Physics 30
Momentum and Impulse

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Unfinished!

Contents

Review

Scalar v/s Vector Quantity

- **Scalar** = Magnitude (size) only
- **Vector** = Magnitude (size) AND direction

Sig Digs

Multiplication & Division

Least number of sig digs in numbers provided by question.

Addition & Subtraction

Least number of decimal places in numbers provided by question.

Unit Analysis

km/h to m/s

$$100\,\mathrm{km/h} \times \frac{1000\,\mathrm{m}}{1\,\mathrm{km}} \times \frac{1\,\mathrm{h}}{3600\,\mathrm{s}}$$

Proportional

$$a \propto b$$

If a variable is proportional to the other, increasing one will increase the other, same with decreasing.

$$a \propto \frac{1}{b}$$

If a variable is inversely proportional to the other, increasing one will decrease the other, and vice versa.

Proportionality Example

If the velocity of a car is doubled and the mass of the car is decreased by $\frac{1}{3}$ determine the new momentum and new Kinetic energy.

Momentum

$$\vec{p} = m\vec{v}$$

$$\left(\tfrac{1}{3}m\right)\left(2\vec{v}\right)$$

The Kinetic energy would be 1.33X greater.

$$\tfrac{1}{3} \times 2 = 0.6666666667$$

The momentum of the car would be 0.667X greater.

Kinetic Energy

$$E_K = \tfrac{1}{2}mv^2$$

$$\left(\tfrac{1}{3}m\right)\left(2v\right)^2$$

We only include the variable values that have changed

$$\left(\tfrac{1}{3}\right) \times \left(2\right)^2 = 1.\overline{333}$$

A truck has a momentum of $2.00 \times 10^5\ \mathrm{Kg \cdot \frac{m}{s}}$, E. Calculate the momentum the truck would have if it's velocity was tripled and it's mass was halved.

$$\vec{p} = m\vec{v}$$

$$\vec{p} = \left(\tfrac{1}{2}m\right)\left(3\vec{v}\right) \qquad \underline{\underline{1.5}}$$

$$\vec{p}_{new} = \vec{p}_{ORIGINAL} \times 1.5$$

$$\vec{p}_{new} = 2.00 \times 10^5\ \mathrm{Kg \cdot \tfrac{m}{s}} \times 1.5$$

E

$$\vec{p}_{new} = 3.00 \times 10^5\ \mathrm{Kg \cdot \tfrac{m}{s}}\ (E)$$

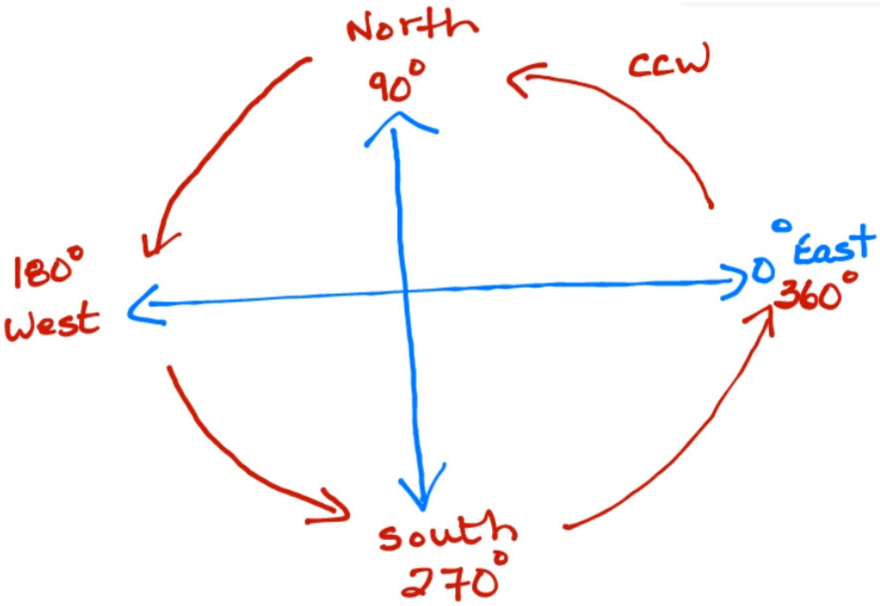
Tip: When determining how a variable changes proportional to the others in an equation, isolate the variable to one side.

