Stat 311 Homework 4

This assignment does not use any data sets but will include a few parts of problems where you will copy/paste R code with answers. Parts of a few problems are modified from McClave and Sincich, *Statistics*, Ninth Edition.

- You may handwrite or type up this assignment and submit as .docx or .pdf. Do not do this assignment in Rmarkdown.
- If you handwrite your assignment, you can write down your R code with the answers in the few places where you are to show your R code.
- Be sure to show work as needed. If you simply put down a numeric answer it will be marked wrong, even if the number is correct.
- All requested interpretations should be in complete sentences.

1. A fair six-sided die is tossed and the up face is noted. If the number is ≤ 2, the die is tossed again; if the number is > 2, a fair coin is tossed. Define the events:

   A: {A tail appears on the coin} and B: {The die is tossed only one time}

   a) List the sample points in the sample space [Hint: there are 20 sample points].

   b) Give the probability for each of the sample points.

   c) Find $P(A)$ and $P(B)$.

   d) Identify the sample points in A^c, B^c, $A \cap B$, and $A \cup B$.

   e) Find $P(A^c)$, $P(B^c)$, $P(A \cap B)$, $P(A \cup B)$, $P(A \mid B)$, and $P(B \mid A)$.

   f) Are A and B mutually exclusive events? Independent events? Why? Justify

   g) Are the events A and B independent? Justify your answer using an equation.

2. A balanced six-sided die is thrown once. If a 3 appears, a ball is drawn from urn 1; otherwise, a ball is drawn from urn 2. Urn 1 contains six red and four white balls. Urn 2 contains four red, three white, and three black balls.

   a) Find the probability that a red ball is drawn.

   b) Find the probability that urn 1 was used given that a red ball was drawn.

3. Seventy-eight percent of all women who submit to pregnancy tests are pregnant. A new pregnancy test gives a false positive result with probability 0.03 and a correct positive result with probability 0.98. Define the events: P: {a woman is pregnant} and +: {the pregnancy test is positive}.

   a) Using the events P and + as defined, write out the information given in this problem.

   b) What is the sensitivity of this test? Explain in layperson terms what this means?

   c) What is the specificity of this test? Explain in layperson terms what this means?

   d) What is the false negative rate (FNR) for this test?

   e) What is the unconditional probability that this test produces a positive result?

   f) For a randomly chosen woman who receives a positive result using this test, what is the probability that she is pregnant? Interpret this result.

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4. City officials estimate that a dangerous intersection in the city has 0.4 accidents per month. Let X be the number of accidents that occur at this intersection over the next three months.
   a) What probability distribution can you use to model X?
   b) Calculate the probability that are at most two accidents occur at this intersection in the next three months. Write out the equation you need with numbers substituted but solve using R. Copy/paste your R code here, with the answer.
5. One way to get out of jail in MONOPOLY is to roll a double (two of the same up faces when rolling two six-sided dice, such as 1 and 1).
   a) Let X be the number of rolls needed to get out of jail. What probability distribution can you use to model X?
   b) What is the probability that you get out of jail within 4 rolls of the dice? Write down the formula you would use with numbers substituted but use R to calculate the answer. Copy and paste your R code here with the answer.
   c) What is the expected number of rolls before the roll for getting out of jail?
6. Let X be a random variable that denotes the number of heads in a set of 6 tosses.
   a) What is the sample space for X?
   b) Assume that the coin is unfair with $P(H) = 0.2$. Assuming independent toss outcomes, what probability distribution can you use to model X? Make sure to name the distribution and the values of any parameters.
   c) What are $E(X) = \mu_X$ and σ_X based on the distribution named in part (b) above? Interpret these numbers.
   d) What is the exact probability that $X > 4$ in any given case? Write down the formula you would use with numbers substituted but use R to calculate the answer. Copy and paste your R code here with the answer.
7. It is thought that there are 100 moose in the Yellowstone Park moose population. Last year, wildlife officials captured and tagged 10 moose to monitor their movement throughout the park. Six months later 20 moose were captured to gather some health data. Define the RV, X, to be the number of tagged moose in the group of 20 most recently captured moose. We will assume that all moose are still living and that the population total has not changed during the six months.
   a) Name a probability distribution that you could use to find probabilities of X. Do not forget to include values of any parameters.
   b) What is the probability that 3 of the captured moose are tagged? Write out the equation you need with numbers substituted but solve using R. Copy/paste your R code here, with the answer.