AI Planning for Autonomy

5. Delete Relaxation Heuristics

It's a Long Way to the Goal, But How Long Exactly?
Part I: *Acting As If the World Can Only Get Better*

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Agenda

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Motivation: For Planning Systems

Remember?

Imagine . . .

You are a planning system.

Somebody: *Hey you, here's that PDDL input. Solve it.*

You (thinking): *Hm, the only method I have is heuristic search.*

You (thinking): *Hm, I need a heuristic function.*
[Note: You programmer is presently in Honolulu.]

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You: I already tried goal counting, and it performed badly. *What now?!?*

Motivation

$\rightarrow$ Delete relaxation is a method to relax planning tasks, and thus automatically compute heuristic functions h.

$\rightarrow$ Every h yields good performance only in some domains! (Search reduction vs. computational overhead)
$\rightarrow$ We must come up with as many alternative methods as possible!

We cover the 4 different methods currently known:

- Critical path heuristics:
- Delete relaxation. Soon to be Done.
- Abstractions.
- Landmarks.

$\rightarrow$ Delete relaxation is very wide-spread, and highly successful for satisficing planning!

We introduce the method in STRIPS.

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Reminder: Relaxing the World by Ignoring Delete Lists

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Real world: (before)

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The Delete Relaxation

Definition (Delete Relaxation).

(i) *For a STRIPS action a, by a^+ we denote the corresponding delete relaxed action, or short relaxed action, defined by $pre_{a^+} := pre_a$, $add_{a^+} := add_a$, and $del_{a^+} :=$*

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(iii) *For a STRIPS planning task $\Pi = (F, A, c, I, G)$, by $\Pi^+ := (F, A^+, c, I, G)$ we denote the corresponding (delete) relaxed planning task.*

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Definition (Relaxed Plan). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, and let s be a state. An (optimal) relaxed plan for s is an (optimal) plan for Π_s^+. A relaxed plan for I is also called a relaxed plan for Π.*

$\rightarrow$ Anybody remember what Π_s is?

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$\rightarrow$ Anybody remember what Π_s is? $\Pi_s = (F, A, c, s, G)$

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State Dominance

Definition (Dominance). *Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states. We say that s'^+ dominates s^+ if $s'^+ \supseteq s^+$.*

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Proposition (Dominance). *Let $\Pi^+ = (F, A^+, c, I, G)$ be a STRIPS planning task, and let s^+, s'^+ be states where s'^+ dominates s^+. We have:*

(i) *If s^+ is a goal state, then s'^+ is a goal state as well.*

(ii) *If $\vec{a}^+$ is applicable in s^+, then $\vec{a}^+$ is applicable in s'^+ as well, and $appl(s'^+, \vec{a}^+)$ dominates $appl(s^+, \vec{a}^+)$.*

Proof. (i) is trivial. (ii) by induction over the length n of $\vec{a}^+$. Base case $n = 0$ is trivial. Inductive case $n \rightarrow n + 1$ follows directly from induction hypothesis and the definition of $appl(.,.)$.

$\rightarrow$ It is always better to have more facts true.

The Delete Relaxation and State Dominance

Proposition. *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let $a \in A$. Then $appl(s, a^+)$ dominates both (i) s and (ii) $appl(s, a)$.*

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$\Longrightarrow$ Optimal relaxed plans admissibly estimate the cost of optimal plans:

Proposition (Delete Relaxation is Admissible). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task, let s be a state, and let $\vec{a}$ be a plan for Π_s. Then $\vec{a}^+$ is a relaxed plan for s.*

Proof. Prove by induction over the length of $\vec{a}$ that $appl(s, \vec{a}^+)$ dominates $appl(s, \vec{a})$. Base case is trivial, inductive case follows from (ii) above.

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$\Longrightarrow$ It is now clear how to find a relaxed plan:

- Applying a relaxed action can only ever make more facts true ((i) above).
- That can only be good, i.e., cannot render the task unsolvable (dominance proposition).

$\rightarrow$ So?

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$\rightarrow$ So? Keep applying relaxed actions, stop if goal is true (see next slide).

Greedy Relaxed Planning

Greedy Relaxed Planning for Π_s^+

$s^+ := s; \vec{a}^+ := \langle\rangle$
while $G \not\subseteq s^+$ **do**:
  if $\exists a \in A$ s.t. $pre_a \subseteq s^+$ and $appl(s^+, a^+) \neq s^+$ **then**
    select one such a
    $s^+ := appl(s^+, a^+); \vec{a}^+ := \vec{a}^+ \circ \langle a^+ \rangle$
  else return "Π_s^+ is unsolvable" **endif**
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Proposition. *Greedy relaxed planning is sound, complete, and terminates in time polynomial in the size of* Π.

Proof. Soundness: If $\vec{a}^+$ is returned then, by construction, $G \subseteq appl(s, \vec{a}^+)$. Completeness: If "$\Pi_s^+$ is unsolvable" is returned, then no relaxed plan exists for s^+ at that point; since s^+ dominates s, by the dominance proposition this implies that no relaxed plan can exist for s. Termination: Every $a \in A$ can be selected at most once because afterwards $appl(s^+, a^+) = s^+$.

$\Longrightarrow$ It is easy to decide whether a relaxed plan exists!

Greedy Relaxed Planning to Generate a Heuristic Function?

Using greedy relaxed planning to generate h

- In search state s during forward search, run greedy relaxed planning on Π_s^+.
- Set $h(s)$ to the cost of $\vec{a}^+$, or ∞ if "Π_s^+ is unsolvable" is returned.

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$\rightarrow$ Is this heuristic admissible? Would be if the relaxed plans were optimal; but they clearly aren't. So h isn't consistent either.

$\rightarrow$ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. Greedy relaxed planning doesn't do this because it may select arbitrary actions that aren't relevant at all.

h^+: The Optimal Delete Relaxation Heuristic

Definition (h^+). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task with state space $\Theta_\Pi = (S, A, c, T, I, G)$. The optimal delete relaxation heuristic h^+ for Π is the function $h^+ : S \mapsto \mathbb{R}_0^+ \cup \{\infty\}$ where $h^+(s)$ is defined as the cost of an optimal relaxed plan for s.*

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Corollary (h^+ is Admissible). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. Then h^+ is admissible, and thus safe and goal-aware.* (By admissibility of delete relaxation.)

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$\rightarrow$ To be informed (accurately estimate h^*), a heuristic needs to approximate the *minimum effort* needed to reach the goal. h^+ naturally does so by asking for the cheapest possible relaxed plans.

[$\rightarrow$ You might rightfully ask "But won't optimal relaxed plans usually under-estimate h^*?" Yes, but that's just the effect of considering a relaxed problem, and arbitrarily adding actions useless within the relaxation does not help to address it.]

h^+ in "TSP" in Australia

- P: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Planning vs. Relaxed Planning:

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- $h^*(I) = 20$; $h^+(I) = 10$.

