Bonus Question:
Does Flexible Incentive Pay Dampen Unemployment Fluctuations?

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Motivation

- Sluggish wage adjustment over the business cycle is important in macro
  - Unemployment dynamics (Hall 2005, Hagedorn & Manovskii 2008, Gertler & Trigari 2009)
  - Inflation dynamics (Christiano et al 2005, 2016)

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- Recent evidence: estimates of wage adjustment depend on measure (Grigsby, Hurst & Yildirmaz 2021)
  - Base wages are sluggish (rarely change, weakly pro-cyclical)
  - But bonuses seem flexible (change frequently, strongly procyclical in some studies/contexts)

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- **This paper:** *how does flexible incentive pay affect unemployment dynamics?*
  - Incentive pay: piece-rates, bonuses, commissions, stock options or profit sharing
  - 30-50% of US workers get incentive pay (Lemieux, McLeod and Parent, 2009; Makridis & Gittelman 2021)
  - Including 25-30% of low wage workers

Wage Cyclicality from Incentives Does Not Dampen Unemployment Fluct's

This paper: incentive pay + unemployment dynamics

- Flexible incentive pay = dynamic incentive contract with moral hazard (Holmstrom 1979; Sannikov 2008)
- Unemployment = standard labor search model (Mortensen & Pissarides 1994)
- Allows flexible + cyclical incentive pay consistent with microdata

Wage Cyclicality from Incentives Does Not Dampen Unemployment Fluct's

This paper: incentive pay + unemployment dynamics

Result #1: Wage cyclicality from incentives does not dampen unemployment fluctuations

Unemployment dynamics first-order identical in two economies calibrated to same steady state:

1. Economy #1: labor search model with flexible incentive pay + take-it-or-leave-it offers
2. Economy #2: labor search model with perfectly rigid wages as in Hall (2005)

Intuition: lower incentive pay raises profits, but worse incentives reduces effort + lowers profits

- **Optimal contract:** effect of wage + effort on profits cancel out

Wage Cyclicality from Incentives Does Not Dampen Unemployment Fluct's

This paper: incentive pay + unemployment dynamics

Result #1: Wage cyclicality from incentives does not dampen unemployment fluctuations

Result #2: Wage cyclicality from bargaining + outside option does dampen unemployment fluctuations

- Similar mechanism to standard model

→ Empirical work should separately measure wage cyclicality **due to bargaining vs. incentives**

Wage Cyclicality from Incentives Does Not Dampen Unemployment Fluct's

This paper: incentive pay + unemployment dynamics

Result #1: Wage cyclicality from incentives does not dampen unemployment fluctuations

Result #2: Wage cyclicality from bargaining + outside option does dampen unemployment fluctuations

Result #3: Calibrated model: ≈60% of wage cyclicality due to bargaining + outside option
→ Calibrate simple models without incentive pay to wage cyclicality that is 40% lower than raw data

▸ Literature

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Frictional labor markets

- Measure 1 of workers begin unemployed and search for jobs; remain unemployed if unmatched
- Firms post vacancies v at cost κ to recruit workers
- Vacancy-filling rate is $q(\theta) \equiv \Psi\theta^{-\nu}$ for $\theta \equiv v/u$ market tightness

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Workers' preferences

- Workers derive utility from consumption c and labor effort a with utility $u(c, a)$
- Employed workers consume wage w and supply effort a
- Unemployed workers have value $U \equiv u(b, 0)$

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Technology

- Firm-worker match produces output $y = z(a + \eta)$
  - z: aggregate labor productivity, always common knowledge
  - η: i.i.d., mean zero output shock with distribution $\pi(\eta)$
- Firms pay workers wage w, earn expected profits from a filled vacancy:

$$J(z) = \mathbb{E}_\eta \left[z(a + \eta) - w \right]$$

Employment Fluctuations in Static Model

- Free entry to vacancy posting guarantees zero profits in expectation:

$$\kappa = \underbrace{q(v)}_{Pr\{\text{Vacancy Filled}\}} \cdot \underbrace{J(z)}_{\text{Value of Filled Vacancy}}$$

- Employment fluctuations: Derivation

$$\boxed{\frac{d\log n}{d\log z} = constant + \left(\frac{1-\nu}{\nu}\right) \cdot \frac{d\log J(z)}{d\log z}}$$

- Next: solve for dJ/dz to determine employment fluctuations

First Order Effect of Change in Labor Productivity z

Consider effect of small shock to z on expected profits $J(z)$:

$$
\begin{aligned}
\frac{dJ(z)}{dz} &= \frac{d\mathbb{E}_\eta\left[z(a+\eta)-w\right]}{dz} \\
&= \mathbb{E}_\eta\left[\underbrace{\frac{\partial[z(a+\eta)-w]}{\partial z}}_{\text{Direct Productivity}} + \underbrace{\frac{\partial[z(a+\eta)-w]}{\partial w}\cdot\frac{dw}{dz}}_{\text{Marginal Cost}} + \underbrace{\frac{\partial[z(a+\eta)-w]}{\partial a}\cdot\frac{da}{dz}}_{\text{Incentives}}\right]
\end{aligned}
$$

If labor productivity shocks change effort, incentives can partially offset marginal cost effect

Next: different models of a and w

Two Models of a and w

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a - \frac{dw}{dz} + z\frac{da}{dz} \right]$$

Model	a	w	$\frac{dJ(z)}{dz}$
Fixed effort and wage (Hall 2005)			
Optimal incentive contract (Holmstrom 1979)			

Two Models of a and w

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[\bar{a} - \overbrace{\frac{dw}{dz}}^{=0} + z \overbrace{\frac{da}{dz}}^{=0}\right]$$

Model	a	w	$\frac{dJ(z)}{dz}$
Fixed effort and wage (Hall 2005)	$\bar{a}$	$\bar{w}$	$\bar{a}$
Optimal incentive contract (Holmstrom 1979)			

Moral Hazard, Optimal Contract with Incentive Pay

- Moral hazard: firm cannot distinguish effort a from idiosyncratic shock η (Holmstrom 1979)

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- Firm meets worker and offers contract to maximize value of filled vacancy

$$J(z) \equiv \max_{a(z), w(z,y)} \mathbb{E}[z(a(z) + \eta) - w(z, y)]$$

subject to

$$\begin{aligned} \text{incentive compatibility constraint:} \quad & a(z) \in \arg\max_{\tilde{a}(z)} \mathbb{E}\left[u(w(z,y), \tilde{a}(z))\right] \\ \text{participation constraint w/ bargaining:} \quad & \mathbb{E}\left[u(w(z,y), a(z))\right] \geq \mathcal{B}(z) \end{aligned}$$

Moral Hazard, Optimal Contract with Incentive Pay

- Moral hazard: firm cannot distinguish effort a from idiosyncratic shock η (Holmstrom 1979)
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- Properties of the contract:
  1. Incentives—pass through of y into w
  2. Reduced form "bargaining rule" $\mathcal{B}(z)$ (Michaillat 2011)
  3. When $\mathcal{B}(z) = U$ no bargaining power + constant outside option $\rightarrow$ all wage cyclicality due to incentives

