42
36
6 33.875

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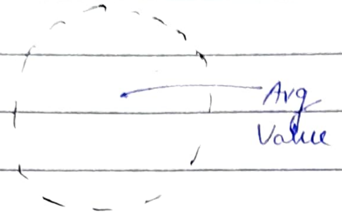
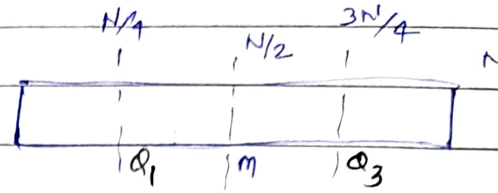
Quartile Deviation :-





Question :

		f	cf	Exp. Valu
	15.5 – 20.5	1	1	18
	20.5 – 25.5	2	3	23
	25.5 – 30.5	7	10	28
$Q_1 = N/4 = 14$	30.5 – 35.5 Q_1	10	20	33
	35.5 – 40.5	16	36	38
$Q_1 = 3N/4 = 42$	40.5 – 45.5 Q_3	9	45	43
	45.5 – 50.5	8	53	48
	50.5 – 55.5	2	55	53
	55.5 – 60.5	1	56	58

$Q_1 = L + \frac{C}{f} \times (N/4 - cf\ pred)$

$Q_1 = \frac{5}{10} \times (14 - 10)$

$= \frac{1}{\cancel{2}} \times \cancel{4}^{2} + 30.5$

$Q_1 = 32.5$

Exact value = cf × exp. value
Ex = 1×18 or 2×23

$Q_3 = L + \frac{C}{f} \times (3N/4 - cf) = 40.5 + \frac{5}{9} \times (42 - 36)$

$= 40.5 + \frac{5}{\cancel{9}_{3}} \times \cancel{6}^{2} = 40.5 + 3.33 = 43.83.$

The divide the data into 100 parts.

$Q_k = \frac{KN}{100} = \frac{55 \times 56}{100} = 30.8$

$Q_k = 35.5 + \frac{5}{16}(30.8 - 20)$

$Q_k = 35.5 + 3.375$

$Q_k = 38.875$

$$y = \frac{x-a}{c} \Rightarrow x = cy + a \quad \text{--- (1)}$$

$$\Sigma x = c\Sigma y + an$$

Dividing this by n.

$$\frac{\Sigma x}{n} = c\frac{\Sigma y}{n} + a.$$

$$\boxed{\mu_x = c\mu_y + a} \quad \text{--- (2)}$$

Substracting (2) from (1)

$$\Sigma(x-\mu_x)^2 = \Sigma c^2(y-\mu_y)^2$$

$$\boxed{\sigma_x^2 = c^2\sigma_y^2} \quad \text{--- (3)}$$

$\Rightarrow$ if $a = \mu_x$ and $c = \sigma_x$.

(from (2)) $\mu_y = 0$ and $\sigma_y = 1$ (from (3))

$\Rightarrow$ if a is not exactly μ_x but close to μ_x.

$\Rightarrow$ $(a \approx \mu_x)$ (for prev.) $N/2 = 26$.

Lies in $35.5 - 40.5$.

$\Rightarrow$ Expected Mean is 38 for that range

$\Rightarrow$ Mean $= -4 \times 16 + -3 \times 12 + (-2)7 + 0(16) + 4 \times (+1) + 8 \times (+2) + 3 \times 2 + (+1) \times 9$

$$= \frac{-26 + 29}{56} = \frac{1}{56} = 0.018$$

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$\Rightarrow \therefore$ Mean (X) = (Mean y) + 9.

$\Rightarrow$ $= 5 \times 0.018 + 38$

$= \underline{\underline{38.09}}$

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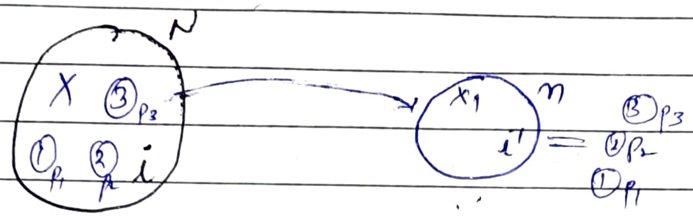

$x_1 \subseteq x$

Mean $= \mu$ $\bar{X}$

σ^2 S_1^2

σ S

n is proper representation of N.

-1					
4	B B B	5	$P = 0.008$		$P(4) = 0.2 \times 0.2 \times 0.2$
9	A A A	10	$P = 0.008$	[ ] [ ] [ ]	$P(9) = 0.2 \times 0.2 \times 0.2$
14	\$ \$ A	15	$P = 0.006$		$3 \rightarrow P(14) = (0.1 \times 0.1 \times 0.2)\, 3$
19	\$ \$ \$	20	$P = 0.001$		$P(19) = 0.1 \times 0.1 \times 0.1$

$P(-1) = 1 - [P(4) + P(9) + P(14) + P(19)]$

$P(\$) = 0.1$ $P(A) = P(B) = 0.2$ $P(\text{other}) = 0.5$

Total no. of instances are 5.