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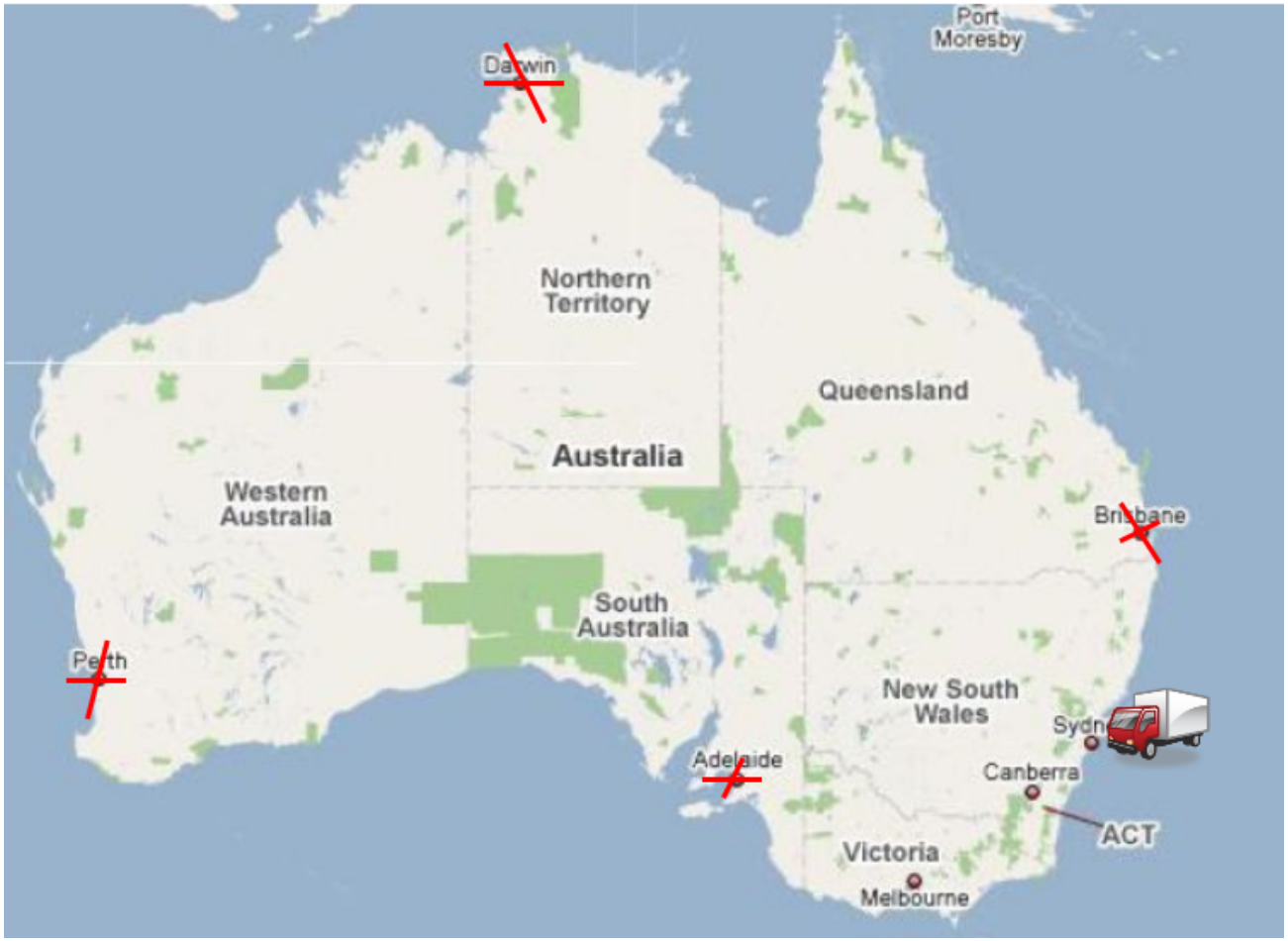

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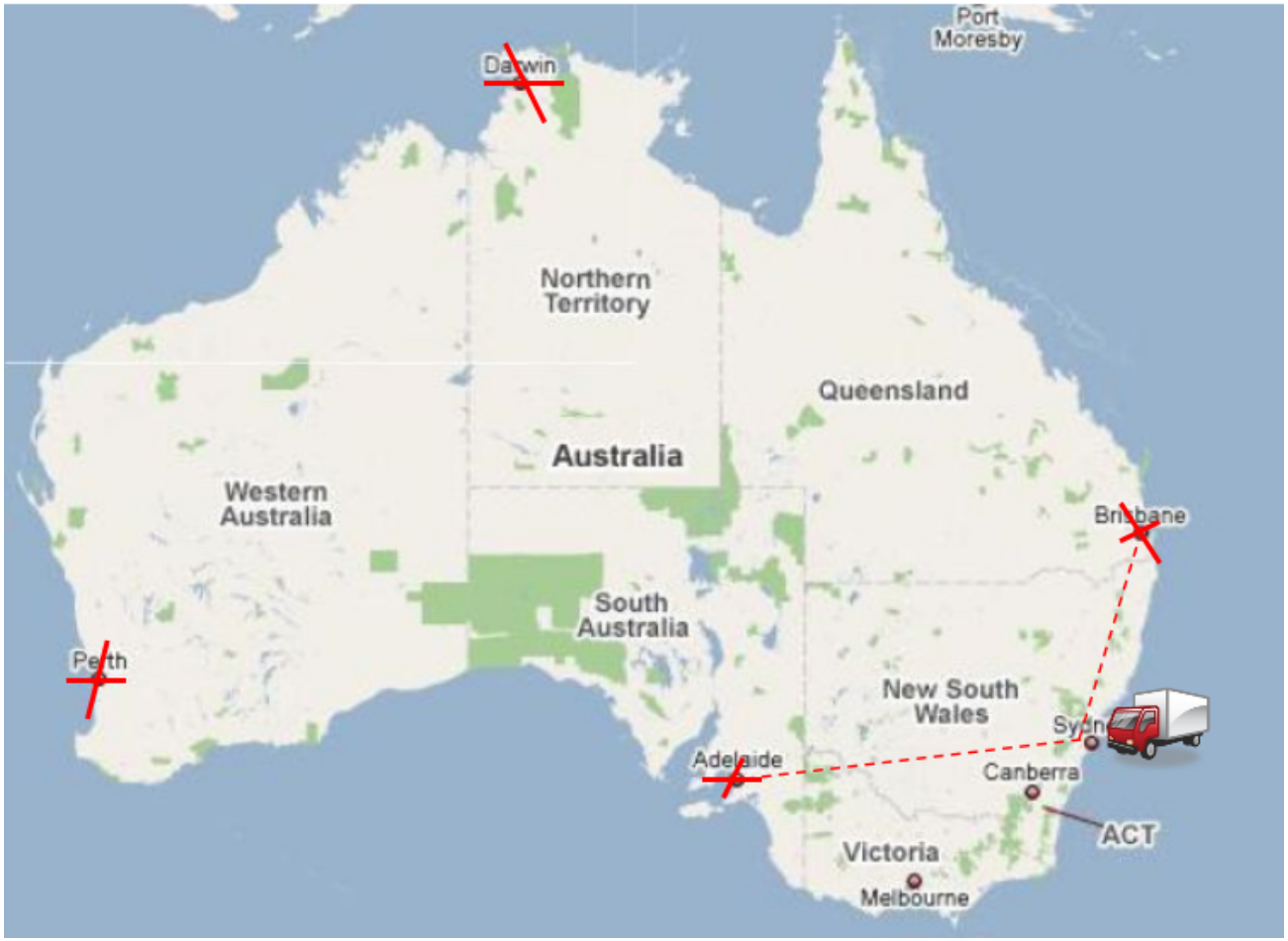
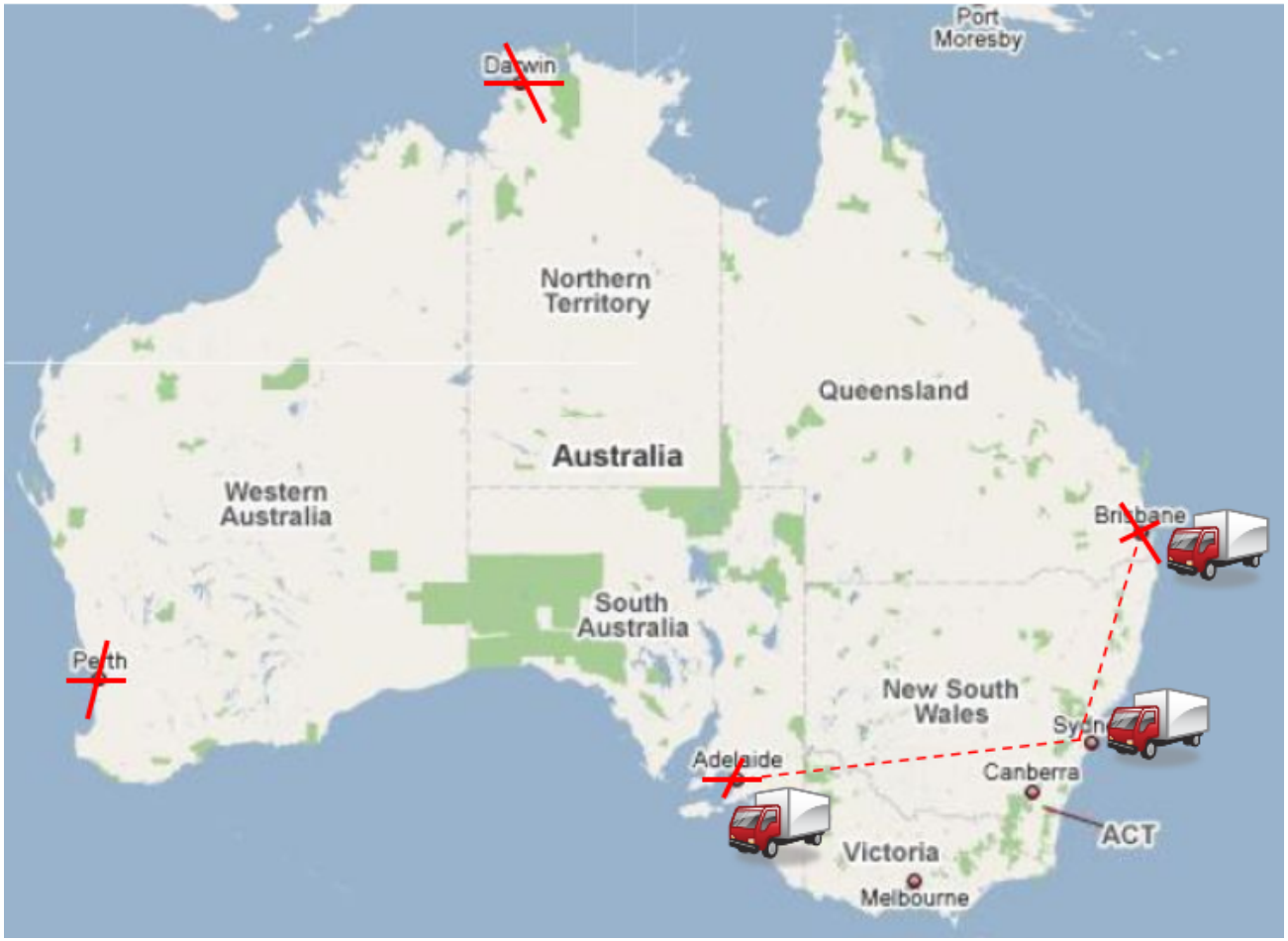


