Context-Free Grammar

COMP90042

Natural Language Processing

Lecture 14

Semester 1 2022 Week 7

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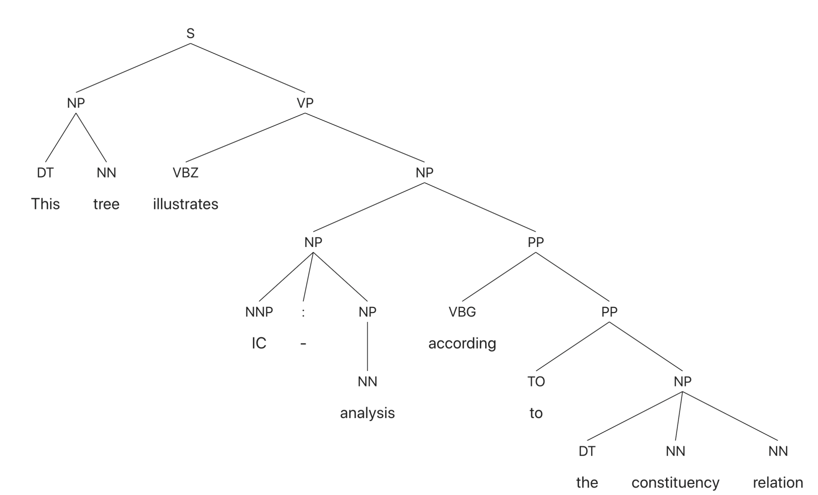


Recap

- Center embedding
  - The cat loves Mozart
  - The cat the dog chased loves Mozart
  - The cat the dog the rat bit chased loves Mozart
  - The cat the dog the rat the elephant admired bit chased loves Mozart
- Cannot be captured by regular expressions (S^nV^n)
- **Context-free grammar!**

Basics of Context-Free Grammars

- **Symbols**
  - **Terminal**: word such as *book*
  - **Non-terminal**: syntactic label such as NP or VP

convention:
lowercase for terminals
uppercase for non-terminals

- **Productions** (rules)
  - W → X Y Z
  - Exactly one non-terminal on left-hand side (LHS)
  - An ordered list of symbols on right-hand side (RHS); can be **terminals** or **non-terminals**
- **Start symbol:** S

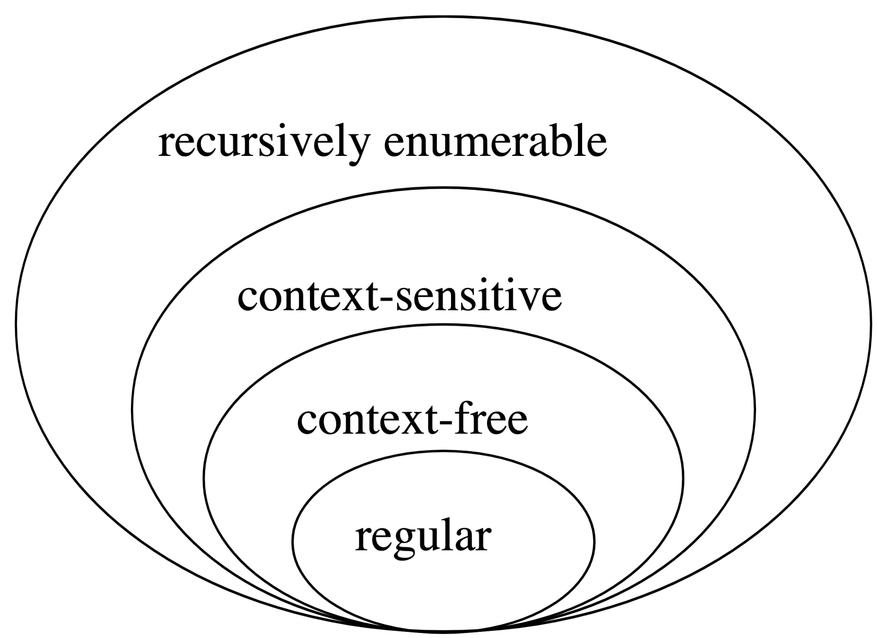
Why “Context Free”

$$W \rightarrow X\ Y\ Z$$

- Production rule depends only on the LHS (and not on ancestors, neighbours)
  - Analogous to Markov chain
  - Behaviour at each step depends only on current state

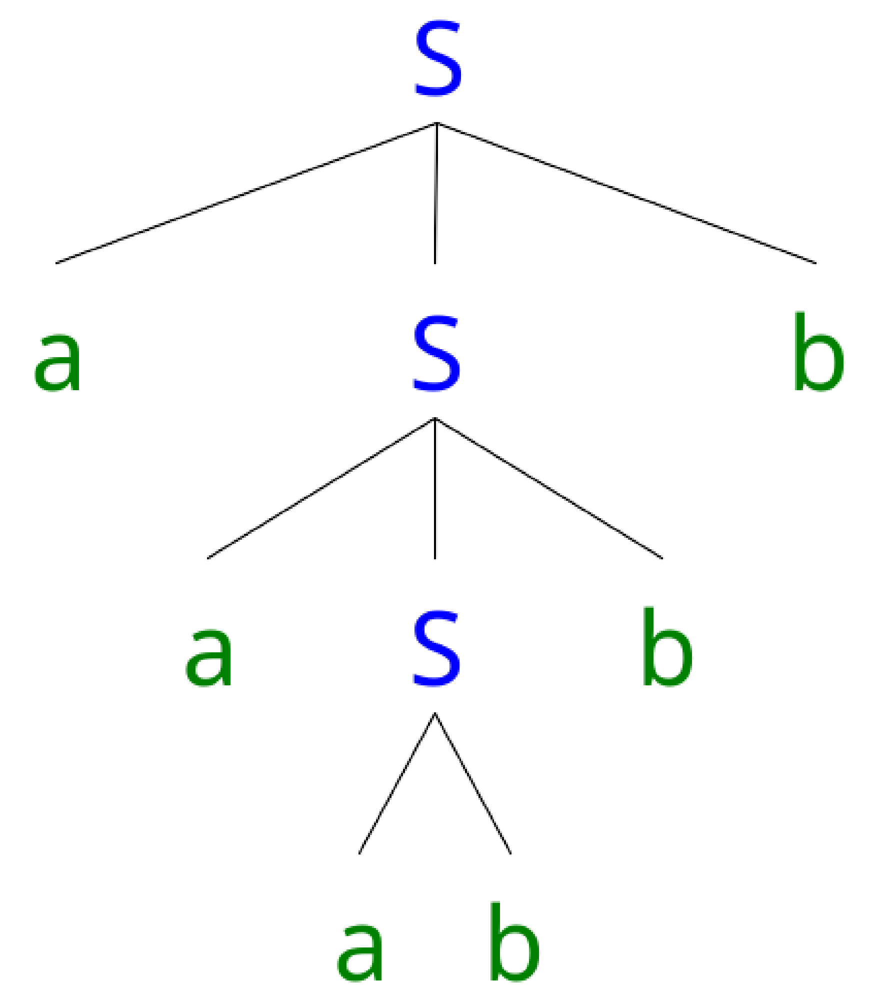
Context-Free vs. Regular

- Context-free languages more general than regular languages
  - Allows recursive nesting



CFG Parsing

- Given production rules
  - S → a S b
  - S → a b
- And a string
  - aaabbb
- Produce a valid parse tree



What This Means?

