**PHYS UN1402 Final Exam**
**Thursday, May 12$^{th}$, 2022**
**Prof. Jeremy Dodd**

Answer **all eight** questions. Point allocations for each question are shown in parentheses. You may answer the questions in any order, but **start each problem on a new page**. Be sure to **show all of your work** to receive (partial) credit, and to **give units** where appropriate. Answers alone will receive no credit.

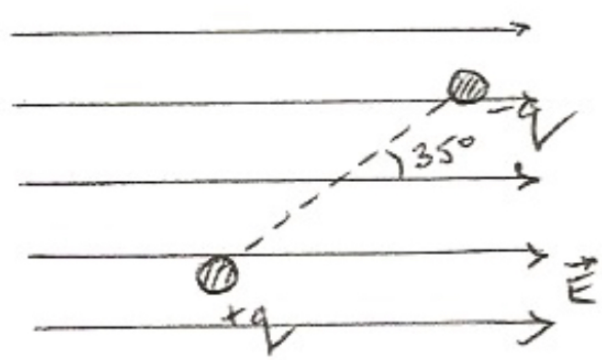
Please **write clearly and, if possible, in ink**. A formula sheet is provided separately. You may use a calculator. The test is **no-books, no-notes, and you are not permitted to access the internet or collaborate with any other student**.

You have 180 minutes to complete the exam. Good luck!

**Question 1 (15 points)**

The figure below shows an electric dipole in a uniform electric field of magnitude $5.0 \times 10^5$ N/C directed parallel to the plane of the figure. The dipole consists of two charges of $\pm 1.6 \times 10^{-19}$ C, also in the plane of the figure, separated by a distance of $1.25 \times 10^{-10}$ m.

a). What is the net force exerted on the dipole by the electric field?
b). What is the magnitude and direction (with respect to the electric field direction) of the electric dipole moment?
c). What is the magnitude and direction of the torque?
d). What is the potential energy of the system in the orientation shown below?
e). If the dipole is now allowed to rotate to its stable equilibrium position, what is the potential energy of the system in this final state?

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**Question 2 (10 points)**

A metal sphere with radius $R_1$ has a charge $Q_1$. Take the electric potential to be zero at an infinite distance from the sphere.

a). What are the electric field and electric potential at the surface of the sphere?

This sphere is now connected by a long, thin conducting wire to another sphere of radius $R_2$ that is several meters from the first sphere. Before the connection is made, this second sphere is uncharged. After electrostatic equilibrium has been reached, what are:

b). the total charge on each sphere;

c). the electric potential at the surface of each sphere;

d). the electric field at the surface of each sphere?

Assume that the amount of charge on the wire is much less than the charge on each sphere. (Express your answers in terms of $k$, $Q_1$, $R_1$ and $R_2$, as needed.)

**Question 3 (15 points)**

Two current segments are shown in the figure below. The lower segment carries a current of $i_1 = 0.40$ A and includes a semicircular arc with radius 4.0 cm, angle 180°, and center point $P$. The upper segment carries current $i_2 = 2.5i_1$ and includes a circular arc with radius 3.0 cm, angle 120°, with the same center point $P$.

a). *Derive* an expression for the magnitude of the magnetic field at the center of a circular arc of current-carrying wire subtended by an angle θ

What are the:

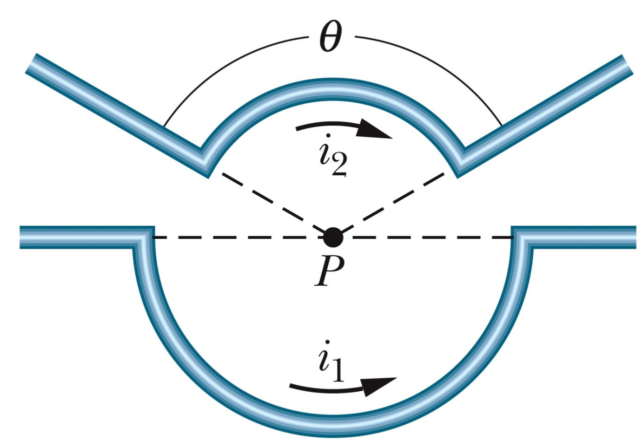
b). magnitude and

c). direction of the net magnetic field $\vec{B}$ at P for the current directions shown?

