence Relations, Social Processes, Motivation, and Rationality and Administrative Decision Making. The book is dedicated "to Harold Guetzlow, friend and companion in adventure." The term "adventure" suggests very well the character of the book and of Simon's work in general. Simon is a courageous, possibly even rash at times, innovator in the construction of theories of human behavior in the broadest sense. The mathematical reader will find this book exciting and provocative. But he must not expect complete and elegant exposition from a mathematical point of view. He may amuse himself by finding mathematical errors, which, however, Simon would rightly consider trivial in relation to the main purpose of his work.

The following observations concern points of this type. In discussing causal relations, especially on pages 5 and 65, Simon refers to functions as though they were one-to-one correspondences, i.e. as though they all had formal inverses. This idea is used as the basis for rejecting the association

of causality with functional determination. While this reviewer agrees that the notion of functional determination is not synonymous with causality, it does seem that the distinction between functions and non-functional relations (single-valued and multiple-valued functions) could be used somehow in the theory of causation. While it is perfectly true that the distinction between dependent and independent variables is a matter of symbolism, it does not follow that a unique value of the independent variable corresponds to each value of the dependent value merely because a unique value of the dependent variable corresponds to each value of the independent variable. In order to "simplify" the notation, Simon uses the same letter to stand for a function and its Laplace transform, distinguishing between them by the use of different arguments. Mathematics has been defined as the art of calling the same thing by different names, but this reviewer finds it jarring to call different things by the same name, even though Simon, in doing so, is in good company. In the argument on page 264, the enumerations on which the calculation of probabilities is based do not seem to take account of the possibilities of multiple counting.

A review in this journal is not the place to discuss the many fascinating ideas presented in these articles, which have already become part of the stream of research in their area. Mathematicians will find them interesting and suggestive of the many unworked veins of mathematical gold that lie near the surface in the social sciences.

Carleton College

Kenneth O. May

Economic Models: An Exposition. By E. F. Beach. John Wiley and Sons, New York, 1957. x + 227 pp.

The word "model" is used by many scientists today to stand for any abstract theory or small mathematical system which was designed to reflect some properties of the "real world". In Sciences with a well-established theoretical milieu a model is expected also to provide "rules of interpretation" ("operational definitions") to correlate its theoretical terms with observables. Where theory is still in the pioneering stage, on the other hand, models are often presented with no mention of possible rules of interpretation.

During the past hundred years mathematical models (in the wider sense - nothing usually said about operational definitions) have played an increasingly important part in economic theory. This has resulted in several advanced treatises on the mathematics of economic theory and also in many elementary textbooks in "mathematics for economists". Professor Beach's *Economic Models* is a new departure, both in its explicit recognition of the central concept of "model", and in the readers it is written for. It is explicitly written for economists with limited mathematical knowledge who nevertheless wish to follow and criticize mathematical theory in their fields.

The main body of the book falls naturally under two headings: what the author calls "mathematical models" and what he calls "econometric models". The difference is roughly that in this book "mathematical models" have absolutely nothing to do with empirical fact, while "econometric models" is a name for a part of the book that says nothing whatever about models until the final chapter.

Chapters 3-6 treat four different kinds of systems of equations by describing the systems and certain especially simple methods of solving a few of them. These models are meant to be assessed solely in terms of their "reasonableness" and whether they admit unique solutions for given values of certain parameters. (It is an interesting question, barely broached in this book, why many economists so often insist on the kind of "equilibrium" that comes with a unique solution.) The only suggestion of an intended interpretation for these models is in names like "price", "demand", and so on, for certain variables.

In Chapters 7-9 the viewpoint has suddenly shifted to that of the empiricist seeking to determine one relation among a number of "observed variables". These chapters provide a survey of that part of elementary cookbook statistics which deals with regression and correlation techniques.

The last chapter brings models back into the picture. Here the author states and briefly discusses the two important principles of scientific inference whose formulation in economic contexts is due to Haavelmo and to Koopmans. This section, greatly expanded, might provide a focus for the material of all the rest of this book.

Economic Models suffers from not having enough mathematics. It is hard to say anything sharp about systems of linear equations if you have decided "to avoid advanced mathematical devices such as the manipulation of determinants and matrices". The non-linear models turn out to be "non-linear" in the sense that some functions are of unspecified form (and hence, no doubt, may well be non-linear); in these cases the author has usually just written down the equations.

If this book has too little mathematics it seems sure that it has too much about regression. Anyone who is going to find a regression surface for himself can learn how to do so from standard compendia of statistical methods. Anyone else will find that this material does not bear closely enough on the subject of economic models to sustain his interest.

This reviewer thinks that if any of the economists the author wants to help have sufficient interest and energy to benefit from this book they would gain much more from a more thorough book or from one of the special courses in mathematics for social scientists now proliferating in our universities. On the other hand, it is good to have these economic models recognized as a textbook material. Perhaps Professor Beach or someone else will go on from here to offer us a more thorough treatment of this potentially fascinating subject.

Robert L. Davis

Experimental Designs, Second Edition, By W. G. Cochran and G. M. Cox. New York, John Wiley, 1957, xiv + 616 pp., \$10.75

The purpose of this second edition of a well-known work is "to present the plans of all the useful types of experimental designs, showing the kind of work for which each design is appropriate, and giving illustrations of its practical use."

New sections have been added throughout, and two new chapters have been added in order to make the material complete and up-to-date. The new edition also takes account of the fact that "research workers in the medical and social sciences, in physics and chemistry, and in industrial research have become increasingly attentive to the principles of experimental design."

More than ever, this unique volume is an indispensable tool of the practitioner and student alike.

University of Illinois

Franz E. Hohn

Numerical Analysis. By Kaiser S. Kunz. McGraw-Hill Book Company, Inc., New York, 1958. ix + 381 pp., \$8.00

Since numerical analysis is autochthonous to and symbiotic with applications, it has been fortunate in having a large share of innovations and textbooks contributed by people whose primary interest has lain beyond mathematics. In *Numerical Analysis* Kaiser S. Kunz of the Schlumberger Well Surveying Corporation continues this happy trend in a book whose message is mellifluously borne on a lozenge diagram. Recognizing that most numerical analysis requires a "rather modest knowledge of mathematics" Mr. Kunz digs right into the subject without undue pedantry or rigor.

The large coverage afforded by the book can be seen from the list of chapter headings: **1.** The Real Roots of an Equation, **2.** Roots of Polynominal Equations, **3.** Finite-Difference Tables and the Theory of Interpolation, **4.** Central-Difference Interpolation Formulas, **5.** La Grange's Interpolation Formula and Inverse Interpolation, **6.** Summation of Series, **7.** Numerical Differentiation and Numerical Integration, **8.** Numerical Solution of Ordinary Differential Equations: Methods of Starting the Solution, **9.** Solutions of Ordinary Differential Equations: Methods for Continuing the Solution, **10.** Simultaneous Equations and Determinants, **11.** Interpolation in Tables of Two or More Variables, **12.** Expression of a Partial Differential Equation as a Partial Difference Equation, **13.** Solution of Partial Differential Equations of the Elliptic Type, **14.** Solution of Partial Differential Equations of the Parabolic and Hyperbolic Types, **15.** Integral Equations, Appendix A. Estimation of Error in Numerical Computation, Appendix B. References, Index.

Among the notable features of the book is the careful documentation of theory with judiciously selected and conscientiously worked out examples. For instance, in introducing interpolation possible pitfalls are forcefully demonstrated by a seventh degree polynomial which has zero second differences for the table **0(1)6**. The entire discussion of interpolation and numerical differentiation and integration is illuminated by a clear, full, and in many cases, novel use of lozenge diagrams with adequate proofs for the derivation of both formulas and remainder terms. Chapter **11** presents a good attempt, which as far as it goes, is successful in introducing the elements of multivariable interpolation. Chapter **6** contains an interesting description of a few classical aspects of the summation problem including increasing the rate of convergence of a series. For a survey book of this type Chapter **8** presents an unusually full description of starting the solution of a differential equation. Chapter **10** on linear equations is probably the weakest in the book, since the failure to mention the eigenvalue problem inhibits a proper exploration of this consequential subject. This difficulty again presents itself in the almost one hundred pages devoted to partial differential and integral equations. The list of problems at the end of each chapter is adequate and the book is well indexed.

In any survey book it is easy for a reviewer to compile a long list of complaints: e.g. archaism, "in an electronic computer. . . found inexpedient to build a dividing unit"; deviationism, "pseudo exponent" for 'factorial exponent'; complexity, six lines plus an appeal to the theory of indefinite

summation to show that $\frac{1-x^{i+1}}{1-x} = 1+x+\dots+x^i$; ambiguousness, it

is never stated that if the original series converges then the Euler transformed series converges, and further (here is where the student may go astray) that in some cases the transformed series converges while the original diverges; omission, subtabulation in particular, the influence of Comrie in general; emphasis, is smoothing really "peripheral" when in the modern industrial set-up the "job of getting an answer" from noisy data often falls on the numerical analyst despite or because of statisticians?; etc. However, it is just such complaints which are *Numerical Analysis'* source of charm since they make the book individualistic and original.

Almost no one will be surprised that the author has failed in his claim of including "in one book all the background material needed for obtaining numerical solutions to advanced problems in applied mathematics." Nonetheless, this carefully thought out and well written text will probably succeed in the more modest goal of becoming a deservedly popular introduction to numerical analysis, since it does give the student sufficient background for performing the more routine computations that arise in practice and a firm basis for going on to advanced study.

University of Illinois

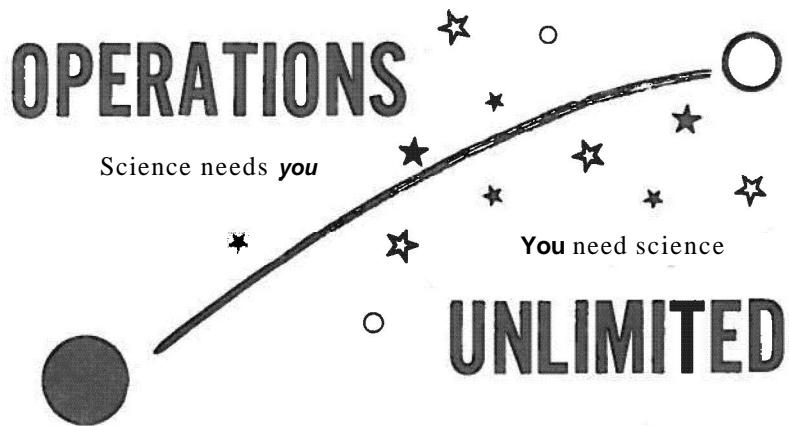
R. B. Kelman

- T. W. Anderson: Introduction to Multivariate Statistical Analysis, New York, John Wiley, **1958, \$12.50.**
- R. Dorfman, P. A. Samuelson, and R. M. Solow: Linear Programming and Economic Analysis, New York, McGraw-Hill, **1958, \$10.00.**
- S. Goldberg: Introduction to Difference Equations, With Illustrative Examples from Economics, Psychology, and Sociology, New York, John Wiley, **1958, \$6.75.**
- W. Hurewicz: Lectures on Ordinary Differential Equations, New York, John Wiley, **1958, \$5.00.**
- E. T. McSwain and R. J. Cooke: Understanding and Teaching Arithmetic in the Elementary School, New York, Holt, **1958, \$5.50.**
- P. M. Morse: Queues, Inventories, and Maintenance, Publications in Operations Research No. **1**, New York, John Wiley, **1958, \$6.50.**
- OR Group, Case Institute: A Comprehensive Bibliography on Operations Research, Publications in Operations Research No. **4**, New York, John Wiley, **1958, \$6.50.** (See review, this issue.)
- L. G. Peck and R. N. Hazelwood: Finite Queuing Tables, Publications in Operations Research No. **2**, New York, John Wiley, **1958, \$8.50.**
- E. J. Purcell: Analytic Geometry, New York, Appleton-Century-Crofts, **1958, \$4.50.** (See review, this issue.)
- J. Riordan: An Introduction to Combinational Analysis, New York, John Wiley, **1958, \$8.50.**
- J. B. Rosenbach, E. A. Whitman, B. E. Meserve, and P. M. Whitman, College Algebra, Fourth Edition, Boston, Ginn and Co., **1958, \$5.25.** (See review, this issue.)
- S. N. Roy: Some Aspects of Multivariate Analysis, New York, John Wiley, **1958, \$8.00.**

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Mathematics possesses not only truth but supreme beauty a beauty cold and austere, like that of sculpture without appeal to any part of our weaker nature, sublimely pure, and capable of a stern perfection such as only the greatest art can show.