Wage Cyclicality from Incentives Does Not Dampen Unemployment Fluct's

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a + \overbrace{z\frac{da}{dz} - \frac{dw}{dz}}^{=0 \text{ by Envelope Thm}} \right]$$

Model	a	w	$\frac{dJ(z)}{dz}$
Fixed effort and wage (Hall 2005)	$\bar{a}$	$\bar{w}$	$\bar{a}$
Incentive contract w/o bargaining or cyclical outside option, $\mathcal{B}(z) = U$	$a^*(z)$	$w^*(z, y)$	$a^*(z)$

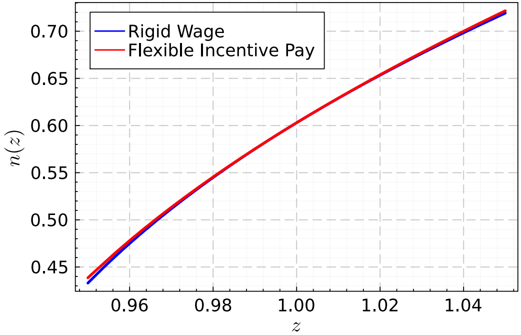
Result #1: wage cyclicality from incentives does not dampen unemployment fluctuations

- NB: Output dynamics not equivalent

Optimal Contract Example

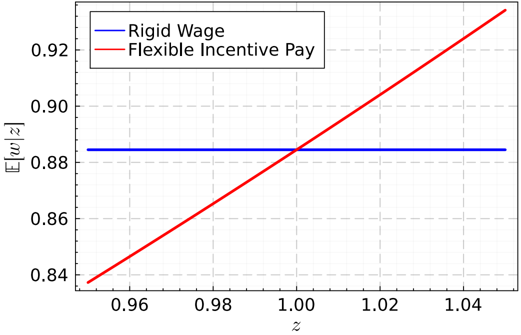
Same Employment Response w/ Rigid Wage or Flexible Incentive Pay

- **Fixed effort, fixed wages** (Hall)
  $\longrightarrow$ Large fluctuations in n when z fluctuates
- **Incentive contract** with $\mathcal{B}(z) = U$
  $\longrightarrow$ No bargaining or cyclical outside option
  $\longrightarrow$ Identical to rigid wage economy!

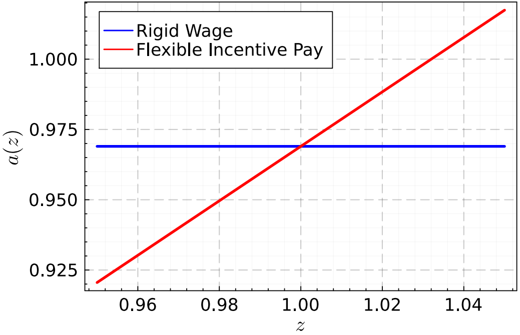


Optimal Contract Details Parameterization Full Information Benchmark Proof Outline

Holds even though average wages can be strongly “pro-cyclical”



$\mathbb{E}_\eta[w^*(z,y)|z]$



$a^*(z)$

▸ Proof Outline ▸ Envelope Intuition ▸ What is a Bonus?

Wage Cyclicality from Bargaining Does Dampen Unemployment Fluct's

Result #2: Wage cyclicality from bargaining or outside option does dampen unemployment fluctuations

$$\boxed{\frac{dJ}{dz} = a^* - \mu^* \mathcal{B}'(z)}$$

- Direct productivity effect a^*
- Cyclical utility from bargaining or outside option $\mathcal{B}'(z)$
- μ^* = Lagrange multiplier on participation constraint

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$$\boxed{\begin{gathered}\frac{dJ}{dz} = a^* - \mu^* \mathcal{B}'(z) \\[2em] \mu^* \mathcal{B}'(z) = \underbrace{\mathbb{E}\left[\frac{dw^*}{dz} - z\frac{da^*}{dz}\right]}_{\text{bargained wage cyclicality}}\end{gathered}}$$

- Direct productivity effect a^*
- Cyclical utility from bargaining or outside option $\mathcal{B}'(z)$
- μ^* = Lagrange multiplier on participation constraint
- $\mu^* \mathcal{B}'(z)$ is bargained wage cyclicality

Intuition: higher wages from bargaining or outside option not accompanied by higher effort

- Same mechanism as standard model (e.g. Shimer 2005)

Summary of Static Model

1. Wage cyclicality **due to incentives** does not dampen employment fluctuations
   - Marginal cost + incentive effects cancel out to a first order on optimal contract

2. Wage cyclicality **due to bargaining or outside option** does dampen employment fluctuations

Summary of Dynamic Model

Diamond-Mortensen-Pissarides labor market

- Firms post vacancies, match with unemployed in frictional labor market w/ tightness θ_t
- Baseline: exogenous separations, extension w/ endogenous separations

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Dynamic incentive contract (Sannikov 2008)

- General production and utility functions $f(z_t, \eta_t)$ and $u(w_t, a_t)$, discount factor β
- Unobservable history of effort a^t shifts distribution of observable persistent idiosyncratic shock η_t
- Firm offers dynamic incentive contract:

$$\left\{w_t\left(\eta^t, z^t\right), a_t\left(\eta^{t-1}, z^t\right)\right\}_{\eta^t, z^t, t=0}^{\infty}$$

1. Sequence of incentive constraints
2. Ex ante participation constraint w/ reduced form bargaining
3. Two sided commitment

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✓ Allows long term contracts (Barro 1977; Beaudry & DiNardo 1991) Details

Result#1: Incentive Wage Cyclicality Doesn't Mute Unemployment Fluct's

Temporarily shut down bargaining power + outside option → all wage cyclicality due to incentives

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Assume: (i) proximity to aggregate steady state (ii) production function is h.o.d. 1 in z, (iii) z is driftless random walk (iv) no worker bargaining power + constant outside option. In incentive pay economy

$$\frac{d\log\theta_0}{d\log z_0} \propto \frac{1}{1-\textit{labor share}}, \qquad \textit{labor share} = \frac{\mathbb{E}_0[\textit{present value wages}]}{\mathbb{E}_0[\textit{present value output}]}$$

The same equations characterize a rigid wage economy with fixed wages + effort.

Implication: incentive wage cyclicality does not mute unemployment fluctuations

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Proof sketch: optimal contract + envelope theorem

- $\rightarrow$ No first order effect of wage + effort changes on profits in response to z_0
- $\rightarrow$ Same profit response as if fixed wages + effort

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Generality: analytical results with general functions, persistent idiosyncratic shocks ▸ Assumptions

- *In paper:* same result w/ efficient endogenous separations

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Result #2 in paper: bargained wage cyclicality does mute unemployment fluctuations

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Numerical Exercise: Overview

Questions

- How much wage cyclicality due to incentives vs bargaining + outside option?
- How to calibrate simpler model of wage setting without incentives?

