[illegible] 1/a/a (order of, linear approx).

highlight the pointillistic nature!!!

$$f'(x_0) = \lim_{x \to x_0} \frac{f(x) - f(x_0)}{(x - x_0)} \equiv f'(x) = \lim_{\substack{h \to 0 \\ x \to x_0 \\ h = x - x_0}} \frac{f(x+h) - f(x)}{h}$$

$$f'(x_0) - \frac{f(x_0+h) - f(x_0)}{\underset{\to 0}{e_1}} \simeq \varepsilon_1$$

$$\left| f'(x_0) - \frac{f(x) - f(x_0)}{\underbrace{x - x_0}_{e_1}} \right| \leq \frac{1}{\underbrace{m}_{e_0}}$$

$$e_1 \geq \underbrace{\left| f'(x_0) - \frac{f(x) - f(x_0)}{\underset{e_0}{x - x_0}} \right|}_{g(e_0)}$$

$$\varepsilon_1 \geq g(\varepsilon_0)$$

$$\left| f'(x_0) - \frac{f(x) - f(x_0)}{x - x_0} \right| \begin{matrix} \simeq 0 \\ \leq \varepsilon_1 \end{matrix}$$

$h = x - x_0$, $x_0 = x - h$

$x = x_0 + h$

$h = x - x_0$ $\quad x = x_0 + h$

$$f'(x_0 + h) - \frac{f(x_0) - f(x_0+h)}{h} \Big\} \begin{matrix} x_0 = x - h \\ \simeq 0 \end{matrix}$$

$$f'(x_0 + \varepsilon) - \frac{f(x_0 + \varepsilon) - f(x_0)}{\varepsilon}$$

$$f'(x_0) - \frac{f(x_0) - f(x_0 - \varepsilon)}{\varepsilon}$$

$$\eta\left[\frac{1}{m}\right] \quad \left[f'(x_0) - \frac{f(x) - f(x_0)}{[x - x_0]} \right]$$

$$f'(x_0) \quad [x - x_0]\} \quad \left[f'(x_0) - \frac{f(x) - f(x_0)}{x - x_0} \right]$$

$$\lim$$

$$[x - x_0]\} \rightsquigarrow \left[f'(x_0) - \frac{f(x) - f(x_0)}{[x - x_0]} \right]\}$$

$g(x)$

$f'(x_0)$ exists

$\Updownarrow$

no "$f'(x_0)$" is unique by prop of limits

$(\lim) \quad [x - x_0]\}$

$$[x - x_0]\} \quad \llcorner \; f(x_0) - \frac{f(x) - f(x_0)}{[x - x_0]}$$

$$\llcorner\} \left[\overset{g(x_0)}{f'(x_0)} - \frac{f(x) - f(x_0)}{[x - x_0]} \right] \boxed{(=0)}$$

$$\}(x - x_0) \qquad \}(x - x_0) \; \llcorner\} \left[\overset{d}{f'(x_0)} - \frac{f(x) - f(x_0)}{x - x_0} \right]$$

$\doteq$

$f'(x_0)$ is the der of f at x_0 $\doteq$ ~~$g(x_0)$~~

g do not $f'(x_0)$

d is the der of $f(x)$ at x_0

let $g(x_0) = d$, the [illegible] of f at x_0 den g by f'

absorb ∂x, 11/9/aaaa|2|

$$d - \frac{f(x) - f(x_0)}{x - x_0} \approx 0$$

$$\frac{d(x-x_0) - [f(x) - f(x_0)]}{x - x_0} \approx 0$$

$$\lim_{x \to x_0} \frac{d(x-x_0) - [f(x) - f(x_0)]}{(x-x_0)} = 0$$

$$\lim_{x \to x_0} \frac{g(x)}{x - x_0} = 0$$

$$\forall \quad \left| \frac{g(x)}{x - x_0} \right| \le \frac{1}{n}$$

$\forall$ $\varepsilon = 1/m$ | $\exists$ $x - x_0 = 1/n$

$$\left| \frac{f(x) - f(x_0)}{x - x_0} - d \right| \le \varepsilon$$

$$\left| \frac{f(x) - f(x_0) - d}{x - x_0} \right| \le \varepsilon \to 0$$

$0 < |x - x_0| < 1/n$

the processes (lip for detail)