- If English can be represented with CFG:
  - first develop the production rules
  - can then build a “parser” to automatically judge whether a sentence is grammatical!
- But is natural language context-free?
- Not quite: cross-serial dependencies ($a^m b^n c^m d^n$)

Swiss-German:

...de Karl d’Maria em Peter de Hans laat hälfe lärne schwüme

English:

...Charles lets Mary help Peter to teach John to Swim

But…

- CFG strike a good balance:
  - CFG covers most syntactic patterns
  - CFG parsing is computational efficient
- We use CFG to describe a core fragment of English syntax

Outline

- Constituents
- CYK Algorithm
- Representing English with CFGs

Constituents

Syntactic Constituents

- Sentences are broken into **constituents**
  - word sequence that function as a **coherent unit** for linguistic analysis
  - helps build CFG production rules
- Constituents have certain key properties:
  - movement
  - substitution
  - coordination

Movement

- Constituents can be moved around sentences
  - Abigail gave [her brother] [a fish]
  - Abigail gave [a fish] to [her brother]
- Contrast: [gave her], [brother a]

Substitution

- Constituents can be substituted by other phrases of the same type
    - Max thanked [his older sister]
    - Max thanked [her]
- Contrast: [Max thanked], [thanked his]

Coordination

- Constituents can be conjoined with coordinators like *and* and *or*
  - [Abigail] and [her young brother] brought a fish
  - Abigail [bought a fish] and [gave it to Max]
  - Abigail [bought] and [greedily ate] a fish

Constituents and Phrases

- Once we identify constituents, we use **phrases** to describe them
- Phrases are determined by their **head word:**
  - noun phrase: her younger **brother**
  - verb phrase: greedily **ate** it
- We can use CFG to formalise these intuitions

He gave a lecture and away a pie. Which of the following is a constituent

- a lecture
- gave a lecture
- a pie
- away a pie

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A Simple CFG for English

Terminal symbols: *rat, the, ate, cheese*

Non-terminal symbols: S, NP, VP, DT, VBD, NN

Productions:

S → NP VP

NP → DT NN

VP → VBD NP

DT → *the*

NN → *rat*

NN → *cheese*

VBD → *ate*

Generating Sentences with CFGs

Always start with S (the sentence/start symbol)

S

Apply a rule with S on LHS (*S* → *NP VP*), i.e substitute RHS

NP VP

Apply a rule with NP on LHS (*NP* → *DT NN*)

DT NN VP

Apply rule with DT on LHS (DT → *the*)

the NN VP

Apply rule with NN on LHS (NN → *rat*)

the rat VP

S → NP VP
NP → DT NN
VP → VBD NP
DT → *the*
NN → *rat*
NN → *cheese*
VBD → *ate*

Generating Sentences with CFGs

Apply rule with VP on LHS (VP → VBD NP)

the rat VBD NP

Apply rule with VBD on LHS (VBD → *ate*)

the rat ate NP

Apply rule with NP on LHS (NP → DT NN)

the rat ate DT NN

Apply rule with DT on LHS (DT → *the*)

the rat ate the NN

Apply rule with NN on LHS (NN → *cheese*)

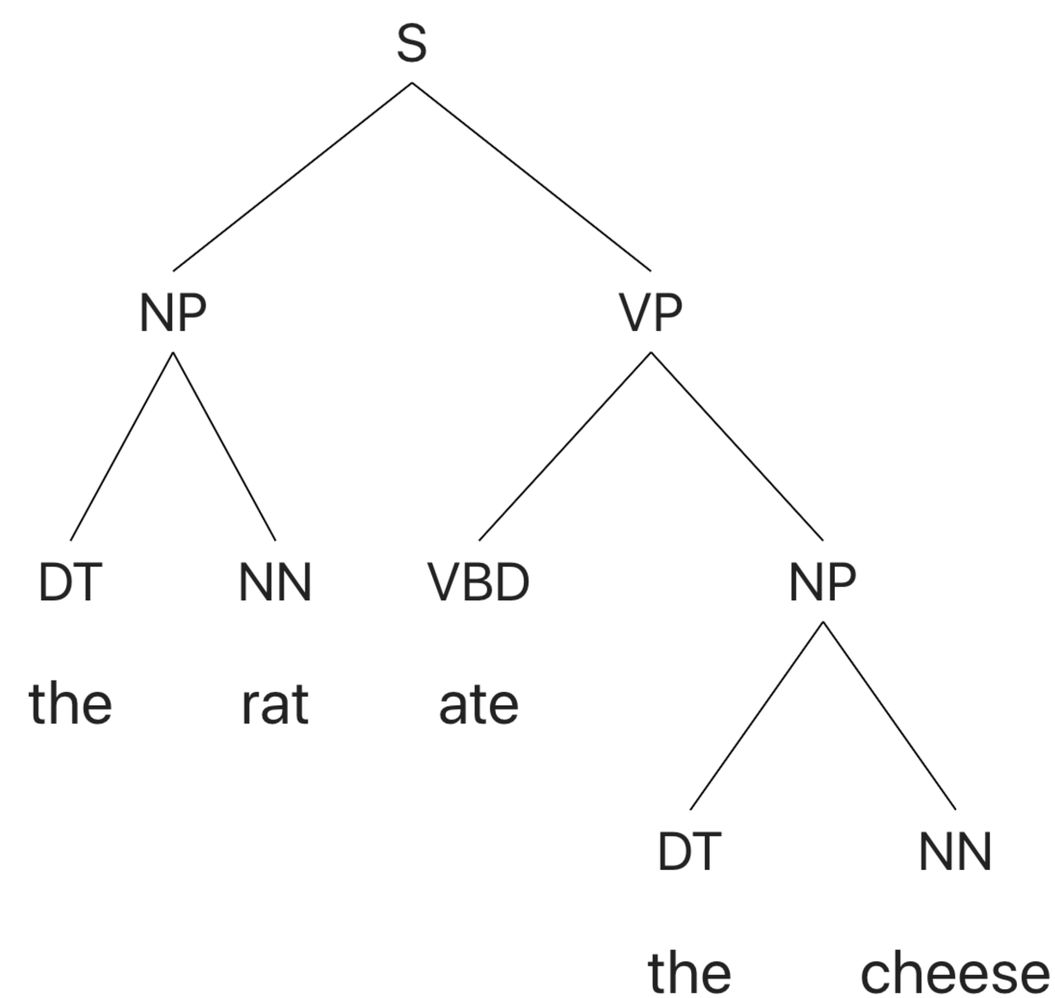
the rat ate the cheese ← No non-terminals left, we're done!

S → NP VP
NP → DT NN
VP → VBD NP
DT → *the*
NN → *rat*
NN → *cheese*
VBD → *ate*

CFG Trees

- Generation corresponds to a syntactic tree
- Non-terminals are internal nodes
- Terminals are leaves

```
(S (NP (DT the)
       (NN rat))
   (VP (VBG ate)
       (NP (DT the)
           (NN cheese))))
```



- CFG parsing is the **reverse** process (sentence → tree)

A CFG for Arithmetic Expressions

$$
\begin{aligned}
\textsc{S} &\rightarrow \textsc{S Op S} \mid \textsc{Num} \\
\textsc{Op} &\rightarrow + \mid - \mid \times \mid \div \\
\textsc{Num} &\rightarrow \textsc{Num Digit} \mid \textsc{Digit} \\
\textsc{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

- S = starting symbol
- | = operator OR
- Recursive, NUM and S can produce themselves

Parsing

$$
\begin{aligned}
\textsc{S} &\rightarrow \textsc{S}\ \textsc{Op}\ \textsc{S} \mid \textsc{Num} \\
\textsc{Op} &\rightarrow + \mid - \mid \times \mid \div \\
\textsc{Num} &\rightarrow \textsc{Num}\ \textsc{Digit} \mid \textsc{Digit} \\
\textsc{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

