**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?

Answer: After the car dropped off and the customer is told that the car will be ready within 1 hour from dropped off. The service manager takes 50 minutes to ready the car, so the probability that the service manager cannot meet his commitment is given by

**P[X> 50]= 0.2676**

1. 0.3875
2. **0.2676**
3. 0.5
4. 0.6987

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1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Z score for 44 =(44-38)/6=1

Z score for 38 =(38-38)/6= 0

probability for age below 44: 0.8413447460685429

probability for age below 38: 0.5

probability for age between 44 and 38: 0.3413447460685429

Here we can see that probability of age below 44 is 84.13% i.e probability for employees older than 44 is 100-84.13= 15.87% that means (15.87\*400)/100=63.48 employees. and the probability that employees age between 44 and 38 is 34.13% that means (34.13\*400)/100=136.52 employees.

**False**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Here probability that employees under the age of 30 is 9.12% that means (9.12\*400)/100= 36.48 employees.

probability for age below 30 is: 0.09121121972586788

**TRUE**

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1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

**Answer:**

If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) then by using additive property of normal distribution X1+X2 ~N(μ1+ μ2, σ12 + σ22)

And 2X1~ N(2 μ1, 4 σ12)

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1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.

P[a< z< b] = 0.99

P[z<b] –P[z<a] = 0.99

1. 90.5, 105.9
2. 80.2, 119.8
3. 22, 78
4. 48.5, 151.5
5. 90.1, 109.9

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1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

1. Specify the 5th percentile of profit (in Rupees) for the company

Answer: Let the 5th percentile of profit for the company is q

I.e P[p<q]= 0.05

i.e P[(p-5)/ < (p-5)/] =0.05

(p-5)/ ~ N(0,1) , it can be said that ) = 0.05 , where represents the CDF of a standard random variable .

From the standard Normal distribution table it is obtained that

1. Which of the two divisions has a larger probability of making a loss in a given year?

Answer:

Probability of first division making a loss=P(profit1<0)

Profit1~ N(5,9)

Z=(Profit1- 5)/9 ~N(0,1)

P(Profir1 < 0)= P((Profit1- 5)/9 < (0-5)/ 9)

= P(Z< 1.67)

= 0.9525

Probability of second division making a loss=P(profit2<0)

Profit2 ~ N(7, 42)

Z=(Profit2- 7)/16 ~N(0,1)

P(Profit2< 0)= P((Profit1- 7)/16 < (0-7)/ 16)

= P(Z< 1.75)

= 0.9599

Thus the second division has a larger probability of making loss