Probabilistic Context-Free Grammar

COMP90042

Natural Language Processing

Lecture 15

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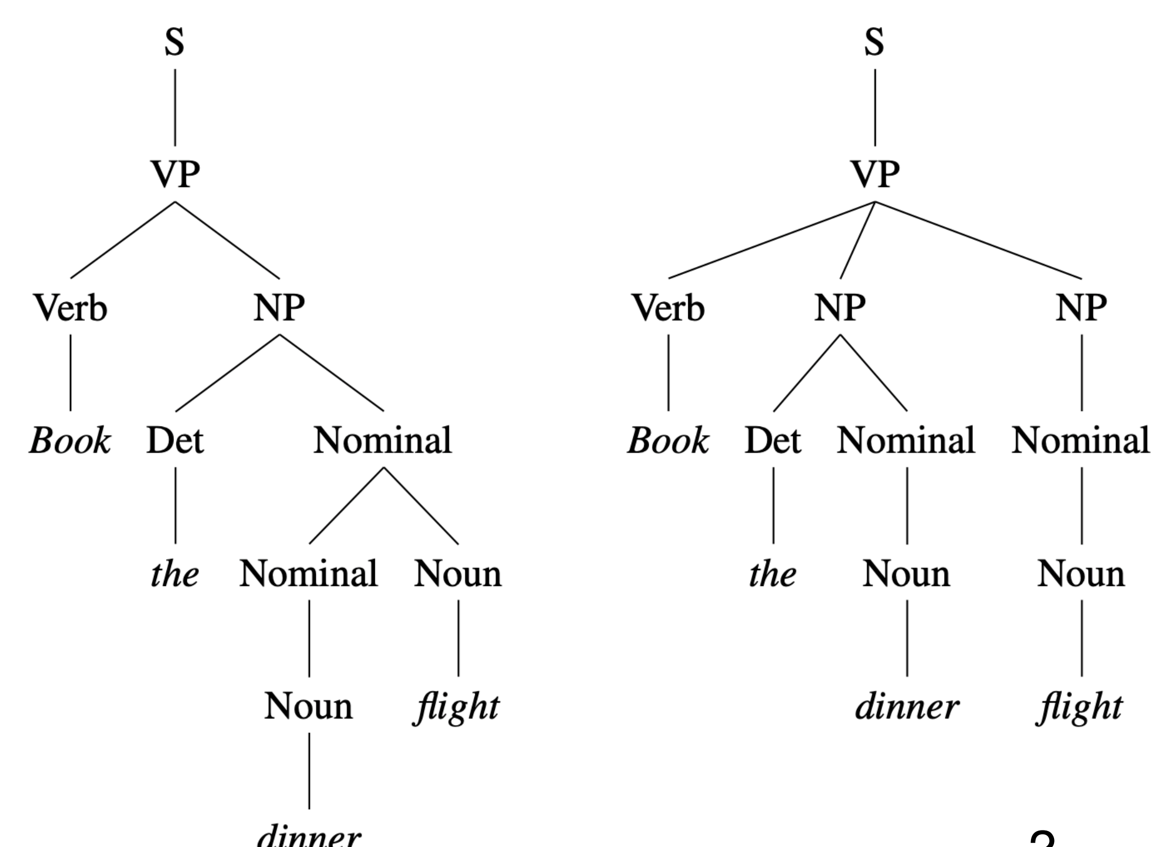
Jey Han Lau

THE UNIVERSITY OF MELBOURNE

Ambiguity In Parsing

- Context-free grammars assign hierarchical structure to language
  - Formulated as generating all strings in the language
  - Predicting the structure(s) for a given string
- Raises problem of ambiguity — which is better?
  - Probabilistic CFG!



Outline

- Basics of Probabilistic CFGs (PCFGs)
- PCFG parsing
- Limitations of CFG

Basics of PCFGs

Basics of PCFGs

- Same symbol set:
  - Terminals: words such as *book*
  - Non-terminal: syntactic labels such as NP or NN
- Same productions (rules)
  - LHS non-terminal → ordered list of RHS symbols
- In addition, store a **probability** with each production
  - NP → DT NN [p = 0.45]
  - NN → cat [p = 0.02]
  - NN → leprechaun [p = 0.00001]
  - …

Basics of PCFGs

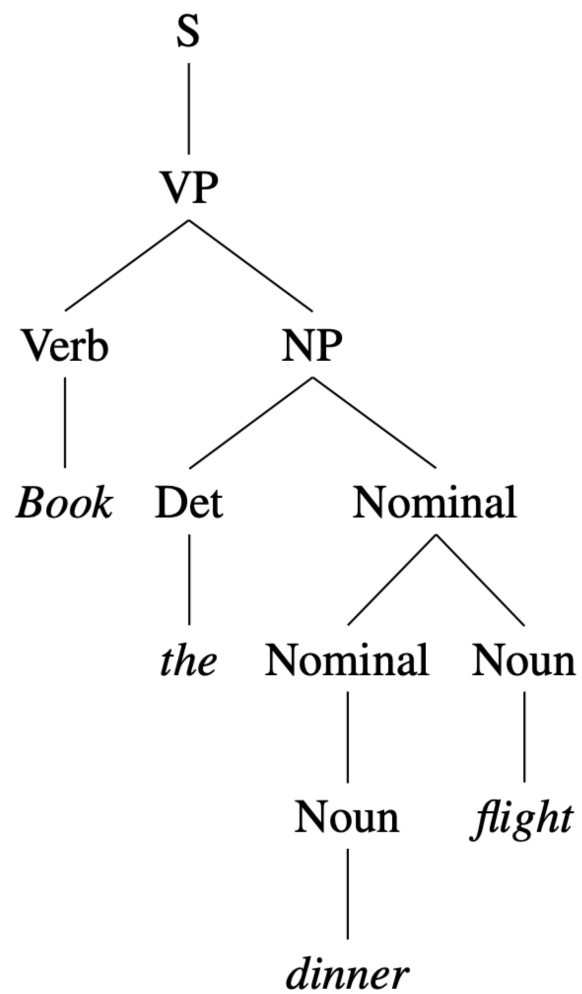
- Probability values denote **conditional**
  - P(LHS → RHS)
  - P(RHS | LHS)
- Consequently they:
  - must be positive values, between 0 and 1
  - must sum to one for given LHS
- E.g.,
  - NN → aadvark [p = 0.0003]
  - NN → cat [p = 0.02]
  - NN → leprechaun [p = 0.0001]
  - $\sum_{x} P(\text{NN} \rightarrow x) = 1$

