Statiscal Methods

The statiscal methods are used for decision making or expecting the knowledge where uncertainity is involved.

Experiment
- Deterministic → output we ~~don~~ known
- Non-deterministic (Random) → output is not known

1) uncertainity involve in Random experiment.
2) Natural phenomenon is Random

3) Wind Power Generation

$$P = \eta_1 \times \eta_2 \times \eta_3 \times \rho \times v^3$$

η_1 → Efficienty of wind turbine
η_2 → efficiency mechanical to transmission power
η_3 → " transmission to electric

$$P \propto \rho \times v^3$$

Eg →
at time v_1 power P_1
Same v_1 power may not same as P_1

ρ → density
v = speed wind

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Following are the steps which are involved in statiscal method

1)- Study of methods to collect & organize numeric data related to phenomenon or exp.
2) Analyse the numeric information.
3) Interpretate the numeric information

Individual → people or object included in the study
Variable → Characteristics of individual
data →

for example
coin → face → variable
head Tail → data

Dice → face - variable
1,2,3,4,5,6 data

Wind Turbine

Individual → Wind Turbine
Variable → Wind Speed, Power Generated
Data - Value of variable

Types of Variable

Qualitative Variable → There is no ordering of data of variable.
Quantitative Variable →

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Ω Sample Space

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Quantitative - Variable

Discrete — data may be listed

Continuous — values lies b/w interval can't be counted or listed

Random Variable - → Real valued function defined of sample space of random whose value depends on chance

Random Variable can be defined on sample space of Random experiment

X = no of heads Random variable on tossing a pair of coin

X = 0, 1, 2 (Instances / Events)

For each instances probability includes

Eg. 2 Tossing a coin

X = H is not a random variable

X = no of head is random variable

Eg-3. Throwing a pair of dice

X = absolute difference of two numbers

S = 36 ordered pair

X = 0, 1, 2, 3, 4, 5

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(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1)
(3,1)
(4,1)
(5,1)
(6,1)

$0 \Rightarrow \quad x=0 \Rightarrow 1/6$

$x=1 \Rightarrow (1,2),(2,1),(2,3),(3,2),(3,4),(4,3),(4,5)(5,4)$
$(5,6)(6,5)$

$x=1 \quad \frac{10}{36}$

$x=2 \quad (1,3)(2,4)(3,5)(4,6)$

$x=2 \quad \frac{8}{36}$

$x=3 \quad (1,4)(2,5)(3,6)$

$x=3 \quad \frac{6}{36}$

$x=4 \quad (1,5)(2,6)$

$\frac{4}{36}$

$x=5 \quad \frac{2}{36} \quad (1,6)$

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Event. Event is also subset of sample space

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Discrete Random Variables — possible instances can be counted.

A probability function defined on X are called instances.

Sum of probability of instances is 1.

For each element of sample space should be output and unique.

Probability Density Function

1) Mutually Exclusive Set
no common elements

$A \& B = \phi$ A and $B = \phi$

2) Collectively Exhaustive — When it covers all element of sample space

describes every possibilities

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average → exact value
mean → expected value

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Probabilities distribution – is a list of outcomes and their associated probability

Probability Density Function (PDF)

A function that represents probility distribution of discrete random variable

$$\boxed{\begin{array}{l} P(x_i) > 0 \\ \sum P(x_i) = 1 \end{array}}$$

Ways to represent probability distribution

1) Histogram

Variance

avg deviation from expected value

$$V(X) = \sum (x_i - \mu)^2 \times P(x_i)$$

$$V(X) = \sum [x_i]^2 \times P(x_i) - \mu^2$$

$$= E(X^2) - [E(X)]^2$$

Mean of Squares – Squares of mean

Assignment

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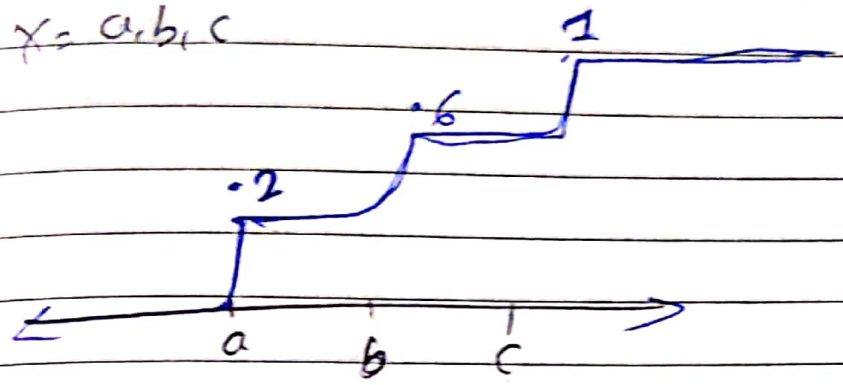
Standard deviation

$$\sigma = \sqrt{V(x)} = \sum (x_i - \mu)^2 P(x_i)$$

$$= \sum x_i^2 P(x_i) - \left(\sum x_i P(x_i)\right)^2$$

Distribution Function of Discrete Random Variable

$X = a, b, c$



Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Note:

1) Probability and Prior Probability are unconditional.

2) Prior Probability - some observation involved.
History

3) Likely hood Probability → conditional
↳ observation is easier.

Eg → Toss winning captain will elected bat first
E2 → Stadium is wankhede.

4) Postier Probability conditional but observation is not easy.

Eg → E_1 → Probability of Stadium is wankhede
E_2 → Toss winning captain will elect bat first

Eg It is given that in a population 1 in 50000 has meningities and 1 in 20 has stiff neck. Have to compute no of meningities patient having stiff neck and no of stiff neck patient having meningities.

$P(A) \rightarrow 1/50000$ → likely hood

$P(B) \rightarrow 1/20$ → postier

$P(B|A)$ → likely hood

If likely hood probability ⇒ 1/2

If one is likelyhood →

$$\boxed{P(A|B) = P(B|A) \times \frac{P(A)}{P(B)}}$$

If both A & B are postier probability then we can't use the above formula.

Exm

$P(B|A) = 1/2$

$\frac{1}{2} \times \frac{1}{5000} \times 2 = P(B|A)$

$\boxed{P(B|A) = 1/5000}$ Ans

$\frac{1}{2} \times \frac{1}{5000} \times 2 = \boxed{1/5000}$ Ans//

Eg - $P(A|B) \rightarrow$ postier probability

$B \rightarrow$ prior

$A \rightarrow$ is divided k mutually exclusive exhaustive

Total Probability Theorem

$$\left[P(B) = \sum_{i=1}^{k} P(B/A_i) \cdot P(A_i) \right]$$

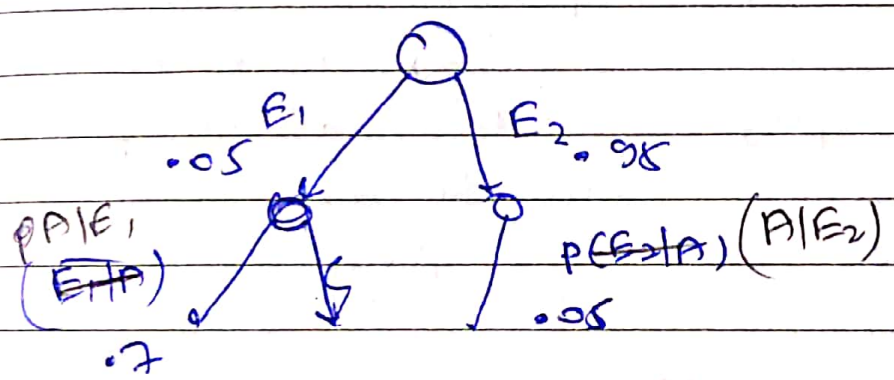
Eg — Prob. of a person choosen at random having a disease is 0.05. The choosen person is administrated a diagonatic test which is not perfect. The probability that the diagonistic test gives a positive result when a disease is prusent 0.7 and negative result when disease is not prusent is 0.95 find out probility of a person having disease, when diagonistic test gives a positive result