Reminder: h^+ in (the real) TSP



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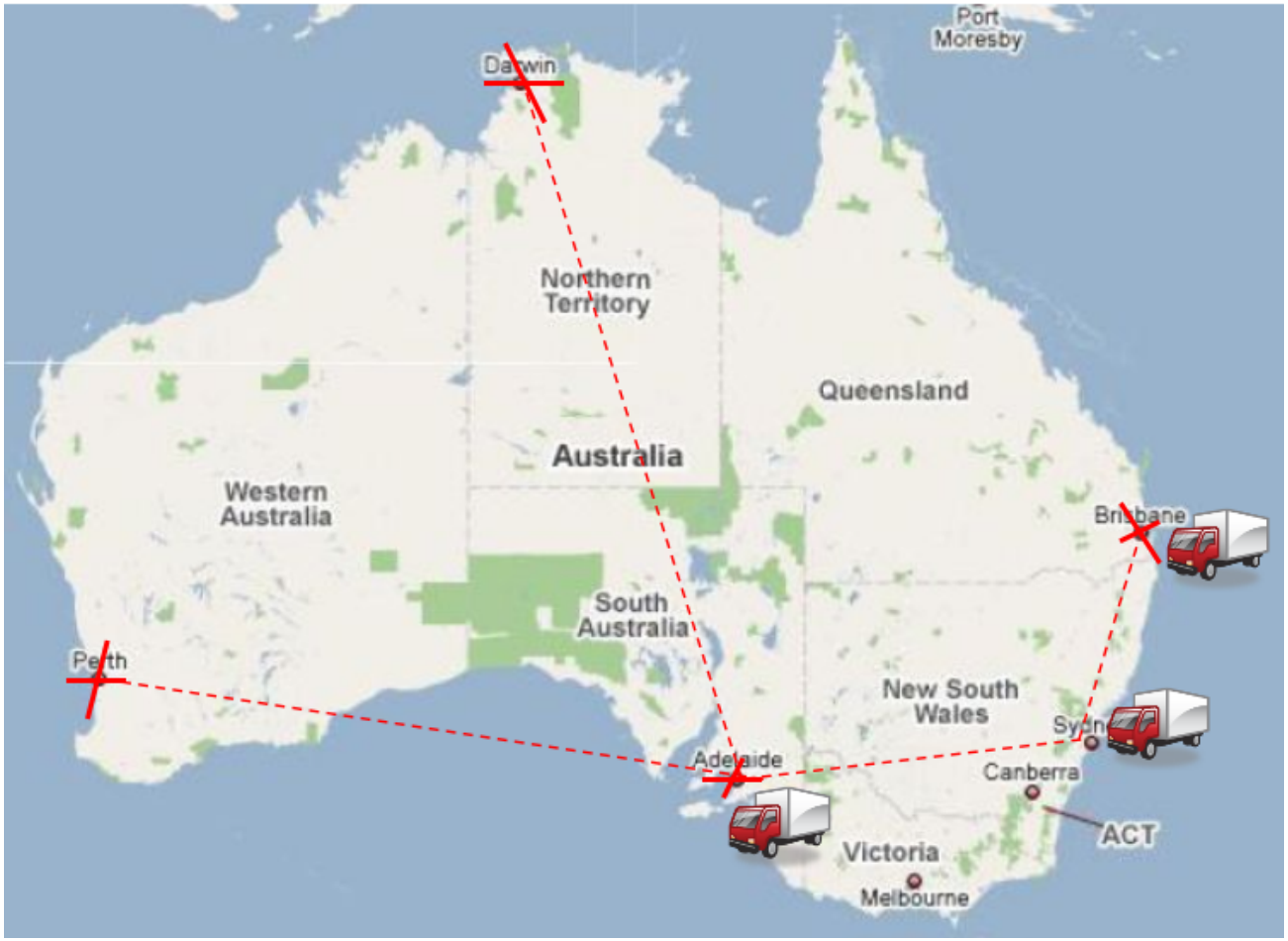

