19.07.2023
Wednesday

Lecture-1

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STATISTICAL METHODS

These methods are used for extracting the info. where uncertainity is involved

If o/p is fixed ⇒ NO uncertainity (ON/OFF Switch)

If o/p is not fixed ⇒ ~~Is~~ uncertainity (Tossing coin)

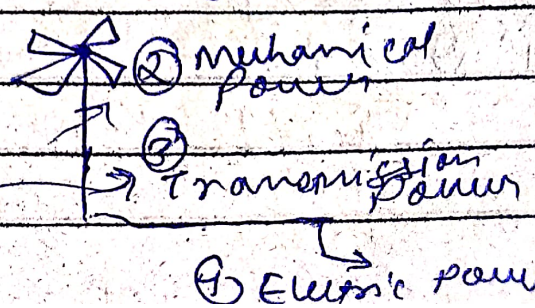
Statistical methods & ~~signs~~ science used for ~~not~~ decision making or extracting the knowledge where uncertainity is involved :-

Experiment

Deterministic (o/p known) — Switch on/off the bulb

Non-deterministic (Random) (o/p not known) — Tossing of a coin ↓ "uncertainity"

Natural Phenomenon ↓ All are random ↓ Current speed of wind = v_1 — What will be after 5 min? → Not known



Efficiency = η_i

η_1 η_2 η_3

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$$\boxed{P = \eta_1 \eta_2 \eta_3 \rho v^3}$$

ρ → air density

v → air speed

η_1, η_2, η_3 Remains constant unless everything is new

ρ & v changes uncertain

Most volatile = v

$\therefore P \propto v^3$ → change ઉમેરવાની ટેવ !

$P_1 = K_1 v_1$
$P_2 = K_2 v_1$

$P_1 \neq P_2$ even when $v_1 = v_1$ other factors can change.

① Study of methods to collect & organise numeric data related to phenomenon & experiment

② Analyze the numeric info.

③ Interpret the → mind

(i) Individual → coin, Tuesday, Tossing a coin

(ii) Variability : People or object included in the study

(iii) Data : → Face of the coin → Mind Speed & Power

Head / Tails

Electric

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Wind → Wind Speed → Value of this

Turbine → Power Electric → Value of this

Variables

- **Qualitative**
  - There is no ordering in this.
  - Tossing a coin :- Head, Tails
  - Religion of a man
- **Quantitative**
  - Rolling a dice
  - **Discrete**
    - Outcome can be listed
  - **Continuous**
    - Values lie betn the interval (in Range)

Random Variable: Whose values are depending on the chance.

Defined on Random experiment.

→ Instances : Sample Space of exp.

Random exp.: Tossing pair of coin

Sample Space : {HH, HT, TH, TT}.

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Random variable = x = No. of head] defination of variable

Can only be defined on sample space of Random experiment.

Real valued function

$x = 0, 1, 2$ → Instances / Events of R.V.

$\{TT\}$ $\{HT, TH\}$ $\{HH\}$

$1/4$ $1/2$ $1/4$ → Probabilities

For each instance there is probability

R.V. is the real valued fn defined on sample space of Random exp.

x = head in a single — Not a R.V. X (Boolean true or false)

x = No. of Heads — R.V. (Counting is involved)

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)

(2,2)

(3,3)

(4,4)

(5,5)

(6,6)

	1	2	3	4	5	6
1	0	1	2	3	4	5
2	1	0	1	2	3	4
3	2	1	0	1	2	3
4	3	2	1	0	1	2
5	4	3	2	1	0	1
6	5	4	3	2	1	0

RV. = x = abs. diff of two numbers

$x = 0, 1, 2, 3, 4, 5$ (6 instances)

$\frac{1}{6}$ $\quad \frac{\cancel{10}\,5}{\cancel{36}\,18}$ $\quad \frac{8}{36}, \frac{6}{36}, \frac{4}{36}, \frac{2}{36}$

Probability f^n can be defined on x these are called instances. Prob. density f^n

If sum of the probabilities of all the instances should be equal to 1

each i/p $\Rightarrow$ unique o/p

element of SS are covered

For each i/p of SS there is unique o/p

~~A~~ ~~Each~~

Each element of SS should be name in a single instance

In this case $\sum P_i = 1$

$P_i > 0$ (atleast one element is considered)

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Lecture-2.

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Random variable is a mapping from a sample space Ω to the relevant space of numbers.

↓ collection of
All possible outcomes

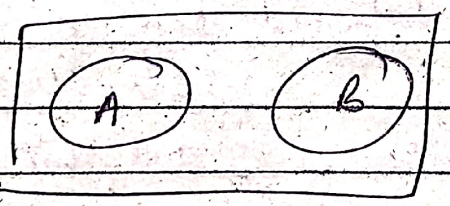
Event :- An evt. associated with a random exp. is a subset of SS
↳ Instance

E : Evt. $\bar{E}$ = evt.

A : Evt. B : Evt. $A \cap B$: Evt.
$A \cup B$: Evt.

Mutually Exclusive

A & B → no two common evts. in A & B.



↓
Random variable's instances always ME
(part of 1 instance)

Collutively Exhaustive ~~Who~~

When it covers all elements of SS & no other possibility.

Probability Distrib^n

list of all valid outcomes with associated probabilities.

Each observation belongs to one instance & all should be covered. $P_i > 0$

ME & CE

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Probability Density f^n A f^n that represents PD of discrete random variable. $\Sigma P_i = 1$, $\Sigma P_i > 0$

Ex:

Observations : 60

Instances :	5 ft.	5.5 ft.	6 ft.
Probability			

Can be represented by

Prob. Histogram

Mean & Standard Deviation of a Discrete RV

→ exact value

Avg. :- When no. of students are fixed.

Mean :- In form of group with probability we calculate mean.

→ expected value

(Group form).

~~Contin~~ interval based

$a < x < b$ $P(x) =$

No. of observations are very large.

Variance :- Avg. deviation of all instances w.r.t. expected value.

Variance of discrete R.V. $= V(x) = \Sigma [x_i - \mu]^2 P(x_i)$

$$\Rightarrow \boxed{\begin{aligned} V(x) &= \Sigma [x_i]^2 P(x_i) - \mu^2 \\ &= \text{Mean of squares} - \text{Sq. of mean} \end{aligned}}$$

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$$P[x_i]^2 + P[\mu]^2 - 2 x_i \mu$$
$$P[x_i]^2 + P[\mu]^2 - 2 \mu \mu$$
$$P[x_i]^2 - P[\mu]^2$$

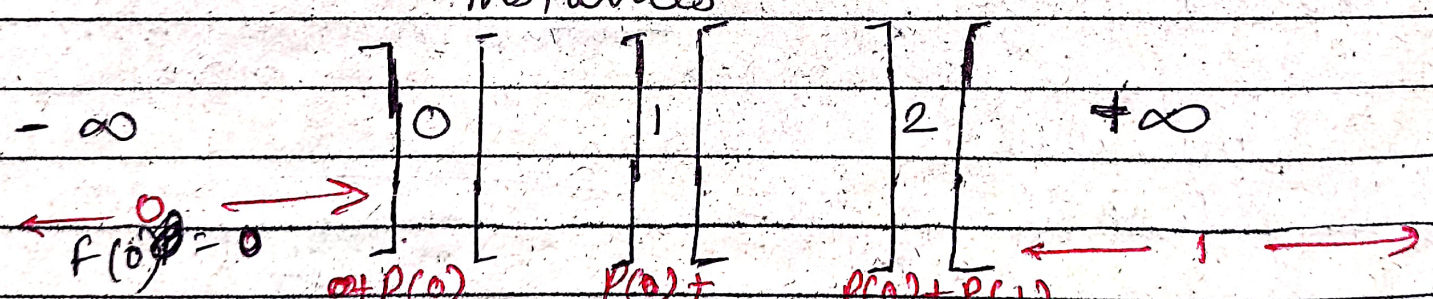
Standard Deviation $\sigma = \sqrt{\Sigma [x_i - \mu]^2 P(x_i)}$

Probability Distribution fn cumulative sum of probabilities from $-\infty$ to ∞
[all points]

valid instances = 0, 1, 2

∴ for all other points make $P = 0$

→ gives prob. of all valid & invalid instances



$$F(a) = P(x \le a) = \sum_{x_i \le a} P(x_i), \quad x_i \in X$$

union of all instances less than a

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Lecture 03

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Sum of 2 number is 5 → valid event
Abs diff. of 2 num. → Not valid
Abs. → is 5 → valid evt.