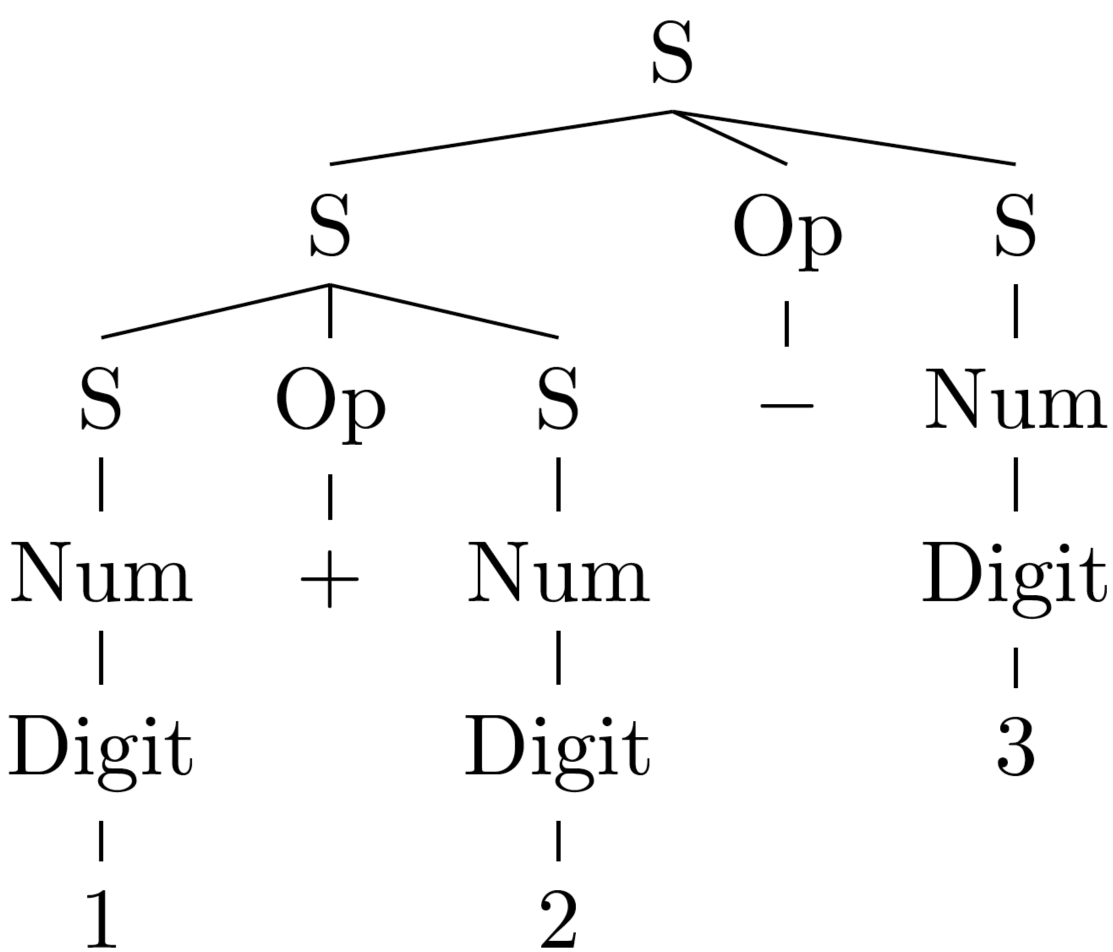
- Is '4' a valid string?

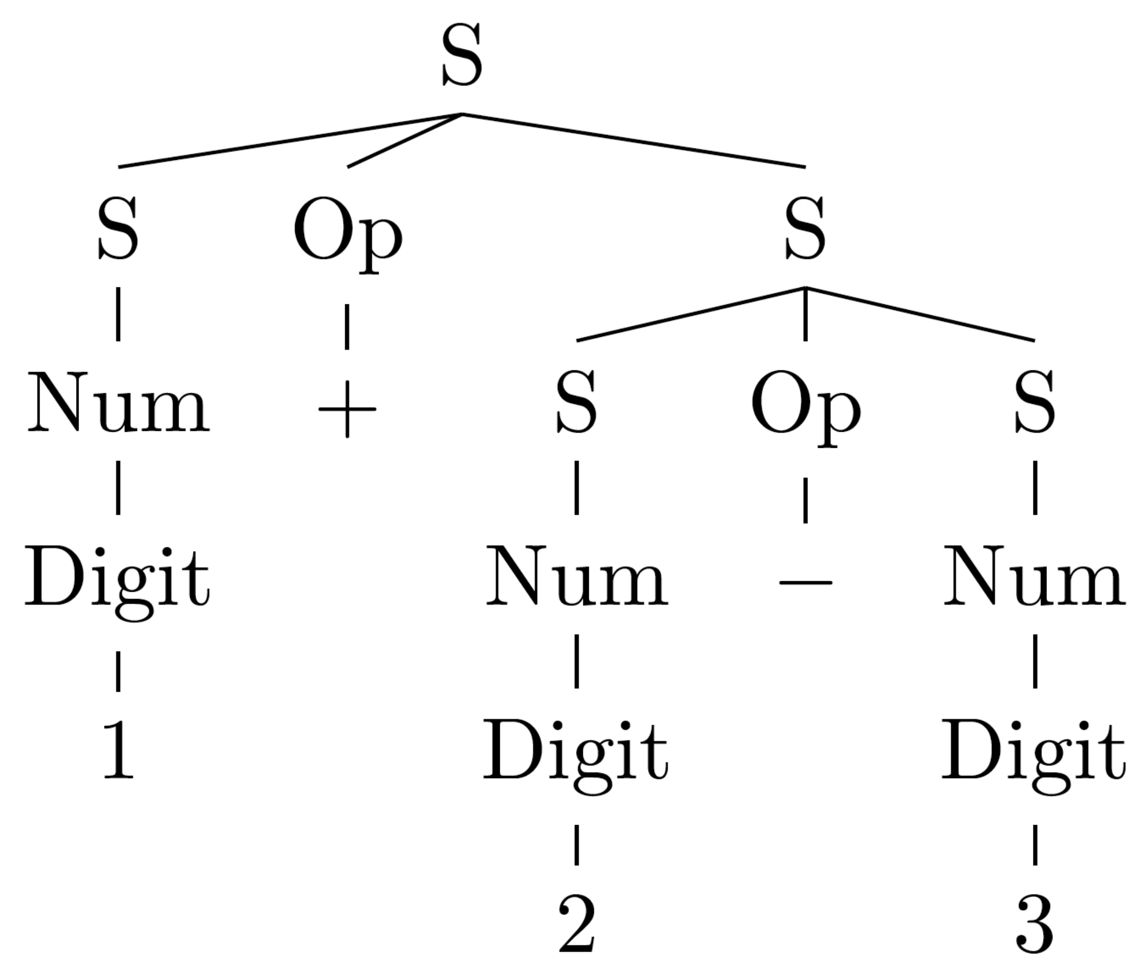
$$
\begin{array}{c}
\mathrm{S} \\
| \\
\mathrm{Num} \\
| \\
\mathrm{Digit} \\
| \\
4
\end{array}
$$

Parsing

$$
\begin{aligned}
\textsc{S} &\rightarrow \textsc{S Op S} \mid \textsc{Num} \\
\textsc{Op} &\rightarrow + \mid - \mid \times \mid \div \\
\textsc{Num} &\rightarrow \textsc{Num Digit} \mid \textsc{Digit} \\
\textsc{Digit} &\rightarrow 0 \mid 1 \mid 2 \mid \ldots \mid 9
\end{aligned}
$$

- Is ‘1+2-3’ a valid string?





