

Tom: howdy
me: im having trouble with sthg while reviewing II.2 before moving on
Tom: what's that?
me: This result supports the intuition that even in higher-dimensional spaces, lines are straight and planes are flat. For any two points in a linear surface, the line segment connecting them is contained in that surface (this is easily checked from the definition). But if the surface has a bend then that would allow for a shortcut (shown here grayed, while the segment from P to Q that is contained in the surface is solid).
pg 41
i thought i understood this
but now im doubting that
Tom: i am aware of that notion, but I didn't think too deeply about that one yet
the non-euclidean geometries
my way of thinking about it is this
me: its supposed to intuitively show us that linear surfaces in R^n are flat
and u said sthg about this when u mentioned u now understand that the line can be 'inside' the surface
and not necessarily on it
Tom: yeah, if the "surface" is more than 2D
me: ok
Tom: but in that case, "on" and "inside" i think become interchangeable terms
me: but i still cannot convince myself that that paragraph supports any intuition
Tom: yeah, you might be right about that
i think we would need more rigorous definitions of "straight" and "flat"
me: even if we had those
i don't see the point
Tom: what do you mean?
me: you take 2 points P and Q on a linear surface (whatever that is)
ok great and then what?
Tom: well, if you think that the "surface" you are dealing with is linear
me: you know that the shortest distance between them is the 'vector' between their endpoints
Tom: it then follows that you can, from those two points, create an equation for a line
me: and a vector in euclidean spaces is a straight flat thing
OK
but .. so?
Tom: and that all points on that line are then also "on" a surface
not same as a vector
the line projects infinitely
me: but what does what u said have to do with the triangle inequality
Tom: not sure about that
I was just talking about my own understanding of what it means for a surface to be linear
me: that's supposed to be the whole point
Tom: I don't have a good explanation for how triangle equality ties in
me: I'm ok with that
i think we need to find out what he means
Tom: my suspicion is that this is a poorly worded glimpse into non-euclidean geometry concepts
much like some of the hints at analysis that we ran into in calculus
me: maybe somebody explains it better somewhere?
ok
my original question was
why is the equation of a line produce a straight line?
Tom: lovely question :)
me: original: first think i was thinking about before reading this chapter and hoping cauchy-schwarz provides some answer
Tom: either there is some really profound shit underlying that, or else it is just something we've agreed upon to be true
me: and i had wrongly convinced myself this paragraph does just that
but now ... i don't know
Tom: no, i don't think anything in the chapter tells us why the equation of a line is straight
me: because if sthg does
Tom: or at least not in any straightforward manner that I can see
me: then that can be generalized
using i think the same idea he is trying to express with that 'curved surface'
Tom: it is an excellent question tho
me: maybe its a very silly question
but im always searching for the link between algebra and geometry
Tom: i don't think so, because it seems very simple, but I don't even know where to begin to answer it
me: and i was hoping the CS inequality was IT
Tom: it could be that there is a clever way to make use of CS to make some headway
you could be intuitively on to something
but I really don't think the paragraph you cited does that at all
me: well I was thinking: this means proving that from any 2 points taken that satisfy the line equation, if you make a line equation, it will turn out to be matching the original line
would that maybe prove something?
compared to let say a parabola where this would not be the case?
not sure...
Tom: could be
if you pick any two random points, there are an infinite number of parabolas to fit to them
but there is only a single choice for a line
i don't know if that addresses "straightness"
me: that's also a good point !
maybe with the help of CS it does.
Tom: but it does say something interesting and unique about lines as opposed to any other structures
they are simpler in that sense
me: i think there are a couple of concepts we first need to 'separate' and then link
Tom: probably they are the only structure that can be uniquely defined in space by two points
what is interesting is that this is true no matter what the dimensionality of the space
me: ok and what is the paragraph trying to say then?
Tom: I think each of the first three sentences is addressing a separate concept (although related)
first talks about "flatness" of lines and planes
second talks about lines being "contained" in the surfaces
third talks about curved surfaces, with the implication of curved space. He uses the word surface, but I think he is really referring to space
it is all a bit muddy when you try to analyze it (or is to me)
me: no i think he means a surface as a linear surface and trying to show that it has to be flat
because it can only contain lines
Tom: but that isn't true
a surface could contain a parabola just as easily
think of a 2D plane in a 3D space - a parabola could live on that just fine
Sent at 10:14 PM on Wednesday
Tom: more specifically, it is saying "pick any two points on/in the surface, the line that passes through these points is itself always contained in the surface"
me: and why is it?
and if that does so what?
Tom: or, to put another way, it is impossible to pick two points on/in the surface, and then create a line for which every point in that line is not in the surface
i think this has to do with the idea of "vector spaces", which linear algebra gets into later
the main thing being that if you have a set of things, you want some operations that give you results that are also in your set
which I think is referred to as being "closed over"
me: ok and how is this related to flat
Tom: take 2 flat things, combine them with operation X. Result is guaranteed to be flat as well
so you can then compose things however many layers deep you want