Bertrand Russell: *The Study of Mathematics*



This section of the Journal is devoted to encouraging advanced study in mathematics and the sciences. Never has the need for advanced study been as essential as today.

Your election as members of Pi Mu Epsilon Fraternity is an indication of scientific potential. Can you pursue advanced study in your field of specialization?

To point out the need of advanced study, the self-satisfaction of scientific achievement, the rewards for advanced preparation, the assistance available for qualified students, etc., it is planned to publish editorials, prepared by our country's leading scientific institutions, to show their interest in advanced study and in you.

We are pleased to announce that Bell Telephone Laboratories will have an editorial encouraging advanced study in the spring (1959) issue. Through these and future editorials it is planned to show the need of America's scientific industries for more highly trained personnel and their interest in scholars with advanced training.

The following lists contributing corporations with the issue in which their editorials appeared or will appear.

Bell Telephone Laboratories	Vol. 2, No. 10
Bendix Aviation Corporation	Vol. 2, No. 8
Emerson Electric Company	Vol. 2, No. 7
General American Life Insurance Company	Vol. 2, No. 9
Hughes Aircraft Corporation	Vol. 2, No. 9
International Business Machines Corporation	Vol. 2, No. 8
McDonnell Aircraft Corporation	Vol. 2, No. 7
Monsanto Chemical Company	Vol. 2, No. 7
North American Aviation, Inc.	Vol. 2, No. 9
Olin Mathieson Corporation	Vol. 2, No. 7

We are grateful to these scientific institutions for their editorials and their material support of this publication.

The future holds a scientific career for those who have the fortitude to seek it. "Carry the ball" for preparation, don't "bawl" for lack of it.

NORTH AMERICAN AVIATION, INC.

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AERONAUTICS, ASTRONAUTICS, AND MATHEMATICS

by
William R. Laidlaw
Chief, Dynamic Science Section
Columbus, Ohio



William R. Laidlaw

Beyond doubt one of the most outstanding accomplishments of the past half century has been the attainment of manned flight. Continuous and intensive development of the aeronautical sciences has brought us from a humble beginning at Kittyhawk to the point where we now contemplate flight to the farthest reaches of outer space. Scientific advancement alone, however, could not have produced the many outstanding aeronautical accomplishments of the past fifty years. The application and incorporation of advanced scientific principles in the design of reliable working hardware has been the job of industry. It is obvious, therefore, that aeronautical advancement is founded on a strong union between the research scientist and the engineer. As the number and complexity of our technical problems is increased by virtue of our ever expanding objectives, it becomes increasingly difficult to separate our thinking in terms of the classical categories of scientific research, engineering application, and production. The production of a modern air vehicle is characterized by comprehensive research and design studies which are virtually indistinguishable.

Aeronautical engineering has always been rigorous. The reasons are obvious: the margin for error is small, and the consequences of error are great. The attainment of rigor is impossible without a firm foundation in mathematics and the physical sciences and the ability to apply these principles to the production of practical and economic hardware. As the scope of our endeavors increases, so also must our rigor. Accordingly, we must emphasize more than ever our reliance on the fundamental scientific principles and the key to their use — mathematics.

In the past, mathematics has made innumerable contributions of great value to the aeronautical sciences. Our understanding of the basic principles of fluid mechanics, hydrodynamics, stress phenomena in complex structures, the mechanics of bodies in free flight, and many others too numerous to mention would have been impossible without the pioneering work of many famous mathematicians. Effective applications of these principles to specific problem areas requires both mathematicians who are well versed in the physical nature of the problem and engineers with a solid mathematical foundation.

The continuing influence of mathematics in the aeronautical field is evident from a brief examination of several typical modern problems. In order to evaluate the stability and control characteristics of a typical aircraft or missile system, it is necessary to solve a set of non-linear coupled differential equations of high order. The derivation of these equations follows from the principles of Euler and involves consideration of detailed structural and aerodynamic reactions. Solutions to the equations are normally possible only through the medium of high-speed analog and digital computing equipment. The use of such machines involves a full knowledge of their operational techniques and the ability to code programs and operate them.

Design studies of control, guidance and navigation systems involve the application of network theory, operational calculus, and operational techniques including the **Laplace** and Fourier transform methods.

The analysis of highly redundant structures frequently involves the solution of a multitude of simultaneous differential or algebraic equations. The construction of appropriate mathematical representations involves the application of strain energy techniques to deflection and stress analyses of non-uniform, heat-conducting bodies with multiple load paths and high-order indeterminacy.

The study of aeroelastic phenomena such as flutter, dynamic response, and static aeroelastic behavior requires the solution of complex algebraic equations with as many as 50 to 100 unknowns.

Statistical techniques are finding increased application in the areas of rough-air flight, buffeting, acoustic response, and environmental prediction. These methods are also being applied to error analysis and operational analysis. In the latter case, the analysis is devoted to the determination of the operational suitability of new types of military weapons.

In order to account adequately for aerodynamic heating effects at high speed, it is necessary to conduct mathematical investigations of the behavior of heat-conducting turbulent boundary layers and to determine the nature of the phenomena associated with heat-flow into a heat absorbing and conducting structure.

In addition to the above problem areas, there exist many other areas of research and development activity which rely upon a strong foundation in mathematics. It is apparent from the above brief listing that practically every area involved in the development of a modern air vehicle requires the ability to describe a physical situation in mathematical language and to conduct intelligent and exhaustive solutions of the resulting equations in an attempt to understand and control the physical phenomena involved.

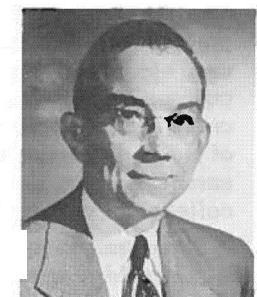
As we proceed into the future, we may look forward to increasing technical challenges. Space flight or astronautics will pose a multitude of questions which can only be answered through the medium of mathematical analysis. Typical of these are the optimization studies necessary for the determination of optimum rocket and missile configurations, trajectories and orbits, and navigational problems of all types.

Here at North American we are justifiably proud of our activities in many fields of science and engineering. These include: the development and production of modern high-speed aircraft, missiles, advanced rocket power plants, atomic energy, automation, electronics, space flight, and a multitude of other fields. We realize full well that our attainments in these many diverse activities would have been impossible without the highly skilled team of mathematicians, scientists and engineers who make up our many technical departments. Mathematics is the foundation in every activity in which they engage, and it is apparent that the success of our company and every other like it engaged in furthering the aeronautical sciences will rely, for its future success, on the common denominator to all science - mathematics.

GENERAL AMERICAN LIFE INSURANCE CO.

MATHEMATICS AND THE ACTUARIAL PROFESSION

by
Otto J. Burian, Vice President and Actuary
Fellow of the Society of Actuaries



Otto J. Burian

"The work of science is to substitute facts for appearances and demonstrations for impressions". This quotation by **Ruskin** has been adopted by the Society of Actuaries as its basic theme and is stated in each of its official publications. Such a statement is particularly applicable to actuarial science as it deals with the behavior of, or occurrences to, human beings -- areas where mathematical laws seem quite remote.

The mathematician is indispensable to actuarial science but, unfortunately, many mathematics students are unaware of this important field for their talents. For this reason it seems well to discuss here just what an actuary is, where he works, what he does, and finally, his education and training.

An actuary is basically a mathematician, but may use his skill in a manner ranging from that of strictly research to that of a businessman. He is a "social mathematician", as is the economist and the statistician, because he deals with human problems. The problems he deals with can roughly be divided into two broad areas, each handled by a different type of actuary. The "life" actuary handles the need for insurance against medical expenses and death, and the need for retirement benefits. The "casualty" actuary also works with insurance

against medical expenses but in addition he deals with entirely different lines of insurance, such as for fire and property damage, automobile and property liability, workmen's compensation, marine, etc.

When one considers that all types of insurance and retirement plans (including Social Security) are based on the mathematical techniques of the actuary, the importance of his work in our society can be recognized and the reason why his actual work can be so varied is easily understood. The actuary may be employed by a life insurance company, a casualty insurance company, various types of industrial firms, the federal government in one of several agencies, a state government, by a consulting actuarial firm, or may be self-employed (doing consulting work).

Because he is basically a mathematician, the actuary must be both interested in, and have an aptitude for, mathematics. Furthermore, in most cases he should be more interested in practical mathematics than theory or pure mathematics. His work fundamentally deals with probabilities of certain occurrences (sickness, death, property damage, etc.) and their financial impact on society. The college math courses he probably would take beyond differential and integral calculus would be theory of probability, mathematical statistics, calculus of finite differences, graduation, compound interest, and special insurance mathematics. These latter courses are given in relatively few colleges or universities, and some on only a graduate level.

Since many actuaries enter the business world, their formal education should also include business and economics courses such as accounting, business law, money and banking, and investments. It is the college student who prefers to supplement his math courses with courses in the social sciences rather than with courses in the physical sciences who is the type of individual who would find an actuarial career an interesting one.

Many times the actuarial student finds it desirable or necessary to get the mathematics courses he needs by means of graduate work. The additional courses he needs are probably not more "advanced" but are merely more specialized, such as a course in "Life Contingencies" dealing with probabilities of people living and dying and the financial effects thereof.

At this point it would be well to point out that the actuary's training is in three stages, with some overlapping of each. His formal academic schooling has been discussed and is the first stage. Passing of national professional examinations is the second stage. The third stage is the common one of learning from practical experience.

The passing of national professional examinations is quite a rigorous training program in itself. Both the Society of Actuaries (for life actuaries) and the Casualty Actuarial Society (for casualty actuaries) give annual examinations. There are eight examinations for either type of actuary, the first ones being very similar for the two Societies and the later ones differing because of the different nature of the work.

The early examinations for the life actuary are each three hours in length and cover the college mathematics discussed above. The later examinations are each six hours in length, and cover such areas as mortality rate and disability rate tables and investigations, underwriting the insurance risk, calculation of policy values, life insurance law, agency problems, policy liabilities and insurance accounting, investment of life insurance funds, group insurance, pension plans, accident and sickness insurance, and social insurance. The casualty actuarial exams are all three hours in length and cover comparable areas. Both Societies publish information on the content of the examinations. In addition, they furnish study materials and a list of reading references to be used in preparing for the exams.

It often requires eight to twelve years for an actuary to pass all of these examinations. Many times he starts taking them while still in college, if he has decided to enter the profession by then. Since many of the areas of study are closely connected with the actuary's actual work, his working experience aids in the preparation for the examinations, and vice versa. On completion of all eight examinations he is designated a "Fellow" in the Society. Because of such intensive training in so many areas of the insurance business, actuaries are often named to high administrative positions in the industry.

