- Head first statistics Dawn Griffiths
- Introductory statisics Neil A Weiss

Chap 1 → Introductory statistics

- Collecting data → summarize → vizualization → analysis & making inferences

Descriptive statistics v/s Inferential statistics

- Abt summarizing data set & visualization
- Predection of some data in data set

Ex Flower classification → (Inferential statistics)

- distinguish rose & lotus 50 roses & 50 lotus
- we have patel length & width for each recorded
- we now have a new flower given patel length = a units & patel width = b units, determine species of flower

- We find object with nearest data and this flower belong to this category.

optimize

- find centroid of data for features & find nearest catego

Mean Median Mode

- Just compare it with centroid of diff. category & interprete.

Descriptive Statistics →

- Measures of central tendency → mean, median, mode
- Measure of varience → range, IQR, percentile.
- Data visualization → piechart, Box plot, Bar charts, Stem & leaf diagram, surface plots. → use matplotlib library in python

- Observational study → based on observation only
- Experimental study → based on experiment done on some data grp

Inferential Statistics →

- regression - relationship b/w dependent & independent variable
- naive Bayes classification

$$y = w_0 + w_1 x + w_2 x^2$$

↳ Polynomial regression of order 2

Ex. Need to predict age given value of Blood suger, weight, height
∴ we need to establish a function for that we make assumption known as hypothesis

let Age = $w_0 + w_1 \cdot \underset{x_1}{BS} + w_2 \cdot \underset{x_2}{Weight} + w_3 \cdot \underset{x_3}{height}$.

or y

let $y = w_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 + w_4 \cdot x_1^2 + w_5 x_2^2 + w_6 x_3^2 + w_7 x_1 x_2 + w_8 x_2 x_3 + w_9 x_1 x_3$

- optimal order = 1 more than number of turns

- Sample should be with no bais & ideal group should be selected from population
- sampling technique → random sampling, systemetic random sampling, cluster based sampling.

- Discrete variable → table (Prob. dist.)
- Random variable → probo distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

Tally mark :-

Ex 1 1 2 2 2 3 3 3 4 4 4 5 5 1 1 2 2 3 5 6

→ One pass

1	||||
2	𝍸 |
3	||||
4	|||
5	|||
6	|

- Write a prog to compute mean & standard deviation (σ) ?

- We just need 1 pass.

$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n} = \frac{\sum (x_i^2 - 2x_i\mu + \mu^2)}{n}$$

$$= \sum_{i=1}^{N} \frac{x_i^2}{N} - 2\mu \sum_{i=1}^{N} \frac{x_i}{N} + \frac{N\mu^2}{N}$$

$$= \sum_{i=1}^{N} \frac{x_i^2}{N} - 2\mu^2 + \mu^2$$

$$= \sum_{i=1}^{N} \frac{x_i^2}{N} - \mu^2$$

$$= \sum_{i=1}^{N} \frac{x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$$

→ Computation formula for standard deviation

- If we use above methode upon adding data our previous computation are not lost

- An algorithm is said to be incremental when u can update the model on availibility of additional data.

Design experiment →

- Experimental units
- treatment ⟶ 8
- factor ⟶ Irr. regin, polymer
- level of factor ⟶ Irr. regin ⇒ 4, polymer ⇒ 2
- response variable ⟶ wt. gain

Ex Impact of irregation regin & use of a particular polymer? for max weight gain of cactus plant

irregation regin → none low moderate high

polymer → yes no

	N	L	M	H
Y	T1	T2	T3	T4
N	T5	T6	T7	T8

Q. A ML algorithm named SPM has 2 free parameters named regularization parameter and kernal width parameter, σ. In orderto find optimal values of parameter we perform a grid search

$C \in \{2^{-18}, 2^{-16}, \dots 2^{0} \dots 2^{50}\}$ = 35 terms

$\sigma \in \{2^{-18}, 2^{-16}, \dots 2^{0} \dots 2^{20}\}$ = 20 terms

for which accuracy is maximum

factors → C, σ

level of factor → C = 35, σ = 20

Response var → Accuracy

treatment → 35 X 20

Q. An exp. was conducted to study the impact of follic acid on birth defect, to perform this study a grp of 200 women considered, 100 women were given 10mg follic acid tablet & the rest of women were given trace element find out the factors, number of treatment experimental variable & response variable!

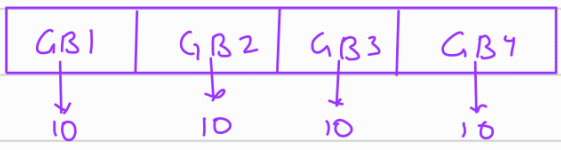
Exp. unit ⇒ subject (Biology) 200
Res. var ⇒ Child birth
factor ⇒ Acid
level ⇒ 2
Treatment ⇒ 2

Sampling → with replacement
Sampling → Without replacement

How to distribute experimental unit among treatments →

- Complete randomized design
- Randomized block design

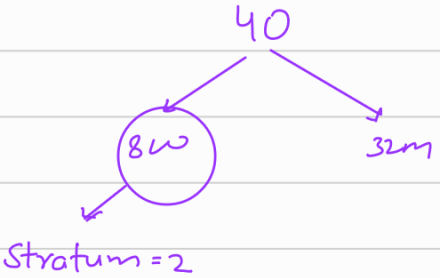
Q. Driving distance of golf ball - 4 Brands golf balls
their are 40 golfers 8 are women & 32 are men



8+2 10M 10M 10M

GB2 driving dist Less
As sample is not good.

- Sample should be representative of population
- For good same their shd be 8M + 2W



Sampling

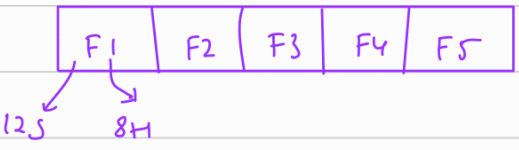
- SRS (simple random sampling)
  - without replacement
  - with replacement
    - same no more than once
- Cluster based
  - Packets instead of individual
- Systematic random sampling.
  - Sample size = ss
  - population size = ps
  - $m = \frac{ss}{ps}$
  - Generate random no k from 1 to m
  - Now my samples are k, k+m, k+2m, ...
- Stratified random sampling.
  - Population of subconcepts
  - 4 types of colors in trophy

- Cross validation used stratified random sampling
-

100 Emails → 80 training, 20 testing
- 60 S
- 40 H

Then how??

data of 5 equal parts & maintain ratio



	Train	Test
T1:	F1	F2 F3 F4 F5
T2:	F2	F1 F3 F4 F5
T3:	F3	F1 F2 F4 F5
T4:	F4	F1 F2 F3 F5
T5:	F5	F1 F2 F3 F4

m-classes

C_1	n_1
C_2	n_2
C_3	n_3
⋮	⋮
C_m	n_m

Find unique no. of experimental set up as per using k-fold cross validation.

$\underbrace{x_1\ x_2\ x_3\ x_4}_{R}\quad \underbrace{y_1\ y_2\ y_3\ y_4}_{L}$ = 8 instances

2-fold cross validation
↳ divide in 2 equal part

4 instances	4 instances
fold 1	fold 2

maintaing ratio.

Training	Test
fold 2	fold 1
fold 1	fold 2

- I/P in machine learning is data
- In regression target var. contineous & in classification target is discrete

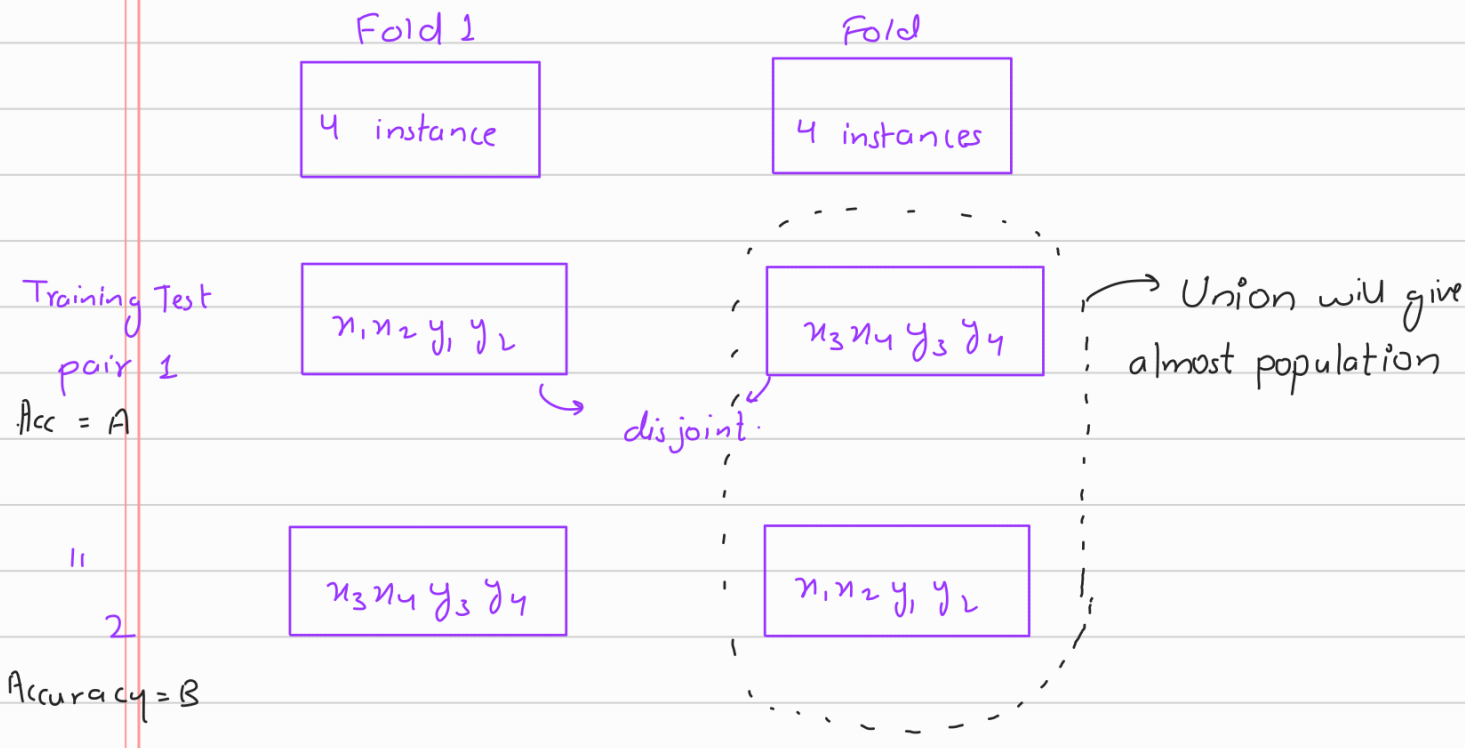
Training	
T	n
S	5'
T	6'
T	7'
S	4'

