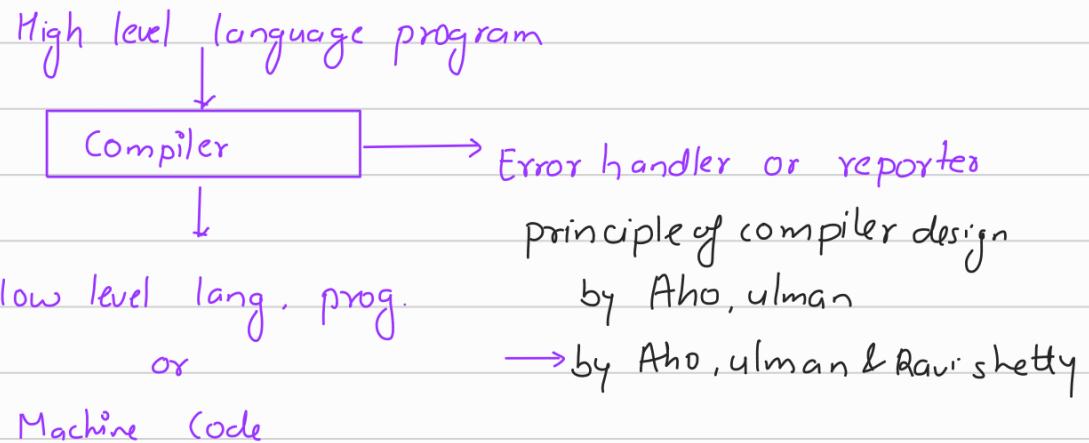


- Compiler is a system program that reads program of one language and translates it in low level progr. or machine code
- High level lang  $\xrightarrow{\text{compiler}}$  low level language



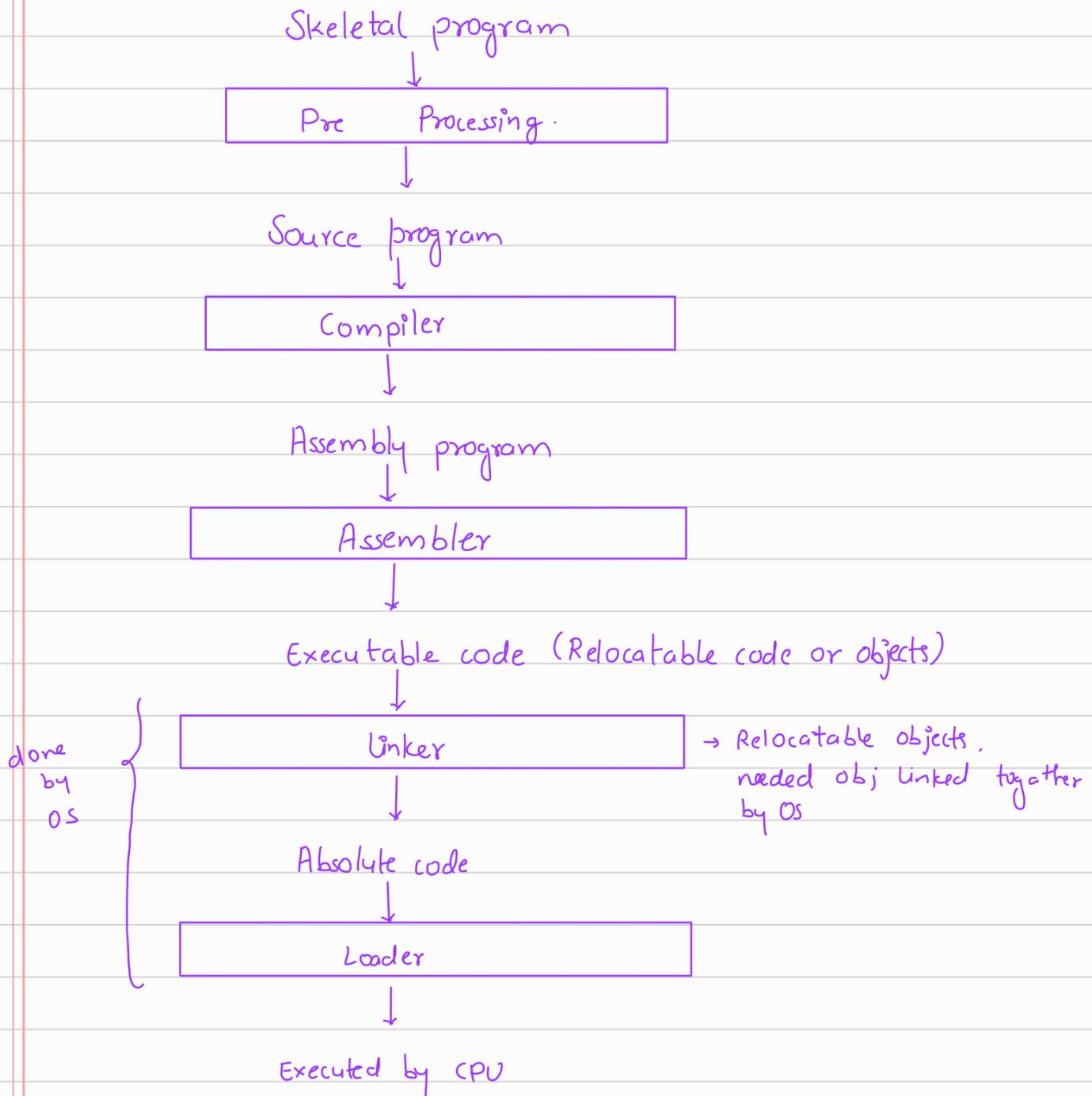
- Interpreter is also sys. prog that translate & execute prog line by line

Compiler	Interpreter
- It translates HLL prog to low level or machine code	- It translates & executes prog. line by line
- It requires complete prog. at once for compilation	- No such requirement
- It reports on errors after compilation	- It stops execution when an error occurs during execution of any line
- It requires additional memory to store intermediate or assembly code	- It executes code line by line so no additional memory is required
- It takes less time to execute the program as compared to interpreter	- It takes more time to execute the program bcz it always starts with fresh execution from the begining.

- Ex - C, C++, Java  
(source prog)

Ex - Python, Java, Visual basic      byte code

## Execution of program



## Phases of compiler →

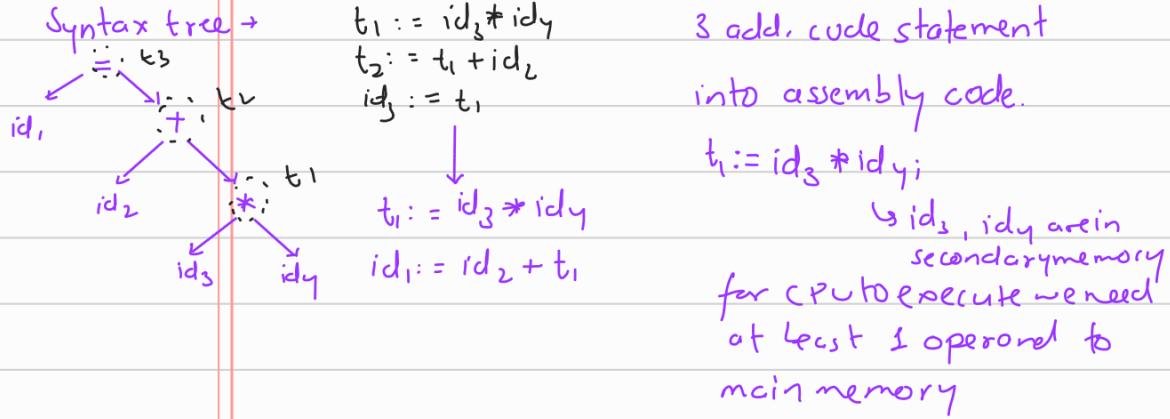
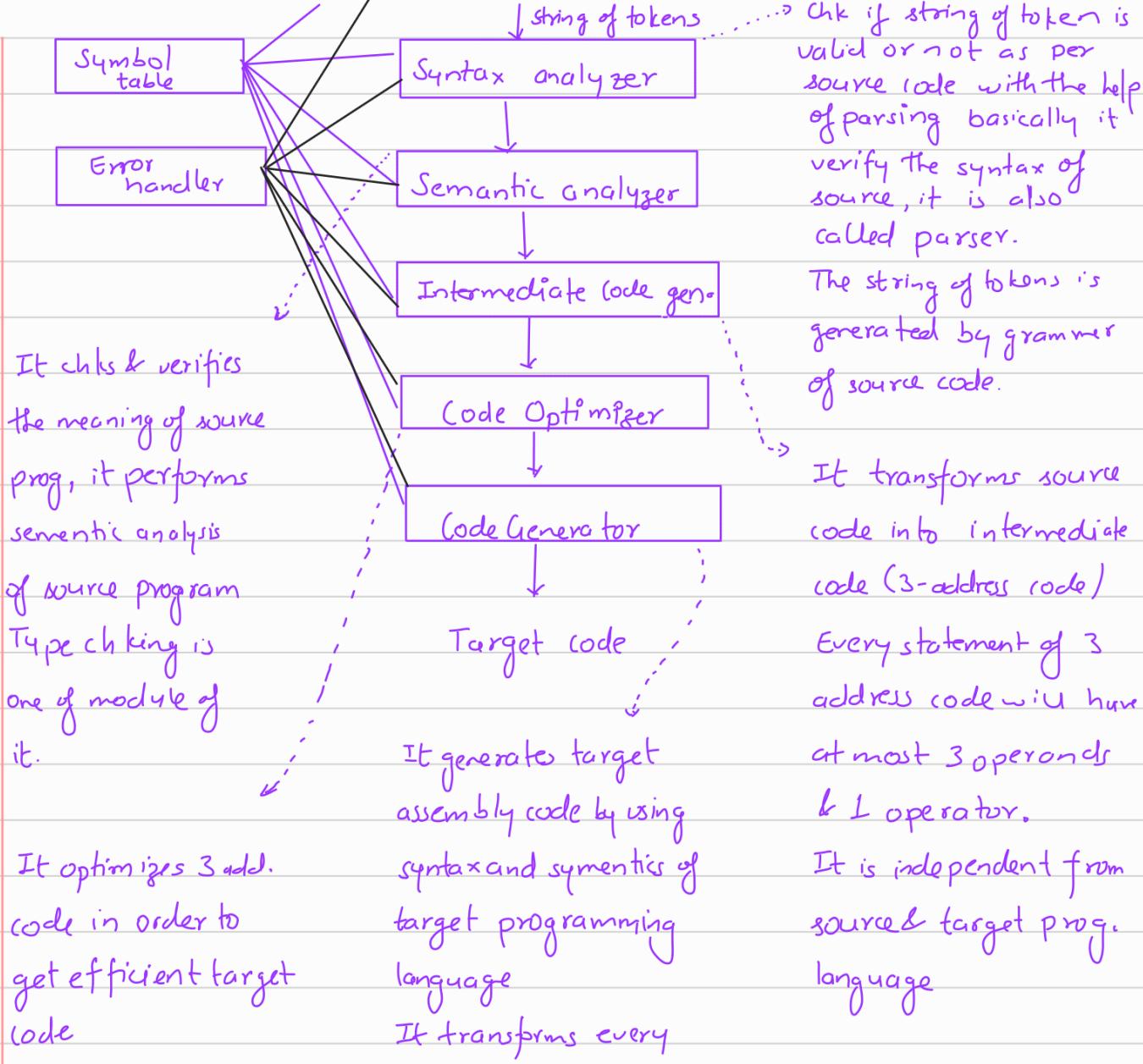
Meaning  
shd rem.  
same

~~shd remain  $\leftarrow \text{id}_1 := \text{id}_2 + \text{id}_3 * \text{id}_4;$~~

$$z = a + b * c$$

## Source program

... $\rightarrow$  It scans the source program  
and generates string of tokens  
It is also known as scanner



```

MOV R1, id3   R1 ← id3
MUL R1, idy   R1 ← R1 * idy
MOV t1, R1;   t1 ← R1
  
```

```

id1 = t1 + id2
MOV R1, id2
ADD R1, t1
MOV id1, R1
  
```

### Symbol table →

- It is a data structure that keep track of scope , binding , type , value , token and some other information of all variable names of source program.

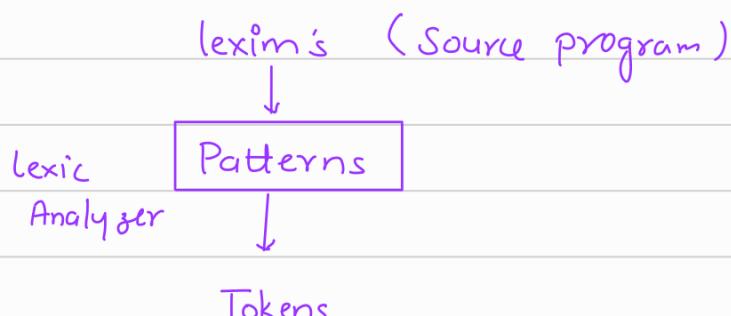
Identifiers	tokens	Scope	binding	type	Value
a	id <sub>1</sub>	local	static	int	5
b	id <sub>2</sub>	global	dynamic	real	5.5
c	id <sub>3</sub>	local	dynamic	real	9.8

### Error handler →

- It reports error that is detected at any phase of the compiler

### Lexical Analysis →

- It scans source prog. and generates strings of tokens
- lexic analyzer has set of rules called pattern which maps lexems to tokens

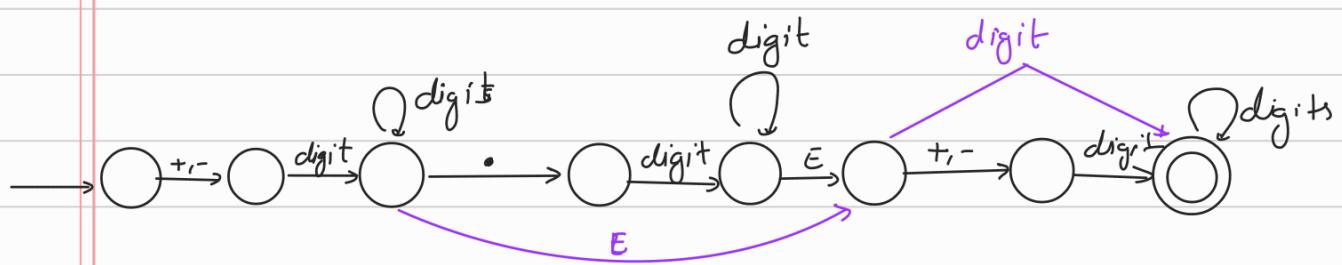


- lexim: Sequence of character with collective meaning in source program is called a lexim Ex: int
- token: Output of lexical analyzer is token
- Patterns are designed using regular expression
- Identifiers: letter (letter + digit)\*