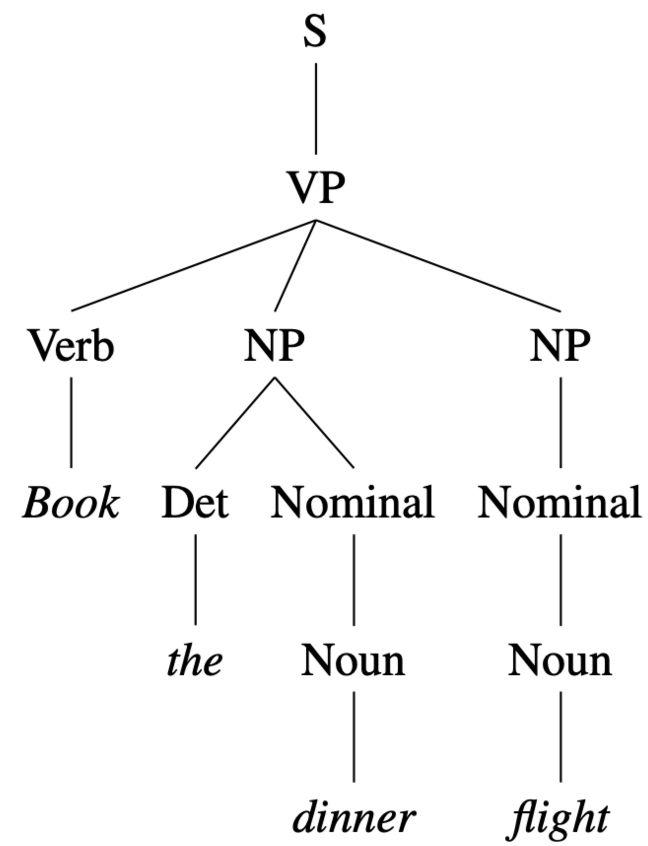
Stochastic Generation with PCFGs

Almost the same as for CFG, with one twist:

1. Start with S, the sentence symbol
2. Choose a rule with S as the LHS
   - **Randomly select a RHS** according to P(RHS | LHS) e.g., S → VP
   - Apply this rule, e.g., substitute VP for S
3. Repeat step 2 for each non-terminal in the string (here, VP)
4. Stop when no non-terminals remain

Gives us a tree, as before, with a sentence as the yield





Rules			P	Rules			P
S	→	VP	.05	S	→	VP	.05
VP	→	Verb NP	.20	VP	→	Verb NP NP	.10
NP	→	Det Nominal	.20	NP	→	Det Nominal	.20
Nominal	→	Nominal Noun	.20	NP	→	Nominal	.15
Nominal	→	Noun	.75	Nominal	→	Noun	.75
				Nominal	→	Noun	.75
Verb	→	book	.30	Verb	→	book	.30
Det	→	the	.60	Det	→	the	.60
Noun	→	dinner	.10	Noun	→	dinner	.10
Noun	→	flight	.40	Noun	→	flight	.40

How Likely Is a Tree?

- Given a tree, we can compute its probability
  - Decomposes into probability of each production
- P(tree) =

  P(S → VP) ×

  P(VP → Verb NP) ×

  P(Verb → *Book*) ×

  P(NP → Det Nominal) ×

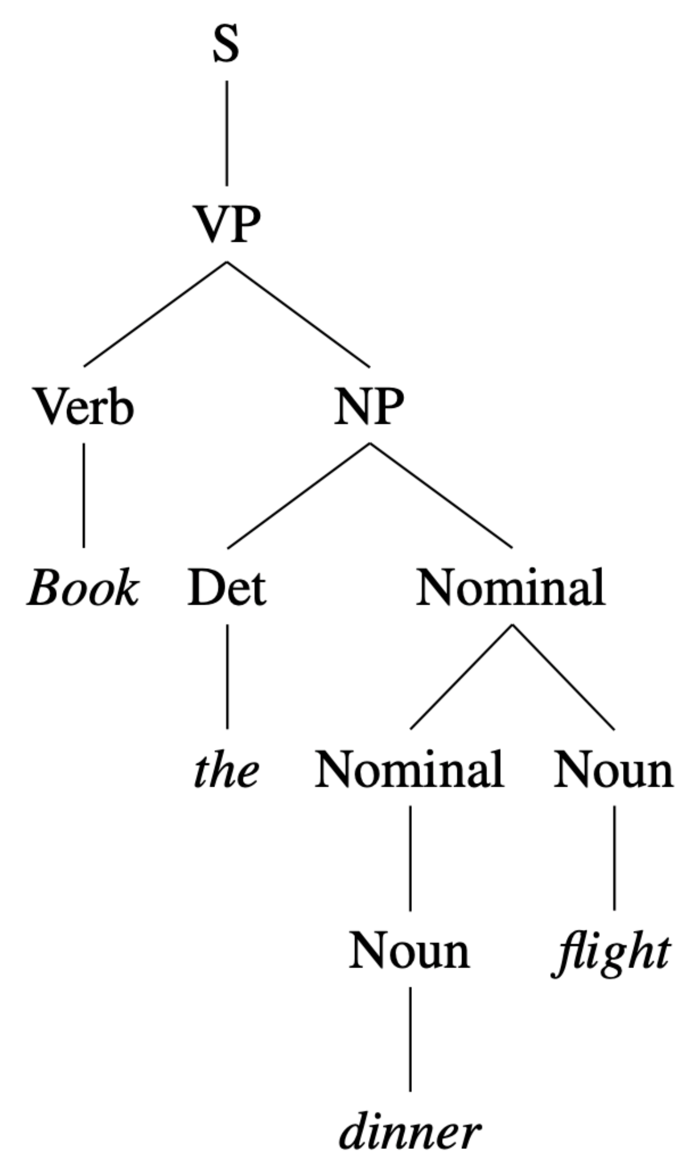
  P(Det → *the*) ×

  P(Nominal → Nominal Noun) ×

  P(Nominal → Noun) ×

  P(Noun → *dinner*) ×

  P(Noun → *flight*)



How Likely Is a Tree?