Approach

1. Explicit and tractable optimal contract building on Edmans et al (2012) Details
2. Reduced form bargaining: take-it-or-leave it with cyclical value of unemployment
3. Calibrate parameters targeting micro moments of wage adjustment

Heuristic Identification: Disentangling Bargaining from Incentives

1. **Ex post wage pass through informs incentives**
   - Key moments: pass-through of firm-specific profitability shocks to wages, variance of wage growth
   - Key parameter: disutility of effort, variance of idiosyncratic shocks
   - Conservative choices to reduce role of incentives (e.g. target low pass-through)
2. **Ex ante fluctuations in wage for new hires informs bargaining + outside option**
   - Key moment: new hire wage cyclicality
   - Key parameter: cyclicality of promised utility
3. **Externally calibrate standard parameters**
   - Separation rate, discount rate, vacancy cost, matching function (Petrosky-Nadeau and Zhang, 2017)
   - TFP process from Fernald (2014), accounting for capacity utilization of labor + capital

Optimal Contract Identification: Equations

Result#3: Substantial Share of Overall Wage Cyclicality Due to Incentives

Table: Data vs Simulated Model Moments

Moment	Description	Data	Baseline
$\text{std}(\Delta \log w_{it})$	Std. Dev. Log Wage Growth	0.064	0.064
$\partial \mathbb{E}[\log w_0]/\partial u$	New Hire Wage Cyclicality	-1.00	-1.00
$\partial \log w_{it}/\partial \log y_{it}$	Wage Passthrough: Firm Shocks	0.039	0.035
u_{ss}	SS Unemployment Rate	0.060	0.060
$\text{std}(\log u_t)$	Std. Dev. of unemployment rate	0.157	0.103
BWC	Share of Wage Cyclicality Due to Bargaining	–	0.60

- Good match to targeted moments
- Rationalize about 2/3 of unemployment fluctuations in data
- **60% wage cyclicality due to bargaining**, remainder due to incentives

▸ Parameter Estimates ▸ Equivalence Plots ▸ Calculating Bargained Share

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

	Model: source of wage flexibility	
	(1)	(2)
Moment	Incentives + Bargaining	Bargaining only
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-0.60
$\partial \log \theta_0 / \partial \log z_0$	13.3	13.0
std($\log u_t$)	0.103	0.103

- Calibrate baseline model w/ bargaining + incentives and simple/standard model with bargaining only
- Analytical results suggest:
    - Calibrate bargaining + incentives model to overall wage cyclicality
    - Calibrate bargaining only model to bargaining wage cyclicality **which is less procyclical**

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

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$\partial \log \theta_0 / \partial \log z_0$	13.3	13.0
$\text{std}(\log u_t)$	0.103	0.103

- Bargaining only model calibrated to weakly cyclical wages
- Has similar employment dynamics to bargaining + incentives model w/ strongly cyclical wages

User Guide: Calibrate Model w/o Incentives to Less Cyclical Wages

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$\partial \log \theta_0 / \partial \log z_0$	13.3	13.0
$\text{std}(\log u_t)$	0.103	0.103

Takeaway:

- Can study simple models of wage setting without incentives
- But calibrate to relatively rigid wages

All Wage Cyclicality from Bargaining

Static Model

Dynamic Model

Numerical Exercise

Conclusion

Conclusion

- *Does flexible incentive pay dampen unemployment fluctuations?*
- Incentive effect (effort moves) offsets marginal cost effect (wage moves)

Results:

1. Incentive wage cyclicality **does not** dampen unemployment fluctuations
2. Bargained wage cyclicality **does** dampen unemployment fluctuations
   - Important to separately measure bargaining and incentives
3. Numerically: **60%** of wage cyclicality due to bargaining
   - Calibrate simple model without incentives to weakly cyclical wages

Appendix

Why is employment log-linear in expected profits?

Free entry into vacancies

$$\kappa = q(v)J(z)$$

Substitute in for $q(v)$ and re-arrange for equilibrium vacancy posting

$$v^* = \left(\frac{\Psi J(z)}{\kappa}\right)^{\frac{1}{\nu}}$$

Now note that $n = f(v)$ (because initial unemployment $= 1$). Plug in to see

$$f(v) \equiv \frac{m(u,v)}{u} = \Psi v^{1-\nu} \qquad \Longrightarrow \qquad n = \left(\frac{\psi^{\nu+1}}{\kappa}\right)^{\frac{1}{\nu}} J(z)^{\frac{1-\nu}{\nu}}$$

Take logs to obtain result

$$\boxed{\ln n = \mathit{constant} + \left(\frac{1-\nu}{\nu}\right) \cdot \ln J(z)}$$

Technical Assumptions

Theorem

- The utility function u is Lipschitz continuous in the compact set of allocations
- z_t and η_t are Markov processes
- Local incentive constraints are globally incentive compatible
- The density $\pi\left(\eta_i^t, z^t | z_0, a_i^t\right)$ is continuous in the aggregate state z_0

Full Information Benchmark

Employment Responses

- Firm observes aggregate productivity z and offers contract to worker
- Firm observes worker's effort a and idiosyncratic output shock η after production
- Firm offers contract to maximize profits

$$\max_{a(z,\eta),w(z,\eta)} J(z) = z\left(a(z,\eta)+\eta\right) - w(z,\eta)$$

subject to worker's participation constraint

$$\mathbb{E}_\eta\left[u\left(w(z,\eta), a(z,\eta)\right)\right] \geq \mathcal{E}$$

- First order condition implies optimal contract $a^*(z), w^*(z)$
- Yields fluctuations in profits

$$\frac{dJ(z)}{dz} = \mathbb{E}\left[a^*(z) + z\frac{da^*(z)}{dz} - \frac{dw^*(z)}{dz}\right] = a^*(z)$$

Parameterization

- CARA utility

$$u(c,a) = -e^{-r\left(c-\frac{\phi a^2}{2}\right)}$$

- Linear contracts

$$w(y) = \alpha + \beta y$$

  - α: "Base Pay"
  - β: "Piece-Rate" or "Bonus"
- Noise observed after worker's choice of action
- Yields optimal contract

$$\beta = \frac{z^2}{z^2 \phi r \sigma}, \qquad \alpha = b + \frac{\beta^2\left(\phi r \sigma^2 - z^2\right)}{2\phi}, \qquad a = \frac{\beta z}{\phi}$$

Static Model Parameter Values

- Elasticity of matching function $\nu = 0.72$ (Shimer 2005)
- Matching function efficiency $\psi = 0.9$ (Employment/Population Ratio = 0.6)
- Non-employment benefit $b = 0.2$ (Shimer 2005)
- Vacancy Creation Cost $\kappa = 0.213$ (Shimer 2005)
- CARA utility

$$u(c, a) = -e^{-r\left(c - \frac{\phi a^2}{2}\right)}$$

  with $\phi = 1$ and $r = 0.8$
- Linear contracts

$$w(y) = \alpha + \beta y$$

  - α: “Base Pay”
  - β: “Piece-Rate” or “Bonus”
- Profit shocks $\eta \sim \mathcal{N}(0, 0.2)$