→ Union: Abs. diff 2 num. is 5 & sum of 2 number is 5
∴ Valid evt.

All instances are part of SS but vice versa is not true.

Instances are always evts but all evts are not instances

All the ~~cases~~ evts where we can generate real value
→ Random variable.
" boolean value
→ X RV X

For ~~Sam~~ same RV we can't find intersection of instances (ME)

For diff. RV we can find ∩ of instances
~~Cmay~~ :

No. of ways events can occur

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Probability for Equally likely Outcomes = $\frac{f}{N}$

Total no. of possible outcomes

① Random variable notat^n

X = denote total no. of tails in 4 tosses.

$\{X = 3\}$. Total no. of tails obtained in 4 tosses

②

No. of tails

$P(0) = \frac{1}{16}$ — — — —

$^4C_1 = 4$

$P(1) = \frac{4}{16} = \frac{1}{4}$

$P(2) = \frac{6}{16} = \frac{3}{8}$ — — — — $^4C_2 = \frac{4 \times 3}{2} = \frac{6}{16}$

$P(3) = \frac{3}{8}$ $^4C_3 =$

③ $X = N(H) - N(T)$

	1	1	-1
3 HHH	HHT	HTH	HTT
THH	THT	TTH	TTT
1	-1	-1	-3

① $\{X = 3, 1, -1, -3\}$

$P(3) =$

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(i) Probability density fn

$P(X = -3) = \frac{1}{8}$

$P(X = -1) = \frac{3}{8}$

$P(X = 1) = \frac{3}{8}$

$P(X = 3) = \frac{1}{8}$

(ii) Mean $= \frac{1}{8} \times (-1) + \frac{3}{8} \times (-3) + \frac{3}{8} \times 3 + \frac{1}{8} \times 1$

$\Sigma P_i(x_i)$

$= \cancel{\frac{1+9+9+1}{8} = \frac{20}{8} = \frac{5}{2}}$

0

(iii) Standard deviation variance $= \sigma^2 = \Sigma P(x_i) \times (x_i)^2 - \mu^2$

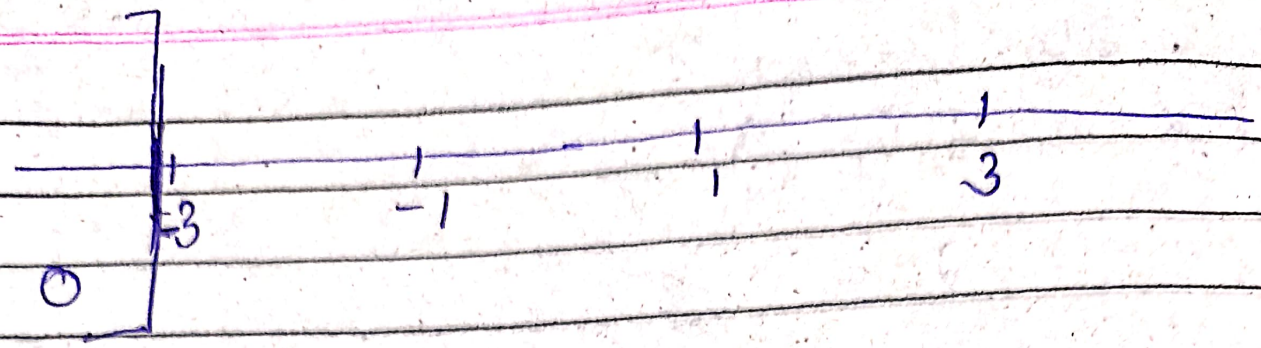
$= \cancel{\frac{9 \times 3}{8}}$

$= (-3)^2 \times \frac{1}{8} + (-1)^2 \times \frac{3}{8} + (1)\frac{3}{8} + \frac{1}{8} \times 3^2$

$= \frac{9+3+3+9}{8} = 3$

(iv) $P(\cancel{x < -3}) =$

~~Distribution fn~~



$F(-3) = P(x \leq -3) = \frac{1}{8}$

$F(-1) = P(x \leq -1) = \frac{1}{8} + \frac{3}{8} = \frac{1}{2}$

$F(+1) = P(x \leq 1) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} = \frac{7}{8}$

$F(+3) = P(x \leq 3) = \frac{1}{8} + \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = 1$

$$F(x) \begin{cases} 0 & x < -3 \\ 1/8 & -3 \leq x < -1 \\ 1/2 & -1 \leq x < 1 \\ 7/8 & 1 \leq x < 3 \\ 1 & x \geq 3 \end{cases}$$

Q)

$$P(x) = \begin{cases} k & \text{if } x = 0 \\ 2k & \text{if } x = 1 \\ 3k & \text{if } x = 2 \\ 0 & \text{otherwise} \end{cases}$$

① $\sum P_i = 1$

$k + 2k + 3k = 1 \Rightarrow 6k = 1 \Rightarrow k = \frac{1}{6}$

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(2) $P(x=0) = \frac{1}{6}$

$P(x=1) = \frac{2}{6}$

$P(x=2) = \frac{3}{6}$

(i) $P(x<2) = \underset{x=0}{\frac{1}{6}} + \underset{x=1}{\frac{2}{6}} = \frac{3}{6} = \frac{1}{2}$

(ii) $P(x \leq 2) = 1$ (all the points

(iii) $P(0 < x \leq 2) = \frac{2}{6}$ $(x=1)$

(3) Distribution Function:-

$F(0) = P(x \leq 0) = \frac{1}{6}$

$F(1) = P(x \leq 1) = \frac{2}{6} = \frac{1}{3}$

$F(2) = P(x \leq 2) = \frac{3}{6} = \frac{1}{2}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{6} & 0 \leq x < 1 \\ \frac{1}{3} & 1 \leq x < 2 \\ \frac{1}{2} & x \geq 2 \end{cases}$$

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Q)

$$P(x) = \begin{cases} 1/4 & x = -2 \\ 1/4 & x = 3 \\ 1/2 & x = 6 \\ 0 & \text{Otherwise} \end{cases}$$

(i) $P(1 < x \leq 4) = P(x=3) = \frac{1}{4}$

(ii) $P(x > 5) = P(6) = \frac{1}{2}$

(iii) $P(2x - 3 \geq 1) = P(3) + P(6) = \frac{1}{4} + \frac{2}{4}$

$2x - 3 \geq 1$ $\qquad = \frac{3}{4}$

$x \geq 2$

(iv) $P(x^2 - 2x \leq 3) = P(3) = \frac{1}{4}$

$x^2 - 2x - 3 \leq 0$

$(x-3)(x+1) \leq 0$

$x \in [-1, 3]$

Prob. Mass Distribution : Integration (Area)
Prob density Distribu^n : Isolated Pts.

Discrete RV.

2 Coin Toss : x (No of Heads) =

0	1	2
$\frac{1}{4}$	$\frac{2}{4}$	$\frac{1}{4}$

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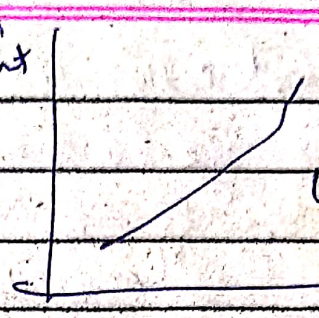
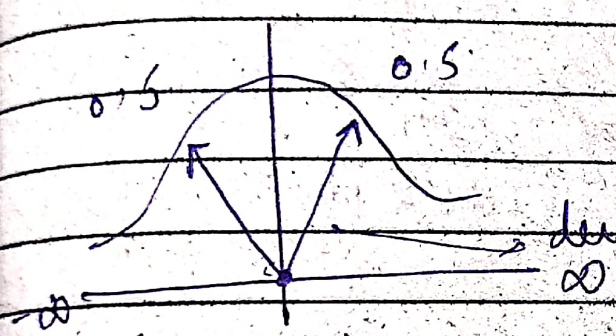
$r \in [-1, 1]$
correlation coefficient

x	y
2	1
5	3
7	9
⋮	⋮

+ve relation
$x \uparrow$ $y \uparrow$



Linear Regression



Continuous RV
$\int_{-\infty}^{\infty} f(x)\, dx = 1$

density Discrete RV

Symmetric
Normal

B.D. $^nC_r\, p^r q^{n-r}$
$p + q = 1$

Poisson D. $p \to 0$
$n \to \infty$

$y = mx + c$
Single variable

$y = m_1 x_1 + m_2 x_2 + \dots + m_n x_n + c$

Q) Three coin are tossed simultaneously
① find the P(getting atleast 2 Heads)
$\Rightarrow \frac{4}{8} = \frac{1}{2}$