CYK Algorithm

CYK Algorithm

- Bottom-up parsing
- Tests whether a string is valid given a CFG, without enumerating all possible parses
- Core idea: form small constituents first, and merge them into larger constituents
- Requirement: CFGs must be in **Chomsky Normal Forms**

Convert to Chomsky Normal Form

- Change grammar so all rules of form:
  - A → B C
  - A → a
- Convert rules of form A → B c into:
  - A → B X
  - X → c

Convert to Chomsky Normal Form

- Convert rules A → B C D into:
  - A → B Y
  - Y → C D
  - E.g. VP → VP NP NP
    for ditransitive cases, "*sold [her] [the book]*"
- X, Y are new symbols we have introduced

Convert to Chomsky Normal Form

- CNF disallows unary rules, A → B.
- Imagine NP → S; and S → NP … leads to infinitely many trees with same yield.
- Replace RHS non-terminal with its productions
    - A → B, B → *cat*, B → *dog*
    - A → *cat*, A → *dog*

The CYK Parsing Algorithm

- Convert grammar to Chomsky Normal Form (CNF)
- Fill in a parse table (left to right, bottom to top)
- Use table to derive parse
- S in top right corner of table = success!
- Convert result back to original grammar

we	eat	sushi	with	chopsticks
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]

S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

we	eat	sushi	with	chopsticks
NP [0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	V [1,2]	[1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

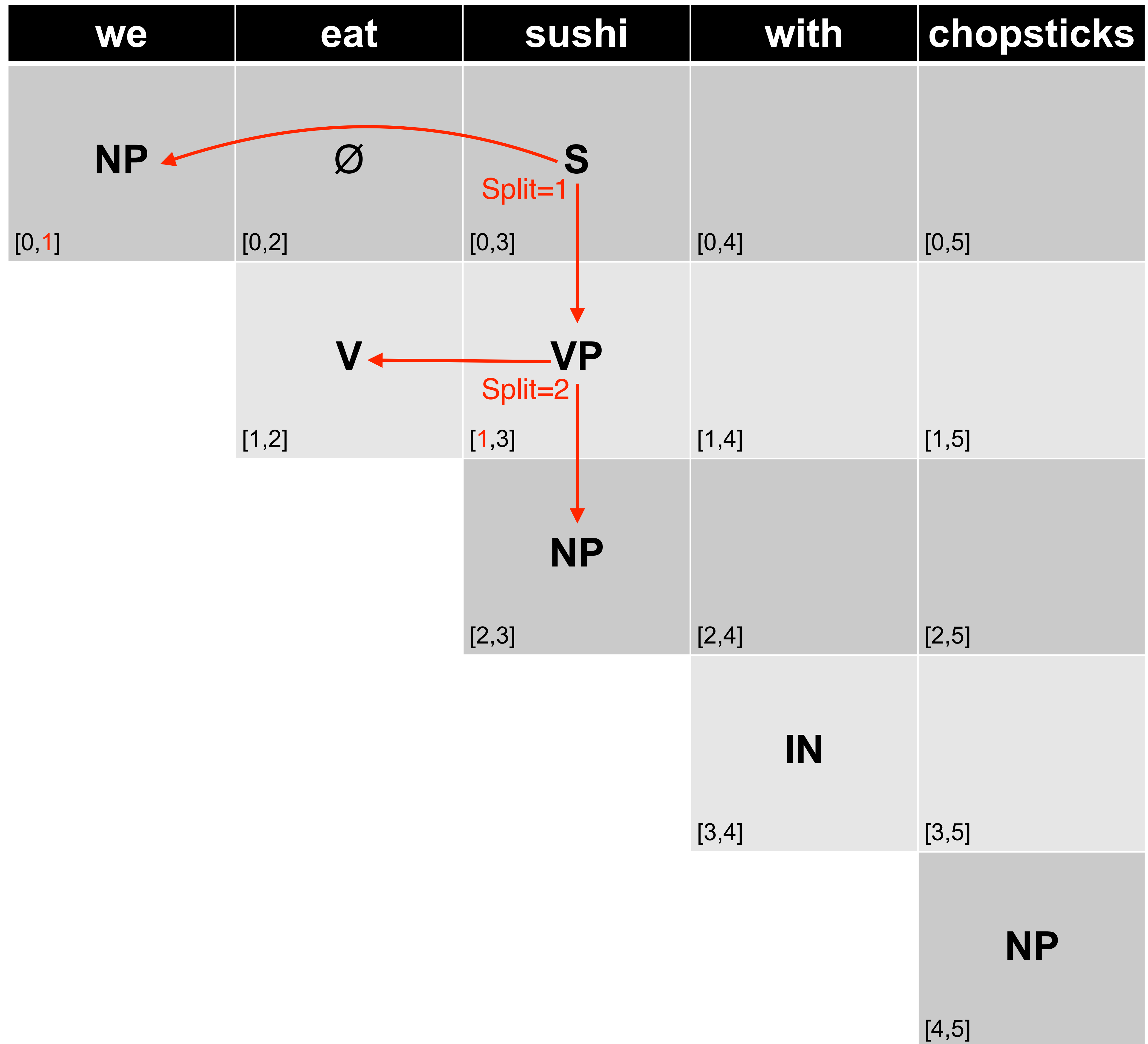
S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	[0,3]	[0,4]	[0,5]
	V [1,2]	[1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

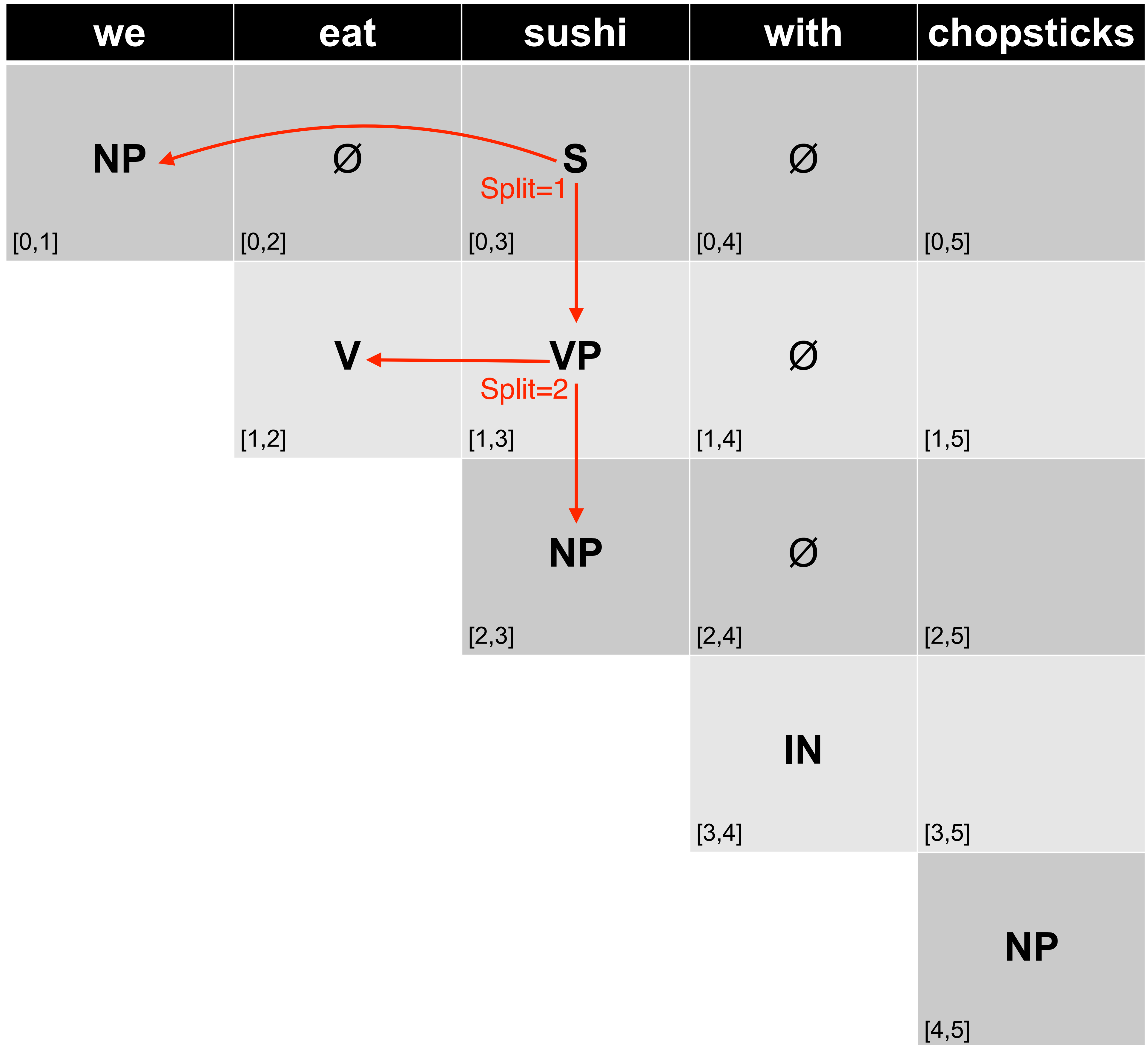
S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	[0,3]	[0,4]	[0,5]
	V [1,2]	VP Split=2 [1,3]	[1,4]	[1,5]
		NP [2,3]	[2,4]	[2,5]
			IN [3,4]	[3,5]
				NP [4,5]