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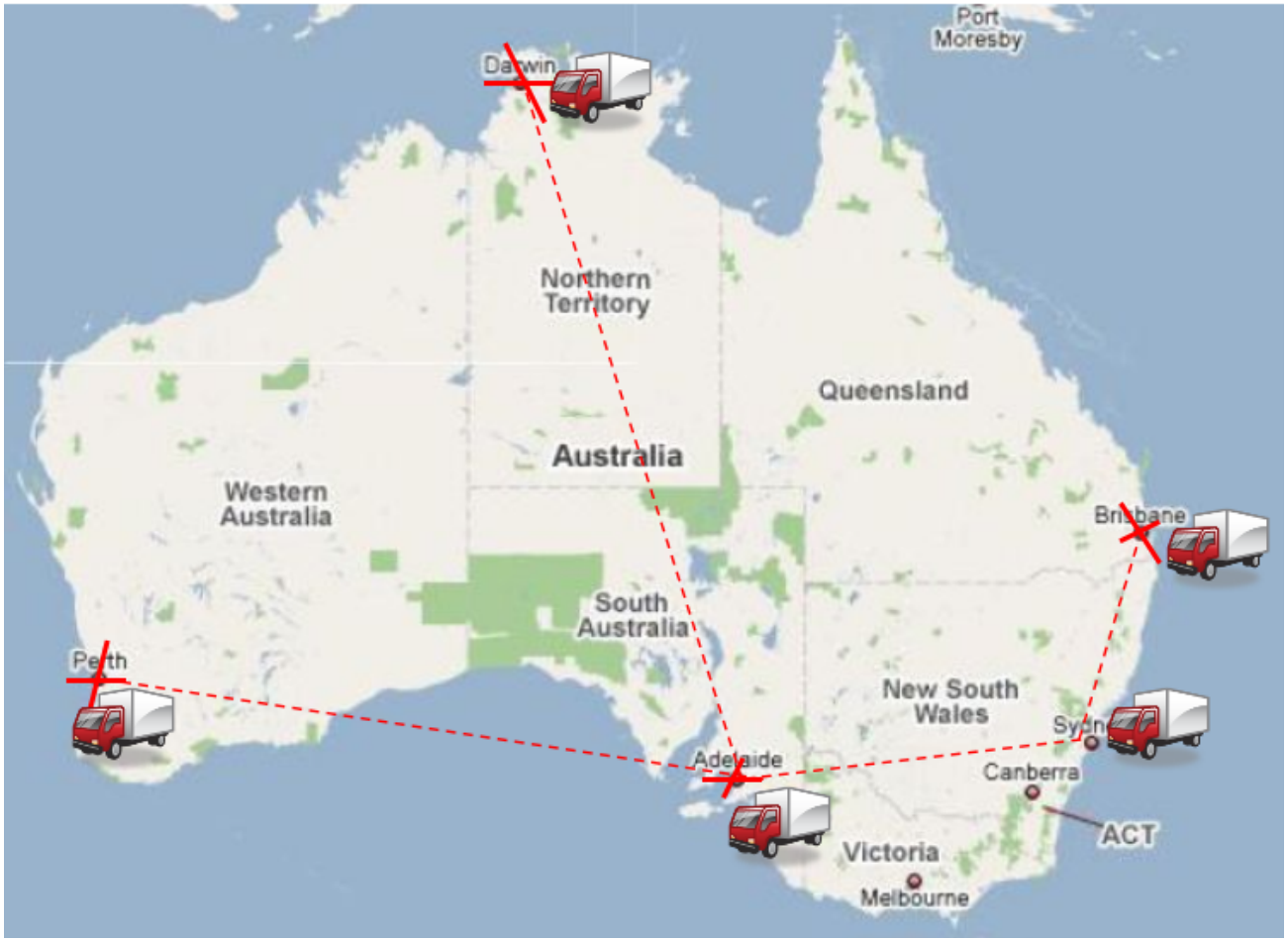

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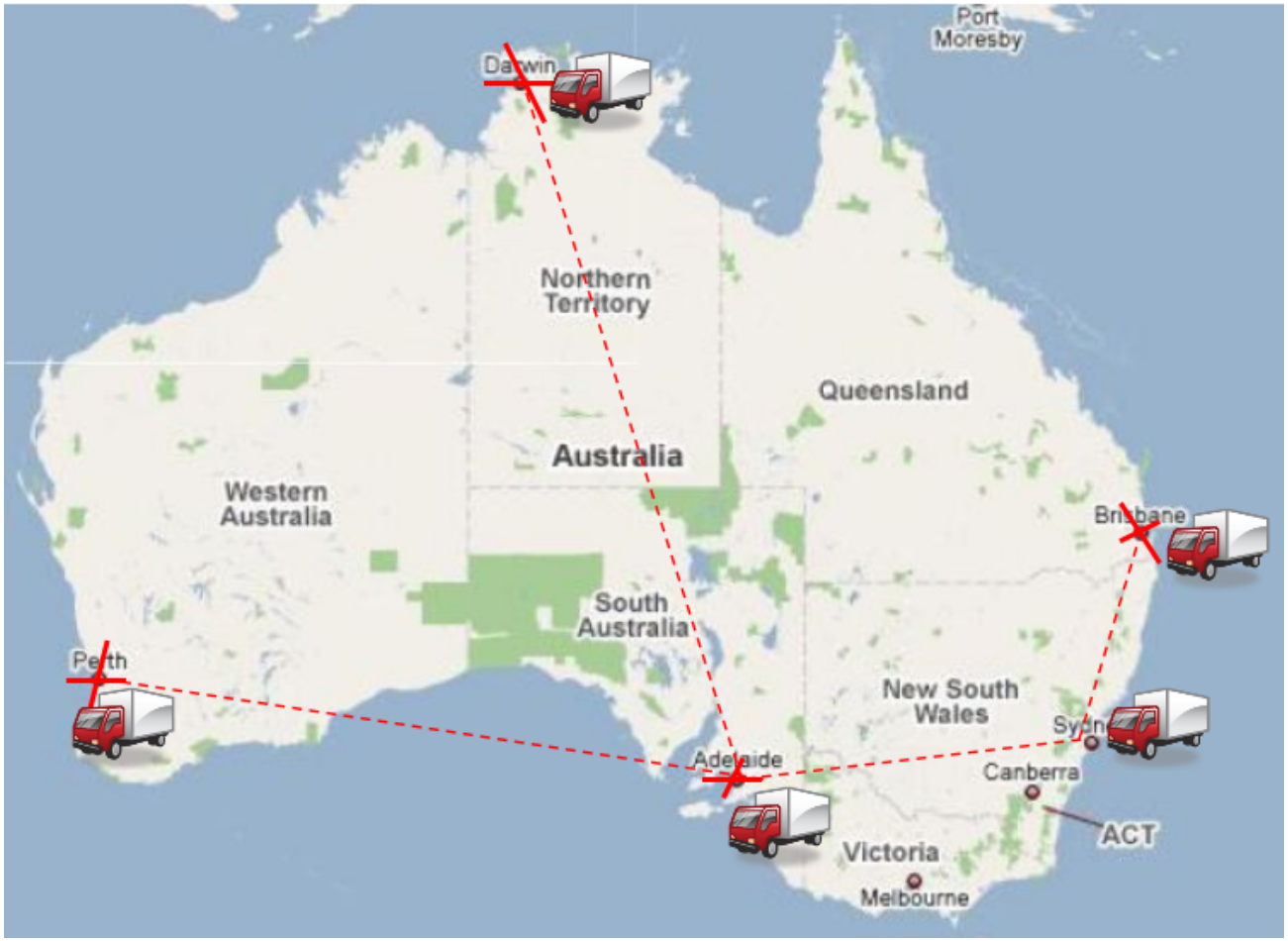
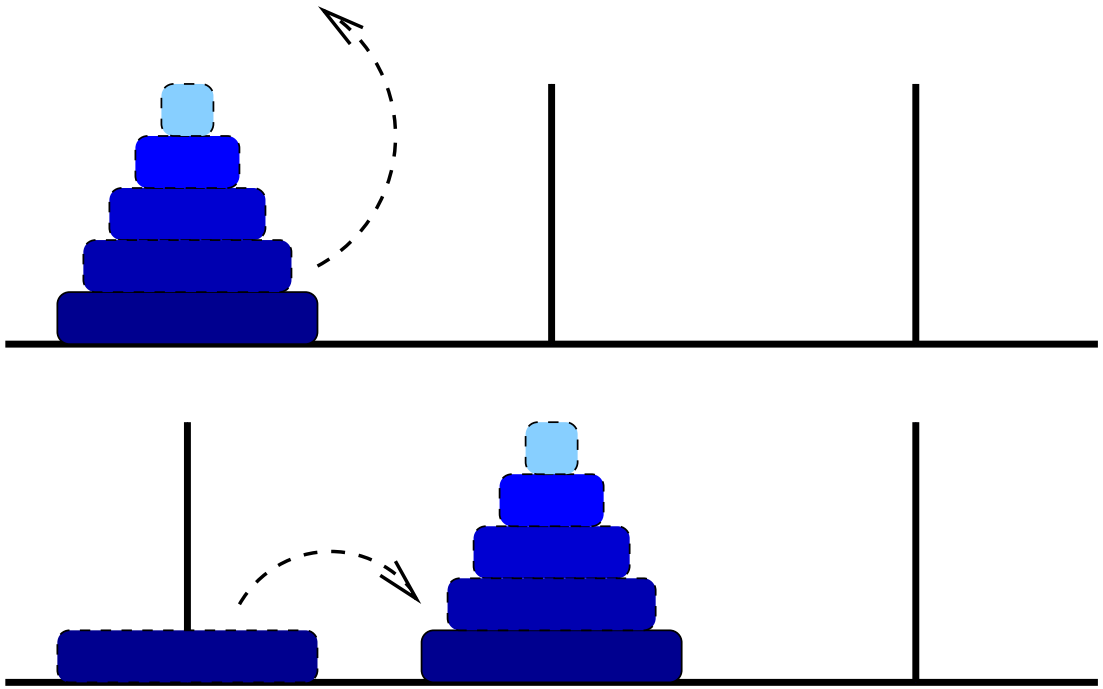


$h^+(\text{TSP}) = \text{Minimum Spanning Tree!}$

Reminder: h^+ in Hanoi



$h^+(\text{Hanoi}) = n$, not 2^n

But How to Compute h^+?

