Thermodynamics Sample Problem 1

This is problem 1.2 on the 2023 QE.

In a certain industrial process, a heat engine operates between two reservoirs: a high-temperature reservoir at 600°C and a low-temperature reservoir at 150°C. The engine takes in 1500 J of heat from the high-temperature reservoir and delivers 900 J of work while rejecting heat to the low-temperature reservoir. Assume that the heat engine operates in a steady-state manner and there are no other sources of energy losses or inefficiencies in the engine.

Part A

Calculate the efficiency of the heat engine.
The efficiency (η) of a heat engine is defined as the ratio of the work output (W) to the heat input (Q_{in}):

$$\eta = \frac{W}{Q_{in}} \tag{1}$$

Substituting the given values:

$$\boxed{\eta = \frac{900\,\mathrm{J}}{1500\,\mathrm{J}} = 0.6\,\mathrm{or}\,60\%} \tag{2}$$

Part B

Determine the amount of heat rejected to the low-temperature reservoir.
The heat rejected (Q_{out}) to the low-temperature reservoir can be calculated using the first law of thermodynamics for a heat engine:

$$Q_{in} = W + Q_{out} \tag{3}$$

Rearranging the equation to solve for Q_{out}:

$$Q_{out} = Q_{in} - W \tag{4}$$

Substituting the given values:

$$\boxed{Q_{out} = 1500\,\mathrm{J} - 900\,\mathrm{J} = 600\,\mathrm{J}} \tag{5}$$

Part C

Calculate the change in entropy of the high-temperature reservoir, low-temperature reservoir, and the overall entropy change for the entire process.

The change in entropy (ΔS) for a reservoir can be calculated using the formula:

$$\Delta S = \frac{Q}{T} \tag{6}$$

Where Q is the heat exchanged and T is the absolute temperature in Kelvin. First, we need to convert the temperatures from Celsius to Kelvin:

$$T_{high} = 600 + 273.15 = 873.15\,\mathrm{K} \tag{7}$$
$$T_{low} = 150 + 273.15 = 423.15\,\mathrm{K} \tag{8}$$

Now, we can calculate the change in entropy for each reservoir:

$$\Delta S_{high} = \frac{-Q_{in}}{T_{high}} = \frac{-1500\,\mathrm{J}}{873.15\,\mathrm{K}} \approx -1.72\,\mathrm{J/K} \tag{9}$$
$$\Delta S_{low} = \frac{Q_{out}}{T_{low}} = \frac{600\,\mathrm{J}}{423.15\,\mathrm{K}} \approx 1.42\,\mathrm{J/K} \tag{10}$$

The overall change in entropy for the entire process is the sum of the changes in entropy of both reservoirs:

$$\Delta S_{total} = \Delta S_{high} + \Delta S_{low} \approx -1.72\,\mathrm{J/K} + 1.42\,\mathrm{J/K} \approx -0.30\,\mathrm{J/K} \tag{11}$$
$$\boxed{\Delta S_{total} \approx -0.30\,\mathrm{J/K}} \tag{12}$$

Part D

Discuss the feasibility of achieving a higher efficiency for the given temperature reservoirs.

To evaluate the feasibility of achieving a higher efficiency, we can compare the calculated efficiency of the heat engine to the Carnot efficiency, which represents the maximum possible efficiency for a heat engine operating between two temperature reservoirs. The Carnot efficiency (η_{Carnot}) is given by:

$$\eta_{Carnot} = 1 - \frac{T_{low}}{T_{high}} \tag{13}$$

Substituting the temperatures in Kelvin:

$$\eta_{Carnot} = 1 - \frac{423.15\,\mathrm{K}}{873.15\,\mathrm{K}} \approx 0.515\,\mathrm{or}\,51.5\% \tag{14}$$

Since the calculated efficiency of the heat engine (60%) exceeds the Carnot efficiency (51.5%), it indicates that the engine is operating beyond the theoretical maximum efficiency, which is not feasible according to the second law of thermodynamics. Therefore, achieving a higher efficiency than the Carnot efficiency is impossible for the given temperature reservoirs.