Test		
P	T	n
S	S	5'1"
S	S	5'2"
T	T	6'3"
T	T	6'7"

S* 4.5' T* 6.5'

S* 5'1.5" T* 6'5"

- Here test instance is easy
- To get better dct we use k (2) fold cross validation



$$\text{Net Accuracy} = \frac{A+B}{2}$$

$$\text{Unique} = \frac{{}^4C_2 \times {}^4C_2}{2}$$

Ass. →

$x_1 \text{ ---- } x_{10} \Rightarrow$ Class 1

$y_1 \text{ ------ } y_{10} \Rightarrow$ Class 2

find out unique experimental setups using 2&5-flod cross validation prove that no. of unique experimental setups for k-fold cross validation where the $C_1, C_2 \text{ --- } C_m$ are the m classes with $n_1, n_2, \text{ --- } n_m$ instances respectively is

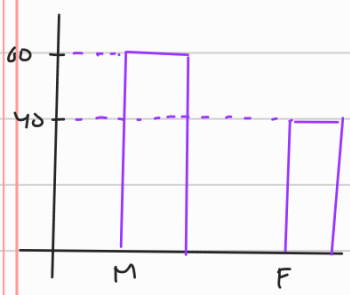
$$\frac{\prod_{i=1}^{m} {}^{n_i}C_{n_i/k}}{k}$$

	Not suitable
Mean	In presense of outliers (Teacher ↑ avg age of class)
Medians	(Parent with kid in swimming class)
Mode	(5 5 36 36)

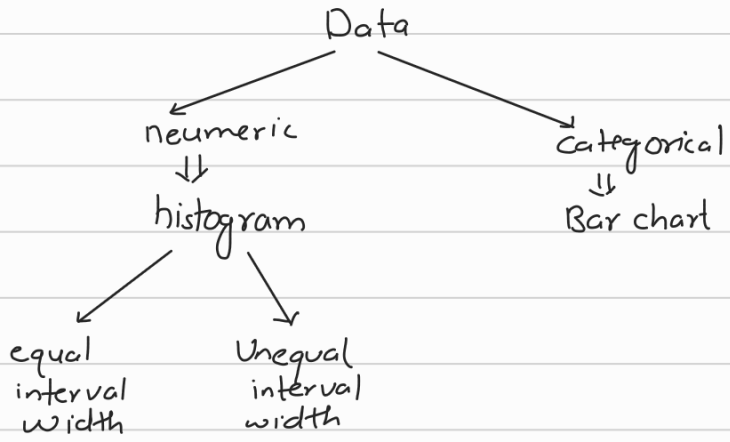
mode = $\frac{36+5}{2}$ = 20.5

Distribution →

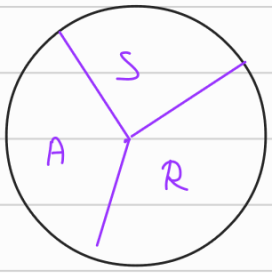
- bar-chart →



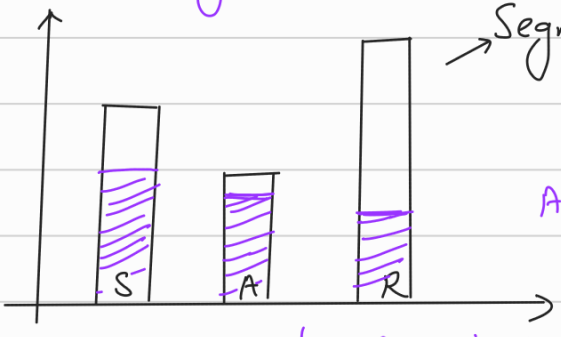
Height of bar ∝ Freq.



- Pie-chart →



How to plot a add on data
- Using barcharts



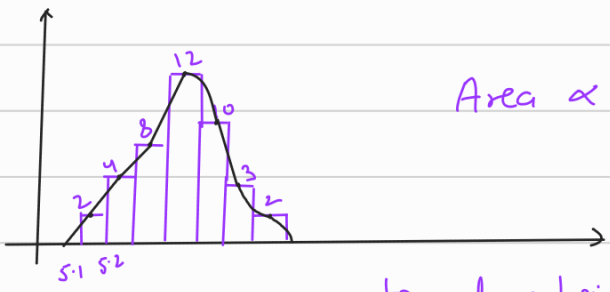
Add. data by coloring.

↳ Gaps in b/w barchart

- Histogram →

	freq
5 → 5.1	2
5.1 → 5.2	4
5.2 → 5.3	8

6.1 → 6.2	8
6.2 → 6.3	3



Area ∝ Freq.

- No gap b/w bar

- to reduce height we can use fraction
- join centre of histogram

Histogram →

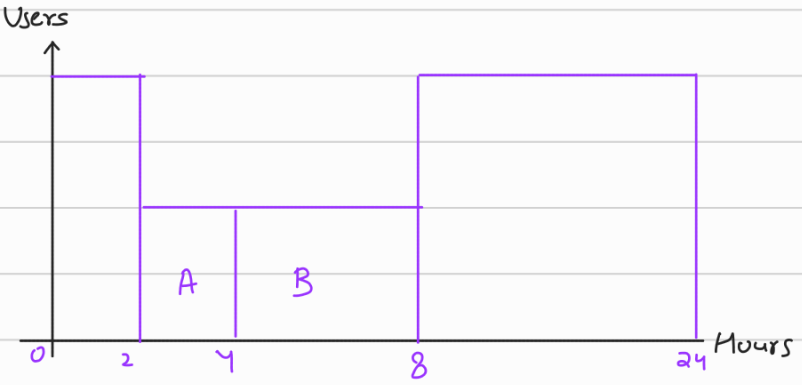
3000 Customers

0 → 2	1000
2 → 4	500
4 → 8	500
8 → 24	1000

lower limit, upper limit

[8, 24)

Bar chart →



B > A ?? NO Barchart drawback

(For Nominal or Categorical attribute)

Histogram ↓

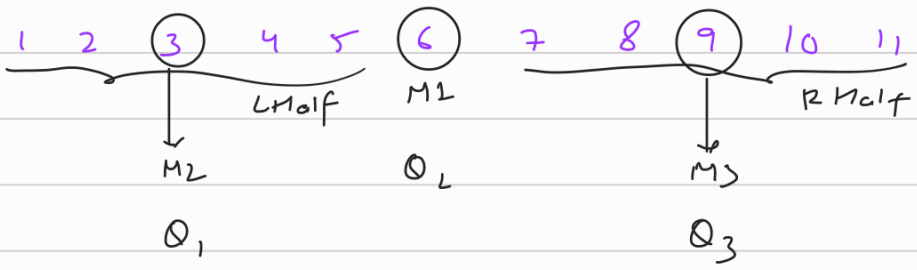
Class	freq	Class width	height
0 → 2	1000	2	500
2 → 4	500	2	250
4 → 8	500	4	125
8 → 24	1000	16	62..

Box plot →

Central tendency	Variance
→ Mean (fail in outliner)	→ trimmed range (Range - Outliner)
→ Median (fail 2 major class) (kid + parent)	→ range (It contaiant outliner)
→ Mode	→ IQR (for median) - Interqurtile range ⇒ 25% (Q_1), 50% (Q_2), 75% (Q_3); $IQR = Q_3 - Q_1$
	→ Varience

- both collectively ct & var. provide much more info about data

IQR →



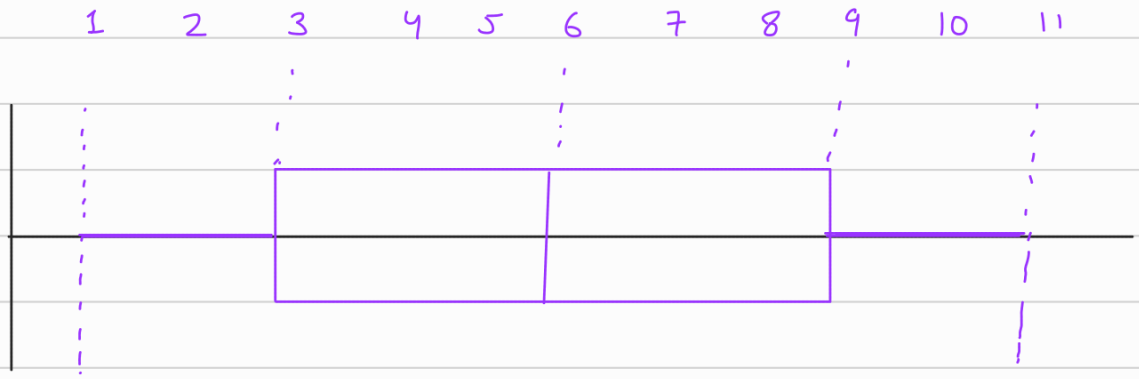
25% data lie below Q_1

lower bound = 1

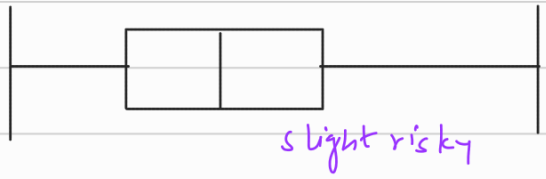
Upper bound = 11

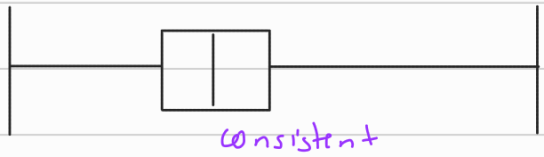
$Q_1\ Q_2\ Q_3$ = 3, 6, 9

Median = 6

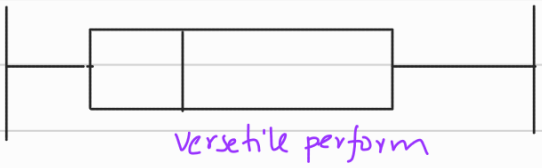


Q.





Whom to select??



Q Compare the profit of 5 companies for the month Jan-Apr 2023.