- Frictional labor market: vacancy filling rate $q_t = \Psi \theta_t^{-\nu}$, market tightness $\theta_t \equiv v_t / u_t$
- Production function $y_{it} = f\left(z_t, \eta_{it}\right)$
  - Density $\pi\left(\eta_i^t | z^t, a_i^t\right)$ of idiosyncratic shocks $\eta_i^t = \{\eta_{i0}, ..., \eta_{it}\}$
  - Affected by **unobservable** action $a_i^t = \{a_{i0}, ..., a_{it}\}$ + **observable** aggregate shocks z^t
- Dynamic incentive contract:
  $\{\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}\} = \left\{w_{it}\left(\eta_i^t, z^t; z_0, b_{i0}\right), a_{it}\left(\eta_i^t, z^t; z_0, b_{i0}\right), c_{it}\left(\eta_i^t, z^t; z_0, b_{i0}\right), b_{i,t+1}\left(\eta_i^t, z^t; z_0, b_{i0}\right)\right\}_{t=0,\eta_i^t,z^t}^{\infty}$
- Value of filled vacancy at time zero:

$$V \equiv \sum_{t=0}^{\infty} \int \int \left(\beta\left(1-s\right)\right)^t \left(f\left(z_t, \eta_{it}\right) - w_{it}\left(\eta_i^t, z^t; z_0, b_{i0}\right)\right) \pi\left(\eta_i^t, z^t | z_0, b_{i0}, a_i^t\right) d\eta_i^t dz^t$$

s: exogenous separation rate, β: discount factor

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$\text{[PC]} \quad \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^{t} \mathbb{E}\left[u\left(c_{it}, a_{it}\right) + \beta s \mathcal{E}\left(b_{i,t+1}, z_{t+1}\right) | z_{0}, b_{i0}, a_{i}^{t}\right] = \mathcal{E}\left(b_{i0}, z_{0}\right)$$

s.t. $\quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{\text{b}}$ assuming r fixed

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$\text{[PC]} \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{E}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t\right] = \mathcal{E}(b_{i0}, z_0)$$

$$\text{s.t.} \quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{b} \quad \text{assuming } r \text{ fixed}$$

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

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$$\text{[PC]} \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(c_{it}, a_{it}) + \beta s \mathcal{E}(b_{i,t+1}, z_{t+1}) \,|z_0, b_{i0}, a_i^t \right] = \mathcal{E}(b_{i0}, z_0)$$

s.t. $b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{\mathrm{b}}$ assuming r fixed

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$$\text{[IC]} \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(\tilde{c}_{it}, \tilde{a}_{it}) + \beta s \mathcal{E}\left(\tilde{b}_{i,t+1}, z_{t+1}\right) |z_0, b_{i0}, \tilde{a}_i^t \right] \leq \mathcal{E}(b_{i0}, z_0)$$

s.t. $\tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{\mathrm{b}}$ assuming r fixed

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- Loosely denote constraints as $PC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) = 0$, $IC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \leq 0$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

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s.t. $\quad \tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{\mathrm{b}}$ assuming r fixed

- Maximized value of a filled vacancy:

$$J(z_0, b_{i0}) \equiv \max_{\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})}_{\text{vacancy value}} + \underbrace{\langle \boldsymbol{\mu}, PC(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})\rangle}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b}; z_0, b_{i0})\rangle}_{\text{incentive compatibility}}$$

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$$\text{s.t.} \quad b_{i,t+1}(\eta_i^t, z^t) + c_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, b_{it}(\eta_i^t, z^t), \quad b_{it}(\eta_i^t, z^t) \geq \underline{\text{b}} \quad \text{assuming } r \text{ fixed}$$

- Incentive compatibility constraints: for all $\tilde{a}_i^t \in [\underline{a}, \overline{a}]^t$, $\tilde{c}_i^t \in [\underline{c}, \overline{c}]^t$, $\tilde{b}_i^{t+1} \geq [\underline{b}]^t$

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$$\text{s.t.} \quad \tilde{b}_{i,t+1}(\eta_i^t, z^t) + \tilde{c}_{it}(\eta_i^t, z^t) = w_{it}(\eta_i^t, z^t) + (1+r)\, \tilde{b}_{it}(\eta_i^t, z^t), \quad \tilde{b}_{it}(\eta_i^t, z^t) \geq \underline{\text{b}} \quad \text{assuming } r \text{ fixed}$$

- Maximized value of a filled vacancy:

$$J(z_0, b_{i0}) \equiv \max_{\mathbf{w},\mathbf{a},\mathbf{c},\mathbf{b},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0})}_{\text{vacancy value}} + \underbrace{\langle \boldsymbol{\mu}, PC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \rangle}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w}, \mathbf{a}, \mathbf{c}, \mathbf{b}; z_0, b_{i0}) \rangle}_{\text{incentive compatibility}}$$

- Free entry condition pins down market tightness: $\mathbb{E}_b[J(z_0, b_{i0})] = \frac{\kappa}{q(\theta_0)}$

Static Model Proof Outline

- Firm's value given by Lagrangian

$$J(z) = \mathbb{E}[z(a^*(z) + \eta) - w^*(z,y)] + \lambda \cdot \left(\mathbb{E}\left[u(w^*(z,y), a^*(z))\right] - \mathcal{E}\right) + \mu \cdot [IC]$$

  for λ and μ Lagrange multipliers on PC and IC, respectively.
- Take derivative w.r.t. z

$$\frac{dJ}{dz} = \mathbb{E}[a^*(z)] + z\frac{d\mathbb{E}[a^*(z,y)]}{dz} - \frac{d\mathbb{E}[w^*(z,y)]}{dz} + [PC] \cdot \frac{d\lambda}{dz} + [IC] \cdot \frac{d\mu}{dz} + \lambda\frac{\partial PC}{\partial z} + \mu\frac{\partial IC}{\partial z}$$

- Blue terms sum to zero by envelope theorem
- Red terms equal to zero as z does not appear in them
- Thus only direct term left

Intuition for Envelope Result

- Firm is trading off incentive provision and insurance
- Suppose z rises $\Rightarrow$ changes desired effort
- If z and a complements (as here), increase desired effort
- Incentivize worker $\Rightarrow$ steeper output-earnings schedule $\Rightarrow$ expose worker to more risk
- Must pay worker more in expectation to compensate for more risk
- Mean wage and effort move together
- Optimal contract $\Rightarrow$ marginal incentive and insurance motives offset

Aside: Interpretation of Bonus vs. Base Pay in Incentive Model

What is a bonus payment?