$f(x) = f(\,)$

$f(x_0 + h) = f(x_0) + d(h) + o(x_0, h)$

$[f(x_0 + h) - (f(x_0) + d(h))]$

$o(x_0, h)$

$f(x) - [f(x_0) + d(x - x_0)]$

exp $<<(x - x_0)$

$f(x) - [f(x_0) + d(x - x_0)]$

$f(x) - [f(x_0) - d(h)]$

exp

$o(h)$

$\lim_{h \to 0} o(x_0, h)$

$\varepsilon_0 \,|\, \varepsilon_1$ $\quad \frac{f(x) - f(x_0)}{x - x_0}$, d

$\delta x\, \varepsilon$ $\quad \frac{f(x) - f(x_0)}{x - x_0}$, d

$f_a'(b) = \frac{a}{b^2}$

$f_a(x) = a\left(\frac{1}{x}\right) = a(-1)x^{-2} = -\frac{a}{x^2}$

$f_a''(x) = (-a)(-2)x^{-3} = \frac{2a}{x^3}$

$f_a(x) - f_a(a) + \dots$

red(x, x_0) is $\to$

red$\left(\frac{\text{exp}}{x - a}, 0\right)$ is $\to$

red(exp, $x - x_0$) is red $\to$

but red(exp, $x - x_0$) is 'dominated'

it's $o(\,)$

d

instead of

behave $\to$

and 'dominated'

$\delta x\, \varepsilon$ $\quad \frac{f(x) - f(a)}{x - a}$, d $\qquad \frac{f(x) - f(a) - d(x-a)}{x-a}$ exp $\qquad \delta x\, \frac{1}{\varepsilon}$ exp subtends

$$d\left(\frac{f(x) - f(a)}{x - a}, d\right) < \varepsilon$$

$\left| \frac{1}{x-a} \text{exp} \right| = \frac{|1|}{|x-a|} |\text{exp}| < \varepsilon$ $\quad d(a, b) < \varepsilon \quad |a - b| < \varepsilon$

$|\cdots - d| < \varepsilon, \quad \left| \frac{\text{exp}}{x - a} \right| < \varepsilon, \quad |\text{exp}| < \varepsilon |x - a|$ $\quad$ fixed $o(\,)$??

ebrook 2a, 1/a/af31

little oh is $\boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $g(x)$, $f(x)$

lit oh is byd f/g!

$\boxed{\delta \rightsquigarrow \varepsilon \; \div}$ — $f(x)$, $g(x)$

$\boxed{\delta \rightsquigarrow \varepsilon \; \frac{\cdot}{\cdot}}$ — $f(x)$, $g(x)$

$\frac{f(x)}{g(x)} < \varepsilon$

$f(x) << g(x)$

$\boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $g(x)$, $f(x)$

ε — g — smaller

(like $1/x$)