Definition (Optimal Relaxed Planning). *By PlanOpt$^+$, we denote the problem of deciding, given a STRIPS planning task $\Pi = (F, A, c, I, G)$ and $B \in \mathbb{R}_0^+$, whether there exists a relaxed plan for Π whose cost is at most B.*

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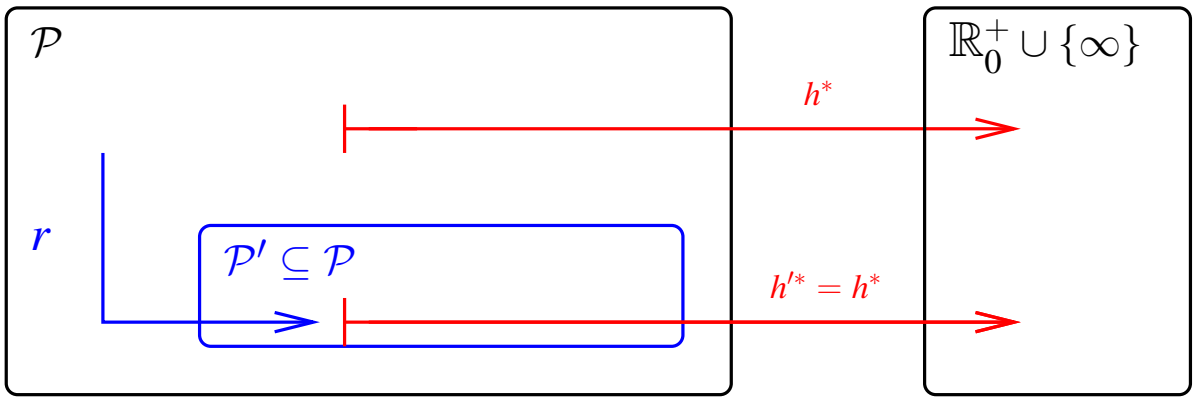
Proof. Membership: Guess action sequences of length $|A|$ – in a relaxed plan, each action is applied at most once!

Hardness: By reduction from SAT.

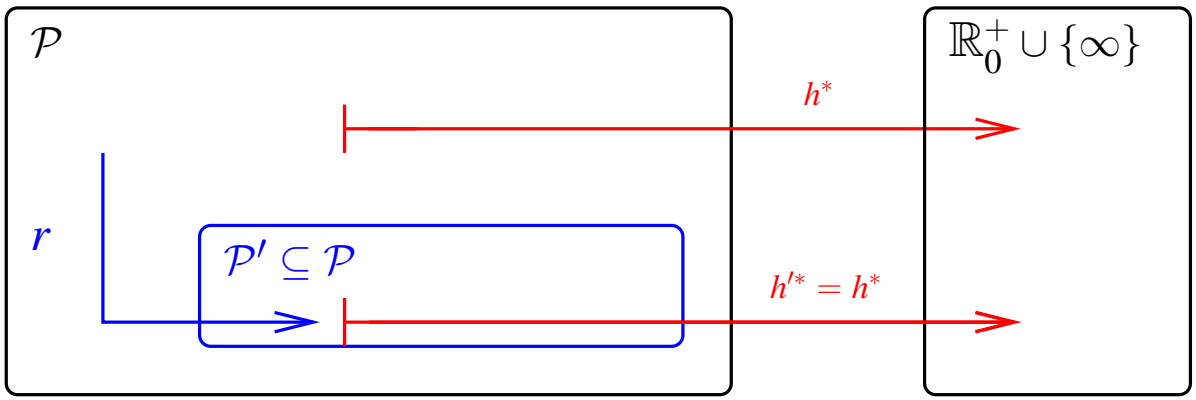
- For each variable $v_i \in \{v_1, \ldots, v_m\}$ in the CNF, three facts v_i, $notv_i$, and $setv_i$; for each clause $c_j \in \{c_1, \ldots, c_n\}$ in the CNF, one fact $satc_j$.
- Actions $setvtrue_i$: $(\emptyset, \{v_i, setv_i\}, \emptyset)$ and $setvfalse_i$: $(\emptyset, \{notv_i, setv_i\}, \emptyset)$.
- Actions $makesatc_j$: $(\{v_i\}, \{satc_j\}, \emptyset)$ where v_i appears positively in clause c_j; $(\{notv_i\}, \{satc_j\}, \emptyset)$ where v_i appears negatively in clause c_j.
- Initial state $\emptyset$, goal $\{setv_1, \ldots, setv_m, satc_1, \ldots, satc_n\}$; $B := m + n$.

h^+ as a Relaxation Heuristic



where, for all $\Pi \in \mathcal{P}$, $h^*(r(\Pi)) \leq h^*(\Pi)$.

h^+ as a Relaxation Heuristic



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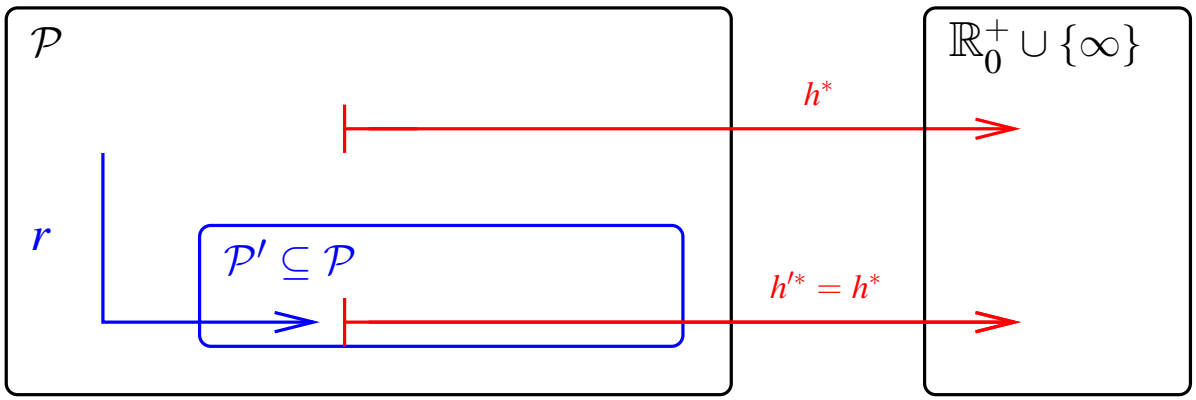
For $h^+ = h^* \circ r$:

- Problem $\mathcal{P}$: All STRIPS planning tasks.
- Simpler problem $\mathcal{P}'$: All STRIPS planning tasks with empty deletes.
- Perfect heuristic h'^* for $\mathcal{P}'$: Optimal plan cost $= h^*$ on $\mathcal{P}'$.
- Transformation r: Drop the deletes.

