Math 31
Limits and the Derivative

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Unfinished!

Contents

Factoring Brief Review

Differences of Square

$x^2 - 4 = (x+2)(x-2)$

Polynomial

$2x^2 + 3x - 2$
$\longrightarrow (2x^2 + 4x)(-x - 2)$
$\longrightarrow\ 2x(x+2) - 1(x+2)$
$\longrightarrow (2x-1)(x+2)$

Radical Fractions

- Multiply everything by monomial denominator
  $\frac{2}{\sqrt{2x}} \longrightarrow \frac{2\sqrt{2x}}{2x} \longrightarrow \frac{\sqrt{2x}}{x}$
- Multiply everything by conjugate for polynomial denominators
  $\frac{3}{2+\sqrt{x}} \times \frac{2-\sqrt{x}}{2-\sqrt{x}} = \frac{6-3\sqrt{x}}{4-2\sqrt{2}+2\sqrt{x}-x} = \frac{6-3\sqrt{x}}{4-x}$

Mixed Radicals

$\sqrt{162} \longrightarrow \sqrt{9^2 \times 2} \longrightarrow \sqrt{9^2} \times \sqrt{2} \longrightarrow 9\sqrt{2}$

Absolute Polynomial

$|x-1| = 3$

$x - 1 = 3,\ x = 4$
$x - 1 = -3,\ x = -2$

Adding/Subtracting Fractions

Multiply both terms so that the denominators are the same, then add/subtract.
$\frac{2}{x-1} - \frac{3}{x+3}$
$\longrightarrow \frac{2(x+3)}{(x-1)(x+3)} - \frac{3(x-1)}{(x-1)(x+3)}$
$\longrightarrow \frac{(2x+6)-(3x-3)}{(x-1)(x+3)}$
$= \frac{-x+3}{(x-1)(x+3)}$

Piecewise Functions

Piecewise functions are functions with multiple inequalities/restrictions that dictate which function to use at specific x values.

When graphing...

- if an inequality is less/greater than a value, the plot point is not filled in
- if an inequality is less/greater than OR equal to a value, the plot point is filled in
- if x of different functions equal the same value, the graphs are continuous, and are filled in if one of the functions is inclusive

If the inequalities do not state a function for a specific x value (e.g. $x = 2$ for $2 < x < 2$) then that value **DNE**. (does not exist)

Rational Function

A function with a polynomial in the numerator and denominator.

Vertical Asymptotes

Zeros of the denominator of a rational function.

x may approach these values, but never touch them.

Point of Discontinuity

Any vertical asymptote (zeros of denominator) before simplifying a rational function.

These vertical asymptotes only applies to the unsimplified form; this makes it a point of discontinuity.

Horizontal Asymptotes

Horizontal asymptotes describe the trend of a function.

The graph line can cross over it fine, as opposed to vertical asymptotes.

Determining Horizontal Asymptotes

- degree of numerator < degree of denominator
  $\longrightarrow y = 0$
- degree of numerator = degree of denominator
  $\longrightarrow y = \frac{\text{leading coefficient of numerator}}{\text{leading coefficient of denominator}}$
- degree of numerator > degree of denominator
  $\longrightarrow$ Divergent (no horizontal asymptote)

Limits

$$\lim_{x \to a} f(x) = b$$

The limit of $f(x)$ as x approaches a is b.