$1/x^2$

$1/x^3$

$\delta \rightsquigarrow \frac{1}{\varepsilon}$ — $g(x)$, $f(x)$ smaller

dist, asymp

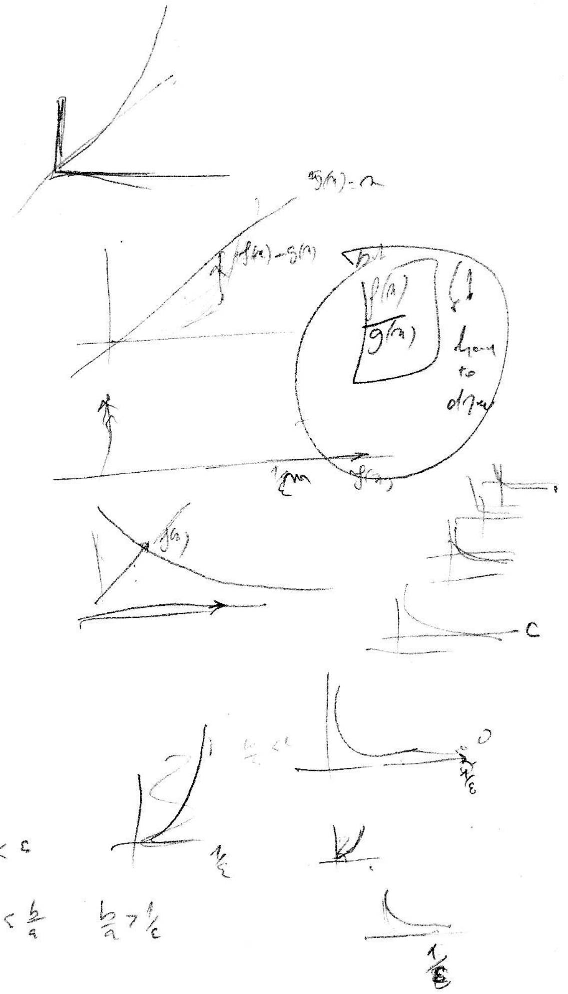
$\frac{f(x)}{g} < \varepsilon$

$g(x) > 1/x$

$\frac{f(x)}{g} < \varepsilon$

$\frac{f(x)}{g(x)} < \varepsilon$

$\frac{1}{\varepsilon} < \frac{g}{f}$ $\quad \frac{g}{f} > \frac{1}{\varepsilon}$



$\boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $|x-a|$, $1-\dots$ — $f(x)-f(a)$, $d(x-a)$

f 2. d is $o(|x-a|)$ approx of f

1. is $o(x-a)$ as $o(x)$

great

TODO $o(1)$ for cardinality

TODO $|x^2-a|$?? for $f'()$, $f''()$??

$f(x) - f(a) + d(x-a) + e(x-a)^2$

e = d' !!

To 'prove'

what is $f''(a)$ when $f'(x)$ only exists at a !! no $d'(x)$ exists

right! [cwot] p177

$f'(a) \cdot \lim \frac{f'(x) - f'(a) - d'(x-a)}{}$

$\boxed{\delta \sim \varepsilon}$ — $\frac{f'(x) - f'(a)}{x-a}$ — d'

$\boxed{\delta \sim \varepsilon}$ — $x-a$, d, $\frac{f'(x)-f'(a)-d'(x-a)}{x-a}$

TBC

Vec calc

absorb 2n, 1/a/a | 4 |

to prove that using $f(x) - f(a) + d(x-a) + e(x-a)^2$ is $o(|x-a|^2)$,
we need to define $f''(a)$ in terms of both $f(x)$ and $f'(x)$

for $f(x)$ we have $f''(a) = e$, where $f(x) - f(a) + d(x-a) + e(x-a)^2$ is $o(|x-a|^2)$ (1)

but for $f'(x)$, we need $\boxed{!!}$ to talk about $f'(a)$ as a fn of x, not only as one value $f'(a) = d$,

$$f''(x) = \lim_{x \to a} \frac{f'(x) - f'(a)}{x-a} \quad (2)$$

so this requires that $f'(a)$ exist in some interval around a, and that is correct!!! we need to recognise this, see [TWoA] (p 177)

given this assumption, we have

(1) $x, a \to \boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $|x-a|^2$ assumed; $f'(x)$; $d^?(x-a) + f''(a)$ and (2) $x, a \to \boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $(x-a)^2$; $f(x)$; $f(a) + d(x-a) + e(x-a)^2$; $f'(a)$, $f''(a)$

$a \to$ some fn of δ

$$\left| \frac{f(a+\delta) - f(a)}{(a+\delta) - a} - d \right| = \varepsilon$$

$\frac{f(a+\delta) - f(a)}{\delta_\varepsilon} - d = \varepsilon$

[4] make role explicit: $\left| \frac{f(a + \delta(\varepsilon)) - f(a)}{d} \right| = \varepsilon$

[does (1) $\Rightarrow$ (2)? assuming d is $f'(a)$, e is $f''(a)$] X — or rather, prove that $f''(a) = e$

starting from (0),

but we have

$x, a \to \boxed{\delta \rightsquigarrow \varepsilon}$ — $\frac{f(x) - f(a)}{x-a}$; $f'(a)$; $f(x)_a$

(0) $x, a \to \boxed{\delta \rightsquigarrow \frac{1}{\varepsilon}}$ — $x - a$; $f(x)$; $f(a) + f'(a)(x-a)$

$\left| d\tilde{a} - da + e(\tilde{a}-a)^2 \right|$, $f(\tilde{a}) - f(a)$

$\left| d(\tilde{a}) + e(\tilde{a}^2) \right|$, $f(\tilde{a}) - f(a)$, $\tilde{a} = a + \varepsilon$

$x, a \to \boxed{\delta \rightsquigarrow \varepsilon}$ — $f(x) - f(a)$; $\left| d\tilde{a} - da \right|$, $f(\tilde{a}) - f(a)$; [4'] (back page)

$x, a \to \boxed{\ }$ — $f(x)$; $f(a) + f'(a)(x-a)$

note this regarding requiring MVT

note: how to pass to f'' from second difference! $\Delta_h(\Delta_h f)(x) = \Delta_h(f(x+h)) - \Delta_h f(x) = f(x+h+h) - \dots$ [TWoA] (p 180) ??