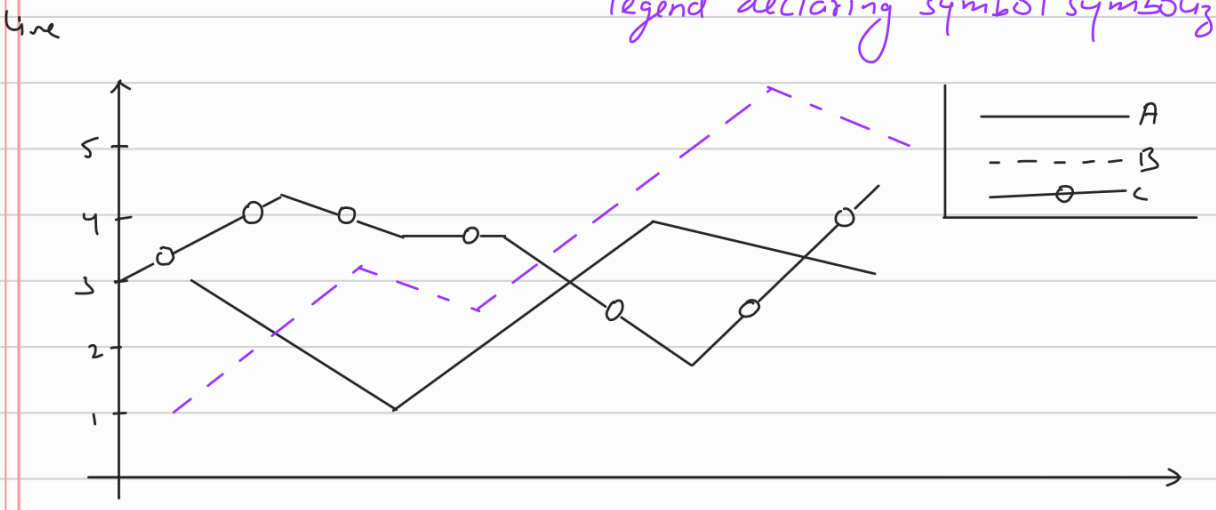
- What to use

(best)

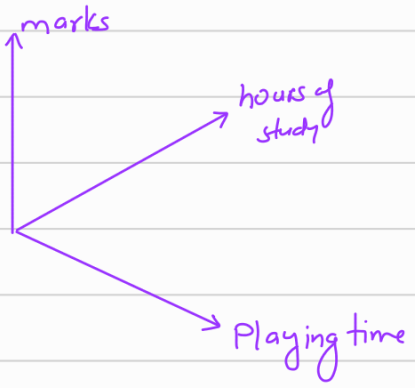
Piechart — line Chart → (diff line similar to below one)

Barchart — Scatter blots → (diff symbol for diff people)

Just plot point on x-y & we create a box on right corner called legend declaring symbol symbolization.



Surface plot 3D →



Normal distribution →

- Can be used to summarize numerical data.

↗ Parametric study (Assume data follow a particular prob. density function)
↘ Non-parametric

data ↗ Categorical/Nominal ⇒ prob. distribution table (coin HT table)
data ↘ Numeric ⇒ prob. density function

Temp-play →

Yes	No
Mild	Mild
Mild	
Mild	Mild
Mild	
Mild	hot
Hot	

summarize →

	Hot	Mild	Cool
yes (9)	2/9	4/9	3/9
No (5)	3/5	2/5	0/5

→ likelihood

Hot	hot
cool	
cool	hot

$P\left(\frac{\text{Hot}}{\text{Yes}}\right)$??

Numeric

Temp - Play	
Yes	No
31	41
32	42
33	43
⋮	44
39	45

Assume data follow normal or gaussian distribution

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

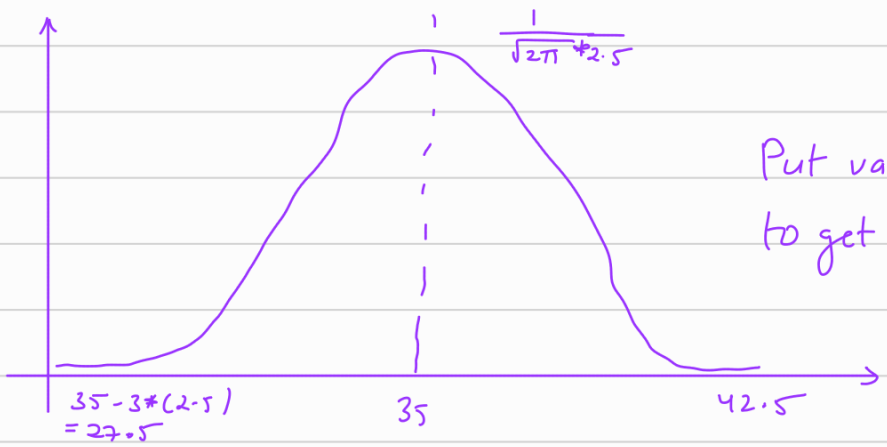
μ, σ = ??

$\Rightarrow$ temp = 36.5 play = ??

$\mu = 35$, $\sigma = \sqrt{\frac{60}{9}} = \frac{7.745}{3} = 2.5$

X = random var.

x = value of random var.



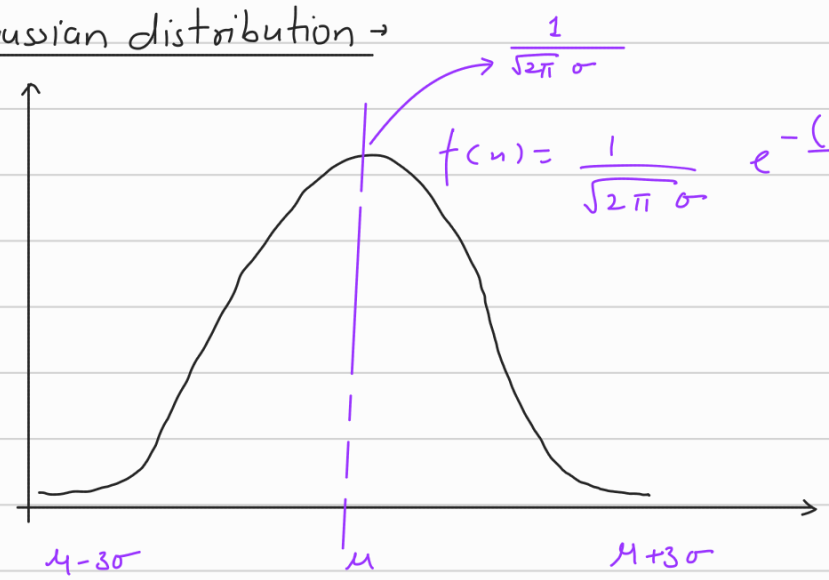
Put value in function to get ans.

Q. Compute the likelyhood of play = yes & play = NO when temprature is 40°c assume that the data follows normal/gaussian pdf

→ P (X < 40 & Play = Yes) = Area under curve till x = 40

- To solve Area under curve we convert this distribution into standard normal distribution ($\mu = 0$, $\sigma = 1$)

Gaussian distribution →



Emperical rule → (65-95-99.7 rule)

65% of entire population lie in range of $\mu-\sigma$ to $\mu+\sigma$
95% of entire population lie in range of $\mu-2\sigma$ to $\mu+2\sigma$
99.7% of entire population lie in range of $\mu-3\sigma$ to $\mu+3\sigma$

Standard normal distribution → (Z-Table)

Mean = 0 , Standard deviation = 1

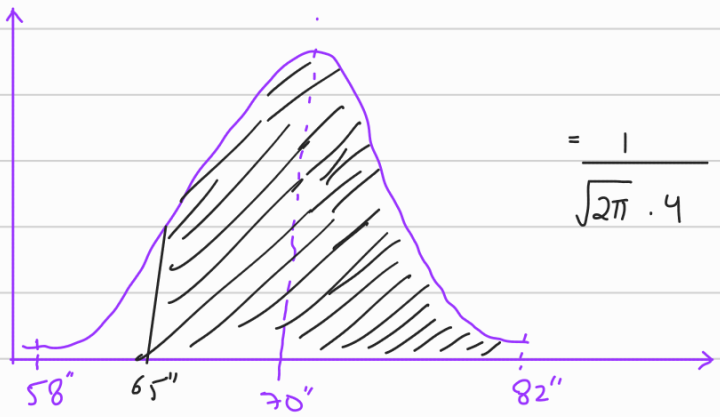
$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}} \xrightarrow[\sigma=1]{\mu=0} \frac{1}{\sqrt{2\pi}}\, e^{-x^2/2}$$

→ eqn of standard normal distribution

Q. She want to marry a man who is taller than her. The dis. of the height of man is given by $N(70'', 16'')$ Given girl height = 65".

μ σ^2

$\sigma = 4$



$$= \frac{1}{\sqrt{2\pi}\cdot 4}\, e^{-(x-70)^2/2(16)}$$

↓ integrate in range

or

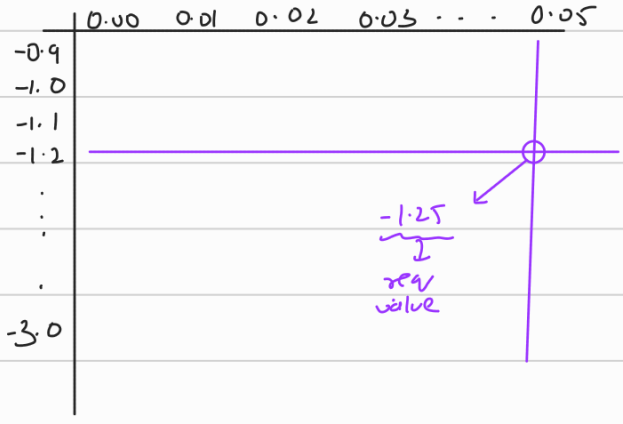
transform it to standard normal distribution

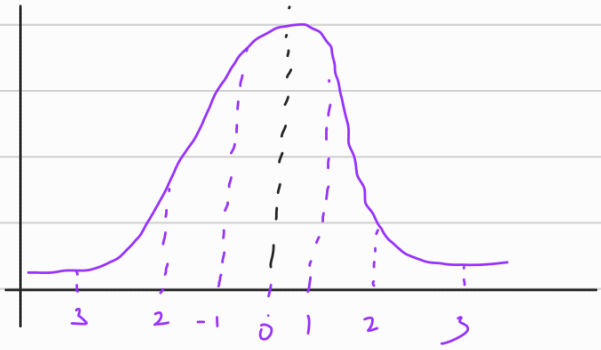
x	$x-\mu$
1	-4.5
2	-3.5
3	-2.5
4	-1.5
5	.5
6	.5
7	1.5
8	2.5
9	3.5
10	4.5
$\mu = 5.5$	$\mu = 0$

to convert σ to 1 divide $x-\mu/\sigma$

$$z = \frac{65-70}{4}$$

- z-table give area on left hand side of curve



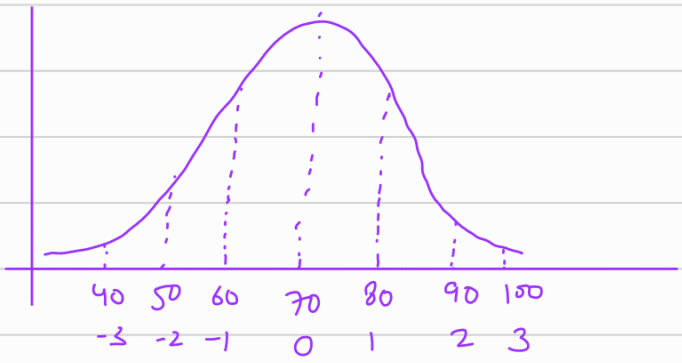


- This will give are upto this portion
- We need area on right ∴ 1 - Area of left.

