# Econometrics Test Exercise 5

Jáder Martins Camboim de Sá (jader.martins@ipea.gov.br)

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**1. Show that $\frac{\partial Pr[\mathbf{resp}_i=1]}{\partial \mathbf{age}_i} + \frac{\partial Pr[\mathbf{resp}_i=0]}{\partial \mathbf{age}_i} = 0$.**

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$$\frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} + \frac{\partial Pr[\text{resp}_i = 0]}{\partial \text{age}_i} = \frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} + \frac{\partial (1 - Pr[\text{resp}_i = 1])}{\partial \text{age}_i} =$$

$$\frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} + \frac{\partial 1}{\partial \text{age}_i} - \frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} = \frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} + 0 - \frac{\partial Pr[\text{resp}_i = 1]}{\partial \text{age}_i} = 0$$

**2. Assume that you recode the dependent variable as follows: $\mathbf{resp}_i^{\mathbf{new}} = -\mathbf{resp}_i + 1$. Hence, positive response is now defined to be equal to zero and negative response to be equal to 1. Use the odds ratio to show that this transformation implies that the sign of all parameters change.**

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$$\frac{Pr[\text{resp}_i{}^{new} = 1]}{Pr[\text{resp}_i{}^{new} = 0]} = \frac{1}{Pr[\text{resp}_i = 1]/Pr[\text{resp}_i = 0]} = \frac{Pr[\text{resp}_i = 0]}{Pr[\text{resp}_i = 1]}$$

So we have:

$$\frac{Pr[\text{resp}_i{}^{new} = 0]}{Pr[\text{resp}_i{}^{new} = 1]} = \frac{Pr[\text{resp}_i = 1]}{Pr[\text{resp}_i = 0]}.$$

With the definition of $Pr[\text{resp}_i = 1]$:

$$\frac{Pr[resp_i = 1]}{Pr[resp_i = 0]} = exp\left(\beta_0 + \beta_1 * male_i + \beta_2 * active_i + \beta_3 * age_i + \beta_4 * (age_i/10)^2\right),$$

and then:

$$\frac{Pr[resp_i = 0]}{Pr[resp_i = 1]} = \frac{1}{exp\left(\beta_0 + \beta_1 * male_i + \beta_2 * active_i + \beta_3 * age_i + \beta_4 * (age_i/10)^2\right)},$$

with $\frac{1}{exp(x)} = exp(-x)$ we have:

$$\frac{1}{exp\left(\beta_0 + \sum_{j=2}^{k} \beta_j * x_{ji}\right)} = exp\left(-\beta_0 - \sum_{j=2}^{k} \beta_j * x_{ji})\right),$$

opening the expression:

$$exp\left(-\beta_0 - \beta_1 * male_i - \beta_2 * active_i - \beta_3 * age_i - \beta_4 * (age_i/10)^2\right).\square$$

**3. Consider again the odds ratio positive response versus negative response:**

$$\frac{Pr[resp_i = 1]}{Pr[resp_i = 0]} = exp\left(\beta_0 + \beta_1 * male_i + \beta_2 * active_i + \beta_3 * age_i + \beta_4 * (age_i/10)^2\right) .$$

**During lecture 5.5 you have seen that this odds ratio obtains its maximum value for age equal to 50 years for males as well as females. Suppose now that you want to extend the logit model and allow that this age value is possibly different for males than for females. Discuss how you can extend the logit specification.**

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We can split the sample in two (male-only and female-only) groups, and estimate the logit model seperately for each group.