It will also be of interest to many to know that the insurance industry is one of the foremost industries to utilize electronic computers in their operations. Both because of their many complex actuarial calculations and because of their vast volumes of paper work, insurance companies have quickly taken to electronic computers to help with their daily work. The actuary, because of his ability and training, is closely connected to such operations, both in their planning and operating stages.

Through lack of knowledge about the actuarial profession on the part of many students and because of the constant competition for good mathematics students from other professions (teaching, engineering, statistics, etc.), there is a real shortage of trained actuaries. For example, there are currently only 1,743 members of the Society of Actuaries to handle actuarial problems for over 800 life insurance companies, thousands of large industries and federal and state governmental agencies. There is a comparable shortage in the casualty field.

Because of the shortage of actuaries, because of the high standards set, because of the intensive training received, and because of the demand for actuaries in administrative positions, a career as an actuary offers *Opportunities Unlimited* to the student with mathematical ability.

HUGHES AIRCRAFT COMPANY

THE USES OF MATHEMATICS IN INDUSTRY

Leo A. Aroian, Head, Mathematics Section



Leo A. Aroian

I have frequently been asked by high school students and teachers just how and what mathematics is used in industry. This question can only be clarified by asking two other questions. What is mathematics? And what industry do you have in mind? By mathematics I wish to be very broad and to include in the meaning of that term all its methods and all of its applications. As for industry I can speak from first hand observation of two at least: life insurance, since I worked for a short period of time at the Metropolitan Life Insurance Company, and in the electronic industry for the last eight years at Hughes Aircraft Company.

In the company I am Head of the Mathematics Section of the Systems Analysis Laboratory, which is one of the Laboratories in the Systems Development Laboratories. The Mathematics Section, with about forty people, has essentially two duties - consultation in mathematics, pure and applied in all its aspects, and the solution of problems by the use of IBM machines. The numerical work is handled on an IBM 704, a tremendously powerful machine, and it is expected that this will be replaced next year by an IBM 709. I wish to make it clear that we service the whole company in the two areas above. If it is a mathematical problem we will try to solve it. If it is a bookkeeping problem keeping track, for instance, of all the different sizes of wires for a radar set, we defer to the Data Processing groups in the company.

I have taught in high school for a year, half a year in Upper Michigan, and half a year at Cass Technical High School. I have taught college at Colorado State University (it was Aggies then) and at Hunter College of the City of New York.

The most important ingredient in mathematics is reasoning - and the same is true in industry:

1. What is the problem, and how can we simplify it?
2. What are the essential relationships or the equations?
3. How can we solve the equations?

Of these three points, the really fundamental ones are 1 and 2. In high school algebra and geometry we have the exact counterparts of the problems in industry, the word problems in algebra, the constructions in geometry, and the proofs of original theorems. Word problems are exceedingly important since the emphasis is on reasoning, something most of us do not enjoy doing. But once we start to think, we are really fascinated to see how wonderfully simple a seemingly complicated problem can be, and, how the many pages of words may be reduced ultimately to a set of equations which contains all of the ideas in their simplest forms. Word problems challenge the reading ability of a student as well as his ability to do just what the book tells him to do. There is nothing silly about A, B, C, and D doing a job together such as A does a job in four hours, B in five hours, C in six hours, and D in twelve hours. How much time will it take for them to do the job together? Certainly the problems involving time, rate, and distance to an organization concerned with range, velocity, and acceleration are just as cogent now as forty years ago.

Let us look at a typical systems problem, that of a missile and interceptor attempting to kill a target. The mathematics of this problem involves:

- (a) the geometry of space, the transformation of co-ordinates,
- (b) trigonometry in all of its aspects, and
- (c) the solution of 9 second order linear differential equations using boundary value conditions.

Frequently spherical and cylindrical co-ordinates are used.

How about the use of trigonometry? The cosine law takes first place here. The other laws for the solution of triangles are used infrequently. Ordinarily we do the calculations in radians and change to degrees later. But the most important parts of trigonometry are the identities and the analytic side involving the formulas for the sum of two angles, the double angle, the half angle formulas, and so forth. It is evident that for navigation of any kind trigonometry is a necessity.

As far as advanced collegiate mathematics is concerned we use every type of higher mathematics except for the abstract parts of modern algebra, topology, and metamathematics. Symbolic logic and number theory are quite helpful in the design of computing machines. Analysis, particularly advanced calculus, complex variables, and differential equations are essentials. If I should list the problems by type, we would have a table:

Data reduction:

5%

Evaluation of functions including complex arithmetic:

20%

Analysis:

30%

Evaluation of integrals, infinite series, Fourier series, both mathematical and numerical.
 Solutions of differential equations and systems of differential equations, usually numerically.
 Integral equations numerically.
Laplace transform and its uses.
 Partial differential equations of parabolic, hyperbolic, and elliptic types.
 Continued fractions.

Statistics and Monte Carlo techniques:

20%

Industrial statistics, quality control, and so forth.
 Autocorrelation and power spectrum.

Polynomial equations, systems of linear equations, matrix theory, and eigenvalues of matrices:

25%

For the high school teacher this suggests that complex numbers should be as natural as the real numbers.

The analysis tends strongly to numerical methods **i.e.**, most definite integrals cannot be evaluated by elementary functions. Even after we add to the elementary functions, the functions defined by second order linear differential equations such as the **Bessel** functions, the **Legendre** functions, the hypergeometric functions, still many of the definite integrals cannot be readily expressed in terms of these functions. Quite frequently the modern machine will complete the problem while the mathematician is manipulating these functions. However, if the machine runs into difficulties in special cases, this indicates strongly that a little mathematical analysis will help. Before **any** problem is **done**, be **sure** that you **have** correctly transferred the physical situation to the mathematical system of equations. This is called the simulation of a physical system.

Now it would be ideal if the physicist in a complex system could state all his equations at one time, preferably at the beginning of the problem without making any changes thereafter. This would be wonderful and would save lots of time and money. More often the engineer makes an approximation, gets some numerical answers, and then if the answers look wrong, he next tries to correct the equations. Where strong physical intuition exists, this method appears to be satisfactory. Remember if experimentation were cheaper, there would be little need for mathematical solutions to problems.

Let's look at another of the typical problems. Consider a complicated system Φ depending on ten parameters. What values of these parameters will maximize Φ ? Generally in such problems calculus is of little value and what is usually done is to investigate a few cases and attempt a sequential ascent to the maximum. The attitude generally is to get the absolutely last bit of usefulness from all equipment. Consequently, we must optimize; if we do not, some other company will optimize and thus obtain our business.

The theory of probability and its applied counterpart - the theory of statistics, has become increasingly important in science, in quality control, in industrial experimentation, and in psychology. For example the area under a curve may be obtained merely by finding the average of a set of ordinates. This is not practical for single integration, but very important if you have a four fold or five fold integration. The field of statistics, linear programming, dynamic programming, and inventory control are fascinating and new areas of applied mathematics.

If I were to start my life over again as a mathematician I would do two things I have not done as thoroughly as I would have liked to do. That is, take more courses in pure mathematics and, secondly, spend a great deal of time in the physical sciences. I know of very few mathematicians who can actually do this. With a field so vast this is presently impossible.

In all of this arithmetic to be forgotten? Do we still need to know how to add, subtract, multiply, and divide? Unfortunately, yes. With the advent of the high speed machine, practically all the operations of mathematical analysis have been reduced to these four operations, with the addition of one concept - the limit concept. The high speed machine also can make comparisons, and forms an ideal vehicle for mathematical logic and the theory of games. Thus, we complete a circle from the simple to the profound and back again to the simple.

In the years to come, say twenty-five years from now, as life grows ever more complicated with increasing automation, the uses and the need for mathematics will increase. The ability to think clearly and abstractly will exist in the future even though today we cannot predict accurately the problems of tomorrow.

PROFESSIONAL OPPORTUNITIES IN MATHEMATICS

Third Edition, January 1958

This third edition of a booklet of which more than 20,000 have previously been sold was prepared by H. M. Gehman with the assistance of A. S. Householder and others. It is available at 25 cents per copy, or 20 cents each in orders for five or more, from The Mathematical Association of America, University of Buffalo, Buffalo 14, New York.

This is an extremely well written pamphlet whose purpose is two-fold; "to suggest the types of positions available to young people with mathematical aptitude and training; and to urge such young people to broaden their horizons, and secure as sound and as deep a general education as the opportunities available to them will permit."

The pamphlet tells about career opportunities, necessary and desirable training, and possible salaries in five major areas: (I) teaching in high school, college, or university; (II) mathematical and applied statistics; (III) industrial laboratories; (IV) government agencies; (V) insurance companies and actuarial work. It lists companies and agencies employing mathematicians and it gives a bibliography for further reading.

This is a booklet which every adviser of mathematics majors should study with care and have at hand for loan to students. A meeting open to all students and devoted to reports on the five sections of this pamphlet might well be made an annual project by each chapter of Pi Mu Epsilon. Such a meeting, properly advertised, would indeed be a service to the cause of mathematics.

University of Illinois

Franz E. Hohn

SPEAKER FOR MEETINGS

Here is an excerpt of a letter from IBM whose editorial encouraging advanced study appeared in "Operations Unlimited" in the spring (1958) issue. It should be of interest to you.

"It seems to me that there are many ways in which we can encourage advance study on the part of math students and that one of the most effective means is through this type of article. Certainly we should keep this series going. Another way in which this can be accomplished is to have speakers from industry talk to groups of students. I know we would be very happy to try to arrange for representatives of IBM to talk at local chapter meetings of the Pi Mu Epsilon Fraternity on this general subject. If any chapter is interested in doing this, I would be glad to send you the name of the appropriate person to contact in each college location.

International Business Machines Corporation
Data Processing Division
White Plains, New York
M. A. Shrader, Manager
University & Research Institute Program"

Edited By
Mary L. Cummings, University of Missouri

On April 8, 1958, the Oregon State College chapter of Pi Mu Epsilon, jointly with the Oregon State College Department of Mathematics, sponsored two competitive examinations, for freshmen and for sophomores, respectively. For freshmen, first prize award of \$30 went to Chin Shing Hoo, while honorable mention book awards were given to Dale Leroy Harmer and Stephen Kerron Prothero. For sophomores, the \$30 first prize award went to Tom Edward Lok, with honorable mention book awards going to Norman Ray Franzen and George Eyre Andrews.

Kenneth Kalantar, New Jersey Alpha, '55, is now the secretary-treasurer of the Pi Mu Epsilon chapter at Rutgers University.

Army Second Lieutenant William R. Kampfe, University of Nebraska, '57, was recently graduated from the field artillery officer basic course at The Artillery and Missile School, Fort Sill, Oklahoma. Lt. Kampfe was formerly a mechanical engineer with Chance Vought Aircraft in Dallas.

At the annual meeting of Missouri Gamma chapter (St. Louis University), Miss Mary Lou Krug and Brother Martin Flynn, C. R., received James W. Garneau Awards of \$25 each for being outstanding senior students in mathematics.

On February 28, the birthday of Robert O. Moe, Missouri Gamma, '54, whose death occurred February 1 while he was on duty with the Air Force at Wheelas AFB, North Africa, his widow, Mrs. Elaine Moe, established The Robert O. Moe Memorial Scholarship Fund. Bob was graduated with honors from Parks Air College of St. Louis University in 1954.

Missouri Alpha, at the University of Missouri, was honored by having two members receive their degrees with distinction in mathematics at the June commencement. They are Mrs. Anne Kercheval Duncan and Donald Barnett. Donald also was awarded a Woodrow Wilson scholarship for graduate study, and has elected to pursue his work in mathematics at the University of Chicago.

Second Lieutenant Elmer W. Galbiatti, Rutgers University, '56, and the University of Illinois, '57, recently completed the ten-week officer basic course at the Army Ordnance School, Aberdeen Proving Ground, Maryland. Before entering the army, he was a student at Georgetown University Law Center, Washington, D. C.