$(-1)\,P(-1) + 4\,P(4) + 9(P(9))$

$E(X) = -0.77$ $V(X) = 2.6971$

New Variable $Y = 5X - 2$. -2

$P(5) = 23$ $5 \times 4 - 2$

$P(10) = 48$ $5 \times 10 - 2$

$P(15) = 73$ $5 \times 15 - 2$

$P(20) = 98$ $20 \times 5 - 2$

$E(Y) = -0.85$ $V(Y) = 67.4275$.

$\sigma \; \sigma \; \sigma$

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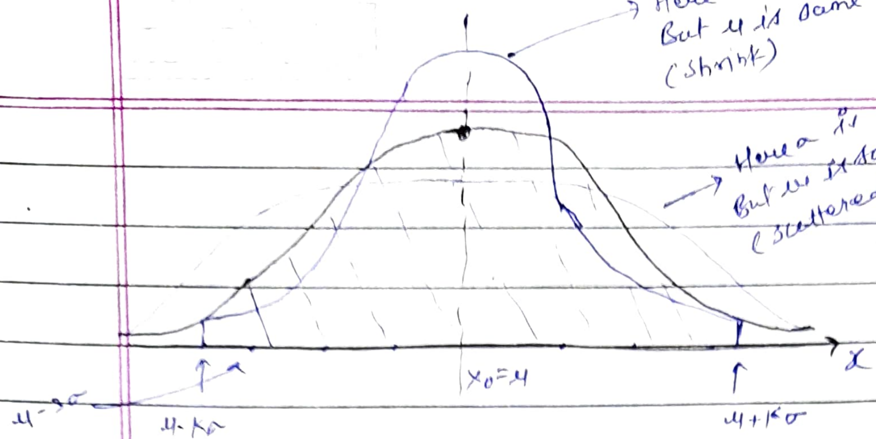

$F(x) = \dfrac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$

$P(\mu - K\sigma < x < \mu + K\sigma)$

⇒ If there is linear transformation then its no change in Normal curve

$$\int_{\mu-K\sigma}^{\mu+K\sigma} \left(\frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2} \right) dx$$

Area under the curve is 1 under $-\infty$ to $+\infty$.

if $z = \dfrac{x-\mu}{\sigma}$

$$\int_{-K}^{K} \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz = \int_0^K \frac{2}{\sqrt{2\pi}} e^{-\frac{1}{2}z^2} dz.$$

if $y = \left(\frac{z}{\sqrt{2}}\right)^2$

$$\Rightarrow \int_0^{K/\sqrt{2}} \frac{2}{\sqrt{\pi}} e^{-y^2} dy$$

For any value of μ and σ the area under the curve is same.

if σ change then shape of curve changes.

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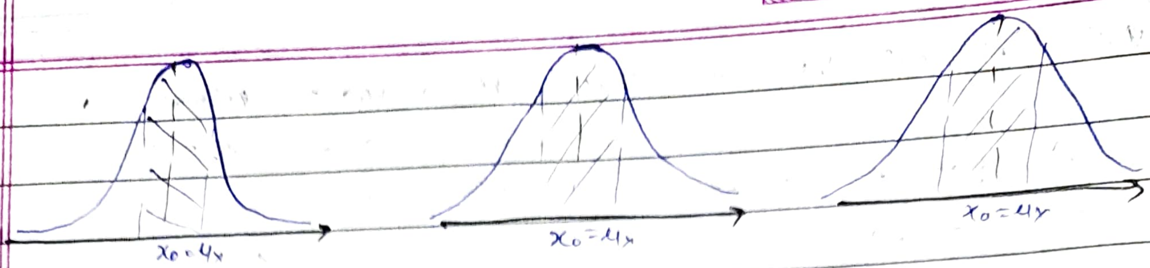
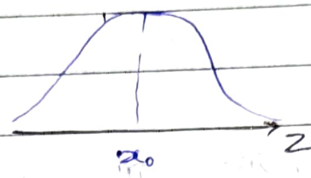


Area under the curve in all these 3 curve will be same

$z = \dfrac{x - \mu_x}{\sigma}$



$\Rightarrow x = \sigma_x z + \mu_x$ — (1)

$\Rightarrow \Sigma x = \Sigma \sigma_x (z) + \Sigma \mu_x$.

$\Rightarrow$ Dividing by n.

$\Rightarrow \dfrac{\Sigma x}{n} = \sigma_x \left(\dfrac{\Sigma z}{n}\right) + \dfrac{\Sigma \mu_x}{n}$.

$\Rightarrow \mu_x = \sigma_x \mu_z + \dfrac{\cancel{n} \mu_x}{\cancel{n}}$ — (11)

from (1) and (11)

$x - \mu_x = \sigma_x (z - \mu_z)$

$(x - \mu_x)^2 = \sigma_x^2 (z - \mu_z)^2$

$\Sigma (x - \mu_x)^2 = \sigma_x^2 \Sigma (z - \mu_z)^2$

$\cancel{\sigma_x^2} = \cancel{\sigma_x^2} \sigma_z^2$

$\sigma_z^2 = 1$.

$\Rightarrow 0.682 = \displaystyle\int_0^{K/\sqrt{2}} \frac{2}{\sqrt{\pi}} e^{-y^2} dy$.