{HHH THH
HHT THT
HTH TTH
HTT TTT}

② at most 2 heads
$\Rightarrow \frac{7}{8}$

Q) A bag contain 4 red balls, 6 yellow balls & 3

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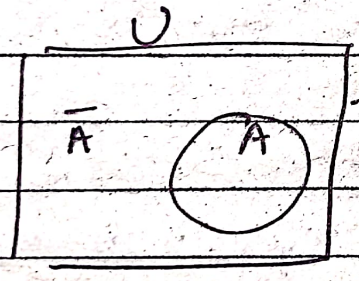
balls are drawn randomly then what is probability that are of same color?

3

$$= \frac{{}^4C_3 + {}^6C_3}{{}^{10}C_3}$$

$$= 4 + \frac{\frac{6 \times 5 \times 4}{3 \times 2}}{\frac{10 \times 9 \times 8}{3 \times 2}} \qquad \frac{24}{10 \times 12} = \frac{1}{5}$$



→ can't define RV here

Qualitative outs

↓

Qualitative ← Toss winning captain will won the match

T F

n no. of matches how many times tossing winning won the match

n+1 instances $\{0, 1, 2, 3, \ldots, n\}$

Quantitative

Tossing a coin equally likely

P(batting if batting is preferred) $> \frac{1}{2}$

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Indian captain choosing batting after winning toss (good in bat)

→ collect historical data (observations)

Unconditional Prob. :- No. extra cond^n.
Calculated w.r.t. Universe

conditional Prob. :- $P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\frac{P(A \cap B)}{P(B)}$

P(A) w.r.t. P(B)

Looking $A \cap B$ w.r.t. B

~~P(A)~~ $A \cap B$ w.r.t. U

① Prob.] uncond^n
② Prior Prob.]
③ Likelihood] cond^n
④ Posterior Prob.]

Some obs. are required (historical data)
Parameters
Observation needs to be done.

Toss winning captain will elect the bat first. → (I)
The stadium is Wankhedi stadium. → (II)
P(that (I) when the stadium (II))

Sample space: All people played in Wankhedi.
easy observation → Check how (I) satisfies in this & this observed.
tough to interrogate (We can interrogate)
P(that match is played on Wankhedi with that toss winning captain elect the bat first) → Collect all info and record

When given is less we can interrogate

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Q) It is given that in a population one in 50,000 has meningitis. 1 in 20 has stiff neck. How to compute no. of M. patients having stiff neck and no. of stiff neck patients having M.

$\rightarrow M$

$P(M) = \frac{1}{50000}$ $\quad P(N) = \frac{1}{20}$

$\cancel{P(M|N) = \frac{P(M \cap N)}{P(N)} = \frac{\frac{1}{50000} \times \frac{1}{20}}{\frac{1}{20}}}$ = Given

likelihood

$P(N|M) =$ M is small. If likelihood is $\frac{1}{2}$

$\rightarrow$ A given B.

$P(A|B) = \frac{P(A \cap B)}{P(B)}$ $\quad \cancel{P(A|B)} \quad \frac{1}{P(A \cap B)} = \frac{1}{1}$

$P(B|A) = \frac{P(A \cap B)}{P(A)}$

Bayes formula

$$\boxed{P(A|B) = \frac{P(B|A) \times P(A)}{P(B)}}$$

about one should be likelihood

$P(N|M) = \frac{1}{2}$ $\quad$ Neck given meningitis

$P(M|N) = \frac{P(N|M) \times P(M)}{P(N)}$ $\quad \frac{1}{2} \times \frac{1}{50000}$

$= \frac{1}{2} \times \frac{1}{50000}$

$= \frac{\frac{1}{100000}}{\frac{1}{20}} = \frac{1}{5000}$

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$P(A|B) \rightarrow$ Post erior

$P(B) = x$. $P(A) = ?$

highly populated.
(nearly equal to Uniform)

$A \rightarrow A_1$
$\rightarrow A_2$
$\vdots$
A_K

ME & CE

'R.V.

$P(A_i) = ?$

$$\sum P(A_i|B) = \sum \frac{P(B|A_i) \times P(A_i)}{P(B)}$$

$P(A_i|B) > 0$ $\quad \sum P(A_i|B) = 1$

$$1 = \frac{P(B|A_1) * P(A_1)}{P(B)} + \frac{P(B|A_2) * P(A_2)}{P(B)} + \dots \frac{P(B|A_K) * P(A_K)}{P(B)}$$

$$\Rightarrow \boxed{\sum P(B|A_i) * P(A_i) = 1}$$

Total Probability

① ~~Every~~ all B should be covered by A.

#) Blood test

+ve Res $\frac{1}{2}$ $\quad$ -ve Res $\frac{1}{2}$

in general

on diseased person
$P(+ve\ res) \approx 1$

on not diseased person
$P(+ve\ res) \approx 0$

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Q) The Prob. of a person choosen at random having a disease is 0.05. The choosen person is administr-ated a diagonistic test which is not perfect. The prob. that the diagonostic test gives a positive result when the disease is present is 0.7 & a negative result when disease is not present is 0.95. Find out the P(person) having the disease when the diagnostic test gives a +ve result.

Sol:

$P(D) = 0.05 \Rightarrow P(\bar{D}) = P(ND) = 1 - 0.05 = 0.95$

~~P(+ve) = 0.7 P(~~

$P(+ve \mid D) = P(P \mid D) = 0.7$ 70% accuracy

$P(N \mid ND) = 0.95$ 95% chances (better)

$P(P \mid ND)$

$P(P)$ $P(N)$ → ~~dy~~ unknowns

$$P(D \mid P) = \frac{P(P \mid D) \times P(D)}{P(P)}$$

$P(P)$ → Total Probability → P → D, ND

$$P(P) = P(P/D) \times P(D) + P(P/ND) \times P(ND)$$

$$= \cancel{0.05 \times 0.0} \; 0.7 \times 0.05 + (0.05) \times 0.95$$

$$= 0.0825$$

$$P(D \mid P) = \frac{0.7 \times 0.05}{0.0825} = 0.4242$$

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Sol:

100%

30%	30%	40%
P_1	P_2	P_3
Defective rate 2%	3%	4%

~~4% → Defective rate~~

$P(P_2|D)$

$P = P(P_2 \cap D)$

$$P(P_2|D) = \frac{P(P_2 \cap D)}{P(D)} = \frac{0.02 \times 0.3}{0.02 \times 0.3 + 0.03 \times 0.3 + 0.04 \times 0.4}$$

Total probability

$$= \frac{3 \times 3}{\underset{6}{2 \times 3} + \underset{9}{3 \times 3} + \underset{16}{4 \times 4}} = \frac{9}{31}$$

$$x^*|c = \underset{(y_i)}{\operatorname{argmax}} \left[P(x_i/c) \right] \forall \text{ values of } x$$

(Always Posterior)

$$= \underset{(y_i)}{\operatorname{argmax}} \left[\frac{P(c/x_i) \times P(x_i)}{P(c)} \right] \forall \text{ values of } x$$

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BINOMIAL COEFFICIENT

Two events successes & failures.

Out of n trials (n+1) successes are possible.

Instance (0, 1, 2, 3, ..., n-1)

BD is used to describe the probability distribution of the no. of successes in n independent trial of some experiment.

$$\boxed{P(X=x) = {}^nC_x (p)^x (1-p)^{n-x}}$$

$$\boxed{\begin{array}{l} \text{mean} = np \\ \text{Variance} = npq \end{array}}$$

POISSON

$$\frac{e^{-\lambda}\lambda^x}{x!}$$

Q) In a particular minority 40% students smoke, 6 minority students are selected to know their views on smoking. Find Prob. that

(i) None of those selected smoke

(ii) All those selected smoke

(iii) Atleast half of the 6 smoke.