[MVT; FTC] [RRi; C] leads to MVT?

but why $f'(a)(\tilde{a} - a)$ not $f'(?)(\tilde{a} - a)$; $a < ? \leq$ more $(\tilde{a} - a)$

$\left| \frac{f(a) + d(\tilde{a}-a)}{f(\tilde{a})} \right| = \tilde{\varepsilon}$, $\tilde{a} - a$

so d is '$f'(a)$' f' varying $a \to$ varying d

maybe better.
if (2) exists, then (1) is true where $f''(a) = e$.
or do we need 'mean value the?'

$f(g(x)a)$ $\left| \frac{\frac{f(\tilde{a}) - f(a)}{\tilde{a} - a}}{d} \right| = \varepsilon$, $\left| \frac{f(\tilde{a}) - [f(a) + d(\tilde{a} - a)]}{\tilde{a} - a} \right| = \varepsilon$

a is ε^+ is $\tilde{a}$

$x, a \to \boxed{\delta \rightsquigarrow \varepsilon}$ — $\frac{f(x) - f(a)}{x-a}$; d

$$\left| \frac{\frac{f(a\varepsilon^+) - f(a)}{x\varepsilon^+ - a}}{d} \right| \leq \varepsilon$$

$\left| \frac{\dots}{d} \right| \approx 0$

$$\left| \frac{\frac{f(\tilde{a}) - f(a)}{\tilde{a} - a}}{d} \right| = \tilde{o}$$

$f(x)$ $\frac{f(\frac{g(x)}{a}) - f(a)}{\frac{g(x)}{a}}$ $f^{g(x)} - f^{a}$

$f(x a)$ $\frac{f(x g a) - f(a)}{(x g a) - a} = \infty 0$

absorb dx, 11a1a | 5 |

* in the MVT bookmarks, we see that, ~~[illegible]~~ remembering 'pointillistic', the passage from 'axless' pointillistic calculus to ~~[illegible]~~ C^1 'smooth' calculus, it can be bootstrapped by some 'property' of 'not being pointillistic with derivatives'. this can be provided by the MVT, but also, according to those W. teacher, by the IFT. [RRiC].

* note that in

$x, a \to \boxed{\text{fn eq}} \to \frac{f(x)-f(a)}{x-a}$, $\to d \to f'(a)$

$\hookrightarrow$ f' is <u>not</u> a function on ~~[illegible]~~ $\mathbb{R}$, or on any interval, since it is only defined at a. We could say that this is 'not' a function. We call call a function 'proper' when doing 'calculus' (in the smooth way), only when it <u>is</u> def on <u>intervals</u>.

(measure > 0 ??)

or at the <u>same interval as f</u>.

if f is def on I then we ~~call~~ only consider a 'function' f', if it also is defined on I.

* we continue with <u>[RRiC]</u>. <u>especially thm 2</u>.

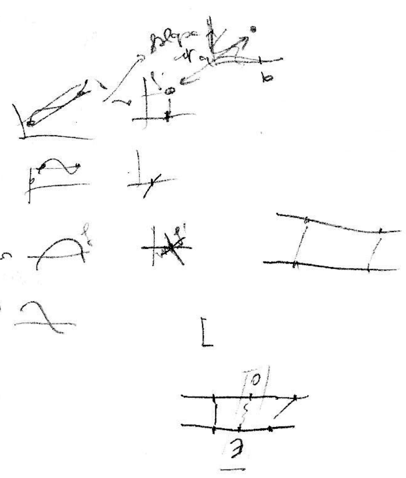
faces

MVT

Rolle

EVT →

IVT →



$\int_a^b$

$\overset{[a,b]}{f} \Rightarrow f(a) \le f(c) \le f(b)$

$\forall\, f(a) \le d \le f(b)$

$\exists c,\ f(c) = d$

$f(a) \le c^* \le f(b)$

capen C[dd b(cA)] [L o H]

critical pt th

f max at m, th

$f(m + \Delta m) \le f(m)$

$\varepsilon > 0,\ \delta < 0$

$f'(m) \simeq \dfrac{f(m+\varepsilon) - f(m)}{\varepsilon} \le 0 \le \dfrac{f(m+\delta) - f(m)}{\delta} \simeq f'(m)$

