COMP3121 Assignment 2 – Q1

1.1

A sequence of actions is minimal if the company can demolish all the buildings in the shortest sequence of actions. Consider 4 buildings with heights 2, 3, 5 and 1 in the array $H[2, 3, 5, 1]$ and then company will also begin with \$1000.

The minimal sequence of actions will include:

1. Small explosive on index 2 on $[2, 3, 5, 1] = [2, 3, 4, 1]$.
2. Spend \$1000 on a wrecking ball on $[2, 3, 4, 1] = [1, 2, 3, 0]$, gain \$1000 for destroying building.
3. Spend \$1000 on a wrecking ball on $[1, 2, 3, 0] = [0, 1, 2, 0]$, gain \$1000 for destroying building.
4. Spend \$1000 on a wrecking ball on $[0, 1, 2, 0] = [0, 0, 1, 0]$, gain \$1000 for destroying building.
5. Spend \$1000 on a wrecking ball on $[0, 0, 1, 0] = [0, 0, 0, 0]$, gain \$1000 for destroying building.

This sequence of actions is minimal as it mainly uses the wrecking ball. The wrecking ball is the most efficient in reducing building height in terms of how much height is removed per action (maximum n buildings).

The company can continue to use the wrecking ball for 4 consecutive actions because each use of the wrecking ball would finish the demolition of a building, rewarding them with \$1000.

This is sequence is minimal as it uses mainly wrecking balls to destroy building height and the result is the shortest number of actions which can finish the demolition of all buildings in the array.

1.2

A sequence of actions is minimal if the company can demolish all the buildings in the shortest sequence of actions. As briefly demonstrated in question 1.1, a minimal sequence in most scenarios would mostly rely on wrecking balls when applicable as it is the most effective and efficient in removing the most height from the buildings in a single action.

The main limitation of using wrecking balls is that it will cost the company \$1000 for each use. If the company looks to use a wrecking ball in the next consecutive actions, there will need to be at least one building with height 1 so that the company receives money back before the next action.

The small explosives are used to alter the building heights to create a condition which when a wrecking ball is used, it can ensure the use of a wrecking ball in the next action. To reiterate, each use of the wrecking ball should completely destroy at least 1 building so that a reward of \$1000 is returned and a wrecking ball can be used in the next action.

Without the use of small explosives before the use of wrecking balls, there may be cases where the use of a wrecking ball does not reward the company with at least \$1000 to use the wrecking ball again in the following actions.

For some cases, such as the one from question 1.1, it is a special case where the wrecking balls can be used to its entirety before the use of small explosives can be used to finish off the last building. However, there are more cases where the set of buildings will not have the proper conditions for the wrecking ball to be used continuously due to the limitation of money.

For example, consider the array of buildings $[3, 3, 6, 2, 5]$. If the company was to first use a wrecking ball on this set of buildings, they would not receive the \$1000 reward to allow them to use the wrecking ball again as no building would be completely destroyed. Thus, small explosives must be used to create an environment where the company can consecutively use the wrecking ball by destroying a building each action and receiving the \$1000 reward per building destroyed. The small explosives can be used to shorten buildings to a preferrable height to where at least 1 building is destroyed per use of the wrecking ball.

Therefore, there is always a minimal sequence of actions which results in the buildings being completely destroyed where all uses of small explosives occur before any uses of a wrecking ball.

1.3

Consider a given array $H[1 \dots n]$. A criterion can be used to determine if H has a minimal sequence of actions to demolish all buildings using only the wrecking ball.

The criterion should test the given H array as follows:

1. H should have at least one building with a height of 1.
2. When sorted in ascending order, the height difference between each consecutive building is no more than 1.
    a. This means that the height of a building can be the same as the next building, allowing duplicates.

If the given array satisfies the conditions of the criterion, the criterion will answer YES, indicating that there is a minimal sequence using only the wrecking ball or NO if otherwise.

By having the first criteria that there is initially at least one building with a height of 1, it can be ensured that the wrecking ball can be used in the next action.

Moreover, the second criteria ensures that with each use of the wrecking ball after the first, there will be at least 1 building destroyed during each action (as there is no height difference greater than 1 between each consecutive building). This will guarantee that the use of the wrecking ball in the next move is valid as it rewards them with money by destroying a building with a wrecking ball.

Thus, this criterion will be able to determine whether there is a minimal sequence of actions to demolish all buildings only using the wrecking ball.

1.4

Given an array $H[1 \dots n]$ which represents the heights of buildings needing to be demolished, consider an algorithm which runs in $O(n \log n)$ time and determines a minimal sequence of actions to reduce each building's height to 0.

This algorithm will be partially based on the criteria from question 1.3. The algorithm will begin by creating a new array, A, to store a sorted array. Next, it will array H into ascending order using a merge sort into the new array, A. This will have a time complexity of $O(n \log n)$**.** A merge sort will have a worst-case time complexity of $O(n \log n)$ as it will have to consider a maximum of n elements multiplied by the height of the tree created by the merge sort, $\log n$.

To begin, the algorithm will run an if statement to check if the first element has the value of 1. If it does not, subtract 1 from the value until it is equal to 1. This will have a time complexity of $O(n)$.

Once this array of heights is sorted into A, the algorithm will run a *for loop*.

The *for loop* will check the element of the current index and compare it with the element of the next index. The comparison will check if the difference between the next element and the current element has a difference greater than 1.

If the difference is greater than one, the algorithm will reduce the value of the next element until the difference is 1. This sequence of actions will represent the use of small explosives and each unit removed from the value is each small explosive used.

If the difference is not greater than one, nothing is to be done.

The *for loop* will continue to the next iteration. This loop will have a time complexity of $O(n)$.

Once the loop has iterated through the entire array and the values have been updated to match the criterion from question 1.3, another *for loop* will be run to determine how many wrecking balls have to be used to completely demolish the set of buildings.

This loop will have a condition that, while each value in the array is not 0, the loop will subtract 1 from every element in the array.

Since the array has been modified to match the criteria of question 1.3, the wrecking ball will be able to be continuously used without the limitation of not having enough money.

The loop will also keep a continuous counter to determine how many times the wrecking ball will be used until the job is complete.

Overall, this will determine the minimal sequence of actions to reduce each building's height to 0.

This algorithm will have a total time complexity of $O(n \log n) + O(n) + O(n)$ which is simplified to an overall time complexity of $O(n \log n)$.