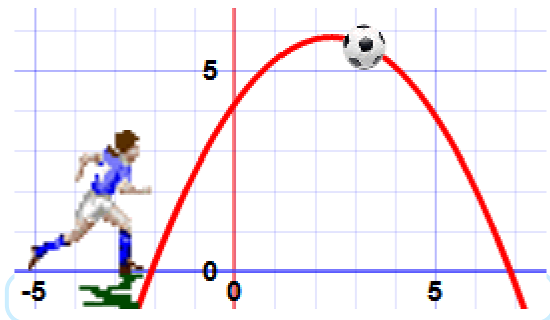
An example of a **Quadratic Equation**:

this makes it Quadratic

$$5x^2 + 3x + 3 = 0$$

Quadratic Equations make nice curves, like this one:



Name

The name **Quadratic** comes from "quad" meaning square, because the variable gets squared (like $\mathbf{x^2}$).
It is also called an "Equation of Degree 2" (because of the "2" on the **x**)

Standard Form

The **Standard Form** of a Quadratic Equation looks like this:

$$ax^2 + bx + c = 0$$

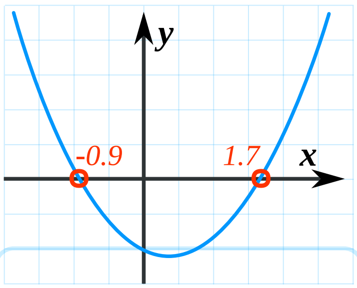
- **a**, **b** and **c** are known values. **a** can't be 0.
- "**x**" is the **variable** or unknown (we don't know it yet).

Here are some examples:

$2x^2 + 5x + 3 = 0$	In this one **a=2**, **b=5** and **c=3**
$x^2 - 3x = 0$	This one is a little more tricky: • Where is **a**? Well **a=1**, as we don't usually write "$1x^2$" • **b = −3**

- And where is **c**? Well **c=0**, so is not shown.

$5x - 3 = 0$	**Oops!** This one is **not** a quadratic equation: it is missing **x^2** (in other words **a=0**, which means it can't be quadratic)



Have a Play With It

Play with the "[Quadratic Equation Explorer]" so you can see:

- the graph it makes, and
- the solutions (called "roots").

Hidden Quadratic Equations!

As we saw before, the **Standard Form** of a Quadratic Equation is

$$ax^2 + bx + c = 0$$

But sometimes a quadratic equation doesn't look like that!

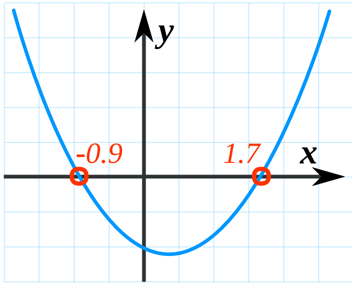
For example:

In disguise		In Standard Form	a, b and c
$x^2 = 3x - 1$	Move all terms to left hand side	**$x^2 - 3x + 1 = 0$**	a=1, b=−3, c=1
$2(w^2 - 2w) = 5$	Expand (undo the brackets), and move 5 to left	**$2w^2 - 4w - 5 = 0$**	a=2, b=−4, c=−5
$z(z-1) = 3$	Expand, and move 3 to left	**$z^2 - z - 3 = 0$**	a=1, b=−1, c=−3

How To Solve Them?

The "**solutions**" to the Quadratic Equation are where it is **equal to zero**.

They are also called "**roots**", or sometimes "**zeros**"



There are usually 2 solutions (as shown in this graph).

And there are a few different ways to find the solutions:

We can Factor the Quadratic (find what to multiply to make the Quadratic Equation)

Or we can Complete the Square

Or we can use the special **Quadratic Formula**:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Just plug in the values of a, b and c, and do the calculations.

We will look at this method in more detail now.

About the Quadratic Formula

Plus/Minus

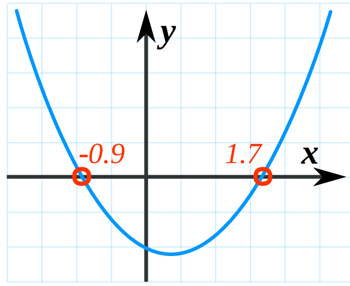
First of all what is that plus/minus thing that looks like ± ?

The ± means there are TWO answers:

$$x = \frac{-b + \sqrt{(b^2 - 4ac)}}{2a}$$

$$x = \frac{-b - \sqrt{(b^2 - 4ac)}}{2a}$$

Here is an example with two answers:



But it does not always work out like that!

- Imagine if the curve "just touches" the x-axis.
- Or imagine the curve is so **high** it doesn't even cross the x-axis!

This is where the "Discriminant" helps us ...

Discriminant

Do you see $\mathbf{b^2 - 4ac}$ in the formula above? It is called the **Discriminant**, because it can "discriminate" between the possible types of answer:

- when $\mathbf{b^2 - 4ac}$ is positive, we get two Real solutions
- when it is zero we get just ONE real solution (both answers are the same)
- when it is negative we get a pair of Complex solutions

Complex solutions? Let's talk about them after we see how to use the formula.

Using the Quadratic Formula

Just put the values of a, b and c into the Quadratic Formula, and do the calculations.

Example: Solve $5x^2 + 6x + 1 = 0$

Coefficients are: $a = 5, b = 6, c = 1$

Quadratic Formula: $x = \dfrac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$

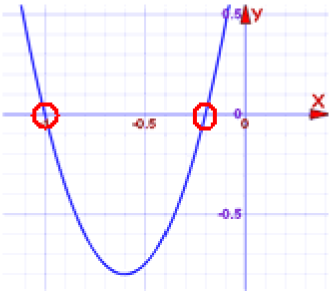
Put in a, b and c: $x = \dfrac{-6 \pm \sqrt{(6^2 - 4\times5\times1)}}{2\times5}$

Solve: $x = \dfrac{-6 \pm \sqrt{(36 - 20)}}{10}$

$x = \dfrac{-6 \pm \sqrt{(16)}}{10}$

$$x = \frac{-6 \pm 4}{10}$$

$x = -0.2$ **or** -1



Answer: $x = -0.2$ **or** $x = -1$

And we see them on this graph.

Check **-0.2**: $5\times(\mathbf{-0.2})^2 + 6\times(\mathbf{-0.2}) + 1$
$= 5\times(0.04) + 6\times(-0.2) + 1$
$= 0.2 - 1.2 + 1$
$\mathbf{= 0}$

Check **-1**: $5\times(\mathbf{-1})^2 + 6\times(\mathbf{-1}) + 1$
$= 5\times(1) + 6\times(-1) + 1$
$= 5 - 6 + 1$
$\mathbf{= 0}$