$\rightarrow$ Is this a native relaxation?

h^+ as a Relaxation Heuristic



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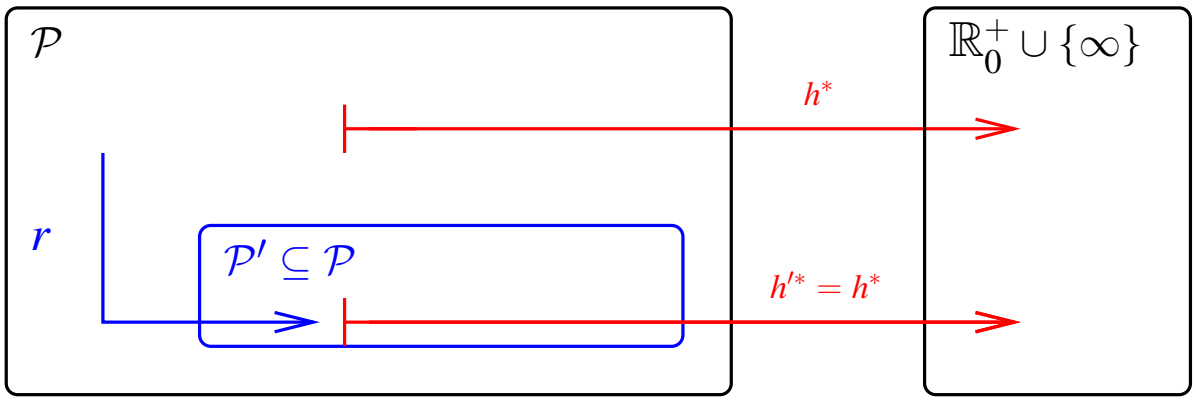

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$\rightarrow$ Is this a native relaxation? Yes.
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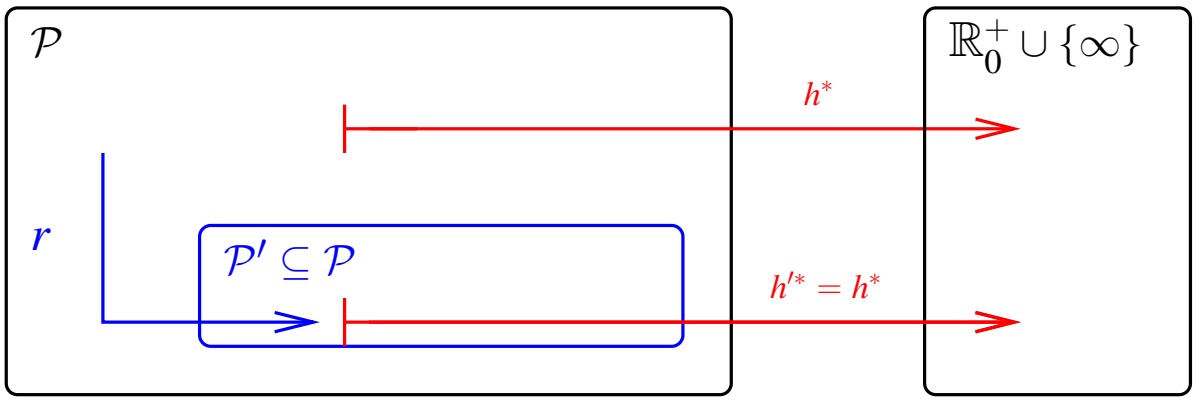
h^+ as a Relaxation Heuristic



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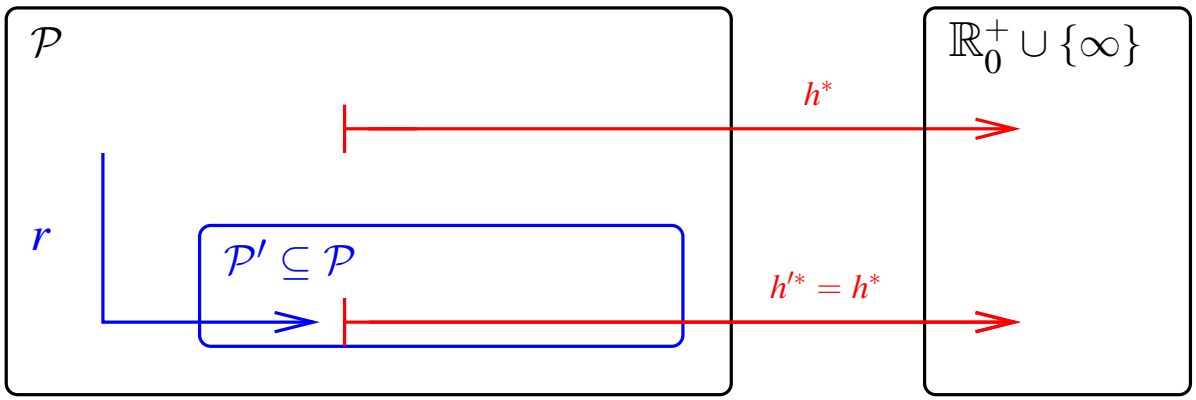

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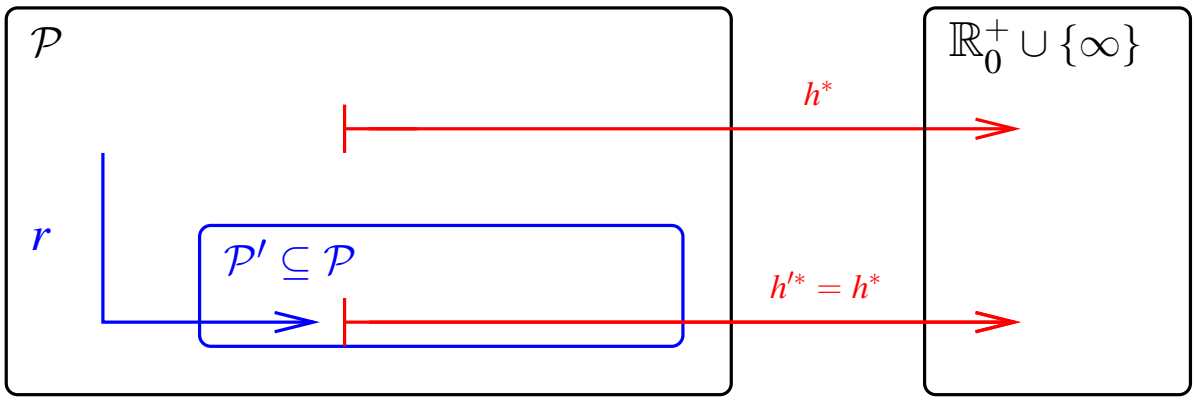

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$\rightarrow$ Is this a native relaxation? Yes.
$\rightarrow$ Is this relaxation efficiently constructible? Yes.
$\rightarrow$ Is this relaxation efficiently computable? No.

What shall we do with this relaxation?

Reminder: → **Lecture 4**

What if $\mathcal{R}$ is not efficiently computable?

- Either (a) approximate h'^*, or (b) design h'^* in a way so that it will typically be feasible, or (c) just live with it and hope for the best.
- Many known relaxations (in planning) are efficiently computable, some aren't (like h^+). The latter use (a); (b) and (c) are not used anywhere right now.

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- Many known relaxations (in planning) are efficiently computable, some aren't (like h^+). The latter use (a); (b) and (c) are not used anywhere right now.

→ The delete relaxation heuristic we want is h^+. Unfortunately, this is hard to compute so the computational overhead is very likely to be prohibitive. All implemented systems using the delete relaxation approximate h^+ in one or the other way. We now look at the the most wide-spread approaches to do so.