Conventions

Signs

- $+$ positive: right, up, north, east
- $-$ negative: left, down, south, west

Direction



Uniform Velocity

$$v = \frac{d}{t}$$

Uniform = constant: velocity does not change over time.

If velocity changes (starts from rest, acceleration exists) then this formula CANNOT be used. There are others on your formula sheet.

Formula Review

$$g = 9.81\,\text{m/s}^2$$

$$\sum E_{top} = \sum E_{bottom}$$

$$E_p + E_k = E_p + E_k$$

$$\text{Newton's 2nd Law (Force, in N)} = \vec{F} = m\vec{a}$$

$$\text{Weight (N)} = \vec{F_g} = m\vec{g}$$

Inertia

The tendency of an object to resist acceleration.

Object wants to maintain the velocity it was at — e.g. stay at rest if suddenly moved, or keep moving forward if suddenly stopped.

e.g. Pulling the cover under a table fast enough doesn't drag everything with it. A large boulder being transported in a truck will smash into the driver if the truck suddenly stops.

Momentum

$$\vec{p} = m\vec{v}$$

Momentum is conserved between colliding objects in a isolated/closed system.

- $\vec{p} =$ momentum, product of mass and velocity
  vector quantity, kg · m/s or N · s
- $m =$ mass
  scalar quantity, kg
- $\vec{v} =$ velocity
  vector quantity, m/s

$$\vec{p} \propto m$$

$$\vec{p} \propto \vec{v}$$

Examples

A bowling pin is dropped from 2.50 m above the ground. If the bowling pin has a mass of 1.50 kg find the momentum of the bowling pin as it strikes the ground.

$\vec{p} = m\vec{v}$

$v_1 = 0 \frac{m}{s}$

$h = d = -2.50\,m$

$g = a = -9.81 \frac{m}{s^2}$

$\Sigma E_{TOP} = \Sigma E_{BOTTOM}$

$E_p + \cancel{E_k} = \cancel{E_p} + E_k$

$mgh = \frac{1}{2}mv^2$

$v = \sqrt{\frac{2\cancel{m}gh}{\cancel{m}}}$

$\vec{p} = m\vec{v}$

$v_f^2 = v_1^2 + 2ad$

$v_f = \sqrt{2ad}$

$v_f = \sqrt{2 \times -2.50\,m \times -9.81 \frac{m}{s^2}}$

$v_f = 7.003570518 \frac{m}{s}$

$\vec{p} = 1.50\,kg \times 7.003570518 \frac{m}{s}$ (down)

$\vec{p} = 10.5\,kg \cdot m/s$ down

An object has a velocity of $5.00\,\frac{m}{s}$ East and a momentum of $42.0\ kg\cdot\frac{m}{s}$, East. What is the weight of the object.

$\vec{p} = m\vec{v}$ $\quad m = \frac{\vec{p}}{\vec{v}} = \frac{42.0\ kg\cdot\cancel{\frac{m}{s}},\ \cancel{E}}{5.00\ \cancel{\frac{m}{s}},\ \cancel{E}}$

$m = 8.40\ kg$

$\vec{F_g} = 8.40\ kg \times -9.81\ \frac{m}{s^2}$

$\vec{g} = \frac{\vec{F_g}}{m}$

$\vec{F_g} = -82.404\ N$

$\vec{F_g} = m\vec{g}$

$\vec{F_g} = 82.4\ N$ towards the Earth

WEIGHT (N)