NEW CHAPTER APPROVED

The United Chapters of Pi Mu Epsilon have voted to grant a charter for a chapter at Montana State College, Bozeman, Montana. The date of the formal installation has not been set:

News items should be sent to Mary L. Cummings, Mathematics Department, University of Missouri, Columbia, Missouri.

DEPARTMENT DEVOTED TO CHAPTER ACTIVITIES

Edited by
Houston T. Karnes, Louisiana State University

EDITORS NOTE: According to Article VI, Section 3 of the Constitution: "The Secretary shall keep account of all meetings and transactions of the chapter and, before the close of the academic year, shall send to the Secretary General and to the Director General, an annual report of the chapter activities including programs, results of elections, etc." The Secretary General now suggests that an additional copy of the annual report of each chapter be sent to the editor of this department of the Pi Mu Epsilon Journal. Besides the information listed above, we are especially interested in learning what the chapters are doing by way of competitive examinations, medals, prizes and scholarships, news and notices concerning members, active and alumni. Please send reports to Chapter Activities Editor Houston T. **Karnes**, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana. These reports will be published in the chronological order in which they are received.

REPORTS OF THE CHAPTERS

GAMMA OF PENNSYLVANIA, Lehigh University

The Pennsylvania Gamma Chapter held six meetings during the **1957-58** year. This included the annual banquet on May **7, 1958**. The following papers were presented at the five program meetings:

- "Continued Fractions" by Dr. Clarence A. Shook
- "Digital Computers" by Gerhard Payne
- "Satellite Orbits" by Ralph Van **Arnam**
- "Electronic Computers" by Guy Benson of IBM
- "The Concept of Parity" by Dr. James **McLennan**

The meeting featuring Mr. Benson was held jointly with the Lehigh Chapter of the American Institute of Physics.

Officers for **1958-59** are: Director, Willard Kauffman; Secretary, **Andris Suna**; Treasurer, Hugh Schwandt.

ALPHA OF NORTH CAROLINA, Duke University

The North Carolina Alpha Chapter held two meetings during the **1957-58** year. At the first meeting nine students were initiated, and twenty-eight students were initiated at the second meeting. The following papers were presented:

- "Job Opportunities" by Mr. Eugene Smith of the Appointments Office
- "Problems Associated with a **650** IBM Computer" by Mr. Jerome Clutter of the Computing Staff

Officers for **1958-59** are: Director, Carol Annette Cleave; Vice-Director, Edward Dennis **Theriot**; Secretary, Sarah Thomas; Treasurer, Frances Elaine Fowler.

GAMMA OF MISSOURI, St. Louis University

The Missouri Gamma Chapter held five meetings during the **1957-58** year. This included the Twenty-first Annual Banquet. The following papers were presented:

- "Envelopes of Certain Families of Conics" by Miss Katherine Lipps
- "The Use of Gravity Meters in Oil Prospecting" by Mr. William B. Herr
- "**Sylow** Groups" by Mr. J. James **Malone**
- "Mathematical Models of Physical Systems" by Professor Franz E. Hohn, University of Illinois

CHAPTER ACTIVITIES

Miss Katherine Lipps was the chapter delegate to the national convention of Pi Mu Epsilon held in August, **1957**.

At the final meeting of the year ninety-two members were initiated. For the Twenty-first Annual Banquet Mr. Edwin Eigel, Jr. was toastmaster. Professor Hohn was a guest at the banquet.

Dr. Waldo Vezeau presented the following awards:

The Mathematical Award of the Chemical Rubber Company to Miss Marguerite Van Flanders. The Pi Mu Epsilon Junior Contest to Mr. John L. Martin. This was an award of **\$10.00**. The annual James W. Garneau award to Miss Mary Lou Krug and Mr. Martin C. Flynn, C. R.. This is an award of **\$25.00** for being the highest ranking seniors majoring in mathematics. This was the first time there were dual recipients of this award.

Officers for **1958-59** are: Director, James J. **Malone**; Secretary, Sister Gregory Marie **Meyer**, O.S.F. Faculty Adviser and Permanent **Secretary-Treasurer**, Dr. Francis **Regan**.

BETA OF PENNSYLVANIA, Bucknell University

The Pennsylvania Beta Chapter held two special meetings during the year. The first one was an initiation meeting wherein twenty-four new members were initiated. At a later date, two additional members were initiated who were not able to be present for the first ceremony. Following initiation ceremony Mr. Stanley F. Dice gave the address. The second meeting was the annual Pi Mu Epsilon Picnic.

In addition the Pennsylvania Beta Chapter sponsored five meetings of the Bucknell Mathematics Club. At these program meetings the following papers were presented:

- "The Development of Logarithms" by **Georgie Ann McKay**
- "Direct Line to Decision", an IBM Public Relations movie
- "Empirical Formulas" by Henry **Farrell**
- "Linear Programming", Professor William A. Beck
- "Can Concepts from Higher Mathematics Be Taught in Elementary Schools?" by Robert C. **Appleman**

The chapter was also invited to have a part in two other lectures during the year. These lectures were by Dr. Richard **Schafer**, Mathematical Association of America lecturer, and Dr. Harlow **Shapley**, Phi Beta Kappa Visiting Scholar.

Officers for **1957-58** were: Director, Professor William A Beck; Vice-Director, James C. Biedleman; Secretary, Barbara Stech; Treasurer, William Sponaugle; Corresponding Secretary, Professor William I. Miller.

Officers for **1958-59** are: Director, Mr. Boyd Earl; Vice-Director, Gary K. **Munkelt**; Secretary, Mary Emma Fetter; Treasurer, Dorothea L. Bell.

GAMMA OF KANSAS, University of Wichita

The Kansas Gamma Chapter sponsored two picnics for all faculty and students interested in mathematics in the **1957-58** year. The Chapter also furnished travel expense to take a car load of students to the state meeting of the Mathematical Association of America.

Five meetings were held during the year, one of which was the annual initiation dinner. Eighteen new members were initiated on this occasion. It is interesting to note that the Chapter for the first time initiated the daughter of a previous member (Miss Carolyn Hildyard, daughter of Vida Grace Hildyard). Among the papers presented were the following:

- "The Hyperbolic Paraboloid as a Design for Roofs of Buildings".
- "A General **Legendre** Transformation".

A considerable amount of money was added to the principal of the Pi Mu Epsilon Scholarship Fund which is granted each year to a mathematics major. This year the holder of the scholarship was Mr. Calvin Schwartzkopf. The award now amounts to approximately **\$75.00** each year.

Officers for **1957-58** were: Director, Ralph Bargen; Vice-Director, Calvin Schwartzkopf; Secretary, David Henderson; Treasurer, Agnes Nibarger; Corresponding Secretary and Faculty Representative, Professor C. B. Read.

BETA OF KANSAS, Kansas State College

The Kansas Beta Chapter held seven meetings during the 1957-58 year. This included the initiation and banquet, on which occasion eleven new members were initiated. The following papers were presented during the year:

"Concerning Spirals in the Plane" by Dr. B. J. Pearson

"The Laplace Transformation as an Applied Mathematical Tool" by Dr. G. J. van der Maas, Department of Physics

"Analysis of Sandwich Materials" by Dr. W. R. Kimel, Department of Engineering

"Integral Equations in Representation Theory of Functions of a Complex Variable" by Dr. Evelyn Kinney

"Orbit Calculations" by Professor P. G. Kirmser, Department of Applied Mechanics

"The Impact of Computers on Mathematics" by Mr. Herbert Lechner, a representative of IBM.

DELTA OF NEW YORK, New York University

The New York Delta Chapter held four meetings during the 1957-58 year including an initiation meeting. Eleven new members were initiated. The following papers were presented:

"Mathematical Sociology" by Professor Joseph B. Keller of the Institute of Mathematical Sciences

"Goedel's Theorem" by Professor John van Heijenoort.

"Maximum Problems Without Calculus" by Professor Hollis R. Cooley

Officers for 1957-58 were: Director, Stanley Kleinman; Vice-Director, Paul Voynow; Secretary-Treasurer, Veronica Gainer; Faculty Advisor, Professor John van Heijenoort.

ALPHA OF NEVADA, University of Nevada

The Nevada Alpha Chapter held seven business meetings and three program meetings during the 1957-58 year. The following papers were presented:

"Linear Inequalities" by Ed Wagner

"Diophantine Equations" by Dr. E. M. Beesley

"The Six Color Theorem and Fibonacci Numbers" by Dr. H. S. M. Coxeter of the University of Toronto.

During the year seventeen new members were initiated.

The Nevada Alpha Chapter also co-sponsored a state wide mathematics contest which was given to junior and senior high school students by the University of Nevada.

Officers for 1958-59 are: Director, Carl Fisher; Vice-Director, Hans Lindblom; Secretary-Treasurer, Ed Wishart.

ALPHA OF ARIZONA, University of Arizona.

The Arizona Alpha Chapter held five meetings during the year. This included the initiation banquet which was held on March 28, 1958. The banquet speaker was Dr. H. S. M. Coxeter of the University of Toronto.

The Arizona Alpha chapter has a continuing contest throughout the year. Every two weeks a new problem is placed on the bulletin board. The person who submits the first correct solution is awarded \$1.00.

GAMMA OF ILLINOIS, De Paul University

The Illinois Gamma Chapter held seven meetings during the 1957-58 year. The following papers were presented:

"Introduction to Infinity" by Mr. E. Merkes

"What is an Equation?" by Dr. J. De Cicco

"Elementary Probability Theory" by Dr. W. Caton

"Introduction to Elementary Topology" by Dr. W. Darsow

"Introduction to Cardinal Numbers" by Dr. G. Weiss

"Introduction to the Theory of Mathematical Induction" by L Aquila

CHAPTER ACTIVITIES

Officers for 1957-58 were: President, Charles Mitchell; Vice-president, Anthony Behof; Secretary and Treasurer, Louis Aquila.

Officers for 1958-59 are: Director, Professor Wilis Caton; Permanent Secretary, Professor Guido Weiss; President, Louis Aquila; Vice-president, Gerard Lietz; Secretary, Jon Laboda; Treasurer, John Synowiec.

BETA OF WASHINGTON, University of Washington

The Washington Beta Chapter held eight meetings during the year which included the annual initiation. A total of twenty-four new members were initiated. The following papers were presented during the year.

"What is a Transcendental Number?" by Mr. Karl R. Stromberg

"Facts and Fallacies of Farthing Flipping" by Professor R.K. Getoor

"Convex Sets" by Professor V. L. Klee

"The Theory of Games" by Professor J. R. Isbell

"Mathematics of Automata" by Professor R. S. Pierce

"The Golden Section and Fibonacci Numbers" by Professor

H. S. M. Coseter, University of Toronto

As another activity, the chapter also sponsored a problem Seminar for undergraduate students and a spring quarter picnic for the entire mathematics department.

Officers for 1957-58 were: Director, Robert R. Phelps; Vice-Director, Colin W. Clark; Acting Secretary, Gloria C. Hewitt; and Treasurer, James D. Reid.

Officers for 1958-59 are: Director, James W. Armstrong; Vice-Director, Kenneth A. Ross; Secretary, Gloria C. Hewitt, and Treasurer, Theodore Mueller.

ALPHA OF LOUISIANA, Louisiana State University

The Louisiana Alpha Chapter held five meetings during the 1957-58 year. One of these meetings was a group tour of the Delta Tank Corporation. At the initiation meeting thirty-three new members were inducted. Following the initiation ceremony, Professor Richard V. Andree, secretary-treasurer general from the University of Oklahoma spoke on the subject, "Perfect Numbers".