P(tree)

= P(S → VP) × P(VP → Verb NP) × P(Verb → Book) × P(NP → Det Nominal) × P(Det → the) × P(Nominal → Nominal Noun) × P(Nominal → Noun) × P(Noun → dinner) × P(Noun → flight)

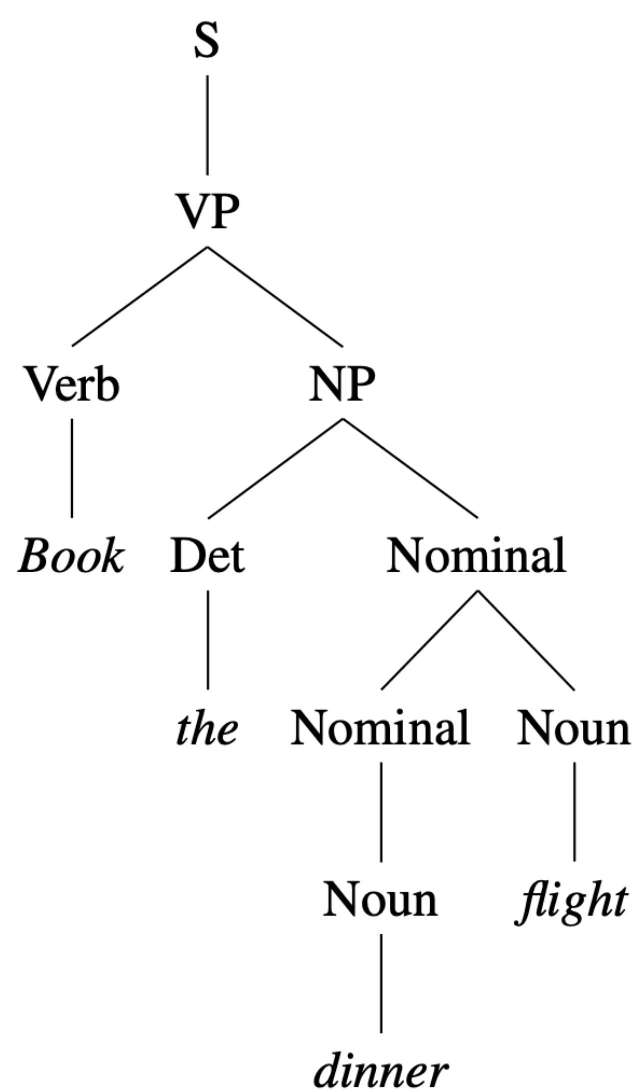
= 0.05 × 0.20 × 0.30 × 0.20 × 0.60 × 0.20 × 0.75 × 0.10 × 0.40

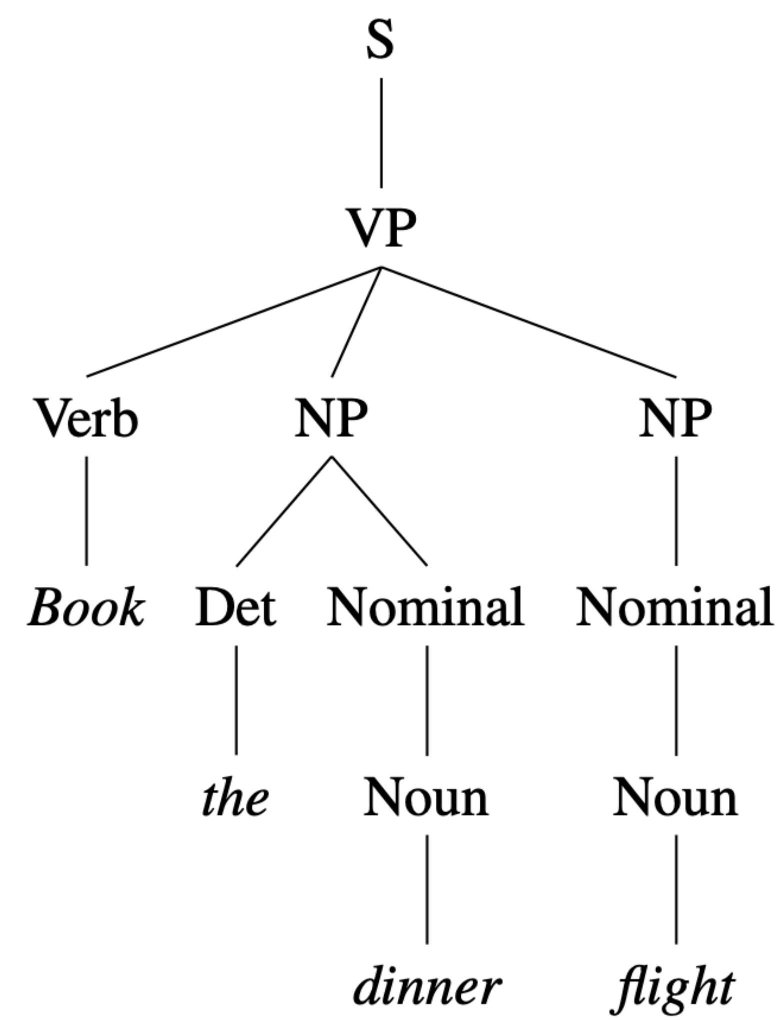
= 2.2 × 10-6

Rules			P
S	→	VP	.05
VP	→	Verb NP	.20
NP	→	Det Nominal	.20
Nominal	→	Nominal Noun	.20
Nominal	→	Noun	.75
Verb	→	book	.30
Det	→	the	.60
Noun	→	dinner	.10
Noun	→	flight	.40

Resolving Parse Ambiguity

- Can select between different trees based on P(T)
- $P(T_{left}) = 2.2 \times 10^{-6}$ $P(T_{right}) = 6.1 \times 10^{-7}$





PCFG Parsing

Parsing PCFGs

- Before we looked at
  - CYK
  - for unweighted grammars (CFGs)
  - finds **all possible trees**
- But there are often 1000s, many completely nonsensical
- Can we solve for the **most probable tree**?