Q. The marks of students of CSE follow gaussian distribution with mean 70 marks & variance of 100 marks find out the following prob.
$P(X \geq 80)$, $P(60 \leq X \leq 80)$, $P(X \leq 50)$

- $\mu = 70$, $\sigma = 10$

(i)



$P(X \geq 80) = 1 - (z \text{ at } 1)$

$P(60 \leq X \leq 80) = (z \text{ at } 1) - (z \text{ at } -1)$

$P(x \leq 50) = (z \text{ at } -2)$

Binomial distribution →

$P(X = r) = {}^{n}C_{r}\, p^{r} q^{n-r}$

$P(X = 2) = {}^{n}C_{4} (1/2)^{2} (\tfrac{1}{2})^{2}$

$n = 4$

coin toss

Head 1, 2, 3, 4 first

Distribution to learn
- Geometric
- Normal or gaussian
- Poisson
- Binomial

Q. There are 100 questions with 4 possible ans A, B, C, D in a test all the options are equally likely to be correct, each question carry 1 mark, find out the probability that the no. of ans successfully marks < 70.

→ Binomial is over calculative ∴ we will transform it to other distribution

Expectation & varience of random variable:-

Ex- lottery system → 3-window & D1 symbol → $, C, L, other

0.1 0.2 0.2 0.5

same for D2 & D2, say if we get $ $ $ we win 20 Rs, $ $ C = 15 Rs (in any seq.), CCC = 10 Rs, LLL = 5 Rs

we need to find expected amount we will win, entry fee: 1 Rs

we need to make prob dist. table

X	Prob. $(X=x)$	
19	$0.1 * 0.1 * 0.1$	$= 0.001$
14	$0.1 * 0.1 * 0.2 * 3$	$= 0.006$
9	$0.2 * 0.2 * 0.2$	$= 0.008$
4	$0.2 * 0.2 * 0.2$	$= 0.008$
-1	$1 - AU$	$= 0.977$

$$\text{Expectation } [x] = \sum_x X P(X=x)$$

$$= 19 * 0.001 + 14 * 0.006 + 9 * 0.008 + 4 * 0.008 - 1 \times 0.977$$

$$E[x] = -0.77$$

$$\text{Varience } [x] = E[(x-\mu)^2]$$

$$\because \quad E[f(x)] = \sum_x f(x) \cdot P(X=x)$$

$$= \sum_x (x-\mu)^2 P(X=x)$$

$$= \sum_x (x^2 + \mu^2 - 2\mu x) P(X=x)$$

$$= \sum_x x^2 P(X=x) + \mu^2 \sum_x P(X=x) - 2\mu \sum x P(X=x)$$

(μ^2 and $2\mu^2$)

$$= E(X^2) - E^2(X)$$

X	Prob. $(X=x)$		$X-\mu$	$(x-\mu)^2$
19	$0.1 * 0.1 * 0.1$	$= 0.001$	19.77	390.85
14	$0.1 * 0.1 * 0.2 * 3$	$= 0.006$	14.77	218.15

9	$0.2 * 0.2 * 0.2 = 0.008$	9.77	95.45
4	$0.2 * 0.2 * 0.2 = 0.008$	4.77	22.75
-1	$1 - AU = 0.977$	-0.23	0.05

$$var = 0.001 * 390.85 + 0.006 * 218.15 + 0.008 * 95.45 + 0.008 * 22.75 + 0.977 * 0.05$$

$$= 2.69$$

$$\sigma = \sqrt{2.69} = 1.64$$

- In general we lose 0.77 vs by gussian dist.

$\therefore$ Our win range lie in $[-0.77 - 3 * 1.64, -0.77 + 3 * 1.64]$

- Price money for the lottery is made 5 times & fee is doubled.

Y	P(x)
98	0.001
73	0.006
48	0.008
23	0.008
-2	0.977

Original win - fee = X

original win = X + fee = (X+1)

$Y = 5(X+1) - 2$

$Y = 5X + 3$

↓

Linear dependency

- If $Y = aX + b$; $E(x)$ is known

then $E[y] = aE[x] + b$

$Var[y] = a^2 V[x]$

Derive → $E(x) = \sum_x f(x) \cdot P(X = x)$

$E(aX + b) = \sum_x (ax + b) \cdot P(X = x)$

$$= \sum_n a \cdot n \cdot P(X=n) + \sum_n b\, P(X=n)$$

$$= a \sum_n n\, P(X=n) + b$$

$$= a\, E[n] + b$$

$$\text{Var}(aX+b) = E[(an+b)^2] - (E[an+b])^2$$

$$= E[a^2n^2 + b^2 + 2abn] - (aE[n]+b)^2$$

$$= a^2E[X^2] + \cancel{2ab\,E[x] + b^2} - a^2E^2[x] - \cancel{2ab\,E[x] - b^2}$$

$$= a^2\left(E[x^2] - E^2[n]\right)$$

$$= a^2\, \text{Var}[x]$$

Note: $2X$ is diff from $X+X \Rightarrow 2X \Rightarrow$ price money doubled

$X+X$ is 2 lottery ticket purchase

$2X \rightarrow$ linear dependency

X	15	-2
$P(X=n)$	0.4	0.6
$Y = 2X$	30	-4
$P(Y=y)$	0.4	0.6

$X+X \rightarrow$ independent observation

X	$P(n)$
30	0.16
13	0.48
-4	0.36

both differ

- $E(X+X) = 2E(X)$ $\quad Var[X+X] = 2\,Var[X]$
- $E(2X) = 2E(X)$ $\quad Var[2X] = 4\,Var[X]$

Derive:

$E(aX+bY) = aE[X] + bE[Y]$

$Var[aX+bY] = a^2 Var[X] + b^2 Var[Y]$

$E[aX-bY] = aE[X] - bE[Y]$

$Var[aX-bY] = a^2 Var[X] + b^2 Var[Y]$

} For independent var

- Chap. 5 Head first statistics

Expectation & Varience of geometric distribution:

P = Prob. win r = trails r = n to success ?! q = lose prob

$E(X) = \sum X P(X=n)$

$= \sum_{r=1}^{\infty} r\,P(X=r)$

$E(x) = 1.P + 2.qP + 3q^2P + 4q^3P + \cdots$

$qE(x) = 1.q.P + 2q^2P + 3q^3P + \cdots$

$(1-q)E(x) = P + Pq + Pq^2 + \cdots Pq^5 + \cdots$

$PE(x) = P[1 + q + q^2 + \cdots q^{\infty}]$

$$\boxed{E(x) = \frac{1}{1-q} = \frac{1}{P}}$$

$Var[x] = E(x^2) - (E[x])^2$

$= E[x^2] - \frac{1}{P^2}$

$= E[x(x-1)] + E[x] - (E[x])^2$

$E[x(x-1)] = \sum_{k=1}^{\infty} k(k-1)P(X=k)$

$= \sum_{k=1}^{\infty} k(k-1)\,P.q^{k-1}$

$= P\sum_{k=1}^{\infty}(k-1)k\,q^{k-1} = P.\frac{d}{dq}\left(\sum_{k=1}^{\infty}(k-1)q^k\right)$

$= P\frac{d}{dq}\left(q^2\sum_{k=1}^{\infty}(k-1)q^{k-2}\right) = P\frac{d}{dq}\left(q^2\sum_{k=2}^{\infty}(k-1)q^{k-2}\right)$

$= P\frac{d}{dq}\left(q^2\frac{d}{dq}\left(\sum_{k=2}^{\infty}q^{k-1}\right)\right) = P\frac{d}{dq}\left(q^2\frac{d}{dq}\left(\sum_{k=1}^{\infty}q^k\right)\right)$

$= P\frac{d}{dq}\left(q^2\frac{d}{dq}\left(\frac{1}{1-q}-1\right)\right) = P\frac{d}{dq}\left(q^2\left(\frac{-1}{(1-q)^2}\right)\right)$

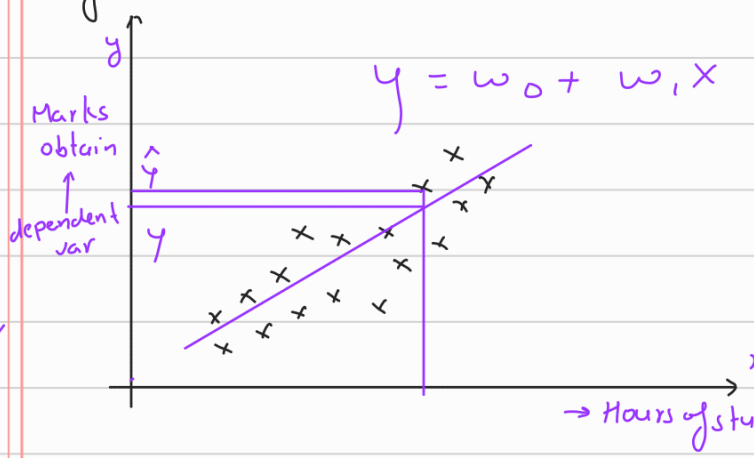
$= P\frac{d}{dq}\left(\frac{-q^2}{(1-q)^2}\right) = P.\frac{2q}{(q-1)^3} = P.\frac{2q}{P^3} = \frac{2q}{P^2}$

$\therefore Var[x] = \frac{2(1-P)}{P^2} + \frac{1}{P} - \frac{1}{P^2}$

$= 1-P/P^2$

$= q/P^2$

Regression →



$y = w_0 + w_1 x$

Machine learning
- Supervised → Regression
- Unsupervised

$Y = w_1 X + w_0$ (Assume linear relation)

let Assume sleep also affect then

$Y = w_1 X_1 + w_2 X_2 + w_0$ (Multiple linear relation)

polynomial reg → let 1 ind. var but 2nd order relationship

$Y = w_2 x^2 + w_1 x + w_0$ (reg with 2nd order poly)

let 2 ind. var →

$Y = w_0 + w_1 X_1 + w_2 X_2 + w_3 X_1^2 + w_4 X_2^2 + w_5 X_1 X_2$

$\hat{y} - y$

$\hat{Y}$	X	let $y = x$	Residual error	$E = \frac{1}{2}\Sigma$ sq	let $y = 2x$	RE	sq
2	1	1	1 → sq	1	2	0	0
4	2	2	2 → sq	4	4	0	0
6	3	3	3 → sq	9	6	0	0
8	4	4	4 → sq	16	8	0	0

$E = 15$ ↓ this need to min.