Sol: ① Identify the success

smoking

T

40% S — 60% NS

② $p = 0.4$ $q = 0.6$

③ $n = 6$

(i) ${}^nC_x p^x q^{n-x} = 1 \times (0.6)^6$

$= 0.046656$

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(ii) $P(x=6) = {}^{n}C_{6} (0.4)^{6} = 4.096 \times 10^{-3}$

(iii) $P(x \geq 3) = P(x=3) + P(x=4) + P(x=5) + P(x=6)$

Q) In a family of 3 children. What is the prob. that there will be exactly 2 girls

Sol: ① G → Success

② $p = 0.5$ $q = 0.5$

③ $n = 3$ $x = 2$

${}^{3}C_{2} (0.5)^{2} (0.5)^{1} = 0.375$

Q) A multiple choice test consists of 4 questions each questions has 5 possible answers of which only 1 is correct. A student guesses on every question. Find the probability distribution of x : the num. of questions she answered correctly.

Sol: ① Success → correct ans.

② $p = 0.2 = \frac{1}{5}$ $q = 0.8$ single trial ↓ single question

③ $n = 4$

x	$P(x)$
0	${}^{n}C_{x} p^{x} q^{x}$
1	
2	
3	
4	

5 instances

For

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Q) In one city 23% of population is under 25 years of the age. 3 people are selected at random from the city. Find the prob. distribution of x where x is the no. among the 3 that are under 25 yrs of age.

Sol: (1) success → " < 25 yrs

(2) $p = 0.23$ $q = 1 - p$

(3) $n = 3$

x	$P(x)$	${}^nC_x p^x q^{1-x}$
0		
1		
2		
3		

For Binomial R.V. ~~is binom~~ has prob. distributio table → Binomial distribution table.

$(-\infty, \infty)$

↳ only values of ~~vals~~ p, n ⟵ p ⟶ should be given

		0.1	0.2	0.3
$n=1$	$x=0$	a_1		
	$x=1$	a_2		
$n=2$	$x=0$			
	$x=1$			
	$x=2$			

$P(x \le 0)$

$f(x=0) = a_1$

$= P(x=0)$

$f(x \le 1) = a_2$

$= P(x \le 1)$

$P(x=1) = a_2 - a_1$

$= f(x=1) - f(x=0)$

a = Any valid Instance of Binomial Random Variable

① $P(x=a) = F(a) - F(a-1)$ → Previous Instance

② $P(x<a) = F(a-1) = P(x \le a-1)$

③ $P(x>a) = 1 - F(a)$

④ $P(a_1 \le x < a_2) = F(a_2-1) - F(a_1-1)$

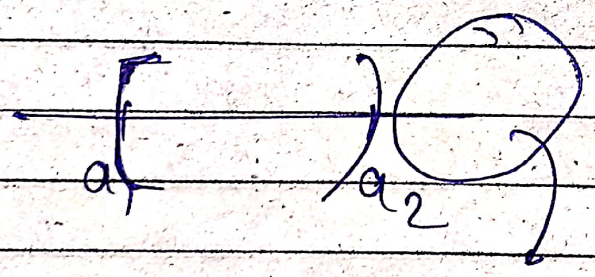
$F(a_2-1)$

$F(a) = P(0) + P(1) + \dots P(a)$

$F(a-1) = P(0) + P(1) + \dots P(a-1) + P(a)$

$F(a) - F(a-1) = P(a)$

$P(a+1) + P(a+2) + \dots + P(n)$



$1 - F(a_2)$

$1 - F(a_2) - F(a_1-1)$

$1 - (1 - F(a_2)) - F(a_1-1)$

$F(a_2-1) - F(a_1-1)$

$P(0) + P(1) + P(2) + \dots +$
$P(a_1-1) + P(a_1) + \dots P(a_2-1) + P(a_2) + \dots + P(n) = 1$

$F(a_2-1) - F(a_1-1)$

Date: _____ Page: _____

Q) A large lot of apples is expected to contain 20% defectives. A retailer before accepting the lot selects at random a sample of 12 apples & he will purchases the lot if it contains not more that 2 defective apples. Determine the probability that the retailer will accept the lot.

Sol:

① Success = Defective

② $p = 20\% = 0.2$ $\quad q = 0.8$

③ $n = 12$

$P(x \leq 2) = F(x=2) = \cancel{0.253}\ 0.558$

↳ look in table

$$P(x) = {}^nC_x\, p^x\, q^{n-x}$$

$$P(x+1) = {}^nC_{x+1}\, p^{x+1}\, q^{n-(x+1)}$$

$$\frac{P(x)}{P(x+1)} = \frac{{}^nC_x}{{}^nC_{x+1}} \cdot \frac{q}{p}$$

$$\Rightarrow \frac{P(x+1)}{P(x)} = \frac{{}^nC_{x+1}}{{}^nC_x} \cdot \frac{p}{q} = \frac{\frac{n!}{(x+1)!\,(n-(x+1))!}}{\frac{n!}{x!\,(n-x)!}} \cdot \frac{p}{q}$$

$$\Rightarrow \boxed{\frac{P(x+1)}{P(x)} = \frac{(n-x)}{(x+1)} \cdot \frac{p}{q}}$$

Date: Page:

Expected value = Mean

$$= \sum_{i=0}^{n} x_i p(x_i)$$

$\boxed{\text{Mean} = np}$ → Prove this

↳ The expected value of BRV is np

$$\sum_{x=0}^{n} x \cdot {}^nC_x p^x q^{n-x}$$

No relation betn n & p

n : No. of trials

p : success

There is no relationship betn n & p

$\boxed{\text{Variance} = npq}$

$$\sum_{i=0}^{n} p(x_i)(x_i - \text{Mean})$$

$$\sum x \, {}^{n-1}C_{x-1} p^x (1-p)^{n-x}$$

$$\left[{}^nC_k = \frac{n}{k} \, {}^{n-1}C_{k-1} \right]$$

$$\Rightarrow np \sum {}^{n-1}C_{x-1} p^{x-1} (1-p)^{(n-1)-(x-1)}$$

let $n-1 = m$

$x-1 = j$

$$\Rightarrow np \underbrace{\sum_{j=0}^{m} {}^mC_j p^j (1-p)^{m-j}}_{\text{Binomial PMF}} = np.$$

Date: Page:

Q) Suppose 60% of males in a certain group are smokers & that a sample of 25 members are selected from random these groups.

(i) What are the expected no. of smokers.

(ii) What is the most likely no. of smokers in the selected group of 25?

(iii) What is the SD of no. of smokers in the group of 25 selected. (Assuming BD is approximated in the experiment)

Sol:

T

60% → S Smoker

40% → NS

① Success = Smokers

② $p = 0.6$

③ $n = 25$

(i) $np = 0.6 \times 25 = 15$

(ii) Instance with highest probability

$\boxed{x = 15}$ Mode of

(iii) $\sqrt{npq} = \sqrt{25 \times 0.6 \times 0.4}$

$= 2.449$

SD

0.034 44

0.78

~~0.154 76~~

~~0.268 114~~

~~0.414 148~~

0.575 161

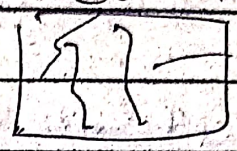
03.08.2021 Thursday

Date: Page:

Discrete RV, Binomial R.V., Poisson R.V. explain with examples. (Pages 31 Explain)

Instances of DRV

elements which are ME & CE & real value no.



A	$\bar{A}$

No. of elements in 2 part should be countable.

$$P(A) = \frac{n(A)}{n(A)+n(B)}$$

DRV can't be used for Boolean

If we use success

These are not time relevant nor space relevant

3 cases are reported

Accidents (Success)

total type of No. of cases are reported.

$n = 3$

$p =$ not decide

Suppose 4.