CYK for PCFGs

- CYK finds **all trees** for a sentence; we want **best** tree
- Prob. CYK follows similar process to standard CYK
- Convert grammar to Chomsky Normal Form (CNF)
  - VP → Verb NP NP [0.10]
  - VP → Verb NP+NP [0.10]
    NP+NP → NP NP [1.0]
  - where NP+NP is a new symbol.

we	eat	sushi	with	chopsticks
[0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	[1,2]	[1,3]	[1,4]	[1,5]
		[2,3]	[2,4]	[2,5]
			[3,4]	[3,5]
				[4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	[0,2]	[0,3]	[0,4]	[0,5]
	V 1 [1,2]	[1,3]	[1,4]	[1,5]
		NP 1/8 [2,3]	[2,4]	[2,5]
			IN 1 [3,4]	[3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	[0,3]	[0,4]	[0,5]
	V 1 [1,2]	[1,3]	[1,4]	[1,5]
		NP 1/8 [2,3]	[2,4]	[2,5]
			IN 1 [3,4]	[3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	[0,3]	[0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 (1/2 * 1 * 1/8) [1,3]	[1,4]	[1,5]
		NP 1/8 [2,3]	[2,4]	[2,5]
			IN 1 [3,4]	[3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	**S** 1/64 (1 * 1/4 * 1/16) [0,3]	[0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 [1,3]	[1,4]	[1,5]
		NP 1/8 [2,3]	[2,4]	[2,5]
			IN 1 [3,4]	[3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	**S** 1/64 [0,3]	Ø [0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 [1,3]	Ø [1,4]	[1,5]
		NP 1/8 [2,3]	Ø [2,4]	[2,5]
			IN 1 [3,4]	[3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	**S** 1/64 [0,3]	Ø [0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 [1,3]	Ø [1,4]	[1,5]
		NP 1/8 [2,3]	Ø [2,4]	[2,5]
			IN 1 [3,4]	**PP** 1/2 (1 * 1 * 1/2) [3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	**S** 1/64 [0,3]	Ø [0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 [1,3]	Ø [1,4]	[1,5]
		NP 1/8 [2,3]	Ø [2,4]	**NP** 1/128 (1/8 * 1/8 * 1/2) [2,5]
			IN 1 [3,4]	**PP** 1/2 [3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

we	eat	sushi	with	chopsticks
NP 1/4 [0,1]	Ø [0,2]	**S** 1/64 [0,3]	Ø [0,4]	[0,5]
	V 1 [1,2]	**VP** 1/16 [1,3]	Ø [1,4]	**?** [1,5]
		NP 1/8 [2,3]	Ø [2,4]	**NP** 1/128 [2,5]
			IN 1 [3,4]	**PP** 1/2 [3,5]
				NP 1/2 [4,5]

S	→	NP VP	1
NP	→	NP PP	⅛
	→	we	¼
	→	sushi	⅛
	→	chopsticks	½
PP	→	IN NP	1
IN	→	with	1
VP	→	V NP	½
	→	VP PP	¼
	→	MD V	¼
V	→	eat	1

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Prob CYK: Retrieving the Parses

- S in the top-right corner of parse table indicates success
- Retain back-pointer to best analysis
- To get parse(s), follow pointers back for each match
- Convert back from CNF by removing new non-terminals

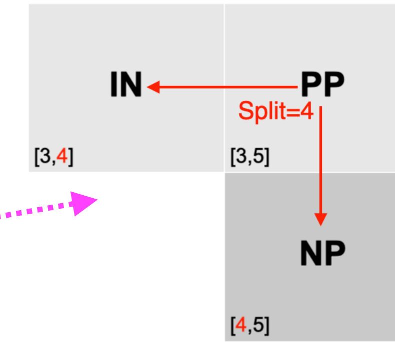
Prob. CYK

function PROBABILISTIC-CKY(*words,grammar*) **returns** most probable parse and its probability
  for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**
    for all $\{ A \mid A \rightarrow words[j] \in grammar \}$
      $table[j-1, j, A] \leftarrow P(A \rightarrow words[j])$
    for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**
      for $k \leftarrow i+1$ **to** $j-1$ **do**
        for all $\{ A \mid A \rightarrow BC \in grammar,$
          and $table[i,k,B] > 0$ **and** $table[k,j,C] > 0 \}$
          if $(table[i,j,A] < P(A \rightarrow BC) \times table[i,k,B] \times table[k,j,C])$ **then**
            $table[i,j,A] \leftarrow P(A \rightarrow BC) \times table[i,k,B] \times table[k,j,C]$
            $back[i,j,A] \leftarrow \{k, B, C\}$
  return BUILD_TREE(*back*[1, LENGTH(*words*), *S*]), *table*[1, LENGTH(*words*), *S*]

Source: JM3 Ch 14

function CKY-PARSE(*words, grammar*) **returns** *table*

for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**
  for all $\{A \mid A \rightarrow words[j] \in grammar\}$
    $table[j-1, j] \leftarrow table[j-1, j] \cup A$
  for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**
    for $k \leftarrow i+1$ **to** $j-1$ **do**
      for all $\{A \mid A \rightarrow BC \in grammar$ **and** $B \in table[i,k]$ **and** $C \in table[k,j]\}$
        $table[i,j] \leftarrow table[i,j] \cup A$



CYK

Both CYK and prob. CYK store all possible NTs

validity test now looks to see that the child chart cells have non-zero probability

function PROBABILISTIC-CKY(*words,grammar*) **returns** most probable parse and its probability

for $j \leftarrow$ **from** 1 **to** LENGTH(*words*) **do**
  for all $\{A \mid A \rightarrow words[j] \in grammar\}$
    $table[j-1, j, A] \leftarrow P(A \rightarrow words[j])$
  for $i \leftarrow$ **from** $j-2$ **downto** 0 **do**
    for $k \leftarrow i+1$ **to** $j-1$ **do**
      for all $\{A \mid A \rightarrow BC \in grammar,$
        and $table[i,k,B] > 0$ **and** $table[k,j,C] > 0\}$
          if $(table[i,j,A] < P(A \rightarrow BC) \times table[i,k,B] \times table[k,j,C])$ **then**
            $table[i,j,A] \leftarrow P(A \rightarrow BC) \times table[i,k,B] \times table[k,j,C]$
            $back[i,j,A] \leftarrow \{k,B,C\}$
return BUILD_TREE($back$[1, LENGTH(*words*), S]), $table$[1, LENGTH(*words*), S]

Instead of storing set of symbols, store the probability of best scoring tree fragment covering span [i,j] with root symbol A

Overwrite lower scoring analysis if this one is better, and record the best production

Probabilistic CYK

Time complexity in terms of sentence length n?