- Number : int  $(\text{digit})^+$   
 real  $(\text{digit})^+.\text{(digit)}^*$

$\pm 25.57 \times (\text{E})^{\pm 38}$  FA ??  
 dig Radix dig exp digits



Lexems	Patterns	Token
1. Identifier	$\text{letter}.\text{(letter+digit)}^*$	id
2. Number		
int	$(\text{digit})^+$	Number
real	$(\text{digit})^+.\text{(digit)}^*$	Number
3. Operators	Operators	Operators
+	+	+
4. Keywords	Keywords	Keywords
int	int	int
5. Scope	Scope	Scope

Regular expression  $\rightarrow$

Operation  $\{ +, *, \cdot, \emptyset \}$

property  $\{ \epsilon \Rightarrow \text{Empty string}, \emptyset = \text{Empty set} \}$

$$(i) R + \epsilon = \epsilon + R$$

$$(ii) R + \emptyset = \emptyset + R = R$$

$$(iii) R \cdot \emptyset = \emptyset \cdot R = \emptyset$$

$$(iv) R \cdot \epsilon = \epsilon \cdot R = R$$

$$(v) R \cdot R^* = R^* \cdot R = R^+$$

$$(vi) \epsilon + RR^* = R^*$$

$$(vii) (R_1 + R_2)^* = (R_1^* + R_2^*)^* = (R_1^* + R_2)^* = (R_1^* R_2^*)^*$$

$$(viii) (R_1^* \cdot R_2)^* \neq (R_1 \cdot R_2^*)^*$$

$$(ix) R_1 \cdot (R_2 + R_3) = R_1 R_2 + R_1 R_3$$

$$(x) R_1 R_2 + R_1 R_3 = R_1 (R_2 + R_3)$$

$$(xi) (R_1^* \cdot R_2)^* \cdot R_1^* = R_1^* (R_2 \cdot R_1^*)^*$$

$$(xii) \emptyset^* = \epsilon^*$$

lexical analyzer  $\longrightarrow$  Set of pattern

- It is design by finite automata

- Designed by regular expression

Finite automata  $\rightarrow$

$$FA (\emptyset, Q_0, \Sigma, \delta, F)$$

$\delta$  - transition function

Deterministic

FA

$$\emptyset \times \Sigma \rightarrow Q$$

Total function

from each state

for each I/P there

must be one

transition

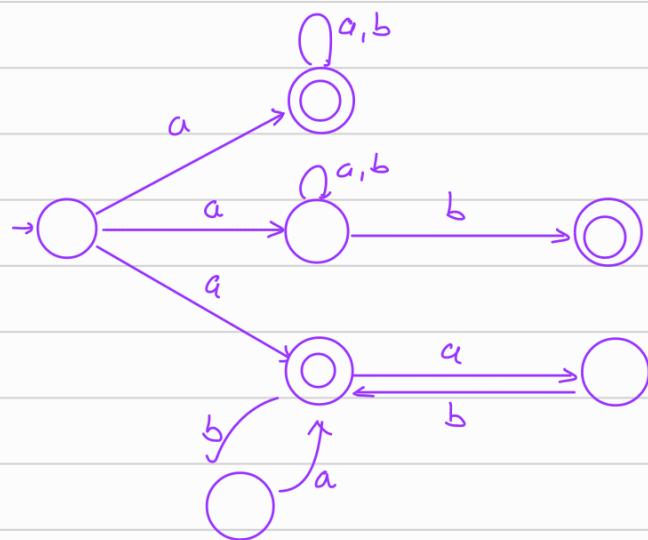
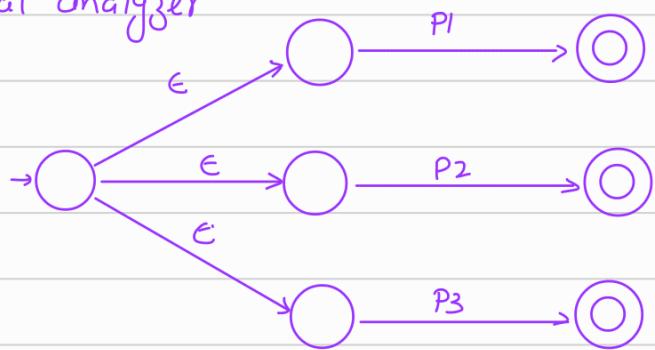
Non-deterministic

FA

$$\emptyset \times (\Sigma \cup \epsilon) \rightarrow 2^Q$$

- Q. Design lexical analyzer that can recognise the following patterns
- $a \cdot (a+b)^*$
  - $a \cdot (a+b)^* \cdot b$
  - $(ba+ab)^*$

lexical analyzer



States	I/P	
	a	b
$\rightarrow A$	B	C
B	D	E
C	F	I
D	D	G
E	B	H
F	I	C
G	D	G
H	J	G
I	I	F
J	B	H
T	T	T

$$G\text{-closer}(\{v_0\}) = \{v_0, v_4\} = A$$

$$\delta(A, a) = \delta(\{v_0, v_4\}, a) = \{v_1, v_2, v_6\}$$

$$\delta(A, b) = \{v_5\}$$

$$\delta(B, a) = \{v_1, v_2\}$$

$$\delta(B, b) = \{v_1, v_2, v_3, v_4\}$$

$$\delta(C, a) = \{v_4\}$$

$$\delta(C, b) = \emptyset$$

$$\delta(D, a) = \{v_1, v_2\}$$

$$\delta(D, b) = \{v_1, v_2, v_3\}$$

$$\delta(E, a) = \{v_1, v_2, v_5\}$$

$$\delta(E, b) = \{v_1, v_2, v_3, v_5\}$$

:

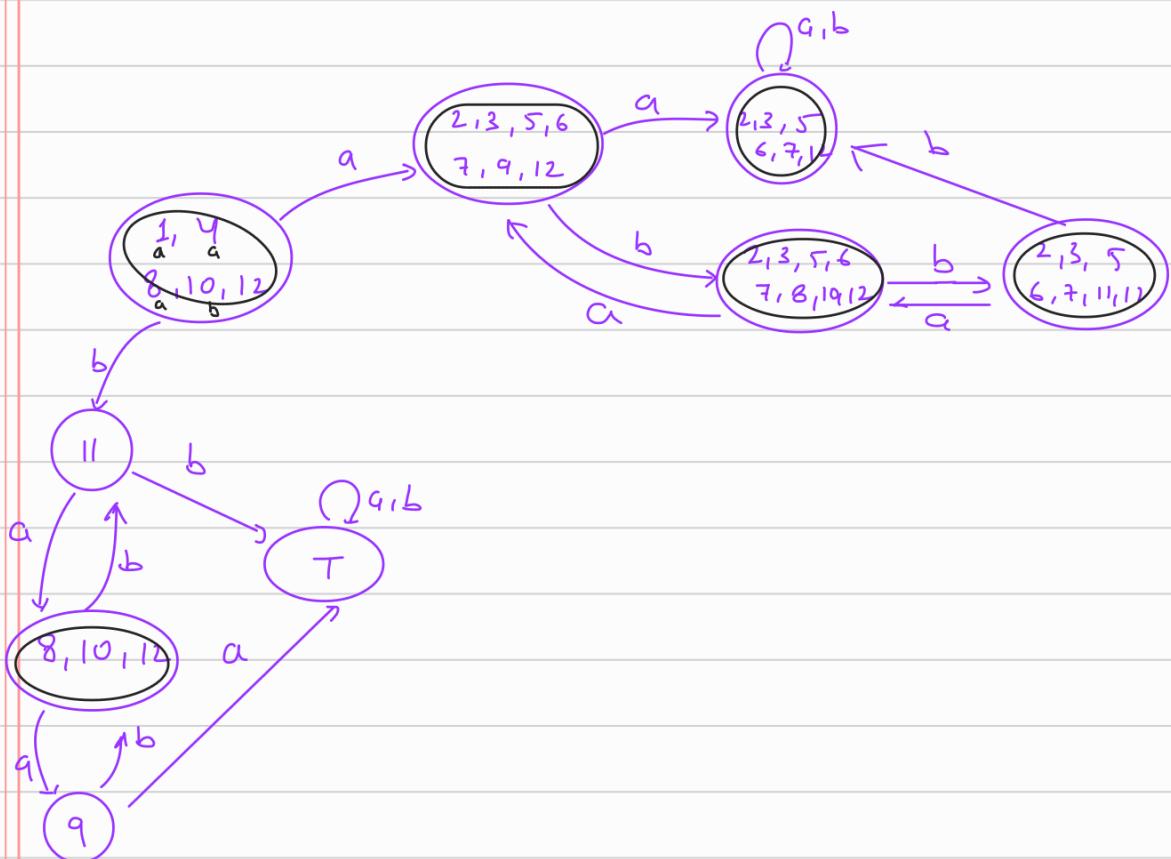
:

Using follow pos

$$(a \cdot (a+b)^*) + a(a+b)^*b + (ab+b^*a)^* \#$$

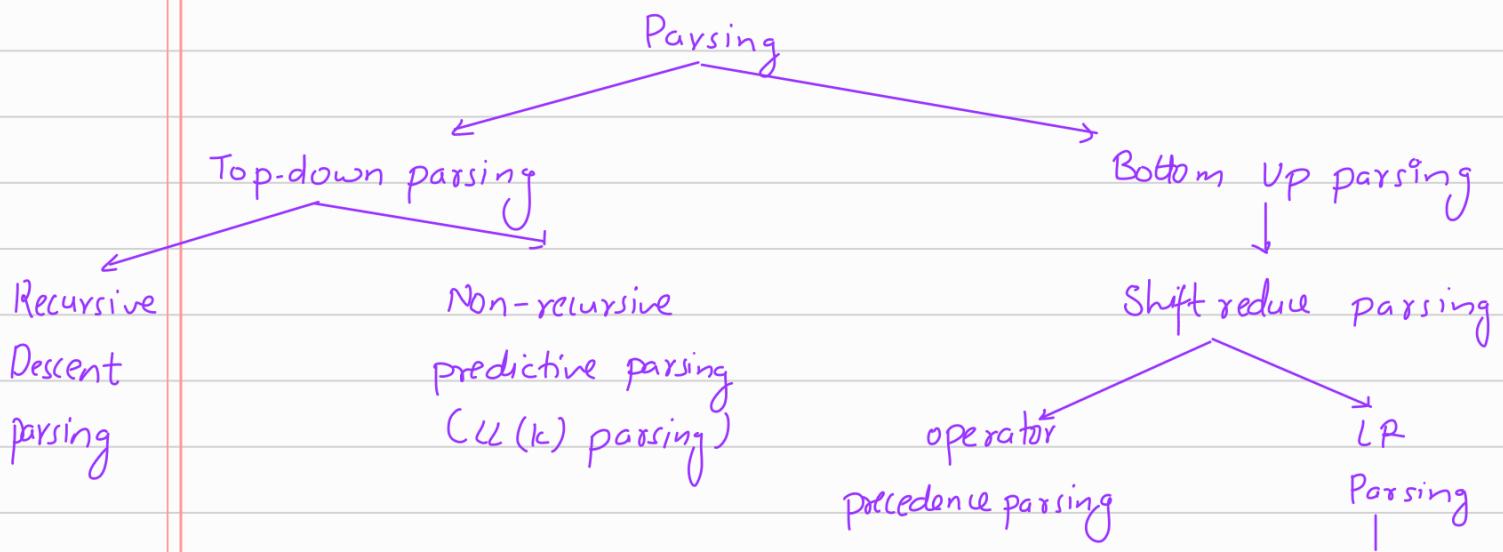
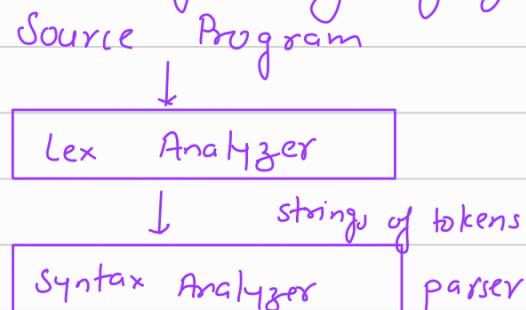
1 2 3 4 5 6 7 8 9 10 11 12

Pos	Follow pos	
1	2, 3, 12	initial state
2	2, 3, 12	1, 4, 8, 10, 12
3	2, 3, 12	
4	5, 6, 7	
5	5, 6, 7	
6	5, 6, 7	
7	12	
8	9	
9	8, 10, 12	
10	11	
11	8, 10, 12	
12	-	



### Syntax analysis →

- It checks syntax of source program
- It is also known as parsing
- Parsing is to check whether given string of tokens generated by grammar of source programming language



## Grammer of language →

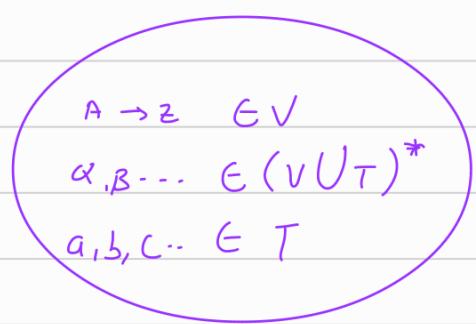
$G(V, T, S, P)$

$V$ : set of finite variables

$T$ : set of finite terminals

$S$ : starting variable

$P$ : set of finite production.



### - Regular language



### - Context free grammar

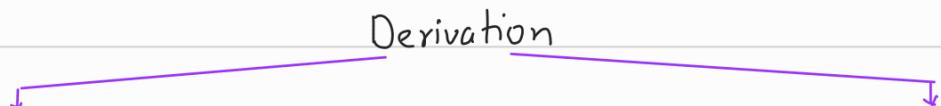
$\# P : A \rightarrow \alpha$

$A \in V, \alpha \in (V \cup T)^*$

Ex -  $S \rightarrow aSbS / bSaS / \epsilon$  for  $n_a(\omega) = n_b(\omega), \omega \in \{a, b\}^*$

## Derivation (Parsing) →

- Set of step to generate set of string from the grammer is called derivation



Left Most Derivation

- left most variable is replaced by its production in each step of derivation

Right Most Derivation

- rightmost variable is replaced by its production in each step of derivation

left sentential form  $\rightarrow$

- Each step to generate left most derivation is called left sentential form similar for right sentential form.