"n will not decide p"

$p = \frac{1}{4}$ deviated

$P_0 + P_1 + P_2 + \cdots P_n = 1$

$P_0 + P_1 + P_2 +$

$^4C_1 \left(\frac{1}{2}\right)$ $\frac{4}{16}$ 4

$^4C_2 = \frac{4 \times 3}{2}$ 6

3 coins

0	$\frac{1}{8}$	0	$\frac{1}{16}$
1	$\frac{3}{8}$	1	$\frac{1}{4}$
2	$\frac{3}{8}$	2	
3	$\frac{1}{8}$	3	
		4	1/16

Date: Page:

$$P(\text{event}) = \frac{n(\text{success event})}{n(\text{success}) + n(\text{failure})}$$

frequency of success depends on no. of trials.

$$f(p) + f(q) = n$$

If ~~to~~ Accidental case = 1

$\Rightarrow$ No. of Non Accidental case = 2

P (Accidental case) = ?

type of cases should be given

Time Relevant : In a time duration no. of accidents in a time duration

Success : Accidents cases reported in the hospital

Failure : ~~Accidents~~ Cases not reported

Can't calculate

Probo & no. of cases

Isolated event

No. of telephone cases received by me.

No. of defects on fabric.

Amlignous.

p = can't find / calculate

n = no. of trials not given (How many times we can iterogate?)

04.08.2023
Friday

np = constant
$n \uparrow$ $p \downarrow$

Date: Page:

It can be used, mini sec.,

Poisson Distⁿ

No. of trials are infinite & events are isolated event.

Expected no. of success is 2 per hr.,
Will the prob. of each will increase or decrease if increase no. of trials
→ decreas.

$np = 2$
mean =

Isolate event
↳ No. of failure not known.

Poisson Discrete R.V.

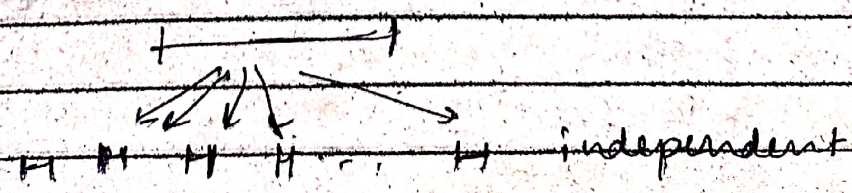
A RV x is said to follow PDistⁿ if following condⁿ are satisfied

i) the no of occurences of the evt. into 2 disjoint intervals are independent.

ii) the probability of occurence of ~~evt~~ event in a small interval 'h' is approximately 'mh'

where m: Avg. rate at which the evt. occurs per expected interval/mean unit of time & space.

iii) The probability of 2 or more occurences of evt. in a very small interval 'h' is negligible in comparision with the probability with one occurence ~~in~~ during this small interval



Date: Page:

~~events~~

Mean = np

n (trials) intervals

1 interval duration = 1/n

P = mean × 1 interval duration

interval ↓ P ↓

= (np) × (1/n)

= mh

mean

$$P(X=x) = \frac{e^{-\lambda} \times \lambda^{x}}{x!}$$

$x \in [0, \infty]$

Stop when $P \approx 0$

Q) Defects in Yarn manufactured by local mill can be approximated by Poisson Distⁿ with a mean of 1.2 defects for every 6 metres of length. If length of 6 m needs to be inspected. Find P(less than 2 defects) = ?

Sol: $\lambda = 1.2$ $x = 0, 1$

$P(x < 2) = 0.8795$ (from table)

Q) For 12 m, Find P(less than 2 defects) = ?

Sol: $\lambda = 1.2 \times \frac{12}{6} = 2.4$ $x < 2$

$P(x < 2) =$

Date: Page:

Q) If the no. of telephone calls that an operator received in a half an hour interval follows PD. with avg. of 3 calls. What is the

(i) P(operator will be idle in an half an hour) = ?

(ii) P(< 4 calls) = ?

(iii) What is the most likely no. of calls that an operator received.

Sol. $\lambda = 3$ half an hour

(i) $P(x=0) = 0.0498$ → No calls $x=0$ [From Table]

(ii) $P(x<4) = 0.6472$

↳ $P(x \le 3) = F(3) = 0.2241$

(iii) 2 calls and 3 calls

(consider upto 3

Mean $= \lambda$

Variance $= \sqrt{\lambda}$

$$P(x) = \frac{e^{-\lambda} \times \lambda^{x}}{x!}$$

$$P(x+1) = \frac{e^{-\lambda} \times \lambda^{x+1}}{(x+1)!}$$

$$\frac{P(x)}{P(x+1)} = \frac{(x+1)!}{x!} \times \frac{e^{-\lambda} \lambda^{x}}{e^{-\lambda} \times \lambda^{x+1}}$$

np ↳ should be finite no.

$n > 100$

$np < 10$

$$\Rightarrow \frac{P(x)}{P(x+1)} = \frac{(x+1)}{\lambda}$$

$$\Rightarrow \boxed{P(x) = \frac{(x+1)}{\lambda} P(x+1)}$$

$$\boxed{P(x+1) = \frac{P(x)\,\lambda}{x+1}}$$

for poisson
n is large
np is finite } → use

$n > 100$
$np < 10$

Date: Page:

Q) Trucks arrived at a dockyard at a mean rate of 2 per hrs. using a poisson approximation determine the 30min.
(i) no. trucks arrived
(ii) 2 trucks arrived

Sol: $\lambda = 2$ (1 hour interval) $\frac{2 \times 3}{60}$
$\Rightarrow \lambda = 1$ (1/2 hour interval)

(i) $P(x=0) = 0.367879$
(ii) $P(x=2) = 0.18394$

Q) The probability that the call received by certain switch board will be the wrong no. is 0.01 use Poisson approx. to Binomial distr^n to find the probability that among 120 calls received by the switch board there are no wrong no.

Sol: ① Success = wrong no.
② $p = 0.01$
③ $n = 120$ $n > 100 \rightarrow \therefore$ Poisson

Mean $= \lambda = np = 1.2$ finite

$P(x=0) = 0.3012$

$\lambda = np = 2.8$ (finite)

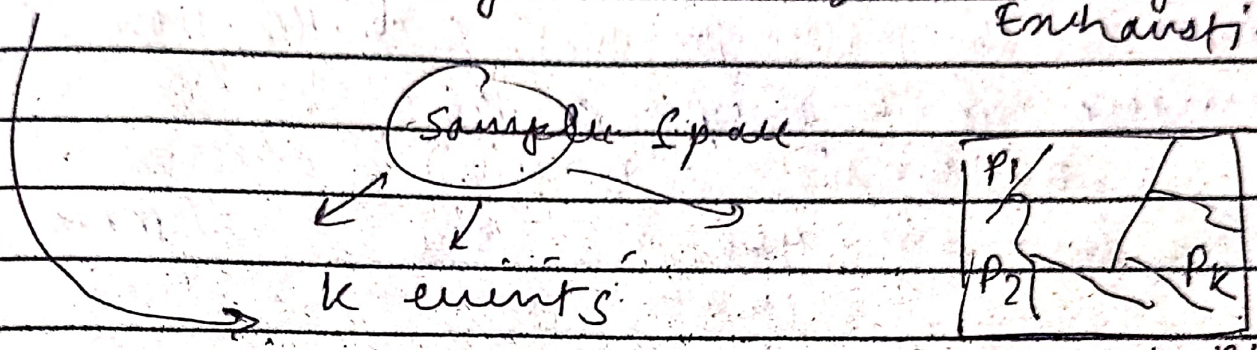
Q) The pr $p = 0.02$ $n = 140$ calls $n > 100$ → Poisson

$P(\text{atleast 2 wrong no.}) =$

$P(x \geq 2) = P(2)$

MULTINOMIAL DISTRIBUTION

Events → Mutually exclusive & collectively Exhaustive



Probability p_i

But we are not considering real values for p_i

$x_1 + x_2 + \cdots x_k = n$

↳ freq. of 1st event success

BD

$^nC_x \, p^x q^{n-x}$

↓

$\frac{n!}{x!(n-x)!} p^x q^{n-x}$

Multinomial Theorem

$$\boxed{\frac{n!}{x_1!\, x_2!\, x_3! \cdots x_k!} p_1^{x_1} \cdot p_2^{x_2} \cdots p_k^{x_k}}$$

Q) A hospital receives an avg. of 4 emergency call in 10 min. interval. What is the prob. that there are atmost 2 emergency call.

Sol: Poisson Dist $\lambda = 4$ (in 10 min.)