$\mu_x - K\sigma_x$ ...	$\mu_x - \sigma_x$	μ_x	$\mu_x + \sigma_x$	$\mu_x + 2\sigma_x$	... $\mu + K\sigma_x$
$-K$	-1	0	1	2	K

$\Rightarrow$ Area under -2 to 2 then is 0.97.

If $(\mu = 0)$.

$\Rightarrow$ If $K = 3$ then area is 0.997.

Ques: find out the area under the standard normal. $\mu_x = 0$ and $\sigma_x = 1$.

(i) b/w $z = 0$ to 1.65 — $P(z_1) - P(z_0) = 0.95053 - 0.5 = 0.45053$

(ii) $z = -1.36$ to 0 — $P = 0.41$

(iii) $z = -0.65$ to 2.35 — $P = 0.42$

(iv) $z = 1.15$ — 2.56

(v) greater than 2.15

(vi) -0.66 to 0.66

Ques: IQ level measured on 80 students are normally dist, mean = 100 and sd = 16. Determine the IQ b/w 115 and 140

$$z_1 = \frac{x - \mu_x}{\sigma} = \frac{115 - 100}{16} = \frac{15}{16}$$

$$z_1 = 0.94$$

$$z_2 = \frac{140 - 100}{16} = \frac{40}{16} = 2.5$$

$\Rightarrow P(2.5) - P(0.94) = 0.1674$

$\Rightarrow 80 \times 0.1674 = 13.4 \approx 13$

(ii) obtain 90th percentile of IQ.

if Area is 0.90

$$z = \frac{x - \mu_x}{16} \Rightarrow 1.28 = \frac{x - 100}{16}$$

$\Rightarrow x = 1.28 \times 16 + 100 = 120.48.$

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Ques: Data collected on domestic water consumption from houses as Normal dist $\mu = 105$ and s.d $= 69.8$

(I) Determine b/w 50 to 100 l of water.
(II) Determine % of house that consumes less than 200 l.
(III) Determine & interpret the 40% consumption

(I)

$$z_1 = \frac{50-105}{69.8} = \frac{55}{69.8} = -0.78$$

$$z_2 = \frac{100-105}{69.8} = \frac{5}{69.8} = -0.07$$

$$P(z_2) - P(z_1) = 0.25 = 25\%.$$

(II)

$$z = \frac{200-105}{69.8} = \frac{95}{69.8} = 1.36 = P(z) = 91\%$$

$$P(z) = 0.08694$$

(III)

-0.25 for 40% ile.

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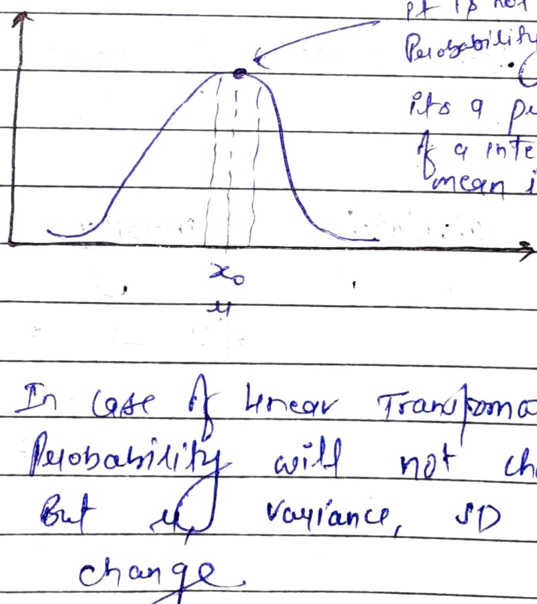
Heart beat Rate

Class Interval. Mid. f

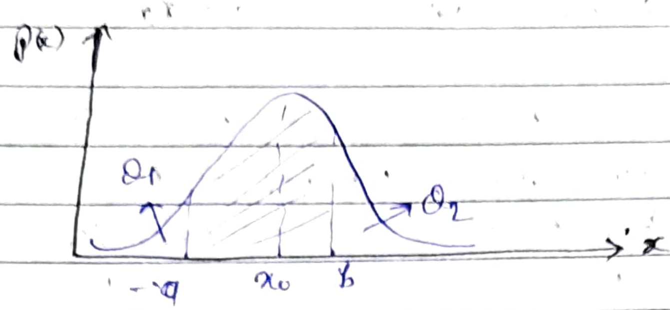
50.5 - 55.5	53	4
55.5 - 60.5	58	14
60.5 - 65.5	63	24
65.5 - 70.5	68	40
70.5 - 75.5	73	54
75.5 - 80.5	78	43
80.5 - 85.5	83	36
85.5 - 90.5	88	20
90.5 - 95.5	93	11
95.5 - 100.5	98	3
100.5 - 105.5	103	1



at μ not the Probability of μ.
its a probability of a interval where mean is present.

In case of Linear Transformation, Probability will not change. But μ, variance, SD will change.

Outlier → which behaves as abnormal.



suppose

find the probability b/w -a to b.

