

# Math 31

## Derivatives of Trig and Exponential Functions

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ALL DERIVATIVES ARE ON YOUR FORMULA SHEET!

## Review

You need to review the following for this unit. There are good review resources in the notes booklet.

- Trigonometry
- Exponents and Logs (notably log laws)
- The graphs of the above two

## Exponent Appearance

$$\sin(x+2)^3 = \sin(x^3 + 2^3)$$

$$\sin^3(x+2) = (\sin(x+2))^3$$

## Derivatives of Trigonometric Functions

### Primary

$$\frac{d}{dx} \sin u = \cos u \cdot u'$$

$$\frac{d}{dx} \cos u = -\sin u \cdot u'$$

In simpler terms...

- Swap between sin and cos
- cos to sin prepends negative
- Multiply by derivative of trig function argument

### Reciprocal

$$\frac{d}{dx} \csc u = -\csc u \cot u \cdot u'$$

$$\frac{d}{dx} \sec u = \sec u \tan u \cdot u'$$

$$\frac{d}{dx} \tan u = \sec^2 u \cdot u'$$

$$\frac{d}{dx} \cot u = -\csc^2 u \cdot u'$$

## Warning

The derivative is multiplied to the entire trig function, **not inside the argument**.

$$\frac{d}{dx} \sin u = \cos(u) \cdot u'$$

$$\frac{d}{dx} \sin u \neq \cos(u \cdot u')$$

## Related Rates with Trigonometric Functions

Related rates is done the same way as before. Here are some formulas you could derive and plug in.

- **Cosine Law:** For non-right triangles, on your formula sheet

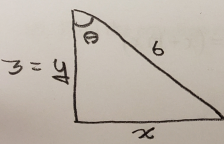
$$c^2 = a^2 + b^2 - 2ab \cos C$$

(remember that angle C is opposite to side c)

- Use trigonometric ratios for right triangles (stuff like  $\sin \theta = \frac{x}{2}$ )

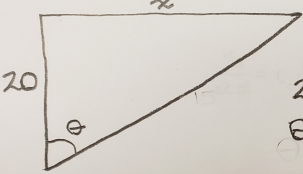
In addition, use either pythagorem theorem or trigonometric functions to solve for any missing variables in the derived function when necessary. (often theta)

6. A 6 m ladder is leaning against a wall and begins to slide. The foot of the ladder slides outward at a rate of 0.2 m/s. At what rate is the angle between the ladder and the wall changing when the top is 3 m from the ground?



$x' = 0.2$     $\theta' = ?$     $\cos \theta = \frac{3}{6} = \frac{1}{2}$   
 $\sin \theta = \frac{x}{6}$   
 $6 \sin \theta = x$   
 $6 \cos \theta \cdot \theta' = x'$   
 $\theta' = \frac{x'}{6 \cos \theta}$   
 $\theta' = \frac{0.2}{6 \cdot \frac{1}{2}}$   
 $\theta' = \frac{1}{15} \text{ rad/sec}$

7. A vehicle moves along a straight road at a speed of 4 m/s. A searchlight is located on the ground 20 m from the road and is focused on the vehicle. At what rate (in rad/sec) is the searchlight rotating when the vehicle is 15 m from the point on the road closest to the searchlight?



$\theta' = ?$     $x' = 4$     $x = 15$   
 $\tan \theta = \frac{x}{20}$   
 $20 \tan \theta = x$   
 $20 \sec^2 \theta \theta' = x'$   
 $\theta' = \frac{x'}{20 \sec^2 \theta}$   
 $\theta' = \frac{4}{20 \cdot \frac{25}{16}}$   
 $\theta' = 0.128 \frac{\text{m}}{\text{s}}$

tan, Fox and Lane Notes 2020.docx

Figure 1: Two of the most common types of questions: ladder sliding, and searchlight tracking straight path

## Revolutions

Related rates questions often involve radians per second/minute for rate of change of theta.

Some, however, have revolutions per minute (rpm). To convert, multiply it by  $2\pi$ .

$$32 \frac{\text{rev}}{\text{min}} \cdot \frac{2\pi \text{rad}}{\text{rev}} = 64\pi \frac{\text{rad}}{\text{min}}$$

## Euler's Number

$$e \approx 2.71828$$

$$\ln x = \log_e x$$

## Derivative of Exponential Functions

$$\frac{d}{dx} e^u = e^u \cdot u'$$

$$\frac{d}{dx} a^u = a^u \cdot \ln a \cdot u'$$

## Derivatives of Logarithmic Functions

$$\frac{d}{dx} \log_b u = \frac{u'}{u \ln b}$$

$$\frac{d}{dx} \ln u = \frac{u'}{u}$$

## Restrictions

Remember that the argument of a log function must be greater than zero.

$$\log x, x > 0$$

You'll need to be able to determine the restrictions as well as derive log functions. Get the restrictions of the original function, before deriving.

If you do get the log of a value 0 or lower, the answer is DNE.

## Applications of Logarithmic Functions

$$y = y_i e^{kt}$$

You do have to memorize this. (wasn't on unit exam, though...)

- $y$ : final population
- $y_i$ : initial population
- $k$ : growth period
  - $k > 0$  is exponential growth
  - $k < 0$  is exponential decay
- $t$ : time