$P(x \leq 2) = \cancel{0.6767}\ 0.2381$

(from table)

Date: Page:

$$P(x \le 2) \frac{e^{-\lambda} \lambda^x}{x!} = e^{-\lambda} \sum_{x=0}^{2} \frac{\lambda^x}{x!} = e^{-4}\left(\frac{4^0}{1} + \frac{4^1}{1} + \frac{4^2}{2}\right)$$

$$= 13 e^{-4} = 0.2381$$

Q) The mortality rate for certain disease is 7 per 1000. What is the prob. for just 5 deaths from these diseases in a group of 400.

Sol: $n = 1000$ $(n > 100)$

$$\lambda = \frac{7 \times 400}{1000} = 2.8 \quad \text{(mean)}$$

$$P(x=5) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$= \frac{e^{-2.8} (2.8)^5}{5!} = 0.08721$$

Q) If the prob. that an individual suffers from a bad reaction is 0.001. Determine the prob that out of 2000 individual

(i) exactly 3 (ii) more than 2 (iii) none (iv) more than 1 individual will suffer from bad reaction

Sol: Poisson distribution $n > \cancel{2000}\ 100$ $n = 2000$

$\lambda = 0.001$

$$P(x=1) = \frac{e^{-\lambda} \lambda^x}{x!}$$

$$\Rightarrow 0.001 = \frac{e^{-\lambda} \lambda}{1!}$$

Date: Page:

$\lambda = 2000 \times 0.001 = 2$

(i) $P(x=3) = \frac{e^{-\lambda}\lambda^x}{x!} = \frac{e^{-2}2^3}{3!} = 0.1804$

(ii) $P(x>2) = 1 - P(x \le 2) = 1 - \frac{e^{-2}2^2}{2!} - \frac{e^{-1}2^2}{1!} - \frac{e^{-0}2^2}{0!}$
$= 0.3233$

(iii) $P(x=0) = \frac{e^{-2}2^0}{0!} = e^{-2}$
$= 0.1353$

(iv) $P(x>1) = 1 - P(x \le 1) = 1 - \frac{e^{-1}2^1}{1!} - \frac{e^{-0}2^0}{0!}$
$= 0.594$

Q) A customer has center receives on avg. 3.5 calls every hour. What is the prob. that it will receive at most 4 calls every hour
(i) P(atleast 5 calls every hr)
~~(iii)~~

Sol: $\lambda = 3.5$

(i) $P(x \le 4) = 0.7254$
(ii) $P(x \ge 5) = 1 - P(x \le 4) = 1 - 0.7254 = 0.2746$

Multinomial

Q) Prodn manager finds that 80% of the ~~boat~~ days output is of good quality, 15% good require some working & the balance 5% needs to be destroyed. In a sample lot of 10 units, what is the probability obtaining 9 good units with 1 require reworking & none must be destroyed.

Date: Page:

Q01. $P_1 = 0.8$ $x_1 = 9$ $P(x) = \frac{n!}{x_1!\, x_2! \ldots x_k!} p_1^{x_1} p_2^{x_2} \ldots p_k^{x_k}$

$P_2 = 0.15$ $x_2 = 1$

$P_3 = 0.05$ $x_3 = 0$

$P(x_1 = 9, x_2 = 1, x_3 = 0) = \frac{10!}{9! \times 1! \times 0!} \times (0.8)^9 \times (0.15) \times 0.05$

$= 0.225$

Mean of Multinomial theorem (Not in course)

Date: Page:

Uniform Discrete R.V.

For each & every instance prob. is constant

n instances 1 to n with uniform probo

$\Sigma P = 1$

$\Rightarrow P_1 + P_2 + \cdots P_n = 1 \Rightarrow n P_1 = 1 \Rightarrow P_1 = 1/n$

$(P_1 = P_2 = \cdots P_n)$.

$$\boxed{P(x = x) = \frac{1}{N}}$$

$$\mu = \Sigma x \, P(x) \rightarrow \text{constant}$$

$$= \frac{1}{N} \Sigma x = \frac{1}{N} (1 + 2 + 3 + \cdots + n)$$

$$= \frac{n(n+1)}{2n} = \frac{n+1}{2}$$

$$Var. = \Sigma x^2 P(x) - \left(\frac{N+1}{2}\right)^2$$

$$= \frac{1}{N} \Sigma x^2 - \left(\frac{N+1}{2}\right)^2$$

$$= \frac{1}{N} \frac{N(N+1)(2N+1)}{6} - \left(\frac{(N+1)}{2}\right)^2$$

$$= \frac{(2N^2 + N + 2N + 1)}{6} - \frac{(N^2 + 1 - 2N)}{4}$$

$$\boxed{Var. = \frac{N^2 - 1}{12}}$$

Date: Page:

3) In a rolling of 6 fair die Determine the expected value & variance where x is outcome of experiment.

Sol: $N = 6$ $\mu = \left(\frac{N+1}{2}\right)^2 - \left(\frac{5}{2}\right)^2$

$\{1, 2, 3, 4, 5, 6\}$ equall likely $\left(\frac{N+1}{2}\right) = \frac{7}{2} = 3.5$

$$Var. = \frac{N^2 - 1}{12}$$

$$= \frac{6^2 - 1}{12} = 2.916$$

Discrete → countable (on scale)

CONTINUOUS RANDOM VARIABLE

I drive my car from speed 0 to 100 km/h

$\therefore P(0 < \text{speed} \leq 100) = 1$

$P(60.999) = ?$ → Given in interval

infinite points (Not countable)

density function is continuous.

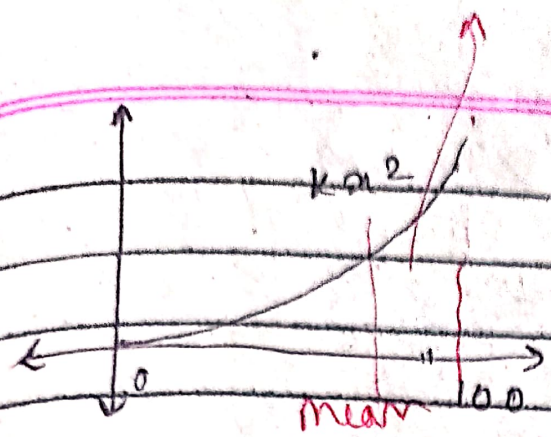
Discrete R.V. → density fn independent of x

dependent on x (when variable)

independent on x (when constant)

In Non-Uniform curve: Half Area (Median)

Symmetric: Mean (Half)



$P(x) = kx^2 \quad 0 < x \leq 100$

P is distributed over area

$P(x_1) = P(x_2)$

either consider boundary pts. $\left[\begin{array}{ll} 0 < x_1 \leq 100 & 0 \leq x_2 \leq 100 \\ \text{No yet due point} & \end{array} \right.$

$$\int_0^{100} kx^2\,dx \qquad \left.\frac{kx^3}{3}\right]_0^{100} = 1$$

$$k = \frac{3}{10^6}$$

density f^n

$$\int_0^{100} 3\times10^{-6}\, x^2\,dx$$

upto 50 km/h $\quad P(0 \leq x \leq 50) = \int_0^{50} kx^2\,dx$

$$= \frac{x^3}{3} \times 3\times10^{-6}$$

$$= 125\times10^{-3}$$

$$= 0.125$$

~~Mean = $\int x\,P(x)\,dx$~~

$$\int_0^{100} kx^3\,dx = \left.\frac{kx^4}{4}\right]_0^{100} = \frac{3\times10^{-6}\times100^4}{4}$$

$$= \frac{3\times10^2}{4} = 75 \text{ km/h}$$

Var. $= \int x^2\,P(x)\,dx$

$$= \int kx^4\,dx = \left.\frac{kx^5}{5}\right]_0^{100} = \frac{3\times10^{-6}\times100^5}{5\times10^4}$$

$$= 6000$$

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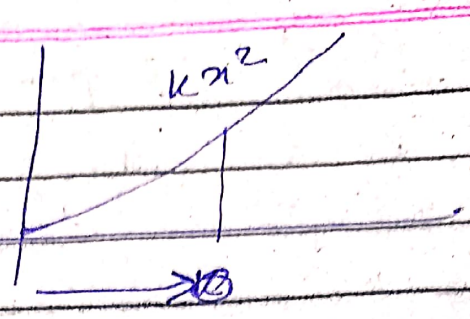
$F(a) = P(x \leq a)$



$$F(x) \begin{cases} 0 & x < 0 \\ kx^2 & \\ 1 & \end{cases}$$

Q) $f(x) = kx^2 e^{-x}$ interval $x \geq 0$

$k = ?$ $\mu = ?$ $var = ?$

Sol: $\int_0^\infty kx^2 e^{-x} = 1$ ILATE

$\Rightarrow k \int x^2 e^{-x}$ $\quad I \int II - \int I' \int II$

$\quad I \quad II$

$-x^2 e^{-x} + \int 2x \, e^{-x}$

$\quad I \quad II$

$+ 2 [-x e^{-x} - e^{-x}]$

$= -x^2 e^{-x} - 2x e^{-x} - e^{-x} = 1$

$k(-1 - 1) = 1$

$= \boxed{k = \frac{1}{2}}$

Mean $= \int_0^\infty x k x^2 e^{-x} \, dx$

$= \int_0^\infty k x^3 e^{-x} \, dx = 3$

Var. $= \int_0^\infty k x^4 e^{-x} \, dx - 9 = 12$

Date: Page:

Split ~~interval~~ domain → different internal with distribution f^n

$$f(x) = \begin{cases} k & 0 \le x < 100 \\ kx & 100 \le x \le 200 \\ kx^2 & 200 \le x \le 300 \end{cases}$$

$$\int_0^{100} k\,dx + \int_{100}^{200} kx\,dx + \int_{200}^{300} kx^2\,dx = 1$$

Non-Split domain

पहले वाले।

$f(x) = kx^3$ $\quad 2 < x < 4$

0 otherwise

Find k, μ, σ^2 $\quad P(1 < x < 3)$

(Sol) $\frac{kx^4}{4} = 1$ $\quad k\left(\frac{8-1}{3}\right)$ $\quad k\frac{(64-8)}{3} = 1$

$k \cdot \frac{56}{3} = 1 \Rightarrow k =$

$k\left(\frac{4^4 - 2^4}{4}\right) = 1$ $\quad k = \frac{1}{60}$

$\mu = \int_1^3 kx^3 \cdot x = \left(\frac{kx^5}{5}\right)\Big|_1^3$

$= \int_1^2 0 + \int_2^3 kx^4\,dx$

$\frac{kx^5}{5} = \frac{60}{5}(3^5 - 2^5)$

$= 2532$

$$\int_2^{\text{median}} \frac{x^3}{60}\,dx = \frac{1}{2}$$

$$\left(\frac{x^4}{4\times 60}\right) = \frac{16}{4} = \frac{2\times 60}{2}$$

$$x^4 = 18\times 60 \quad 1536$$

$$\boxed{x = 32.86}$$

median.

$f(x) = \int f(x)\,dx$

$f(x)$ density f^n	P	$a < x < b$
	Q	$b < x < c$
	R	~~$c < x$~~ $x > c$

$$F(x) \text{ distribution } f^n = \begin{cases} 0 & x < a \\ \int_a^x P\,dx & a < x < b \\ \int_a^b P\,dx + \int_b^x Q\,dx & b < x < c \\ \int_a^b P\,dx + \int_b^c Q\,dx + \int_c^x R\,dx & x > c \end{cases}$$

If $R = 0$

last term 1 $x > c$

Q) $f(x) \begin{cases} ax & 0 < x < 1 \\ a & 1 < x < 2 \\ 3a - ax & 2 < x < 3 \\ 0 & \text{otherwise} \end{cases}$

Sol

$$\int_0^1 ax\,dx + \int_1^2 a\,dx + \int_2^3 (3a - ax)\,dx = 1$$

$$\Rightarrow \frac{a}{2} + a + 3ax\Big]_2^3 - \left[\frac{ax^2}{2}\right]_2^3 = 1$$

$$\Rightarrow 3\frac{a}{2} + 3a - \frac{a}{2}(9-4) = 1$$

$3a + \frac{3a}{2}$ $\quad \frac{3a - 2a}{2}$

Date: Page:

$5a - \frac{5a}{2} = 1 \Rightarrow \cancel{\boxed{a = \frac{2}{5}}} \quad a = \frac{1}{2}$

$$F(x) \begin{cases} 0 & x < 0 \\ \int_0^x a\,dx & 0 < x < 1 \\ \frac{1}{4} + \int_1^x a\,dx & 1 < x < 2 \\ \frac{1}{4} & \end{cases}$$

$a = \frac{1}{2}$

$\int_0^1 \frac{1}{2} x\,dx = \frac{1}{2} \frac{x^2}{2} \Big]_0^1 = \frac{1}{4}$

$ax \quad ax \Big]_1^2 = a = \frac{1}{2}$

$\frac{1}{4} + \frac{1}{2} = \frac{3}{4}$

$\frac{1}{2}(3 - x)\,dx = \frac{1}{2}\left(3x - \frac{x^2}{2}\right)\Big]_1^x$

$= \frac{1}{2} \quad 3x - \frac{x^2}{2} - 3 + \frac{1}{2}$

$\left(\frac{1}{2}\left(3x - \frac{x^2}{2} - \frac{5}{2}\right)\right)$

$$F(x) \begin{cases} 0 & \\ \frac{x^2}{4} & 0 < x < 1 \\ \frac{1}{4} + \frac{x}{2} - \frac{1}{2} & 1 < x < 2 \\ \frac{3x}{2} - \frac{x^2}{4} - \frac{5}{4} & 2 < x < 3 \\ 1 & x > 3 \end{cases}$$

$= \frac{x}{2} - \frac{1}{4}$

$\cancel{\frac{2}{4}} + \frac{3x}{2} - \frac{x^2}{4} - \cancel{\frac{2}{4}} - \cancel{\frac{2}{4}} \quad \frac{1}{2}$

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Find μ of RV x

Sol:

$$\int_0^1 x^2 \, dx + \int_1^2 x \, dx + \int_2^3 3x - \int_2^3 x^2 \, dx$$

$$= \frac{1}{3} + \frac{4}{2} - \frac{1}{2} + \frac{27}{2} - \frac{12}{2} - \left[\frac{27}{3} - \frac{8}{3}\right]$$

$$= \frac{1}{3} + \frac{3}{2} + \frac{27}{2} - \frac{12}{2} - \frac{27}{3} + \frac{8}{3}$$

$$= \frac{19}{2} - \frac{27}{3} + 6 = \frac{19}{2} - \frac{19}{3} = 19\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$= \frac{19}{6}$$

$$\frac{1}{2} \times \frac{19}{6} = \frac{19}{12}$$

$$1.5$$

Median $\frac{x}{2} - \frac{1}{4} = \frac{1}{2} \Rightarrow \frac{x}{2} = \frac{1}{4} \Rightarrow \boxed{x = \frac{1}{2}}$

$$\frac{x}{2} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$x = \frac{6}{4} = \frac{3}{2} = 1.5 \text{ Median}$$

Interval size [1, 2.00]

Possible values so only

↳ many no. have more than 1 freq.