$P(D) = .05$

$P(\bar{D}) = .95$

$P(+/D) = .7$

$P(-/\bar{D}) = .95$



$\cancel{P(A/E_1) = P(E_1/A) \times P(E}$

$$P(E_1/A) = \frac{P(E_1) \times P(A/E_1)}{P(E_1) * P(A/E_1) + P(E_2)\ P(A/E_2)}$$

$$= \frac{.05 \times .7}{.05 \times .7 + .95 \times .05} \qquad \frac{.035}{.0525}$$

$$\boxed{P(E_1/A) = .424}$$

Ans

Posterior likelihood

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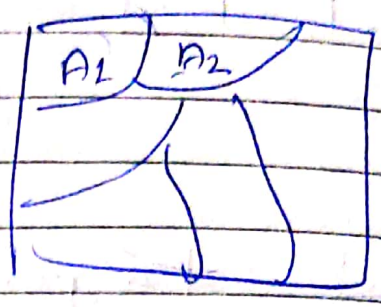
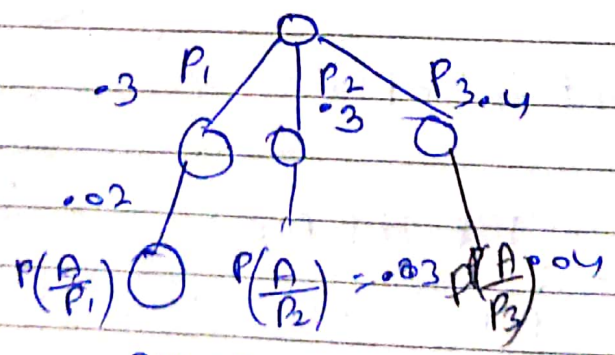
Probability of boss winning exp team select the best first

$P\left(\frac{A_1}{Total}\right) \rightarrow$ Posterior

$P\left(\frac{E}{A_1}\right) =$ Likelihood



Q- A firm producing Spark plugs have 3 plants in city the production of all three plants is store in a single centrally located warehouse without any distinction among the products out of the total production 30% comes from P_1 30% plant P_2 and remaing Plant 3. The past production record indicate that the defective rate of P_1 P_2 P_3 are 2%, 3%, 4% respectively. Prior to any shipment of box of 100 spark plugs to a dealer a spark plugs selected at random from the box. was found to be defective what is the probability produced by P_2?



$$P\left(\frac{P_2}{A}\right) = \frac{.3 \times .03}{.009 + .006 + .06} = \frac{.09}{.031} \Rightarrow \left(9/31\right) = \boxed{.29}$$

Ans

$[P(x_i/c) \rightarrow$ positive always$]$

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$$x_1\ x_2\ x_3 \cdots x_n \xrightarrow{F(x)} y_1\ y_2\ y_3 \cdots y_n$$

$$y_i = F(x)$$

$$x^* = \max[x_1, x_2 \cdots x_n]$$

$$y^* = \max[F(x_1)\ F(x_2)\ F(x_3) \cdots]$$

$f \rightarrow$ Probability

Value of x for which y is maximum.

$$x^* = \underset{\text{over all value of } x}{\operatorname{argmax}} [f(x_1)\ F(x_2) \cdots F(x_n)]$$

argmax will return corresponding x value of max $F(x_i)$ or Y_i.

Q2 [which value of x output is maximum $F(x)$].

$$x_1/c,\ x_2/c,\ x_3/c \cdots x_n/c \longrightarrow y_1\ y_2\ y_3 \cdots y_n$$

$y_i = F(x_i)$ $\quad f \rightarrow$ Probability

$$x^*/c = \underset{y_i}{\operatorname{argmax}} [F(x_1/c)\ F(x_2/c) \cdots F(x_n/c)]$$

$$x^*/c = \underset{y_i}{\operatorname{argmax}} [\cdot\ p(x_i/c)] \quad \text{for all values of } x$$

$$= \operatorname{argmax} \left[\frac{P(c/x_i) * (P x_i)}{P(c)} \right]$$

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Postier Pro-bability $P(x_i/c)$ Suppose c has two part but in two part quite difficult to solve

$$\left[x^*/c = \operatorname{argmax}\left[P(c/x_i) \times P(x_i)\right]\right] \text{ Ans}$$

~~BION~~ BINOMIAL Distribution

Two events Success Failure

out of n ~~successes~~ trials (n+1) successes are possible

Instance (0, 1, 2, ---, n+1)

BD used to described the probability distribution of the no of successes of n independent trial of same experiment

$$\boxed{P(X=x) = {}^nC_x (p)^x (1-p)^{n-x}}$$

mean = np

variance = npq

Poisson $\dfrac{e^{-\lambda}\lambda^x}{x!}$

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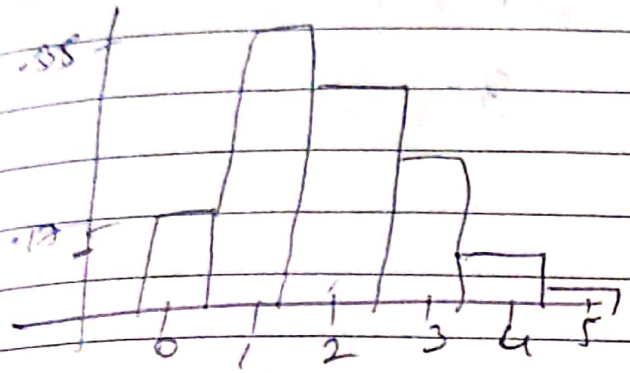
Histogram is way to represent Probability distribution / Function (prob) distribution function

$f_0(p) = (1-p)^5$



if cummulative sum of probability covers (0-5) then that instance is median.

for median we require cummulative sum distribution function

mean and mode we require probability distribution function

Q1

$S = \{BB, BG, GB, GG\}$

$E = \{BB, B, G\}$ $\Rightarrow {}^nC_r (p)^r (q)^{n-r}$

$F = \{BB\}$ $\quad {}^3C_2 \left(\frac{1}{3}\right)$

$$\boxed{P(F/E) = \frac{1}{3}}$$

Q-2 Out of 800 families with 4 children each how many families will be expected to have

i) 2 G

${}^4C_2 \cdot {}^4C_2$

$800\left({}^4C_2 \times \left(\frac{1}{2}\right)^2 \left(\frac{1}{2}\right)^2\right) \Rightarrow 2 \cancel{\frac{4\times3}{2}} \Rightarrow \frac{6\times}{16} = 3/8 \times 800 = 300$

(ii) atleast 1 boy

$800\left(1 - {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4\right) = 800\left(1 - \frac{1}{16}\right) = 50 \times 15$

$= \boxed{750}$

(iii) at most 2 girls

$\left({}^4C_0 \left(\frac{1}{2}\right)^4 + {}^4C_1 \left(\frac{1}{2}\right)^4 + {}^4C_2 \left(\frac{1}{2}\right)^4\right) 800$

$= \frac{800}{16}\left[1 + 4 + 6\right]$

$= 50 \times 11 = \boxed{550}$

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$$\frac{P(X=2)}{P(X=1)} = \frac{2C_2\left(\frac{1}{2}\right)^2}{2C_1\left(\frac{1}{2}\right)^2 + 2C_2\left(\frac{1}{2}\right)^2}$$

$$= \frac{2C_2}{2C_1+2C_2} = \frac{1}{2+1} = \boxed{\frac{1}{3}}$$

Binomial distribution

Q: In a particular university 40% of student are found to be smoking. 6 university students are selected find their views on smoking

(i) Find probability?

(i) None of those selected smoke?

(ii) All those selected smoke

(iii) atleast half of the 6 smoke?

