Math 31
Derivatives of Trig and Exponential Functions

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Unfinished!

Contents

ALL DERIVATIVES ARE ON YOUR FORMULA SHEET!

Review

You need to review the following for this unit. There are good review resources in the notes booklet.

- Trigonometry
- Exponents and Logs (notably log laws)
- The graphs of the above two

Exponent Appearance

$$\sin(x+2)^3 = \sin(x^3+2^3)$$

$$\sin^3(x+2) = (\sin(x+2))^3$$

Derivatives of Trigonometric Functions

Primary

$$\frac{\mathrm{d}}{\mathrm{d}x}\sin u = \cos u \cdot u'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cos u = -\sin u \cdot u'$$

In simpler terms...

- Swap between sin and cos
- cos to sin prepends negative
- Multiply by derivative of trig function argument

Reciprocal

$$\frac{\mathrm{d}}{\mathrm{d}x}\csc u = -\csc u \cot u \cdot u'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\sec u = \sec u \tan u \cdot u'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\tan u = \sec^2 u \cdot u'$$

$$\frac{\mathrm{d}}{\mathrm{d}x}\cot u = -\csc^2 u \cdot u'$$

Warning

The derivative is multiplied to the entire trig function, not inside the argument.

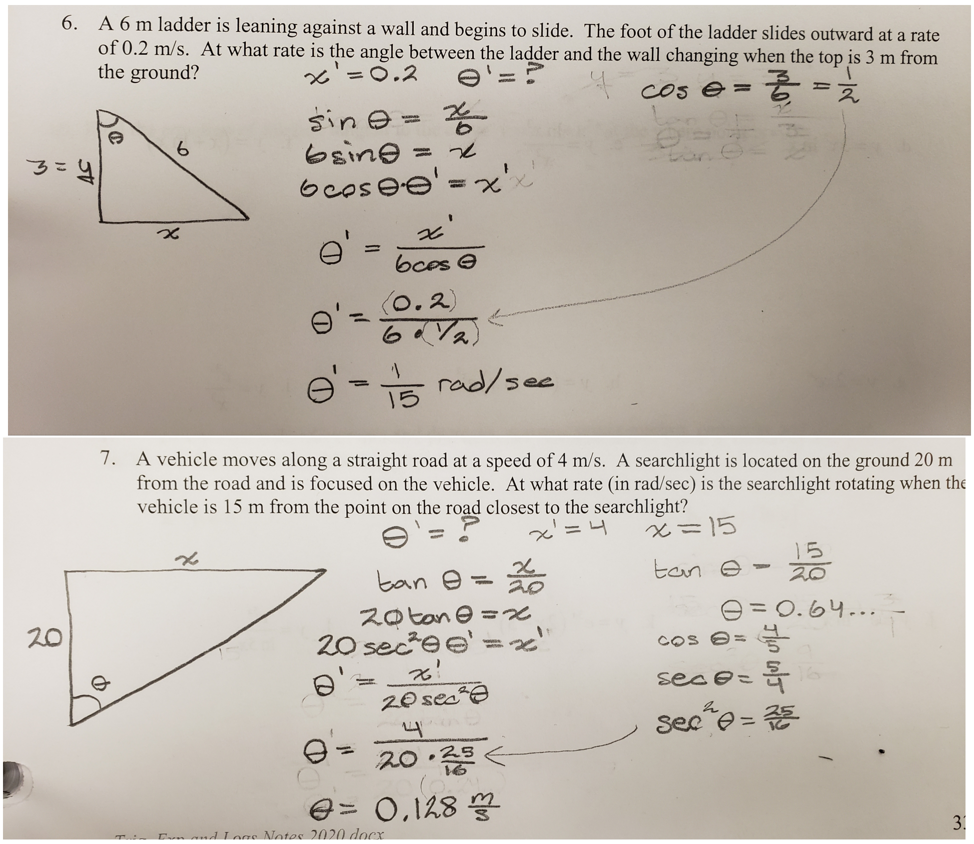
$$\frac{\mathrm{d}}{\mathrm{d}x}\sin u = \cos(u) \cdot u'$$
$$\frac{\mathrm{d}}{\mathrm{d}x}\sin u \neq \cos(u \cdot u')$$

Related Rates with Trigonometric Functions

Related rates is done the same way as before. Here are some formulas you could derive and plug in.