$\therefore E = 0$ → min

- In regression we minimize least squares errors

$$E = \frac{1}{2}\sum_{i=1}^{N} r_i^2 = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2$$

$$= \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - w_0 - w_1 x_i)^2$$

↓ minimize

$$\frac{\partial E}{\partial w_0} = \frac{1}{2} \times 2 \sum_{i=1}^{N}(\hat{y}_i - w_0 - w_1 x_i)(-1) \Rightarrow 0$$

$$= \sum_{i=1}^{N} \hat{y}_i - w_0 N - w_1 \sum_{i=1}^{N} x_i = 0$$

$$= w_0 N + w_1 \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \hat{y}_i \quad - (1)$$

$$\frac{\partial E}{\partial w_1} = \frac{1}{2} \times 2 \sum_{i=1}^{N}(\hat{y}_i - w_0 - w_1 x_i)(-x_i) \Rightarrow 0$$

$$= \sum_{i=1}^{N} \hat{y}_i x_i - w_0 \Sigma x_i - w_1 \sum_{i=1}^{N} x_i^2 = 0 \quad - (2)$$

Use 1, 2 & data, substitute it ⇒

y	x	x^2
2	1	1
4	2	4
6	3	9
8	4	16
20	10	30

(1) ⇒ $w_0(4) + w_1(10) = 20$

(2) ⇒ $w_0(10) + w_1(30) = 60$

$w_0 = 0 \quad w_1 = 2 \Rightarrow$ solution optimal

Ex-

$\hat{y}$	x
4	1
8	2
13	3
32	4

Fit in the following poly.

(i) $y = w_0 + w_1 x$

(ii) $y = w_0 + w_1 x + w_2 x^2$

(i) ① $\Rightarrow$ $w_0 4 + w_1 10 = 57$

② $\Rightarrow$ $187 = w_0 10 + w_1 30$

$19 w_0 + 30 w_1 = 171$

$10 w_0 + 30 w_1 = 187$

$2 w_0 = -16$

$w_0 = -8 \Rightarrow w_1 = 57 - 4(-8)$

$= 57 + 32 = 89$

(ii) Solve diff.

Order = curve + 1 (Change) → n U

Q

$\hat{y}$	x	$\hat{y} - y$
2	1	$2 - w$
5	2	$5 - 2w$
5	3	$5 - 3w$
7	4	$7 - 4w$
11	5	$11 - 5w$

$y = wx$ → Given

$$\frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2 = \frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - wx)^2$$

$$\frac{dE}{dw} = \frac{1}{2} \times \sum_{i=1}^{N} 2(\hat{y}_i - w x_i)(-x_i) = 0$$

$$= \sum_{i=1}^{N}(\hat{y}_i - w x_i) x_i = 0$$

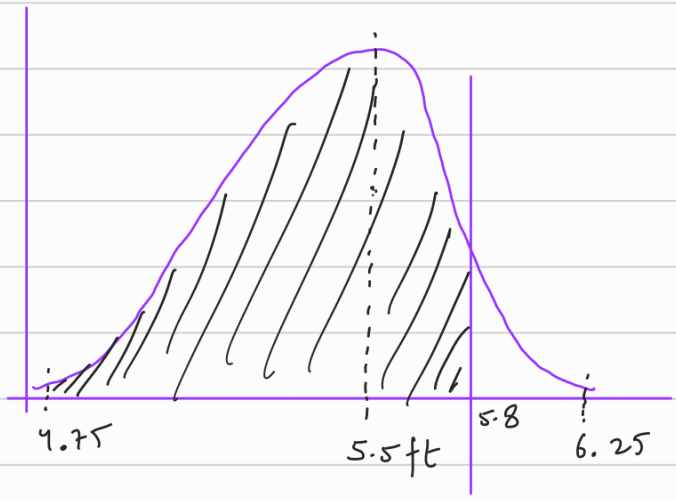
$$= \sum_{i=1}^{N} y_i x_i - w \sum_{i=1}^{N} x_i^2 = 0$$

$= 110 - 55w = 0$

$\Rightarrow w = 2$

$\therefore y = 2x$

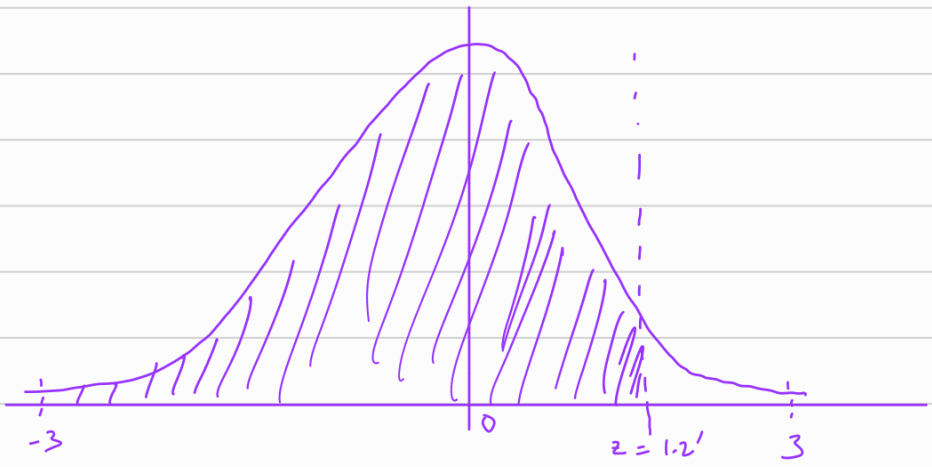
Q. The height of students of class follow normal distribution with avg height of 5.5ft and varience of 1/16 ft find out prob that student will have height less then 5.8 ft?



$\sigma^2 = 1/16$

$\sigma = 1/4$

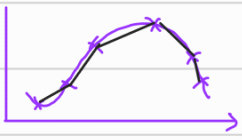
Standard. $z = \frac{x-\mu}{\sigma}$

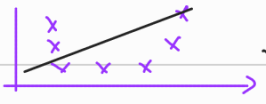


$z = \frac{5.8' - 5.5'}{0.25} = 1.2'$

Ans = z-table (1.2') =

SEE IN PYTHON

Curve fitting → Overfitting ⇒  taken deg. > req. deg.

Curve fitting → Underfitting.  req. poly degree > taken poly

- To resolve both we use regulaization

Regression with regulation →

- Here we modify error function to incorporate regularization term :-

We need to minimise ⇒ $\frac{1}{2}\sum_{i=1}^{N}(\hat{y}_i - y_i)^2 + C \cdot \frac{1}{2} w^T w$

$C \in [2^{-20}, 2^{-18} \ldots\ldots, 2^{50}]$ ← Regularization parameter

$[w_1\ w_2]\begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$ ← regulaization

$= \frac{1}{2}(w_1^2 + w_2^2)$

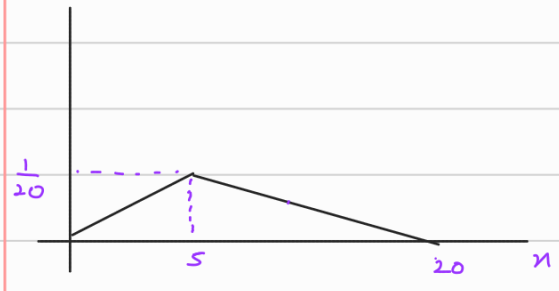
- In regulaization we need to tune C & d (chk all poss. for best result)

⇒ KEEL DATASET Repo. → regression prob.

- Find 3-4 reg. prob. & use suitable tool box & solve these problems find optimal value of degree of polynomial & regularization polynomial

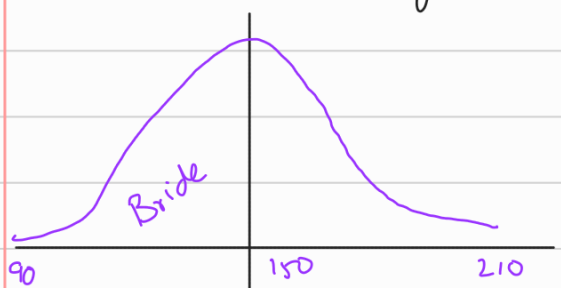
Chap-9 Beyond normal distr.→

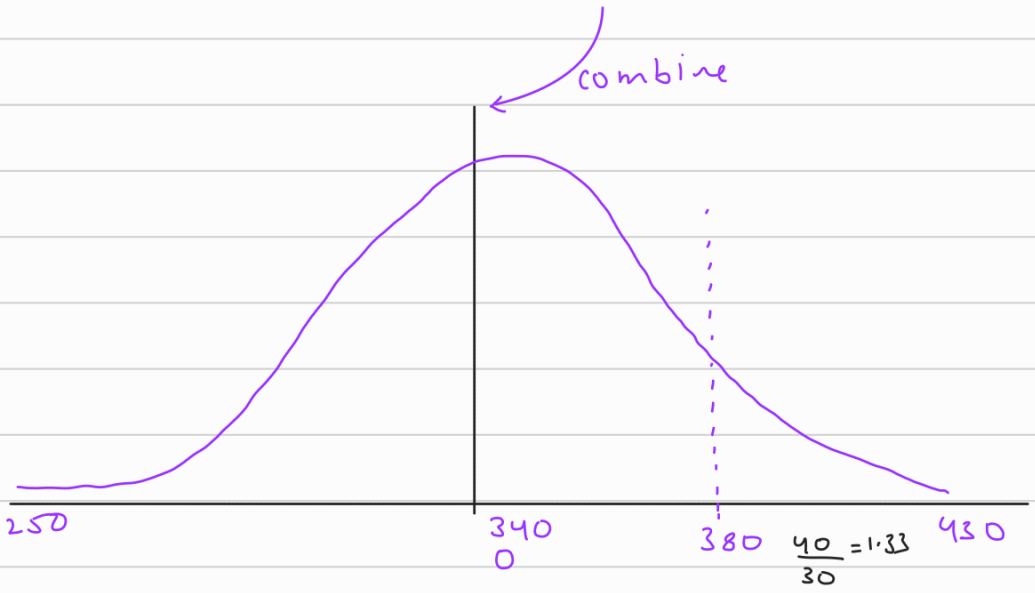
Q.



Find prob $n > 10$

Q. Roller coster ride where X rep. weight of bride $X \sim N(150, 400)$ (μ, σ^2) & weight of groom is given by $Y \sim N(190, 500)$, they can ride if their combined weight is <=380





- When we sum up 2 distribution follow gaussian distribution it will also follow gaussian dist.

