Mechanical Design - QE Equation Sheet

Design Philosophy and Safety

Factor of Safety

Based on Strength:

$$n = \frac{S_{\text{strength}}}{S_{\text{stress}}}$$

Based on Load:

$$n = \frac{P_{\text{failure}}}{P_{\text{applied}}}$$

Typical values:

- Known materials, certain loads: $n = 1.5$ to 2.5
- Uncertain loads or materials: $n = 3$ to 4
- Life-critical applications: $n > 4$

Design Process

1. Define requirements and constraints 2. Perform preliminary analysis 3. Select materials and components 4. Detailed stress analysis 5. Check for failure modes 6. Iterate and optimize

Static Failure Theories

Ductile Materials

Maximum Shear Stress Theory (Tresca):

$$\frac{\sigma_1 - \sigma_3}{2} \leq \frac{S_y}{n}$$

or

$$\sigma_1 - \sigma_3 \leq \frac{S_y}{n}$$

Distortion Energy Theory (von Mises):

For 3D stress state:

$$\sigma' = \sqrt{\frac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} \leq \frac{S_y}{n}$$

For plane stress ($\sigma_3 = 0$):

$$\sigma' = \sqrt{\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2} \leq \frac{S_y}{n}$$

For simple stress states:

$$\sigma' = \sqrt{\sigma^2 + 3\tau^2} \leq \frac{S_y}{n}$$

Note: von Mises theory generally more accurate for ductile materials

Brittle Materials

Maximum Normal Stress Theory (Rankine):

$$|\sigma_1| \leq \frac{S_{ut}}{n} \quad \text{or} \quad |\sigma_3| \leq \frac{S_{uc}}{n}$$

where S_{ut} is ultimate tensile strength and S_{uc} is ultimate compressive strength

Modified Mohr Theory:

Accounts for different tensile and compressive strengths

Most conservative and commonly used for brittle materials

Fatigue Failure

S-N Curve (Stress-Life)

Endurance Limit:

For steel: $S'_e \approx 0.5 S_{ut}$ (up to $S_{ut} = 1400$ MPa or 200 ksi)

For non-ferrous materials: No true endurance limit; use strength at $N = 5 \times 10^8$ cycles

Modified Endurance Limit:

$$S_e = k_a k_b k_c k_d k_e k_f S'_e$$

where: - k_a = surface finish factor - k_b = size factor - k_c = load factor - k_d = temperature factor - k_e = reliability factor - k_f = miscellaneous effects factor

Surface Finish Factor:

$$k_a = a S_{ut}^b$$

Surface Finish	a	b
Ground	1.34	-0.085
Machined	4.51	-0.265
Cold-drawn	4.51	-0.265
Hot-rolled	57.7	-0.718
As-forged	272	-0.995

(S_{ut} in MPa for these values)

Size Factor:

$$k_b = \begin{cases} \left(\frac{d}{7.62}\right)^{-0.107} & 2.79 \leq d \leq 51 \text{ mm} \\ 0.91d^{-0.157} & d > 51 \text{ mm} \end{cases}$$

For non-rotating round: $d_{eq} = 0.808\sqrt{bh}$ (rectangular $b \times h$)

Load Factor: - Bending: $k_c = 1$ - Axial: $k_c = 0.85$ - Torsion: $k_c = 0.59$

Temperature Factor:

$$k_d = \begin{cases} 1 & T \leq 450řC \\ 1 - 0.0058(T - 450) & 450 < T < 550řC \end{cases}$$

Reliability Factor:

Reliability	k_e
50%	1.000
90%	0.897
95%	0.868
99%	0.814
99.9%	0.753

Fluctuating Stress

Mean and Alternating Stress:

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Stress Ratio:

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Completely Reversed: $R = -1$ ($\sigma_m = 0$)

Zero-to-Max: $R = 0$ ($\sigma_m = \sigma_a$)

Fatigue Failure Criteria

Goodman Criterion (conservative):

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{n}$$

Gerber Criterion (less conservative):

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = \frac{1}{n}$$

Soderberg Criterion (most conservative):