- Incentive contract is $w^*(\eta)$ = mapping from idiosyncratic shocks to wages
- Base wage = "typical" value of $w^*(\eta)$
- Bonus wage = $w^*(\eta)$ – base wage

Example 1: two values of idiosyncratic shock $\eta \in \{\eta_L, \eta_H\}$

- Base = $\min_\eta w(\eta)$, Bonus = $w(\eta) -$Base

Example 2: continuous distribution of η

- Base = $\mathbb{E}_\eta[w(\eta)]$, Bonus = $w(\eta) -$Base

$\rightarrow$ Specific form will depend on context but does not affect equivalence results

Isomorphism of Bargaining to TIOLI w/ cyclical unemp. benefit

Suppose worker and firm Nash bargain over promised utility $\mathcal{E}$ when meet

$$\mathcal{E}(z) \equiv arg \max_{E} J(z, E)^{\phi} \cdot (E - U(z))^{1-\phi}$$

Key: firm profits still determine employment fluctuations and defined as

$$\begin{aligned} J(z, \mathcal{E}) = & \max_{\mathbf{a}, \mathbf{w}} EPDV(\text{Profits}) \\ s.t. \quad & \mathbf{a} \text{ is incentive compatible} \\ & \text{Worker's expected utility under contract } \geq \mathcal{E} \end{aligned}$$

Under TIOLI contract offers, $\mathcal{E}(z) = U(z)$ so that

$$\mathcal{E}(z) = U(z) = b(z) + \beta \mathbb{E}[\mathcal{E}(z')|z]$$

whether $\mathcal{E}(z)$ moves due to bargaining or $b(z)$ moves is first-order irrelevant to $J(z)$ and thus unemployment

Optimal Contract

▸ Expressions ◂ Environment ◂ ID: Main Slides ◂ ID: Equations

- Wages are a random walk

$$\ln w_{it} = \ln w_{it-1} + \psi h'(a_t) \cdot \eta - \frac{1}{2}(\sigma_\eta h'(a_t))^2$$

  initialized at

$$w_{-1}(z_0) = \psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right)$$

  for $\psi \equiv (\beta(1-s))^{-1}$ dubbed the "pass-through parameter" and $Y(z_0)$ the EPDV of output

- Effort increasing in z_t and satisfies

$$a_t(z_t) = \left[\frac{z_t a_t(z_t)}{\psi \left(Y(z_0) - \frac{\kappa}{q(\theta_0)} \right)} - \frac{\psi}{\varepsilon}(h'(a_t)\sigma_\eta)^2 \right]^{\frac{\varepsilon}{1+\varepsilon}}$$

- Worker utility under the contract equals $\mathcal{E}(z_0)$, the EPDV of unemployment utility
  - Cyclical $b(z) \implies w_{-1}(z)$ cyclical so influence new hire wages

Quantitative Contract: More Expressions

- EPDV of output

$$Y(z_0) \equiv \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}\left[z_t(a_t + \eta_t) | z_0\right]$$

- Worker utility under contract

$$\frac{\log w_{-1}}{\psi} - \mathbb{E}_0 \left[\sum_{t=0}^{\infty} (\beta(1-s))^{t-1} \left(\frac{\psi}{2} (h'(a_t)\sigma_\eta)^2 + h(a_t) + \beta s \mathcal{E}(z_{t+1}) \right) | z_0 \right] = \underbrace{\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \ln b(z_t) | z_0 \right]}_{\mathcal{E}(z_0)}.$$

Identification: Some Equations ▸ Optimal Contract ◂

Variance of log wage growth is

$$Var(\Delta \ln w_t) = \psi^2 Var(h'(a)\eta) \approx (\psi h'(a))^2 \sigma_\eta^2$$

Pass through of idiosyncratic firm output shocks to wages is

$$\frac{d \ln w_{it}}{d \ln y_{it}} = \frac{d \ln w}{d\eta} \cdot \left(\frac{d \ln y}{d\eta}\right)^{-1} = \psi h'(a) \cdot \left(\frac{1}{a+\eta}\right)^{-1}$$

Wages martingale $\implies$ new hire wages equal to w_{-1}/ψ in expectation, and $\ln w_{-1}$ equal to outside option:

$$\frac{\log w_{-1}}{\psi} - \mathbb{E}_0\left[\sum_{t=0}^{\infty}(\beta(1-s))^{t-1}\left(\frac{\psi}{2}(h'(a_t)\sigma_\eta)^2 + h(a_t) + \beta s \mathcal{E}(z_{t+1})\right)|z_0\right] = \mathbb{E}_0\left[\sum_{t=0}^{\infty}\beta^t(\ln\gamma + \chi \ln z_t)|z_0\right]$$

Differentiating both sides w.r.t. z shows clear relationship between χ (RHS) and $d \ln w_{-1}/d \ln z_0$

Externally Calibrated Parameters

Parameter	Description	Value	Source
β	Discount Factor	$0.99^{(1/3)}$	Petrosky-Nadeau & Zhang (2017)
s	Separation Rate	0.031	Re-computed, following Shimer (2005)
κ	Vacancy Cost	0.45	Petrosky-Nadeau & Zhang (2017)
ι	Matching Function	0.8	Petrosky-Nadeau & Zhang (2017)
ρ_z	Persistence of z	0.966	Fernald (2012)
σ_z	S.D. of z shocks	0.0056	Fernald (2012)

Estimated Parameters

Parameter	Description	Estimate	Bargain Estimate
σ_η	Std. Dev. of Noise	0.52	0^*
χ	Elasticity of unemp. benefit to cycle	0.49	0.63
γ	Steady State unemp. benefit	0.43	0.48
ε	Effort Disutility Elasticity	3.9	1^*

Equivalence Theorem Numerically

Figures/EGSS-Simulations/fix_hbar1/tightness.pdf

Figures/EGSS-Simulations/fix_hbar1/profit

Wage Differences: Full model vs Incentives Only

Figures/EGSS-Simulations/fix_hbar1/wages.pdf

Figures/EGSS-Simulations/fix_hbar1/effort

Calculating Share of Wage Cyclicality due to Bargaining

1. Calculate total profit cyclicality in full model $\dfrac{dJ}{dz}$
2. Calculate direct productivity effect

$$(\mathbf{A}) = \sum_{t=0}^{\infty} (\beta(1-s))^t \mathbb{E}_0 f_z(z_t, \eta_{it}) \frac{\partial z_t}{\partial z_0}$$

3. Calculate "$(\mathbf{C})$ term" as difference between profit cyclicality and direct productivity effect

$$(\mathbf{C}) = \frac{dJ}{dz} - (\mathbf{A})$$

4. Bargained wage cyclicality share is share of profit fluctuations due to (C) term

$$BWS = -\frac{(\mathbf{C})}{dJ/\,dz}$$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} \left(\beta\left(1-s\right)\right)^{t} \mathbb{E}\left[u\left(w_{it}, a_{it}\right) + \beta s \mathcal{E}\left(z_{t+1}\right) | z_0, a_i^t\right] = \mathcal{E}\left(z_0\right)$$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s \mathcal{E}(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{E}(z_0)$$

- Incentive compatibility constraints: for all $\left\{\tilde{a}\left(\eta_i^{t-1}, z^t; z_0\right)\right\}_{t=0,\eta_i^t,z^t}^{\infty}$

$$[\text{IC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, \tilde{a}_{it}) + \beta s \mathcal{E}(z_{t+1}) \,|z_0, \tilde{a}_i^t\right] \leq \mathcal{E}(z_0)$$

- Loosely denote constraints as $PC(\mathbf{w}, \mathbf{a}; z_0) = 0$, $IC(\mathbf{w}, \mathbf{a}; z_0) \leq 0$

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[\text{PC}] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s \mathcal{E}(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{E}(z_0)$$

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- Maximized value of a filled vacancy:

$$J(z_0) \equiv \max_{\mathbf{w},\mathbf{a},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V(\mathbf{w},\mathbf{a};z_0)}_{\text{vacancy value}} + \underbrace{\mu PC(\mathbf{w},\mathbf{a};z_0)}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w},\mathbf{a};z_0)\rangle}_{\text{incentive compatibility}}$$

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- Free entry condition pins down market tightness: $J(z_0) = \frac{\kappa}{q(\theta_0)}$

A Dynamic Incentive Contract Equivalence Theorem

Assume (i) local constraints are globally incentive compatible (ii) unemployment benefits b are constant.

