Math 31
Antiderivatives and Integration

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Unfinished!

Contents

Antiderivative

The antiderivative is the opposite of a derivative.

- $F(x)$ = antiderivative
- $f(x)$ = original
- $f'(x)$ = derivative

C

The derivative of any constant is zero. Therefore, even if the original function has no constant, it could have had it in the antiderivative.

This is accounted for by adding the constant variable C.

$$f(x) = 2x$$

$$F(x) = x^2 + C$$

Antiderivative Tips

Polynomials

- Deriving involves subtracting the exponent by 1
- Therefore, antideriving always involves adding 1 to the exponent
- Determine a coefficient for the antiderivative that, if derived, would equal the original
  - Keep the original coefficient
  - Divide the term by the new exponent (after adding 1)
  - Simplify
  - e.g. $f(x) = 6x^2$
    $F(x) = 6x^3 = \frac{6x^3}{3} = 2x^3$
  - $F(x) = 2x^3 + C$
- Imagine constants in the original function having x^0 on them. Therefore, the antiderivative of -5 would be $-5x$

e.g.

$$f(x) = 8x, F(x) = x^8 + C$$

$$f(x) = 2x + 5, F(x) = x^2 + 5x + C$$

$$f(x) = 6x^3, F(x) = \frac{3}{2}x^4 + C$$

$$f(x) = 2x^2 - x + 7, F(x) = \frac{2}{3}x^3 - \frac{1}{2}x^2 + 7x + C$$

Trigonometry with angle x

Follow the normal trigonometry derivative rules, but just in reverse. These are also on your formula sheet.

$$f(x) = \cos x - \sin x$$
$$F(x) = \sin x + \cos x + C$$

Trigonometry with angle ax

For all trignometric functions, do the "reverse" derivative like the previous section. To account for the coefficient, divide the term by the derivative of the trigonometric function's argument.

This is to get rid of the coefficient on the whole term if you derive $F(x)$, therefore making it correct.

$$f(x) = \cos 6x$$
$$F(x) = \frac{1}{6} \sin 6x + C$$

Exponential Functions

- Add 1 to the exponent
- Divide everything by the new exponent

$$f(x) = \sqrt[3]{x} = x^{\frac{1}{3}}$$
$$F(x) = \frac{x^{\frac{4}{3}}}{\frac{4}{3}} = \frac{3}{4} x^{\frac{4}{3}} + C$$

Variable in Denominator

The following is in your formula sheet.

$$\frac{\mathrm{d}}{\mathrm{d}x} \ln u = \frac{1}{u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

Using this, you can convert a fraction derivative into a natural log function antiderivative.

$$f(x) = \frac{-3}{x}, F(x) = -3 \ln x + C$$
$$f(x) = \frac{2x}{x^2 + 1}, F(x) = \ln(x^2 + 1) + C$$

Euler's Number

- Divide everything by the derivative of the exponent

$$f(x) = e^{3x}$$
$$F(x) = \frac{1}{3}e^{3x} + C$$

$$f(x) = xe^{x^2}$$
$$F(x) = \frac{xe^{x^2}}{2x} = \frac{1}{2}e^{x^2} + C$$

Reciprocal Trigonometric Functions

Like the derivatives of trig functions, the antiderivatives of trig functions is on your formula sheet under "INTEGRALS & ANTIDERIVATIVES."

$$f(x) = \sin^2 x \cos x$$
$$F(x) = \frac{1}{3}\sin^3 x + C$$

Kinematics

Recall that,

- $s =$ displacement
- $s' = v =$ velocity
- $s'' = v' = a =$ acceleration

It works backwards as well with antiderivatives.

Differential Equations with Initial Conditions

When getting the antiderivative, you'll have the unknown value of C.

In questions with initial conditions, you can solve for C, then write the equation.

e.g. *Solve for the differential equation* $\frac{\mathrm{d}s}{\mathrm{d}t} = 2t$ *with the initial conditions of* $s = 3$ *when* $t = 0$

$$s(0) = 3, v(t) = 2t$$

$$s(t) = t^2 + C$$

$$s(0) = 3 = 0^2 + C, C = 3$$

$$s(t) = t^2 + 3$$

More examples page 7-9 in booklet.

Integrals

The integral of a curve function is the area under the curve.

Indefinite Integrals

Indefinite integrals have no bounds, and are the same as antiderivatives.

Take the antiderivative of $f(x)$ with respect to x.

$$F(x) = \int f(x)dx$$

Definite Integrals

The definite integral from a to b is,

$$\int_b^a f(x)dx = F(a) - F(b)$$

which is the area under the curve between an interval/range of two x points.

Find $F(x)$, then substitute $F(x)$ with a subtracting b.

You can ignore the C, since both functions have it, it will always be cancelled out.

e.g.

$$\int_1^3 xdx = F(3) - F(1)$$

$$F(x) = \frac{1}{2}x^2 + C$$

$$(\frac{3^2}{2} + C) - (\frac{1^2}{2} + C)$$

$$-15 - (-6) = -9$$

Properties of Integrals

$$\int_a^a f(x)dx = 0$$

$$\int_b^a f(x)dx = -\int_a^b f(x)dx$$

$$\int_b^a c \cdot f(x)dx = c \cdot \int_b^a f(x)dx$$

$$\int_b^a \Big(f(x) \pm g(x)\Big)dx = \int_b^a f(x) \pm \int_b^a g(x)$$