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Limitations of CFG

CFG Problem 1:
Poor Independence Assumptions

- Rewrite decisions made independently, whereas inter-dependence is often needed to capture global structure.
  - NP → DT NN [0.28]
  - NP → PRP [0.25]
  - Probability of a rule independent of rest of tree
  - No way to represent this contextual differences in PCFG probabilities

	Pronoun	Non-Pronoun
Subject	91%	9%
Object	34%	66%

NP statistics in the Switchboard corpus

Poor Independence Assumptions

	Pronoun	Non-Pronoun
Subject	91%	9%
Object	34%	66%

NP statistics in the Switchboard corpus

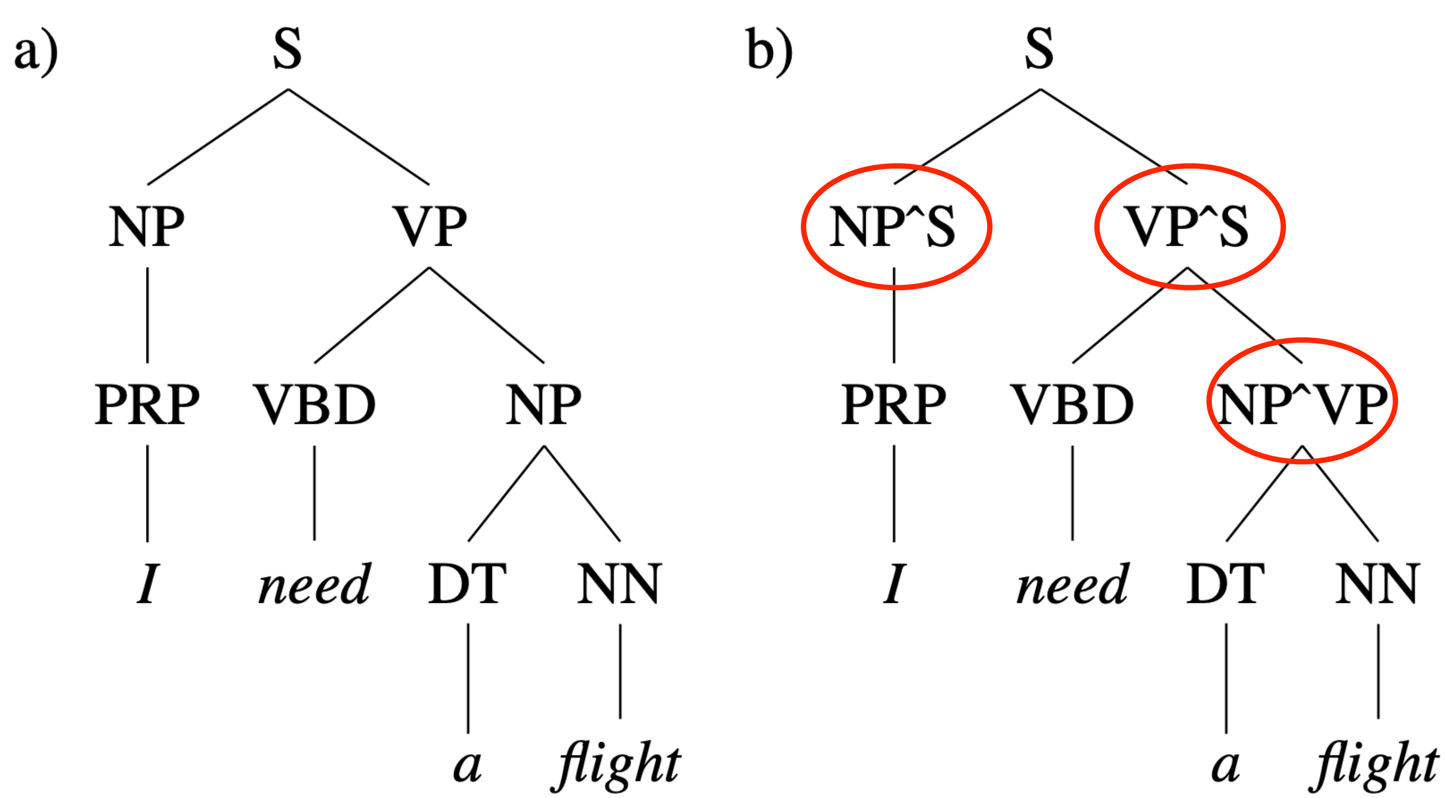
$$NP \rightarrow DT\ NN \quad .28$$
$$NP \rightarrow PRP \quad .25$$

PCFG probabilities based on Switchboard corpus

- NP → PRP should go up to 0.91 as a subject
- NP → DT NN should be 0.66 as an object
- Solution: add a condition to denote whether NP is a subject or object