$E(x+y) = E(x) + E(y)$ $\quad x, y$ = independent

$Var[x,y] = Var[x] + E[y]$

$\therefore (x,y) \sim N(340, 900)$

$$Z = \frac{380-340}{30} = 1.33$$

$\therefore$ Area = Prob = 0.908

Q. In town male height ~ $N(71, 20.25)$, & for women $\sim N(64, 16)$ what is prob that man will be at least 5 inches taller than women

$X : N(71, 20.25)$ $\quad X - Y \geq 5$

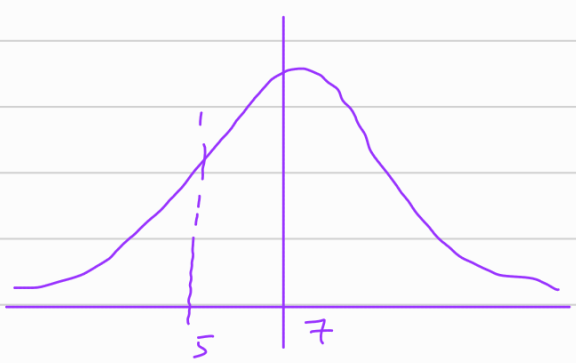
$Y : N(64, 16)$ $\quad x, y \in$ independent

$E(X-Y) = E(X) - E(Y) = E_N$

$V[X-Y] = V[X] + V[Y] = V_N$

$E_N = 7$, $V_N = 36.25$

$\sigma_N = 6.02$



$$P(<5) = z\left(\frac{5-7}{6.02}\right) = z\left(\frac{-2}{6.02}\right) = z(-0.33) = 0.37070$$

$$\therefore P(>=5) = 1 - 0.37070$$
$$\simeq 0.63$$

Till chap 9 Head first stat.

Binomial distribution →

- 'n' - Toss head success
- $P(X=r) = {}^nC_r\, p^r q^{n-r}$

Q. Ques. paper 50 Q → 4 possible ans (A, B, C, D) one of them is correct prob ≤30 correct?

$$p = \frac{1}{4}, \quad q = \frac{3}{4}, \quad n = 50$$

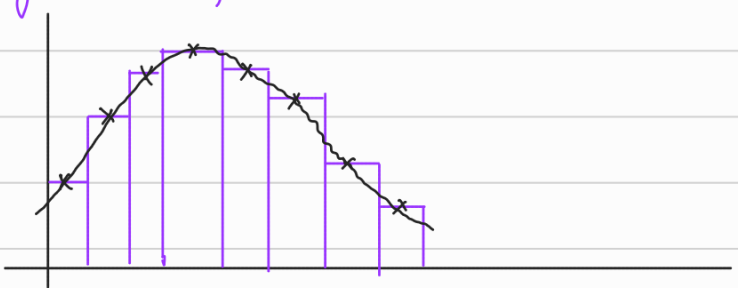
$$P(X \leq 30) = \sum_{n=0}^{30} {}^nC_r\, p^r . q^{n-r}$$

→ Calculative

∴ If $np \leq 10$ we plot a histogram for 'i' we plot bar from i-0.5 to i+0.5

→ but if $np > 10$ then this histogram tends to follow gaussian distribution.

- Then we normalize freq. of histogram by dividing freq/Total freq.
- Then we apply regression to found the curve



- let by req we get Pdf if $\int_{-\infty}^{\infty} pdf \cdot dx = p$ divide it by p $\therefore pdf = \frac{1}{p}(pdf)$

Else we can use parzen window estimator.

- let 3 student height 5', 5.5', 5.6' & i want to find overall pdf
- we will first find standard deviation 'σ'
- we will take diff function

$$Pdf_{Parzen} = \frac{1}{3}(GF('5', 'σ') + GF(5.5', σ) + GF(5.6', σ))$$

- The shape of window can be anything

$E[x] = np$

$Var[X] = npq$

X	0	1
P(x)	q	p

$$E[x]_{\text{single trail}} = \sum x P(x=n) = 0 \cdot q + 1 \cdot p = p$$

$E[x_1 + x_2 \cdots + x_n] = np$ ∵ trails are independent

$$E[x] = np$$

$Var[x_1 \cdots + x_n] = n\, Var[x]$

$$Var_{\text{single trail}}[x] = E[x^2] - (E[x])^2$$

$$= \sum x^2 P(x=n) - p^2$$

$$= 0 \cdot q + 1 \cdot p - p^2$$

$$= p - p^2 = p(1-p) = pq$$

$\therefore$ Var [x] = pq
for 1 trail

So. $\text{var}_n[x] = npq$

Q. Their are 20 Question the prob of getting 4 or less question correct (single MCQ ans 2 option)

$$P(X=4) = {}^{20}C_4 \left(\frac{1}{2}\right)^4 \left(\frac{1}{2}\right)^{16}$$

$$P(X=3) = {}^{20}C_3 \left(\frac{1}{2}\right)^{20}$$

$$P(X=2) = {}^{20}C_2 \left(\frac{1}{2}\right)^{20}$$

$$P(X=1) = {}^{20}C_1 \left(\frac{1}{2}\right)^{20}$$

$$P(X=0) = {}^{20}C_0 \left(\frac{1}{2}\right)^{20}$$

$$\Sigma = \frac{1}{2^{20}} \left[{}^{20}C_0 + {}^{20}C_1 + {}^{20}C_2 + {}^{20}C_3 + {}^{20}C_4 \right]$$

$$= \frac{1}{2^{20}} \left\{ 1 + 20 + 190 + 1140 + 4845 \right\}$$

$$= \frac{1}{2^{20}} \cdot (6196) = 0.0059$$

- Expectation is mean.

Transform

$\text{Exp}[x] = np = 20 \cdot \frac{1}{2} = 10 \geq 10$ $\therefore$ Gaussian distribution

$\mu = 10$, $\text{var} = npq = 5 \Rightarrow SD = \sqrt{5}$

$X = 4$ transform

$$Z = \frac{4-10}{\sqrt{5}} = -2.68$$

Area $P(X \leq 4) = 0.0037$ Wrong

we need to apply distribution correction

for i histogram goes till i+0.5

$\therefore$ $X = 4.5$ transform

$$Z = \frac{4.5-10}{\sqrt{5}} = \frac{-5.5}{\sqrt{5}} = -1.1 * \sqrt{5} = -2.46$$

Area = Area(Z) = 0.0069 $\simeq$ 0.0059 $\therefore$ Right

Questions →

1.98 Lifetimes of Flashlight Batteries. Two different options are under consideration for comparing the lifetimes of four brands of flashlight battery, using 20 flashlights.

a. One option is to randomly divide 20 flashlights into four groups of 5 flashlights each and then randomly assign each group to use a different brand of battery. Would this statistical design be a completely randomized design or a randomized block design? Explain your answer.

b. Another option is to use 20 flashlights—five different brands of 4 flashlights each—and randomly assign the 4 flashlights of each brand to use a different brand of battery. Would this statistical design be a completely randomized design or a randomized block design? Explain your answer.

- Complete random no generator when ∃ same unit (cactus) we will divide them by random no without replacement

- randomize block design when ∃ more then 1 species say out of 16 ∃ (8, 8), 2 species then we divide them equally both species

	T1	T2	T3	T4
2SP1	2	2	2	2
2SP2	2	2	2	2

1.91 Treating Heart Failure. In the journal article "Cardiac-Resynchronization Therapy with or without an Implantable Defibrillator in Advanced Chronic Heart Failure" (*New England Journal of Medicine*, Vol. 350, pp. 2140–2150), M. Bristow et al. reported the results of a study of methods for treating patients who had advanced heart failure due to ischemic or nonischemic cardiomyopathies. A total of 1520 patients were randomly assigned in a 1:2:2 ratio to receive optimal pharmacologic therapy alone or in combination with either a pacemaker or a pacemaker–defibrillator combination. The patients were then observed until they died or were hospitalized for any cause.

a. How many treatments were there?
b. Which group would be considered the control group?
c. How many treatment groups were there? Which treatments did they receive?
d. How many patients were in each of the three groups studied?
e. Explain how a table of random numbers or a random-number generator could be used to divide the patients into the three groups.

In Exercises 1.92–1.97

a. 1: optimal therapy
2: Peacemaker optimal therapy
3. Peacemaker-defibrillator optimal therapy
} 3 treatment

Jisko desired treatment denge

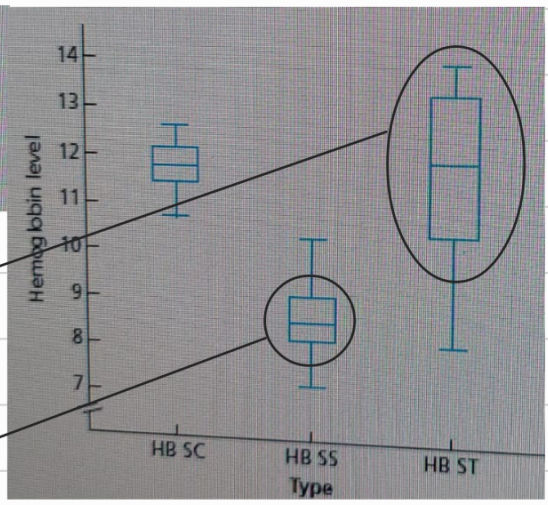
b. Optimal therapy

c. $\frac{1520}{5}$, $\frac{1520 \times 2}{5}$, $\frac{1520 \times 2}{5}$

d.

e.

3.181 Sickle Cell Disease. A study published by E. Anionwu et al. in the *British Medical Journal* (Vol. 282, pp. 283–286) examined the steady-state hemoglobin levels of patients with three different types of sickle cell disease: HB SC, HB SS, and HB ST. Use the following boxplots to compare the hemoglobin levels for the three groups of patients, paying special attention to center and variation.



High variation

low suger level

23. Millionaires. Refer to Problem 20. The ages of the 36 millionaires sampled are arranged in increasing order in the following table.

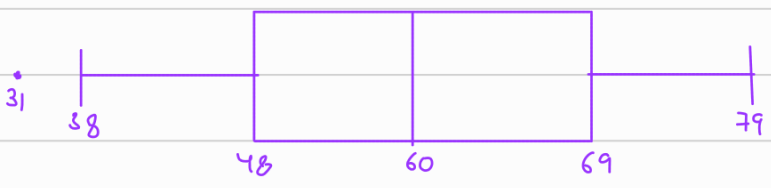
31	38	39	39	42	42	45	47	48
48	48	52	52	53	54	55	57	59
60	61	64	64	66	66	67	68	68
69	71	71	74	75	77	79	79	79

a. Determine the quartiles for the data.
b. Obtain and interpret the interquartile range.
c. Find and interpret the five-number summary.
d. Calculate the lower and upper limits.
e. Identify potential outliers, if any.
f. Construct and interpret a boxplot.

Outlier represented by dot in boxplot isko ignore karo our median nikalo

(M) median = 60 (35 element)
median (1→M) = 48 (17 element)
median (M+1→last) = 69 (17 element)

} Quartile



Outlier : 31.

Interquartile range: 69-48 = 21 LB = 38, UP = 79.

3.195 Fill in the following blanks.
a. A standardized variable always has mean _____ and standa deviation _____.
b. The z-score corresponding to an observed value of a variable tel you _____.
c. A positive z-score indicates that the observation is _____ th mean, whereas a negative z-score indicates that the observatio is _____ the mean.