Technical Assumptions

The elasticity of market tightness with respect to aggregate shocks is to a first order

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t E_{0,a^*} f_z\left(z_t,\eta_{it}\right)\frac{\partial z_t}{\partial z_0}z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\left(E_{0,a^*} f\left(z_t,\eta_{it}\right) - E_{0,a^*} w_{it}^*\right)},$$

where a_{it}^* and w_{it}^* are effort and wages under the firm's optimal incentive pay contract.

The elasticity of market tightness in a rigid wage economy with $w=\bar{w}$ and $a=\bar{a}$ is

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu}\frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t E_{0,\bar{a}} f_z\left(z_t,\eta_{it}\right)\frac{\partial z_t}{\partial z_0}z_0}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^t\left(E_{0,\bar{a}} f\left(z_t,\eta_{it}\right) - E_0\bar{w}\right)}.$$

Equivalence in Richer Models

- **Private savings and borrowing constraints** (Aiyagari 1993; Krusell et al 2010)
  - Equivalent impact elasticities
- **Endogenous separations** (Mortensen & Pissarides 1994)
  - Equivalent impact elasticities

Numerical Results: Assuming All Wage Cyclicality is Bargaining

▸ Return

	Model: source of wage flexibility		
Moment	(1) Incentives + Bargaining	(2) Bargaining Only	(3) Bargaining: BWF only
$\partial \mathbb{E}[\log w_0]/\partial u$	-1.00	-1.00	-0.60
$\partial \log \theta_0 / \partial \log z_0$	13.3	10.4	13.0
$\text{std}(\log u_t)$	0.103	0.078	0.103
$\text{std}(\log \mathbb{E}[y_{it} \mid \text{emp}])$	0.07	0.02	0.02
W_0 / Y_0	0.96	0.96	0.96
$\partial \log Y_0 / \partial \log z_0$	0.68	0.51	0.51
$\partial \log W_0 / \partial \log z_0$	0.44	0.32	0.26

