# Manufacturing

## Metal Casting

### Solidification and Cooling

**Chvorinov's Rule:**

$$t = C\left(\frac{V}{A}\right)^n$$

where: - $t$ = total solidification time - $V$ = volume of casting - $A$ = surface area of casting - $C$ = mold constant (depends on material and mold properties) - $n$ = exponent (typically $n = 2$)

**Modulus of Casting:**

$$M = \frac{V}{A}$$

Larger modulus → slower cooling → larger grain size

**Riser Design:**

For riser to solidify last:

$$\left(\frac{V}{A}\right)_{\text{riser}} > \left(\frac{V}{A}\right)_{\text{casting}}$$

### Fluidity and Filling

**Fluidity:** Ability of molten metal to flow and fill mold cavities

Factors affecting fluidity: - Temperature (higher → better fluidity) - Composition (lower melting point → better fluidity) - Surface tension - Oxide formation

**Pouring Basin and Sprue:**

Flow rate through sprue:

$$Q = A_2 v_2 = A_2\sqrt{2gh}$$

where $h$ is height of molten metal above sprue base

### Shrinkage

**Total Shrinkage = Liquid shrinkage + Solidification shrinkage + Solid shrinkage**

Typical total shrinkage: 3-8% depending on material

Must account for shrinkage in pattern making

# Metal Forming

## True Stress and True Strain

**True Strain:**

$$\epsilon_T = \ln\left(\frac{L}{L_0}\right) = \ln(1 + \epsilon_E)$$

where $\epsilon_E$ is engineering strain

**True Stress:**

$$\sigma_T = \sigma_E(1 + \epsilon_E)$$

For plastic deformation (constant volume):

$$\sigma_T = \frac{F}{A} = \frac{FL}{A_0 L_0}$$

## Flow Stress

**Flow Curve (Power Law):**

$$\bar{\sigma} = K\epsilon^n$$

where: - $\bar{\sigma}$ = flow stress (true stress) - $K$ = strength coefficient - $\epsilon$ = true strain - $n$ = strain hardening exponent

**Average Flow Stress:**

$$\bar{Y}_f = \frac{K\epsilon^n}{1 + n}$$

Used to calculate forces in forming operations

## Rolling

**Draft:**

$$d = t_0 - t_f$$

where $t_0$ is initial thickness, $t_f$ is final thickness

**Reduction:**

$$r = \frac{d}{t_0} = \frac{t_0 - t_f}{t_0}$$

**True Strain:**

$$\epsilon = \ln\left(\frac{t_0}{t_f}\right)$$

**Roll Force (approximate):**

$$F = \bar{Y}_f w L$$

where: - $\bar{Y}_f$ = average flow stress - $w$ = width of strip - $L$ = contact length $\approx \sqrt{R \cdot d}$ - $R$ = roll radius

**Roll Torque:**

$$T = 0.5FL$$

**Roll Power:**

$$P = 2\pi NT$$

where $N$ is rotational speed (rev/time)

### Forging

**Forging Force (open die, no friction):**

$$F = \bar{Y}_f A_f$$

where $A_f$ is final area

**With Friction (disk approximation):**

$$F = \bar{Y}_f A_f \left(1 + \frac{2\mu r}{3h}\right)$$

where: - $\mu$ = coefficient of friction - $r$ = radius of workpiece - $h$ = height of workpiece

### Extrusion

**Extrusion Ratio:**

$$r_x = \frac{A_0}{A_f}$$

where $A_0$ is initial billet area, $A_f$ is final extrudate area

**True Strain:**

$$\epsilon = \ln(r_x)$$

**Extrusion Force (Johnson Formula):**

$$F = A_0 \bar{Y}_f \left[a + b \ln(r_x)\right]$$

where: - $a = 0.8$ (typical) - $b = 1.2$ to $1.5$ (depends on friction and die angle)

**Ram Pressure:**

$$p = \frac{F}{A_0}$$

**Maximum Extrusion Ratio:**

Limited by: - Material strength - Press capacity - Buckling of billet

## Wire and Tube Drawing

**Drawing Stress (ideal, no friction):**

$$\sigma_d = \bar{Y}_f \ln\left(\frac{A_0}{A_f}\right)$$

**With Friction:**

$$\sigma_d = \bar{Y}_f \left[1 + \frac{\mu}{\tan\alpha}\right] \ln\left(\frac{A_0}{A_f}\right)$$

where $\alpha$ is die semi-angle

**Drawing Force:**

$$F = \sigma_d A_f$$

**Drawing Power:**

$$P = Fv$$

where $v$ is drawing velocity

**Maximum Reduction per Pass:**

Limited by tensile strength of material:

$$\sigma_d \leq \sigma_{UTS}$$

## Sheet Metal Working

**Bend Allowance:**

$$L_b = \alpha(R + kt)$$

where: - $\alpha$ = bend angle (radians) - $R$ = bend radius - $t$ = sheet thickness - $k$ = factor (0.33 for $R < 2t$, 0.5 for $R > 2t$)