$\Rightarrow f'(m) = 0$

choose 'm' interals, find ... c cannot ...

cap

cap

C[0,A]

p 130

p 161, 156

IZS $f[a,b]$ ⊃ cave

$\frac{a+b}{2}$

$a, \frac{a+b}{2}, b$

$f[-]$ $f[-]$

N

$a + \frac{(a-b)}{}$

$\lambda a + (1-\lambda) b$

$\frac{1}{n} a + (1 - \frac{1}{n}) b$

a, b

$[0,1]$

$\frac{[a,b] - a}{(b-a)}$

$\frac{a+b}{n}$ $\frac{a+b}{n} - a$ $\frac{a+b-an}{n} = \frac{a(1-n)+b}{n} =$

$a + \frac{(b-a)}{n} =$

$\frac{an + b - a}{n} = \frac{a(n-1)+b}{n}$

$f|[a,b] = f|d$

$f|c \in f|[a,b]$

$[a,b]$ $[\cancel{\ }b]$ $[a, \frac{a+b}{n}, b]$

$\frac{a+b}{n} \le b$

$a+b \le nb$

$\frac{a+b}{b} \le n$

$n \ge \frac{a+b}{n}$

$f|c \in f[a,b]$

$f|c \in f|[a,b]$

1) Why is $[a,b]$ interval??

this is what we are proving??

interpolation

cont. f at a

$[_a \delta f] [\delta_{\varepsilon^*}] \overset{\varepsilon}{=} 0$

$|_a \Delta f|^{\delta_{\varepsilon^*}} \overset{\varepsilon}{=} 0$

part. continuity

IVT

$f(c) = d$

$|\alpha'| = \frac{1}{2^n}|\alpha'|$

$|\alpha| = \frac{1}{2}$

$[\delta + \text{}^{a}_{\delta\text{-}\varepsilon^*} - \delta]^{a} - [d] = \varepsilon^*$

$\exists\, d \mid \varepsilon \rightsquigarrow 0 \mid \exists\, \delta_{\varepsilon^*} \mid {}_a\Delta\delta(\cdot) \rightsquigarrow {}_a d[\cdot] \mid \delta(\varepsilon^*)$

$\boxed{\exists\, d \mid \varepsilon \rightsquigarrow 0 \mid \exists\, \delta_{\varepsilon^*} \mid {}_a\Delta\delta(\cdot)\ \varepsilon^*\ \delta(\varepsilon^*) \rightsquigarrow {}_a d[\cdot]}$

${}_a\Delta\delta(\cdot) \rightsquigarrow \delta(a) \rightsquigarrow {}_a d[\cdot]$

$[d_{at\,a}]$

$\forall\, \varepsilon$ $\exists\, \delta$

${}_a\Delta\delta(\cdot)$

small $\varepsilon \rightsquigarrow \delta_\varepsilon$ sub.

${}_a d[\cdot]$

fixed

δ, a

$\delta(\varepsilon^*)$

$\delta + \text{}_a - \delta + a \over \delta(\varepsilon^*)$

$\left| \dfrac{\delta + {}_a - \delta + a}{\delta(\varepsilon^*)} \right| = \varepsilon^*$

d

$\delta(a^*) - \delta(a)$

δ, d

$\delta_a [+ - \delta \quad \varepsilon^*$

$\varepsilon^* \rightsquigarrow 0 \mid \exists\, \delta$

$\exists\, d \mid \varepsilon \rightsquigarrow 0 \mid \exists\, \delta(\varepsilon^*) \mid {}_a\Delta\delta(\cdot) \rightsquigarrow {}_a d[\cdot] \mid \delta(\varepsilon^*)$

