MATH 1231 Assignment

Applied mathematics

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Question 1

The function *T*: is defined by

*T* = for all

Show that *T* is linear.

Let , . Where

= and = ,

Then,

*T* () = *T*  = and *T* () = *T* = .

Consider *T* (,

*T* ( = *T*

=

=

=

=

= *T* () + *T* ()

Therefore, *T* is closed under addition.

Let α , and consider *T* (α),

*T* (α) = *T*

=

=

= α

= α *T* ()

Therefore, *T* is closed under scalar multiplication.

Consider the zero vector, *T* (0),

*T* (0) = *T*

=

=

Since 0 = 0, There exists a zero vector.

Since, *T* is closed under addition, scalar multiplication and there exists a zero vector.

Therefore, the function of *T* is linear.

Question 2

Show that

*S* = {

Is a subspace of

is a known vector space.

Let , *S*. Where

= and = ,

Then,

and .

Consider + ,

+ = ,

= ,

= ,

= ((

= 0 + 0.

Therefore, *S* is closed under addition.

Let α and consider α,

α = ,

= ,

= α(,

= α(0),

= 0.

Therefore, *S* is closed under scalar multiplication.

Consider the zero vector,

0 = ,

= ,

= 0.

Therefore, there exists a zero vector for *S*.

Since *S* is closed under addition, scalar multiplication and exists a zero vector. Therefore, *S* is a subspace of .

Question 3

Let be the set of real polynomials at most *n,* and write *p'* and *p''* for the first and second derivatives of *p*. Show that

*S* = {

Is a subspace of .

Let , . Where

and .

Consider, (

(

= ,

= 0 + 0,

= 0.

Therefore, *S* is closed under addition.

Let α and consider (α 2(α

(α 2(α,

.

Therefore, *S* is closed under scalar multiplication.

Consider the zero polynomial

By considering the first and second derivative we can determine,

.

Therefore, there exists a zero polynomial in S.

Since *S* is closed under addition, scalar multiplication and exists a zero vector. Therefore, *S* is a subspace of

Question 4

The air in a 86 cubic metre kitchen is initially clean, but when Derek burns his toast while making breakfast, smoke is mixed with the room’s air at a rate of 0.06mg per second. An air conditioning system exchanges the mixture of air and smoke with clean air at a rate of 4 cubic metres per minute. Assume that the pollutant is mixed uniformly throughout the room and that burnt toast is taken outside after 30 seconds. Let *S*(t) be the amount of smoke in mg in the room at time t (in seconds) after the toast first began to burn.

1. Find a differential equation obeyed by *S*(t).
2. Find *S*(t) for 0 t 30 by solving the differential equation in (a) with an appropriate initial condition.
3. What is the level of pollution in mg per cubic metre after 30 seconds?
4. How long does it take for the level of pollution to fall to 0.007mg per cubic metre after the toast is taken outside?
5. To find a differential equation for *S*(t), we must consider the rate of change of smoke in the room’s air.
6. To solve the differential equation, we can write the first order linear ODE in the form

Therefore,

Let .

Therefore,

This allows us to calculate

To calculate c we must use the initial condition where .

Therefore,

1. Sub in t = 30.

Therefore, the level of pollution in mg per cubic metre after 30 seconds is .

1. Since the toast is taken outside, we must use a different equation for because it only considers .

By considering the rate of change of smoke in the room air after 30 seconds we can determine a new equation for for t .

Since the toast is taken outside there is no longer a smoke source being emitted into the house. Therefore, smoke in = 0.

Therefore,

By integrating both sides we can determine .

We can determine c by using

Therefore, for t

We can now sub and solve for t.

Therefore, the time it takes for the level of pollution to fall to 0.007mg per cubic metre after the toast is taken outside is seconds.