Solution: Parent Conditioning

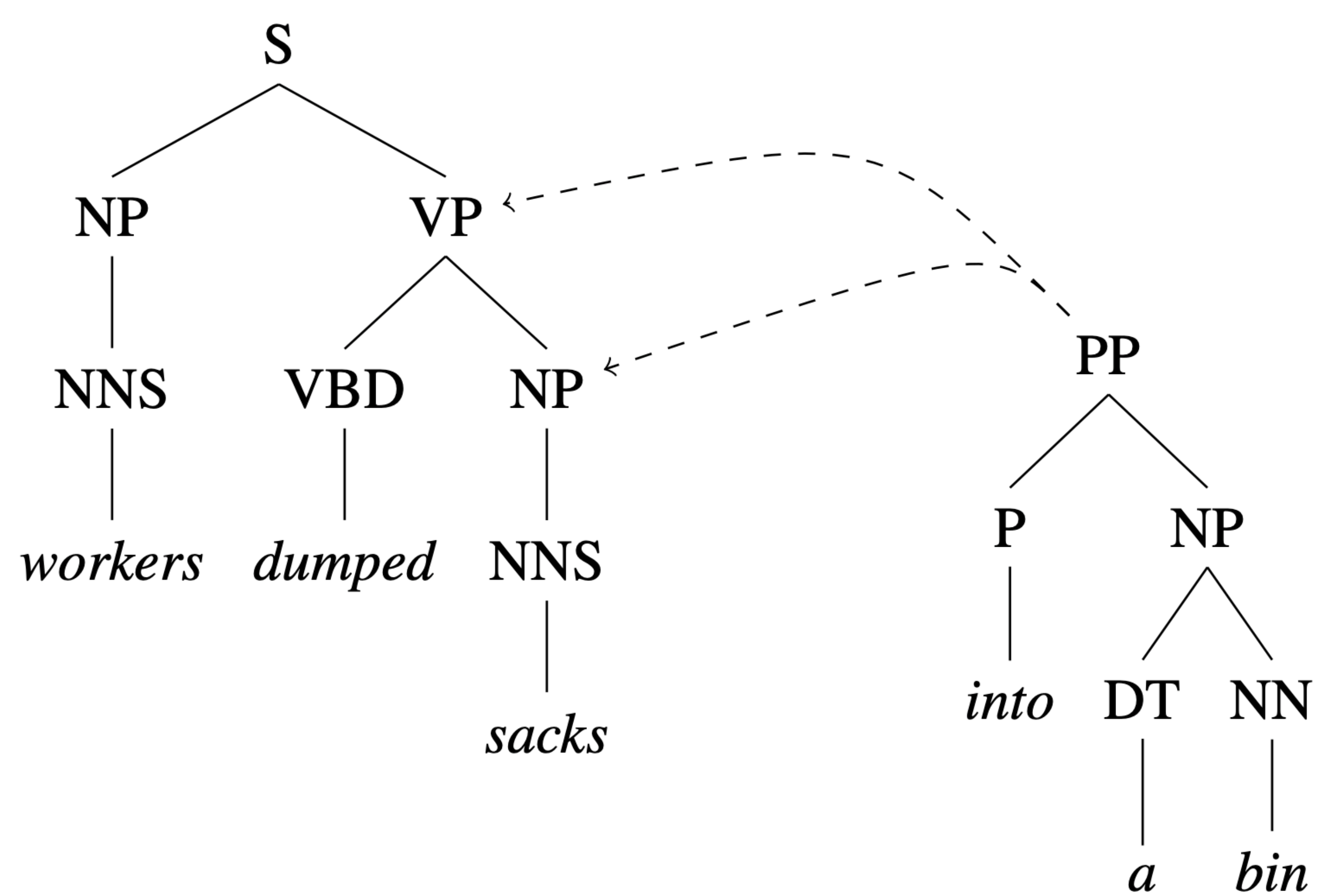
- Make non-terminals more explicit by incorporating parent symbol into each symbol



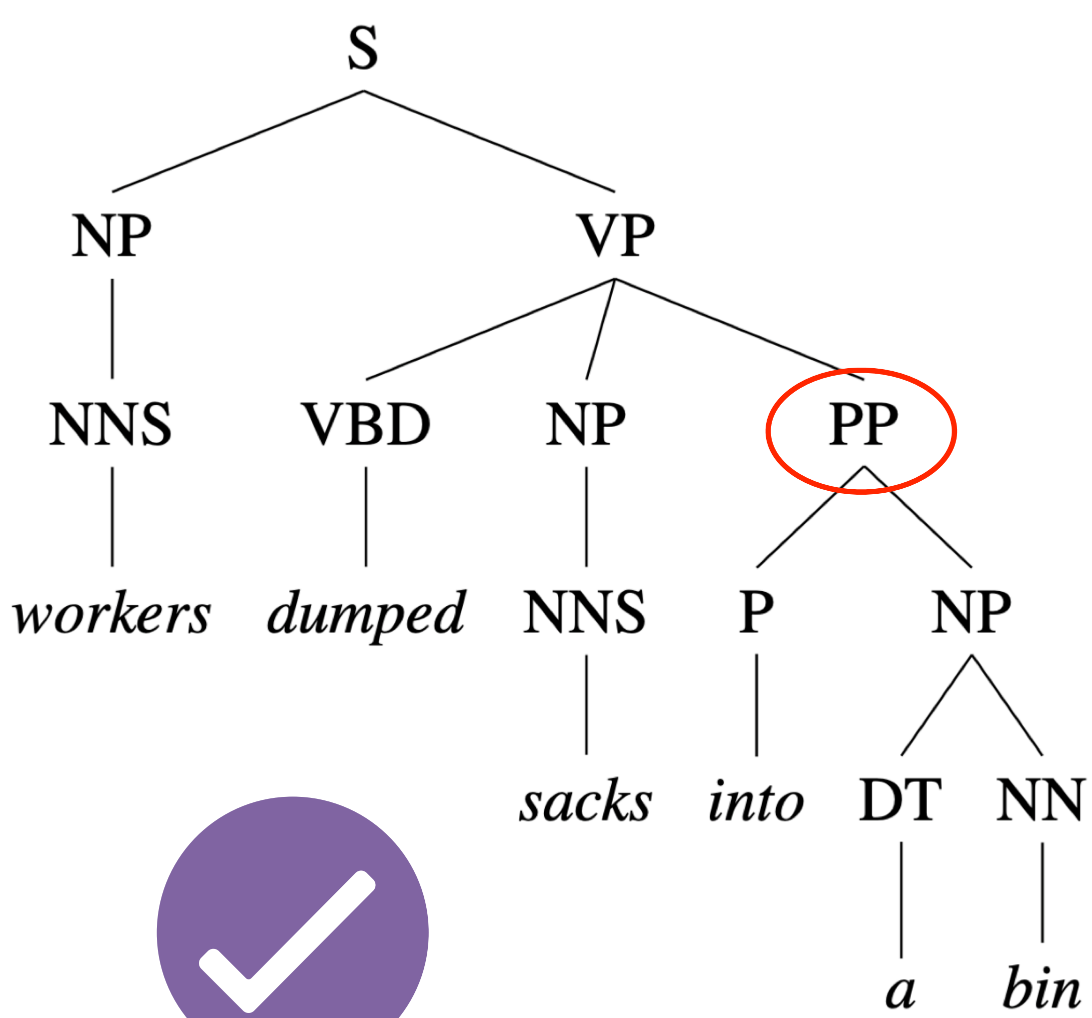
- NP^S represents subject position (left)
- NP^VP denotes object position (right)