$p = 4/10 \quad q = 6/10$

$n = 6$

(i) ${}^6C_0(4/10)^0 (6/10)^6 \quad (X=0) \Rightarrow (.6)^6$

(ii) $(X=6) \Rightarrow {}^6C_6(.4)^6(.6)^0 \Rightarrow$

(iii) $(X \geq 3)$ ~~+~~

${}^6C_3(.4)^3(.6)^3 + {}^6C_4(.4)^4(.6)^3$

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Q- In a family of three children what is the probability that there will be exactly 2 girls.

$(X=2) = {}^3C_2\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^1$

$= \frac{3\times\cancel{2!}}{\cancel{2!}} \times \frac{1}{8} = (3/8)$ Ans

Q- a multiple choice consist of 4 questions each question has 5 possible answers of which only 1 is correct. a student guess on every question? Find probability distribution of X no of question she answers correctly.

$p = 1/5 \quad q = 4/5$

	Probability
$X=0$	${}^4C_0\,(1/5)^0\,(4/5)^4$
$X=1$	${}^4C_1\,(1/5)^1\,(4/5)^4$
$X=2$	
$X=3$	
$X=4$	

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Q. In one city 23% of the population is under 25 year age. 3 people are selected at random from the city find probability distribution of X where X is number among the 3 that are under 25 years of age

$P = .23 \quad n = 3 \quad X = 0 - - - 3$

Binomial Distribution Table

~~d~~ binomial distribution function given then find

$P(X=a) = F(a) - F(a-1)$

$P(X<a) = F(a-1)$

$P(X>a) = 1 - F(a)$

$P(a_1 \leq x \leq a_2) = \cancel{F(a_2) - F(a_1 - 1)}$

$P(a_1 \leq x < a_2) \quad \boxed{F(a_2-1) - F(a_1-1)}$ Ans

Q. $P(0) + P(1) - - - \underset{P(a_1-1)}{\downarrow} \boxed{P(a_1) - -} P(a_2) - - P(n)$

$F(a_2-1) - F(a_1-1)$

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Q A large lot of apples is expected to contain 20% of defectives. A Retailer before excepting a lot select at random sample of 12 apples and he will purchase a lot if it contains not more than 2 defective apples. Determine the probability that the retailer will accept the lot.

~~P = 1.2~~ $p = .2$ $n = 12$ $X = 2$

$P(X \leq 2) = \cancel{{}^{12}C_2 (.2)^2 (.8)^{10}}$ $\boxed{.558}$

$=$

$P(2) = P(X=0) + P(X=1) + P(X=2)$

$P(X)$ and $P(X+1)$

$P(X+1) = {}^nC_{X+1} (p)^{x+1} (q)^{n-x-1}$

$P(X) = {}^nC_X (p)^x (q)^{n-x}$

$$\frac{P(x+1)}{P(x)} = \frac{{}^nC_{X+1}}{{}^nC_X} \frac{p}{q}$$

$$= \frac{X!(n-x)!}{(X+1)!(n-X-1)!} \frac{p}{q}$$

$$= \frac{(n-X)}{X+1} \frac{p}{q}$$

$$\boxed{\frac{P(x+1)}{P(x)} = \frac{(n-x)}{(x+1)} \frac{p}{q}}$$

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Assignment

$\boxed{\text{Mean} = np}$ $\boxed{\text{Variance} = npq}$

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$$\text{Mean} = \sum_{i=0}^{n} x_i \, p(x_i)$$

$$= \sum_{i=0}^{n} x_i \, {}^nC_{x_i} (p)^{x_i} (1-p)^{n-x_i}$$

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Suppose that 60% of male members in certain age group are smokers and that sample of 25 male members selected at random what is expected no of smokers among selected.

1) What is expected no of smokers $\boxed{np}$ = mean np
2) What is most likely no of smokers → mode
3) What is standard deviation of no of smokers in group of 25 selected assuming binomial distribution is approximated ~~in~~

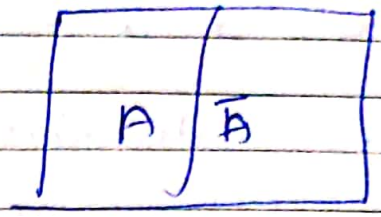
→ find from cummulative ~~pro~~ binomial table and find instance with maximum probability

3) $\sqrt{npq} = \sqrt{25 \times .6 \times .4} = \sqrt{.6} \Rightarrow \sqrt{6} = 2.54$

discrete Random Variable
Binomial Random Variable
Poission Random Variable

4C2

No of element in $\bar{A}$ should be defined.



$$P(A) = \frac{n(A)}{n(A)+n(\bar{A})}$$

$n=3$

$P(X=0) = 1/8$

$P(X=1) = 3/8$

$P(X=2) = 3/8$

$P(X=3) = 1/8$

$n=4$

$P(X=0) = 1/16$

$P(X=1) = 4/16$

$P(X=2) = 6/16$

$P(X=3) = 4/16$

$P(X=4) = 1/16$

(probability of success ~~on~~ increasing n decreased)

(n = success + failure)
freq freq

Q- In a hospital 3 cases are ~~admitted~~ reported find of accidental case. then there should be given types of case.

<u>Isolated Events</u> →

relevant to time and space and isolated events of success can't be calculate

when n is not defined

Poission distribution applied when we have isolated events and no of trials are not defined

Poission Distribution -

$$P(X=x) = \frac{e^{-\lambda} \times \lambda^{x}}{x!}$$

where $\lambda \rightarrow$ mean

Eg - Defects in Yarn manufactured by — — can be approximated by poission distribution with mean ~~1.2~~ 1.2 defects per meter 6 if length of 6 meter are to be inspected find the probability of less than 2 defects.

$P(x=0) + P(x=1)$ $\lambda = 1.2$

$$\Rightarrow \frac{e^{-1.2} \times (1.2)^{0}}{0!} + \frac{e^{-1.2} \times (1.2)^{1}}{1!}$$

$$\Rightarrow e^{-1.2} + 1.2\, e^{-1.2}$$

$$= 2.2\, e^{-1.2}$$

$$= \boxed{0.6626} \text{ Ans}$$

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Eg- If no of telephone calls that operator receives in an half an hour receives follows poission poissin distri:
with an avg of 3 calls

i) what is the probability the operator will be idle
ii) less than 4 calls
iii) most likely number of calls operator will recive

$\lambda = 3$

(i) $P(x=0) = \frac{e^{-3}(3)^0}{0!} \Rightarrow e^{-3} \Rightarrow \boxed{.0498}$ Ans

(ii) $P(x=0) + P(x=1) + P(x=2) + P(x=3)$

$\Rightarrow e^{-3}\left[1 + 3 + (3)^2\right]$

$\boxed{.6472}$ Ans

(iii) most likely $P(2)$, $P(3)$

Formula -

Mean $\rightarrow \lambda$
Variance $= \sqrt{\lambda}$

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$$P(X=x) = \frac{e^{-\lambda}\lambda^{x}}{x!}$$

$$P(x+1) = \frac{e^{-\lambda}\lambda^{x+1}}{(x+1)!}$$

$$\frac{P(x+1)}{P(x)} = \frac{\lambda \quad \cancel{x!}}{(x+1)\,(\cancel{x!})}$$

$$\boxed{P(x+1) = \frac{P(x)\times\lambda}{(x+1)}}$$

Binomial to Poission

if n is large and np (mean) is finite

Eg Trucks arrived at a dockyard at a mean rate of 2 per hours using poission approximation determine if 30min

In 30 min approx 1 truck arrives

$\boxed{\lambda = 1}$

(i) No truck arrived

(ii) 2 trucks arrives

Soln

(i) $P(x=0) = \frac{e^{-1}(1)^{0}}{0} \Rightarrow$ $F(0) = \boxed{0.3670}$

(ii) $P(x=2) = \frac{e^{-1}(1)^{2}}{2!} =$ $F(2)-F(1)$

Q- Probability that call received by certain switch board will be a wrong number is 0.01, use the poission approximation to the binomial distribution to find P that

(i) among 120 calls received by switch board there are no wrong no.