- **Cosine Law**: For non-right triangles, on your formula sheet
  $c^2 = a^2 + b^2 - 2ab\cos C$
  (remember that angle C is opposite to side c)
- Use trigonometric ratios for right triangles (stuff like $\sin\theta = \frac{x}{2}$)

In addition, use either pythagorem theorem or trigonometric functions to solve for any missing variables in the derived function when necessary. (often theta)



6. A 6 m ladder is leaning against a wall and begins to slide. The foot of the ladder slides outward at a rate of 0.2 m/s. At what rate is the angle between the ladder and the wall changing when the top is 3 m from the ground?

$x' = 0.2 \quad \theta' = ?$

$\cos\theta = \frac{3}{6} = \frac{1}{2}$

$3 = y$, 6, x, θ

$\sin\theta = \frac{x}{6}$

$6\sin\theta = x$

$6\cos\theta \cdot \theta' = x'$

$\theta' = \frac{x'}{6\cos\theta}$

$\theta' = \frac{(0.2)}{6 \cdot (1/2)}$

$\theta' = \frac{1}{15}$ rad/sec

7. A vehicle moves along a straight road at a speed of 4 m/s. A searchlight is located on the ground 20 m from the road and is focused on the vehicle. At what rate (in rad/sec) is the searchlight rotating when the vehicle is 15 m from the point on the road closest to the searchlight?

$\theta' = ? \quad x' = 4 \quad x = 15$

x, 20, θ

$\tan\theta = \frac{x}{20}$

$20\tan\theta = x$

$20\sec^2\theta\,\theta' = x'$

$\theta' = \frac{x'}{20\sec^2\theta}$

$\theta' = \frac{4}{20 \cdot \frac{25}{16}}$

$\theta' = 0.128 \frac{m}{s}$

$\tan\theta = \frac{15}{20}$

$\theta = 0.64\ldots$

$\cos\theta = \frac{4}{5}$

$\sec\theta = \frac{5}{4}$

$\sec^2\theta = \frac{25}{16}$

Trig, Exp and Logs Notes 2020.docx

Figure 1: Two of the most common types of questions: ladder sliding, and searchlight tracking straight path

Revolutions

Related rates questions often involve radians per second/minute for rate of change of theta.

Some, however, have revolutions per minute (rpm). To convert, multiply it by 2π.

$$32\,\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\text{rad}}{\text{rev}} = 64\pi\,\frac{\text{rad}}{\text{min}}$$

Euler's Number

$$e \approx 2.71828$$

$$\ln x = \log_e x$$

Derivative of Exponential Functions

$$\frac{\mathrm{d}}{\mathrm{d}x} e^u = e^u \cdot u'$$

$$\frac{\mathrm{d}}{\mathrm{d}x} a^u = a^u \cdot \ln a \cdot u'$$

Derivatives of Logarithmic Functions

$$\frac{\mathrm{d}}{\mathrm{d}x} \log_b u = \frac{u'}{u \ln b}$$

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln u = \frac{u'}{u}$$

Restrictions

Remember that the argument of a log function must be greater than zero.

$$\log x, x > 0$$

You'll need to be able to determine the restrictions as well as derive log functions. Get the restrictions of the original function, before deriving.

If you do get the log of a value 0 or lower, the answer is DNE.

Applications of Logarithmic Functions

$$y = y_i e^{kt}$$

You do have to memorize this. (wasn't on unit exam, though...)

- y: final population
- y_i: initial population
- k: growth period

  $k > 0$ is exponential growth

  $k < 0$ is exponential decay
- t: time