$\delta, a \quad [\ (\cdot) + (\cdot) - (\cdot)\ ] - [\ \cdot\ ] = \cdot$

$\delta(\varepsilon^*), d$

$[\ \cdot) + \ - \cdot\ ]$

$\varepsilon^* \rightsquigarrow 0 \quad {}_a\Delta\delta\,\cdot \rightsquigarrow {}_a d\,\cdot \quad \delta(\varepsilon^*)$

$\varepsilon^* \rightsquigarrow 0 \quad {}_a\Delta\delta\,\cdot \rightsquigarrow {}_a d \cdot [\cdot] \quad \delta(\varepsilon^*)$

$\delta a \quad \delta a$

$\varepsilon^* \rightsquigarrow 0 \quad {}_a\Delta\delta\,\cdot \rightsquigarrow {}_a d \cdot [\cdot] \quad \delta(\varepsilon^*)$

${}_a\Delta\delta\,\delta(\varepsilon^*) \rightsquigarrow {}_a d \cdot \delta(\varepsilon^*) \quad \varepsilon^* \rightsquigarrow 0$

$-\delta\langle^{a}_{a}$

$-\rangle\langle^{-\delta\rangle\langle^{a}_{\delta(\varepsilon^*)}}_{d\,\delta(\varepsilon^*)} = \varepsilon^*$

$\left| \begin{matrix} {}_a\Delta\delta - [\cdot] \\ {}_a d \cdot [\cdot] \end{matrix} \right|_{\varepsilon^* \rightsquigarrow 0}$ with $\delta(\varepsilon^*)$

$\delta($

$\delta_a\delta(\delta_{\varepsilon^*}) - {}_a d\, \delta_{\varepsilon^*}$

$\boxed{[\delta\delta][\delta_{\varepsilon^*}] \overset{\varepsilon^*}{\simeq} [d][\delta_{\varepsilon^*}]}$

$\left| \begin{matrix} {}_a\Delta\delta - [\cdot] \\ {}_a d \cdot [\cdot] \end{matrix} \right| \overset{\varepsilon^*}{=} 0$ with δ_{ε^*}, $\varepsilon^* \rightsquigarrow 0$

$\left| \begin{matrix} {}_a\Delta\delta_{\delta_{\varepsilon^*}} \\ {}_a d\,. \end{matrix} \right.$

${}_a\delta\delta_{\delta_{\varepsilon^*}} \overset{\varepsilon^*}{=} {}_a d \cdot \delta_{\varepsilon^*}$

${}_a\Delta\delta_{\delta_{\varepsilon^*}} \overset{\varepsilon^*}{=}$

$|\ {}_a\Delta\delta_{\delta_{\varepsilon^*}},\ {}_a d \cdot \delta_{\varepsilon^*}$

[CPaS] (p252)

$f^n|_a$

$\int_0^\infty$ $d^i m^i_i|_a$ → conv $\mathcal{E}$ — immediately? can be $\emptyset$ → div $\mathcal{D}$ — $R \setminus \mathcal{E}$

$\lim_{n\to\infty} R_{n+1}(a) = 0$ (radius of convergence)

probably $\neq \infty$

infeasible → ? $f(\mathcal{E}_a) = (d^i m^i_i)(x)$ only??

feasible → analytic

$f(\mathcal{E}) = (d^i m^i_i)(\mathcal{E})|_a$

$f^n|_a = \sum^m d_i m^i_i + R_{m+1}(\xi)|_a$

ξ is some point between a and m

we can prove this

Lagrange Remainder (p60)

(1)

satisfies

$f_1 > d^i m^i_i$ — non-analytic?

note: spread for power series one of these holds:

1. conv at 0
2. conv on $\mathbb{R}$
3. conv only $|R|$ and div outside it

in general test ratio, cauchy [within radius-of-convergence] e.g. root test

Solution: mot:1885997

also, in [KRIC] (pp 236) we only need exp at h, so δx a.