$n=120 \quad p=0.01 \qquad \lambda = np = 120 \times .01 = \boxed{1.2}$

$$P(X=0) = \frac{e^{-1.2}(1.2)^0}{0} = e^{-1.2}$$

$$= \boxed{0.3012} \text{ Ans,}$$

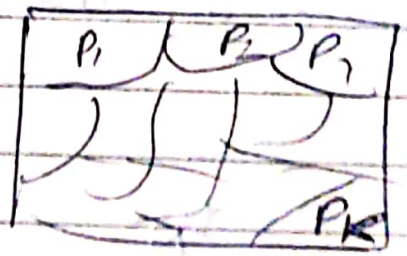
Q- The probability that

Multinomial Distribution

in binomial

$$P(X=x) = {}^nC_x (P)^x (1-P)^{n-x}$$

$$= \frac{n!}{x!(n-x)!} (P)^x (1-P)^{n-x}$$



k events

n trials

in multinomial

$$\left[P(X=x) = \frac{n!}{x_1!\,x_2!\cdots x_k!} \times (P_1)^{x_1} (P_2)^{x_2} \cdots (P_k)^{x_3} \right] \text{ Ans}$$

mean of multinomial distribution

Q. A production manager finds that 80% of the days output is of good quality, 15% would require some working and balance 5% needs to be destroyed. In a sample lot of 10 unit what is the probability of obtaining 9 good units with 1 requiring reworking and none must be destroyed.

$n = 10$

$$P(X) = \frac{n!}{x_1!\, x_2!\, x_3!} (P_1)^{x_1} (P_2)^{x_3} (P_3)^{x_3}$$

$$= \frac{10!}{9!\, 1!\, 0!} (.8)^9 (.15)^1 (.05)^0$$

$$= 10 \times (.8)^9 \times .15$$

$$= \boxed{0.2013}$$

Uniformaly distributed ~~mult~~ probability

$X \to 1, 2, 3 --- N$

$P(X = x) = 1/N$

$$\text{mean} = \sum x P(x)$$

$$= \frac{1}{N} \sum x$$

$$= \frac{1}{N} \times \frac{N(N+1)}{2}$$

$$= \left(\frac{N+1}{2}\right) \text{Ans}$$

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mean = $\frac{n+1}{2}$

Variance = $\frac{n^2-1}{12}$]

Variance →

$$V = \sum x^2 P(x) - \left(\sum x P(x)\right)^2$$

$$= \frac{1}{N} \sum x^2 - \left(\frac{(n+1)}{2}\right)^2$$

$$= \frac{1}{N} \times \frac{N}{6}(n+1)(2n+1) - \left(\frac{n+1}{2}\right)^2$$

$\boxed{\frac{n^2-1}{12}}$ ⇒ Variance, $= \left(\frac{n+1}{2}\right)\left(\frac{2n+1}{3} - \frac{n+1}{2}\right)$

$$\frac{n+1}{2}\left(\frac{4n+2-3n-3}{6}\right)$$

$$\frac{(n+1)(n-1)}{12} \Rightarrow \boxed{\frac{n^2-1}{12}} \text{ Ans}$$

Q- Determine expected value & expected value of x
x is defined as outcome of experiment

In rolling of 6 sided dice exper

determine expected value and Variance of Random value X, outcome of experiment

$P(X) = 1/6$

mean = $\left(\frac{6+1}{2}\right) = 3.5$ = Ans

Variance = $\frac{n^2-1}{12} = \frac{35}{12}$ = Ans.

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Continuous Random Variable

$$\left(\frac{kx^3}{3}\right)_0^{100} = 1$$

$$\frac{k \times 1000000}{3} = 1$$

$$k = \frac{3 \times 10^{-6}}{1}$$

$$\boxed{k = 3 \times 10^{-6}}$$

$$P = \int_0^{50} kx^2 \, dx$$

$$= 3 \times 10^{-6} \times \left(\frac{x^3}{3}\right)_0^{50}$$

$$= 3 \times 10^{-6} \times \frac{125 \times 10^{3}}{3}$$

$$= 125 \times 10^{-3}$$

$$= \boxed{.125} \text{ Ans}$$

mean $\int x P(x)$

$$\int_0^{50} kx^3 \Rightarrow 3 \times 10^{-6} \times \left(\frac{x^4}{4}\right)_0^{50}$$

$$= 3 \times 10^{-6} \times \frac{25 \times 25 \times 10^{4}}{4}$$

$$= \frac{625 \times 3}{4} \times 10^{-3}$$

$$= \frac{1875}{4} = \boxed{4.6875}$$

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Variance ⇒

$$V = \int_0^{100} x^2 P(x) - (xP(x))^2$$

$$\int_0^{100} kx^3 \Rightarrow \left[\frac{kx^4}{4}\right]_0^{100}$$

$$= \frac{3\times10^{-6} \times 10^{10}}{4}$$

$$= \frac{300}{4}$$

$$\boxed{\text{mean} \Rightarrow 75}$$ Ans

$$V = \int_0^{100} (kx^4)_{dx} - (75)^2$$

$$= \left[\frac{kx^5}{5}\right]_0^{100} - (75)^2$$

$$= \frac{3\times10^{-6} \times 10^{10}}{5} - (75)^2$$

$$= \frac{30000}{5} - (75)^2$$

$$= 6000 - (75)^2$$

$$\boxed{V = 375}$$

Distribution fun

$$P(X=x) = \begin{cases} 0 & x<0 \\ \int kx^2\,dx & 0 \le x < 100 \\ 1 & x > 100 \end{cases}$$

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Q- $f(x) = kx^2 e^{-x}$ $\quad x \geq 0$ $\quad$ mean variance

$x(0, \infty)$

mean $\Rightarrow \int_0^\infty x f(x)\, dx$

$= kx^3 e^{-x}$

$\int_0^\infty kx^2 e^{-x}\, dx = 1$

$k\left[-x^2 e^{-x} + \int 2x e^{-x}\, dx\right]_0^\infty$

$k\left[-x^2 e^{-x} - 2x e^{-x} + 2e^{-x}\right]_0^\infty = 1$

$= k[0 \cdot 2 = 1]$

$\boxed{k = 1/2}$

mean $= \int_0^\infty x f(x)\, dx$

(9) $= \int_0^\infty \frac{x^3 e^{-x}}{2}\, dx$

Varian $= \int_0^\infty \frac{x^4 e^{-x}}{2}\, dx - (9)^2$

(3)

$\Rightarrow$

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$f(x) = kx^3 \qquad 2 < x < 4$

Find $k, \mu, \sigma^2, P(1 < x < 3)$

$$\int_2^4 kx^3 dx = 1$$

$$k \times \left[\frac{x^4}{4}\right]_2^4 = 1$$

$$k \times \left[\frac{16 \times 16}{4} - \frac{4 \times 4}{4}\right] = 1$$

$$k \times [64 - 4] = 1$$

$$\boxed{k = 1/60}$$

$$\mu = \int_2^4 x f(x)$$

$$\mu = \int_2^4 \frac{x^4}{60} dx$$

$$= \frac{1}{60}\left[\frac{x^5}{5}\right]_2^4$$

$$= \frac{1}{60}\left[\frac{256 \times 4}{5} - \frac{32}{5}\right]$$

$$= \frac{1}{60 \times 5}\left[1024 - 32\right]$$

$$= \frac{992}{60 \times 5}$$

$$= \frac{9 \cdot 92}{3}$$

$$= \boxed{3 \cdot 36}$$

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Variance $\sigma^2 \Rightarrow \int_2^4 k x^5 \, dx - (3.3)^2$

