Dynamics

Kinematics

Particle Motion

$$\mathbf{v} = \dot{\mathbf{r}}, \quad \mathbf{a} = \dot{\mathbf{v}}$$

Rectilinear motion:

$$v = \frac{dx}{dt}, \quad a = \frac{dv}{dt} = v\frac{dv}{dx}$$

Curvilinear (normal–tangential):

$$\mathbf{a} = \dot{v}\,\hat{t} + \frac{v^2}{\rho}\,\hat{n}$$

Polar coordinates:

$$\mathbf{v} = \dot{r}\,\hat{e}_r + r\dot{\theta}\,\hat{e}_\theta$$

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\hat{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{e}_\theta$$

Rigid Body Kinematics

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\omega} \times \mathbf{r}_{B/A}$$

$$\mathbf{a}_B = \mathbf{a}_A + \boldsymbol{\alpha} \times \mathbf{r}_{B/A} + \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r}_{B/A})$$

Rolling without slipping:

$$v = \omega R, \quad a = \alpha R$$

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Mass Properties

Center of mass:

$$\mathbf{r}_G = \frac{1}{M}\int \mathbf{r}\,dm$$

Parallel-axis theorem:

$$I_O = I_G + Md^2$$

Common moments of inertia:

$$I_{\text{rod, center}} = \frac{1}{12}mL^2, \quad I_{\text{rod, end}} = \frac{1}{3}mL^2$$

$$I_{\text{disk}} = \frac{1}{2}mR^2, \quad I_{\text{ring}} = mR^2$$

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Kinetics (Newton–Euler)

Particle:

$$\sum \mathbf{F} = m\mathbf{a}$$

Rigid Body (Planar):

$$\sum \mathbf{F} = m\mathbf{a}_G$$

$$\sum M_G = I_G \boldsymbol{\alpha}$$

About a fixed point O:

$$\sum M_O = I_O \boldsymbol{\alpha}$$

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Work, Energy, and Power

Kinetic energy:

$$T = \frac{1}{2}mv^2 + \frac{1}{2}I_G\omega^2$$

Potential energy:

$$V_g = mgh, \quad V_s = \frac{1}{2}kx^2$$

Work–energy principle:

$$T_1 + V_1 + W_{nc} = T_2 + V_2$$

Power:

$$P = \mathbf{F} \cdot \mathbf{v}, \quad P = M\omega$$

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Linear and Angular Momentum

Linear momentum:

$$\mathbf{p} = m\mathbf{v}$$

Impulse–momentum:

$$\int_{t_1}^{t_2} \sum \mathbf{F}\, dt = m(\mathbf{v}_2 - \mathbf{v}_1)$$

Angular momentum about point O:

$$\mathbf{H}_O = I_G \boldsymbol{\omega} + \mathbf{r}_{G/O} \times m\mathbf{v}_G$$

Angular impulse–momentum:

$$\int_{t_1}^{t_2} \sum M_O \, dt = \mathbf{H}_{O2} - \mathbf{H}_{O1}$$

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Special Cases and Conservation Laws

Conservation of energy:

$$T_1 + V_1 = T_2 + V_2 \quad \text{(no non-conservative work)}$$

Conservation of linear momentum:

$$\sum \mathbf{F}_{\text{ext}} = 0$$

Conservation of angular momentum:

$$\sum M_O^{\text{ext}} = 0$$

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