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{n}$$

ASME Elliptic (for shafts):

$$\left(\frac{\sigma_a}{S_e/n}\right)^2 + \left(\frac{\sigma_m}{S_y/n}\right)^2 = 1$$

Combined Stresses in Fatigue

von Mises Approach:

$$\sigma'_a = \sqrt{\sigma_a^2 + 3\tau_a^2}$$
$$\sigma'_m = \sqrt{\sigma_m^2 + 3\tau_m^2}$$

Then apply Goodman or other criterion with σ'_a and σ'_m

Stress Concentration in Fatigue

Fatigue Stress Concentration Factor:

$$K_f = 1 + q(K_t - 1)$$

where: - K_t = theoretical stress concentration factor - q = notch sensitivity ($0 \leq q \leq 1$) - $q = 0$: no sensitivity (ignore K_t) - $q = 1$: full sensitivity (use full K_t)

Notch Sensitivity:

$$q = \frac{1}{1 + a/\sqrt{r}}$$

where a is Neuber constant (material property) and r is notch radius

Applying K_f:

For alternating stress only:

$$\sigma_a = K_f \sigma_{a,nominal}$$

Mean stress usually not affected by K_f

Cumulative Damage (Miner's Rule)

$$\sum_{i=1}^{k} \frac{n_i}{N_i} = C$$

where: - n_i = number of cycles at stress level i - N_i = cycles to failure at stress level i - C = damage sum (failure when $C \geq 1$)

Typically use $C = 1$ for design

Shaft Design

ASME Code for Transmission Shafts

For ductile materials with yield strength:

$$d = \left[\frac{16n}{\pi S_y} \sqrt{(K_t M_a + K_{tm} M_m)^2 + \frac{3}{4}(K_{ts} T_a + T_m)^2} \right]^{1/3}$$

where: - d = shaft diameter - n = factor of safety - M_a, M_m = alternating and mean bending moments - T_a, T_m = alternating and mean torques - K_t, K_{tm}, K_{ts} = fatigue stress concentration factors

Simplified for steady loading:

For rotating shaft with steady torque:

$$d = \left[\frac{16n}{\pi S_y} \sqrt{4(K_t M)^2 + 3(K_{ts} T)^2} \right]^{1/3}$$

For infinite life (fatigue):

$$d = \left[\frac{16n}{\pi} \sqrt{\left(\frac{K_f M_a}{S_e}\right)^2 + \frac{3}{4}\left(\frac{K_{fs} T_a}{S_e}\right)^2} \right]^{1/3}$$

Critical Speed

First Critical Speed (simply supported):

$$\omega_c = \sqrt{\frac{g}{\delta_{static}}}$$

where δ_{static} is static deflection at center

For design, operating speed should be:

$$\omega_{operating} < 0.8\omega_c \quad \text{or} \quad \omega_{operating} > 1.2\omega_c$$

Springs

Helical Compression Springs

Shear Stress:

$$\tau = K_s \frac{8FD}{\pi d^3}$$

where: - F = axial force - D = mean coil diameter - d = wire diameter - K_s = shear stress correction factor (Wahl factor)

Wahl Correction Factor:

$$K_s = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

where $C = D/d$ is spring index (typically $4 \leq C \leq 12$)

Spring Rate (Stiffness):

$$k = \frac{F}{\delta} = \frac{Gd^4}{8D^3 N_a}$$

where: - G = shear modulus - N_a = number of active coils

Deflection:

$$\delta = \frac{8FD^3 N_a}{Gd^4}$$

Solid Height:

$$L_s = d(N_t + 1)$$

where N_t is total number of coils

Free Length:

$$L_f = L_s + \delta_{max} + \text{(clash allowance)}$$

Clash allowance typically 10-15% of maximum deflection

Spring Design Considerations

End Conditions:

End Type	N_t	**Solid Height**
Plain	N_a	dN_t
Plain and ground	N_a	dN_t
Squared	$N_a + 2$	$d(N_t - 1)$
Squared and ground	$N_a + 2$	$d(N_t - 1)$

Buckling:

For $L_f/D > 4$, check for buckling

Helical Extension Springs

Similar formulas to compression springs, but: - Initial tension F_i often present - Hooks/loops add stress concentration - K_s includes hook effects