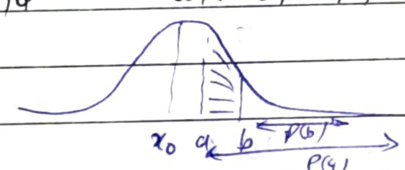
$Q_1 + Q_2 + \text{Area}(-a \text{ to } b) = 1$

it is same as that -a to a

$P(a)$ $\quad \overline{P}(b) + P(x \geq a) = 1$

$$\boxed{P(a \geq x) = 1 - P(b) - P(a)}$$

- If a and b are on the +ve side.



$P(a \leq x \leq b) = P(a) - P(b)$

Mean value of Heart Rate $= 74.8 = 75$
SD is $= 9.98 \approx 10$.

- Calculating Manually the value of Z.

$P(Z \geq z) = \frac{1}{2} e^{(bz + az^2)}$

where $a = -0.416$ and $b = -0.717$

$z = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$, where $c = -\log_2(2P)$

① SP

① Binomial.
⑪ Poission
⑫ Normal/
⑬ Central M, Median, mode

$$z = \frac{-(-0.717) - \sqrt{(.519) - 4\times(.416)\times \log_2(2\times 0.02)}}{2\times(.416)}$$

$z = 2.06$.

⇒ If $z = \dfrac{x-\mu}{\sigma}$ $[\mu = 75$ and $\sigma = 10]$

$x = 95.6$ and

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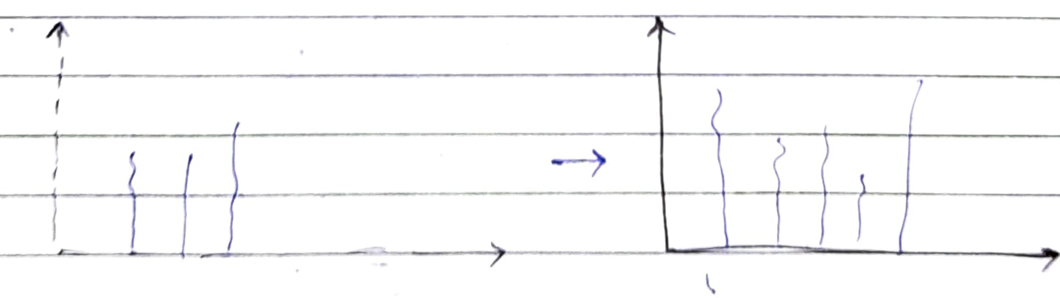
x_1	0	5	10	15	20
$X = X_1 - 1$	-1	4	9	14	19

$Y = 5X_1 - 2$
$= 5X_2 - 3$

In this mean will change, becoz linear transformation variance will not change.

Variance will change only ↑ size of interval, or dec size of interval.

if $c > 0$, var ↑ by value c^2
$c < 0$, variance ↓ by value c^2.



As we increases the interval size then variance also increases

$\mathrm{Var}[x+y] \xrightarrow{\text{variance}} \mathrm{Var}[x+y]$

$\mathrm{Var}[x-y] = \mathrm{Var}[x+y]$

variance

$x_i P_i \quad x_i P_j \quad x_j^2 E_x$

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$2\sum P_i$

X	x_1	x_2	x_3
	P_1	P_2	P_3

X	x_1	x_2	x_3	$x_4 \ldots x_n$
X				
x_1	$P_1 P_1$	$P_1 P_2$	$P_1 P_3$	$P_1 P_4$
x_2	$P_2 P_1$	...		
x_3				
x_4				
x_n				

$X + X$

$\Rightarrow$ $x_i + x_j = P_i \cdot P_j$

$x_i + x_j = P_i \cdot P_j$

Merge this

$$E[x] = \sum x_i P_i$$

$$E[ax+b] = \sum (a x_i + b) \cdot P_i$$

$$= a \sum x_i \cdot P_i + b \sum P_i$$

$\rightarrow$ $$E[x+x] = \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i + x_j) P_i \cdot P_j$$

$x_i P_i \sum P_j = 1$

$$= \sum_{i=1}^{n} [x_i P_i + P_i E x]$$

$\sum P_i = 1$

$$= Ex + Ex$$

$\rightarrow$ $$\text{Variance}[x+x] = E(x+x)^2 - [E(x+x)]^2$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} (x_i + x_j)^2 P_i P_j - [2Ex]^2$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} [x_i^2 P_i P_j + x_j^2 P_i P_j + x_i x_j P_i P_j] - [2Ex]^2$$

$$2Ex^2 + 2[Ex]^2$$

$$= 2Ex^2 + 4[Ex]^2 - 4[Ex]^2 = 2\,\text{Var}[x]$$

$\rightarrow$ $$\text{Variance}[x+y] = \text{Variance}[x] + \text{Variance}[y]$$

after solving same as above.