without worrying that you all of a sudden ended up with a non-flat thing to which the rules no longer apply
an example
if you have two gaussian distributed random variables
and you add them
me: how is this related to 'This result supports the intuition that even in higher-dimensional spaces, lines are straight and planes are flat.'
Tom: the result is also a gaussian
it doesn't - I have no explanation for that sentence :)
me: so i think we are missing something
Tom: well, the second part of the sentence is true
but there is no direct link explaining how the result supports it
me: ok second part
why is it true?
Tom: you can prove it, I think one of the exercises does something similar to that
although I'd have to dig through a bit to find which one again
me: ok
i think my main problem
i don't see why a linear surface (an algebraic concept) would have to be flat (a geometric thing)
Tom: maybe here is another way to think about it
me: or in other words
is there a algebraic definition of flatness?
and how is it linked to geometry
Tom: I think (and this is just speculation on my part)
that you must first define "straightness" of a line
me: or the really basic question: why the hell is there no algebraic proof of pythagoras hahahaha
Tom: and then if you can do that, and you say that your objects must fit that definition of lines being contained in the surface, you say they are flat
me: lets define a line first
Tom: I suspect that "straight" is a more fundamental definition than "flat"
one way to define it, and I suspect it is the correct way, is as being the shortest distance between two points (maybe this is just for a line segment, rather than a full line)
from that standpoint, perhaps CS inequality speaks to that issue
me: is that algebraic or geometric?
Tom: well, CS appears to me to be purely algebraic
but then again, I suppose we have to be very aware of anything pythagorean slipping in..
me: Definition 2.
A line is breadthless length.

Definition 3.
The ends of a line are points.

Definition 4.
A straight line is a line which lies evenly with the points on itself.
:)

Sent at 10:26 PM on Wednesday
me: compare that to : http://en.wikipedia.org/wiki/Hilbert%27s_axioms
Tom: it may be that we are dancing very close to axioms with these questions
which have no basis beyond "because we say so" :P

.....

me: ok i have at least defined what i would like to be 'proved' in terms of straightness at least
can i tell you?
Tom: yes, please
me: ok so you have 2 things
one a purely algebraic definition of a line in Rn
A+Bt, with t a scalar
Tom: right
me: the other thing is a barely acceptable definition of straightness
it comes from 'donkey straight' as in if you put a donkey on pt A and its food on pt B it will know that the shortest way is a straight line to B.
but i provide this using the triangle inequality and i say
Tom: heheheh "barely acceptable"
me: straight means
in Rn
if you take 3 vectors
u v and w
such that w = u + v as vector additions
w is straight
hmmmm
i stumped myself now :)
Tom: i don't think that defines straight
me: nope
Tom: I think it is a result of straightness
how about this?
we think of a line as being a set of points
me: how about : a vector is straight
duh?
Tom: but does that define straight?
pretend we couldn't use the word "straight"
a vector is foo
me: if its one vector its straight
Tom: how do we define "foo"?
so we can't use any intuitive or visual notions
me: maybe we could use the inverse idea
Tom: a "line" is a set of points with the property "foo"
inverse idea?
me: the addition of 2 vectors is foo when the length of that addition is exactly equal to the sum of the lengths
not less
Tom: hmmmm, that's interesting
me: enter the circular dancing pattern
Tom: i think maybe that works if we can define "distance" without reference to straight/foo
:)
me: it is interesting to ask if a vector is straight by definition
yes
distance is the pythagoras theorem equation in Rn
(length)
Tom: it is, however, not the only possible measure of distance
me: nope
Tom: Euclidean distance is also called an L2 norm
me: but that's how im trying to define foo
Tom: but you can have L1 norm, L-infinite norm
it makes sense
me: so why L2 norm? donno you tell me
Tom: i wonder if/how the definition of "straight" changes if you change your definition of distance
well, one advantage of L2 norm over the others
is that it is rotationally invariant
me: hey i didn't define anything yet, im trying to convince of sthg that i have no clue about myself
Tom: with respect to the overall coordinate system

