Homework 1
↳ CSC1301

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1. Explicitly write out the contents of the following set:

$X \in \mathcal{P}(\{1,2,3\}) : 2 \in X$

→ $\mathcal{P}$ denotes powerset, collection of all subsets of the set.

→ $\mathcal{P}(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \{1,2,3\}\}$.

Thus, X must be of the set $\mathcal{P}(\{1,2,3\})$.

→ But $2 \in X$ should be satisfied. The elements which have 2 as a member of the set is the answer.

$\therefore \{\{2\}, \{1,2\}, \{2,3\}, \{1,2,3\}\}$

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2. If x is a rational number and $x \neq 0$, then tan(x) is not a rational number.

The above sentence can be written as:

$$((x \in \mathbb{Q}) \wedge (x \neq 0)) \Rightarrow (\tan(x) \notin \mathbb{Q})$$

OR

$$\neg((x \in \mathbb{Q}) \wedge (x \neq 0)) \wedge \neg(\tan(x) \notin \mathbb{Q})$$

The negation of this statement is:

$$((x \in \mathbb{Q}) \wedge (x \neq 0)) \wedge \neg(\tan(x) \notin \mathbb{Q})$$

$$((x \in \mathbb{Q}) \wedge (x \neq 0)) \wedge \neg(\tan(x) \in \mathbb{Q})$$

→ HW 1 cont...

Hence, the negation of the given statement is:

If x is a non-zero rational number, then tan(x) is a rational number.

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3. Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd.

Proof. (contrapositive)
The contrapositive is: Suppose $a \in \mathbb{Z}$. If a is even, then a^2 is divisible by 4.

Since a is even $a = 2k$ for some integer k.
Then $a^2 = (2k)^2 = 4k^2$ which is a multiple of 4. So it is divisible by 4.

$\therefore$ Since the contrapositive is true, the original statement "Suppose $a \in \mathbb{Z}$. If a^2 is not divisible by 4, then a is odd" is proven.

→ HW1 cont...

4. Suppose $a, b \in \mathbb{Z}$. If $4 \mid (a^2 + b^2)$, then a and b are not both odd.

Proof. (contradiction)

As $4 \mid (a^2+b^2) \Rightarrow a^2+b^2 = 4m$, where $m \in \mathbb{Z}$.

Let a & b be odd.

$\Rightarrow a = 2x+1, \quad b = 2y+1$ for some integer x, y.

$$\Rightarrow a^2 + b^2 = 4x^2 + 1 + 4x + 4y^2 + 1 + 4y$$
$$= 4(x^2+y^2) + 4(x+y) + 2$$
$$= 4(x^2+y^2+x+y) + 2$$

Since $4(x^2+y^2+x+y)+2$ is not a multiple of 4, we cannot write this only in terms of 4 only.

So, $a^2+b^2 \neq 4m, \quad m \in \mathbb{Z}$.

Hence it is contradictory to our given statement. So, a and b both must not be odd.

$\therefore$ Using proof by contradiction we have proved that if $4 \mid (a^2+b^2)$, then a and b are not both odd.

HW1 cont...

5. Given an integer a and b, then $a^2(b+3)$ is even if and only if a is even or b is odd.

Proof. (Direct)

Let us assume that a is even. (1)

Here as a is even so it can be written as $(2 \cdot i)$ where i is an integer.

Now,

$$
\begin{aligned}
a^2(b+3) &= (2 \cdot i)^2 (b+3) \\
&= (4 \cdot i^2)(b+3) \\
&= 2((2 \cdot i^2)(b+3))
\end{aligned}
$$

Here it can be seen that $a^2(b+3)$ is reduced to $2((2i^2)(b+3))$ meaning any number that is a multiple of 2 is an even number.

If a is even then $a^2(b+3)$ is also even. Hence proving assuming a is even. (1)

Let us assume that b is odd. (2)

Here as b is odd it can be written as $(2i+1)$ where i is some integer.

→ # 5. cont...

Now,

$$\begin{aligned} a^2(b+3) &= a^2((2i+1)+3) \\ &= a^2(2i+4) \\ &= a^2(2(i+2)) \\ &= 2(a^2(i+2)) \end{aligned}$$

Here it can be seen that $a^2(b+3)$ is reduced to $2(a^2(i+2))$. Any number that is a multiple of 2 is an even number. Again we have even plausibility.

This means that if b is odd then $a^2(b+3)$ is even. Hence proving (2).

∴ By direct proof we have proven $a^2(b+3)$ is even iff a is even or b is odd.