Torsion Springs

Bending Stress:

$$\sigma = K_b \frac{32M}{\pi d^3}$$

Angular Deflection:

$$\theta = \frac{64MDN_a}{Ed^4}$$

where M is applied moment and E is Young's modulus

Screws and Fasteners

Power Screws

Torque to Raise Load:

$$T_R = \frac{Fd_m}{2}\left(\frac{l + \pi\mu d_m}{\pi d_m - \mu l}\right)$$

Torque to Lower Load:

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi\mu d_m - l}{\pi d_m + \mu l}\right)$$

where: - F = load - d_m = mean diameter - l = lead ($l = p$ for single thread, $l = np$ for n threads) - p = pitch - μ = coefficient of friction

Efficiency:

$$e = \frac{Fl}{2\pi T_R}$$

Self-Locking Condition:

$$\mu > \frac{l}{\pi d_m}$$

or equivalently: $\alpha < \phi$ where $\alpha = \tan^{-1}(l/(\pi d_m))$ is lead angle and $\phi = \tan^{-1}(\mu)$ is friction angle

Threaded Fasteners (Bolts)

Tensile Stress Area:

$$A_t = \frac{\pi}{4}\left(\frac{d - 0.9382p}{1}\right)^2$$

For Unified threads, approximately:

$$A_t \approx 0.7854 \left(d - \frac{0.9743}{n} \right)^2$$

where n is threads per inch

Preload:

Typical preload: $F_i = 0.75 F_{proof}$

$$F_{proof} = A_t S_p$$

where S_p is proof strength

Joint Stiffness:

Bolt stiffness:

$$k_b = \frac{A_t E_b}{L_t}$$

where L_t is grip length (threaded length under load)

Member stiffness (more complex):

$$k_m = \frac{E_m A_e}{L}$$

External Load Distribution:

Fraction to bolt:

$$C = \frac{k_b}{k_b + k_m}$$

Bolt force under external load P:

$$F_b = F_i + CP$$

Member force:

$$F_m = F_i(1 - C)P$$

Fatigue Loading:

Alternating stress in bolt:

$$\sigma_a = \frac{CP_a}{A_t}$$

Mean stress:

$$\sigma_m = \frac{F_i + CP_m}{A_t}$$

Apply fatigue criteria with these stresses

Bolt Torque

Tightening Torque:

$$T = KFd$$

where: - K = nut factor (typically 0.15-0.20 for lubricated) - F = desired bolt tension - d = nominal diameter

Gears

Gear Nomenclature

Basic Relationships:

$$d = \frac{N}{P_d} = Nm$$

where: - d = pitch diameter - N = number of teeth - P_d = diametral pitch (teeth/inch) - m = module (mm/tooth)

Velocity Ratio:

$$VR = \frac{\omega_1}{\omega_2} = \frac{N_2}{N_1} = \frac{d_2}{d_1}$$

Circular Pitch:

$$p = \frac{\pi d}{N} = \frac{\pi}{P_d}$$

Center Distance:

$$C = \frac{d_1 + d_2}{2} = \frac{N_1 + N_2}{2P_d}$$

Spur Gears

Transmitted Force (tangential):

$$W_t = \frac{T}{r} = \frac{P}{\omega r} = \frac{2P}{\omega d}$$

where T is torque, P is power, ω is angular velocity

Radial Force:

$$W_r = W_t \tan\phi$$

where ϕ is pressure angle (typically 20° or 25°)

Lewis Equation (bending stress):

$$\sigma = \frac{W_t}{FmY}$$

where: - F = face width - m = module - Y = Lewis form factor (depends on number of teeth)

AGMA Bending Stress:

$$\sigma = W_t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_B}{J}$$

where K factors account for: - K_o = overload - K_v = dynamic - K_s = size - K_m = load distribution - K_B = rim thickness - J = geometry factor

Contact Stress (Hertzian):

$$\sigma_c = C_p \sqrt{\frac{W_t K_o K_v K_s K_m}{F d_1} \frac{C_f}{I}}$$

where: - C_p = elastic coefficient - C_f = surface condition factor - I = geometry factor