Ex  $S \rightarrow aSbs / bSaS / \epsilon$  "Ababba"

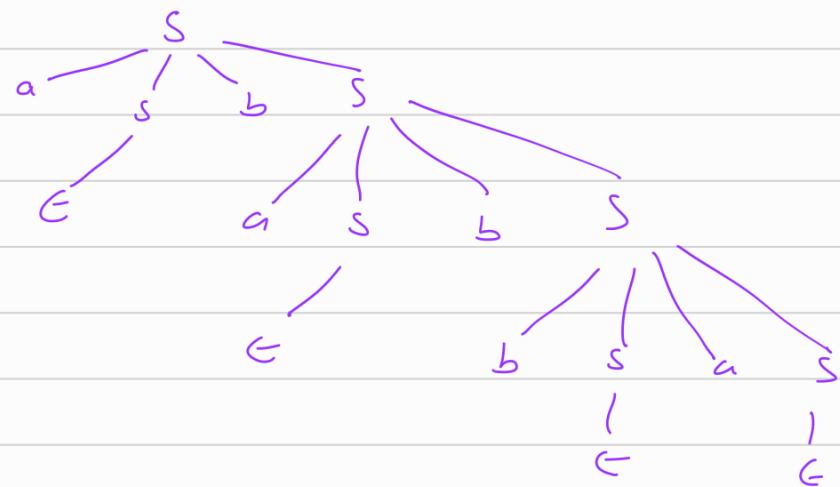
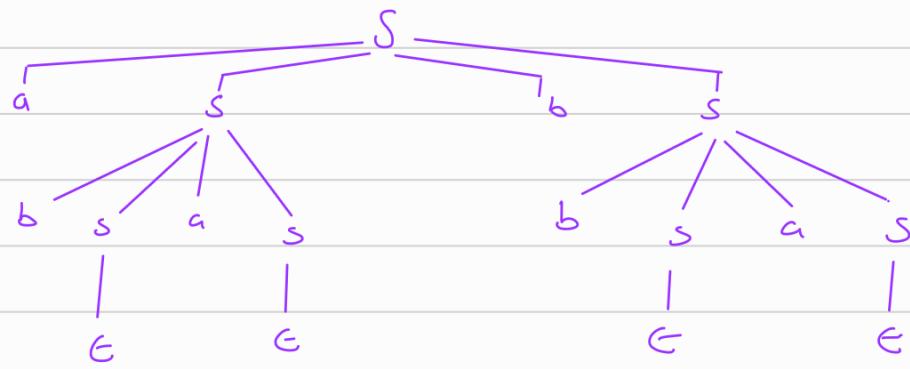
LMD $\rightarrow$	Step	Sentential form	Production
	1.	S	$S \rightarrow aSbs$
	2.	aSbs	$S \rightarrow \epsilon$
	3.	abs	$S \rightarrow aSbs$
	4.	ababs	$S \rightarrow \epsilon$
	5.	ababS	$S \rightarrow bSaS$
	6.	ababbSaS	$S \rightarrow \epsilon$
	7.	ababbas	$S \rightarrow \epsilon$
	8.	ababba	

RHD $\rightarrow$	Step	Sentential form	Production
	1.	S	$S \rightarrow aSbs$
	2.	aSbs	$S \rightarrow aSbs$
	3.	aSbaSbs	$S \rightarrow bSaS$
	4.	aSbaSbbSaS	$S \rightarrow \epsilon, S \rightarrow \epsilon, S \rightarrow \epsilon, S \rightarrow \epsilon$
	5.	ababba	

Parse tree  $\rightarrow$  (Derivation tree)

- It is a graphical way to generate derivation in the form of a tree
- The root of the tree is S (starting variable)
- All the non leaf nodes are represented by variables while all leaf nodes are represented by terminal

Ex  $S \rightarrow aSbs / bSaS / \epsilon$  "Ababba"



$\therefore$  More than 1 LMD, RMD or parse tree  $\therefore$  Ambiguous grammar

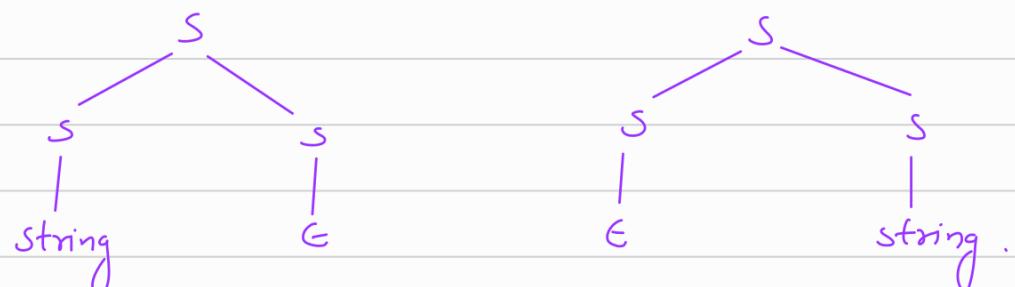
### Ambiguous grammar

- When  $\exists$  more than one LMD, RMD, parse tree for any string of the grammar then the grammar is called ambiguous grammar.

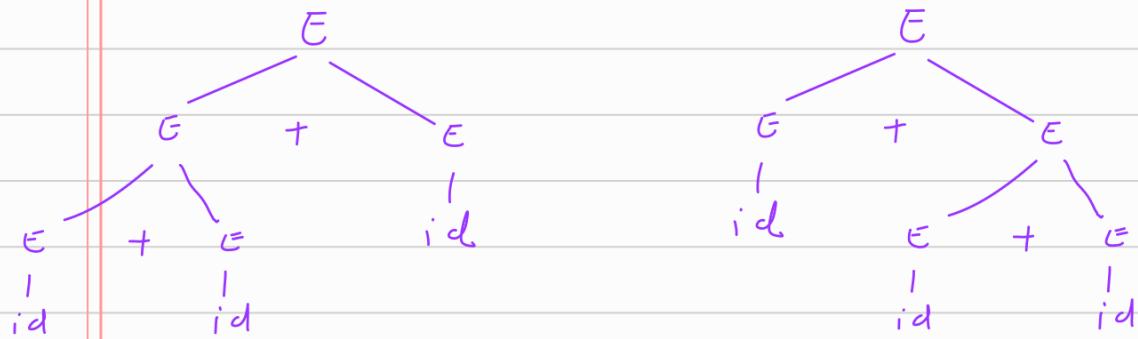
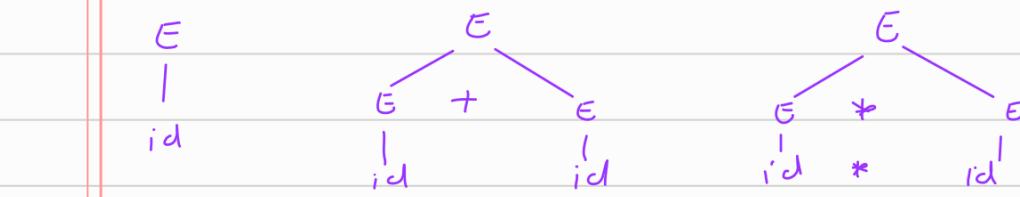
### Inherently ambiguous grammar

- When  $\exists$  more than one LMD, RMD, parse tree for every string of the grammar then the grammar is called ambiguous grammar.

Ex-  $S \rightarrow aSbS / bSaS / \epsilon / ss$



E  $\rightarrow E \rightarrow E + E / E * E / id$  identify it as ambiguous or inherently ambiguous?



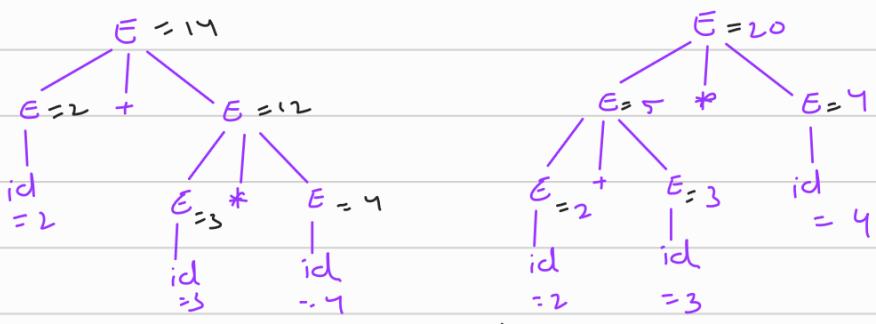
∴ Ambiguous grammar

$E \rightarrow E + E / E - E / E * E / E / E^E / (E) / id$

This is also ambiguous

- Ambiguous grammar can not be converted to unambiguous grammar
- But for some languages, we can have both ambiguous & unambiguous grammar

$2 + 3 * 4$



More than 1 result ∴ Ambiguous.

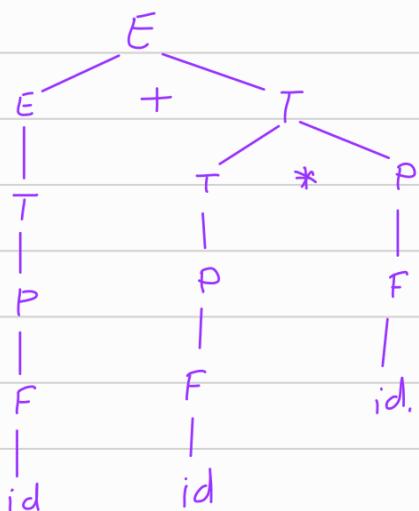
- As we move deeper operator precedence must increase as subtree will be solved first.

Unambiguous grammar for the Expression →

Precedence ↑	Operator	Associativity
(1)	+, -	left associativity
(2)	*, /	left associativity
(3)	^	Right - .. -

$$\begin{array}{ll}
 E \rightarrow E + T / E - T / T & - \\
 T \rightarrow T * P / T / P / P & - \\
 P \rightarrow F ^ P / F & - \\
 F \rightarrow (E) / \text{id.} & -
 \end{array}
 \quad \left. \begin{array}{l} - \\ - \\ - \end{array} \right\} \text{Alg to associativity}$$

Checking if it is now ambiguous or not chk for  $\text{id} + \text{id} * \text{id}$



only one ∴ not ambiguous

Ex-  $E \rightarrow E * T / T$       left ass.

$T \rightarrow F + T / F$       right ass.

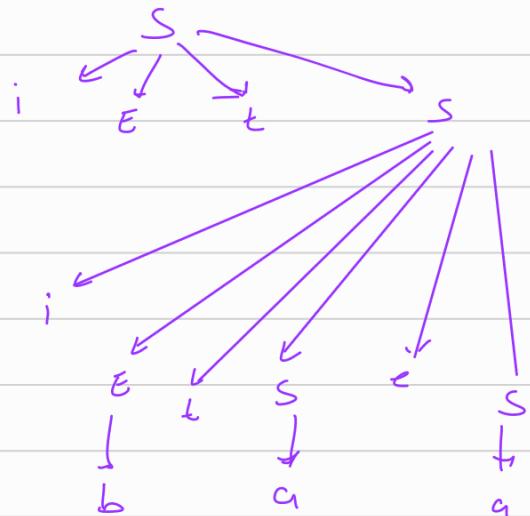
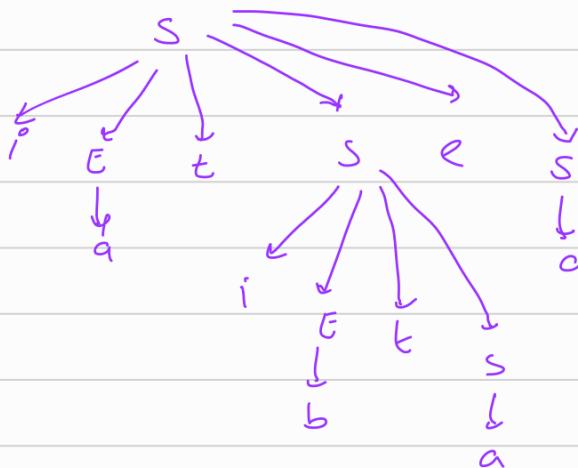
$F \rightarrow \text{id}$

$\text{prec. } + > \text{prec. } *$

Dangling if else structure  $\rightarrow$

$$S \rightarrow iETs / iETseS / a$$
$$E \rightarrow b$$

Ambiguous or not



$\therefore$  Ambiguous

parsing  $\rightarrow$

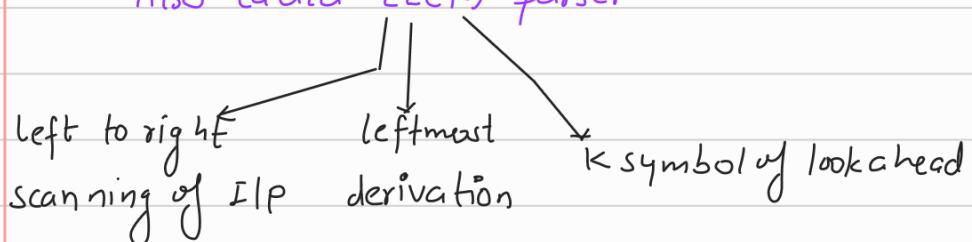
1. Top-down parser  $\rightarrow$  (left Most Derivative)

(a) Recursive descent parser  $\rightarrow$

To design this parser we write recursive subroutine code for all variables of the grammar.

(b) Non Recursive predictive parser  $\rightarrow$

Also called LL(k) parser



Before designing parser 1st simplify grammar.

(1) Remove left recursion

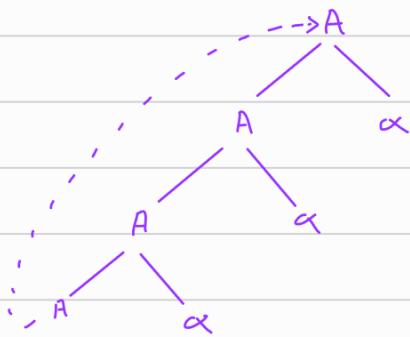
(2) Left factoring

(1) left recursion  $\rightarrow$

$$A \rightarrow A\alpha / \beta$$

(Compiler ko nahi bola ki  $\beta$  le to vo nahi lega)

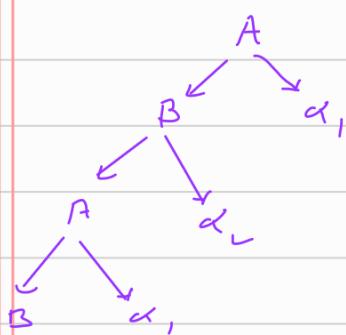
direct left recursion



$$A \rightarrow B\alpha_1 / \beta_1$$

$$B \rightarrow A\alpha_2 / \beta_2$$

indirect left recursion



Remove direct left recursion  $\rightarrow$

$$A \rightarrow A\alpha / \beta \Rightarrow L = \{ \beta\alpha^n \mid n \geq 0 \}$$

- We convert left recursion to right recursion

$$A \rightarrow \beta A'$$

$$\Rightarrow L = \{ \beta\alpha^n \mid n \geq 0 \}$$

$$A' \rightarrow \alpha A' / \epsilon$$

Ex.  $A \rightarrow A\alpha_1 / A\alpha_2 \dots / A\alpha_n / \beta_1 / \beta_2 \dots / \beta_m$

↓ convert to right rec.

$$A \rightarrow \beta_1 A' / \beta_2 A' \dots / \beta_m A'$$

$$A' \rightarrow \alpha_1 A' / \alpha_2 A' \dots / \alpha_n A' / \epsilon$$

## Remove indirect left recursion

- Convert indirect left recursion to direct left recursion
- Now remove direct left recursion.