$= k \left[\frac{x^6}{6}\right]_2^4 - (3.3)^2$

$= k \left[\frac{2^{12} - 2^6}{6}\right] - (3.3)^2$

$= \frac{k}{60} \times \frac{2^6}{6} \left[2^6 - 1\right] - (3.3)^2$

$= \frac{64}{60 \times 6} \left[63\right] - (3.3)^2$

$= \frac{16}{15 \times 6} \times 63 - (3.3)^2$

$= \boxed{.31}$

$P(x < x < 3) = F(3) - F(1)$

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1) $f(x) = \begin{cases} P & a<x<b \\ Q & b<x<c \\ R & x>c \end{cases}$

$F(x)$ ~~distribution~~ density function

$F(x) \Rightarrow$ $F(x) = 0 \quad x<a$

$F(x) = \int_a^x P\,dx \quad a<x<b$

$b<x<c \quad F(x) = \int_a^b P\,dx + \int_b^x Q\,dx$

$x>c$

$F(x) = \int_a^b P\,dx + \int_b^c Q\,dx + \int_c^x R\,dx$

Q2) $f(x) = \begin{cases} ax & 0<x<1 \\ a & 1<x<2 \\ 3a-ax & 2<x<3 \\ 0 & \text{otherwise} \end{cases}$

Find a? and $F(x)=?$

$1 = \int_0^1 ax\,dx + \int_2^3 (a-ax)\,dx + \int_1^2 a\,dx$

$1 = a\left[\frac{x^2}{2}\right]_0^1 + a\left[3x - \frac{x^2}{2}\right]_2^3 + a\left[x\right]_1^2$

$1 = \frac{a}{2} + a\left[9 - \frac{9}{2} - 6 + 2\right] + a$

$1 = \frac{a}{2} + -\frac{9}{2}a + a + a\left[\frac{9}{2} - 4\right]$

$\cancel{2 = a - 3a + 2a}$ $\quad 1 = \frac{a}{2} + a + \frac{a}{2}$

$\boxed{a = 1/2}$

$$f(x) = \begin{cases} x/2 & 0<x<1 \\ 1/2 & 1<x<2 \\ 3/2 - x/2 & 2<x<3 \end{cases}$$

Density function

$$F(x) = \begin{cases} 0 & x<0 \\ \int_0^x \frac{x}{2}\,dx & x<1 \\ \int_0^1 \frac{x}{2}\,dx + \int_1^x \frac{1}{2}\,dx & x<2 \\ \int_0^1 \frac{x}{2}\,dx + \int_1^2 \frac{1}{2}\,dx + \int_2^x \left(\frac{3}{2} - \frac{x}{2}\right) dx & x<3 \\ 1 & x>3 \end{cases}$$

$$F(x) = \begin{cases} 0 & x<0 \\ \frac{x^2}{4} & x<1 \\ \frac{x}{2} - 1/2 + 1/4 & x<2 \\ -\frac{x^2}{4} + \frac{3x}{2} - \frac{5}{4} & x<3 \\ 1 & x>3 \end{cases}$$

$\frac{x}{2} - \frac{1}{4}$

$1 - \frac{1}{4}$ $3/4$

$-\frac{9}{4} + \frac{9}{2} - \frac{5}{4}$

$\frac{9}{4} - \frac{5}{4}$ 1

$\frac{1}{2}\left[3x - \frac{x^2}{2} - 6x + 2\right] + \frac{3}{4}$

$\frac{1}{2}\left[-\frac{x^2}{2} - 3x + 4\right] + \frac{3}{4}$

$-\frac{x^2}{4} - \frac{3x}{2} + \frac{1}{2} + \frac{3}{4} = \left(-\frac{x^2}{4} - \frac{3x}{2} - \frac{5}{4}\right)$

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Mean — $\int_0^1 x^2/2\,dx + \int_1^2 \frac{x}{2}\,dx + \int_2^3 \left(\frac{3}{2}x - \frac{x^2}{2}\right) dx$

$= \left[\frac{x^3}{6}\right]_0^1 + \left[\frac{x^2}{4}\right]_1^2 + \left[\frac{3x^2}{4} - \frac{x^3}{6}\right]_2^3$

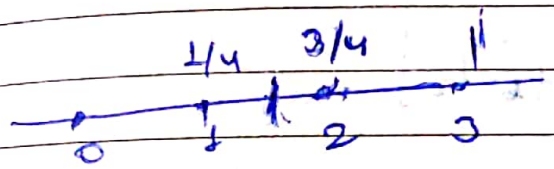
$= \frac{1}{6} + 1 - \frac{1}{4} + \frac{27}{4} - \frac{27}{6} - 3 + \frac{8}{6}$

$= \frac{26}{4} - 3 - 2$

$= \frac{26}{4} - 5$

$= 6.5 - 5$

$= \boxed{1.5}$



2 **median**

$\int_2^x \frac{x^3}{60}\,dx = \int_x^4 \frac{x^3}{60}\,dx$

$\left[\frac{x^4}{240}\right]_2^x = \left[\frac{x^4}{240}\right]_x^4$

$\frac{x^4}{240} - \frac{16}{240} = \frac{256}{240} - \frac{x^4}{240}$

$\frac{x^4}{120} = \frac{272}{240}$

$x^4 = 136$

$\boxed{x = (136)^{1/4}}$

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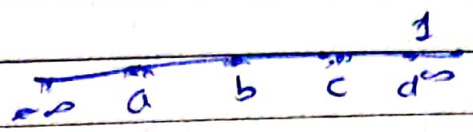
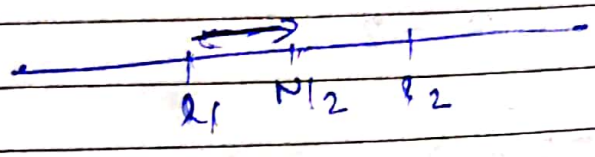
Median –



$$\text{Median} = l + \frac{h}{f}\left(\frac{n}{2} - cf\right)$$



Mode

$$\text{Mode} = l + h\left[\frac{f_0 - f_1}{2f_1 - f_0 - f_2}\right]$$

	f	
0–10	2	
10–20	5	f_0
20–30	10	f_1
30–40	7	f_2
40–50	3	

$$\frac{(f_2 - f_1) - (f_1 - f_0)}{f_2}$$

$$f_1 - f_2 \quad f_1 - f_0$$

$$\frac{f_1 - f_0}{2f_1 - f_0}$$

$$(f_1 - f_0)\, f_2 - f_1$$

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Q. following table give distribution of mars

Marks	No of Students	cf	Interval
0-20	42	42	0-20.5
21-30	38	80	20.5-30.5
31-40	120	200	30.5-40.5
41-50	84	284	40.5-50.5
51-60	48	332	50.5-60.5
61-70	36	368	60.5-70.5
71-80	31	399	70.5-80

$\frac{n}{2} = \frac{399}{2} = 199.5$

30.5-40.5 $f = 120$ $cf = 80$

$\text{Median} = 30.5 + \frac{10}{120}(199.5 - 80)$

$= 30.5 + \frac{119.5}{12}$

$= \boxed{40.45} \Rightarrow \boxed{40.45}$

Q. passing marks is 35% calculate percentage of failure

~~cf at 35 ⇒ $\frac{399 \times 35}{100} = 35 \times 3.99 \Rightarrow 139.65$~~

~~X=~~

assume $X = 34.5 \simeq 35$ (34.5 is also consider as pass)

