

Solving a Quadratic Equation Using Sample Points

Quadratic Equation General Form

The general form of a quadratic equation is:

$$y = ax^2 + bx + c$$

Given Sample Points:

- Point 1: (1, 6)
- Point 2: (2, 11)
- Point 3: (3, 18)

Step 1: Set Up Equations

Substituting each point (x, y) into the quadratic equation $y = ax^2 + bx + c$:

1. For Point 1 (x = 1, y = 6):

$$6 = a(1^2) + b(1) + c \Rightarrow a + b + c = 6$$

2. For Point 2 (x = 2, y = 11):

$$11 = a(2^2) + b(2) + c \Rightarrow 4a + 2b + c = 11$$

3. For Point 3 (x = 3, y = 18):

$$18 = a(3^2) + b(3) + c \Rightarrow 9a + 3b + c = 18$$

Step 2: Solve the System of Equations

We now solve the system of equations:

1. $a + b + c = 6$

2. $4a + 2b + c = 11$

3. $9a + 3b + c = 18$

Eliminate 'c':

Subtract Equation 1 from Equation 2:

$$(4a + 2b + c) - (a + b + c) = 11 - 6$$

$$\Rightarrow 3a + b = 5$$

Subtract Equation 2 from Equation 3:

$$(9a + 3b + c) - (4a + 2b + c) = 18 - 11$$

$$\Rightarrow 5a + b = 7$$

Solve for 'a' and 'b':

Subtract the results:

$$(5a + b) - (3a + b) = 7 - 5$$

$$\Rightarrow 2a = 2 \Rightarrow a = 1$$

Substitute $a = 1$ into $3a + b = 5$:

$$3(1) + b = 5 \Rightarrow b = 2$$

Solve for 'c':

Substitute $a = 1$, $b = 2$ into $a + b + c = 6$:

$$1 + 2 + c = 6 \Rightarrow c = 3$$

Step 3: Final Quadratic Equation

The quadratic equation is:

$$y = x^2 + 2x + 3$$

Verification:

Substituting the points:

1. For (1, 6): $y = 1^2 + 2(1) + 3 = 6$ (Correct)
2. For (2, 11): $y = 2^2 + 2(2) + 3 = 11$ (Correct)
3. For (3, 18): $y = 3^2 + 2(3) + 3 = 18$ (Correct)

Thus, the equation $y = x^2 + 2x + 3$ is correct.