What are the:

d). magnitude and

e). direction of $\vec{B}$ if $i_1$ is reversed?

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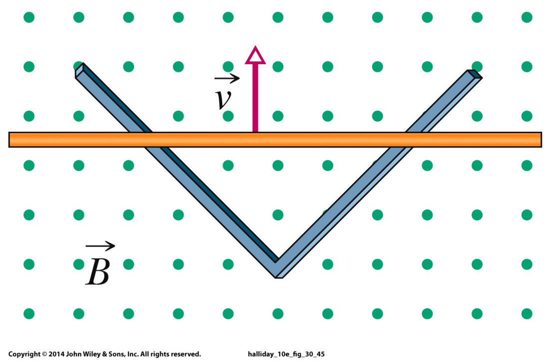
**Question 4 (10 points)**

In the figure below, two straight conducting rails form a right angle. A conducting bar in contact with the rails starts at the vertex at time $t = 0$ and moves with a constant velocity of 5.20 m/s along them as shown. A magnetic field with $\vec{B} = 0.350T$ is directed out of the page. Calculate:

a). the flux through the triangle formed by the rails and bar at $t = 3.00$ s and
b). the emf around the triangle at that time.
c). If the emf is $\varepsilon = at^n$, where $a$ and $n$ are constants, what is the value of $n$?

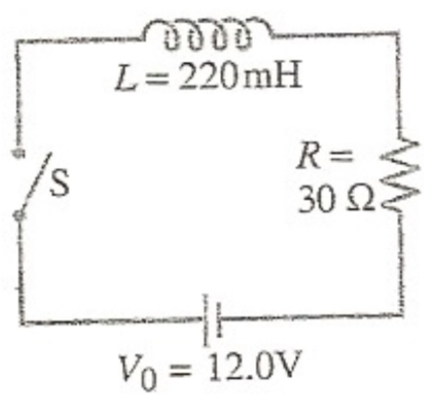
(Note that the area $A$ of a triangle is given by $A = \frac{1}{2}(base)(height)$. Think about the "height" of the triangular area enclosed by the rails and bar in terms of $v$, and how the "base" depends on the "height".)




**Question 5 (15 points)**

At time $t = 0$, a 12.0 V battery is connected in series with a 220 mH inductor and a resistor of 30 Ω resistance, as shown below.

a). What is the current at $t = 0$?
b). What is the time constant for the circuit?
c). What is the maximum current?
d). How long will it take the current to reach half its maximum value?
e). At this instant (from part d. above), at what rate is energy being delivered by the battery?
f). At what rate is energy being stored in the inductor's magnetic field?

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**Question 6 (10 points)**

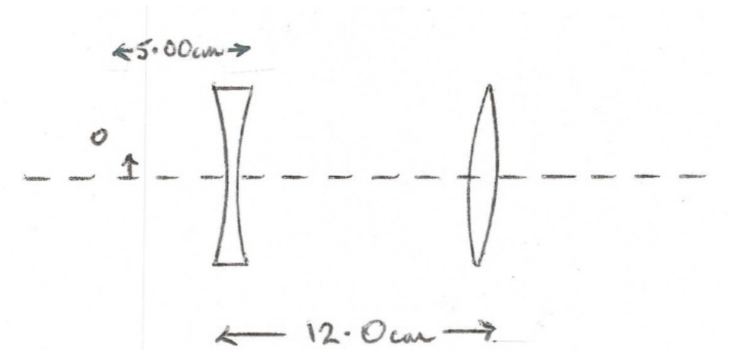
The electric component of a sinusoidal (i.e. plane) electromagnetic wave is:

$$E_y = \left(5.0\frac{V}{m}\right) sin[(1.0 \times 10^6 m^{-1})z + \omega t]$$

a). Write an expression for the magnetic field component of the wave, including a value for ω.
b). What is the wavelength of this light?
c). What is the period of the wave?
d). Parallel to which axis does the magnetic field oscillate?
e). In which region of the electromagnetic spectrum is this wave?