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If n Trials of same exp

→ 1 Machine we perform 2 exp one after the other
= 2 Machine we perform simultaneously at same time

⇒ Linear Trans = n^2 Var [x]
performing / increasing the trials = n Var (x).
Exp [x - Y] = Exp [x] - Exp [Y]
Variance [x + Y] = Var [x] + Var [Y]

→ x 1 2 3.
deviation 1 0 1 mean = 2
x+x: → {2 .... 6} mean = 4.
E[x] → increase
var [x] → will also increase

x - x {-2, -1, 0, 1, 2} mean = 0
Var (x+x) = will increase [interval ↑]

E [x.Y] and Var [x.Y].

census → each and every individual is to be studied to know the characteristics

sampling → studying a group of people ~~to know the characteristics~~

6 sept 2018

$a_1, a_2, a_3 \ldots a_N$

$x_1, x_2, x_3 \ldots x_N$

statistical characteristics → mean, σ^2, σ, P
(μ) variance, S.D, ↳ proportion

N → No. of individual in total population
n → sample size

Random

Random with Replacement

- → same individual can be repeated / used.
- → Two individual have same characteristics
- N^n → samples

Random samp. without Replacement

- → same individual can't be repeated / used.
- → Two individual can't have same characteristics.
- ${}^N C_n$ → samples

$x \rightarrow \mu_x, \sigma_x^2, \sigma_x, P$

$n \quad (x_1, x_2, x_3 \ldots x_n)$

$\downarrow \quad \downarrow \mu_x \quad \downarrow \mu_x \quad \downarrow \mu_x$

expected value
μ_x

$Var(x_1 = x_2 = x_n) = \sigma_x^2$

Mean of sample $= \dfrac{x_1 + x_2 + x_3 + \ldots x_n}{n}$

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$\bar{x} \rightarrow$ Mean of sample

Mean of diff samples

Sampling Mean $= \bar{x} = \frac{1}{n}\left[x_1 + x_2 + x_3 + \ldots x_n\right]$

$\bar{x}_1 = x$ bar one

$\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots\ldots$

Eg: 1, 2, 3 $\rightarrow$ 2 $\rightarrow$ estimating these two $\rightarrow$ point estimation

samples
1, 2 $\rightarrow$ 1.5
1, 3 $\rightarrow$ 2
2, 3 $\rightarrow$ 2.5

It may or may not be same

Mean of samples $\rightarrow \bar{x}$ is not same as $\bar{X}, X_1$

Statistical Distribution of samples

Sample statistics :- from one Individual.
Sampling statistics :- from all the samples
↳ same as population

⇒ Standard Error of Sampling Mean
$$\frac{1}{M}\sum(\bar{x}-\mu)^2$$

→ If we use interval estimation then probability of μ to be

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Player	Height
A	76
B	78
C	79
D	81
E	86

Mean Height $= \frac{76+78+79+81+86}{5}$

$= 80$

$\Rightarrow$ $^5C_2 = 10$

(A,B) $\frac{76+78}{2} = 77$, $(A,C) = \frac{76+79}{2} = 77.5$ $(A,D) = \frac{76+81}{2} = 79$

$(A,E) = \frac{76+86}{2} = 81$, $(B,C) = \frac{78+79}{2} = 78.5$ $(B,D) = \frac{78+81}{2} = 79.5$

$(C,D) = \frac{79+81}{2} = 80$, $(B,E) = \frac{78+86}{2} = 82$ $(C,E) = \frac{79+86}{2} = 82.5$

$(D,E) = \frac{81+86}{2} = 83.5$

Here $\mu = 80$

Error of Approximation $= \frac{9}{10}$

Probability of Correct Approximation $= \frac{1}{10}$

$\Rightarrow$ Group of 4 → samples.

$A+B+C+D = \frac{76+78+79+81}{4} = 78.5$

$A+B+C+E =$

$= 79.5$ $= 81$

80.25

80.50

$\frac{\sigma^2}{n} = 4$

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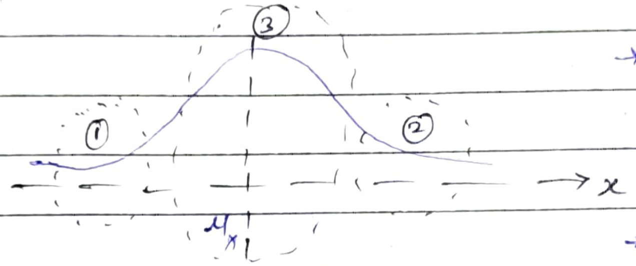
Probability of correct Approximation = 0.

$\Rightarrow$ If we allow 0.5 margin
Probability of correct Approximation = 3/5 [79.5, 80.5, 80.15]

As we increase the size of sample → Approximation correctness increases.

$\rightarrow \bar{x} = \frac{1}{n}[x_1 + x_2 + \dots x_n]$ — ①

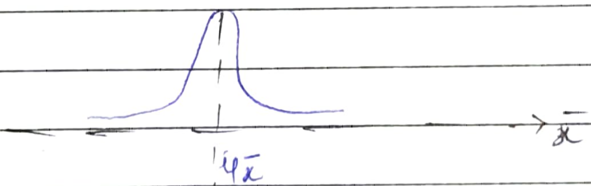
Normal curve



→ if we take ① and ② then the sampling is same

→ if we take only ① then sampling shift toward left and vice versa for ②.

