The 'Time of Impact' Argument

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I Introduction

Consider the idealized setup in the plane, consisting of a point particle that is subject to inelastic collisions with a of number static segments (of infinite mass). Given an initial situation at time t_0, we are interested in correctly predicting the position of the particle at time $t_0 + h$, where h is the duration of a 'frame'.

We constrain ourselves by requiring that the prediction method must model each collision with a segment using a collision with some half-space instead of the segment itself. The half-space can be chosen at will by the method. Half-spaces can only be combined by union of their disallowed areas, which means that a half-space's disallowed half cannot be reduced by the addition of any other half-space. We call any such method a 'half-space method'. The motivation for this method is the usage of 'Linear Complementarity Problem' solvers for the simulation of rigid-body dynamics.

The 'Time of Impact' argument we present asserts that any half-space method cannot guarantee a correct prediction if it is 'one-shot', that is, if it does not iterate over the times at which the actual collisions happened, resolving them sequentially.

To simplify the argument, we work within a subset of all possible situations chacaterized by two assumptions:

1. The segments are so set up that one can connect (not necessarily in a single straight streak) any point to any other point in the plane without intersecting any segment, viz. there are no closed regions.
2. Collisions with segment end-points do not occur.

Since any method must also handle this subset, there is no loss of generality in working within it in an impossibility argument like this one.

II Argument

1. A 'ghost' collision is a predicted collision with a point that is not on any segment, a ghost point.

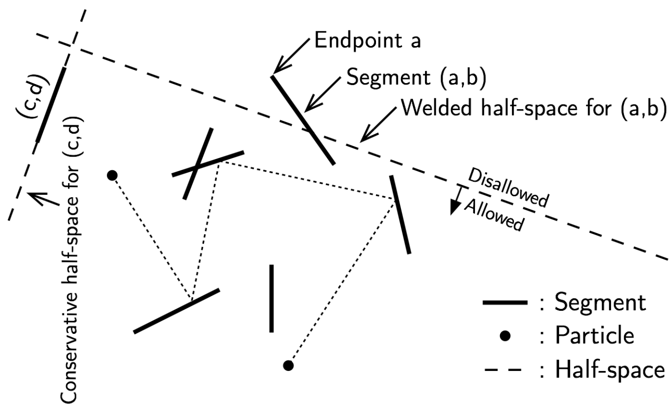

Figure 1: An illustration of the terms used in the argument.

2. A 'tunneling' is a part of the predicted trajectory that passes through a segment without colliding with it, tunneling through it.

3. This list exhausts the possible relations between a segment and possible configurations of the half-space used to stand in for it in the prediction of a single collision, that is, intersection with a single straight fragment of the particle's trajectory. It can easily be verified graphically, geometrically or analytically. In it, we identify all cases where the particle is in the disallowed half-space with case 4, since then, the half-space has no effect.

   (a) Both points of the segment are either on the border or within the disallowed half: Ghosting can occur.

   (b) Both points are within the allowed half: Both ghosting and tunneling can occur.

   (c) None of the above: Both ghosting and tunneling can occur.

   (d) Additionally to the above, we consider the option of discarding the half-space fully. In this case, only tunneling can occur.

4. In the list, the possibility of occurence of an artifact is determined by the trajectory of the particle. If, within the frame, the segment joining the particle's positions at some t_i and t_j crosses the half-space boundary before crossing or without crossing the source segment, ghosting occurs. If it crosses the source segment before or without crossing the boundary, tunneling does. Otherwise, no artifact occurs.

5. From the list we see that every item comes with the potential occurence of at least one of the two artifacts.

6. A 'conservative half-space' is a half-space that contains its source segment in its boundary (line). If only such half-spaces are considered and discarding them is not allowed, the resulting method is called a 'conservative half-space method'. By 3a we see that such a method can only suffer from ghosting but not tunneling.

7. Any method that is not 'conservative' is called a 'welding half-space method'. The process of choosing a half-space that stands for a specific segment (other than the conservative one) is called 'welding'.

8. By the list, any 'welding half-space method' can, in addition to ghosting, potentially also suffer from tunneling if no measure is taken against it.

9. Consider any one-shot method. It must choose a half-space to stand in for every segment. For every choice of half-space, an artifact can occur if the correct particle trajectory is unfavorable as described in 4. Nevertheless, consider the method found a way to choose the half-spaces such that no artefact occurs. There exists another situation with a particle with the same initial conditions but with a higher speed, extending the trajectory covered by the first situation. By I.1, there exists an arrangement of additional segments that do not interfere with the first part of the trajectory, but that are such that the trajectory extension induces an unfavorable situation for some segment in the initial set of segments, producing an artefact in the extended trajectory, mispredicting it.

10. One can counter the above by claiming that the method, given the additional set of segments, would produce a different choice of stand-in half-spaces for the segments involved in the initial part of the trajectory. But we argue that this means that the method must then consider the effect of every segment on every other segment, and this is impossible without considering the way the particle interacts with the segments, which contradicts the one-shot assumption. In other words, a correct half-space method cannot be one-shot.

III Contributors

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