The following papers were presented during the year:

"The Five Color Problem" by Dr. Irwin Stuart Kruse

"Future of Mathematics and the Need of Mathematical Training" by Dr. Henry G. Jacob

"Bridge Problem" by Mr. Sam Lott

The annual Pi Mu Epsilon Senior Award was won by Melvin E. L. Oakes, and the Freshman Award was won by John C. Wiese.

Officers for 1958-59 are: Director, Paul M. Brown; Vice-Director, Ronald Folsom; Secretary, Sandra Passantino; Treasurer, Patrick J. Haddican; Faculty Adviser, Dr. Haskell Cohen; Corresponding Secretary, Dr. Houston T. Kames.

GAMMA OF OHIO, University of Toledo

The Ohio Gamma Chapter held three meetings during the 1957-58 year. Two of these were business meetings followed by short talks. The third was a business meeting followed by a social period.

Officers for 1958-59 are: Director, Professor C. W. Thomson; Vice-Director, Richard Marlowe; Treasurer, Michael Kelley; Secretary, Duane Biglin.

GAMMA OF CALIFORNIA, Sacramento State College

The California Gamma Chapter held eight meetings during the 1957-58 year. This included the initiation banquet, four business meetings and three program meetings. The following papers were presented at the program meetings:

"Numerical Short-Cuts" by Dr. Ching-Hwa Meng

"Some Curiosities from the Theory of Numbers" by Dr. Henry L. Alder, University of California, at Davis.

"Moore-Smith Limits" by Dr. Charles Hayes, Jr., University of California, at Davis.

The chapter engaged in an activity which did much to further interest in mathematics. On several occasions high school students interested in mathematics from the El Camino High School attended meetings wherein short talks were given by members of Pi Mu Epsilon. Those who participated in these meetings were Bill Patterson, Brandon Wheeler, Larry Bryans, Ross Brown, and Marcella May.

Officers for 1958-59 are: Director, Lawrence Bryans; Vice-Director, Ross Brown; Secretary, Marcella May; and Treasurer, William Patterson.

DELTA OF OHIO Miami University

The Ohio Delta Chapter held nine meetings during the 1957-58 year. This included business meetings, program meetings and the annual initiation banquet. The following papers were presented during the year.

"Computers, Their Applications and Vocational Aspects" by Mr. Jack Ramsen, from I. B. M.

"Space Filling Curves" by Professor Richard DeMar

"Topological Tricks and Puzzles"

"Mathematical Modeling" by Professor Philip A. Macklin, Department of Physics

"Interesting Mathematical Problems" by Professor Eric E. Erickson

The speaker for the annual banquet was Dr. Paul R. Rider, chief statistician of Wright Air Base. His subject was "Probability and Statistics".

The officers for 1957-58 were: Director, Joseph F. Tinney; Vice-Director, Charles F. Dugan; Secretary, Sue Lashley; Treasurer, C. Richard Cothorn.

BETA OF NORTH CAROLINA, University of North Carolina

The North Carolina Beta Chapter held eight meetings during the 1957-58 year. One of these meetings was an initiation ceremony wherein eleven new members were inducted. At one of the springmeetings an additional member was initiated in absentia. This initiate was Miss Inez Belleza of the University of the Phillipines. The final meeting was a banquet which was attended by the faculty of the Department of Mathematics as guests. The following papers were presented during the year.

"Equivalence Classes of Ordered Pairs" by Professor D. W. Wall.

"Simple Games" by Professor B. J. Pettis

"Structure and Structures" by Professor E. A. Cameron

"Installment Buying" by Professor A. T. Brauer

"Taylor's Series for Some Elementary Functions" by Professor J. S. MacNerney

"The Cryptogram of Euler" by Professor Hans Rohrbach.

"Fun with Lattice Points" by Mr. H. W. Gould.

BETA OF FLORIDA, Florida State University

The Florida Beta Chapter held six program meetings during the year

1957-58. The following papers were presented:

"Some Elementary Set Theory" by Bobby Sanders

"Some Recent Developments in Undergraduate Mathematics" by Professor Howard Taylor.

"Hit or Miss" Dr. Alan S. Galbraith of Elgin Air Force Base

"Game Theory" by Professor James W. Ellis

"Mathematical Paradoxes" by Joe Neggers

"Numerical Analysis and the Digital Computer" by Professor H. C. Griffith

The chapter also cooperated with the Mathematics Department in sponsoring a talk by Professor Ky Fan of Notre Dame and the Oak Ridge National Laboratory; his visit was made possible by the Oak Ridge Institute of Nuclear Studies.

In addition to the program meetings, several business meetings and a picnic were held. A total of twenty-six new members were elected and initiated. The speaker for the annual initiation banquet was Dr. Werner A. Baum, Director of University Research, whose talk was entitled "Remarks on Graduate Study and Research in Mathematics".

Officers for 1957-58 were: Faculty Adviser, Professor Charles W. McArthur; Director, Bobby Sanders; Vice-Director, Al LeDuc; Secretary-Treasurer, Richard Chandler; Corresponding Secretary, Professor James W. Ellis.

Officers for 1958-59 are: Director, Al LeDuc; Vice-Director, Joe Neggers; Secretary-Treasurer, Barbara Toney; Faculty Adviser, Professor Charles W. McArthur; Corresponding Secretary, Professor James E. Snover.

ALPHA OF MISSOURI, University of Missouri

The Missouri Alpha Chapter held four program meetings during the 1957-58 year. The following papers were presented:

"Some Curious Results from Space Geometry" by Professor L. M. Blumenthal

"The Role of Fundamental Research in Modern Society" by Professor John Schopp

"The Closure of the Zuider Zee" by Christiaan Jan Penning

"The Idea of a University" by Professor John Cutts

In addition there were two initiation meetings. Twenty-three new members were initiated in the fall and twenty-two new members were initiated in the spring. The final meeting of the year was the annual banquet. Ninety people were in attendance. At this meeting announcement was made of the winners of the Pi Mu Epsilon prizes in Calculus. Winners were: First, William Brinkman, Jr.; Second, Robert Wenski; Third, a tie, Norman Asbridge and James Delmore.

Officers for 1958-59 are: Director, David Lee; Vice-Director, Daniel Hutchinson; Secretary, Lauralon Schaper; Treasurer, James Monsees.

ALPHA OF IOWA, Iowa State College

The Iowa Alpha Chapter held three business meetings, one program meeting, and the annual initiation banquet during the 1957-58 year. For the program meeting Dr. W. W. Sawyer of the University of Illinois addressed the chapter on "Intuition in Mathematics". Dr. D. E. Sanderson gave the banquet address.

Officers for 1958-59 are: Director, William Frank Ward; Vice-Director, Gary Zuck; Secretary, Patricia Walter; Treasurer, Kenneth Kopecky; Corresponding Secretary, Dr. D. E. Sanderson; Faculty Adviser, Dr. Marshall Ruchte.

ALPHA OF NEW JERSEY, Rutgers University

The New Jersey Alpha Chapter held three program meetings during the 1957-58 year. The following papers were presented:

Address by Mr. George Cherlin of the Mutual Benefit Life Insurance Company
"Representatives of Real Numbers" by Professor Harold Grant

"Mathematical Structures" by Professor Abe Shenitzer

In addition to the above meetings there was the annual initiation banquet. The banquet speaker was Robert E. Luce who spoke on the topic, "Cultural Aspects of Mathematics".

Three awards were made on the annual State Mathematics Day. These awards were books. The first prize was dedicated to Professor Robert E. Luce whose passing at the end of the first semester was a great loss to the chapter.

GAMMA OF NEW YORK, Brooklyn College

The New York Gamma Chapter held thirteen meetings during the **1957-58** year. This included two initiation meetings. The following papers were presented at the program meetings of the year:

- "**Theory of Games**" by Professor Moses Richardson
- "**Some Aspects of Probability**" by Professor James Singer
- "**Some Properties of Vectors**" by Professor Walter Prenowitz
- "**Calculus of Variation**" by Mr. Joseph Shpiz
- "**Integral Domains**" by Professor Samuel Borofsky

There **were two** initiation meetings during the year. Professor Harriet M. Griffin and Professor Samuel Borofsky gave addresses following these two initiation ceremonies.

The officers for **1957-58** were: Director, Paul Goldstein; Vice-Director, Barbara King; Secretary, Ellen Lorber; Treasurer, Diane Cantor; Faculty Adviser, Harriet Griffin.

The officers for **1958-59** are: Harold Engelsohn, Director; Vice-Director, Myra Cohen; Secretary, Rochelle Fuhr; Treasurer, Alfred Brandstein.

BETA OF OREGON, Oregon State College

The Oregon Beta Chapter held three program meetings during the **1957-58** year. The following papers were presented:

- "**Accumulation of Round-Off Error In Machine Calculation**" by George C. Town

- "**Fibonacci Sequences**" by Professor Robert Stalley

- "**An Isoparametric Problem**" by Professor Arvid T. Lonseth

Two social events were held during the year. In the fall there was a picnic and in the spring the annual initiation banquet. The banquet speaker was Professor John C. Decius who spoke on "**Some Mathematical Aspects of Music**". On this occasion two annual awards were presented. These awards are based upon examinations given to freshmen and sophomores. Chin Shing Hoo won first place on the freshman examination and Tom Edward Lok won first place on the sophomore examination. These awards were \$30.00 each. Those receiving honorable mention on the freshman examination were Dale Leroy Harmer and Stephen Kerron Prothero. Those receiving honorable mention on the sophomore examination were Norman Ray Franzen and George Eyre Andrews.

Nineteen new books were added to the chapter library.

Officers for **1958-59** are: Director, Don Amos; Vice-Director, Sheldon Rio; Secretary, John Vinson.

ALPHA OF CALIFORNIA, University of California at Los Angeles

The California Alpha Chapter held five program meetings during the **1957-58** year. The following papers were presented:

- "**A Survey of Finite Geometries**" by Raymond Killgrove
- "**On Guard**" a film shown by Mr. Ed Smith of I.B.M. which deals **with the SAGE system**

- "**Mathematics and Management**" by Professor James R. Jackson, Department of Business **Administration**

- "**The New Spitz Planetarium**" by Professor George Abell of the Department of Astronomy

- "**Laplace Transform**" by Professor Raymond Redheffer

There were two initiation ceremonies during the year. In addition, there **were two** social events. At the first one, Professor A. E. Taylor gave an illustrated lecture on mountain climbing in Switzerland. The second social event was a picnic.

Winners of the annual Calculus Prize examination were as follows: First Place: Chung Wa Wong and Jan Gizesik. The second prize winner was Keith Kendig. The awards were mathematics books and membership in the Mathematical Association of America.

CHAPTER ACTIVITIES

Officers for **1957-58** were: Director, Afton H. Cayford; Vice-Director, Earline Madsen; Secretary, Ernest M. Scheuer; Treasurer, Professor P. C. Curtis; Faculty Adviser, Professor R. Steinberg.

Officers for **1958-59** are: Director, Andrew Bruckner; Vice-Director, Herbert Gindler; Secretary, Raymond Killgrove; Treasurer, Professor T. Ferguson; Faculty Adviser, Professor R. Steinberg.

ALPHA OF FLORIDA, University of Miami

The Florida Alpha Chapter held five program meetings and a banquet during the **1957-58** year. The following papers were presented:

- "**Fixed Points**" by Dr. Charles E. Capele

- "**The Engineer's Use of the Digital Computer**" by Dr. Wayman Strother

- "**Topological Transformation Groups**" by Dr. Deane Montgomery of the Institute for Advanced Study at Princeton

- "**Spectral Theory for Ordinary Differential Operations**" by Dr. Marshall H. Stone of the University of Chicago

- "**Russian Education in Mathematics and the Sciences**" by Dr. Herman Meyer

The annual initiation banquet was held in May. Dr. J. Riis Owre, Dean of the Graduate School, spoke on "**The Proposed Ph.D. program in the University**.