CFG Problem 2: **Lack of Lexical Conditioning**

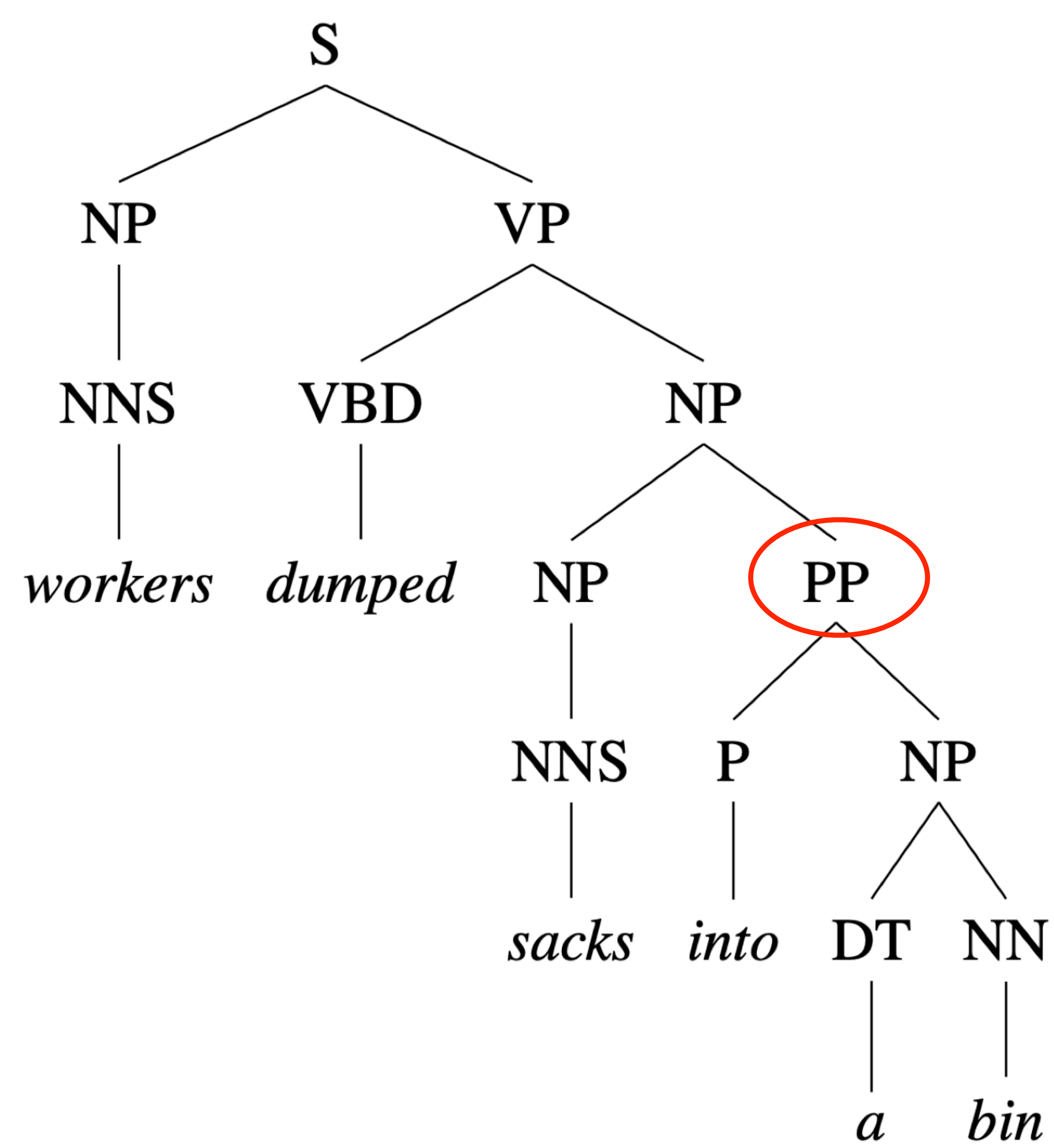
- Lack of sensitivity to words in tree
- Prepositional phrase (PP) attachment ambiguity
  - *Worker dumped sacks into a bin*



PP Attachment Ambiguity



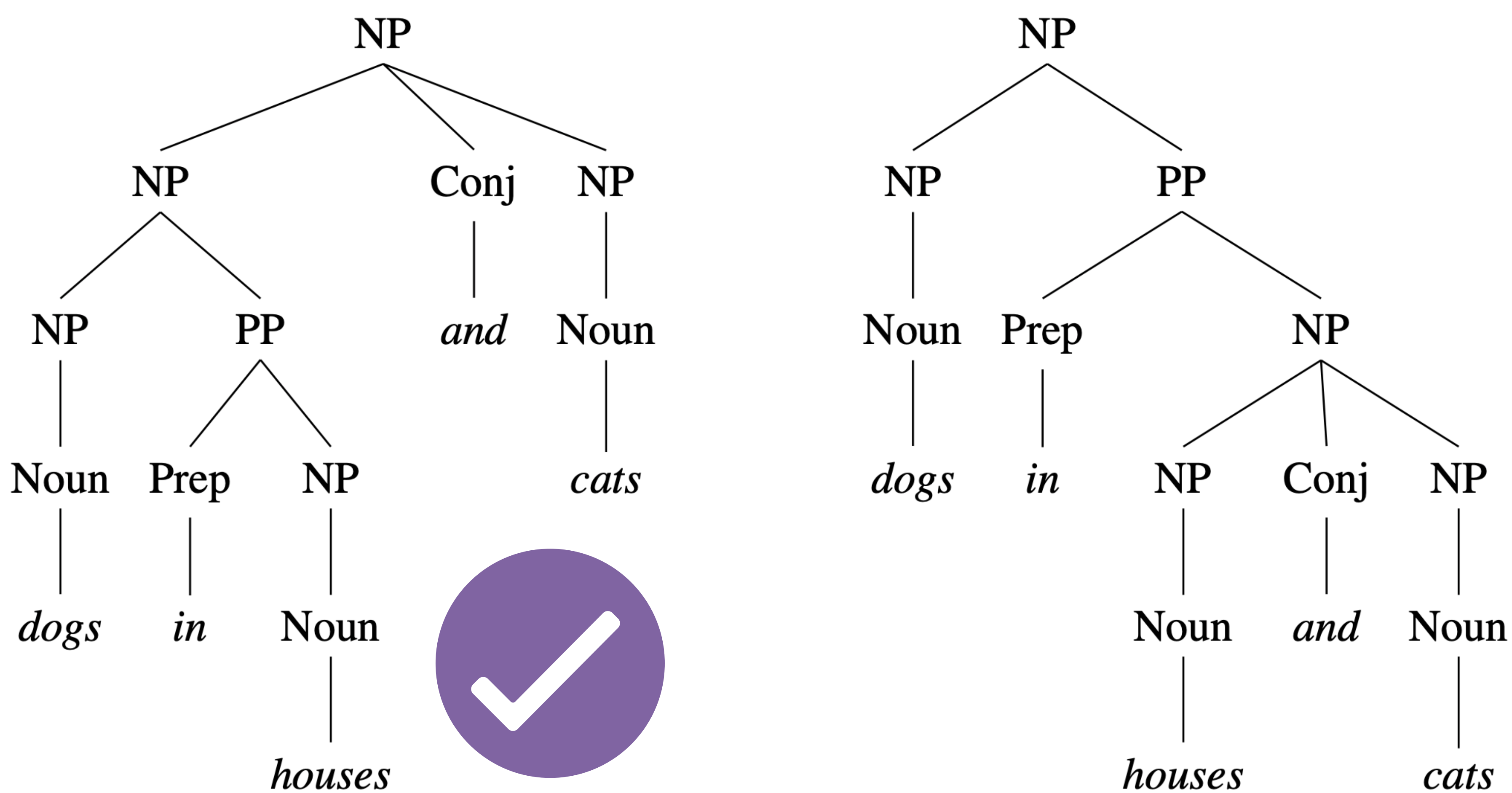
“into a bin” describes the resulting location of the sacks