Sampling Normal curve



From eqn ① $E[\bar{x}] = \frac{1}{n}[\mu_1 + \mu + \mu \dots \mu] = \frac{n(\mu)}{n} =$

$= \mu.$

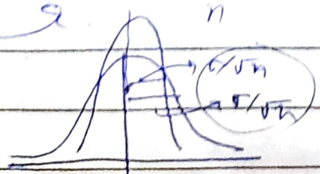
$Var[\bar{x}] = \frac{1}{n^2}[\sigma^2 + \sigma^2 + \sigma^2 + \dots \sigma^2]$

$= \sigma^2/n$ → sampling size

$SD = \sigma/\sqrt{n}$

sample size < 10%

$= \frac{\sigma^2}{n} \times \frac{N-n}{N-1}$ [It is valid only for small value of n]



$\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}$

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For standardization of x $\quad z = \frac{x-\mu}{\sigma}$, $E(z)=0$, $Var=1$

For standard of $\bar{x}$ $\Rightarrow z = \left(\frac{\bar{x}-\mu}{\sigma^2}\right)\sqrt{n}$, $E(z)=0$, $Var=1$.

Standard Deviation $\bar{x} \pm \bar{y} = (\sigma)\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$

Statistic	Standard Error
Sample Mean $\bar{x}$	$\sigma/\sqrt{n}$
Sample SD, s	$\sqrt{\sigma^2/2n}$
Sample Variance, s^2	$\sigma^2\sqrt{2/n}$
Sample proportion, p	$\sqrt{PQ/n}$
Sample Correlation, r	$(1-\rho^2)/\sqrt{n}$
Mean $(\bar{X}_1-\bar{X}_2)$	$\sqrt{\frac{\sigma_1^2}{n_1}+\frac{\sigma_2^2}{n_2}}$

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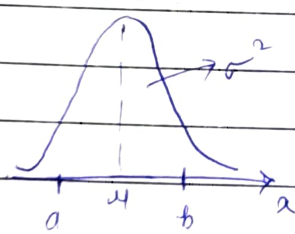
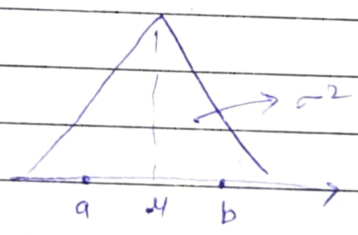
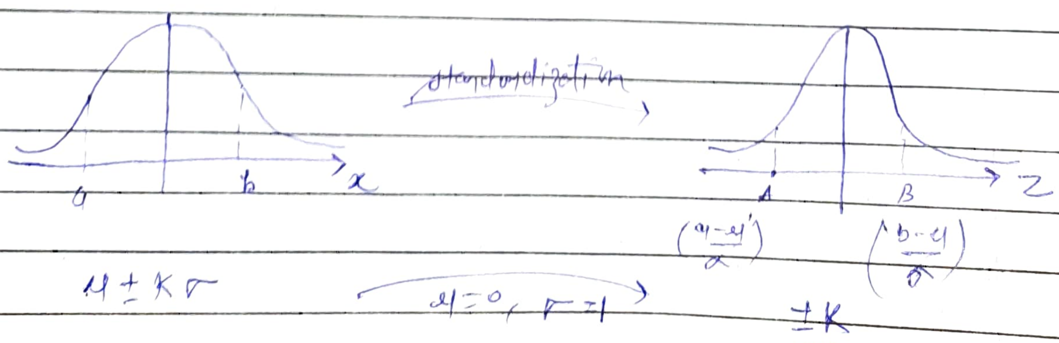




These are not symmetric curve and not standard Normal curve
For these two we can't use z-table.

→ Only for standard curve we can find the area and apply z-table.



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→ For any single point in curve, the probability is zero.

Central Limit Theorem

→ For any non-normal curve, the sampling distribution always be normal distribution curve

iff $n \geq 30$ then only normal distribution even if the curve is not normal.

Sample size	No. of possible sample	No. of samples corr. estimated with ±1	% of correctly estimated
1	5	2	40%
2	10	3	30%
3	10	5	50%
4	5	4	80%
5	1	1	100%