DELTA OF PENNSYLVANIA, Pennsylvania State University

The Pennsylvania Delta Chapter held five regular meetings during the **1957-58** year, in addition to a fall banquet and a spring banquet. The following papers were presented:

- "**Some Modern Algebraic Systems**" by Robert Blefko

- "**The Perpetual Calendar**" by Francis Felix

- "**The Fundamental Theorem of Algebra**" by Paul F. Henning

- "**I = 0**" by Barry F. Kramer

Guest speaker for the annual banquet was Dean Ben Euwema of the College of Liberal Arts. The title of his speech was "**The Place of Mathematics in a Liberal Education**".

At a special February meeting Honor Books were presented to outstanding freshmen who had just completed Analytic Geometry.

Officers for **1958-59** are: Director, James Leitzell; Vice-Director, Mary Oliver; Secretary, Barry Kramer; Treasurer, Frank Kocher.

EVENDALE AFFILIATE CHAPTER of Ohio

The Evendale Affiliate Chapter held two regular meetings since the chapter was installed on November **29, 1957**. At the first meeting four new members were initiated. The following papers were presented:

- "**Some Geometric Concepts**" by John Ridout

- "**Theory of Function Approximations**" by Albert Marvin

At the second regular meeting one new member was initiated. The guest speaker on this occasion was Dr. E. F. Trombley of the Jet Engine Department of General Electric Company. The subject of his address was "**Theory of Distribution**".

Officers for **1958-59** are: Director, George J. Marks; Vice-Director, Martha S. Creal; Secretary-Treasurer, Robert Fortier.

ALPHA OF INDIANA, Purdue University

The Indiana Alpha Chapter held six meetings during the **1957-58** year. This included an initiation meeting and installation of new officers. The following papers were presented:

- "**One Dimensional World**" by Dean W. L. Ayres

- "**Mathematical Logic**" by Professor Jack E. Forbes

- "**Hexaflexagons**" by Jim Yost

- "**Turing Theory**" by Dr. Randolph

- "**Mathematical Recreations**" by Dan Hodge and Marie Hudson

- "**What Are the Odds?**" by Dr. Irving Budd

Officers for **1958-59** are: Director, Junius West Frazier; Vice-Director, Dan Hodge; Secretary, Hugh Geary; Treasurer, Jim Yost.

BETA OF WISCONSIN, University of Wisconsin

The Wisconsin Beta Chapter held eleven meetings during the year **1957-58**, including picnics, a Christmas dinner, and the initiation banquet. Professor R. E. Langer, guest speaker at the initiation banquet spoke on "The Biography of an Ancient Mathematical School". Thirty-seven new members were initiated during the year. The following papers were presented:

"What is a Line?" by Dr. Louis F. McAuley

"New Patterns in Geometry" by Professor Richard H. Bruck

"Cylindrical Algebras" by Mr. Joop Doorman

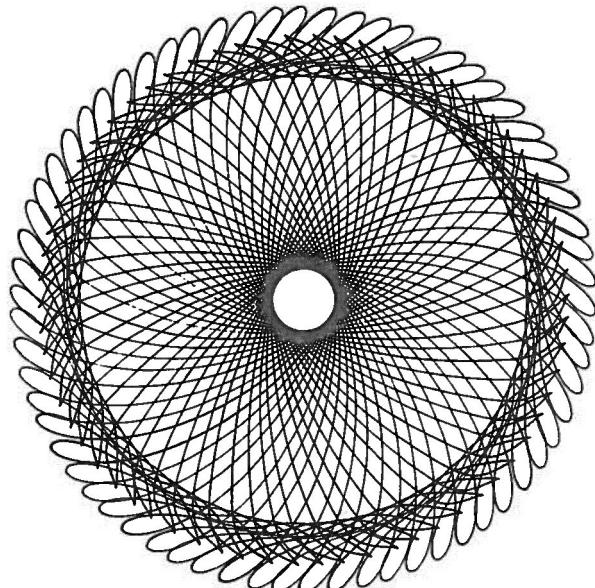
"Regular Polyhedra and Their Symmetry Groups" by Dr. J. Marshall Osborn

"Integration in Finite Terms" by Dr. E. C. Posner

"Disclosure on Foreclosures" by Professor Preston C. Hammer

Officers for **1958-59** are: Director, Jules Vieaux; Vice-Director, Mr. Joop Doorman; Secretary-Treasurer, Patricia Tucker; Faculty Adviser, Dr. J. M. Marshall Osborn.

After seeing the drawings which appeared in the Spring issue of the Journal, one of the members of Pi Mu Epsilon sent us some of his own drawings which he made in St. Louis on a home-made machine. One of these is reproduced below.

**INITIATES****ALABAMA ALPHA**, University of Alabama (May 6, 1958)

Joe H. Bailey	Sidney N. James	Paik Woo Shin
William Bentley	Josh D. Johnson	Sarah Joyce Smelley
Melvin Brown	John C. Jones	Joe H. Sox
Robert B. Cardy	Edith H. Livingston	Waights M. Taylor
James L. Coggins	Elmer M. Mitchell	Roy C. Tew, Jr.
Philip W. Coulter	Walter F. Moody	William S. Thomas
Kenneth E. Harwell	Sarah Ella Mullins	Uthman B. Uthman
Gene E. Huckabee	Ray L. Reid	James E. Wamble, Jr.

ALABAMA BETA, Alabama Polytechnic Institute (April 29, 1958)

Barry Leroy Benner	Robert Douglas Donovan	Tim D. Slogh
Lonnie Julian Clayton	Jessie Sue Green	David J. Vance
Helena Ann Cochran	Charles D. Montgomery	Evelyn A. Wheeler
Earl Lewis Cook	Paul John O'Brien	Joseph Porter Wilson
	Aubrey E. Pippin	

ARIZONA ALPHA, University of Arizona (March 28, 1958)

Paul Aizley	David C. Hensley	Dino Natta
Thomas C. Andrews	Johnny J. Marietti	Joseph Palais
Thomas E. Caldwell	R. Donnell McArthur	Jerome M. Seiler
John K. Gardner	David N. Montgomery	Jerry Stewart
Kenneth L. Grant		Jonathan Tannenbaum

ARKANSAS ALPHA, University of Arkansas (April 3, 1958)

Alfred Boxley	Jimmy Fay Gadberry	John Cleveland Sallis
Lynn Brueggeman	William Irving Heaston	John Saunders
James Harvey Colvert	Louis M. Heerwagen, Jr.	William Farrar Sherman
Fred Stovall Crum	Neil Barton Ingels, Jr.	John Robert Stovall
Larry Gene David	Theodore Phillip Kaufman	William Harrel Sullivan
Edward Chapline Denham	James Revell Kimzey	Maribelle Blew Williams
James Marvin Fawcett	James Marvin Moring	Glen David Wilson
Oscar Lee Fletcher	Jerry Cecil Perciful	Charles Edward Yates
	James Carroll Sadler	

CALIFORNIA ALPHA, U C L. A. (December 14, 1957)

J. Michael Bossert	Ira M. Green	Judith Caroline Olson
Prapat Chandaket	David Hestenes	Louis A. Raphael
Aileen L. Cohen	Richard Dean Larson	Edward A. Sallin
Floyd A. Cohen	Charles Lawson	Albert Henry Samuels
Jack Davis	Curt Marcus	Donald R. Stover
Paul DePascals	Farley Gray McKinney	George W. Soules, Jr.
Ralph P. Emig II	Dimitri M. Mihalas	Georges G. Weill
Kimiye Lois Fujinaka	Joseph F. Mount	Robert A. Walsh

(April 19, 1958)

Julien L. Borden	Louis Jaekel	Daihachiro Sato
George Clinton Bramblett	George W. Kimble	Wilford Neil Shelton
Wilma F. Casner	Dalia Kochba	Jeanne M. Templeton
Max L. Eaton	George S. McCarty, Jr.	Richard John Werner
Phyllis Gaylord	Gregory Merenbach	Ralph H. Wessner
Charles William Hastings	Jimmy Y. Miyamoto	Jack E. Winter
	Robert Phelan	

CALIFORNIA BETA, University of California, Berkeley, Calif. (May 25, 1957)

Maurice L. Bandy	Leah Horwitz	G. Aaron Paxon
Berton Barker	Robert Jaffa	Paul Rosenthal
Richard Clow	Norbert Kauffman	Luiza Rothstein
Tom Creese	Laura Ketchum	Mrs. Ceres Schroer
William Cunnea	Philip Meads	Henry Stone
C. T. Draper	David Monheit	Alvin Swimmer
Alfred Hexter	Farouk Odeh	Charuwan Thirawat
Barbara Hochman		Douglas Underwood

CALIFORNIA GAMMA, Sacramento State College (May 29, 1958)

Linda May Gilbert	Martin Levy	Barbara Ann Reina
Bobby Ray Green	John M. Lewis	Harold Howard Roach
Paul Albert Kiser	George Davison Ralph	Bryce J. R. Vernon

FLORIDA ALPHA, University of Miami (May, 1958)

Douglas Richard Anderson	Alfred Last	Paul Siegel
Donald Bellows	Miles B. Lawrence	Jack Warren Smith
Lloyd Dale Davis	Neil Franklin Michelsen	Clarence B. Stortz
John Frederick Firkens	Boyd Byron Oellerich	Margareta Elaine Tripp
Daniel Clarke Hoagland	Mitchell Philip Rubin	Clifton George Wrestler, Jr.

FLORIDA BETA, Florida State University (May 1, 1958)

Rita Ann Bailey	Sidney D. Calkins	Grace Marjorie Lennon
Avery Dean Baker	John Cobh	Walter Clint Pine
Daniel A. Ball	Robert S. Fouch	Gary G. Smith
James Edward Bayfield	Robert Kalin	Barbara M. Toney
William Martin Boyce	Linda Ann Langston	Herbert S. Willcox

GEORGIA ALPHA, University of Georgia (April 23, 1958)

Gregory L. Bernard	James C. Fortson	Richard O. Kuehne
Thomas C. Collins	Joseph F. Goddard	Edward D. Schmidt
Robert F. Cote	Odell Hamilton	Kenneth R. Williams

ILLINOIS ALPHA, University of Illinois (April 25, 1958)

Rao Channapragada	Jon Laible	Wallace Sanders
Patricia Clafin	Karen Lemme	Everett Schleter
Jo Ann Colvin	Archie Lytle	Herbert Stein
Earl H. Dowell	Ralph W. Magin	Robert Thomas
K. Norman Easley	David B. Marblestone	Nick Thomopoulos
Ross Flenner	Daniel A. Moran	Guy Torchinelli
John Fixmer	Clement Morell	M. E. Van Valkenburg
Caesar Ghilarducci	Anna L. McConnel	Charles Weir
Sarah Jane Hofas	Kenneth McCord	Nancy Whitman
Peter Kozodoy	Richard Plumer	Martin S. Wolfe
Donald Kubose	Allan Stanley Rehm	Tai Te Wu
Thomas Lahey		Charles Zartman

ILLINOIS BETA, Northwestern University (June, 1958)

Burt O. Anderson	Elliott Edward Johnson	Douglas Lee Phyne
LeRoy William Cooper	Patricia Ruth Ludeman	Ronald B. Schwab
James Arthur Cunningham	Martin H. Lundquist	James Michael Webb
William W. Hough	Robert Charles Petrof	Bernard Thomas Westapher

ILLINOIS DELTA, Southern Illinois University (April 25, 1958)

Richard R. Allen	Henry L. Gill	Carl Woodrow Mitchell
Barbara Lee Craig	Frank O. Keller	Samuel Moses
Michael Oliver Flatt	Churl Suk Kim	Joseph F. Urekar
	John Henry Klinger	