**Question 7 (15 points)**

Two thin lenses, each with focal length of magnitude 15.0 cm, are placed 12.0 cm apart. The first lens is diverging while the second is converging. An object 4.00 mm tall is placed 5.00 cm to the left of the first (diverging) lens.
a). Where is the image formed by the first lens located?
b). How far from the object is the final image formed?
c). Is the final image real or virtual?
d). What is the height of the final image?
e). Is the final image inverted or non-inverted?




**Question 8 (10 points)**

Two slits spaced 0.450 mm apart are placed 75.0 cm from a screen.
a). Where is the distance between the second and third dark lines of the interference pattern on the screen when the slits are illuminated by coherent light with a wavelength of 500 nm?
(This question has only one part.)

Once you have completed the exam, check your work, and write on the cover of each blue book: **"I certify that the content of this booklet is entirely my own work"** and **sign your name**. Also, make sure that you have written: your **name**, **UNI**, and **"PHYS UN1402 Final"** on the front of each blue booklet used.

PHYSUN1402 (Spring 2022) Final Formula Sheet

$$\vec{F} = k\frac{q_1 q_2}{r^2}\hat{r} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r^2}\hat{r}$$ (Coulomb's Law)

$$i = \frac{dq}{dt}$$

$$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\vec{E} = k\frac{q}{r^2}\hat{r}$$ (point)

$$E = \frac{1}{2\pi\varepsilon_0}\frac{p}{z^3}$$ (dipole, approximation for point on $z$ axis)

$$p = qd$$ (electric dipole moment)

$$E = k\frac{qz}{\left(z^2+R^2\right)^{3/2}}$$ (charged ring, radius $R$, distance $z$ along perpendicular axis through ring center)

$$E = \frac{\sigma}{2\varepsilon_0}\left(1-\frac{z}{\sqrt{z^2+R^2}}\right)$$ (charged disk, radius $R$, distance $z$ along perpendicular axis through disk center)

$$E = \frac{\sigma}{2\varepsilon_0}$$ (infinite sheet)

$$\vec{F} = q\vec{E}$$ (point)

$$\vec{\tau} = \vec{p}\times\vec{E}$$ (electric dipole in uniform E field)

$$U_E = -\vec{p}\cdot\vec{E}$$ (electric dipole in uniform E field)

$$\Phi_E = \int \vec{E}\cdot d\vec{A}$$ (electric flux)

$$\Phi_E = \oint \vec{E}\cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$ (Gauss' Law, electric)

$$E = \frac{\sigma}{\varepsilon_0}$$ (*close* to *any* conducting surface)

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}$$ (infinite line)

$$E = \frac{\sigma}{2\varepsilon_0}$$ (infinite nonconducting sheet)

$$E = k\frac{q}{r^2}$$ (outside uniform spherical shell)

$$E = 0$$ (inside uniform spherical shell)

$$\Delta U_E = -W^{elec} = -q\int \vec{E}\cdot d\vec{s}$$
$$U_E = -W_\infty^{\ elec}$$
$$\Delta V = \frac{-W^{elec}}{q}$$
$$V = \frac{-W_\infty^{\ elec}}{q}$$
$$U_E = qV$$
$$\Delta K = W^{appl} + W^{elec}$$
$$\Delta V = -\int \vec{E}\cdot d\vec{s}$$
$V = k\dfrac{q}{r}$ (point)

$V = k\dfrac{p\cos\theta}{r^2}$ (dipole, distance $r$ away from dipole center, $r >> d$)

$$V = \frac{1}{4\pi\varepsilon_0}\int \frac{dq}{r}$$
$V = \dfrac{\sigma}{2\varepsilon_0}\left(\sqrt{z^2+R^2} - z\right)$ (charged disk, radius $R$, distance $z$ along perpendicular axis through disk center)

$$E_s = -\frac{\partial V}{\partial s}$$
$U_E = k\dfrac{q_1 q_2}{r}$ (two charged particles)

$$q = CV$$
$C = \dfrac{\varepsilon_0 A}{d}$ (parallel-plate)

