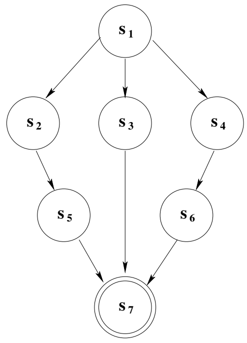
AI Planning for Autonomy

Problem Set II: Heuristic Search

1. Consider the following state space S, where $s_0 = s_1$ and $S_G = \{s_7\}$



where actions changing a state s into another state s' are given by the edges. The cost to transition from state s to s' is given by the following table:

s	s'	$c(s,s')$	s	s'	$c(s,s')$
s_1	s_2	2	s_3	s_7	10
s_1	s_3	2	s_4	s_6	1
s_1	s_4	1	s_5	s_7	3
s_2	s_5	2	s_6	s_7	4

and heuristic estimates for each state:

s	$h_1(s)$	$h_2(s)$	$h_3(s)$
s_1	4	6	6
s_2	3	5	1
s_3	5	10	1
s_4	3	5	5
s_5	2	3	3
s_6	2	4	4
s_7	0	0	0

- Which heuristics are admissible?
  All 3
- Which are consistent?
  h_1 *and* h_2 *are consistent,* h_3 *is not because* $h_3(s_2) < h_3(s_2) + c(s_1, s_2)$
- Does any heuristic *dominate* any other?
  $h_2 = h^*$ *and therefore dominates all other admissible heuristics.* $h_1(s_1) < h_3(s_1)$ *and* $h_3(s_2) < h_1(s_2)$ *therefore neither of* h_1 *and* h_3 *dominate each other*

Describe the execution of one of the following algorithms in this problem using one of the heuristics above. Fill in a table like the one below, showing the contents of the Open and Closed lists at the end of each iteration.

Choose one of: A*, WA* ($w = 5$), or Greedy Best-First Search.

Using A and h_2, labeling each node n_i whose parent is n_p as $n_i = \langle s, g(n_i) + h(s), g(n_i), n_p\rangle$*

	Iteration 1	Iteration 2	Iteration 3	Iteration 4
OPEN	$n_1 = \langle s_1, 6, 0, nil\rangle$ *	$n_2 = \langle s_2, 7, 2, n_1\rangle$ $n_3 = \langle s_3, 12, 2, n_1\rangle$ $n_4 = \langle s_4, 6, 1, n_1\rangle$ *	n_2 n_3 $n_5 = \langle s_6, 6, 2, n_4\rangle$ *	n_2 n_3 $n_6 = \langle s_7, 6, 6, n_5\rangle$ *
CLOSED		n_1	n_1 n_4	n_1 n_4 n_5

- Which is the path returned as a solution?
  $s_1 \to s_4 \to s_6 \to s_7$
- Is this the optimal plan? Has the algorithm proved this?
  Yes, since n_6 is at a goal state and has the lowest admissible cost estimate of any node in the open list in iteration 4, all other paths from any open node must be longer, given h_2 is both admissible and consistent.

2. Consider an $m \times m$ manhattan grid, and a set of coordinates G to visit in any order.
   - Formulate a state-based search problem to find a tour of all the desired points (i.e. define a state space, applicable actions, transition and cost functions).

$$
\begin{aligned}
S &= \{\langle x, y, V\rangle \mid x, y \in \{0, \ldots, m-1\} \wedge V \subseteq G\} \\
A(\langle x, y, V\rangle) &= \{(dx, dy) \mid \ dx, dy \in \{-1, 0, 1\} \\
&\quad \wedge |dx| + |dy| = 1 \\
&\quad \wedge x + dx \in \{0, \ldots, m-1\} \\
&\quad \wedge y + dy \in \{0, \ldots, m-1\}\} \\
t((dx, dy), \langle x, y, V\rangle) &= \langle x + dx, y + dy, V \setminus \{(x + dx, y + dy)\}\rangle \\
c(a, s) &= 1
\end{aligned}
$$

   - What is the branching factor of the search?
     approx. 4
   - What is the size of the state space in terms of m and G.
     approx. $m^2 \cdot 2^{|G|}$
   - Define an admissible heuristic function.

$$h(\langle x, y, V\rangle) = \max_{(x_g, y_g) \in V} (|x - x_g| + |y - y_g|)$$