3.196 Identify the statistic that is used to estimate
a. a population mean.
b. a population standard deviation.

3.216 Low-Birth-Weight Hospital Stays. Data on low-birth-weight babies were collected over a 2-year period by 14 participating centers of the National Institute of Child Health and Human Development Neonatal Research Network. Results were reported by J. Lemons et al. in the on-line paper "Very Low Birth Weight Outcomes of the National Institute of Child Health and Human Development Neonatal Research Network" (*Pediatrics*, Vol. 107, No. 1, p. e1). For the 1084 surviving babies whose birth weights were 751–1000 grams, the average length of stay in the hospital was 86 days, although one center had an average of 66 days and another had an average of 108 days.
a. Can the mean lengths of stay be considered population means? Explain your answer.
b. Assuming that the population standard deviation is 12 days, determine the z-score for a baby's length of stay of 86 days at the center where the mean was 66 days.
c. Assuming that the population standard deviation is 12 days, determine the z-score for a baby's length of stay of 86 days at the center where the mean was 108 days.
d. What can you conclude from parts (b) and (c) about an infant with a length of stay equal to the mean at all centers if that infant was born at a center with a mean of 66 days? mean of 108 days?

Q. $B \sim N(65, 16)$, $G \sim N(60, 9)$

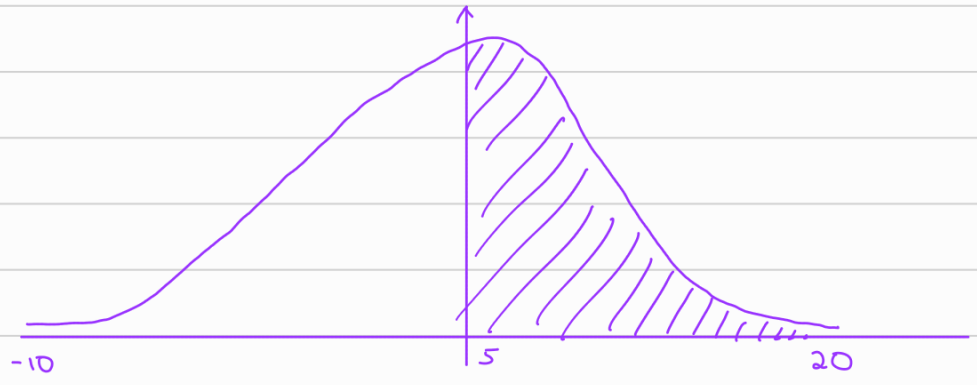
Prob. that a random boy chosen is 5 or more inch taller than a randomly chosen girl

B: $\mu = 65$, $\sigma = 4$

G: $\mu = 60$, $\sigma = 3$

$E(N) = E_B - E_G = 65 - 60 = 5$

$Var(N) = V_B + V_G = 16 + 9 = 25 \Rightarrow \sigma_N = 5$



$Z = \frac{5-5}{5} = 0$

$\therefore$ Area = Prob = z-Score(0) = 0.5

Q. Fit the poly. $y = w_0 + w_1 x$ for the following data. use ridge regularization take $C = 10$, Also do it without regularization

$(Y, X) = (3,1), (4,2), (5,3), (4,4), (5,5), (7,6)$

- Without regularization -

$$\Rightarrow \sum_{i=1}^{N} \hat{y}_i x_i - w_0 \sum x_i - w_1 \sum_{i=1}^{N} x_i^2 = 0 \quad -(2)$$

$$\Rightarrow w_0 N + w_1 \sum_{i=1}^{N} x_i = \sum_{i=1}^{N} \hat{y}_i \quad -(1)$$

$(2) \Rightarrow (3+8+15+16+25+42) - w_0(21) - w_1(91) = 0$

$$109 = 21w_0 + 91w_1$$

$(1) \Rightarrow w_0(6) + w_1(21) = 28$

$$w_0 = \frac{28 - 21w_1}{6}$$

$\hat{y}$	x
3	1
4	2
5	3
4	4
5	5
7	6

$$109 = 21\left(\frac{28-21w_1}{6}\right) + 91w_1$$

$$218 = 196 - 147w_1 + 182w_1$$

$$w_1 = 22/35$$

- With regularization -

$$y = w_0 + w_1 x \quad , C = 10$$

$\hat{y}$	x	y	$y - \hat{y}$
3	1	$w_0 + w_1$	
4	2	$w_0 + 2w_1$	
5	3	$w_0 + 3w_1$	
4	4	$w_0 + 4w_1$	
5	5	$w_0 + 5w_1$	
7	6	$w_0 + 6w_1$	

C. $\frac{1}{2}\sum (y-\hat{y})^2 + \frac{1}{2}\cdot\left(\begin{bmatrix} w_0 \\ w_1 \end{bmatrix}\begin{bmatrix} w_0 & w_1 \end{bmatrix}\right)$ to be minim

$(w_0+w_1-3)^2 + (w_0+2w_1-4)^2 + (w_0+3w_1-5)^2$
$+ (w_0+4w_1-4)^2 + (w_0+5w_1-5)^2 + (w_0+6w_1-7)^2 + \frac{1}{10}(w_0^2+w_1^2)$ to be minim

$(w_0+w_1)^2 + 9 - 6w_0 - 6w_1$
$+ (w_0+2w_1)^2 + 16 - 8w_0 - 16w_1$
$+ (w_0+3w_1)^2 + 25 - 10w_0 - 30w_1$
$+ (w_0+4w_1)^2 + 16 - 8w_0 - 16w_1$
$+ (w_0+5w_1)^2 + 25 - 10w_0 - 50w_1$
$+ (w_0+6w_1)^2 + 49 - 14w_0 - 84w_1$
$+ w_0^2 + w_1^2$ to min

$\Rightarrow (w_0+w_1)^2 + (w_0+2w_1)^2 + (w_0+3w_1)^2 + (w_0+4w_1)^2 + (w_0+5w_1)^2 + (w_0+6w_1)^2$
$+ \frac{1}{10}(w_0^2+w_1^2) + 140 - 56w_0 - 202w_1$ to be mi.

$\Rightarrow w_0^2 + w_1^2 + 2w_0w_1 + w_0^2 + 4w_1^2 + 4w_1w_0 + w_0^2 + 9w_1^2 + 6w_0w_1$
$+ w_0^2 + 16w_1^2 + 8w_0w_1 + w_0^2 + 25w_1^2 + 10w_0w_1 + w_0^2 + 36w_1^2$
$+ 12w_0w_1 + \frac{1}{10}(w_0^2 + w_1^2) - 56w_0 - 202w_1$

$\frac{\partial E}{\partial w_0} = 0$ $\frac{\partial E}{\partial w_1} = 0$

$61w_0^2 + 911w_1^2 + 420w_0w_1 - 560w_0 - 2020w_1$ min

$122w_0 + 420w_1 = 560 \Rightarrow 61w_0 + 210w_1 = 280$

$1822w_1 + 420w_0 = 2020 \Rightarrow 911w_1 + 210w_0 = 1010$

$w_0 = \frac{1010 - 911w_1}{210}$

$$\frac{61\,(1010 - 911\omega_1)}{210} + 210\omega_1 = 280$$

$$61\,(1010 - 911\omega_1) + 44100\omega_1 = 280 \times 210$$

$$(44100 - 911 \times 61)\,\omega_1 = 280 \times 210 - 61 \times 1010$$

$$-11471\ \omega_1 = -2810$$

$$\omega_1 = 0.25$$

$$\omega_0 = 3.725$$

Q 4.72

$\mu = 78, \quad \sigma = 10$

Q. 4.73

$\mu = ?, \quad \sigma = ?$

$$z = \frac{70 - \mu}{\sigma} = -0.6$$

$$z = \frac{88 - \mu}{\sigma} = 1.4$$

$$70 - \mu = -0.6\sigma$$
$$88 - \mu = 1.4\sigma$$

$$18 = 2\sigma$$
$$\sigma = 9 \qquad \therefore \mu = 70 + 0.6(9)$$
$$= 70 + 5.4 = 75.4$$

Q.4.74

(a) $z = -1.20$ to $z = 2.40$

$$z(2.40) - z(-1.20) = 0.99180 - 0.11507$$
$$= 0.87673$$

(b) $z = 1.23$ to $z = 1.87$

$$z(1.87) - z(1.23) = 0.96926 - 0.89065$$
$$= 0.07861$$

(c) $z = -2.35$ to $z = -0.5$

$$z(-2.35) - z(-0.5) = 0.30854 - 0.00939$$
$$= 0.29915$$

Q.4.75

(a) $z(-1.78) = 0.03754$

(b) $z(0.56) = 0.71226$

(c) $1 - z(-1.45) = 1 - 0.07353 = 0.92647$

(d) $1 - z(2.16) = 1 - 0.98461 = 0.01539$

(e) $z(1.53) - z(-0.80) = 0.93699 - 0.21186 = 0.72513$

(f) $z(-2.52) + 1 - z(1.83) = 0.00587 + 1 - 0.96638$

$= 0.03949$

Q.4.76

(a) $1 - z(-1.64) = 1 - 0.05050 = 0.94949$

(b) $z(1.96) - z(-1.96) =$

(c) $z(-1) + 1 - z(1) =$

Q.4.77

(a) $1 - 0.2266 = 0.7734 \Rightarrow z = 0.75$

(b) $0.0314 \Rightarrow z = -1.86$

(c) $z(z) - z(-0.23) = 0.5722$

$z(z) - 0.40905 = 0.5722$

$z(z) = 0.98125$

$= 2.08$

Q.4.78

$1 - z(z_1) = 0.84$

$z(z_1) = 0.16$

$z_1 = 0.99$

4.79

$\mu = 5 \quad , \quad \sigma = 2$

$P(X > 8) = 1 - P(X <= 8) = 1 - z((8-5)/2)$

$= 1 - z(1.5)$

$= 1 - 0.93319$

$= 0.06681$

Q.4.80

$N = 300 \quad , \quad \mu = 68 \quad , \quad \sigma = 3$

(i) $(1-P(X > 72)).N = (1 - z((72-68)/3)).N$

$= (1 - z(1.33)).300$

$= (1 - 0.90824)\ 300$

$= [0.09176 \times 300]$

$= [27.5]$

$= 28$

(ii) $P(X < 64).N = \lceil z((64-68)/3).N \rceil$

$= \lceil 0.09342 \times 300 \rceil$

$= \lceil 28.0267 \rceil = 29$

(iii) $(P(X<71) - P(X<65))N = \left[z\left(\frac{71-68}{3}\right) - z\left(\frac{65-68}{3}\right) \right] N$