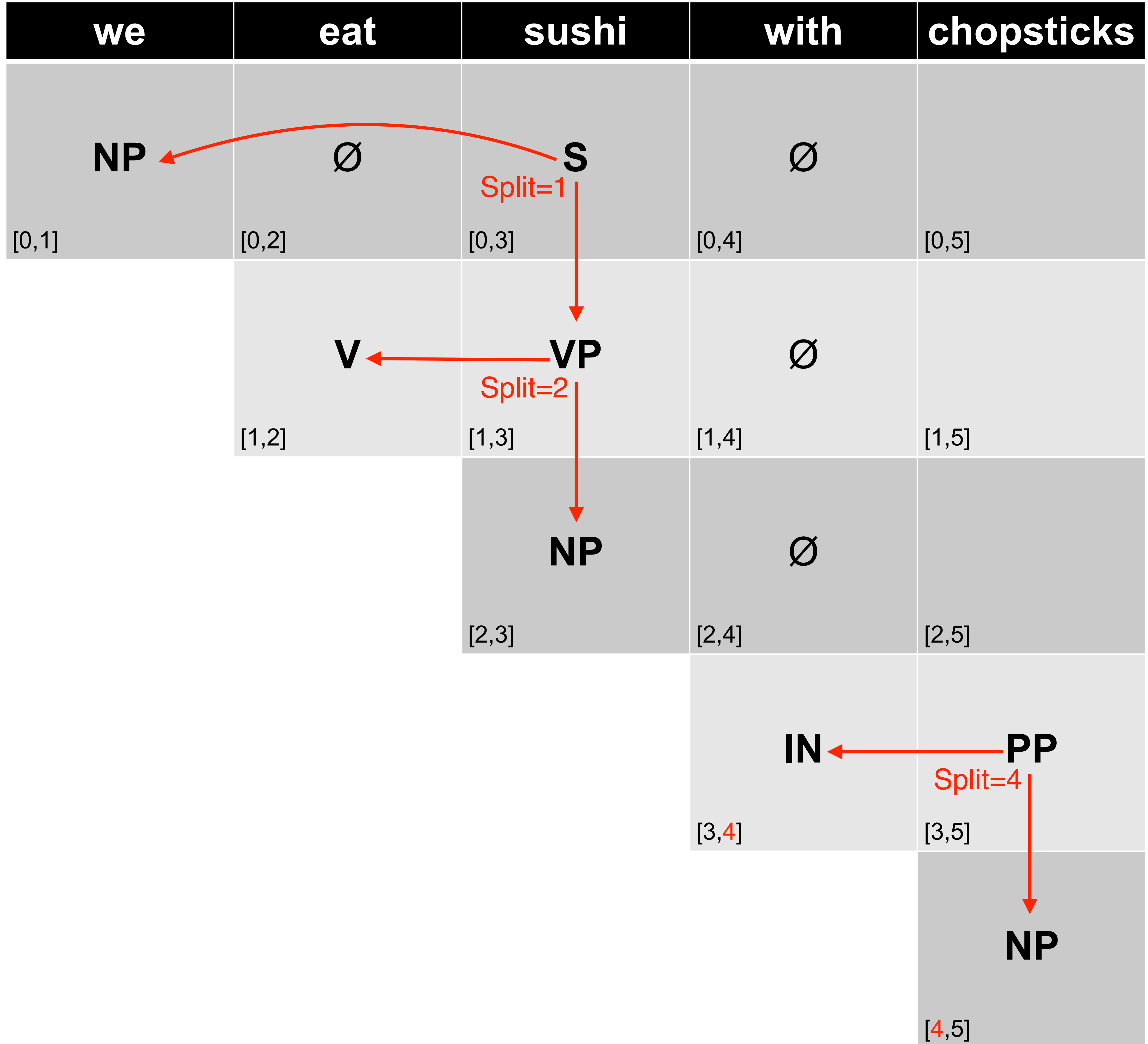
S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat



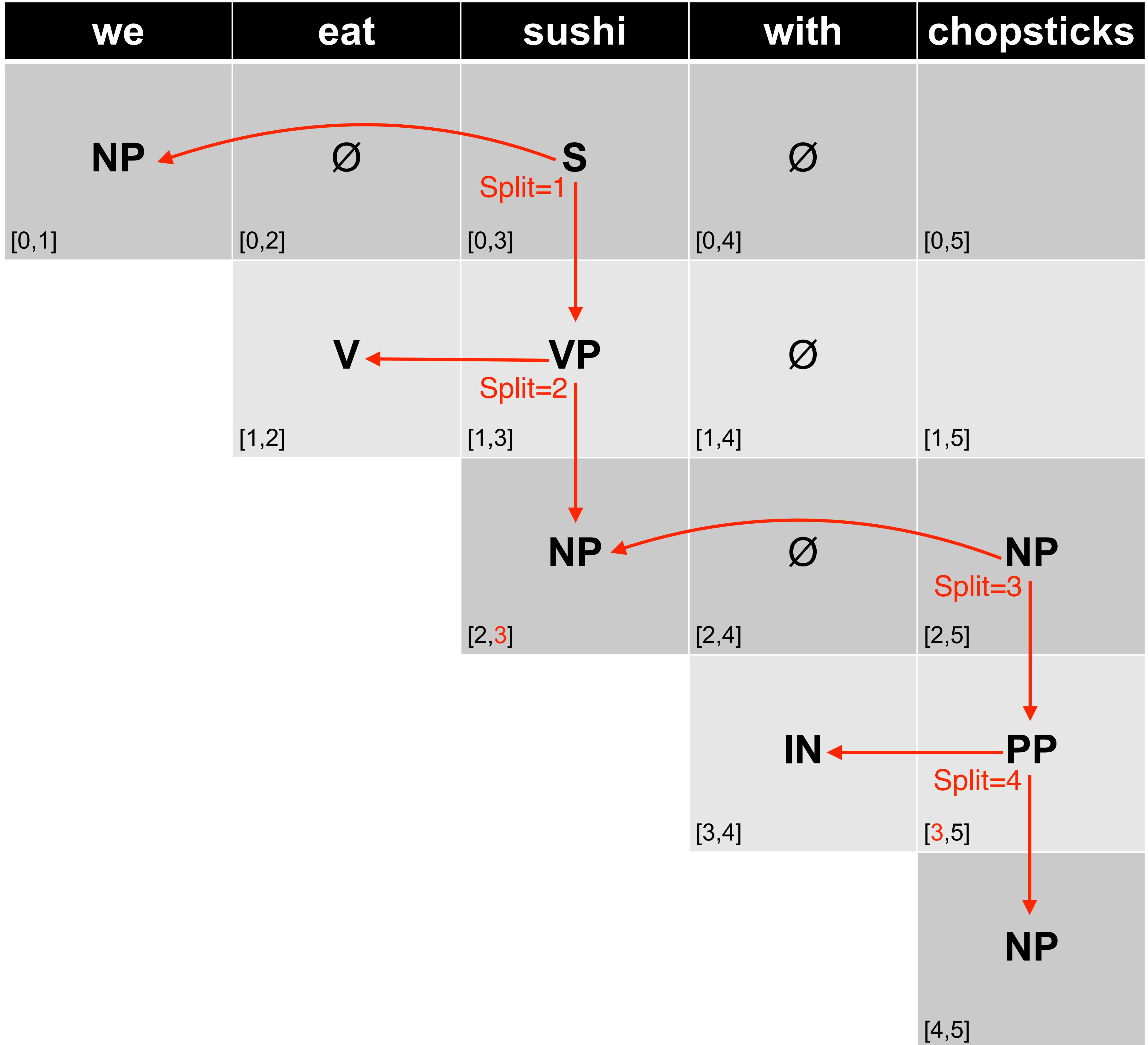
S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat



S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat



S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat



S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

we	eat	sushi	with	chopsticks
NP [0,1]	Ø [0,2]	**S** Split=1 [0,3]	Ø [0,4]	[0,5]
	V [1,2]	**VP** Split=2 [1,3]	Ø [1,4]	**?** [1,5]
		NP [2,3]	Ø [2,4]	**NP** Split=3 [2,5]
			IN [3,4]	**PP** Split=4 [3,5]
				NP [4,5]

S → NP VP
NP → NP PP
PP → IN NP
VP → V NP
VP → VP PP
NP → we
NP → sushi
NP → chopsticks
IN → with
V → eat

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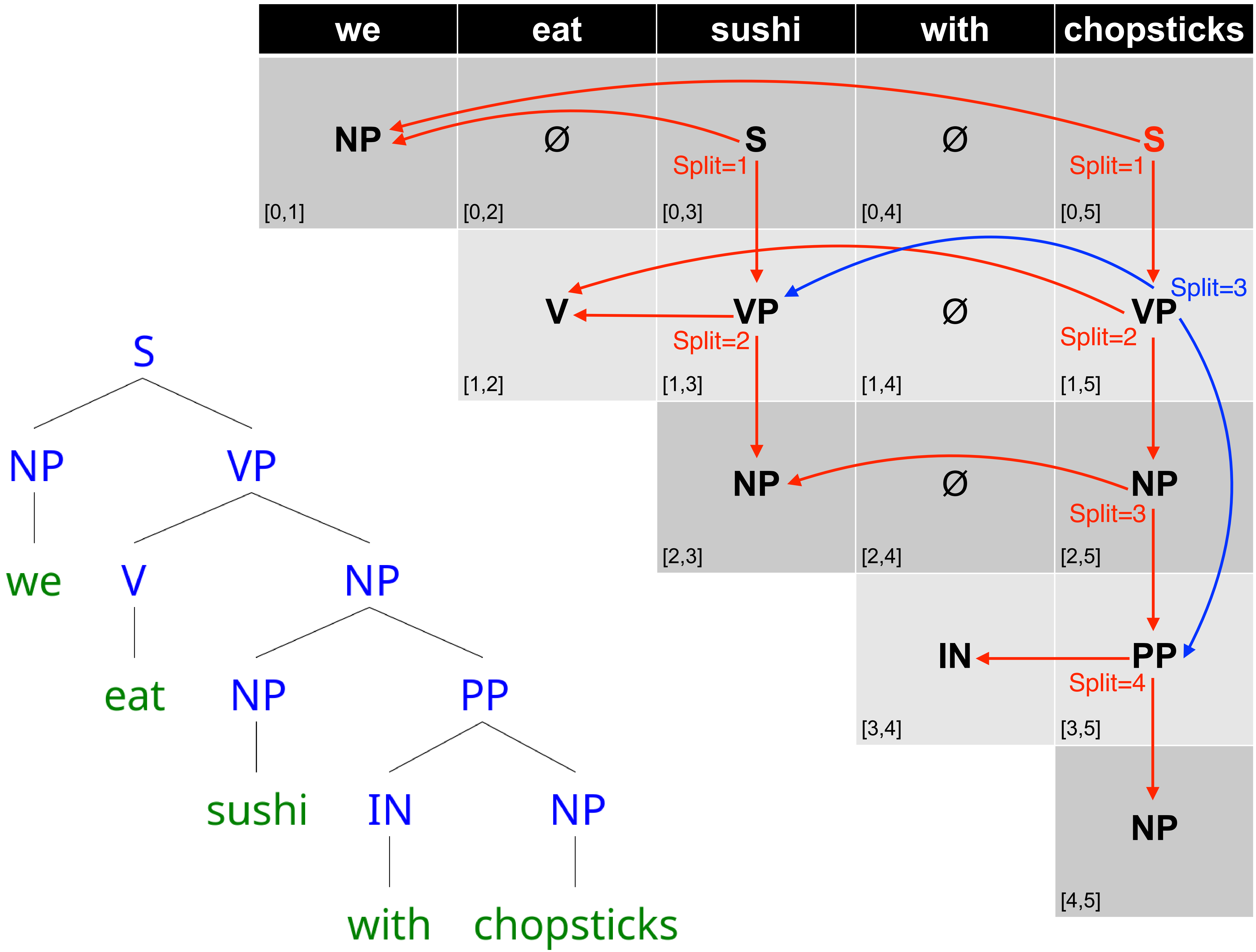
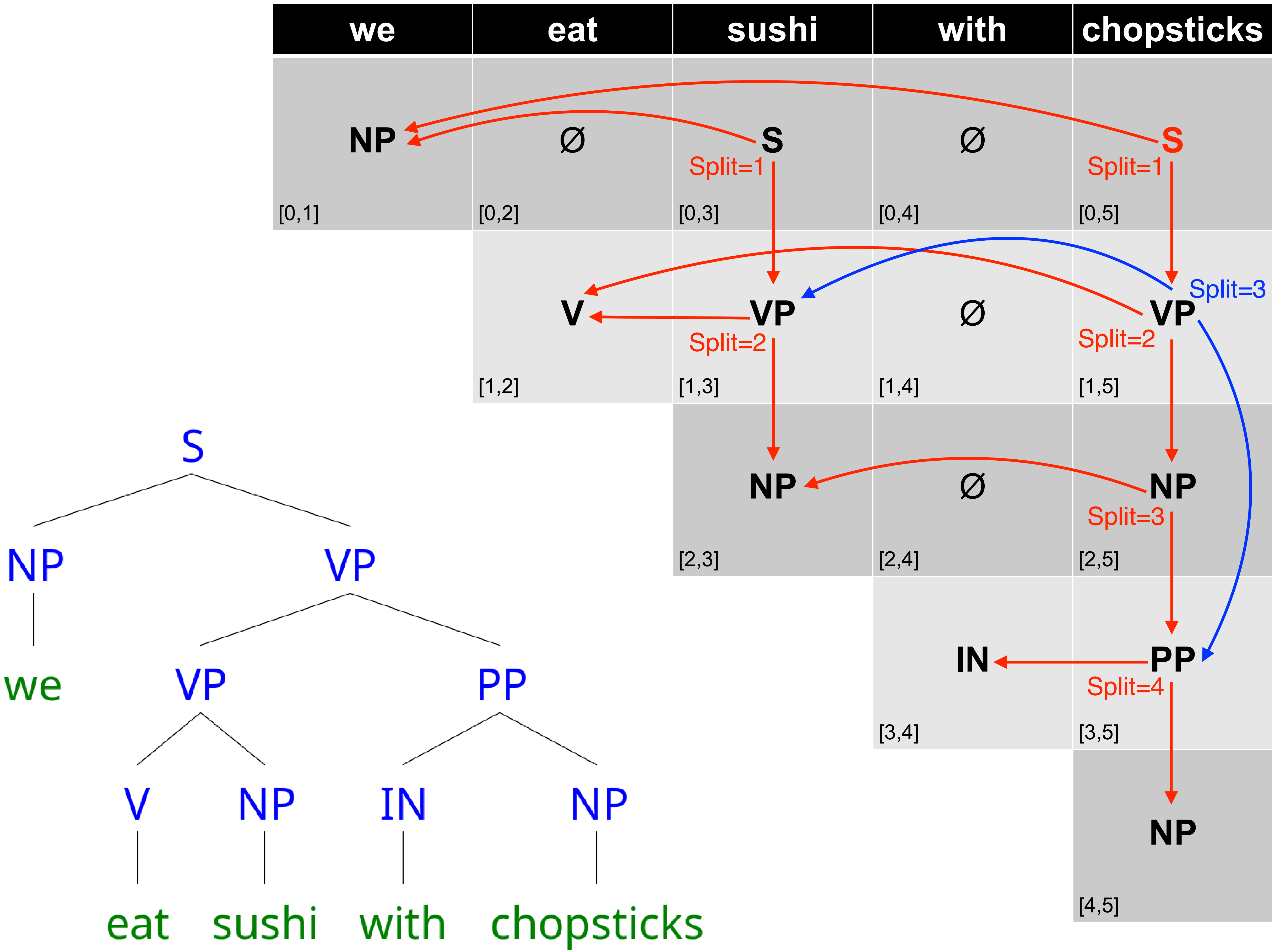
CYK: Retrieving the Parses