sample size ↑ → sampling error ↓

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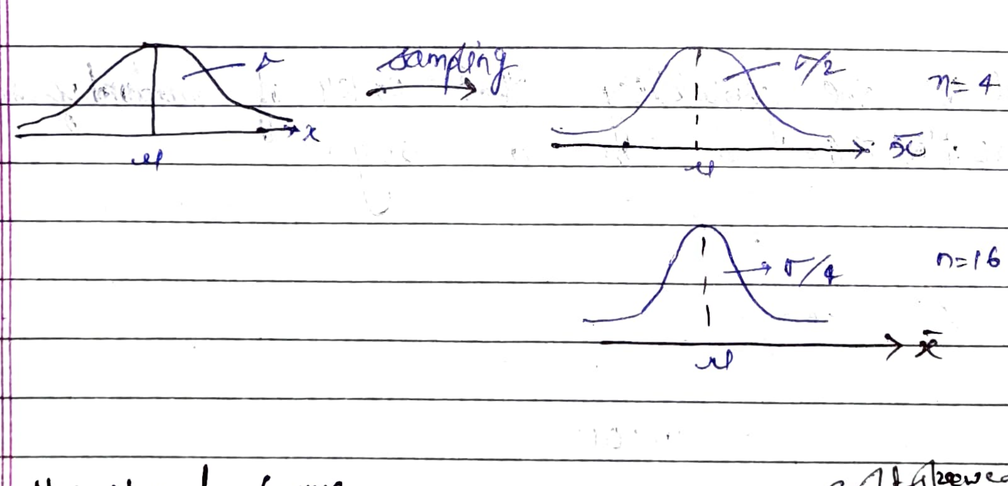
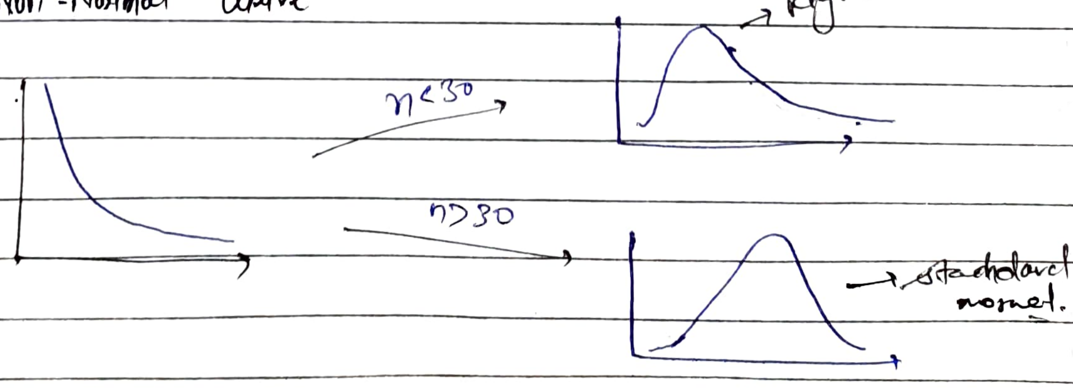


Non-Normal Curve



Mean of sq. − sq. of Mean

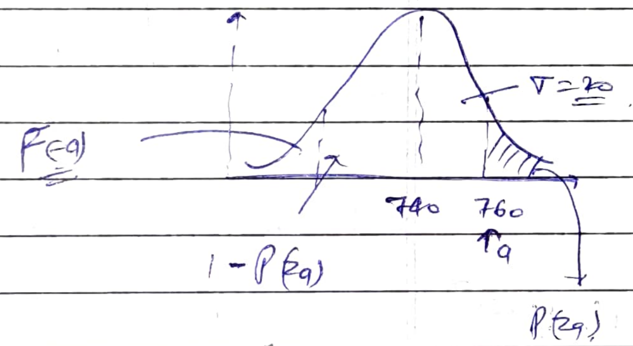
Central Limit Theorem :-
If we have original population

Ques : The no. of calories in an order of pizza in a certain restaurent approximately normally distributed with a mean of 740 cal. SD = 20

(A) What is the probability that a random order have at least 760 calories.

$z = \frac{x - \mu}{\sigma}$, $z = \frac{760 - 740}{20} = \frac{20}{20} = 1$

$P(z=1) = 0.1586$

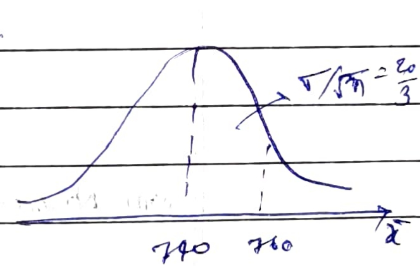


(B) What is the probability that order of randomly selected 9 pizza have at least 760 cal on an avg.

$S.D = \sigma/\sqrt{n} = 20/3$. $\{n = 9\}$

$Z = \frac{760 - 740}{20/3} = \frac{20 \times 3}{20} = 3$.

$P(z \geq 3) = 0.00135$



Sampling Error

scattered ↑ → sampling error ↑.

$$\bar{x} = \frac{1}{n}\left[x_1 + x_2 + x_3 \cdots x_n\right]$$ → Gaussian values

independent, identically

- sum of gaussian variable will be gaussian
- sum of non-Gaussian variable will be gaussian iff $n > 30$

$$s'^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 \quad \text{—(I)}, \qquad s^2 = \frac{1}{n}\sum (x_i - \bar{x})^2 \quad \text{—(II)}$$

$$E(s'^2) = \sigma^2 \qquad\qquad E(s^2) \neq \sigma^2$$

Expected value of s^2

$$\Rightarrow E\, s^2 = \frac{1}{n}\sum E\left(x_i^2 - 2x_i\bar{x} + (\bar{x})^2\right)$$

$$\Rightarrow \cancel{s^2 = \frac{1}{n}\sum\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)}$$

$$\cancel{s^2 = \frac{1}{n}\left[\sum x_i^2 - 2\bar{x}\sum x_i + \sum \bar{x}^2\right]}$$

$$E\, s^2 = \frac{1}{n}\sum E\left(x_i^2 - 2x_i\bar{x} + (\bar{x})^2 + \mu^2 - \mu^2 - 2x_i\mu + 2x_i\mu\right)$$