Helical Gears

Similar to spur gears but with additional axial force component:

Axial Force:

$$W_a = W_t \tan \psi$$

where ψ is helix angle

Bevel Gears

Forces:

Similar analysis but forces act at pitch cone angle

Decompose into tangential, radial, and axial components

Worm Gears

Velocity Ratio:

$$VR = \frac{N_w}{N_g}$$

where N_w is number of threads on worm, N_g is teeth on gear

Efficiency:

$$e = \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda}$$

where ϕ_n is normal pressure angle, f is coefficient of friction, λ is lead angle

Worm gears can be self-locking if $\lambda < \tan^{-1}(f)$

Bearings

Rolling Contact Bearings

Basic Life Equation:

$$L_{10} = \left(\frac{C}{P}\right)^a$$

where: - L_{10} = rating life (millions of revolutions for 90% reliability) - C = basic dynamic load rating - P = equivalent dynamic load - $a = 3$ for ball bearings, 10/3 for roller bearings

Life in Hours:

$$L_{10h} = \frac{L_{10} \times 10^6}{60n}$$

where n is rotational speed (rpm)

Equivalent Dynamic Load:

For radial bearings:

$$P = XF_r + YF_a$$

where: - F_r = radial load - F_a = axial (thrust) load - X, Y = radial and thrust factors (from manufacturer)

Variable Loading:

$$P_{eq} = \left[\frac{\sum(P_i^a n_i)}{\sum n_i}\right]^{1/a}$$

where P_i are loads at different speeds n_i

Journal Bearings (Sliding Contact)

Petroff's Equation (light load):

$$f = \frac{2\pi^2 \mu N}{P}\left(\frac{r}{c}\right)$$

where: - f = coefficient of friction - μ = absolute viscosity - N = shaft speed (rev/s) - P = bearing pressure - r = shaft radius - c = radial clearance

Sommerfeld Number:

$$S = \left(\frac{r}{c}\right)^2 \frac{\mu N}{P}$$

Used to determine bearing performance from charts

Minimum Film Thickness:

$$h_0 = c(1 - \epsilon)$$

where ϵ is eccentricity ratio (from charts based on S)

Brakes and Clutches

Friction Brakes

Torque Capacity:

$$T = \mu F r$$

where: - μ = coefficient of friction - F = normal force - r = effective radius

Power Dissipation:

$$P = T\omega$$

Energy per Stop:

$$E = \frac{1}{2} I \omega^2$$

where I is moment of inertia of rotating mass

Band Brakes

Force Relationship:

$$\frac{F_1}{F_2} = e^{\mu\beta}$$

where β is wrap angle (radians)

Torque:

$$T = (F_1 - F_2) r$$

Keys and Pins

Keys

Shear Stress:

$$\tau = \frac{2T}{dLh}$$

Bearing Stress:

$$\sigma_b = \frac{4T}{dLh}$$

where: - T = torque - d = shaft diameter - L = key length - h = key height

Standard Key Dimensions:

Square key: $h = w = d/4$

Flywheels

Energy Storage:

$$E = \frac{1}{2} I(\omega_{max}^2 - \omega_{min}^2)$$

Coefficient of Fluctuation:

$$C_s = \frac{\omega_{max} - \omega_{min}}{\omega_{avg}}$$

Flywheel Design:

$$I = \frac{E}{C_s \omega_{avg}^2}$$

Material Selection

Common Engineering Materials:

Material	S_y (MPa)	S_{ut} (MPa)	**Applications**
AISI 1020 (HR)	210	380	General purpose
AISI 1045 (CD)	530	625	Shafts, gears
AISI 4140 (Q&T)	655	855	High strength
6061-T6 Al	275	310	Lightweight
Gray Cast Iron	-	170	Machine bases

Quick Reference Formulas

Shaft Diameter (rough estimate):

$$d \approx 2.5 \sqrt[3]{\frac{P}{n}}$$

where P is in kW, n is in rpm, d is in cm

Gear Face Width:

$$F = 3p \text{ to } 5p$$

where p is circular pitch

Spring Index Range:

$$4 \leq C \leq 12$$