**Minimum Bend Radius:**

$$R_{min} = \frac{t}{2}\left(\frac{100}{\%RA} - 1\right)$$

where $\%RA$ is percent reduction in area at fracture

**Bending Force:**

$$F = \frac{K_{bf} TS_{ut} w^2}{D}$$

where: - $K_{bf}$ = bending factor (depends on die geometry) - $TS$ = tensile strength - $w$ = width - $D$ = die opening width

**Deep Drawing:**

Drawing ratio:

$$DR = \frac{D_0}{D_p}$$

where $D_0$ is blank diameter, $D_p$ is punch diameter

Maximum $DR \approx 2.0$ for single draw

**Limiting Drawing Ratio (LDR):**

$$LDR = \frac{D_{0,max}}{D_p}$$

**Drawing Force:**

$$F = \pi D_p t \bar{Y}_f (DR - 0.7)$$

# Machining

## Cutting Speed, Feed, and Depth of Cut

**Cutting Speed:**

$$v = \frac{\pi D n}{1000}$$

where: - $v$ = cutting speed (m/min) - $D$ = workpiece diameter (mm) - $n$ = rotational speed (rpm)

**Feed:**

$$f = n \cdot f_r$$

where $f_r$ is feed per revolution (mm/rev)

**Material Removal Rate (MRR):**

$$MRR = v \cdot f \cdot d$$

where $d$ is depth of cut

For turning:

$$MRR = \frac{\pi D n f_r d}{1000}$$

## Cutting Forces and Power

**Cutting Force:**

$$F_c = K_s \cdot A_c$$

where: - $K_s$ = specific cutting energy (material property) - $A_c$ = cross-sectional area of cut = $f \times d$

**Power:**

$$P_c = F_c \cdot v$$

**Specific Cutting Energy:**

Varies with material: - Aluminum: 0.4-1.1 GPa - Steel: 2.7-9.3 GPa - Titanium: 3.0-4.1 GPa

## Tool Life

**Taylor Tool Life Equation:**

$$vT^n = C$$

or

$$T = \frac{C}{v^{1/n}}$$

where: - $T$ = tool life (min) - $v$ = cutting speed (m/min) - $n$ = exponent (typically 0.1-0.5, 0.125 for HSS, 0.25-0.4 for carbide) - $C$ = constant (depends on material, tool, conditions)

**Extended Tool Life Equation:**

$$vT^n f^m d^p = C$$

**Cost per Part:**

Optimal cutting speed minimizes:

$$C_{part} = C_{machine} t_m + \frac{C_{tool} + C_{change}}{n_{parts}}$$

where $t_m$ is machining time

## Turning

**Machining Time:**

$$t_m = \frac{L}{f_r n} = \frac{L}{f_r} \cdot \frac{1000}{\pi D n}$$

where $L$ is length of cut

**For facing operation:**

$$t_m = \frac{r}{f_r n}$$

where $r$ is radius

## Milling

**Cutting Speed:**

$$v = \frac{\pi D n}{1000}$$

where $D$ is cutter diameter

**Feed per Tooth:**

$$f_t = \frac{f}{n \cdot N_t}$$

where: - $f$ = table feed rate (mm/min) - $N_t$ = number of teeth on cutter

**Material Removal Rate:**

$$MRR = w \cdot d \cdot f$$

where $w$ is width of cut

**Machining Time:**

$$t_m = \frac{L + L_a}{f}$$

where $L_a$ is approach distance

## Drilling

**Feed:**

$$f = n f_r$$

where $f_r$ is feed per revolution

**Material Removal Rate:**

$$MRR = \frac{\pi D^2}{4} \cdot f$$

**Drilling Time:**

$$t = \frac{L + A}{f_r n}$$

where $A$ is approach allowance (typically $A = D/(2\tan\theta)$ for drill point angle $2\theta$)