$34.5 = 30.5 + \frac{10}{120}(X - 80)$

$4 \times 12 = X - 80$

$X = 80 + 48 \Rightarrow \boxed{128}$ Ans //

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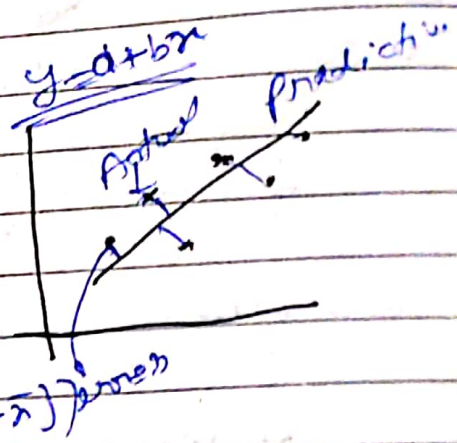
Mode of data - in

$$\text{mode} = l + \frac{h}{c}\left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right)$$

Error & Deviation

~~$y = ax + b$~~ $y = a + bx$



Error → (Actual − predicted)

Mean error

$$= \frac{1}{n}\sum\left[(x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})\right]$$

$$= \frac{1}{n}\sum\left[(x_1 + x_2 + \cdots + x_n) - n\bar{x}\right]$$

$$= \frac{\sum x}{n} - \bar{x}$$

$$= \bar{x} - \bar{x}$$

$$= 0$$

mean error → always (0)

mean absolute error

$$\rightarrow \frac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}| \rightarrow \text{(absolute mean error)}$$

predicted $y = a + bx_i$

actual y_1, y_2, y_3 respon $\hat{y}_1\ \hat{y}_2\ \hat{y}_3$

error ⇒ actual - prediction

$= y_i - a - bx_i$

Mean absolute error / deviation $= \frac{1}{n} \sum |y_i - a - bx_i|$ Mean Error

Square root of mean square error

Standard Error $\sigma = \sqrt{\frac{1}{n} \sum [y_i - a - bx_i]^2}$

Why always we take square of mean instead of mean absolute mean

Eg 0 1 2 1 1 1 0 0 3

mean error ⇒ $\sqrt{5}/3$ 1 1

square mean error ⇒ $\sqrt{2/3}$ $\sqrt{1}$ $\sqrt{3}$

at $x=0$ absolute ~~function~~ function can't be differentiable.

Square mean give minimum error with respect to each and every y_i.

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Coefficient of Variation

$$CV = \frac{\sigma_x}{\mu_x} \times 100$$

Quartile Median

	f	cf			x	y	
					18	-4	-4
15.5 – 20.5	1	1			23	-3	-6
20.5 – 25.5	2	3			28	-2	-14
25.5 – 30.5	7	10			33	-1	-10
30.5 – 35.5	10	20	N/4 ⇒ 14	Q_1	38	0	0
35.5 – 40.5	16	36	N/2 ⇒ 28	Q_2	43	1	9
40.5 – 45.5	9	45	3N/4 ⇒ 42	Q_3	48	2	16
45.5 – 50.5	8	53			53	3	6
50.5 – 55.5	2	55			58	4	4
55.5 – 60.5	1	56					

$$Q_1 \Rightarrow 30.5 + \frac{5}{10}(14-10)$$

$$= 30.5 + \frac{5 \times 4}{10}$$

$$\Rightarrow \boxed{32.5}$$

$$\boxed{Q_1 = 32.5}$$

$$\boxed{Q_2 = 38}$$

$$Q_2 \Rightarrow \cancel{30.5} + 35.5 + \frac{5}{16}(28-20)$$

$$\boxed{\cancel{Q_2 = 45.5}}$$

$$= 35.5 + \frac{5 \times 8}{16}$$

$$= 39 \quad \boxed{38}$$

$$Q_3 \Rightarrow 40.5 + \frac{5}{9}(42-36)$$

$$= 40.5 + \frac{5 \times 6}{9} = \boxed{\cancel{45.5}}$$

$$Q_3$$

$$40.5 + \frac{10}{3} \Rightarrow \boxed{43.83}$$

Ans/

$\boxed{\sigma_x = c\sigma_y}$ Standard Diviation

$\boxed{\mu_x = a + c\mu_y}$

$\frac{KN}{100}$ $K = 55$ $\frac{55}{100} \times 56 = 30.8$

$Q_{55\%} = 35.5 + \frac{5}{16}(30.8 - 20)$

$= 35.5 + \frac{5}{16} \times 10.8$

$\boxed{38.875}$ Ans//

Mean S.D

$Mean = \frac{\Sigma fx}{\Sigma f} = \frac{2133}{56} = \boxed{38.08}$

$y = \frac{x-a}{c}$

$x = cy + a$ — ①

$\Sigma x = c\Sigma y + ca$

$\boxed{\mu_x = c\mu_y + a} \rightarrow 2$

$\boxed{\mu_x = a + c\mu_y}$

$\mu_x - x = c(\mu_y - y)$

$(\mu_x - x)^2 = c^2(\mu_y - y)^2$

$\frac{\Sigma(x-\mu_x)^2}{N} = \frac{c^2\Sigma(y-\mu_y)^2}{N}$

$\sigma_x = \sqrt{\frac{c^2\Sigma(y-\mu_y)^2}{N}}$

$\boxed{\sigma_x^2 = c^2\sigma_y^2}$

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assumed mean

$y = \frac{x-a}{c}$ $\quad a = \mu_x$

$c = \sigma_x$

$y = \frac{x-\mu_x}{\sigma_x}$

$\mu_x = c\mu_y + a$ $\quad \sigma_x^2 = c^2\sigma_y^2$

$\mu_x = c\mu_y + \mu_x$ $\quad \sigma_x^2 = \sigma_x^2\sigma_y^2$

$\boxed{\mu_y = 0}$ $\quad \boxed{\sigma_y = 1}$

If we take approximate value of $a \to 0$ then $\mu_y \to 0$

$\sigma_y \to 1$

~~$\mu_x = 30.5$~~

$\mu_x = 30 - \frac{5}{56}$

Expectation

B B B	5	+4		$P(\$) = 0.1$
A A A	10	+9		$P(A) = 0.2$
$ $ A	15	14		$P(B) = 0.2$
$ $ $	20	19		
otherwise	0	-1		

$X \to$ Total amount $\quad$ each time I must 1 ₹

$X = -1, 4, 9, 14, 19$

$P(X=19) = .001$

$P(X=14) = .002 \times 3 = .006$

$P(X=9) = .008$

$P(X=4) = .008$

$P(X=-1) = \cancel{.125}\ \cancel{.969}\ .977$

Mean → $.012 + .084 + .072 + .032 - .77$

$\rightarrow \boxed{-0.77}$

Variance → $\sum P(x)x^2 - (\sigma)^2$

$\rightarrow 3.29 - (.77)^2$

$= \boxed{2.6971}$

X →	-2	→ ~~.0001~~ .977	.0
	23	→ .001	
	49	→ .006	
	73	→ .008	
	99	→ .008	

(X multiplying by 5 increase and increase amount 2)

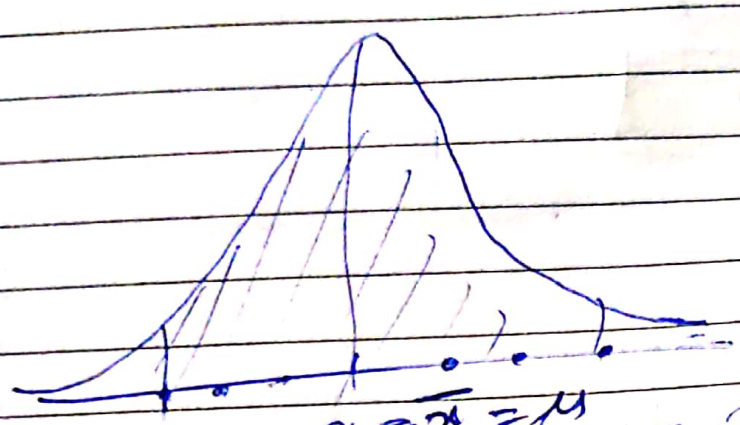
E(Y) Mean = ~~→ 0.5~~ → 0.95

Variance → ~~67.4225~~

Lower the Variance lower the deviation mean observation are certain

[Linear Transformation possible only if probability distribution is not changing]