- S in the top-right corner of parse table indicates success
- To get parse(s), follow pointers back for each match



we
eat
sushi
with
chopsticks
NP
[0,1]
Ø
[0,2]
S
Split=1
[0,3]
Ø
[0,4]
S
Split=1
[0,5]
V
[1,2]
VP
Split=2
[1,3]
Ø
[1,4]
Split=3
VP
Split=2
[1,5]
NP
[2,3]
Ø
[2,4]
NP
Split=3
[2,5]
IN
[3,4]
PP
Split=4
[3,5]
NP
[4,5]
S
NP
VP
we
V
NP
eat
NP
PP
sushi
IN
NP
with
chopsticks


we
eat
sushi
with
chopsticks
NP
[0,1]
Ø
[0,2]
S
Split=1
[0,3]
Ø
[0,4]
S
Split=1
[0,5]
V
[1,2]
VP
Split=2
[1,3]
Ø
[1,4]
VP
Split=3
Split=2
[1,5]
NP
[2,3]
Ø
[2,4]
NP
Split=3
[2,5]
IN
[3,4]
PP
Split=4
[3,5]
NP
[4,5]
S
NP
VP
we
VP
PP
V
NP
IN
NP
eat
sushi
with
chopsticks

CYK Algorithm

function CKY-PARSE(*words, grammar*) **returns** *table*

for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**
  for all $\{A \mid A \rightarrow words[j] \in grammar\}$
    $table[j-1, j] \leftarrow table[j-1, j] \cup A$
  for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**
    for $k \leftarrow i+1$ **to** $j-1$ **do**
      for all $\{A \mid A \rightarrow BC \in grammar$ **and** $B \in table[i,k]$ **and** $C \in table[k,j]\}$
        $table[i,j] \leftarrow table[i,j] \cup A$

fill the diagonals (NP → we)

bottom to top

going through the ‘splits’

Figure 12.5 The CKY algorithm.

create the links if they are in the production rules

Representing English with CFGs

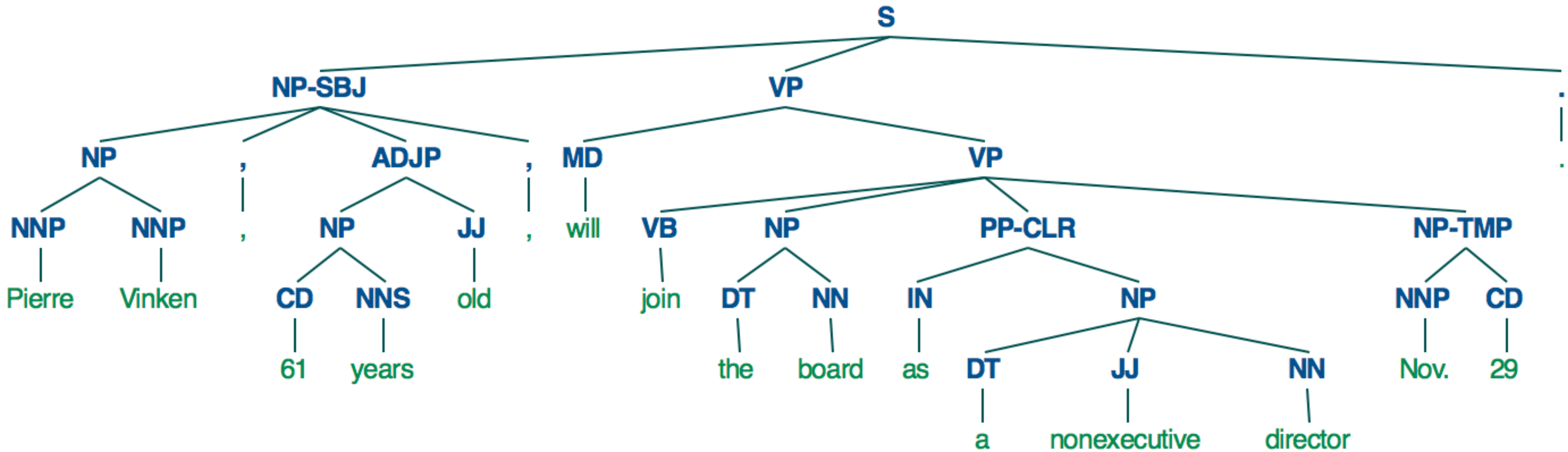
From Toy Grammars to Real Grammars

- Toy grammars with handful of productions good for demonstration or extremely limited domains
- For real texts, we need real grammars
- Many thousands of production rules

Key Constituents in Penn Treebank

- Sentence (S)
- Noun phrase (NP)
- Verb phrase (VP)
- Prepositional phrase (PP)
- Adjective phrase (AdjP)
- Adverbial phrase (AdvP)
- Subordinate clause (SBAR)

Example PTB/0001




```
( (S
    (NP-SBJ
      (NP (NNP Pierre) (NNP Vinken) )
      (, ,)
      (ADJP
        (NP (CD 61) (NNS years) )
        (JJ old) )
      (, ,) )
    (VP (MD will)
      (VP (VB join)
        (NP (DT the) (NN board) )
        (PP-CLR (IN as)
          (NP (DT a) (JJ nonexecutive) (NN director) ))
        (NP-TMP (NNP Nov.) (CD 29) )))
    (. .) ))
```

Basic English Sentence Structures

- Declarative sentences (S → NP VP)
  - *The rat ate the cheese*
- Imperative sentences (S → VP)
  - *Eat the cheese!*
- Yes/no questions (S → VB NP VP)
  - *Did the rat eat the cheese?*
- *Wh*-subject-questions (S → WH VP)
  - *Who ate the cheese?*
- *Wh*-object-questions (S → WH VB NP VP)
  - *What did the rat eat?*

English Noun Phrases

- Pre-modifiers
  - DT, CD, ADJP, NNP, NN
  - E.g. *the two very best Philly cheese steaks*
- Post-modifiers
  - PP, VP, SBAR
  - A delivery from Bob coming today that I don't want to miss

NP → DT? CD? ADJP? (NN|NNP)+ PP* VP? SBAR?

Verb Phrases

- Auxiliaries
  - MD, AdvP, VB, TO
  - E.g *should really have tried to wait*

VP → (MD|VB|TO) AdvP? VP

- Arguments and adjuncts
  - NP, PP, SBAR, VP, AdvP
  - E.g *told him yesterday that I was ready*
  - E.g. *gave John a gift for his birthday to make amends*

VP → VB NP? NP? PP* AdvP* VP? SBAR?