**Torque:**

$$T = \frac{1}{2} F_c \cdot \frac{D}{2}$$

**Power:**

$$P = \frac{2\pi n T}{60}$$

# Surface Finish and Metrology

## Surface Roughness

**Average Roughness ($R_a$):**

$$R_a = \frac{1}{L} \int_0^L |y(x)| \, dx$$

Arithmetic average of absolute deviations from mean line

**Root Mean Square Roughness ($R_q$):**

$$R_q = \sqrt{\frac{1}{L} \int_0^L y^2(x) \, dx}$$

**Maximum Peak-to-Valley Height ($R_t$):**

Distance between highest peak and lowest valley

**Theoretical Surface Roughness (turning, shaping):**

$$R_t = \frac{f^2}{8R_n}$$

where: - $f$ = feed - $R_n$ = tool nose radius

## Geometric Dimensioning and Tolerancing (GD&T)

**Fundamental Tolerance Equation:**

$$\text{Tolerance} = \text{Upper Limit} - \text{Lower Limit}$$

**Bilateral Tolerance:**

$$50.0 \pm 0.1 \text{ mm}$$

**Unilateral Tolerance:**

$$50.0^{+0.2}_{0} \text{ mm}$$

**Fits:**

- Clearance fit: Minimum hole size > Maximum shaft size - Interference fit: Maximum hole size < Minimum shaft size - Transition fit: Can be either clearance or interference

**Standard Tolerance Grades (IT):**

IT01 to IT18, where lower numbers indicate tighter tolerances

# Non-Traditional Machining

## Electrical Discharge Machining (EDM)

**Material Removal Rate:**

$$MRR = K_m \cdot I \cdot V$$

where: - $I$ = discharge current - $V$ = gap voltage - $K_m$ = material removal constant

Advantages: Can machine hard materials, complex shapes

## Electrochemical Machining (ECM)

**Material Removal Rate (Faraday's Law):**

$$MRR = \frac{CIA}{nF\rho}$$

where: - $C$ = current efficiency - $I$ = current - $A$ = atomic weight - $n$ = valence - $F$ = Faraday's constant (96,485 C/mol) - $\rho$ = density

## Laser Beam Machining (LBM)

**Energy Density:**

$$E_d = \frac{P}{A}$$

where $P$ is laser power and $A$ is spot area

High energy density $\rightarrow$ vaporization of material

# Additive Manufacturing (3D Printing)

## Build Time Estimation

**Layer-by-Layer Build:**

$$t = \frac{h}{v_b}$$

where: - $h$ = total build height - $v_b$ = build rate (height/time)

**Material Usage:**

$$m = V \cdot \rho$$

where $V$ is part volume and $\rho$ is material density

### Common AM Processes

- Fused Deposition Modeling (FDM) - Stereolithography (SLA) - Selective Laser Sintering (SLS) - Electron Beam Melting (EBM) - Binder Jetting

## Heat Treatment

### Time-Temperature Transformation

**Cooling Rate:**

$$CR = \frac{\Delta T}{\Delta t}$$

Different cooling rates produce different microstructures: - Slow cool: Pearlite - Medium cool: Bainite - Fast cool (quench): Martensite

### Hardness After Heat Treatment

**Jominy End-Quench Test:**

Measures hardenability by cooling rate

Distance from quenched end correlates with cooling rate

**Tempering:**

Reduces hardness and increases toughness

Temperature and time determine final properties

# TRIBOLOGY

## Friction

### Coulomb (Dry) Friction

**Friction Force:**

$$F_f = \mu N$$

where: - $\mu$ = coefficient of friction - $N$ = normal force

**Static Friction:**

$$F_s \leq \mu_s N$$

**Kinetic Friction:**

$$F_k = \mu_k N$$

Typically $\mu_k < \mu_s$

### Friction Laws

**Amontons' Laws:** 1. Friction force proportional to normal load 2. Friction force independent of apparent contact area 3. Kinetic friction independent of sliding velocity (approximately)

### Friction Mechanisms

**Adhesion:** Bonding at contact points, must be sheared

**Plowing:** Harder asperities plow through softer material

**Total Friction:**

$$\mu = \mu_{adhesion} + \mu_{plowing}$$

### Rolling Friction

**Rolling Resistance Coefficient:**

$$\mu_r = \frac{F_r}{N}$$

Generally $\mu_r \ll \mu_k$

**Rolling Resistance Force:**

$$F_r = \frac{C_r N}{r}$$

where: - $C_r$ = rolling resistance coefficient (length) - $r$ = wheel radius

## Wear

### Archard Wear Equation

**Wear Volume:**

$$V = K\frac{NL}{H}$$

where: - $K$ = dimensionless wear coefficient - $N$ = normal load - $L$ = sliding distance - $H$ = hardness of softer material