A 12.0 kg object travels 8.76 m west in 2.12 s. Assuming uniform velocity, calculate the momentum of the object.

uniform velocity = $v = \frac{d}{t}$

constant

$\vec{v} = \frac{\vec{d}}{t} = \frac{8.76\,m\,(W)}{2.12\ s}$

$\vec{v} = 4.132075472\ \frac{m}{s}\ (W)$

$\vec{p} = m\vec{v} = 12.0\ kg \times 4.132075472\ \frac{m}{s}\ W$

$\vec{p} = 49.58490566\ kg\cdot\frac{m}{s}\ (W)$

$\vec{p} = 49.6\ kg\cdot\frac{m}{s}\ (W)$

Impulse

$$\Delta\vec{p} = \vec{p}_f - \vec{p}_i$$

Impulse is change in momentum; a force applied to an object will change its momentum.

Formula

$$\Delta\vec{p} = \vec{F}\Delta t = m\Delta\vec{v}$$

$$\mathrm{kg} \cdot \mathrm{m/s} = \mathrm{N} \cdot \mathrm{s} = \mathrm{kg} \cdot \mathrm{m/s}$$

Can be reorganized into Newton's 2nd law.

$$\vec{F} = \frac{m\Delta\vec{v}}{\Delta t} = m\Delta a$$

$$\vec{F} \propto \frac{1}{\Delta t}$$

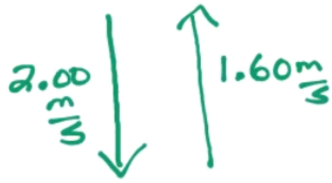
Force is inversely proportional to time; a large force will be in small time (swift execution), a small force will be over large time.

Application

$$\vec{F} = \frac{\Delta\vec{p}}{\Delta t}$$

The greater the time is for an object to, say, fall and hit the ground, the impact will have less force, if all other variables are the same. By this logic, a more cushiony ground would cause the object to take more time to hit the ground, resulting in less force, cushioning their fall. The same applies to airbags.

A 0.625 Kg basketball strikes the floor with a velocity of 2.00 $\frac{m}{s}$. If this basketball bounces up with a velocity of 1.60 $\frac{m}{s}$, what is the ball's change in momentum.



$$\Delta\vec{p} = m\Delta\vec{v}$$

$$\Delta\vec{p} = m\left(v_f - v_i\right)$$

$$\Delta\vec{p} = 0.625\,\text{Kg}\left(+1.60\tfrac{m}{s} - \left(-2.00\tfrac{m}{s}\right)\right)$$

$$\Delta\vec{p} = 2.25\ \text{Kg}\cdot\tfrac{m}{s},\ \text{up}$$

our answer in our calculator is positive so the direction is <u>up</u>

Figure 1: A frictionless disc of mass 0.500 kg is moving in a straight line across an air table top at a speed of 2.40 m/s when the disc bumps into an elastic band stretched between two fixed posts. If the elastic band exerts an opposing force of 1.40 N on the disc for 1.50 s, calculate the final velocity of the disc.

$$\vec{F}\Delta t = m\Delta\vec{v} \qquad \Delta v = v_f - v_i$$

$$\vec{F}\Delta t = m\vec{v}_f - m\vec{v}_i$$

$$\vec{v}_f = \frac{\vec{F}\Delta t + m v_i}{m}$$

$$\vec{v}_f = \frac{(-1.40\,N \times 1.50\,s) + (0.500\,kg \times 2.40\,\frac{m}{s})}{0.500\,kg}$$

$$\vec{v}_f = -1.80\,\frac{m}{s}$$

$\vec{v}_f = 1.80\,\frac{m}{s}$ in the direction of the force

or in the opposite direction to its original motion.

Force as a Function of Time Graphs

- y-axis = Force ($\vec{F}$, N)
- x-axis = Time (t, s)
- Area under line = Change in momentum, aka. Impulse ($\Delta\vec{p}$, N $\cdot$ s)