$= [z(1) - z(-1)]\, N$

$= [0.84134 - 0.15866]\, 300$

$= \lceil 204.8 \rceil = 205$

(iv) $P(X = 68).N = ?$

$$P(x) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}$$

$$P(68) = \frac{1}{\sqrt{2\pi} \times 3}\, e^{\frac{-(68-68)^2}{2\times 9}} = \frac{1}{\sqrt{2\pi} \times 3} = 0.133$$

$$N(X) = P(68).N = 0.133 \times 300 = \lceil 39.9 \rceil = 40$$

Q. 4.81

- $\mu = 0.6140$, $\sigma = 0.0025$

(i) $[P(x < 0.618) - P(x < 0.610)] \cdot 100$

$\Rightarrow [z(1.6) - z(-1.6)] \cdot 100$

$\Rightarrow [0.94520 - 0.05480] \cdot 100$

$\Rightarrow$ 89.04 %

(ii) $[P(x > 0.617)] = 1 - P(x < 0.617)$

$= 1 - P(1.2) \Rightarrow$ 11.507 %

(iii) $[P(x < 0.608)]100 = z(-2.4) \times 100 = 0.00820 \times 100$

$= 0.820\%$

(iv) $P(X = 0.615) \times 100 = 100 \cdot \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

$= 100 \times \frac{1}{\sqrt{2\pi} \times 0.0025} \times e^{-(0.001)^2 / 2(0.0025)^2}$

$= \frac{100 \times 400}{\sqrt{2\pi} \times e^{0.08}} = 14710\%$

Q.4.82

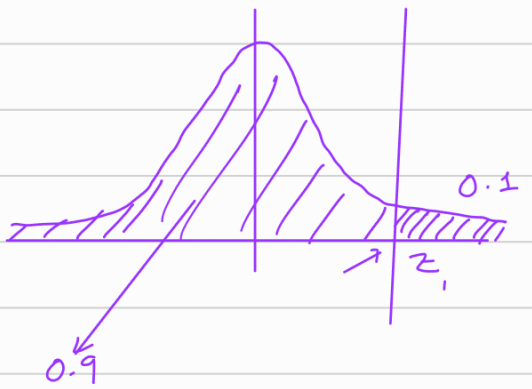
$\mu = 72, \quad \sigma = 9$

Top 10% = A min score A

$z\left(\frac{z_1 - 72}{9}\right) = 0.9$

$z_1 - 72 = 1.29 \times 9$

$z_1 = 83.61$



Q.4.83

(a) $P(X < -0.5) + 1 - P(X > 0.5)$

$0.30854 + 1 - 0.69146$

0.61708

(b) $P(X < 0.75)$

0.77337

Poisson distribution →

$$P(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

Q. 4.84

$\mu = 0, \sigma = 1$

(a) $P(x < 2) - P(x < -2)$

(b) $P(X < -1.2) + 1 - P(x < 1.2)$

(c) $1 - P(x < -1.5)$

Q. 4.85 (a) $P(a) - P(-a) = 0.75$

(b) $P(-a) = 0.22$

Poisonn dist. for diff λ

$\lambda = 1$ Right skewed
$\lambda = 4$ less right skewed
$\lambda = 10$ Almost gaussian

In Binomial distribution $np = \lambda$
(n is very large P is very small)

$X \sim P(\lambda)$
X follows poisions

$np >= 10$
& $npq \sim np$

→ We can transform poisonn to normal distribut -ion.

$E(X) = np$
$V[x] = npq$ $\because$ P is very small $\therefore q \approx 1$

$\therefore E[x] = V[x] = \lambda$

If $\lambda \geq 10$ we can transform poisonn distribution to normal distribution.

Q. A student need to take exam but has not done any rev. for it. he need to guess to ans to each ques. prob. to guess right ans = 0.05
∃ 50 ques in exam what is prob he get 5 Q right Use poisonn approximation to normal distribution?

$n = 50$, $p = 0.05$, $X = 5$
$\lambda = n.p = 2.5$

$$E(X = r) = \frac{e^{-\lambda} \lambda^r}{r!}$$

- Prove that

$$\lim_{n\to\infty} {}^{n}C_{r}\, p^{r}(1-p)^{n-r}$$

$$\lim_{n\to\infty} \frac{n!}{r!(n-r)!}\left(\frac{\lambda}{n}\right)^{r}\left(1-\frac{\lambda}{n}\right)^{n-r}$$

$$\frac{\lambda^{r}}{r!}\lim_{n\to\infty} \underbrace{\frac{n!}{(n-r)!}\left(\frac{1}{n}\right)^{r}}_{I}\underbrace{\left(1-\frac{\lambda}{n}\right)^{n}}_{II}\underbrace{\left(1-\frac{\lambda}{n}\right)^{-r}}_{III} \to 1$$

$$\frac{n(n-1)(n-2)\cdots(n-r+1)}{n\cdot n\cdot n\cdots r \text{ times}}$$

$$\Rightarrow 1$$

$$\frac{\lambda^{r}}{r!}\lim_{n\to\infty}\left(1+\frac{1}{\left(-\frac{n}{\lambda}\right)}\right)^{\left(-\frac{n}{\lambda}\right)(-\lambda)} \Rightarrow \frac{e^{-\lambda}\lambda^{r}}{r!} \quad \because a^{mn} = (a^{m})^{n}$$

$$\because \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{n} = e$$

- Expected value of binomial distribution (long way)

$$E(k) = \sum_{k=0}^{n} k\, {}^{n}C_{k}\, p^{k} q^{n-k} = np \quad \text{(Proof)}$$

$$= \sum_{k=1}^{n} k\, {}^{n}C_{k}\, p^{k} q^{n-k}$$

$$= \sum_{k=1}^{n} k\cdot\frac{n}{k}\, {}^{n-1}C_{k-1}\, p^{k} q^{n-k}$$

$$= \sum_{k=1}^{n} n\cdot {}^{n-1}C_{k-1}\, p^{k-1} q^{n-k}\cdot p$$

$$= np\sum_{k=1}^{n} {}^{n-1}C_{k-1}\, p^{k-1} q^{n-k}$$

$$= np\,(p+q)^{n-1}$$

$$= np$$

Proof :-

- Exp. & Var. of Geometric distribution
- Exp & var of binomial distribution
- Binomial to poisson distribution

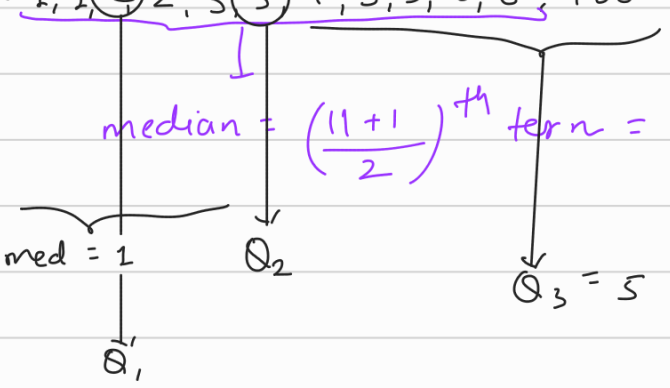
Mini

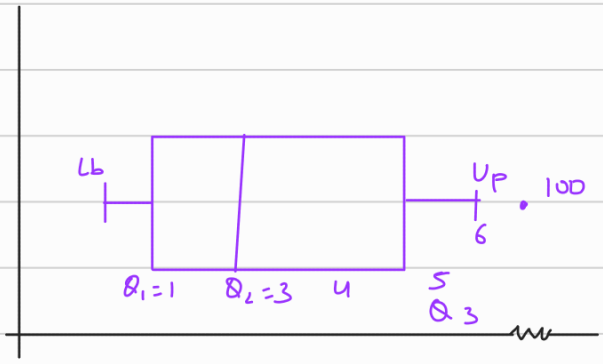
Imp topic :-

Boxplot	1 marks
Histogram	2 marks
Regression	2 marks
Normal distribution , binomial dist. poisson	3 marks
binomial to normal	2 marks

Q. Draw Box plot : 1, 1, 2, 2, 3, 3, 4, 5, 5, 6, 6, 100

Outlier = 100

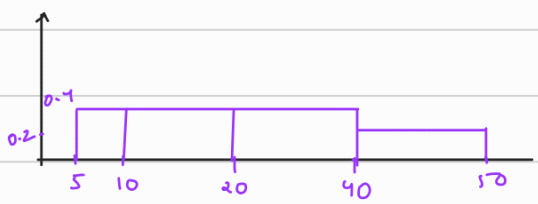




learn how to analyze boxplot

Q. Histogram

Age of mem	of Class	Int. width h:	
5 - 10	2	5	0.4
10 - 20	4	10	0.4
20 - 40	8	20	0.4
40 - 50	2	10	0.2



Q. Regression →

Lasso regularization is also there but it is calculative (used in ML)

Find out normal eqⁿ

$$E = C\frac{1}{2}\sum_{i=1}^{N}(\hat{Y} - y_i)^2 + \frac{1}{2}(w_0^2 + w_1^2)$$

where $y = w_0 + w_1 x$

$$E = \frac{1}{2} C \sum_{i=1}^{N}(\hat{y} - w_0 - w_1 x)^2 + \frac{1}{2}(w_0^2 + w_1^2)$$

$$\frac{\partial E}{\partial w_0} = 0 \quad , \quad \frac{\partial E}{\partial w_1} = 0$$

Regression (Matrix formulation) → (OLS using wikepedia)

using moore penrose pseudo inverse

$Y_{N\times 1} = X_{N\times m}\,\beta_{m\times 1}$ We need to find β

We need to minimize $\|Y - X\beta\|^2 = (X\beta - y)^T_{1\times N}(X\beta - y)_{N\times 1}$

$$L = (\beta^T x^T - y^T)(X\beta - y)$$

$$L = \beta^T x^T X\beta - \beta^T x^T y - y^T X\beta + yy^T$$

$$\frac{\partial L}{\partial \beta} = -2x^T y + 2x^T X\beta = 0$$

$(X^T X)^{-1} X^T$ = Moore penrose pseudo inverse

↳ it minimizes least square error.

With regularization →

$$\beta = (X^T X + I/C)^{-1} X^T y$$

I = Identity of size $m \times m$

C = Reg. param

(1) $\frac{1}{2} \sum_{i=1}^{n} (\hat{y} - y_i)^2 \Rightarrow (Y - X\beta)^2 = (X\beta - Y)(X\beta - Y)^T$

(2) $C \cdot \frac{1}{2} \sum_{i=1}^{n} (\hat{y}_i - y_i)^2 + \beta_0^2 + \beta_1^2 + \cdots$

$$\Downarrow$$

$$\frac{C}{2} (X\beta - Y)^T (X\beta - Y) + \frac{1}{2} \beta\beta^T$$

$$\Rightarrow \beta = (X^T X + I/C)^{-1} X^T Y$$

A1 → Implement regularized regression without using any library function tune the value of regularization parameter C to obtain optimal result

A2 → submit the stepwise proof of regularization regression in matrix form