**Wear Rate:**

$$\frac{dV}{dt} = K\frac{Nv}{H}$$

where $v$ is sliding velocity

**Wear Coefficient:**

Ranges from $10^{-8}$ (mild wear) to $10^{-2}$ (severe wear)

### Types of Wear

**Adhesive Wear:** Material transfer between surfaces

**Abrasive Wear:** Hard particles or asperities scratch surface

- Two-body: Hard surface against soft - Three-body: Hard particles between surfaces

**Fatigue Wear:** Repeated loading causes surface cracks, spalling

**Corrosive Wear:** Chemical reaction forms oxide, then removed

**Fretting Wear:** Small amplitude oscillatory motion

### Specific Wear Rate

$$k = \frac{V}{NL}$$

Units: $\mathrm{mm^3/(N \cdot m)}$

Lower $k$ indicates better wear resistance

## Lubrication

### Viscosity

**Dynamic Viscosity ($\mu$):**

$$\tau = \mu \frac{du}{dy}$$

Units: $\mathrm{Pa \cdot s}$ or $\mathrm{N \cdot s/m^2}$ or poise (1 poise = 0.1 $\mathrm{Pa \cdot s}$)

**Kinematic Viscosity ($\nu$):**

$$\nu = \frac{\mu}{\rho}$$

Units: $\mathrm{m^2/s}$ or stoke (1 stoke = $10^{-4}$ $\mathrm{m^2/s}$)

**Temperature Dependence:**

Viscosity decreases with increasing temperature

## Lubrication Regimes

**Stribeck Curve:** Friction vs. $\frac{\mu v}{P}$

**Boundary Lubrication:** - High load, low speed, thin film - Metal-to-metal contact - High friction ($\mu \approx 0.1$ to $0.15$)

**Mixed Lubrication:** - Partial fluid film, partial contact - Transition regime

**Hydrodynamic (Fluid Film) Lubrication:** - Surfaces completely separated by fluid film - Low friction ($\mu \approx 0.001$ to $0.01$) - Load supported by pressure in fluid

**Elastohydrodynamic (EHL) Lubrication:** - High pressure deforms surfaces - Common in gears, rolling bearings

## Reynolds Equation

**1D Simplified:**

$$\frac{d}{dx}\left(h^3\frac{dp}{dx}\right) = 6\mu U\frac{dh}{dx}$$

where: - $h$ = film thickness - $p$ = pressure - $U$ = velocity

Describes pressure distribution in fluid film

## Minimum Film Thickness

**For journal bearing:**

$$h_{min} = c(1-\epsilon)$$

where: - $c$ = radial clearance - $\epsilon$ = eccentricity ratio

For safe operation: $h_{min} > 3\sigma$ (where $\sigma$ is composite surface roughness)

## Lubricant Selection

**Viscosity Index (VI):**

Measure of viscosity change with temperature

Higher VI $\rightarrow$ less change with temperature

**Pour Point:** Lowest temperature at which oil flows

**Flash Point:** Temperature at which vapor ignites

# Surface Characterization

## Surface Topography

**Peak Count:** Number of peaks per unit length

**Bearing Ratio:** Fraction of surface above a certain depth

**Wavelength:** Distance between repeating features

## Contact Mechanics

**Hertzian Contact (elastic):**

For cylinder on cylinder:

$$p_{max} = \frac{2P}{\pi b L}$$

where: - $P$ = normal load - $b$ = contact width - $L$ = contact length

**Real Contact Area:**

$$A_r = \frac{N}{H}$$

where $H$ is hardness

Real area $\ll$ Apparent area

# Solid Lubricants

Common solid lubricants: - Graphite (requires moisture or gases) - $MoS_2$ (works in vacuum) - PTFE (Teflon) - Soft metals (lead, indium)

Used when liquid lubricants fail (high temp, vacuum, contamination concerns)

# Quick Reference Values

**Typical Friction Coefficients:**

| **Interface** | $\mu$ |
|---|---|
| Steel on steel (dry) | 0.6-0.8 |
| Steel on steel (lubricated) | 0.05-0.1 |
| Steel on bronze (dry) | 0.2 |
| Brake materials | 0.3-0.5 |
| Rubber on pavement | 0.6-0.9 |
| Teflon on steel | 0.04-0.1 |