Normal distribution



deviation

$\mu - k\sigma$ $\quad x_0 = \bar{x} = \mu$

$\mu + k\sigma$

$$F(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{1}{2}\left[\frac{x-\mu}{\sigma}\right]^2}$$

$$\int_{\mu-k\sigma}^{\mu+k\sigma} \frac{1}{\sqrt{2\pi}\,\sigma} e^{-1/2\left[\frac{x-\mu}{\sigma}\right]^2} dx$$

$\frac{x-a}{c}$ $\quad a \to \mu$, $c \to \sigma$

$X = \mu_x + \sigma_x Z$

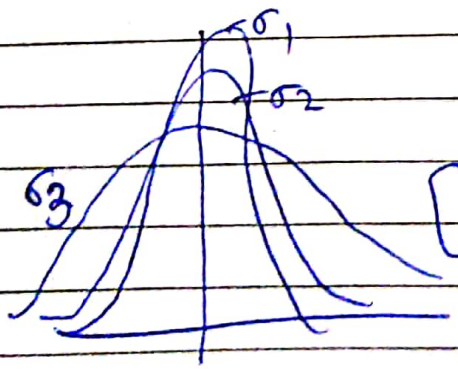
$\frac{\Sigma x}{n} = \mu_x + \sigma_x \frac{\Sigma z}{n}$

$Z = \frac{X-\mu}{\sigma}$

$\mu_x = \mu_x + \sigma_x \mu_z$

$X = \mu + \sigma Z$

$\boxed{\mu_z = 0}$



$[\sigma_1 < \sigma_2 < \sigma_3]$

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$$z = \frac{x-\mu}{\sigma}$$

$$dz = \frac{dx}{\sigma}$$

$$\int_{\mu-k\sigma}^{\mu+k\sigma} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$\boxed{f(y) = \frac{2}{\sqrt{\pi}} \int_0^{k/\sqrt{2}} e^{-y^2} dy}$$

$$\Rightarrow 2\int_0^{k} \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$$

$$y = \frac{z}{\sqrt{2}} \rightarrow dy = \frac{dz}{\sqrt{2}}$$

$$\Rightarrow 2\int_0^{k/\sqrt{2}} \frac{e^{-y^2}}{\sqrt{\pi}} dy$$

$$= \frac{2}{\sqrt{\pi}} \int_0^{k/\sqrt{2}} e^{-y^2} dy$$

$$y = \frac{x-\mu}{\sqrt{2}\sigma}$$

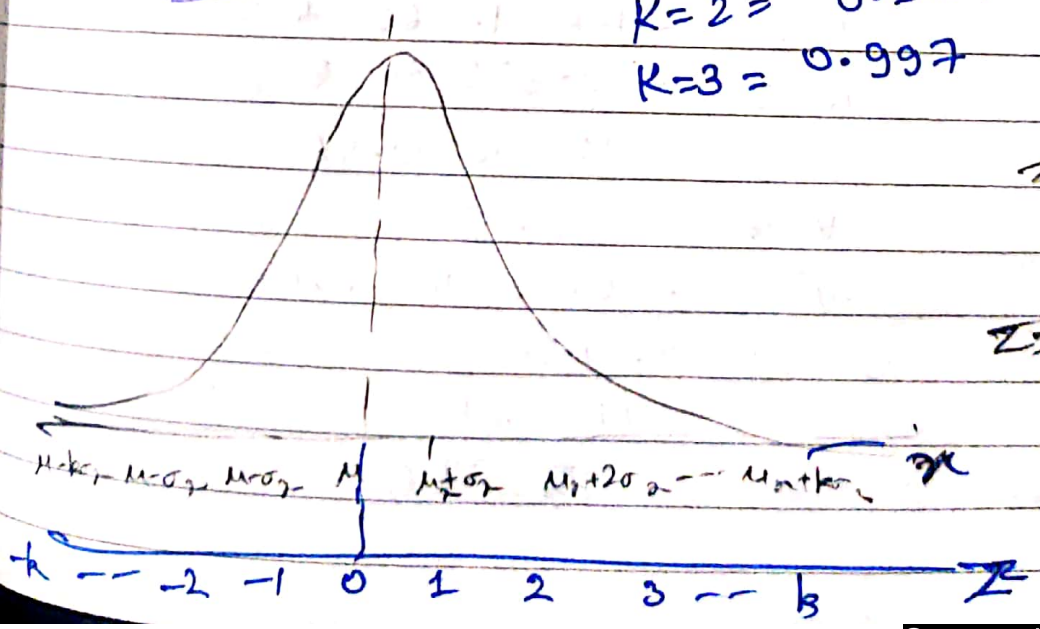
$$\Rightarrow \boxed{\frac{2}{\sqrt{\pi}} \int_0^{k/\sqrt{2}} e^{-y^2} dy} \rightarrow \boxed{k=1 \Rightarrow 0.682}$$