$$E\, s^2 = \frac{1}{n}\sum E\left(x_i^2 + \mu^2 - 2x_i\mu - 2x_i\bar{x} + \bar{x}^2 - \mu^2 + 2x_i\mu\right)$$

$$E\, s^2 = \frac{1}{n}\sum E\left((x_i - \mu)^2 - 2x_i\bar{x} + \bar{x}^2 - \mu^2 + 2x_i\mu\right)$$

$$E\, s^2 = \frac{1}{n}\sum E(x_i - \mu)^2 + \frac{1}{n}E\left[n\bar{x}^2 - n\mu^2 - 2\bar{x}^2(n) + 2\mu\bar{x}(n)\right]$$

$$\rightarrow E\, s^2 = \frac{1}{n}E\sum (x_i - \mu)^2 + \bar{x}^2 - \mu^2 - 2\bar{x}^2 + 2\mu\bar{x}$$

$$\frac{s'^2}{s^2} = \frac{n}{n-1} \Rightarrow \boxed{\frac{s'^2}{s^2} = \frac{n}{n-1}}$$

$$E(s'^2) = \frac{n}{n-1} E(s^2)$$

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$$s^2 = \frac{1}{n}\sum E(x_i-\mu)^2 - E(\bar{x}-\mu)^2$$

Variance ⟷ Mean of sq. of Deviation — Variance of $\bar{x}$ sample

$$s^2 = \sigma^2 - \frac{\sigma^2}{n}$$

$$E s^2 = \sigma^2\left(1-\frac{1}{n}\right) \Rightarrow \sigma^2\left(\frac{n-1}{n}\right) = Es^2$$

⇒

Another way to solve

$$s^2 = \frac{1}{n} E[x_i - \bar{x}]^2$$

$$s^2 = \cancel{\frac{1}{n}E}\; Ex_i^2 - [\bar{x}]^2$$

$$= E[x_i-\mu]^2 - (\bar{x}-\mu)^2$$

If we shift the x_i by μ and $\bar{x}$ by μ there will be no change

Q. A coin is tossed 200 times find the approximate probability that the no. of heads obtain is 80 to 120

Sol^n: $P(80 < x < 120) = 0.9952$

Mean = np = 200×0.5 = 100

$\sigma = \sqrt{npq} = \sqrt{200 \times 0.5 \times 0.5} = \sqrt{50}$.

$$\frac{x-\mu}{\sigma}$$

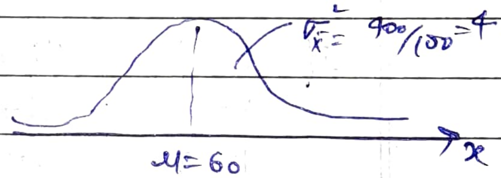
$$P(z \geq 80) = \frac{80-100}{\sqrt{50}} = \frac{-20}{\sqrt{50}}$$

$$P(z \leq 120) = \frac{120-100}{\sqrt{50}} = \frac{20}{\sqrt{50}}$$

Q. A Random sample of size 100 is taken from a population whose mean is 60 and variance is 400. By Central Limit theorem, with what prob. we claim that mean of sample not differ from μ by more than 4.

Soln :

$$\sigma_{\bar{x}}^2 = \frac{400}{100} = 4.$$



$$\sigma_{\bar{x}} = \sqrt{4} = 2.$$

$$|\bar{x} - \mu| \leq 4.$$

$\Rightarrow$ if $\mu = 60$ $\quad P(54 \leq \bar{x} \leq 64)$

$$-4 \leq \bar{x} - \mu \leq +4$$

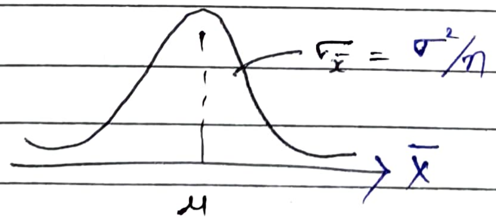
$$P \Rightarrow z_1 = \frac{54-60}{2} = -2.$$

$$z_2 = \frac{64-60}{2} = +2$$

$\mu \pm 4 = 0.95.$

$\sigma = 2.$

$$P(-2 \leq z \leq +2) = 0.95.$$



$\mu \pm$ marginal value

$\mu \pm K\sigma_{\bar{x}}$

Sample Mean $\bar{x} = \mu$.
then it is exat Representation of population

90, 95, .98, .99 → Area

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Page ______

⇒ If the population is normal and variance is given.
interval = $\mu \pm k\, \sigma/\sqrt{n}$

If the population is non-normal and variance is given
if $(n > 30)$
interval = $\mu \pm k\,(\sigma/\sqrt{n})$

If the population and variance is not given
$s = \frac{1}{n-1} \Sigma (x_i - \bar{x})^2$ [n-1].
interval = $\mu \pm k\,(s/\sqrt{n})$

If $k = 1$	then	confidence %	= 68 %
$k = 2$	then	confidence %	= 95 %
$k = 3$	then	confidence %	= 99.7 %
$k \to \infty$	then	confidence %	= 100 %