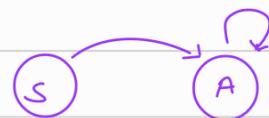
Ex:  $S \rightarrow AS/a$

$$A \rightarrow \boxed{S} A / b$$

LR

$$A \rightarrow (AS/a)A/b$$

$$A \rightarrow AS.A/aA/b$$



∴ new grammar

$$S \rightarrow AS/a$$

$$A \rightarrow \boxed{AS} A / \underbrace{(aA)}_{\alpha} / \underbrace{b}_{\beta}$$

$\alpha$        $\beta$

??

$$A \rightarrow aAA'/bA'$$

$$A' \rightarrow SA'A'/\epsilon$$

Ex:  $E \rightarrow \boxed{E} + T / T$

$\alpha$        $\beta$

$$T \rightarrow T * F / F$$

$$F \rightarrow (E) / id$$

$\alpha$  = recursion se ;  $\beta$  = bino rec.

$$E \rightarrow TE'$$

$$E' \rightarrow +T E'/\epsilon$$

$$T \rightarrow FT'$$

$$T' \rightarrow *FT'/\epsilon$$

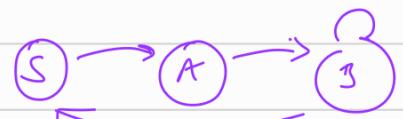
$$F \rightarrow (E) / id$$

Ex:  $S \rightarrow Aa/d$

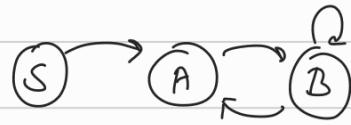
$$A \rightarrow Bb/a$$

$$B \rightarrow \boxed{Sa} B / Bb / b$$

recursive point



convert to direct recursion

$S \rightarrow Aa/d$  $A \rightarrow Bb/a$  $B \rightarrow \boxed{A}aB / daB / Bb/b$  $S \rightarrow Aa/d$  $A \rightarrow Bb/a$  $B \rightarrow B \underbrace{baaB}_{\alpha_1} / \underbrace{aaaB}_{\beta_1} / \underbrace{daB}_{\beta_2} / \underbrace{Bb}_{\alpha_2} / \underbrace{b}_{\beta_3}$  $B \rightarrow aaaBB' / daBB' / bB'$  $B' \rightarrow baB B' / bB' / E$ 

→ We can process right recursion using parser ??

Yes, because LL(k) scan from left to right in non rec. predictive parser.

Left factoring →

- We have to find the left most part common in the set of production of a variable.

$$A \rightarrow \alpha_B / \alpha_{B_1} / \dots / \alpha_{B_n}$$
 $A \rightarrow \alpha_{A'}$  $A' \rightarrow B_1 / B_2 / B_3 / \dots / B_n$ 

Ex  $S \rightarrow iEtSeS / iEts / a$

 $E \rightarrow b$ 

$\downarrow$  left factoring

 $S \rightarrow iEtSS' / a$  $S' \rightarrow eS / E$  $E \rightarrow b$

Ex.  $S \rightarrow aAb/aAd/aB/b$   
 $A \rightarrow bA/bB/b$   
 $B \rightarrow bB/bA/bBd/bSa/a$

$S \rightarrow as'/b$   
 ~~$s' \rightarrow Ab/Ad/B \rightarrow$~~   
 $s' \rightarrow As''/B$   
 $s'' \rightarrow b/d$   
 $A \rightarrow bA'$   
 $A' \rightarrow A/B/C$   
 $B \rightarrow bB'/(AB'')$   
 $B' \rightarrow B/Bd/Sa/BB'''/Sa$   
 $B'' \rightarrow A/E$   
 $B''' \rightarrow d/E$

### Non recursive predictive parser

- First(X) →
- It is a set of all terminals from where 'X' can begin

E	$E \rightarrow TE'$	variables	First	Follow
	$E' \rightarrow +TE'/E$	E	{(, id)}	{\$, )}
	$T \rightarrow FT'$	E'	{+, E}	{\$, )}
	$T' \rightarrow *FT'/E$	T	{(, id)}	{+, \$, )}
	$F \rightarrow (E)/id$	T'	{*, E}	{+, \$, )}
		F	{(, id)}	{*, +, \$, )}

First =

$E \rightarrow \underline{TE'}$

$T \rightarrow \underline{FT'}$

$\therefore E \rightarrow \underline{FT'E'}$

$\hookrightarrow (E) T'E'$

$id T'E'$

Follow

$E \rightarrow E$  ke right dekho in right hand of production.

$T \rightarrow TE'$  aly prod. where  $E'$  can begin  
 $\hookrightarrow T+TE'$

$TE = T$  (follow of E)

$E' \rightarrow$  preeche kuch nahi  $\therefore$  parent ka lo

- Include ' $\epsilon$ ' in first but not in follow

Follow( $X$ )  $\rightarrow$

- It is a set of all terminals that follow variable " $X$ ".
- Add "\$" in starting of starting variable
- " $\epsilon$ " can't be included in the follow of any variable

Ex-  $S \rightarrow A B C$

$$A \rightarrow aA / \epsilon$$

$$B \rightarrow bB / Ad / \epsilon$$

$$C \rightarrow AB / SC / d$$

→  $A = G$  in S

→  $A = G, B = \epsilon$  in C

Variables	First	Follow
S	{a, b, d, $\epsilon$ }	{\$, a, b, d}
A	{a, $\epsilon$ }	{\$, a, b, d}
B	{b, a, d, $\epsilon$ }	{\$, a, b, d}
C	{a, b, d, $\epsilon$ }	{\$, a, b, d}

for Follow  $S \subseteq$

$\therefore$  first of C

for follow A

$A \underline{d}, A \underline{B}$   $\therefore$  first of B, if  $B = \epsilon$   $A \subseteq$

$\therefore$  first of C

follow of C -

$\therefore$  parent as nothing

Ex-  $S \rightarrow aA / BC$

$$A \rightarrow bAd / \epsilon$$

$$B \rightarrow aS / cb / \epsilon$$

$$C \rightarrow Ae / bB / Sd / \epsilon$$

Variable	first	follow
S	{a, b, e, d, $\epsilon$ }	{\$, d, $\overset{\text{follow } B}{a, b, e}$ , $\overset{\text{follow } S}{\epsilon}$ }
A	{ $\epsilon$ , b}	{d, e, \$, $\overset{\text{follow } S}{a, b}$ }
B	{a, $\epsilon$ , b, d, e}	{a, b, e, d, \$}
C	{a, b, e, d, $\epsilon$ }	{a, b, e, d, \$}

First( $X$ )  $\rightarrow$

S1  $\rightarrow$  If  $X$  is a terminal then include it in first( $X$ )

S2  $\rightarrow$  If  $X$  is deriving  $\epsilon$ , then first( $X$ ) includes ' $\epsilon$ '

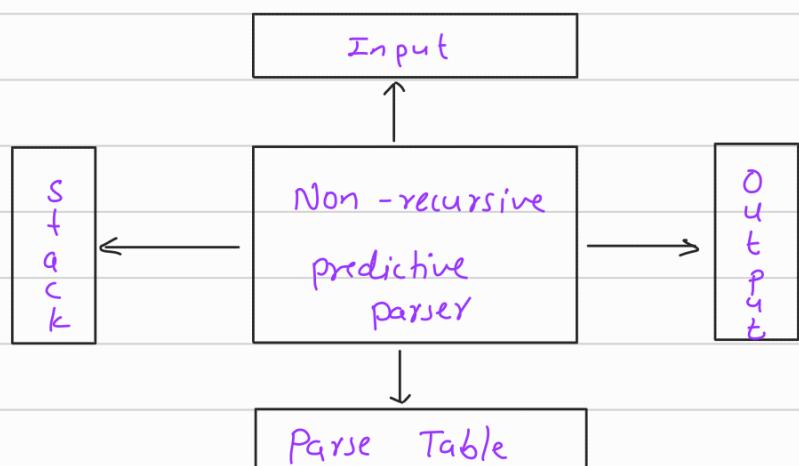
S3  $\rightarrow$  If  $X \rightarrow Y_1 Y_2 \dots Y_n$  is a production then include first of  $Y_1$  in the first of  $X$  except  $\epsilon$ , if  $\epsilon \in \text{First}(Y_2)$  then include first( $Y_2$ ) in first( $X$ ) ...

S4. If  $i \in [1, n] \in \text{first}(y_i)$  then  $\text{first}(x) = \text{first}(y_1) \cup \text{first}(y_2) \dots \cup \text{first}(y_n) \cup \{\epsilon\}$

Follow(x)  $\rightarrow$

- S1 Add "\$" in the follow of starting variable
- S2 If  $A \rightarrow \alpha B \beta$  is a production then include first of  $\beta$  in the follow of  $B$  (except ' $\epsilon$ ')
- S3 If  $A \rightarrow \alpha B \beta$  where ' $\epsilon$ '  $\in \text{first}(\beta)$  then include follow of  $A$  in the follow of  $B$

LL(k) parser  $\rightarrow$



Parse table for LL(1) parser  $\rightarrow$

$$\begin{array}{lll}
 E \rightarrow TE' & T \rightarrow FT' & F \rightarrow (E) / \text{id} \\
 E' \rightarrow +TE'/E & T' \rightarrow *FT'/E &
 \end{array}$$

Variable $\downarrow$	Terminals						$\neq$
	+	*	id	(	)		
E	-	-	$E \rightarrow TE'$	$E \rightarrow TE'$	-	-	-
E'	$E' \rightarrow +TE'$	-	-	-	-	$E' \rightarrow E$	$E' \rightarrow E$
T	-	-	$T \rightarrow FT'$	$T \rightarrow FT'$	-	-	-
T'	$T' \rightarrow E$	$T' \rightarrow *FT'$	-	-	$T' \rightarrow E$	$T' \rightarrow E$	
F	-	-	$F \rightarrow \text{id}$	$F \rightarrow (E)$	-	-	

- All black entry represent error

- Agar  $\epsilon$  hai to follow of parent walo me likho warna first me

## Algorithm to construct parse table

1. Create a table M where all the row headers are represented by variables and terminals represented in column
2. For each production  $A \rightarrow \alpha$  do
  - (i) Compute first of  $\alpha$  and for each  $a \in \text{first}(\alpha)$  do  
 $M[A, a] = \{ A \rightarrow \alpha \}$
  - (ii) If  $\alpha$  derives ' $E'$  then compute follow of ( $A$ ) and for each  $b \in \text{follow}(A)$  do  
 $M[A, b] = \{ A \rightarrow \alpha \}$
3. Return M

<u>LL(L) parsing</u> $\rightarrow$ M [TOS, ip]	Input	Output
\$ $\rightarrow$ TOS Stack	id + id * id \$ ↑ ip	To rep. end of IP string $E \rightarrow TE'$
\$ $E' T \rightarrow$ (in reverse) TOS	id + id * id \$ ↑ ip	$T \rightarrow FT'$
\$ $E' T' F$ TOS	id + id * id \$ ↑ ip	$F \rightarrow id$
\$ $E' T' id$ TOS	id + id * id \$ ↑ ip	id matched
\$ $E' T$ ↑	+ id * id \$ ↑	$T' \rightarrow G$
\$ $E'$ ↑	+ id * id \$ ↑	$E' \rightarrow + TE'$
\$ $E' T +$ ↑	+ id * id \$ ↑	'+' matched
\$ $E' T$ ↑	id * id \$ ↑	$T \rightarrow FT'$
\$ $E' T' F$ ↑	id * id \$ ↑	$F \rightarrow id$
\$ $E' T' id$ ↑	id * id \$ ↑	'id' matched

$\$ E' T'$	$* id \$$	$T' \rightarrow *FT'$
$\$ E' T' F *$	$* id \$$	'*' matched
$\$ E' T' F$	$id \$$	$F \rightarrow id$
$\$ E' T' id$	$id \$$	'id' matched
$\$ E' T'$	$\$$	$T' \rightarrow E$
$\$ E'$	$\$$	$E' \rightarrow E$
$\$$	$\$$	Accepted

Ex- id id

Stack	Input	Output
$\$ E$	id id \$	$E \rightarrow TE'$
$\$ E' T$	id id \$	$T \rightarrow FT'$
$\$ E' T' F$	id id \$	$F \rightarrow id$
$\$ E' T' id$	id id \$	id match
$\$ E' T'$	id \$	Error

Parsing algorithm  $\Rightarrow$  L(L)

Input : Parse table M, input string str

- Push \$ & starting variable in 'S', where 'S' is a starting variable
- Place \$ at the end of str : str = str + \$
- let 'a' is top of stack & 'b' is ip ie input pointer that point to str and initially points to 1st letter of str.
- Repeat following step until a = b = \$
  - If a = b , pop stack & advance input pointer "ip"
  - else if let A  $\rightarrow \alpha$  is a production at location M[c,r]

Then pop the stack & push  $\alpha$  in reverse order,  
& Output is  $A \rightarrow \alpha$

(C) else error

LL(1) Grammer → 1 Char considering at time of parsing each time  
(top 1 in stack)

When each location of parse table is having at most 1 entry / production  
then such grammer is called LL(1) grammer

Eg:  $E \rightarrow TE'$   
 $E' \rightarrow +TE'/E$   
 $T \rightarrow FT'$   
 $T' \rightarrow *FT'/E$   
 $F \rightarrow (E)/id$

} This is LL(1) Lk table above

Eg:  $S \rightarrow iEtSS'/a$  Find given grammer is LL(1) or not!  
 $S' \rightarrow eS/E$   
 $E \rightarrow b$

Variables	First	Follow
$S$	{i, a}	{\$, e}
$S'$	{e, E}	{\$, e}
$E$	{b}	{t}

Parse table →

Variable	Terminal					
	i	t	a	e	b	\$
$S$	$S \rightarrow iEtSS'$		$S \rightarrow a$			
$S'$						$S' \rightarrow E$
$E$					$E \rightarrow b$	

∴ 2 present in same location ∴ Not LL(1) grammer

## LL(1) Test

(i) When each variable of grammar has a single production then such grammar is always LL(1) grammar

(ii) When  $A \rightarrow \alpha / \beta$

$$(i) \text{First}(\alpha) \cap \text{First}(\beta) = \emptyset$$

(ii) If  $\beta \xrightarrow{*} E$  then

$$\text{First}(\alpha) \cap \text{Follow}(A) = \emptyset$$

If both (i) & (ii) are true for the grammar then grammar is LL(1) grammar

(iii)  $S \rightarrow AaBb / BbAa$

$A \rightarrow E$

$$\therefore F(\alpha) = a, F(\beta) = b$$

$B \rightarrow E$

$$\therefore F(\alpha) \cap F(\beta) = \emptyset \therefore \text{LL}(1)$$

(iv)  $S \rightarrow AaBb / BbAb$

$$\therefore F(\alpha) = a, F(\beta) = a$$

$A \rightarrow E$

$$\therefore F(\alpha) \cap F(\beta) = a \neq \emptyset$$

$B \rightarrow E$

$\therefore \text{Not LL}(1)$

(v)  $S \rightarrow AaAb / BbBa$

$A \rightarrow E$

LL(1) ✓

$B \rightarrow E$

(vi)  $S \rightarrow Ab / BC$

$A \rightarrow aA / E$

$$F(\alpha) = a, b$$

$B \rightarrow bB / E$

$$F(\beta) = b, d$$

$C \rightarrow dSb / E$

$$F(\alpha) \cap F(\beta) = b \neq \emptyset \therefore \text{Not LL}(1)$$

(vii)  $S \rightarrow Ab / BC$

$A \rightarrow aA / E$

LL(1)



$B \rightarrow bB / E$

$\xrightarrow{*} E$

$C \rightarrow dSb / E$

$\hookrightarrow \text{Follow}(S) = b$

$$\text{First}(\alpha) = b$$

cond<sup>n</sup> 2 violate

(viii) If a grammar is ambiguous it can never be LL(k) grammar

$$\begin{array}{l} \therefore S \rightarrow iEtSc' / a \\ S' \rightarrow eS/E \\ E \rightarrow b \end{array} \quad \left. \begin{array}{l} \text{is Ambiguous} \\ \therefore \text{not } LL(k) \end{array} \right\}$$

(ix) If grammar is unambiguous then it may be LL(k) grammar

Note: If grammar is LL(1) it will be LL(k) but vice versa need not to be always true.