sacks to be dumped are the ones which are “into a bin”

Coordination Ambiguity

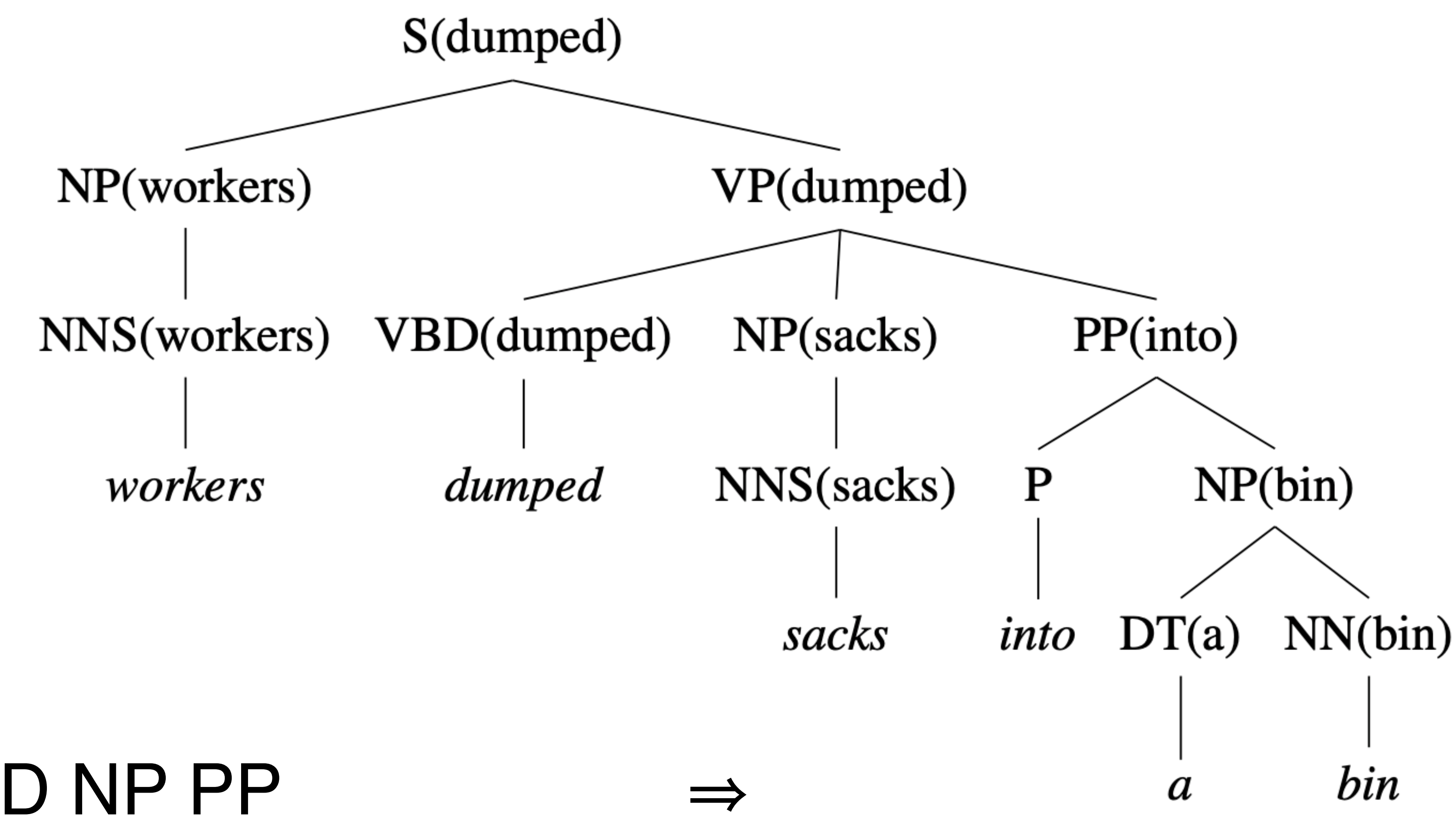
- *dogs in houses and cats*



- *dogs* is semantically a better conjunct for *cats* than *houses* (dogs can't fit into cats!)

Solution: Head Lexicalisation

- Record head word with parent symbols
  - the most salient child of a constituent, usually the noun in a NP, verb in a VP etc



  - VP → VBD NP PP ⇒

VP(dumped) → VBD(dumped) NP(sacks) PP(into)

Head Lexicalisation

- Incorporate head words into productions, to capture the most important links between words
  - Captures correlations between head words of phrases
  - PP(into): VP(dumped) vs. NP(sacks)
- Grammar symbol inventory expands massively!
  - Many of the productions too specific, rarely seen
  - Learning more involved to avoid sparsity problems (e.g., zero probabilities)

A Final Word

- PCFGs widely used, and there are efficient parsers available.
  - Collins parser, Berkeley parser, Stanford parser
  - All use some form of lexicalisation
- But there are other grammar formalisms
  - Lexical function grammar
  - Head-driven phrase structure grammar
  - Next lecture: dependency grammar!

Required Reading

- JM3 Ch. 13-13.2