Agenda

The Additive and Max Heuristics

Definition (h^{add}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The additive heuristic h^{add} for Π is the function $h^{\text{add}}(s) := h^{\text{add}}(s, G)$ where $h^{\text{add}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{add}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{add}}(s, pre_a) & |g| = 1 \\ \sum_{g' \in g} h^{\text{add}}(s, \{g'\}) & |g| > 1 \end{cases}$$

Definition (h^{max}). *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. The max heuristic h^{max} for Π is the function $h^{\text{max}}(s) := h^{\text{max}}(s, G)$ where $h^{\text{max}}(s, g)$ is the point-wise greatest function that satisfies $h^{\text{max}}(s, g) =$*

$$\begin{cases} 0 & g \subseteq s \\ min_{a \in A, g \in add_a} c(a) + h^{\text{max}}(s, pre_a) & |g| = 1 \\ \max_{g' \in g} h^{\text{max}}(s, \{g'\}) & |g| > 1 \end{cases}$$

The Additive and Max Heuristics: Properties

Proposition ($h^{\max}$ is Optimistic). *$h^{\max} \leq h^{+}$, and thus $h^{\max} \leq h^{*}$.*

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Proposition (h^{add} is Pessimistic). *For all STRIPS planning tasks Π, $h^{\text{add}} \geq h^{+}$. There exist Π and s so that $h^{\text{add}}(s) > h^{*}(s)$.*

The Additive and Max Heuristics: Properties

Proposition ($h^{\max}$ is Optimistic). *$h^{\max} \leq h^+$, and thus $h^{\max} \leq h^*$.*

Proposition (h^{add} is Pessimistic). *For all STRIPS planning tasks Π, $h^{\text{add}} \geq h^+$. There exist Π and s so that $h^{\text{add}}(s) > h^*(s)$.*

$\rightarrow$ Both $h^{\max}$ and h^{add} approximate h^+ by assuming that singleton sub-goal facts are achieved independently. $h^{\max}$ estimates optimistically by the most costly singleton sub-goal, h^{add} estimates pessimistically by summing over all singleton sub-goals.

The Additive and Max Heuristics: Properties, ctd.

Proposition ($h^{\max}$ and h^{add} Agree with h^+ on ∞). *For all STRIPS planning tasks Π and states s in Π, $h^+(s) = \infty$ if and only if $h^{\max}(s) = \infty$ if and only if $h^{\text{add}}(s) = \infty$.*

$\rightarrow$ States for which no relaxed plan exists are easy to recognize, and that is done by both $h^{\max}$ and h^{add}. Approximation is needed only for the cost of an optimal relaxed pan, if it exists.

Bellman-Ford for $h^{\max}$ and h^{add}

Bellman-Ford variant computing h^{add} for state s

new table $T_0^{\text{add}}(g)$, for $g \in F$

For all $g \in F$: $T_0^{\text{add}}(g) := \begin{cases} 0 & g \in s \\ \infty & \text{otherwise} \end{cases}$

fn $c_i(g) := \begin{cases} T_i^{\text{add}}(g) & |g| = 1 \\ \sum_{g' \in g} T_i^{\text{add}}(g') & |g| > 1 \end{cases}$

fn $f_i(g) := \min[c_i(g), \min_{a \in A, g' \in add_a} c(a) + c_i(pre_a)]$

do forever:

  new table $T_{i+1}^{\text{add}}(g)$, for $g \in F$

  For all $g \in F$: $T_{i+1}^{\text{add}}(g) := f_i(g)$

  if $T_{i+1}^{\text{add}} = T_i^{\text{add}}$ **then** stop **endif**

  $i := i + 1$

enddo

$\rightarrow$ Basically the same algorithm works for $h^{\max}$, just change $\sum$ for $\max$

Proposition. *Let $\Pi = (F, A, c, I, G)$ be a STRIPS planning task. Then the series $\{T_i^{\text{add}}(g)\}_{i=0,\ldots}$ converges to $h^{\text{add}}(s, g)$, for all g.* (Proof omitted.)

Bellman-Ford for $h^{\max}$ in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^1:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$

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1	0	1.5	1	∞	∞	0	1.5	1	∞	∞

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$\rightarrow h^{\max}(I) =$

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$\rightarrow h^{\max}(I) = 5.5 << 20 = h^*(I)$.

Bellman-Ford for h^{add} in "TSP" in Australia

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- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞

Bellman-Ford for h^{add} in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5

Bellman-Ford for h^{add} in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\text{add}}(I) =$

Bellman-Ford for h^{add} in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x,y)) = \begin{cases} 1 & \{x,y\} = \{Sy, Br\} \\ 1.5 & \{x,y\} = \{Sy, Ad\} \\ 3.5 & \{x,y\} = \{Ad, Pe\} \\ 4 & \{x,y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^+(I)$. But $< 20 = h^*(I)$.

Bellman-Ford for h^{add} in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^+(I)$. But $< 20 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because it counts the cost of $drive(Sy, Ad)$ 3 times:

Bellman-Ford for h^{add} in "TSP" in Australia

- F: $at(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$; $v(x)$ for $x \in \{Sy, Ad, Br, Pe, Ad\}$.
- A: $drive(x, y)$ where x, y have a road.

$$c(drive(x, y)) = \begin{cases} 1 & \{x, y\} = \{Sy, Br\} \\ 1.5 & \{x, y\} = \{Sy, Ad\} \\ 3.5 & \{x, y\} = \{Ad, Pe\} \\ 4 & \{x, y\} = \{Ad, Da\} \end{cases}$$

- I: $at(Sy), v(Sy)$; G: $at(Sy), v(x)$ for all x.

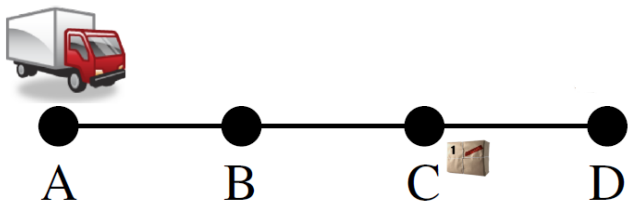
Content of Tables T_i^{add}:

i	$at(Sy)$	$at(Ad)$	$at(Br)$	$at(Pe)$	$at(Da)$	$v(Sy)$	$v(Ad)$	$v(Br)$	$v(Pe)$	$v(Da)$
0	0	∞	∞	∞	∞	0	∞	∞	∞	∞
1	0	1.5	1	∞	∞	0	1.5	1	∞	∞
2	0	1.5	1	5	5.5	0	1.5	1	5	5.5
3	0	1.5	1	5	5.5	0	1.5	1	5	5.5

$\rightarrow h^{\text{add}}(I) = 1.5 + 1 + 5 + 5.5 = 13 > 10 = h^+(I)$. But $< 20 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because it counts the cost of $drive(Sy, Ad)$ 3 times:
As part of $h^{\text{add}}(I, \{v(Ad)\})$, $h^{\text{add}}(I, \{v(Pe)\})$, and $h^{\text{add}}(I, \{v(Da)\})$!

Bellman-Ford for h^{add} in "Logistics"

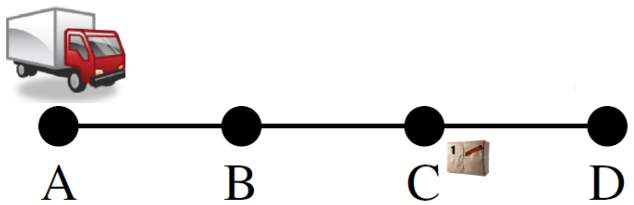


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$

Bellman-Ford for h^{add} in "Logistics"

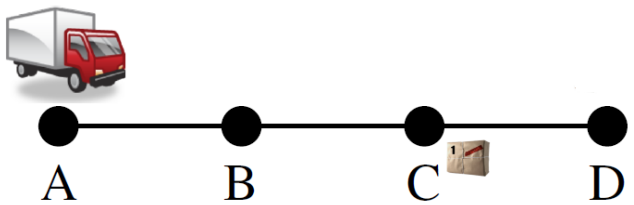


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞

Bellman-Ford for h^{add} in "Logistics"

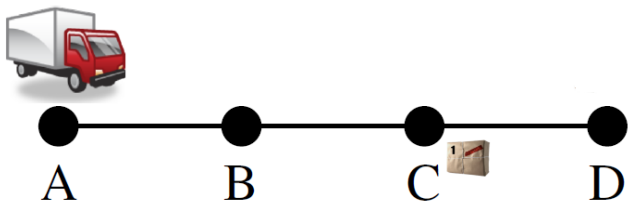


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞

Bellman-Ford for h^{add} in "Logistics"