INDIANA ALPHA, Purdue University (May 14, 1958)

Natalie Carol Bolte	Junius West Frazier	Vincent Paul Pongracz
Florence Lee Firnhaber	Marie Katherine Hudson	Ray J. Williams
	Larry Dean Miller	

IOWA ALPHA, Iowa State College (May 28, 1958)

Henry L. Ablin	Omer D. Erdmann	William J. Meek
Ronald M. Anderson	Howard J. Ewoldt	John N. Mordeson
Mary Kay Berry	James T. Gifford, Jr.	Margaret R. Mundt
Nancy Betz	Robert B. Green	Josephine Proett
Frank H. Binder	Peter M. Hall	John C. Raich
Mary L. Birkholz	Shelby K. Hildebrand	Wesley J. Runyan
Raymond A. Brown	Albert D. Hudek	William Howard Sutherland
Ray L. Buschbom	Richard Warren Hunt	Floyd Lynn Tiffany
Ronald Christensen	S. K. Katti	William M. Wagner
Harry Coonce	Kenneth J. Kopecky	Patricia Ann Walter
Robert F. Cuffel	Constance S. Leslie	Clayton W. Watson
Vernon P. Dorweiler	John Louis McNeley	Gary L. Zuck

INITIATES**KANSAS ALPHA**, University of Kansas (April 25, 1958)

William Eldon Benso	Thomas P. Kezlan	DeWayne Stanley Nymann
Russel G. Bilyeu	Frederick William Koker	Dwight Patton, Jr.
Patricia V. Connack	Mary Laird	Peter C. Patton
Harold B. Hanes, Jr.	David Lane	James C. Pool
Joseph F. Hanna	William M. Lindstrom	Arlan Ramsey
Richard Hinderliter	Charles A. Marsh	Jane Secrest
Buddy Ava Johns	Richard C. McClain	Rhoda Melinda Taylor

KANSAS BETA, Kansas State College (May 8, 1958)

David B. Cline	Darrell R. Pamell	Robert R. Snell
Larry L. Corbet	David Saffry	Benedict Ching-San Sun
Helen Moore	Lorraine Schwartz	Gerard J. van der Mass
Ralph G. Nevins		Isaac Wakabayashi

KANSAS GAMMA, University of Wichita (April 23, 1958)

Gail Eugene Bachman	James R. Crepps	Patsy R. Renfro
Richard Harlan Balay	Lynn Leslie Hershey	Charles R. Schraeder
William Ronald Brigstocke	Vernon O. Hoegne	Ben Arthur Stewart
Michael J. Cambren	Marcella J. Kerrick	Jean Dennis Whitcher

KENTUCKY ALPHA, University of Kentucky (May 2, 1957)

Joyce Eleanor Beals	Gwynneth Mariel Gibson	Edgar Tilden Moore, Jr.
Ralph Everett Beals	Donald Leroy Hartford	Clay Campbell Ross, Jr.
Dwight W. Carpenter	Jerry Porter King	William Tazwell Steed
Betty Charles Detwiler	Dorothy Irene Koehler	Martha Frances Watson
Margaret Louise Ehlen	George Ronald Lester	George Kenneth Williams

(May 6, 1958)

Harvey Kent Brock	Charles Howard Sampson	Sidney Christine Smith
Gunter Brunhart	Christoph Scriba	Joe Warren Vaughn
Hugh Rodman Coomes, Jr.		Han Chang Yang

LOUISIANA ALPHA, Louisiana State University (March 31, 1958)

Marie Louise Babin	Maurice C. Hebert	Alien T. Phillips, Jr.
Jack L. Bahm	Ronald Charles Hyer	Frank Atkinson Rickey, Jr.
Donald S. Bergeron	Donald A. Kniffen	Kenneth O. Rogers
Paul M. Brown	Richard Mark Landry	Geoffrey R. Say
Robert Bruce Butler, III	John Darryl Lieux	Robert L. Say
Richard LeRoy Diener	John T. Moore	Henry M. Troth, Jr.
Bobbie J. Duplantis	Edward J. Nunenmacher	Milan J. Turk
Ronald Courreges Folse	Robert Wayne Ortego	Sandra W. Webb (Mrs.)
Dewitt F. Gordon, Jr.	Sandra Ann Passantino	Neil W. Webre
Jack Paul Guillory	Mary Edith Perkins	Elizabeth H. Wells
Patrick Joseph Haddican	John Walter Perry, Jr.	Marvin H. Williams

MARYLAND ALPHA, University of Maryland (May 26, 1958)

Doris Roberta Aaronson	Thomas Harold Dyer	Young Ho Rhie
Jagjit Singh Bakshi	Dennis Patrick Hanley	Jin-chen Su
Robert Wayne Candi	Patricia Lehman	David Faulds Templeton
Wm Michael Dante	Justin G. MacCarthy	Jackson Yeager

MICHIGAN ALPHA, Michigan State University (February 5, 1958)

Carl William Bell	Lawrence Daniel	Dale Lick
Duane Donald Beyer	Clarence Q. Ford	Harry McClung
Ralph Robert Brown	Roy V. Jackson	Gary D. Thomas
James L. Butler	Richard L. Klinkner	Gerald William Trabbie
Robert George Carol		Thomas P. Wangler

(May 29, 1958)

Charles V. Anderson	Luther W. Rearick	David hi. Tenniswood
Howard T. Cervantes	Richard L. Schmal	Bobbie J. Trantham
Thomas J. McIlrath		Gordon L. Hughes

MISSOURI ALPHA, University of Missouri (May 1958)

Edward Andalafte
Norman **Ashbridge**
Allen E. **Asseelmeier**
Wm Brinkman, Jr.
Anne K. Duncan (Mrs.)
John G. **Innes**
Roy F. **Keller**

Blanca P. Kravcio
Charles W. Miller
Michael Thayer Morgan
Adrian **Nikitins**
Roger Roberts
Richard **Roling**
Robert L. Rurter
Lauralou Schaper

Benjamin **Sims**
Marvin Summers
James **Sundu**
Thos. Swaney
Elizabeth **Swarenengen**
Chas. Thien
Harland **Tompkins**

MISSOURI GAMMA, St. Louis University (April 24, 1958)

Edward P. **Andert**
Donald A. **Baerveldt**
Donald Bean
Paul S. Bielicke
Glyn A. **J. Bindon**
Glen A. **Borman, Jr.**
David A. Brown
Sr. **Marv** Denise Brown
Robert A. **Brungs, S. J.**
Sr. Paul Buser
Herbert Tai Chan **Ching**
Sr. Francerra Coughlin
James G. **Dailey**
Richard T. **Daly, Jr.**
Richard R. David
David M. **Detchmendy**
Sr. Carol Diederich
Rev. B. **Donnelly**, OSB
Charles E. Doolittle
Thomas R. **Etling**
Richard **Farotto**
H. Monica Gallagher
Rev. V. Gillespie, OSB
Miriam Edna Golden
Kenneth E. Grant
John Hayey
Francis J. **Hilbing, Jr.**
Jane E. Hohmann
John Hopkins
Anne C. **Hunfagel**
James J. John

Lawrence S. Joyce
Robert E. **Kaufman**
Donald L. **Hinkle**
Bernadette Kelly
Jerome W. **Kinnison**
Christian N. **Kohlberg**
Gale Kristof
Mary Stuart Lanahan
Martin J. **Lanfranco**
Joseph D. **Laposa**
Patrick Lee
Ronald W. Long
Richard Lumma
Kevin **Lysaght, S. J.**
Carol Ann **McCormick**
Edward J. **McLaughlin**
R. J. **Meier**
Jerry A. **Meyer**
Kenneth R. **Mollering**
William J. Momeno
Ma. Teresa **Mossegeld**
Grattan P. Murphy
Maurice Murray, S. J.
Thomas A. **Musson**
William Nienhaus
Jon F. **Nilan**
Robert H. Neusel
Mother H.A. **Padberg, RSCJ**
Russell A. **Pautler**
George J. Peters

Ronald B. Peterson
Michael **Pohlen**
Lozelle L. **Pratt**
Sr. Margaret Raphael
Jacqueline Rathert
Sr. Ann Cathleen Real
William **Reinwart**
James A. **Renick**
Barbara A. **Resnik**
David Rhoades
Mary Kay Rice
Ronal Rosso
John H. **Rottjakob**
Robert Rownd
Roland **Schoenhoff**
Dennis V. Scholz
Francis Schulre
Arthur Scheer
Gerald Sheldon
Mrs. Shirley A. Smith
William L. Smithisler
Mary A. Swenson
Charles A. **Tapella**
Carmen **Viglino**
Thomas R. **Volkmann**
Virginia Marie **Wegener**
John **Whitmore**
Rev. Leonhard Wiedmayr
Raymond Worseck
Edwin Wright, OSB
Robert A. **Zang**

(July 10, 1958)

Leon **Bankoff****MONTANA ALPHA**, University of Montana (October 15, 1957)

William R. **Ballard**
E. E. Underwood
John J. **McGuire**

Elmer D. Ramer
Howard Reinhardt
Randolph H. Jeppesen
Larry C. **Newell**

James R. Forker
James C. Leary
Jack Silver

NEBRASKA ALPHA, University of Nebraska (April 20, 1958)

James Willard **Adelson**
Rodney James Clifton
Larry **Eugene DeBries**
James Jacobs

Leendert Kersren
Eugene Frederick Loeb
William Arthur Mehrens
James Robert Quick
Marvin Jesse Richardson

Ead Gary **Schmeding**
Don L. Sorensen
Fred David **Swaim**
Leo Alden Tyrrell

NEVADA ALPHA, University of Nevada (April 1, 1958)

David R. Allen
Thomas Buschke
John Cardinalle
Clayton Carlson
Elizabeth Constantino
Evelyn P. Falk

Carl E. Fisher
Harry W. Garner
Hans L. Lindblom
Arlen D. **McGee**
Michelle E. **Moran**

Robert Rose
John W. Traberr
Edwin F. Wagner
Samuel E. Wauchope
Leroy **Wentz**
Edward R. **Wishart**

INITIATES

NEW HAMPSHIRE ALPHA, University of New Hampshire (November 14, 1956)

Gordon Phillip Darling

(January 7, 1958)

Maurice **Dubley**
John **Dubley**
Gerard Nelson **Dionne**

George **Rodney**
John **Legacy**
Armand Lucien Pelletier

Robert Edward Pike
Nancy Wentworth Porter
John **Loring** Ramsey

(May 19, 1958)
William Ernest Bryant
Jean **Macomber**

Wilfred Paul Michaud

Roland Frank Shackford
Lewis Franklin Travis

NEW JERSEY ALPHA, University of New Jersey (December 4, 1958)

Richard C. Allen

Henry E. **Apgar, Jr.**Marvin **Barsky**David **Basson**

John David Bauman

Grafton W. **Bingham**

Courtney P. Brown, Jr.