$C = 2\pi\varepsilon_0 \dfrac{L}{\ln(b/a)}$ (cylindrical)

$C = 4\pi\varepsilon_0 \dfrac{ab}{b-a}$ (spherical)

$C = 4\pi\varepsilon_0 R$ (isolated sphere)

$C_{eq} = \sum_{i=1}^{n} C_i$ (capacitors in parallel)

$\dfrac{1}{C_{eq}} = \sum_{i=1}^{n} \dfrac{1}{C_i}$ (capacitors in series)

$$U_E = \frac{q^2}{2C} = \frac{1}{2}CV^2$$
$C = \kappa C_{air}$ (with dielectric const. $\kappa$)

$\varepsilon_0 \oint \kappa\vec{E}\cdot d\vec{A} = q$ (Gauss' Law with dielectric)

$$i = \frac{dq}{dt}$$
$$i = \int \vec{J} \cdot d\vec{A}$$
$$\vec{J} = ne\vec{v}_d$$
$$R = \frac{V}{i}$$
$$\rho = \frac{1}{\sigma} = \frac{E}{J}$$
$$\vec{E} = \rho\vec{J}$$
$$R = \rho\frac{L}{A}$$
$$P = iV$$
$$P = i^2 R = \frac{V^2}{R} \quad \text{(for resistor)}$$

$$\varepsilon = \frac{dW}{dq}$$
$$\sum i = 0 \quad \text{(Kirchoff junction rule)}$$
$$\sum \Delta V = 0 \quad \text{(closed path, Kirchoff loop rule)}$$
$$i = \frac{\varepsilon}{R}$$
$$P_{emf} = i\varepsilon$$
$$R_{eq} = \sum_{i=1}^{n} R_i \quad \text{(resistors in series)}$$
$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i} \quad \text{(resistors in parallel)}$$
$$q = C\varepsilon(1 - e^{-t/RC}) \quad \text{(charging a capacitor)}$$
$$q = q_0 e^{-t/RC} \quad \text{(discharging a capacitor)}$$
$$i = -\frac{q_0}{RC} e^{-t/RC} \quad \text{(discharging a capacitor)}$$
$$\tau_C = RC$$

$$\vec{F}_B = q(\vec{v} \times \vec{B}) = |q| vB \sin\phi$$
$$\vec{F} = q(\vec{E} + (\vec{v} \times \vec{B})) \quad \text{(Lorentz force)}$$
$$|q| vB = \frac{mv^2}{r} \quad \text{(circular motion in B field)}$$
$$\vec{F}_B = i(\vec{L} \times \vec{B}) \quad \text{(force on current/wire)}$$
$$d\vec{F}_B = i(d\vec{l} \times \vec{B})$$

$\tau = (NiA)B\sin\theta$ (torque on coil)

$|\vec{\mu}| = NiA$ (magnetic dipole moment)

$\vec{\tau} = \vec{\mu} \times \vec{B}$ (magnetic dipole in uniform B field)

$U = -\vec{\mu} \cdot \vec{B}$ (magnetic dipole in uniform B field)

$d\vec{B} = \frac{\mu_0}{4\pi} \frac{id\vec{s} \times \hat{r}}{r^2}$ (Biot-Savart Law)

$B = \frac{\mu_0 i}{2\pi R}$ (long straight wire)

$B = \frac{\mu_0 i \phi}{4\pi R}$ (center of circular arc)

$F_{ba} = \frac{\mu_0 L i_a i_b}{2\pi d}$ (force between parallel currents)

$\oint \vec{B} \cdot d\vec{s} = \mu_0 i_{enc}$ (Ampere's Law)

$B = \mu_0 i n$ (ideal solenoid)

$\vec{B}(z) = \frac{\mu_0}{2\pi} \frac{\vec{\mu}}{z^3}$ (along central axis of coil)

$\Phi_B = \int \vec{B} \cdot d\vec{A}$ (magnetic flux)