?!!

mot:1976275

$R(\tilde{a})$ $R^{\infty}_{|a-a|\to\infty}$

the power series machine

proof missing

$\varepsilon = |m-a|$

f_{gain}

$R(a, \infty)$

independent — $\varepsilon\to 0$ then $K\to 0$ always given. the nature of f_{gain}; $n\to\infty$ then $R \to 0$ analytic, $\not\to 0$ nonanalytic

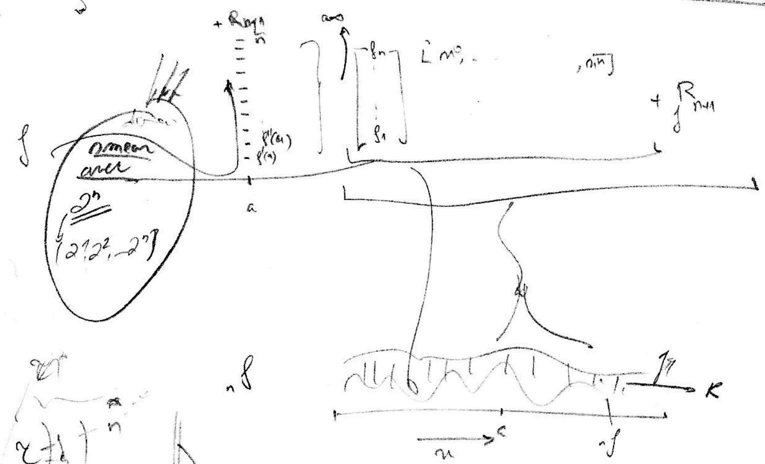
$\varepsilon = |m-a|$

$f\in C^\infty$

Smooth

mean area

$\{2, 2^2, \ldots 2^n\}$



~~$f \in \Theta(g)$~~

1. $f \in o(g)$ $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

2. $\lim_{n\to\infty} \frac{f(n)}{g(n)} = 0$

$\lim_{n\to\infty} \frac{n}{n^2} = 0$ ✓ $n \in o(n^2)$

$\lim_{n\to\infty} \frac{n}{n^2} = \lim_{n\to\infty} \frac{1}{n} = \infty$ ✗ $n \notin \Theta(n)^2$

$n \to \infty$

$n^3 \succ n^2$, $n^2 \prec n^3$

$n \succ$, $n \succ n^2$, $n \prec n^2$

$n^2 \in o(n^3)$

$n \in o(n^2)$

f n / grows slower than n^2

$\boxed{\frac{f(n)}{g(n)}}$

- clear grows → more ~~grows~~ is slower (grows slower or shrinks)
- clear vanishes → ~~now shows~~ is (faster) (shrinks faster)

$\overbrace{f(n) - g(n)}^{h(n)}$ is $o(n \to \infty)$

$h(n)$ vanishes faster than $(n \to \infty)$

$\lim_{n\to\infty} \frac{n^2}{(n)} \to 0$

$\frac{h(n)}{(n\to\infty)}$

$\frac{n^3}{n^2} \cdot \frac{1}{n}$

(n grows as n^2 vanishes faster → math ?!)

$\prec$ $\preceq \prec \prec \succ \succ$

'shrink' 'faster'

'grow' 'slower'

'grow' 'faster'

'shrink' 'slower'

$n^2 \gg n$ $n^2 \gtrsim n$

$n^2 \ll n$ $n^2 \lesssim n$

$n^2 \not\gg n$, $n^2 \not\gg n^2$

$n^2 \not\ll n$, $n^2 \not\ll n^2$

n^2

This is too general!

Once we $g(n) = \frac{1}{n}$, we get 'counter intuitive' results
we probably 'meant' to have $g(n)$ as (eventually) monotonically increasing or increasing.

$(n \to \infty \Leftrightarrow n^{-1} \to 0)$

but then it can signal $n \xrightarrow{\to 0}{\to \infty}$ only

then we have a $\bar{o}$, $\underline{o}$ $\bar{O}$, O Θ, Θ

$\bar{o}_{n\to\infty}$ $\underline{o}_{n\to\infty}$ Θ O Θ Θ Θ

∞

~~$n = \underline{o}(n_2)$~~

$\boxed{n^2 \overset{\infty}{>} n \quad n^2 \overset{0}{<} n}$

$n^3 \overset{\to}{>} n^2$

$n^2 \overset{\leftarrow}{\leq} n^2$

$n^2 \overset{\downarrow}{<} n$

$n^2 = \underline{o}(n)$ n^2 vanishes faster than n

$n = \bar{o}(n^2)$ n explodes slower than n^2

$n \underline{o}\, n^2$ $n^2 \overset{\uparrow}{>} n$