Eg  $S \rightarrow aA/bB$

$A \rightarrow b$

$B \rightarrow d$

LL(2)  $\rightarrow$  Combination of <sup>all</sup> 2 terminal in parse table

is Not LL(1) but LL(2)  $\because$  one prod. placed at ab & other at bd

Bottom Up parser  $\rightarrow$  (Also called Shift reduce parser)

Operator precedence

LR parser

parser

Direct

Indirect

\* For ambiguous grammar

\* For unambiguous grammar

\* Using precedence & associativity of operator.

\* Using leading & trailing

## Operator precedence parsing

- We require operator precedence grammar for operator precedence parsing.
- Operator grammar -
- When right hand part of every production is not having two consecutive variables then such grammar is called operator grammar  
violate operator grammar

Ex-	$A \rightarrow aAbB$	X
	$A \rightarrow a A B b$	✓
	$A \rightarrow A a b B$	X
	$A \rightarrow C$	X
	$A \rightarrow A a B$	X
	$A \rightarrow B C$	✓

## Operator precedence grammar

- $\epsilon$ -free operator grammar is called operator precedence grammar

$$\text{Operator grammar} = \text{Operator precedence grammar} + \epsilon$$

- Why  $\epsilon$  free?

## Operator precedence parsing

- Direct method -
- It is used for the ambiguous grammar, we use associativity & precedence of operator.
- There are 3 types of precedence we have to use  $\cdot >, <, \doteq$

$T_1 < T_2 \rightarrow$  Terminal  $T_1$  has lower precedence than  $T_2$

$T_1 \cdot > T_2 \rightarrow$  Terminal  $T_2$  has more precedence than  $T_1$

$T_1 \doteq T_2 \rightarrow$  Both  $T_1$  &  $T_2$  have same precedence.

## Expression grammar →

- Example of expression grammar

$$E \rightarrow E + E / E * E / (E) / id$$

$$E \rightarrow E + E / E - E / E * E / E / 'E' / E^E / (E) / id$$

## Parse (Precedence) table →

$$E \rightarrow E + E / E - E / E * E / E / 'E' / E^E / (E) / id$$

T2 (R)

	id	+	-	*	/	^	(	)	\$
T1 (L)	id	Error	>	>	>	>	>	>	>
	+	<	> L to R	>	<	<	<	<	>
	-	<	>	>	<	<	<	>	>
	*	<	>	>	>	>	<	>	>
	/	<	>	>	>	>	<	>	>
	^	<	>	>	>	>	<	>	>
	(	<	<	<	<	<	<	=	Error
	)	Error	>	>	>	>	Error	>	>
	\$	<	<	<	<	<	<	Error	E-free No-case

Precedence

↑  
Precedence  
↓

+,-  
\*,/  
^

Associativity

L→R

L→R

R→L

Ex: \$ id + id \* id \$      id → need to be identified first

↓

\$ <. id .> + <. id .> \* <. id .> \$

↑  
Should be valid  
E → id

\$ E + <. id .> \* <. id .> \$

↓  
↓

$\$ E + E \cdot * <. id .> \$$

$\$ E + E * E \$$

After replaced by variable then we remove all variable from expression to identify precedence

$\$ ++ \$$

$\$ <. + .> * > \$$

∴ not error in ++

### Operator precedence parsing →

- Input string 'str' and precedence table or parse table

S1 → Add \$ at the begin and ending of string 'str'

Eg: \$ id + id \* id \$

S2 → Fill the precedence b/w every pair of consecutive terminal in the string String \$ str \$

Eg. \$ <. id .> + <. id .> \* <. id .> \$

S3 → Start from left most '\$' and move toward the right most '\$'.

- find first greater precedence while moving in right dirn & then move left dirn & find first less precedence "<."
- the substring b/w "< . >" should be a valid handle
- If so replace it with left hand variable in production i.e for  $A \rightarrow \text{handle}$ , handle replaced by A

Handle  $\rightarrow$  it is a valid substring present at right side of production set.

$$E \rightarrow E + E / E * E / (E) / id$$

The diagram shows the production rule  $E \rightarrow E + E / E * E / (E) / id$ . Three arrows originate from the right-hand side of the rule and point to the tokens '+', '\*', and ')' respectively, indicating they are part of the handle.

Production on right hand part of production.

S4  $\rightarrow$  Repeat step 3 until we have string or expression with variables and operators

S5  $\rightarrow$  Remove all the variable from the resulting expression  
Now we have to find precedence b/w operator to parse the expression

S6  $\rightarrow$  Now again find the precedence b/w consecutive pair of terminals of expression of step 5

$\$ < . + < . * > \$$

S7  $\rightarrow$  Start from left most  $\$$  & move toward the right most  $\$$  and find  $>$  then move left to find  $<$

$\$ < . + < . * > \$$

The diagram shows the expression  $\$ < . + < . * > \$$  with a horizontal double-headed arrow underneath it, spanning the entire width of the expression, indicating the search range from left to right for the operators  $>$  and  $<$ .

S8  $\rightarrow$  The operator b/w  $< . & . >$  will be parsed first for that substitute all the variable in the expression in same order

$\$ E + E * E \$$

- Now check  $E * E$  is valid handle or not if yes then replace it with left hand variable else error

$\therefore E \longrightarrow E * E \quad \therefore \text{replace}$

exp:  $\$ E + E \$$

Sq → Repeat step 5 to 8 until we get  $\$ S \$$  where  $S$  is starting variable, It indicates end of parsing.

$\$ + \$$   
 $\$ < . + > \$$   
 $\xrightarrow{\leftarrow \rightarrow}$   
 $\$ E + E \$$

$\$ E \$ \longrightarrow \text{end of parsing.}$

(b) Indirect method →

- It is used for unambiguous grammar

(i) leading →

$\text{leading}(A) = \{ a \mid A \xrightarrow{*} \alpha a \beta \text{ where } \alpha \text{ is a single variable or 'E'} \}$ .

or

$\text{leading}(A) = \{ a \mid A \rightarrow a\beta \text{ or } A \rightarrow B a\beta \text{ or } A \rightarrow B \text{ and } \forall a \in \text{leading}(B) \}$

(ii) Trailing →

$\text{Trailing}(A) = \{ a \mid A \xrightarrow{*} \alpha a \beta \text{ where } \beta \text{ is a single variable or 'E'} \}$

or

$\text{Trailing}(A) = \{ a \mid A \rightarrow \beta a \text{ or } A \rightarrow B a \beta \text{ or } A \rightarrow B \text{ and } \forall a \in \text{trailing}(B) \}$

Scan from L→R in production  
& place  $Z^+$  variable in leading  
for trailing just move from R→L

Variables	Leading	Trailing
E	{+, -, *, /, ^, (, id}	{+, -, *, /, ^, ), id}
T	{*, /, ^, (, id}	{*, /, ^, ), id}
P	{^, (, id}	{^, ), id}
F	{(, id}	{), id}

Parse table →

- Move production by product

$$E \rightarrow E + T$$

$\downarrow$

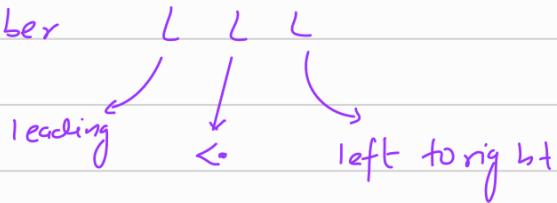
first term of  $E$

E +  
L R

$\therefore$  Select + from Right  
of table right= trail

∴ we need to fix trailing → Always fix greater precedence of E in parse table

for leading remember



$$E \rightarrow E \underset{L}{+} \underset{R}{T} \quad \text{leading}(T) \text{ place less than}$$

- Repeat same for each production

\*  $A \rightarrow \alpha a B b \beta$

where  $B$  may be ' $\epsilon$ ' then  $a = b$

$$\therefore F \rightarrow (E)$$

Algorithm to fill precedence table →

Input: Grammar

S1 → Compute leading & trailing of all variables

S2 → For each production  $A \rightarrow \delta$  do

(a) if  $A \rightarrow \alpha \boxed{a} B \beta$  then

    if  $b \in \text{leading}(B)$  do

        fill  $a < b$  in precedence table

(b) if  $A \rightarrow \alpha \boxed{B} a \beta$  then

    if  $b \in \text{trailing}(B)$  do

        fill  $b > a$  in precedence table

(c) if  $A \rightarrow \alpha a B b \beta$  where  $B$  is a single variable or  $\epsilon$

        fill  $a = b$  in precedence table

S3 → Return precedence table

## Precedence Graph →

- Input: precedence table
  - 1. Assign function name  $f\#$  to all the terminals' '#' present at row index of precedence table & assign  $G\#$  function name to all the terminals '#' present at column index.
  - 2. For each terminal  $f\#$  &  $g\#$  create a separate node and create single node for all  $f\#, g\#$ , which have  $\#_1 \doteq \#_2$  in precedence table
- Eg.  $f_{id}, g_{id}, f_+, g_+$ , & so on for them we create single separate node while  $f_c$  &  $g_c$  will be in a single node
3. Create edges b/w nodes of  $f\#$  &  $g\#$  as follows

(a) if  $f\# \geq g\#$  then

create a edge from



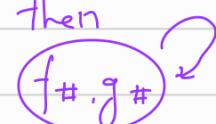
(b) if  $f\# < g\#$  then

create a edge from



(c) if  $f\# \doteq g\#$  then

self loop

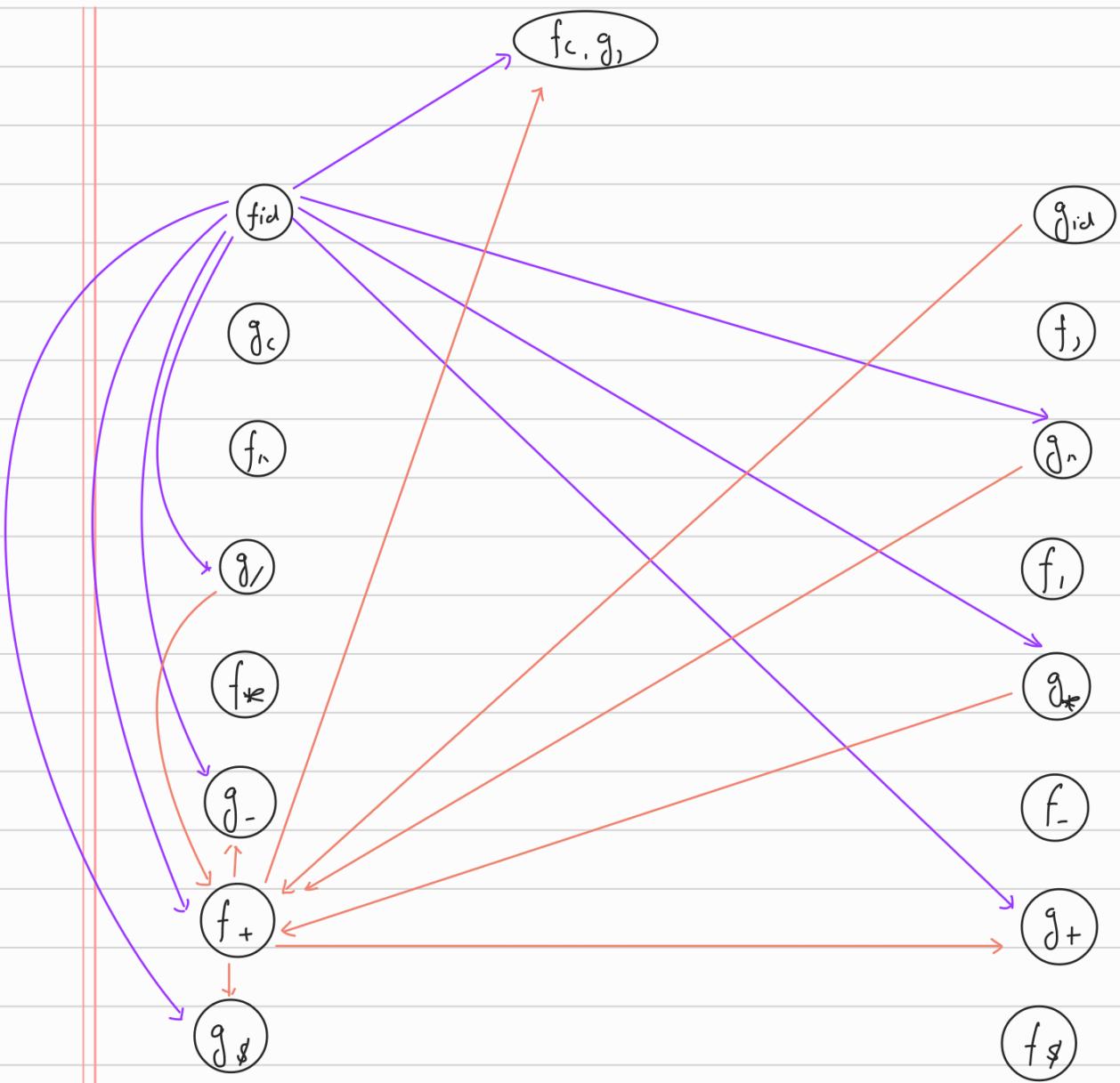


4. Return precedence graph

	id	+	-	*	/	^	(	)	\$
id		>	>	>	>	>		>	>
+	<		>	>	<	<	<	>	>

To construct graph we move from higher precedence to lower precedence & assign the node in alternate way.  $\Rightarrow f \quad g$