$\varepsilon = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

$\varepsilon = \oint \vec{E} \cdot d\vec{s}$

$\oint \vec{E} \cdot d\vec{s} = -\frac{d\Phi_B}{dt}$ (Faraday's Law)

$L = \frac{N\Phi_B}{i}$ (inductance)

$L = \mu_0 n^2 A l$ (solenoid inductance)

$\varepsilon_L = -L\frac{di}{dt}$ (self-induced emf)

$i = \frac{\varepsilon}{R}(1 - e^{-Rt/L})$ (current rise)

$i = i_0 e^{-Rt/L}$ (current decay)

$\tau_L = \frac{L}{R}$

$U_B = \frac{1}{2}Li^2$

$\Phi_B = \oint \vec{B} \cdot d\vec{A} = 0$ (Gauss' Law, magnetic)

$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt}$ (Maxwell's Law)

$\oint \vec{B} \cdot d\vec{s} = \mu_0 \varepsilon_0 \dfrac{d\Phi_E}{dt} + \mu_0 i_{enc}$ (Ampere-Maxwell)

$i_d = \varepsilon_0 \dfrac{d\Phi_E}{dt}$ (displacement current)

$E(x,t) = E_m \sin(kx - \omega t)$ (travel in $+x$ direction)

$B(x,t) = B_m \sin(kx - \omega t)$ (travel in $+x$ direction)

$k = \dfrac{2\pi}{\lambda}$ $\qquad \omega = 2\pi f$ $\qquad c = f\lambda$

$c = \dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$

$\dfrac{E}{B} = c$

$\vec{S} = \dfrac{1}{\mu_0}\left(\vec{E} \times \vec{B}\right)$ (Poynting vector)

$S = \dfrac{1}{\mu_0} EB = \dfrac{1}{c\mu_0} E^2$

$I = S_{avg} = \dfrac{E_m B_m}{2\mu_0} = \dfrac{E_{rms}^2}{c\mu_0}$

$I = \dfrac{P_S}{4\pi r^2}$ (isotropic emission)

$I = \dfrac{1}{2} I_0$ (incident unpolarized light)

$I = I_0 \cos^2 \theta$ (incident polarized light)

$\theta_{inc} = \theta_{refl}$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n = \dfrac{c}{v}$

$i = -o$ (plane mirror)

$\dfrac{1}{o} + \dfrac{1}{i} = \dfrac{1}{f} = \dfrac{2}{r}$ (spherical mirror)

$\dfrac{n_1}{o} + \dfrac{n_2}{i} = \dfrac{n_2 - n_1}{r}$ (spherical refracting surface)

$\dfrac{1}{o} + \dfrac{1}{i} = \dfrac{1}{f} = (n-1)\left(\dfrac{1}{r_1} - \dfrac{1}{r_2}\right)$ (thin lens)

$m = -\dfrac{i}{o}$ and $|m| = \dfrac{h_i}{h_o}$

| | | |
|---|---|---|
| Mirrors | Concave | f,r positive |
| | Convex | f,r negative |

| | | |
|---|---|---|
| Lenses | Convex surf. facing object | f,r positive |
| | Concave surf. facing object | f,r negative |

| Object dist. | Image dist. | |
|---|---|---|
| + | + (real) | - (virtual) |

$n = \frac{c}{v}$ $\lambda_n = \frac{\lambda}{n}$ $v = f\lambda_n$

$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1)$ (assuming $n_2 > n_1$) (phase difference, in wavelengths, between two waves)

$\frac{\Delta L}{\lambda} = 0,1,2\ ...$ (constructive interference)

$\frac{\Delta L}{\lambda} = 0.5,1.5,2.5,\ ...$ (destructive interference)

$d\sin\theta = m\lambda$ $m = 0,1,2,\ldots$ (maxima, 2-slit)

$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ $m = 0,1,2,\ldots$ (min., 2-slit)

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From first semester intro. physics:

$v = v_0 + at$ $x - x_0 = v_0 t + \frac{1}{2}at^2$

$v^2 = v_0^2 + 2a(x - x_0)$ $x - x_0 = \frac{1}{2}(v_0 + v)t$ $x - x_0 = vt - \frac{1}{2}at^2$

