Summary Information for Four Named Discrete Probability Distributions

Distribution	Parameters	Random Variable	Probability Mass Function	Examples
Binomial—may be used in situations with dichotomous outcomes from a fixed number of independent trials with constant probability of success for each trial. When n = 1, called a **Bernoulli trial**	n = number of independent trials p = probability of success (assumed constant over all trials)	X = number of successes in n independent trials. $X \sim \text{Binomial}(n, p)$ $\mu = E(X) = np$ $\sigma^2 = \text{Var}(X) = np(1-p)$ $\sigma = \text{SD}(X) = \sqrt{\sigma^2}$	Probability that an n-trial binomial experiment results in **exactly** x successes: $P(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$ for x = 0, 1, 2, … , n.	Examples of binomial experiments: 1. The experiment is to toss a fair coin 10 times and record the sequence of heads and tails. Let X = the number of tails in 10 tosses. 2. Giving a multiple-choice test with 30 questions, each with 4 options of which only 1 option is correct. Assume random guessing on each question. Let X = number of correct guesses in 30 questions.
Hypergeometric—may be used in situations where you sample n items without replacement from a population of size N. Requires a dichotomous response variable with one outcome designated success and the other failure. Probability of success not constant from trial to trial.	N = size of population M = number of successes in population n = sample size, drawn without replacement.	X = number of successes in a sample of size n. $X \sim \text{Hypergeometric}(n, M, N)$ $\mu = E(X) = n\left(\frac{M}{N}\right)$ $\sigma^2 = \text{Var}(X) = nM(N-M)\frac{N-n}{N^2(N-1)}$ $\sigma = \text{SD}(X) = \sqrt{\sigma^2}$	Probability that an n-trial hypergeometric experiment results in **exactly** x successes, when the population consists of N items, M of which are classified as successes: $P(X = x) = \frac{\binom{M}{x}\binom{N-M}{n-x}}{\binom{N}{n}}$ for x = 0, 1, 2, … , n.	Examples of hypergeometric experiments: 1. Drawing candies from a bowl: you have a bowl with 50 M&M's, 20 blue and 30 not blue. You randomly select 8 M&M's from the bowl without replacement. You count the number of blue M&M's you have drawn. 2. Drawing cards from a standard playing deck: you are randomly dealt 5 cards from the deck. You count the number of spades in your hand.

Summary Information for Four Named Discrete Probability Distributions

Geometric—used to model the number of trials up to and including the first success. Assumes constant probability of success and a dichotomous response.	p = constant probability of success	X = number of Bernoulli trials to get one success. $X \sim$ Geometric(p) $\mu = E(X) = \frac{1}{p}$ $\sigma^2 = \mathrm{Var}(X) = \frac{1-p}{p^2}$ $\sigma = \mathrm{SD}(X) = \sqrt{\sigma^2}$	Probability that the first occurrence of success requires x independent trials each with the constant probability of success, p: $P(X = x) = (1-p)^{x-1}p$ for $x = 1, 2, \ldots$,.	Examples of geometric distributed random variables: 1. Toss a standard six-sided die and count the number of times it takes before you throw a 5. Let X = number of tosses of the die up to and including the first observed 5. 2. A middle school boy attends his first school dance and starts to ask girls to dance. Let X = number of times he must ask a girl to dance before and up to the first yes.
Poisson—used to model events that occur rarely in space or time. Used to calculate the probability of a given number of events occurring in a fixed interval of space or time provided the occurrences are independent of the time or location of the last event and you know the average rate of occurrence.	$\mu = \lambda t$ where λ is the average unit rate or unit area of occurrence, and t is a scaling factor for the number of units.	X = number of occurrences in a specified time frame or area. $X \sim$ Poisson(μ) $\mu = E(X) = \sigma^2 = \mathrm{Var}(X) = \lambda t$	Probability of x discrete occurrences of event X in a given time interval or specified region in space: $P(X = x) = \frac{\mu^x e^{-\mu}}{x!}$, for $\mu > 0$, $x = 0,1,2, \ldots$,	Examples of Poisson distributed random variables: 1. The number of patients that arrive at the Harborview ER during an 8-hour day shift given an average of 3.2 patients per hour. 2. The number of cars sold by a car salesman in one week given an average of 1.2 sales per day.

Other interesting properties:

1. As $n \dashrightarrow \infty$, the binomial distribution converges a to a normal distribution. This is why, when n is large enough, we can use normal approximation to estimate binomial probabilities. For this class, n large enough means $np > 5$ and $n(1-p) = nq > 5$. Use of continuity correction generally recommended.
2. The Poisson distribution can be approximated with a normal distribution when μ is large. Use of continuity correction generally recommended.
3. The Poisson distribution can be used to approximate binomial probabilities when you let $\lambda = np$ and $n \dashrightarrow \infty$, with p very small so that np is essentially fixed.