$g \quad f$



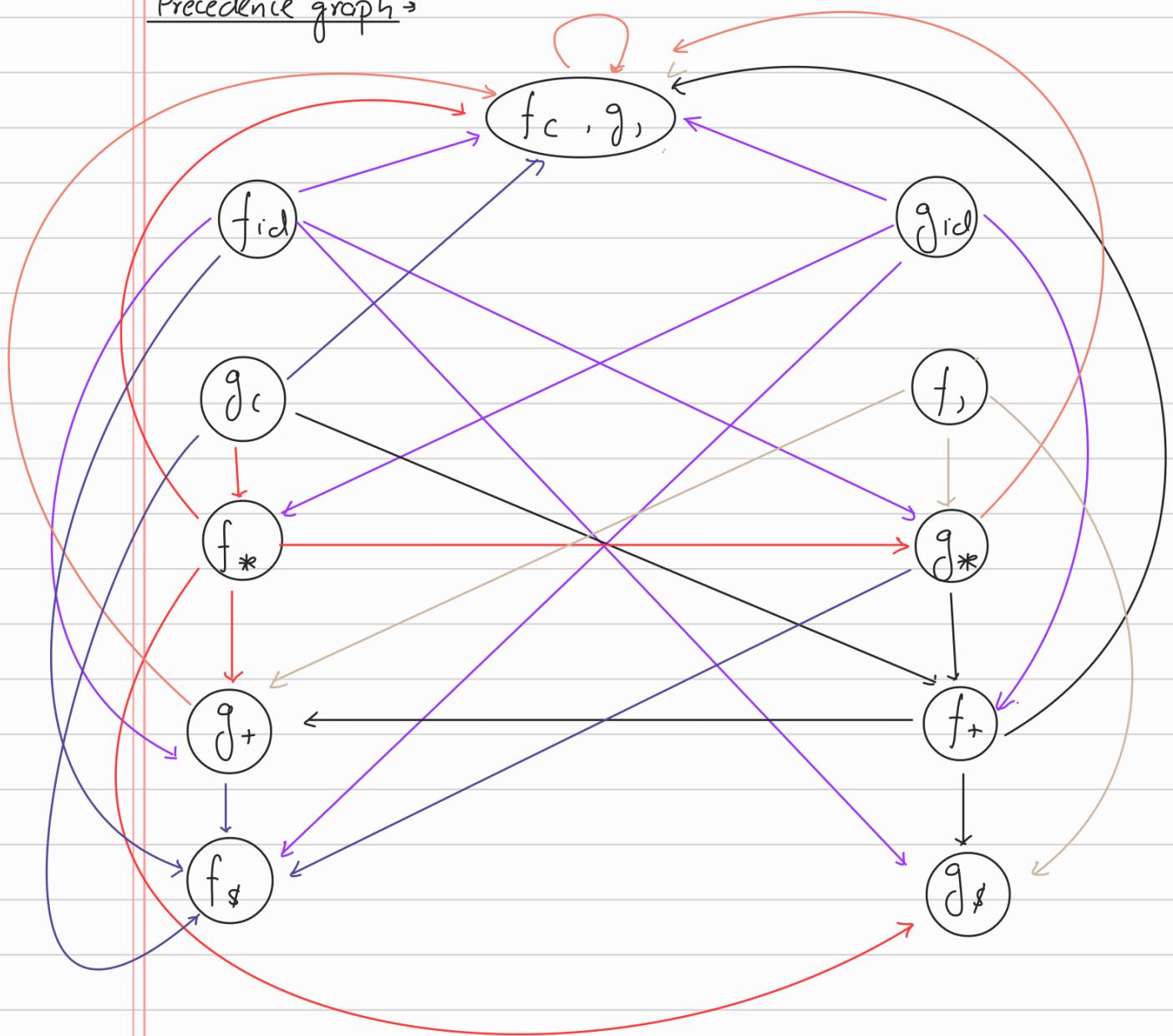
Ex	$E \rightarrow E + T / T$	Variable	Leading	Trailing
	$T \rightarrow T * F / F$	$E$	$+, *, C, id$	$*, + ), id$
	$F \rightarrow (E) / id$	$T$	$*, C, id$	$*, ), id$
		$F$	$C, id$	$), id$

Prec. table →

	$g_{id}$	$g_+$	$g_*$	$g()$	$g()$	$g_()$	$g_*$
$f_{id}$		$>$	$>$		$>$	$>$	
$f_+$	$<$	$>$	$<$	$<$	$>$	$>$	
$f_*$	$<$	$>$	$>$	$<$	$>$	$>$	
$f()$	$<$	$<$	$<$	$<$	$\equiv$		
$f()$		$>$	$>$		$>$	$>$	
$f_*$		$<$	$<$	$<$			
-		$<$	$<$	$<$			

\$E\\$

Precedence graph →



Precedence function →

- longest path from node 'n' represents precedence function of n.

Function	id	(	)	*	+	\$
f	4	0	4	4	2	0 → for sinks
g	5	5	0	3	1	0

longest path length  
chk from upper levels  
par node true - barabar se

If 3 no cycle in graph that mean the grammar is operator precedence grammar.

$$\text{Ex} \rightarrow S \rightarrow (L) / a$$

$$L \rightarrow L, S / S$$

$T_1 \vee T_2$        $T_1 = T_2$

Variable	leading	Trailing
S	{( a }	{ ) a }
L	{ , c a }	{ , ) a }

	$g_a$	$g_c$	$g_{)}$	$g,$	$g_{\$}$
$f_a$			$\Rightarrow$	$\Rightarrow$	$\Rightarrow$
$f_c$	$<.$	$<.$	$\doteq$	$<.$	
$f_{)}$			$\Rightarrow$	$\Rightarrow$	$\Rightarrow$
$f,$	$<.$	$<.$	$\Rightarrow$	$\Rightarrow$	
$f_{\$}$	$<.$	$<.$			

$\$ s \$$   
 ↗ ↘

## - Operator precedence parsing algorithm →

Input: Precedence table and input string

- S1 → Push \$ on the stack and add \$ at the end of the string.
- S2 → a := TOS and b := value of current input pointer that points I/p string, initially 1<sup>st</sup> letter of string
- S3 → Repeat following steps until a = b = \$  
do

Shift Action (a) If a < b or a = b then push b on the stack  
∴ a := b & b := advance i/p pointer value.

Reduce Act. (b) If a > b then repeat pop from stack until current TOS is having  $\Rightarrow$  than recently popped symbol

(c) else error

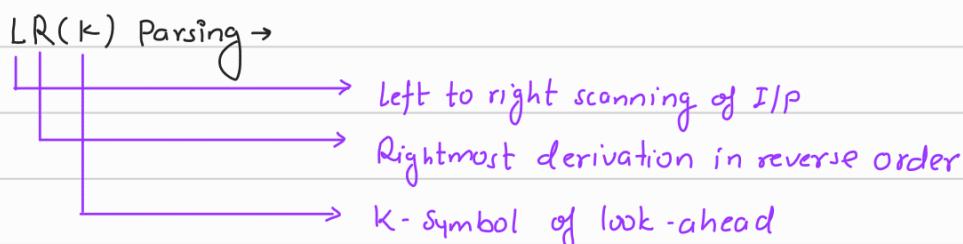
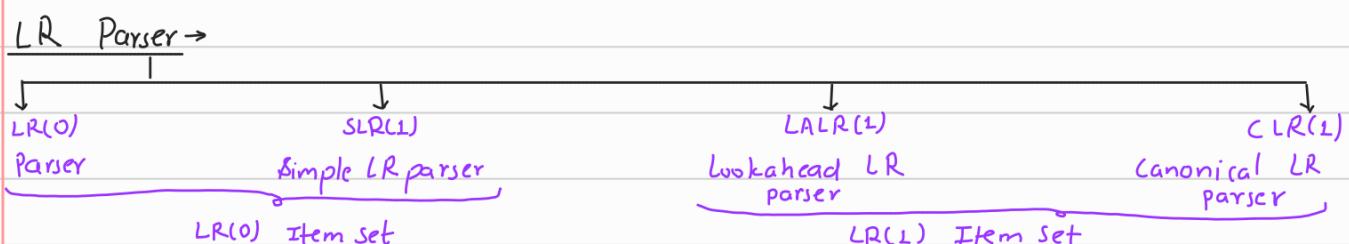
Ex \$ id + id \* id \$

Stack  
\$

Input  
id + id \* id

Output  
Shift id

For understanding $\$ < \cdot id$ $\downarrow$	$+ id + id * id$ $\downarrow$	Reduce
$\$$	$+ id * id \$$	Shift +
$\$ < \cdot +$	$id * id \$$	Shift + id
$\$ < \cdot + < \cdot id$ $\downarrow$	$* id \$$	Reduce
$\$ < \cdot +$	$* id \$$	Shift *
$\$ < \cdot + < \cdot *$	$id \$$	Shift id
$\$ < \cdot + < \cdot * < \cdot id$ $\downarrow$	$\$$	Reduce
$\$ < \cdot + < \cdot *$ $\downarrow$	$\$$	Reduce
$\$ < \cdot +$ $\downarrow$	$\$$	Reduce
$\$$	$\$$	Accepted



LR(0) Itemset → Not stop when we reach starting variable

Sol<sup>n</sup> → Augmented production

## Augumented production

$S' \rightarrow S$  Stops when reach  $S'$  & we can reach  $S^*$  by only 1 times

## Augumented Grammer

$$\begin{array}{ccc} S \rightarrow AA & \Rightarrow & S' \rightarrow S \\ A \rightarrow aA/b & & S \rightarrow AA \\ & & A \rightarrow aA/b \end{array}$$

follow ( $S'$ ) =  $\emptyset$   
↓  
stop

LR(0) ItemSet → (Place ':' before right hand part of production)  
 {':' is very imp}

First Item

$$I_0: S' \longrightarrow .S$$

$$\left\{ \begin{array}{l} I_n: \dots \dots \\ A \rightarrow \alpha \cdot B \beta \\ B \rightarrow \cdot \delta_1 \\ B \rightarrow \cdot \delta_2 \\ \vdots \\ B \rightarrow \cdot \delta_n \end{array} \right.$$

⇒ . before variable then generate all production all of that variable we have to add in that state

$$\begin{array}{l} I_0: S' \longrightarrow .S \text{ (Goto } I_1) \\ S \longrightarrow .AA \text{ (Goto } I_2) \\ A \longrightarrow .aA \text{ (Goto } I_3) \\ A \longrightarrow .b \text{ (Goto } I_4) \end{array}$$

$$\left\{ \begin{array}{l} I_n: \dots \dots \dots \\ A_1 \rightarrow \alpha_1 \cdot B \beta_1 \\ A_2 \rightarrow \alpha_2 \cdot B \beta_2 \\ \vdots \\ A_n \rightarrow \alpha_n \cdot B \beta_n \end{array} \right.$$

• B  
jinti bhi • B wale  
honge vo chale  
jaengi  
A\_n → α\_n · B β\_n

New state →  $I_1: S' \longrightarrow S.$  No scope for growing ∵ close state

$$\begin{array}{l} I_2: S \longrightarrow A \cdot A \text{ (Goto } I_5) \\ A \longrightarrow .aA \text{ (Goto } I_3) \\ A \longrightarrow .b \text{ (Goto } I_4) \end{array}$$

It can grow further

$$\left\{ \begin{array}{l} I_5: \dots \dots \dots \\ A_1 \rightarrow \alpha_1 B \cdot \beta_1 \\ A_2 \rightarrow \alpha_2 B \cdot \beta_2 \end{array} \right.$$

& move '.' over variable  
(one by one production we check)

$$\begin{array}{l} I_3: A \longrightarrow a \cdot A \text{ (Goto } I_6) \\ A \longrightarrow .aA \text{ (Goto } I_3) \\ A \longrightarrow .b \text{ (Goto } I_4) \end{array}$$

Always start from beginning

$I_4: A \rightarrow b.$

$I_5: S \rightarrow AA.$

$$I_6: \begin{array}{l} A \rightarrow a.A \\ A \rightarrow .aA \\ A \rightarrow .b \end{array} \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{Same as } I_3 \Rightarrow A \rightarrow aA.$$

## LR(0) parse table →

### (i) Compute LR(0) itemset

Ex Grammar

$$S \rightarrow AA \quad A \rightarrow aA/b$$

Augumental  
prod

0.  $S' \rightarrow S$
1.  $S \rightarrow AA$
2.  $A \rightarrow aA$
3.  $A \rightarrow b$

Augumented production

States	a	Action $\downarrow$	\$	s	Go to	A
$I_0$	$S_3$	$S_4$		1		2
$I_1$	Acc.	Acc.	Acc.			
$I_2$	$S_3$	$S_4$				5
$I_3$	$S_3$	$S_4$				6
$I_4$	$H_3$	$H_3$	$H_3$			
$I_5$	$R_1$	$R_1$	$R_1$			
$I_6$	$R_2$	$R_2$	$R_2$			

- There are two types of action in LR parser

#### (i) Shift action →

When '.' is shifted over a terminal in any state then we have to place shift ( $S_n$ ) entry for that terminal where  $n$  is a state where '.' is shifted.

#### (ii) Reduce action →

When '.' is reached at end of the production in any state then we place reduce ( $R_n$ ) entry where 'n' is a production number which is going to reduce

When '.' is at the end of augmented production then we have to place accepted entry

For LR(0) parser we have to place reduce  $M_n$  entries corresponding to all the terminals.