me: ok so
length being a (the?) rotationally invariant norm
damn good point wih the rotation invariance
now one must think about preserving straightness or not
Tom: yeah, wondering if there is even a reasonable definition of "straight" if you change the distance function..
me: mammaa help
Tom: heheheh
like can you define "straight" if using Manhattan distance rather than Euclidean??
me: now i guess it all depends
i give up
i dont know why algebraic lines are straight
but good point as u said
Tom: well, its a good question to throw around every once in awhile
me: its better to think: in which axiomatic system are lines these things straight
but then the definition of straight has to come from outside the ssyten
but that makes no sense...
Tom: but that happens in all areas of math at some point
you hit the basement
me: so straight has then to be simply defined
Tom: and there isn't anything below it
me: and then comes why the hell does it then seem to fit reality
Tom: that is a philosophical question, not a mathematical one :)
me: and rthen we do the cool thing
yes exactly
Tom: we aren't allowed to talk about "reality" when doing math! ;)
me: defer to philosophy
Tom: also, according to relativity, the lines in reality aren't quite straight
just mostly, at least if you aren't sitting next to a black hole or going at 0.9999 speed of light
me: ok so we have then no way of proving that algebraic lines are straight on a piece of paper in realityu
in R2
Tom: not sure yet, i think there is definitely more room for discussion on this topic
and given that it took thousands of years to come up with non-euclidean geometry, I don't think we'll solve this in one IM chat
me: ok so then the basic question to hang is why in R3 is A+Bt fit a piece of straightened string in reality
at least for me
Tom: my way of thinking about it is this
Sent at 11:04 PM on Wednesday
Tom: a line is a set A such that you can: (1) pick any two points in the set, (2) create a new equation for a line using those two points, which then creates a set B of points. Sets A and B will always be identical.
if those properties are true, you have a line
me: good enough
still does not explain the connection to reality but good
i think there is some theory of symmetry at play
Tom: 1sec, phone
me: which related to euclids definition actually
A straight line is a line which lies evenly with the points on itself.
i.e: symmetrically.
still it does not help is with reality
except by maybe adding shortest distance (back to distance)
Sent at 11:09 PM on Wednesday
me: this is not bad: http://en.wikipedia.org/wiki/Line_%28geometry%29
Sent at 11:11 PM on Wednesday
Tom: interesting:
For instance, in Coordinate geometry <http://en.wikipedia.org/wiki/Coordinate_geometry> (analytical geometry of high school) lines in a plane are often defined as the set of points whose coordinates satisfy a given linear equation <http://en.wikipedia.org/wiki/Linear_equation>, but in a more abstract setting, such as incidence geometry <http://en.wikipedia.org/wiki/Incidence_geometry>, a line may be an independent object, distinct from the set of points which lie on it.
"distinct from the set of points which lie on it." wtf?
Sent at 11:12 PM on Wednesday
Tom: the first part of that sounds like the linear equation is the assumed thing
so you don't ask - why does a linear equation represent a line - you simply define a line as being the thing that conforms to the linear equation
maybe that is a bit chicken-and-eggy, but oh well..
me: or hehe: <http://www.prep101.com/lSAT/Arguments/Main%20Point.pdf>
yes i think my question boils down to
http://www.google.com/search?q=why+is+real+life+almost+euclidean&ie=utf-8&oe=utf-8&q=t&rls=org.mozilla.en-US.official&client=firefox-a#sclient=psy-ab&hl=en&client=firefox-a&hs=pRu&rls=org.mozilla.en-US%3Aofficial&source=hp&q=is+the+world+euclidean&pbx=1&qo=is+the+world+euclidean&qf=f&aqf=&aqg=&gs_sm=e&gs_upl=48251546512159521515101010121227167712.2.11510&bav=on_2.or_r_ge_r_pw_r_cf.osb&fp=1b8c769a2b41552b&biw=1916&bih=1064
and has nothing to do with math proofs
Tom: yeah, and i think that one is still in the to-be-determined category
with strong leanings toward non-euclidean
me: and this is the 'donkey distance' part : The "straightness" of a line, interpreted as the property that it minimizes distances between its points, can be generalized and leads to the concept of geodesics on differentiable manifolds.
yes see pdf above :)
i guess the question doesnt even make sense at all
a string in real world has very little to do with an algebraic line, its comparing apples to pink elephants
Tom: well, until the two of us start building interstellar space ships, we should be just fine with the Euclidean assumption
me: now what im fine with from now on is: keep the 2 worlds apart
you can still use some intuitions from one into the other
Tom: heheh, yeah I think that is best
me: but that all
Tom: let the physicists deal with that madness :P
me: or use one as a source of definitions and systems (idealized) into the other
i will call it
the math-world interface
need a better word for interface
portal ?:)
the world-math-world portal
the WMWP
Sent at 11:21 PM on Wednesday
Tom: I like it :)
me: it causes dramatic information loss and aberration when crossed
Tom: i think there are many books written on this subject
me: in either direction
Tom: in fact i think i have a couple
years ago that i read about that tho, so the memory is foggy
me: i once read an article about it which i didnt agree with at all
but now we cam e to it from a totally different side
Tom: what aspect did you disagree with?
me: <http://www.dartmouth.edu/~matc/MathDrama/reading/Wigner.html>
all of it
but i cant remember details