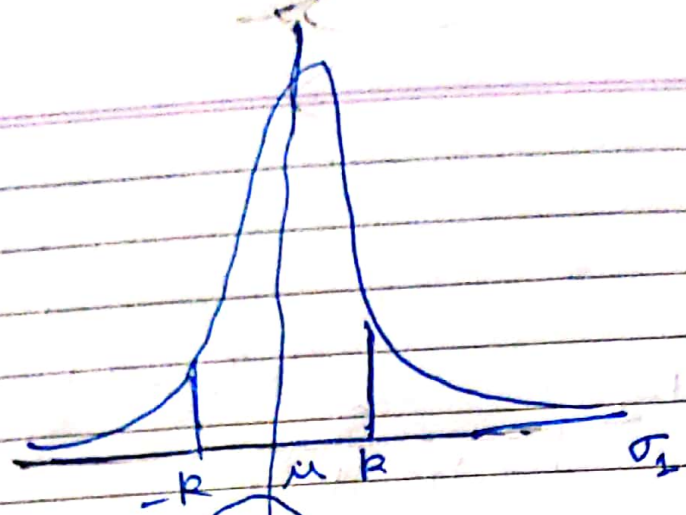
$k = 2 = 0.954$

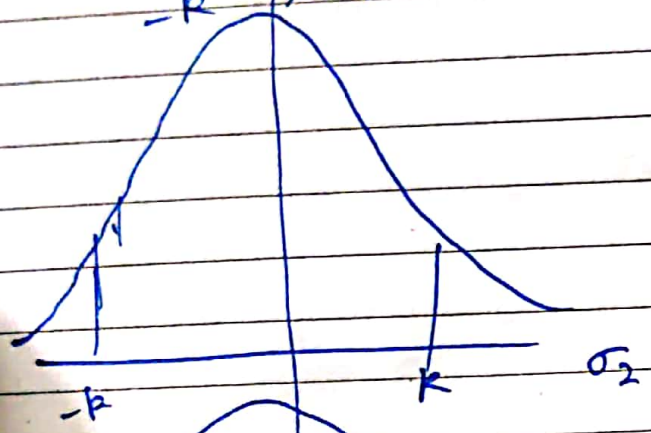
$k = 3 = 0.997$

$$z = \frac{x-\bar{x}}{\sigma}$$

$$z = \frac{\mu-\bar{x}}{}$$



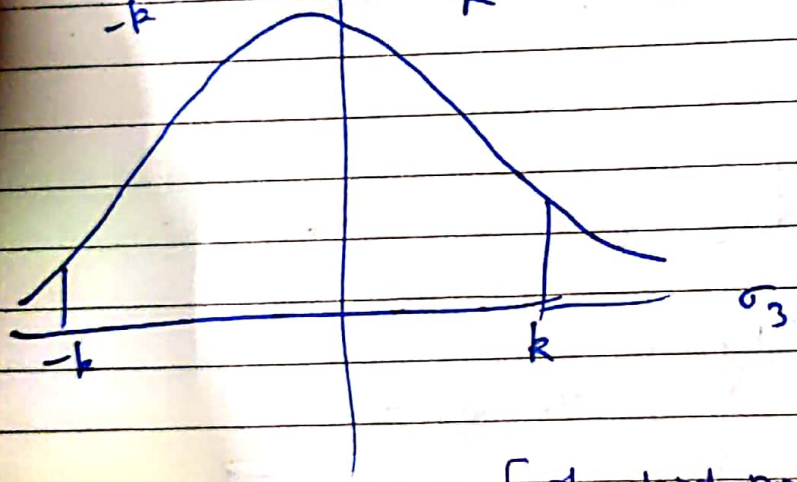




$\sigma_1 < \sigma_2 < \sigma_3$

~~Co~~ gap between 2 value value is

$z_1 < z_2 < z_3$



[standard normal curve] $\mu = 0, \sigma = 1$

Find out area under curve

between i) $z = 0$ to 1.65

ii) $z = -1.6$ 1.362 to 0

iii) $z = -0.652$ to 2.35

iv) $z = 1.15$ to 2.56

v) $z > 2.15$

vi) -0.66 to 0.66

1) $.95053 - 0.5 \Rightarrow 0.45053$

2) $0.5 - .08691 \Rightarrow .41309$

3) $.99061 - .25705 \Rightarrow .79276$

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(v) .98422
.01578 $Z > 2.15$

2) IC level measured on some given scale on 80 students are normally distributed.

$\mu = 100$ $\sigma = 16$ no of student betw (115 to 140)

$Z = \frac{x-\mu}{\sigma}$

$Z_1 = \frac{115-100}{16} = \frac{15}{16} = \frac{1}{4}$

$Z_1 = 0.25$

$\boxed{Z_1 = .94}$

$Z_2 = \frac{140-100}{16}$

$Z_2 = \frac{40}{16} = \frac{2}{3} = .667$

$Z_2 = \frac{40}{16} = 2.5$

.74537
− .59071
$\boxed{.14666}$

.59071 $(2.5 > Z > .94)$

Obtain and 90th percentile for Fqo IQ

$Z = 1.20$

$1.20 \times 16 + 100 = x$

$\boxed{x = 120.48}$

Q- data collected on domestic water consumption from five houses has normal distribution

$\sigma = 69.8 \quad \mu = 105\,l$

1) determine % of houses that consume b/w 50 to 100 l of water

2) determine % of houses that consumes less than 200 liters

3) determine and interpret 40% of water consumption.

$Z_1 = \frac{50-105}{69.8}$ $\qquad$ $Z_2 = \frac{100-105}{69.8}$

$Z = \frac{-55}{69.8}$ $\qquad$ $Z_2 = \frac{-5}{69.8}$

$Z_1 = -.788$ $\qquad$ $Z_2 = -0.072$

$.47210 - .21770$

~~.03~~

$$\begin{array}{r} .47210 \\ -\ .21770 \\ \hline .25440 \end{array}$$

1) $\boxed{.25440}$ Ans

2) $Z = \frac{200-105}{69.8} = \frac{95}{69.8} = 1.36$

$Z < 1.36$

$\boxed{.91309}$

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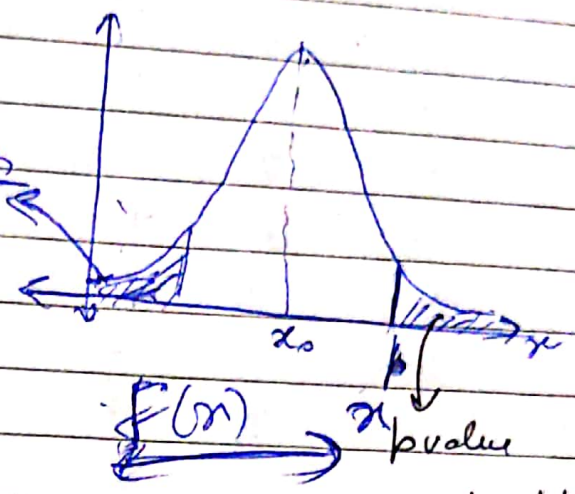
Linear Transformation –

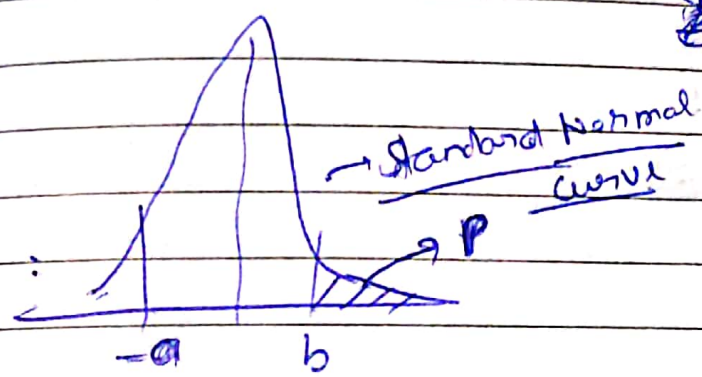
probability of each instance will not change.

abnormal behavior → outer

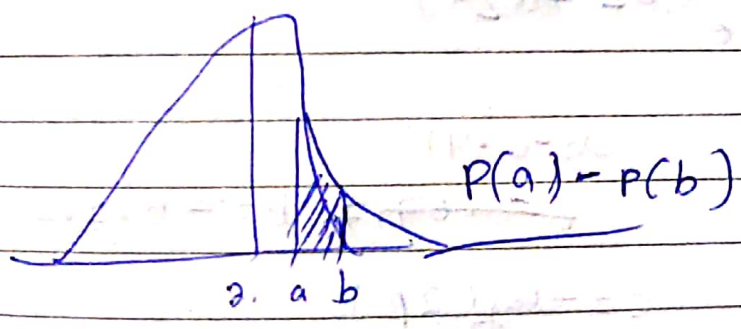


area of outer

$P(X > x)$



$$P(-a < x < b) = 1 - P(b) - P(a)$$



		x	f	x^2
50.5	55.5	53	4	
55.5	60.5	58	14	
60.5	65.5	63	24	
65.5	70.5	68	40	
70.5	75.5	73	54	~~14642~~
75.5	80.5	78	43	
80.5	85.5	83	36	$\bar{x} = 74.8$ 74.9
85.5	90.5	88	20	
90.5	95.5	93	11	
95.5	100.5	98	3	
100.5	105.5	103	1	
			250	

$\bar{x} = 75$

$\sigma = 10$

$$p(z \geq z) = \frac{1}{2} e^{(bz + az^2)}$$

$a = -0.416$

~~$b = 0.0717$~~ $b = -0.717$

$$z = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$c = -\log_2(2p)$

$c = -\log_2(.04)$

$z = +\cancel{0.7}\ \cancel{.0717}$

$$z = \frac{.717 - \sqrt{(.717)^2 - 4 \times (-.416)(4.64)}}{2 \times -.416}$$

$$z = \frac{.717 - \sqrt{.514 + 7.72}}{} \Rightarrow \frac{.717 - 2.869}{2 \times -.416}$$

Bay'
Binomial
Poisson
Normal
Gamma
Median
Mode
Percentile
Standard Deviation

Mean $z = -1$
$z = -2$
$z = -3$

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$\boxed{z = 2.6}$

$2.6 = \frac{x - 75}{10}$

$26 = x - 75$

$\boxed{x = 101}$

$\boxed{z = 2.06}$ $\boxed{x = 95.06 \mid 96}$

find (68 to 77)

$z_1 = \frac{68 - 75}{10} = \frac{-7}{10} = -.07$

$z_2 = \frac{77 - 75}{10} = .02$

P

$1 - P(.02) - P(.07)$

$1 - \frac{1}{2} e^{(-.717 \times .02 + -.416 \times (.02)^2)}$

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