- At most 1 entry in action part of LR(0) parse table  $\therefore$  grammar is LR(0) grammar

### SLR(1) parse table $\Rightarrow$

- (compute LR(0) item set)
- The shift and goto entries of SLR(1) parse table is similar to LR(0) parse table but reduce entries differ

For reduce entries of SLR(1) parse table.

let  $A \rightarrow \alpha.$  is a production in any state then we have to place  $M_n$  entries corresponding to  $\text{follow}(A)$  only where  $\alpha$  is a no. of production

$A$  [Follow]

$\alpha.$  [Follow]

States	a	Action	\$	s	Go to	A
$I_0$	$S_3$	$S_4$		1		2
$I_1$				Acc.		
$I_2$	$S_3$	$S_4$				5
$I_3$	$S_3$	$S_4$				6
$I_4$	$H_3$	$H_3$	$H_3$			
$I_5$				$H_1$		
$I_6$	$H_2$	$H_2$	$H_2$			

Ex- Grammer  $\rightarrow$

$$E' \rightarrow E$$

$$T \rightarrow F$$

$$E \rightarrow E + T$$

$$F \rightarrow (\text{id})$$

$$E \rightarrow T$$

$$F \rightarrow \text{id}$$

$$T \rightarrow T * F$$

LR(0) parse table  $\rightarrow$

States	Action						Goto		
	+	*	(	)	id	\$	E	T	F
I <sub>0</sub>			S <sub>4</sub>		S <sub>5</sub>		1	2	3
I <sub>1</sub>	S <sub>6</sub> Acc.	Acc.	Acc.	Acc.	Acc.	Acc.			
I <sub>2</sub>	g <sub>2</sub>	S <sub>7</sub> g <sub>2</sub>	μ <sub>2</sub>	g <sub>2</sub>	μ <sub>2</sub>	g <sub>2</sub>			
I <sub>3</sub>	μ <sub>4</sub>	μ <sub>4</sub>	μ <sub>4</sub>	μ <sub>4</sub>	μ <sub>4</sub>	μ <sub>4</sub>			
I <sub>4</sub>		S <sub>4</sub>		S <sub>5</sub>			8	2	3
I <sub>5</sub>	μ <sub>6</sub>	μ <sub>6</sub>	μ <sub>6</sub>	μ <sub>6</sub>	μ <sub>6</sub>	μ <sub>6</sub>			
I <sub>6</sub>		S <sub>4</sub>		S <sub>5</sub>				9	3
I <sub>7</sub>		S <sub>4</sub>		S <sub>5</sub>					10
I <sub>8</sub>	S <sub>8</sub>		S <sub>11</sub>						
I <sub>9</sub>	μ <sub>1</sub>	μ <sub>1</sub>	S <sub>7</sub> μ <sub>1</sub>	μ <sub>1</sub>	μ <sub>1</sub>	μ <sub>1</sub>			
I <sub>10</sub>	μ <sub>3</sub>	g <sub>3</sub>	μ <sub>3</sub>	g <sub>3</sub>	μ <sub>3</sub>	g <sub>3</sub>			
I <sub>11</sub>	g <sub>5</sub>	μ <sub>5</sub>	μ <sub>5</sub>	μ <sub>5</sub>	g <sub>5</sub>	μ <sub>5</sub>			

- There are two type of conflict in LR parser

(i) Shift Reduce (SR) conflict

(ii) Reduce Reduce (RR) conflict

SLR(1) Parse table  $\rightarrow$

States	Action						Goto		
	+	*	(	)	id	\$	E	T	F
I <sub>0</sub>			S <sub>4</sub>		S <sub>5</sub>		1	2	3
I <sub>1</sub>	S <sub>6</sub>					Acc.			
I <sub>2</sub>	μ <sub>2</sub>	S <sub>7</sub>		μ <sub>2</sub>		μ <sub>2</sub>			
I <sub>3</sub>	μ <sub>4</sub>	μ <sub>7</sub>		μ <sub>4</sub>		μ <sub>4</sub>			
I <sub>4</sub>		S <sub>4</sub>		S <sub>5</sub>			8	2	3

$I_5$	$H_6$	$H_6$	$H_6$	$S_6$	$S_6$				
$I_6$			$S_4$		$S_5$			9	3
$I_7$			$S_4$		$S_5$				10
$I_8$		$S_6$		$S_{11}$					
$I_9$		$H_1$ , $S_7$		$H_1$		$H_1$			
$I_{10}$	$H_3$	$H_3$		$H_3$		$H_3$			
$I_{11}$	$H_5$	$H_5$		$H_5$		$H_5$			

LR(1) item set  $\rightarrow$

Ex:  $S \rightarrow AA$   $\Rightarrow$  (0)  $S' \rightarrow S$   
 $A \rightarrow aA/b$  Aug. (1)  $S \rightarrow AA$   
(2)  $A \rightarrow aA$   
(3)  $A \rightarrow b$

The computation of LR(1) itemset is similar to LR(0) itemset but in LR(1) we have to add one symbol of look ahead along with the production of itemset.

$I_0 : S' \rightarrow .S, \$ \quad G(I_0)$   
Parent (  $S \rightarrow .AA, \text{first}(E.\$) = \$ \quad G(I_1)$   
 $A \rightarrow .aA, \text{first}(A\$) = a/b \quad G(I_2)$   
 $A \rightarrow .b, \text{first}(A\$) = a/b \quad G(I_3)$

look ahead for newly generated production from parent one  
 $S'$  is highest  $\therefore \$$  is only look ahead  
kharch pani of Aug. prod. is  $\$$

$I_1 :$

$$S' \rightarrow S., \$$$

$I_2 :$

$$S \rightarrow A.A, \$ \quad G(I_5)$$

$$A \rightarrow .aA, \$$$

$$A \rightarrow .b, \$$$

$I_3 :$

$$A \rightarrow a.A, a/b$$

$$A \rightarrow .aA, a/b$$

$$A \rightarrow .b, a/b$$

$I_n : \dots$

$$A \rightarrow a.B\beta, a$$

All prod. with  $.B$  we have to capture in this state only

$$B_1 \rightarrow .S_1, \text{first}(\beta a)$$

$$B_2 \rightarrow .S_2, \text{first}(\beta a)$$

$$\vdots \quad B_n \rightarrow .S_n, \text{first}(\beta a)$$

LALR(1)		<u>LALR(1) itemset →</u>				
$I_4:$	$A \rightarrow b\cdot, a/b$	$I_0:$ Same as LR(1)			(0) $S' \rightarrow S$	
$I_5:$	$S \rightarrow AA\cdot, \$$	$I_1:$ _____    _____			(1) $S \rightarrow AA$	
$I_6:$	$A \rightarrow a\cdot A, \$$ $A \rightarrow \cdot aA, \$$ $A \rightarrow \cdot b, \$$	$I_{36}:$ $A \rightarrow a\cdot A, a/b/\$$ $A \rightarrow \cdot aA, a/b/\$$ $A \rightarrow \cdot b, a/b/\$$	$I_{47}:$ $A \rightarrow b\cdot, a/b/\$$			parse table same as LR(1)
$I_7:$	$A \rightarrow b\cdot, \$$	$I_5:$ Same as LR(1)				reduce differs
$I_8:$	$A \rightarrow aA\cdot, a/b$	$I_{89}:$ $A \rightarrow aA\cdot, a/b/\$$				
$I_9:$	$A \rightarrow aA\cdot, \$$					

Action	S	F
$s_{36}$	1	2
$s_{47}$		
Acc.		
$s_{36}$		
$s_{47}$		
$r_3$		
$r_1$		
$r_2$		

CLR(1) parse table →

↳ Canonical

- The shift & goto entries of CLR(1) parser are similar to SLR(1) parser  
But the reduce entries differs

let in any state  $I_n$  production  $\{A \rightarrow \alpha, a\}$  is going to reduce then we have to place reduce entry  $(y)$  corresponding to look ahead 'a' only where 'y' is the prod<sup>n</sup> number

$I_x: \dots \dots$

$A \rightarrow \alpha\cdot, a$

$(y) A \rightarrow \alpha$

$y$  corr. to  $a$  in row  $n$ .

		Action	Goto
	a	b	\$
$I_0$	$S_3$	$S_4$	(At lookahead) Acc.
$I_1$			
$I_2$	$S_6$	$S_7$	5
$I_3$	$S_3$	$S_4$	8
$I_4$	$R_3$	$R_3$	
$I_5$			$H_1$
$I_6$	$S_6$	$S_7$	9
$I_7$			$H_3$
$I_8$	$H_2$	$H_2$	
$I_9$			$H_2$

NO double entry in action 1 ∵ grammar is CLR(1)

LR(0) item set →

$$\begin{array}{ll}
 \text{Ex} & S \rightarrow L = R / R \\
 & L \rightarrow * R / id \\
 & R \rightarrow L
 \end{array}
 \xrightarrow{\text{Augmented}}
 \begin{array}{l}
 (0) S' \rightarrow S \\
 (1) S \rightarrow L = R \\
 (2) S \rightarrow R \\
 (3) L \rightarrow * R \\
 (4) L \rightarrow id \\
 (5) R \rightarrow L
 \end{array}$$

$S^{LR(1)}$  ✓  
 $L^{LR(1)}$  ✓  
 $(LR(1))$  ✓

$$\begin{array}{ll}
 I_0: & \checkmark S' \rightarrow . S \quad G(I_1) \\
 & \checkmark S \rightarrow . L = R \quad G(I_2) \\
 & \checkmark S \rightarrow . R \quad G(I_3) \\
 & \checkmark L \rightarrow . * R \quad G(I_4) \\
 & \checkmark L \rightarrow . id \quad G(I_5) \\
 & \checkmark R \rightarrow . L \quad G(I_2)
 \end{array}$$

don't open if  
all prod. match

$$I_1: \checkmark S' \rightarrow S.$$

$$I_3: \checkmark S \rightarrow R.$$

$$\begin{array}{l}
 I_2: \checkmark S \rightarrow L = R \\
 \quad \quad \quad \checkmark R \rightarrow . L
 \end{array}$$

$$I_4: L \rightarrow * . R$$

$$R \rightarrow . L$$

$$\begin{array}{l}
 L \rightarrow . * R \\
 L \rightarrow id
 \end{array}$$

$I_5: L \rightarrow \text{id}.$  $I_8: R \rightarrow L.$ 
 $I_6: S \rightarrow L = .R$   
 $R \rightarrow .L$   
 $L \rightarrow . * R$   
 $L \rightarrow . \text{id}$ 
 $I_9: S \rightarrow L = R.$  $I_7: L \rightarrow * R.$ LR(0) Parse table  $\rightarrow$ 

States	Action				Goto		
	=	*	id	\$	S	L	R
$I_0$		$s_4$	$s_5$		1	2	3
$I_1$	Acc	Acc	Acc	Acc			
$I_2$	$s_6$	$h_5$	$h_5$	$h_5$			
$I_3$	$h_2$	$h_2$	$h_2$	$h_2$			
$I_4$		$s_4$	$s_5$		8	7	
$I_5$	$h_4$	$h_4$	$h_4$	$h_4$			
$I_6$		$s_4$	$s_5$		8	9	
$I_7$	$h_3$	$h_3$	$h_3$	$h_3$			
$I_8$	$h_5$	$h_5$	$h_5$	$h_5$			
$I_9$	$h_1$	$h_1$	$h_1$	$h_1$			

SLR(1) parse table  $\rightarrow$ 

States	Action				Goto		
	=	*	id	\$	S	L	R
$I_0$		$s_4$	$s_5$		1	2	3
$I_1$				Acc			
$I_2$	$s_6$	$h_5$			$h_5$		
$I_3$				.	$h_2$		
$I_4$		$s_4$	$s_5$		8	7	
$I_5$	$h_4$			$h_4$			
$I_6$		$s_4$	$s_5$		8	9	

$I_7$	$H_3$	$H_3$
$I_8$	$H_5$	$H_5$
$I_9$		$H_1$

LR(1) itemset →

$$I_{10}: S' \rightarrow .S, \$$$

$$S \rightarrow .L = R, \$$$

$$S \rightarrow .R, \$$$

$$L \rightarrow . * R, =/\$$$

$$L \rightarrow .id, =/\$$$

$$R \rightarrow .L, \$$$

$$L \rightarrow . * R, \$$$

$$L \rightarrow .id, \$$$

within state

$$I_7: S \rightarrow +R., =/\$$$

$$I_8: R \rightarrow L., =/\$$$

$$I_9: S \rightarrow L = R., \$$$

$$I_{10} R \rightarrow L., \$$$

$$I_{11}: L \rightarrow *R., \$$$

$$R \rightarrow .L, \$$$

$$L \rightarrow . * R, \$$$

$$L \rightarrow .id, \$$$

$$I_1: S' \rightarrow S., \$$$

$$I_2: S \rightarrow L. = R, \$$$

$$R \rightarrow L., \$$$

$$I_{12} L \rightarrow id, \$$$

$$I_3: S \rightarrow R., \$$$

$$I_4: L \rightarrow *R., =/\$$$

$$R \rightarrow .L, =/\$$$

$$L \rightarrow *R., =/\$$$

$$L \rightarrow .id, =/\$$$

$$I_{13}: L \rightarrow *R., \$$$

$$I_5: L \rightarrow id, =/\$$$

$$I_6: S \rightarrow L = R, \$$$

$$R \rightarrow .L, \$$$

$$L \rightarrow . * R, \$$$

$$L \rightarrow .id, \$$$

## CLR(1) parse table →

Items	=	*	Action	\$	S	Goto L	Goto R
I <sub>0</sub>		S <sub>4</sub>	S <sub>5</sub>		1	2	3
I <sub>1</sub>				Acc			
I <sub>2</sub>	S <sub>6</sub>			r <sub>5</sub>			
I <sub>3</sub>				r <sub>2</sub>			
I <sub>4</sub>	S <sub>4</sub>	S <sub>5</sub>				8	9
I <sub>5</sub>	r <sub>4</sub>			r <sub>4</sub>			
I <sub>6</sub>	S <sub>11</sub>	S <sub>12</sub>				10	9
I <sub>7</sub>	r <sub>3</sub>			r <sub>3</sub>			
I <sub>8</sub>	r <sub>5</sub>			r <sub>5</sub>			
I <sub>9</sub>				r <sub>1</sub>			
I <sub>10</sub>				r <sub>5</sub>			
I <sub>11</sub>	S <sub>11</sub>	S <sub>12</sub>				10	13
I <sub>12</sub>				r <sub>4</sub>			
I <sub>13</sub>				r <sub>3</sub>			

## - LALR(1) Item set →

1. Find LR(1) item set
2. Find common core in LR(1) item set
- Common core → Same set of production of two or more states with diff. look ahead
  - (a) Merge common core of LR(1) item set as a single state in LALR(1) item set along with look ahead.

$$I_n : \begin{aligned} A_1 &\rightarrow \alpha_1 \cdot B_1 \beta_1, a_1 \\ A_2 &\rightarrow \alpha_2 \cdot B_2 \beta_2, a_2 \\ &\vdots \end{aligned}$$

$$A_n \rightarrow \alpha_n \cdot B_n \beta_n, a_n$$

$$I_y : \begin{aligned} A_1 &\rightarrow \alpha_1 \cdot B_1 \beta_1, b_1 \\ A_2 &\rightarrow \alpha_2 \cdot B_2 \beta_2, b_2 \\ &\vdots \end{aligned}$$

$$A_n \rightarrow \alpha_n \cdot B_n \beta_n, b_n$$

exact same except look ahead



$I_{ng}: A_1 \longrightarrow \alpha_1 \cdot B_1 \beta_1, a_1/b_1$   
 $A_2 \longrightarrow \alpha_2 \cdot B_2 \beta_2, a_2/b_2$   
 $\vdots$   
 $A_n \longrightarrow \alpha_n \cdot B_n \beta_n, a_n/b_n$

(b) States which are not having any common core in multiple states then we have to use these state in LALR(1) item set same as LR(1) item set

let number of states in SLR(1), LALR(1) & CLR(1) parser is  $n_1, n_2, n_3$  respectively then  $n_1 = n_2 \leq n_3$  Emp

④ 'S → S'

Ex  $S \rightarrow {}^1 aA{}^2 d / {}^2 aB{}^3 d / {}^3 bA{}^4 e / {}^4 bB{}^1 e$  LR(0) itemset  
 $A \rightarrow {}^5 d$  LR(1) itemset  
 $B \rightarrow {}^6 d$  LALR(1) itemset

*Ambiguous*

LR(0) itemset →

$I_0: S' \longrightarrow .S (I_0)$   
 $S \rightarrow .aA{}^2 d (I_1)$   
 $S \rightarrow .aB{}^3 d (I_2)$   
 $S \rightarrow ..bA{}^4 e (I_3)$   
 $S \rightarrow ..bB{}^1 e (I_4)$

$I_3: S \rightarrow b. A{}^4 e (I_7)$   
 $S \rightarrow b. B{}^1 e (I_8)$   
 $A \rightarrow .d (I_5)$   
 $B \rightarrow .d (I_6)$