Leo J. **Carling III**Frederick M. **Carlson**

Charles R. Cooper

Donald R. Edmonds

Kenneth L. **Gittings**David **Goltzman**George S. **Greene**James H. **Guerin**Daniel W. **Haines**Frank **Lawton Hindle**Kenneth **Kalanter**

Paul F. Koch

Michell A. **Krasny**

Robert W. Kunze

Wallace T. **Lakin**

Rudolph Landesman

Alonzo W. Lawrence

Lawrence **Lipkin**

Louis S. Napoli

David **Pollison**

Harry Pomeranz

Bruce R. Powell

Robert Rohrschein

Jonel D. Sutton

Thomas M. **Valega**

Thomas A. Van Duyne

Henry W. Weymouth

NEW JERSEY BETA, Rutgers University (March 31, 1958)

Dorothea Bamberger

Mrs. Cora Barry

Margaret Blohm

Elizabeth A. **Bohannon**Sylvia **Bruce**

Carol Crowder

Barbara Decker

Jessica H. Finn

Carole Gorecke

Katherine E. Hazard

Elaine Johnson

Mrs. Marilyn Sherr Leen

Carolyn **Neidig**

Cyril Nelson

Geraldine Nelson

Gilda Parnes

Barbara Sampson

Pauline Scherer

Rose **Silkowitz**

Mrs. Doreen R. Simko

Mrs. Dorothy C. Simons

Diane South

Robert M. Walter

Ceda Watts

Bernice Weinstein

Barbara White

Enid **Willstatter**Amelia **Yetman****NEW YORK BETA**, Hunter College (March 4, 1958)Stanley **Mamangakis**Jane **Matthews**

Elaine Opatowsky

Michael Robin

Linda Rosen

Carole Sloane

Julius **Rosenthal****NEW YORK GAMMA**, Brooklyn College (April 22, 1958)

Ronald Abramoff

Morton **Friedman**

Rochelle Fuhr

Donald **Gelman**

Allan Gewirtz

Robert Greenblatt

Arthur Pessin

Theodora Shuster

Rosalie Steinroth

Myra **Wasserman****NEW YORK DELTA**, New York University (December 2, 1957)

Lucie Bader

Robert Barbieri

Lynn R. Goldthwaite

Alvin I. Greenberg

Doris Heffer

Julian Levy

Gerda **Mendel**

Shirley Roth

Eva Valencia

Milton Oliver **Vassell****NEW YORK ETA**, University of Buffalo (May 14, 1958)Alexander R. **Bednarek**

Samuel Richbart

NORTH CAROLINA ALPHA, Duke University (April 29, 1958)

Sandra Dee Addington

Jason Reid Auman

James Edward **Ballard**Jon Richard **Blyth**

William Forrest Chambers

Hannah E. Flounders

Frances Elaine Fowler

Wayne R. Fox

Bob Winn Gayler

Harmon Thomas Gnuse

Sally Holr Rhodes

Warner Curds **Scott**

Karl David Straub

Edward Dennis Theroir

Sarah Thomas

NORTH CAROLINA ALPHA (Continued)

Shelley Carter Davis	Elliott P. Hinely	Rolf Harry Towe
Charles Bryan Duke	Carroll Ellis Jones	John Heinz Venable, Jr.
Anne Trayton Finegan	Robert Stephens Longdon	Terry Parker Wallace
Fred Paul Fischer, Jr.	Jan Lee Mize	Annette Waters
	Hugh Omega Porter, Jr.	

NORTH CAROLINA BETA, University of North Carolina (March 25, 1958)

Inez G. Belleza

OHIO ALPHA, Ohio State University (May 22, 1958)

Sanford Gilbert Bloom	Walter Ball Laffer II	Alphonso Lehman Smith
Paul B. Chin	Eugene Victor Martin	Josef G. Solomon
Robert Thomas Craig	Stanley Marvin Moss	Donald Charles Stevens
James Marshall Dowdy	Sunirmal Mukherjee	Paul Virgil Titus
Marcellus Duffy, Jr.	Thomas J. Robison	George Veis
Robert Edward Evans	David Ryeburn	Frank Dudley Williams
Raymond Leo Foye	Peter Schmitt	Glenna Gertrude Williamson
Ben Tarver Hagan, Jr.	Carolyn Hall Shelly	Walden Wren
Charles William Hamilton	Gregory Robert Sheridan	Peig Feng Wu
Lyman Sanford Holden	John Edward Shuter	Eugene Carlyle Johnson
Kee Yong Kim	Robert Edwin Smart	John Glenn Maxwell
Alex Kisha		J. Richard Stewart

OHIO BETA, Ohio Wesleyan University (March 11, 1958)

Winifred Kip Burroughs	Keiko Iwashita	Theodore Wesley Smith
Patrick McComb Hemenger	Fouad Ahmed Kukjazada	Carolyn A. Strecker
	Rollin John Morrison	

EVENDALE AFFILIATE, General Electric, Cincinnati, Ohio (November 29, 1958)

Earl L. Auyer	Roy Vincent Holt	Edward L. Nicholson
Frederick B. Banan	Robert M. Kops	Marjorie L. Owen
Sanford Baranow	Jane Lee	Michael Panagopoulos
Arthur Beyer	George J. Marks	Sheb Ray
Edward H. Brooks	Lora Lee Marlette	John Ridout
Blythe Carlson	Albert H. Marvin, Jr.	Rita Anne Rudy
Nina Clark	Jeanne Taylor Martin	Edward J. Shidlowski
Martha S. Creal	Edward B. Masters	William M. Spurgeon
Sam E. Crumpston	Robert R. McCready	Garnett L. Stephens
Andrew Dobrowolski	Joseph P. Miller	Ralph Patsfall
Millard F. Dowell	Max S. Miller	Vonda J. Stratton
Patsy Anne Dyer	Pierce J. Miller	Carol Ann Trupin
Robert E. Fortier	Harold W. Moore	Doneley H. Watson
Kenneth F. Grigg	William R. Morgan	Flora Watson
Doris Harlow	Paul F. Mosher	Morris A. Zipkin
William A. Hendricks		

OHIO GAMMA, University of Toledo (March 26, 1958)

Duane O. Biglin	James Blair Farison	Robert E. Schwarzhak
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OHIO DELTA, Miami University (May 7, 1958)

Paul T. Nugent

OHIO EPSILON, Kent University (February 11, 1958)

Jacqueline Louise Chabot	George Joseph Buta	Everett Charles Short
Dale Lawrence Anderson	Sharon Patricia LeMoine	Kenneth L. Whipkey
	Russell Allen Line	

OKLAHOMA ALPHA, University of Oklahoma (Fall 1957)

Robert V. Clapp	Frank Kern	E. E. Sanmann
Benton Clark	Roger Kleen	James F. Sheppard
John C. Evans	P. K. Nelson	Nancy Tarpley
Jim Hartsuck	James Rhyne	Dennis L. Walker

INITIATES

OKLAHOMA ALPHA (Continued)

(Spring 1958)	
Roy Bernard Adams	Vernon Ernest Hardy
Herbert B. Bebb	Ronnie R. Hart
Charles A. Combs	James Lafe Hill
Jerry W. Evans	Leon Mark Magill
James E. George	Sylvester Miller Neville
	Kenneth C. Ponsor

OKLAHOMA BETA, Oklahoma State University (December 4, 1957)

Ralph Jay Andree	Charles Nuckles
Dean M. DeMoss	James A. Nunley
Jay T. Humphrey	Eddie W. Robinson
Robert R. Kinkade	Kenneth Jess Russell
Jeff B. Krumme	John Ed Sadler
Marylyn Middleton	

OREGON ALPHA, University of Oregon (Spring 1958)

David Boyd Ashby	John Carter Hylen
Duane W. Bailey	Jacqualyn Jay Johnson
Leonard M. Ball	Pete B. King
Wilbert Dale Breshears	Earl D. Kurtz
Giles H. Burgess	Gerald A. Mesecher
Daniel Ch'en	Gordon E. Mills
Robert W. Coffin	Laurence W. Mills
Maurice N. Cox	LeRoy Moen
Terry E. Davis	Ronald R. Morgali
William Ross Doherty	Anne Loreen Moursund
John Meng Sung Fang	Richard D. Neifert
Frederick L. Faw	Charles J. Mickelson
Clifford W. Fountain	Floyd E. Olson
Gretchen B. Glass	Willard A. Parker
Gerald E. Hill	Robert L. Pearson
Quentin A. Holmes	Sebastian Robert Pfau

OREGON BETA, Oregon State College (May 22, 1958)

John C. Anderson	Donald D. Norton
George Eyre Andrews	Stanley J. Huber
Duane Scott Baker	Edward Joseph Kane
Keith Allen Barnes	John Wallace Keizur
John Lawrence Baylor	James Dana Kirkmine
Edward Grant Bennett	Fred Thomas Krough
Mac Waynard Bergstrom	Ralph Robert Laing
Melvin H. Breitsprecher	Rochne Frances Lambert
Glenn Owen Briggs	Gary LeRoy Larsen
Robert Dean Brooks	Albert Lee Laxdal
Clarence Andrew Calder	Robert William Legan
Stuart Leslie Cato	Jimmy Chitto Ling
Peter Peirce Crooker	Tom Edward Karwai Lok
Leroy Mitchell Damewood	Marilyn Ann Lomnický
Ronald Allen Davis	Joseph Anthony Lukes
George Anthony Dubinski	Donald Dean Marsh
George Russell Dunbar	Richard Lee Mayback
Gerald Irene Dunn	William Bruce McAlister
Don Eugene Ehrlrich	Terry George McLarry
Oliver Giovanni Everette	Loren David Meeker
Duncan Owen Faus	Weldon Stuart Meier
Fred Dean Fisher	Chuk Q. Moo
Russell Dale Flannery	Lawrence Leslie Moon
Norman Ray Franzen	Max Joseph Morgan
Dick Howard Gibson	Kenji Nishioka
Eldon Henry Graham	Soren Frederick Norman
Ronald Bernard Guenther	Lawrence Leroy Oden
James Leonard Guthrie	Daniel Odell Oldfather
William L. Hallmark	Edward Lyle Pariseau
Dale Leroy Harmer	Stephen Hathaway Pilcher
Lewis Gregory Hogan	Richard Glen Piller
Leslie H. Holgersen	C. E. Polymeropoulos
Chin Shing Hoo	Stephen Kerron Prothero
	Steve R. Rasmussen

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PENNSYLVANIA GAMMA, Lehigh University (October 9, 1957)

George S. Egeland

Stephen V. Pepper

George S. Stewart

PENNSYLVANIA DELTA, Pennsylvania State University (May 13, 1958)

Martin G. Broadhurst
Leonard A. Casciotti
Robert De Witt Chapman
George A. Clarke
Allen Seymour Cover

Charles Erzen
Howard S. Hall
Donald E. Koontz
Kenneth Magill
Yajiro Morita

Harry Otto
Samuel G. Ross
Walter Sillars
Norman Stemple
Frank Yeatts

(June 1958)

Charles Kaufman

PENNSYLVANIA EPSILON, Carnegie Institute of Technology (May 20, 1958)

Marian O. Chleboski	Melvin J. Hinich	James Rayford Nix
Jan Michael Chaiken	Lawrence Wesley Hutton	Jeremiah David Sullivan
Norman Howard Fuchs	Forrest Traber Miere, Jr.	Ivan E. Sutherland
Richard Leon Hartman	Louis Ralph Ponds	Larry Raymond Turner

VIRGINIA ALPHA, University of Richmond (April 21, 1958)

Jerry Allen Enfield John Forrest Garren William Davis Seward

WASHINGTON BETA, University of Washington (June 4, 1958)

James W. Armstrong	John J. Coleman	John M. Sherwin
Carl R. Austin	David C. Correa	Larry Lee Sleizer
Maryse Bader	William Cummings	Douglas F. Smith
Richard E. Bateman	Albert J. Froderberg	Wallace Smith
Richard F. Bullard	Laurence G. Hoye	Hugh M. Stone
Keith J. Caswell	Beverly M. Neal	Lawrence A. Wold
Fu-Shong Chi	Eduardo I. Pina	Ronald M. Wrona
Sung Chil Choi	James M. Sherman	Paul H. Yearout

WISCONSIN BETA, University of Wisconsin (May 28, 1958)

Conrad Bjerke	Lawrence H. Faust	John Picchiottino, Jr.
Ludwig W. Bruch	John W. Keller	Thomas Rabas
Charles Ehrenpreis	Keith P. LaBudde	Craig C. Work
	Allen Menke	

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