Other Constituents

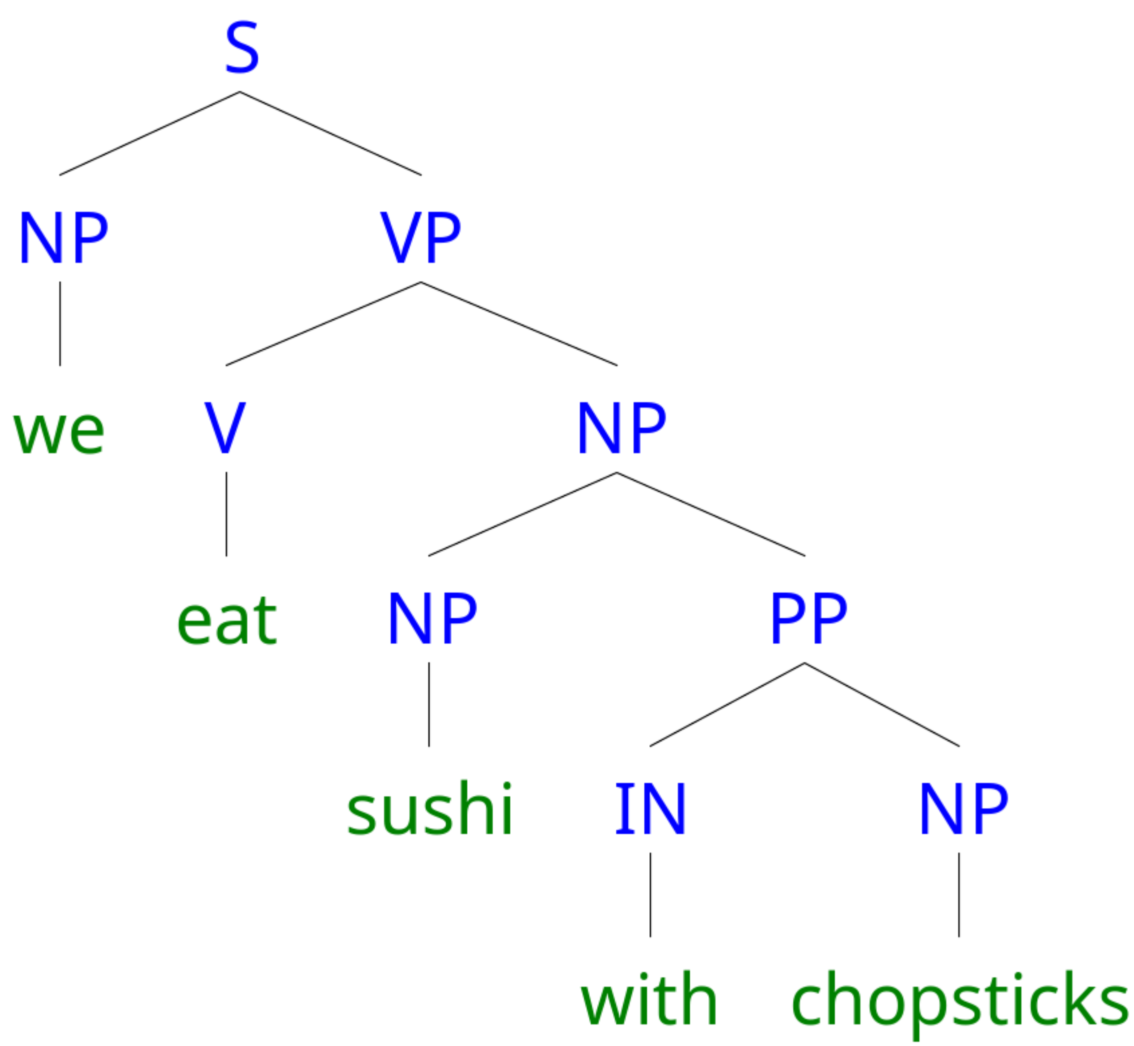
- Prepositional phrase
  - PP → IN NP *in the house*
- Adjective phrase
  - AdjP → (AdvP) JJ *really nice*
- Adverb phrase
  - AdvP → (AdvP) RB *not too well*
- Subordinate clause
  - SBAR → (IN) S *since I came here*
- Coordination
  - NP → NP CC NP; VP → VP CC VP; etc. *Jack and Jill*
- Complex sentences
  - S → S SBAR; S → SBAR S; etc. *if he goes, I'll go*

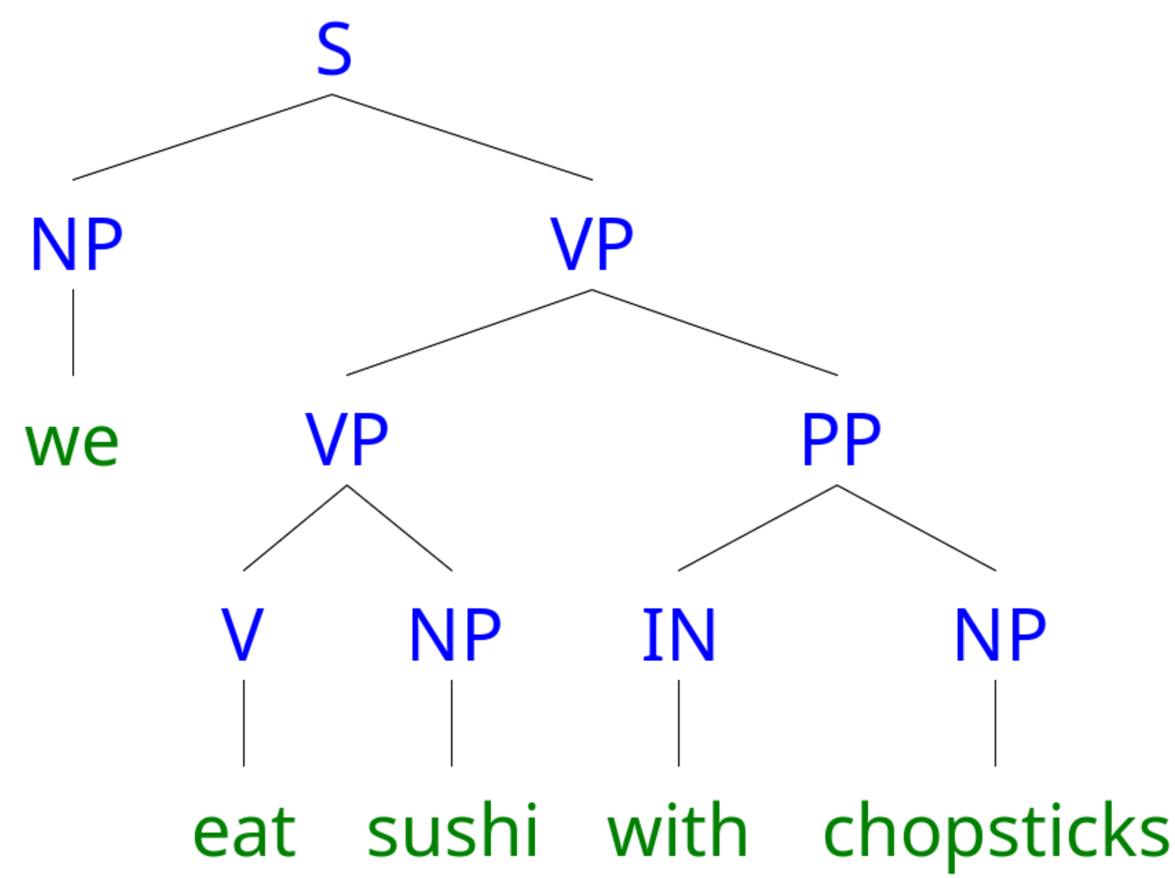
A Final Word

- Context-free grammars can represent most linguistic structures of natural languages
- There are relatively fast dynamic programming algorithms (CYK) to retrieve this structure

Parse Ambiguity

- But what about ambiguity? Often more than one tree can describe a string





Reading

- E18 Ch. 9.2, 10.1