↳ in form of intervals we will make

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	f	cf
1 - 20		
20 - 40		
40 - 60 (L1 L2)		
⋮		
180 - 200		

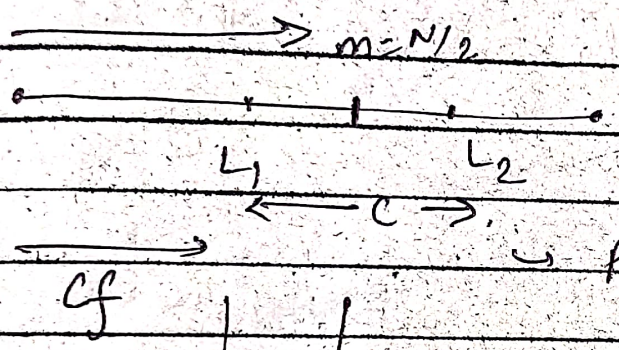
median class

$cf \geq \frac{N}{2}$

$$\text{Median} = L_1 + \frac{c}{f_m}\left(\frac{N}{2} - cf\right)$$

prev

curr



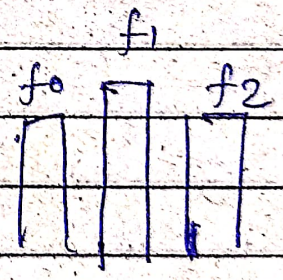
per unit $= \frac{c}{f_m}$

$\left|\frac{N}{2} - cf\right|$ upto m $= \frac{c}{f_m} \times \left(\frac{N}{2} - cf\right)$

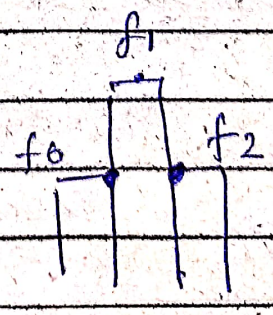
$$\text{Median} = L_1 + \frac{c}{f_m}\left(N/2 - cf\right)$$

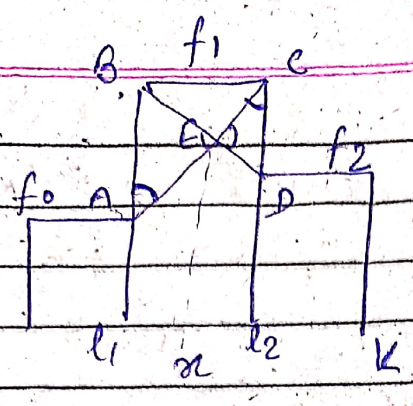
Mode

$$l_0 + \frac{f_1 - f_0}{2f_1 - f_0 - f_2} \times h$$



$$l_0 + \frac{f_1 - f_0}{(f_1 - f_0) + (f_1 - f_2)} \times h$$





$\Delta AEB \sim \Delta \cancel{DEC}\ CED$

$$\frac{AB}{CD} = \frac{BE}{DE}$$

$$\frac{f_1 - f_0}{f_1 - f_2} = \frac{x - l_1}{\cancel{x - l}\ l_2 - x}$$

$$l_2 - x = k(x - l_1)$$

$$l_2 + k l_1 = kx + x$$

$$l_2 + \cancel{f_1} k l_1 = x(1 + k)$$

Q) The following is table distribution for marks scored by some students in the exam

Marks	No. of Students ~~No. of~~
0 - 20	42
21 - 30	38
31 - 40	120
41 - 50	84
51 - 60	48
61 - 70	36
71 - 80	31

Find out the median marks

Class	f	cf
0 - 20.5	42	42
20.5 - 30.5	38	80
30.5 - 40.5	120	200
40.5 - 50.5	84	284
50.5 - 60.5	48	332
60.5 - 70.5	36	368
70.5 - 80.5	31	399

$\frac{N}{2} < cf$ just

$\Sigma f = N = 399$

$\frac{N}{2} = 199.5$

formula of point on $x = \frac{N}{2}$

$$\text{median} = L_1 + \frac{c}{f}\left(\frac{N}{2} - cf\right)$$

$$= 30.5 + \frac{10}{120}(199.5 - 80)$$

$$= 40.45$$

Q) Calculate % of failure if the min. passing marks is 35

Sol:

$$\frac{40.5 - 30.5}{120} \times (35 - 30.5)$$

$$\frac{10}{120} \times 5.5 = \frac{55}{120}$$

$$\frac{120}{10} \times$$

$$\boxed{\text{Point of } x = l_1 + \frac{c}{f}(cf_x - cf)}$$

$$\Rightarrow 35 = 30.5 + \frac{10}{120}(cf_x - 80)$$

$\Rightarrow cf_x = 128$

$\% = 32.08\%$

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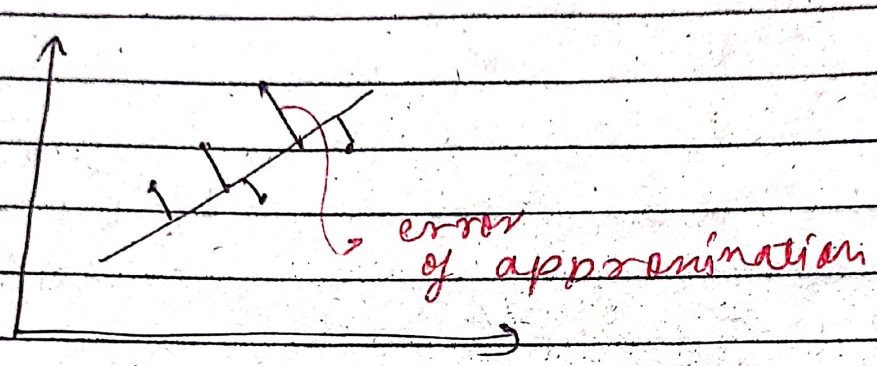
$$\text{Mode} = L_1 + \left(\frac{f_1 - f_0}{f_1 - f_0 + f_2 - f_0}\right) \times h$$

Variation

deviation or error can be +ve or -ve



Error

- mean error (mean absolute error)
- ~~Mean~~ ~~deviation~~ Mean Square ~~dev~~ error

Deviation

- Avg Deviation / Mean Absolute Deviation $\frac{\Sigma|(x_i - \bar{x})|}{\Sigma f \; n}$
- Sq. root of Mean Square deviation

Mean error without absolute $= 0$

$$\frac{1}{n}\left[(x_1 - \bar{x}) + (x_2 - \bar{x}) + \cdots + (x_n - \bar{x})\right]$$

Actual → predicted

$$= \frac{1}{n}\Sigma(x_i) - \frac{1}{n} n\bar{x}$$

$$= \bar{x} - \bar{x}$$

$$= 0$$

Response variable $= y_i$

error in each point = Actual value − Predicted value

$(a + bx)$ = $y_i - \bar{y}_i$

~~value of a & b such that~~

$\boxed{\bar{y}_i = a + bx_i}$

$= y_i - a - bx_i$

mean abs. error

Total error $= \frac{1}{n} \sum |y_i - a - bx_i|$

Mean error or Mean deviation

(Response & predicted value) (only var. x)

diff. for ~~diff. w~~ Actual Response

fixed $\bar{x}$

Mean abs. Deviation $= \frac{1}{n} \sum |(x_i - \bar{x})|$

Sq. root of mean square error $= \sqrt{\frac{1}{n} \sum (y_i - a - bx_i)^2}$

Sq. root of mean square deviation $= \sqrt{\frac{1}{n} \sum (x_i - \bar{x})^2}$

Which is better?

Square deviation is better

(1) For finding mean value of a & b we need to differentiate. But (1) $|a + bx|$ can't be differentiate

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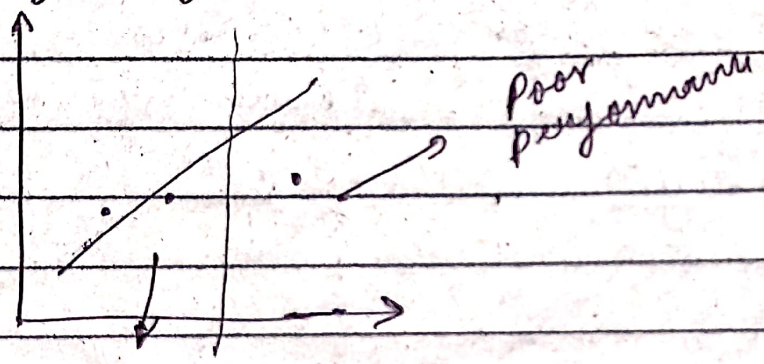
-ialc at point = 0

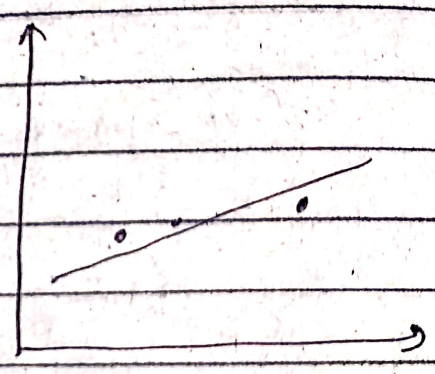
(2) Outlier problem

	1	1	1	0	1	2	0	0	3
Mean abs. error		1			1			1	
~~Sq. root of~~ Mean sq. ~~dev~~ error		1			1.29			1.73	

Maximum

Not good for production model





works good in this area

Good in

Not a good model