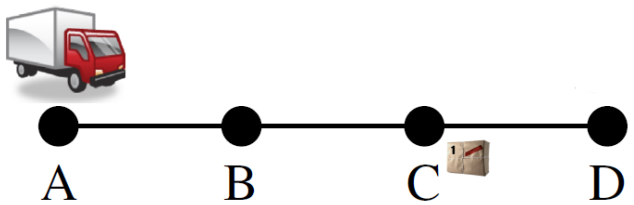


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞

Bellman-Ford for h^{add} in "Logistics"

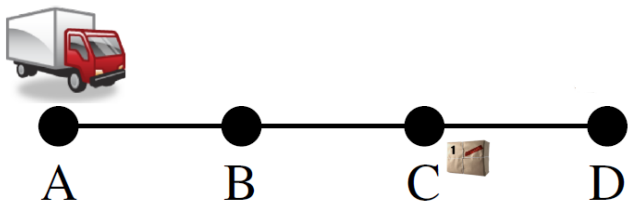


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞

Bellman-Ford for h^{add} in "Logistics"

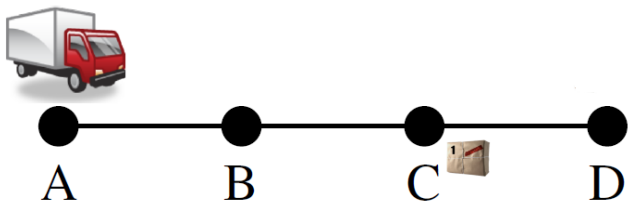


- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)

Bellman-Ford for h^{add} in "Logistics"



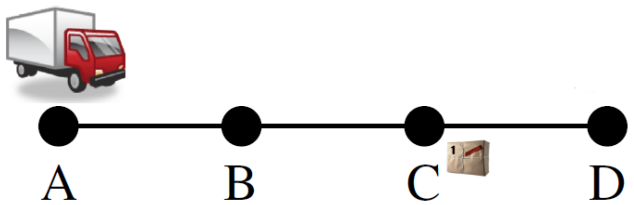
- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) =$

Bellman-Ford for h^{add} in "Logistics"



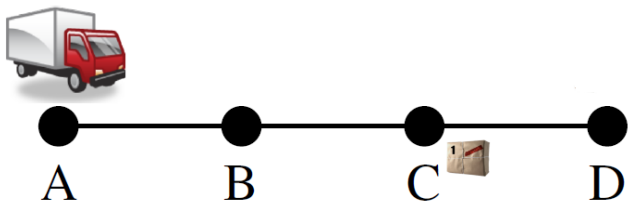
- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

Bellman-Ford for h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

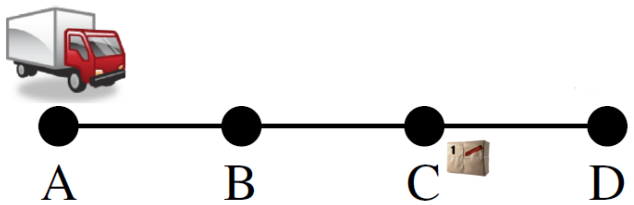
Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because?

Bellman-Ford for h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X, Y), lo(X), ul(X)$.

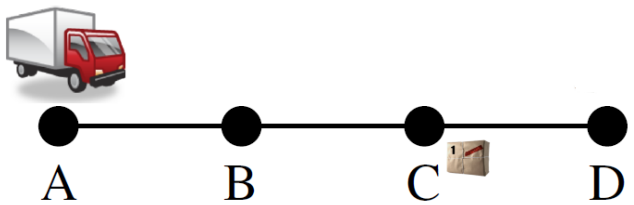
Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because? It counts the cost of $dr(A, B), dr(B, C)$ 2 times, for the two preconditions $p(T)$ and $t(D)$ of achieving $p(D)$.

Bellman-Ford for h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X,Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

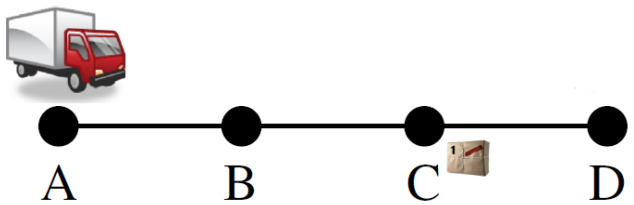
i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because? It counts the cost of $dr(A,B), dr(B,C)$ 2 times, for the two preconditions $p(T)$ and $t(D)$ of achieving $p(D)$.

$\rightarrow$ So, what if $G = \{t(D), p(D)\}$? $h^{\text{add}}(I) =$

Bellman-Ford for h^{add} in "Logistics"



- Initial state I: $t(A), p(C)$.
- Goal G: $t(A), p(D)$.
- Actions A: $dr(X,Y), lo(X), ul(X)$.

Content of Tables T_i^{add}: (Table content T_i^1, where different, given in red)

i	$t(A)$	$t(B)$	$t(C)$	$t(D)$	$p(T)$	$p(A)$	$p(B)$	$p(C)$	$p(D)$
0	0	∞	∞	∞	∞	∞	∞	0	∞
1	0	1	∞	∞	∞	∞	∞	0	∞
2	0	1	2	∞	∞	∞	∞	0	∞
3	0	1	2	3	3	∞	∞	0	∞
4	0	1	2	3	3	4	5 (4)	0	7 (4)
5	0	1	2	3	3	4	5 (4)	0	7 (4)

$\rightarrow h^{\text{add}}(I) = 7 > h^+(I) = 5$. But $< 8 = h^*(I)$.

$\rightarrow h^{\text{add}}(I) > h^+(I)$ because? It counts the cost of $dr(A,B), dr(B,C)$ 2 times, for the two preconditions $p(T)$ and $t(D)$ of achieving $p(D)$.

$\rightarrow$ So, what if $G = \{t(D), p(D)\}$? $h^{\text{add}}(I) = 10 > 5 = h^*(I) = h^+(I)$ because now $dr(A,B), dr(B,C), dr(C,D)$ is counted also as part of the goal $t(D)$.

The Additive and Max Heuristics: So What?

Summary of typical issues in practice with h^{add} and h^{max}:

- Both h^{add} and h^{max} can be computed reasonably quickly.
- h^{max} is admissible, but is typically far too optimistic.
- h^{add} is not admissible, but is typically a lot more informed than h^{max}.
- h^{add} is sometimes better informed than h^{+}, but for the "wrong reasons": rather than accounting for deletes, it overcounts by ignoring positive interactions, i.e., sub-plans shared between sub-goals.
- Such overcounting can result in dramatic over-estimates of h^{*} !!

$\rightarrow$ On slide 28 with goal $t(D)$, if we have 100 packages at C that need to go to D, what is $h^{\text{add}}(I)$?

The Additive and Max Heuristics: So What?

Summary of typical issues in practice with h^{add} and h^{max}:

- Both h^{add} and h^{max} can be computed reasonably quickly.
- h^{max} is admissible, but is typically far too optimistic.
- h^{add} is not admissible, but is typically a lot more informed than h^{max}.
- h^{add} is sometimes better informed than h^+, but for the "wrong reasons": rather than accounting for deletes, it overcounts by ignoring positive interactions, i.e., sub-plans shared between sub-goals.
- Such overcounting can result in dramatic over-estimates of h^* !!

$\rightarrow$ On slide 28 with goal $t(D)$, if we have 100 packages at C that need to go to D, what is $h^{\text{add}}(I)$? $703 >> 203 = h^*(I) = h^+(I)$: For every package, a count of 7 which includes $dr(A,B), dr(B,C)$ for getting the package into the truck, and $dr(A,B), dr(B,C), dr(C,D)$ for getting the truck to D.

$\rightarrow$ Relaxed plans (up next) are a means to reduce this kind of over-counting.