$a_c = \frac{v^2}{r}$ $T = \frac{2\pi r}{v}$

In equilibrium: $\vec{F}_{net} = 0$

$\vec{F}_{net} = m\vec{a}$

$\vec{F}_{A\to B} = -\vec{F}_{B\to A}$

$K = \frac{1}{2}mv^2$ $W = \Delta K = K_2 - K_1$

$\Delta U = -W^{cons}$ $\Delta U = -\int_{x_1}^{x_2} F(x)dx$ $F(x) = -\frac{dU(x)}{dx}$

$E_{mec} = U + K$ $\Delta E_{mec} = \Delta U + \Delta K = 0$

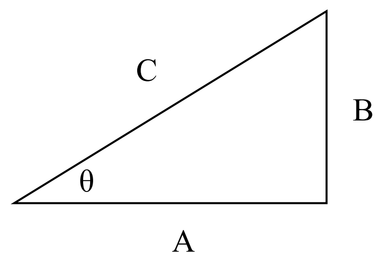
$\vec{\tau} = \vec{r} \times \vec{F}$

$\sin\theta = \dfrac{B}{C}$

$\cos\theta = \dfrac{A}{C}$

$\tan\theta = \dfrac{B}{A}$

$C^2 = A^2 + B^2$




If $ax^2 + bx + c = 0$, then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b = ab\cos\phi$ (either angle between $\vec{a}$ and $\vec{b}$ may be used)

$\vec{a} \times \vec{b} = (a_y b_z - b_y a_z)\hat{i} + (a_z b_x - b_z a_x)\hat{j} + (a_x b_y - b_x a_y)\hat{k} = ab\sin\varphi$ (use smaller angle; direction of resulting vector from Right Hand Rule)

In the following, *u* and *v* are functions are of *x*, and *c* is a constant:

$\dfrac{d}{dx}(uv) = u\dfrac{dv}{dx} + v\dfrac{du}{dx}$ $\qquad$ $\displaystyle\int u\frac{dv}{dx}dx = uv - \int v\frac{du}{dx}dx$

$\dfrac{d}{dx}\sin x = \cos x$ $\qquad$ $\displaystyle\int \sin x dx = -\cos x$

$\dfrac{d}{dx}\cos x = -\sin x$ $\qquad$ $\displaystyle\int \cos x dx = \sin x$

$\dfrac{d}{dx}\sin u = \cos u\dfrac{du}{dx}$ $\qquad$ $\displaystyle\int \frac{dx}{\sqrt{x^2+c^2}} = \ln\left(x + \sqrt{x^2+c^2}\right)$

$\dfrac{d}{dx}\cos u = -\sin u\dfrac{du}{dx}$ $\qquad$ $\displaystyle\int \frac{xdx}{\left(x^2+c^2\right)^{3/2}} = -\frac{1}{(x^2+c^2)^{1/2}}$

$\dfrac{d}{dx}e^u = e^u\dfrac{du}{dx}$ $\qquad$ $\displaystyle\int \frac{dx}{(x^2+c^2)^{3/2}} = \frac{x}{c^2(x^2+c^2)^{1/2}}$

Circle of radius $r$: circumference = $2\pi r$, area = $\pi r^2$

Sphere or radius $r$: area = $4\pi r^2$, volume = $\dfrac{4}{3}\pi r^3$

$g = 9.81$ m/s$^2$
$\varepsilon_0 = 8.85 \times 10^{-12}$ C$^2$/N·m$^2$ = $8.85 \times 10^{-12}$ F/m
$k = 1/4\pi\varepsilon_0 = 8.99 \times 10^9$ N·m$^2$/C$^2$
$\mu_0 = 4\pi \times 10^{-7}$ T.m/A = $4\pi \times 10^{-7}$ H/m
$e = 1.60 \times 10^{-19}$ C
$m_e = 9.11 \times 10^{-31}$ kg
$m_p = 1.67 \times 10^{-27}$ kg
$c = 3.00 \times 10^8$ m/s