Illustration: Wage Cyclicality and Unemployment Responsiveness

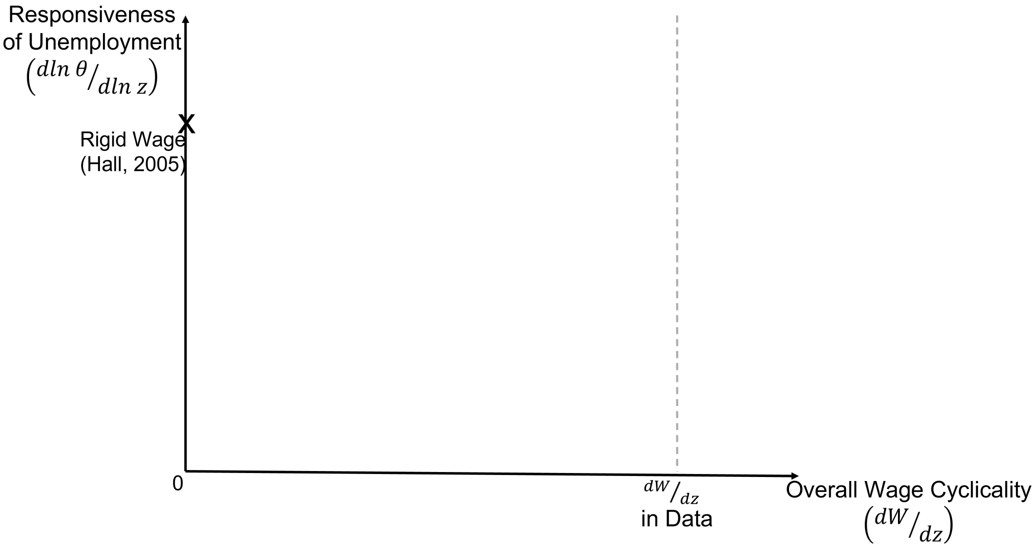


Illustration: Wage Cyclicality and Unemployment Responsiveness

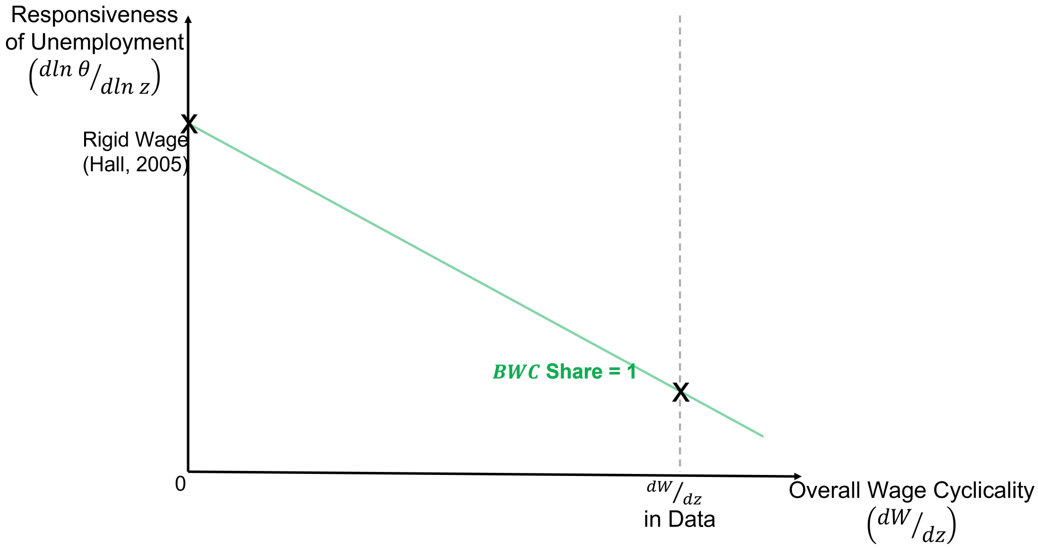


Illustration: Wage Cyclicality and Unemployment Responsiveness

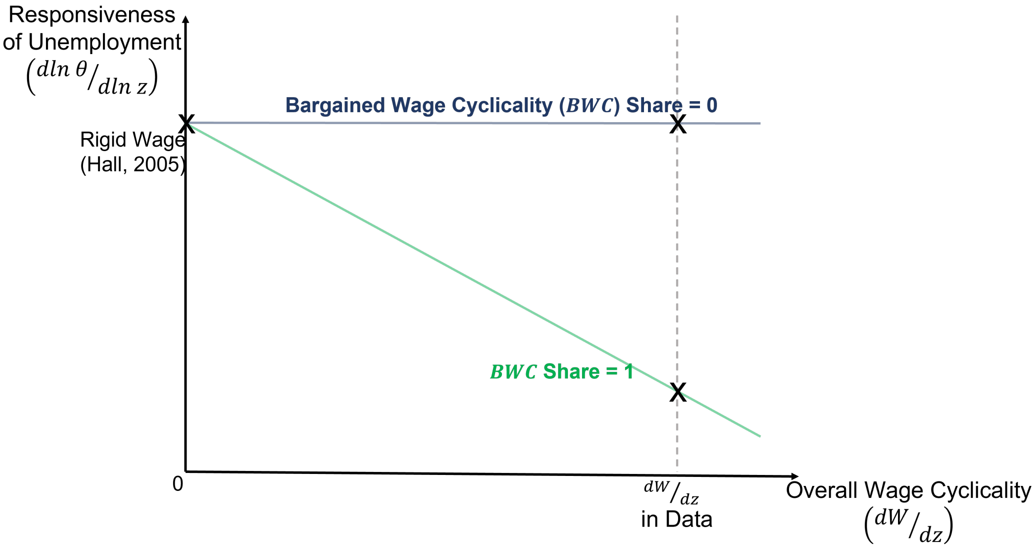
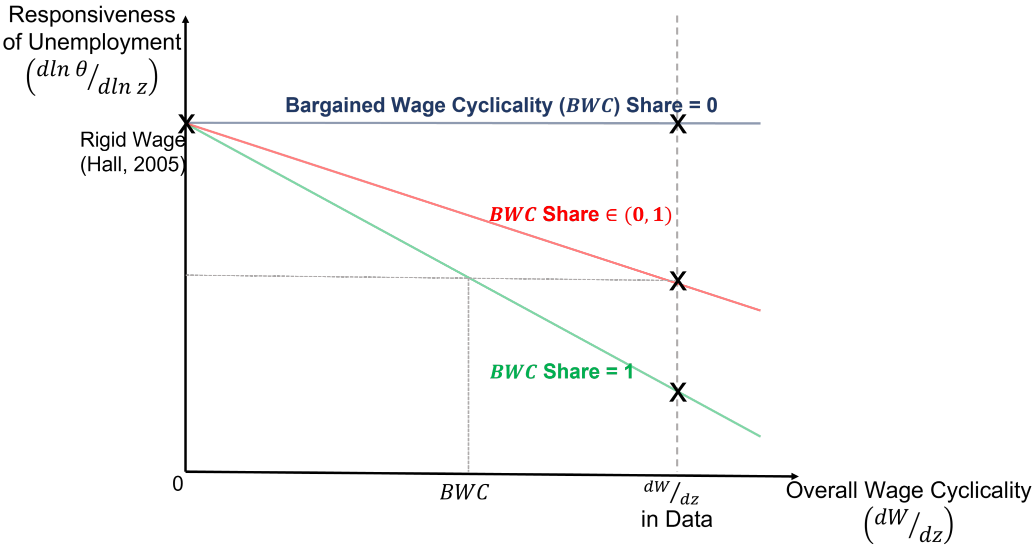


Illustration: Wage Cyclicality and Unemployment Responsiveness



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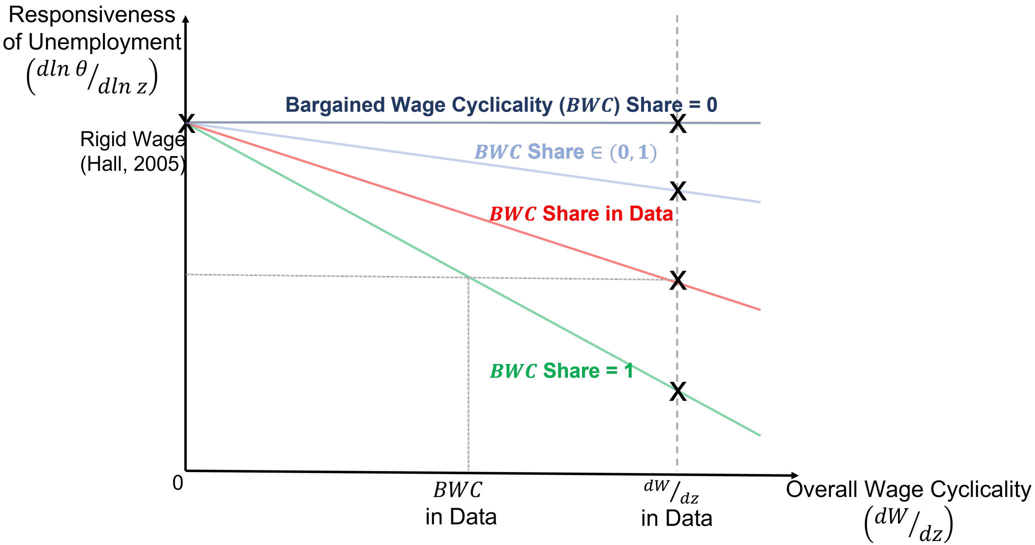
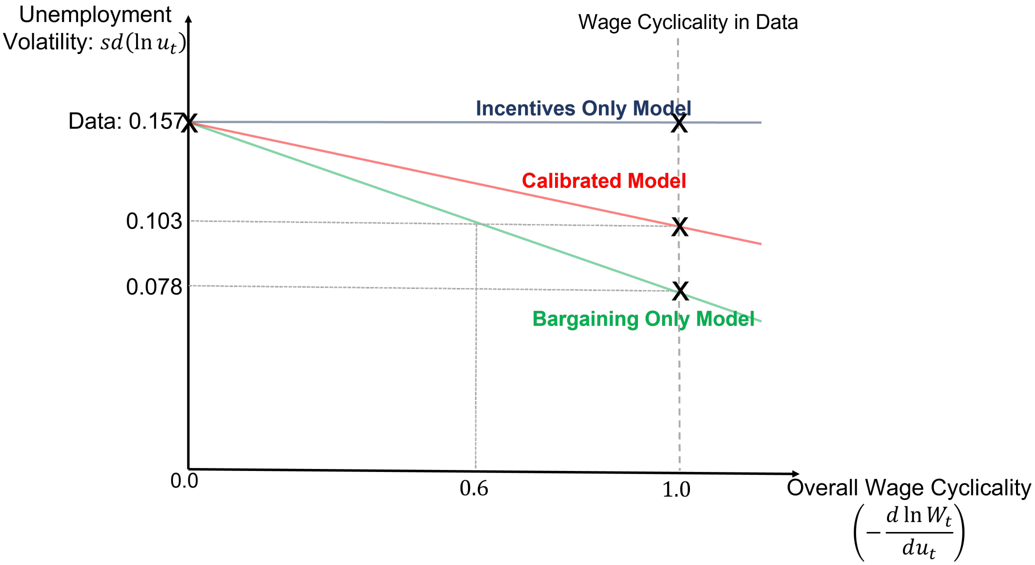


Quantitative Results: Graphical Illustration



Literature

- **Empirics of wage adjustment.** Devereux 2001; Swanson 2007; Shin & Solon 2007; Carneiro et al 2012; Le Bihan et al 2012; Haefke et al 2013; Kudlyak 2014; Sigurdsson & Sigurdardottir 2016; Kurmann & McEntarfer 2019; Grigsby et al 2021; Schaefer & Singleton 2022; Hazell & Taska 2022; Bils et al. 2023