$I_4: S \rightarrow aA.d (I_9)$

$I_1: S' \longrightarrow S.$   
 $I_2: S \rightarrow a.A{}^2 d (I_4)$   
 $S \rightarrow a.B{}^3 d (I_5)$   
 $A \rightarrow .d (I_6)$   
 $B \rightarrow .d (I_6)$

$I_5: S \rightarrow aB.d (I_{10})$

$I_6: A \rightarrow d.$   
 $B \rightarrow d.$

$I_7: S \rightarrow bA.e (I_{11})$  $I_{10}: S \rightarrow aBd.$  $I_8: S \rightarrow bB.e (I_{12})$  $I_{11}: S \rightarrow bAe.$  $I_9: S \rightarrow aAd.$  $I_{12}: S \rightarrow bBe$ 

LR(0): I	Action					Goto		
	a	b	d	e	f	S	A	B
$I_0$	$S_2$	$S_3$				1		
$I_1$	Acc	Acc.	Acc	Acc	Acc			
$I_2$			$S_6$				4	5
$I_3$			$S_6$				7	8
$I_4$			$S_9$					
$I_5$			$S_{10}$					
$I_6$	$r_5$ $r_c$	$r_5$ $r_b$	$r_5$ $r_b$	$r_5$ $r_c$	$r_5$ $r_b$			
$I_7$				$S_{11}$				
$I_8$				$S_{12}$				
$I_9$	$r_1$	$r_1$	$r_1$	$r_1$	$r_1$			
$I_{10}$	$r_2$	$r_2$	$r_2$	$r_2$	$r_2$			
$I_{11}$	$r_3$	$r_3$	$r_3$	$r_3$	$r_3$			
$I_{12}$	$r_4$	$r_4$	$r_4$	$r_4$	$r_4$			

SLR(1): I	Action					Goto		
	a	b	d	e	f	S	A	B
$I_0$	$S_2$	$S_3$				1		
$I_1$				Acc				
$I_2$			$S_6$				4	5
$I_3$			$S_6$				7	8
$I_4$			$S_9$					
$I_5$			$S_{10}$					
$I_6$	$r_5$ $r_6$		$r_5$ $r_c$					
$I_7$				$S_{11}$				
$I_8$				$S_{12}$				
$I_9$					$r_1$			

$I_{10}$	$r_2$
$I_{11}$	$r_3$
$I_{12}$	$r_4$

LR(1) item set  $\rightarrow$

$$I_0: S' \rightarrow .S, \$ (I_1)$$

$$S \rightarrow .aAd, \$ (I_2)$$

$$S \rightarrow .aBd, \$ (I_3)$$

$$S \rightarrow .bAe, \$ (I_4)$$

$$S \rightarrow .bBe, \$ (I_5)$$

$$I_1: S' \rightarrow S., \$$$

$$I_2: S \rightarrow a.Ad, \$ (I_6)$$

$$S \rightarrow a.Bd, \$ (I_7)$$

$$A \rightarrow .d, d (I_8)$$

$$B \rightarrow .d, d (I_9)$$

$$I_9: A \rightarrow d., e$$

$$B \rightarrow d., e$$

$$I_{10}: S \rightarrow aAd., \$$$

$$I_3: S \rightarrow b.Ae, \$ (I_{11})$$

$$S \rightarrow b.Be, \$ (I_{12})$$

$$A \rightarrow .d, e (I_{13})$$

$$B \rightarrow .d, e (I_{14})$$

$$I_{11}: S \rightarrow aBd., \$$$

$$I_{12}: S \rightarrow bAe., \$$$

$$I_{13}: S \rightarrow bBe., \$$$

$$I_4: S \rightarrow aA.d, \$ (I_{15})$$

$$I_5: S \rightarrow aB.d, \$ (I_{16})$$

$$I_6: A \rightarrow d., d$$

$$B \rightarrow d., d$$

$$I_7: S \rightarrow bA.e \$ (I_{17})$$

$$I_8: S \rightarrow bB.e \$ (I_{18})$$

$LR(1)$ :	I	a	b	c	d	e	f	S	Goto	A	B
$I_0$		$S_2$	$S_3$						I		
$I_1$								Acc			
$I_2$				$S_6$					4		5
$I_3$						$S_9$			7		8
$I_4$					$S_{10}$						
$I_5$						$S_{11}$					
$I_6$						$r_5$					
$I_7$						$S_{12}$					
$I_8$						$S_{13}$					
$I_9$						$r_5$					
$I_{10}$							$r_1$				
$I_{11}$							$r_2$				
$I_{12}$							$r_3$				
$I_{13}$							$r_4$				

LALR(1) itemset  $\Rightarrow$

$$\begin{aligned}
 I_0: \quad & S' \rightarrow .S, \$ (I_1) \\
 & S \rightarrow .aA.d, \$ (I_2) \\
 & S \rightarrow .aB.d, \$ (I_2) \\
 & S \rightarrow .bA.e, \$ (I_3) \\
 & S \rightarrow .bB.e, \$ (I_3)
 \end{aligned}$$

$$I_1: \quad S' \rightarrow S., \$$$

$$\begin{aligned}
 I_2: \quad & S \rightarrow a.A.d, \$ (I_4) \\
 & S \rightarrow a.B.d, \$ (I_5) \\
 & A \rightarrow .d, d (I_{6a}) \\
 & B \rightarrow .d, d (I_{6b})
 \end{aligned}$$

$$\begin{aligned}
 I_3: \quad & S \rightarrow b.A.e, \$ (I_7) \\
 & S \rightarrow b.B.e, \$ (I_8) \\
 & A \rightarrow .d, e (I_{6a}) \\
 & B \rightarrow .d, e (I_{6b})
 \end{aligned}$$

$$I_4: \quad S \rightarrow aA.d, \$ (I_{10})$$

$$I_5: \quad S \rightarrow aB.d, \$ (I_{11})$$

$$\begin{aligned}
 I_6: \quad & A \rightarrow d., d/e \\
 & B \rightarrow d., d/e
 \end{aligned}$$

$I_7: S \rightarrow bA.e, \$ (I_{12})$  $I_{11}: S \rightarrow aBd., \$$  $I_8: S \rightarrow bB.e, \$ (I_{13})$  $I_{12}: S \rightarrow bAe., \$$  $I_{10}: S \rightarrow aAd., \$$  $I_{13}: S \rightarrow bBe., \$$ 

LALR(1):	$\Sigma$	a	b	d	e	f	s	Go to	A	B
$I_0$	$S_2$	$S_3$						1		
$I_1$							Acc			
$I_2$				$S_69$					4	5
$I_3$				$S_69$					7	8
$I_4$				$S_{10}$						
$I_5$				$S_{11}$						
$I_69$				$r_5$	$r_5$					
$r_6$				$r_6$	$r_6$					
$I_7$						$S_{12}$				
$I_8$						$S_{13}$				
$I_{10}$							$r_1$			
$I_{11}$							$r_2$			
$I_{12}$							$r_3$			
$I_{13}$							$r_4$			

- Given Grammer is ambiguous as it has multiple way to create add, bde

Ex  $S \rightarrow AaBb / BbBa$       LL(1) parse table

$A \rightarrow E$

SLR(1) parse table

$B \rightarrow E$

LALR(1) parse table

CLR(1) parse table

LL(1):

Var.	First	Follow
S	{a, b, E}	{\\$}
A	{E}	{a, b}
B	{E}	{a, b}

Parse table →

Var.	\$	a	b
S	$S \rightarrow AaAb$ $S \rightarrow BbBa$	$S \rightarrow AaAb$	$S \rightarrow BbBa$
A		$A \rightarrow \epsilon$	$A \rightarrow \epsilon$
B		$B \rightarrow \epsilon$	$B \rightarrow \epsilon$

SLR(1):

$$S \rightarrow AaAb / BbBa$$

$$A \rightarrow \epsilon$$

$$B \rightarrow \epsilon$$

LR(0) itemset →

$I_0$ :

$$(0) \quad S' \rightarrow .S \quad (G1)$$

$$(1) \quad S \rightarrow .AaAb \quad (G2)$$

$$(2) \quad S \rightarrow .BbBa \quad (G3)$$

$$(3) \quad A \rightarrow .$$

$$(4) \quad B \rightarrow .$$

$I_1$ :

$$S' \rightarrow S.$$

$I_2$ :

$$S \rightarrow A.aAb \quad (G4)$$

$I_3$ :

$$S \rightarrow B.bBa \quad (G5)$$

$I_4$ :

$$S \rightarrow Aa.Ab \quad (G6)$$

$$A \rightarrow .$$

$I_5$ :

$$S \rightarrow Bb.Ba \quad (G7)$$

$$B \rightarrow .$$

$I_6$ :

$$S \rightarrow AaA.b \quad (G8)$$

$I_7$ :

$$S \rightarrow BbB.a \quad (G9)$$

$I_8$ :

$$S \rightarrow AaAb.$$

$I_9$ :

$$S \rightarrow BbBa.$$

SLR(1) parse table →

Item	Action			Goto		
	a	b	\$	S	A	B
I <sub>0</sub>				1	2	3
I <sub>1</sub>			Acc.			
I <sub>2</sub>	S <sub>4</sub>					
I <sub>3</sub>	S <sub>5</sub>					
I <sub>4</sub>				6		
I <sub>5</sub>				7		
I <sub>6</sub>		S <sub>8</sub>				
I <sub>7</sub>	S <sub>9</sub>					
I <sub>8</sub>				r <sub>1</sub>		
I <sub>9</sub>				r <sub>2</sub>		

LR(1) item set →

I<sub>0</sub>:

$$\begin{aligned} S' &\rightarrow .S, \$ \\ S &\rightarrow .AaAb, \$ \\ S &\rightarrow .BbBa, \$ \\ A &\rightarrow ., a \\ B &\rightarrow ., b \end{aligned}$$

I<sub>5</sub>:

$$\begin{aligned} S &\rightarrow Bb.Ba, b \\ B &\rightarrow ., a \end{aligned}$$

I<sub>6</sub>:

$$S \rightarrow AaA.b, b$$

I<sub>1</sub>:

$$S' \rightarrow S., \$$$

I<sub>7</sub>:

$$S \rightarrow BbB.a, a$$

I<sub>2</sub>:

$$S \rightarrow A.aAb, a$$

I<sub>8</sub>:

$$S \rightarrow AaAb., b$$

I<sub>3</sub>:  $S \rightarrow B.bBa, b$

I<sub>9</sub>:

$$S \rightarrow BbBa., a$$

I<sub>4</sub>:

$$S \rightarrow Aa.Ab, a$$

$$A \rightarrow ., b$$

CLR(1) parse table →

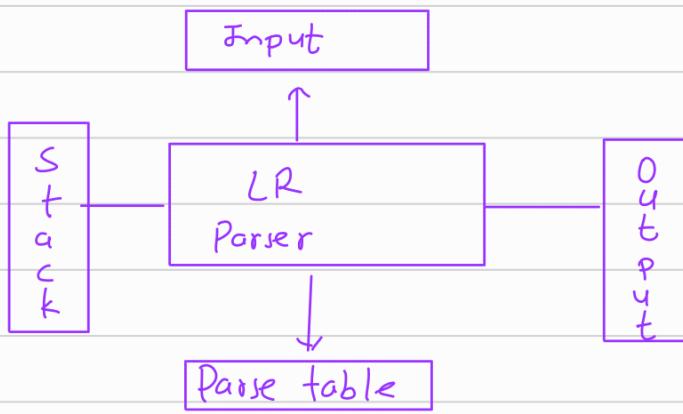
Item	Action			S	Goto	
	a	b	\$		A	B
$I_0$						
$I_1$						
$I_2$						
$I_3$						
$I_4$						
$I_5$						
$I_6$						
$I_7$						
$I_8$						
$I_9$						

LALR(1) itemset →

LALR(1) parse table →

Item	Action			Goto		
	a	b	\$	S	A	B
$I_0$						
$I_1$						
$I_2$						
$I_3$						
$I_4$						
$I_5$						
$I_6$						
$I_7$						
$I_8$						
$I_9$						

## LR parsing →



$E' \rightarrow E$  (0)

$\underline{E} \rightarrow E * T$  (1)

$E \rightarrow T$  (2)

$T \rightarrow T * F$  (3)

$T \rightarrow F$  (4)

$F \rightarrow (E)$  (5)

$F \rightarrow id$  (6)

SLR(1) parse table →

States	Action						Goto		
	+	*	(	)	id	\$	E	T	F
$I_0$			$S_4$		$S_5$		1		
$I_1$	$S_6$								
$I_2$	$H_2$	$S_7$		$H_2$		$H_2$			
$I_3$	$H_4$	$H_7$		$H_7$		$H_7$			
$I_4$			$S_4$		$S_5$		8		
$I_5$	$H_6$	$H_6$		$H_6$		$H_6$			
$I_6$			$S_4$		$S_5$			9	
$I_7$			$S_4$		$S_5$				10
$I_8$	$S_6$			$S_{11}$					
$I_9$	$H_1$	$S_7$		$H_1$		$H_1$			
$I_{10}$	$H_3$	$H_3$		$H_3$		$H_3$			
$I_{11}$	$H_5$	$H_5$		$H_5$		$H_5$			

After reduce

refers to goto of top 2 symbols of stack

Stack	Input	Output
$\emptyset$	$id + id * id \$$	Shift $id \circlearrowleft 5$
$0 id \circlearrowleft 5$	$+ id * id \$$	remove $2   \alpha$ element frm stk
$0 F 3$	$+ id * id \$$	reduce $F \rightarrow id$
$0 T 2$	$+ id * id \$$	reduce $T \rightarrow F$
$0 E 1$	$+ id * id \$$	reduce $E \rightarrow T$
$0 E 1 + 6$	$id * id \$$	shift $+ 6$
$0 E 1 + 6 id 5$	$* id \$$	shift $id 5$
$0 E 1 + 6 F 3$	$* id \$$	reduce $F \rightarrow id$
$0 E 1 + 6 T 9$	$* id \$$	reduce $T \rightarrow F$
$0 E 1 + 6 T 9 * 7$	$id \$$	Shift $* 7$
$0 E 1 + 6 T 9 * 7 id 5$	$\$$	Shift $id 5$
$0 E 1 + 6 T 9 * 7 F 10$	$\$$	reduce $F \rightarrow id$
$0 E 1 + 6 T 9$	$\$$	reduce $T \rightarrow T * F$
$0 E 1$	$\$$	reduce $E \rightarrow E + T$
		Accepted

Ex	$id * id + id$	
$0$	$id * id + id \$$	Shift $id 5$
$0 id 5$	$* id + id \$$	reduce $F \rightarrow id$
$0 F 3$	$* id + id \$$	reduce $T \rightarrow F$
$0 T 2$	$* id + id \$$	shift $* 7$
$0 T 2 * 7$	$id + id \$$	Shift $id 5$
$0 T 2 * 7 id 5$	$+ id \$$	reduce $F \rightarrow id$
$0 T 2 * 7 F 10$	$+ id \$$	reduce $T \rightarrow T * F$