  Contribution: model of wage setting consistent with micro evidence on bonuses

- **Wage adjustment and unemployment dynamics.** Shimer 2005; Hall 2005; Gertler & Trigari 2009; Christiano et al 2005; Gertler et al 2009; Trigari 2009; Christiano et al 2016; Gertler et al 2020; Blanco et al 2022

  Contribution: Flexible incentive pay does not dampen unemployment fluctuations

- **Incentive contracts.** Holmstrom 1979; Holmstrom & Milgrom 1987; Sannikov 2008; Edmans et al 2012; Doligalski et al. 2023

  Contribution: Characterize aggregate dynamics with general assumptions
  (E.g. non-separable utility, persistent idiosyncratic shocks, no reliance on "first order approach")

- **Sales + Rigidity.** e.g. Nakamura & Steinsson 2008; Klenow & Kryvtsov 2008; Kehoe & Midrigan 2008; Eichenbaum et al 2011

  Contribution: incentive pay does not affect aggregate rigidity even if bonuses are cyclical

- Frictional labor market: vacancy filling rate $q_t \equiv q(\theta_t)$, market tightness $\theta_t \equiv v_t / u_t$
- Production function $y_{it} = f\left(z_t, \eta_{it}\right)$
  - Density $\pi\left(\eta_i^t | a_i^t\right)$ of idiosyncratic shocks $\eta_i^t = \{\eta_{i0}, ..., \eta_{it}\}$
  - Affected by **unobservable** action $a_i^t = \{a_{i0}, ..., a_{it}\}$, $a_{it} \in [\underline{a}, \overline{a}]$
- Dynamic incentive contract: $\{\mathbf{a}, \mathbf{w}\} = \left\{a\left(\eta_i^{t-1}, z^t; z_0\right), w\left(\eta_i^t, z^t; z_0\right)\right\}_{t=0,\eta_i^t,z^t}^{\infty}$
- Value of filled vacancy at time zero:

$$V(\mathbf{a}, \mathbf{w}; z_0) \equiv \sum_{t=0}^{\infty} \int \int \left(\beta\left(1-s\right)\right)^t \left(f\left(z_t, \eta_{it}\right) - w_{it}\left(\eta_i^t, z^t; z_0\right)\right) \pi\left(\eta_i^t, z^t | a_i^t\right) d\eta_i^t dz^t$$

s: exogenous separation rate, β: discount factor

- Ex-ante participation constraint: at start of match firm offers worker value of unemployment

$$[PC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, a_{it}) + \beta s U(z_{t+1}) \,|z_0, a_i^t\right] = \mathcal{B}(z_0)$$

  - "Reduced form" **bargaining power** if $\mathcal{B}'(z_0) > 0$
  - Formulation of bargaining power nests e.g. Nash w/ cyclical outside option, Hall-Milgrom bargaining

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- Incentive compatibility constraints: for all $\left\{\tilde{a}\left(\eta_i^{t-1}, z^t; z_0\right)\right\}_{t=0,\eta_i^t,z^t}^{\infty}$

$$[IC] \quad \sum_{t=0}^{\infty} (\beta(1-s))^t \, \mathbb{E}\left[u(w_{it}, \tilde{a}_{it}) + \beta s U(z_{t+1}) \,|z_0, \tilde{a}_i^t\right] \leq \mathcal{B}(z_0)$$

- Loosely denote constraints as $PC(\mathbf{w}, \mathbf{a}; z_0) = 0$, $IC(\mathbf{w}, \mathbf{a}; z_0) \leq 0$

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- Maximized value of a filled vacancy:

$$J\left(z_0\right) \equiv \max_{\mathbf{w},\mathbf{a},\boldsymbol{\mu},\boldsymbol{\lambda}} \underbrace{V\left(\mathbf{w}, \mathbf{a}; z_0\right)}_{\text{vacancy value}} + \underbrace{\mu PC(\mathbf{w}, \mathbf{a}; z_0)}_{\text{participation}} + \underbrace{\langle \boldsymbol{\lambda}, IC(\mathbf{w}, \mathbf{a}; z_0)\rangle}_{\text{incentive compatibility}}$$

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- Free entry condition pins down market tightness: $J(z_0) = \frac{\kappa}{q(\theta_0)}$

Incentive Wage Cyclicality Doesn't Mute Unemployment Fluct's

Temporarily shut down bargaining power $\rightarrow$ all wage cyclicality is due to incentives

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Assume: (i) proximity to non-stochastic steady state (ii) production function is h.o.d. 1 in z, (iii) **contracts offer constant promised utility $\mathcal{B}$.** *Then in the flexible incentive pay*

$$\frac{d\log\theta_0}{d\log z_0} = \frac{1}{\nu_0}\frac{1}{1 - \textit{labor share}}$$

where

$$\textit{labor share} = \frac{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^{t} E_0 w_{it}}{\sum_{t=0}^{\infty}\left(\beta\left(1-s\right)\right)^{t} E_{0,a} f\left(z_0, \eta_{it}\right)}$$

The same equations characterize a rigid wage economy with $w_{it} = \bar{w}$, $a_{it} = \bar{a}$

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Implications: incentive wage cyclicality does not mute unemployment fluctuations

- In an incentive pay economy with **flexible** dynamic incentive pay
- Unemployment dynamics behave "as if" wages are **rigid**

Parameterized Dynamic Incentive Contract Model

- Linear production
- Normally distributed noise $\eta \sim \mathcal{N}(0, \sigma_\eta)$, agg. productivity AR(1) in logs
- Log and isolastic utility

$$u(c, a) = \ln c - \frac{a^{1+1/\varepsilon}}{1+1/\varepsilon}$$

- Agent observes η before deciding action
- Worker's flow consumption during unemployment is $b(z) \equiv \gamma z^{\chi}$
- Firm makes take-it-or-leave-it offers to worker so

$$\mathcal{E}(z_0) = \sum_{t=0}^{\infty} \beta^t \mathbb{E}\left[\ln \gamma + \chi \ln z_t | z_t\right]$$

  - First-order equivalent to fixed b and bargaining over surplus
  - χ governs cyclicality of promised utility and thus "bargained wage cyclicality"

Optimal Contract

Regularity Conditions

1. The distribution of innovations to aggregate productivity does not depend on initial productivity z_0

$$z_t = \mathbb{E}[z_t|z_0] + \varepsilon_t, \qquad \varepsilon^t \sim G_t(\varepsilon^t)$$

2. Utility and production functions are differentiable functions of z, a and c

3. At least one of the following conditions holds
   3.1 The set of feasible contracts is convex and compact. The worker's optimal effort choice is fully determined by the first order conditions to their problem. Finally, idiosyncratic shocks η_{it} follow a Markov process: $\pi_t(\eta_t|\eta^{t-1}, a^t) = \pi_t(\eta_t|\eta_{t-1}, a_t)$
   3.2 Feasible contracts are continuous and twice differentiable in their arguments (z^t, η^t) with uniformly bounded first and second derivatives.