

# Math 6640

## Introduction to Numerical Methods for Partial Differential Equations

### Course Information (Fall 2013)

Time and Place: 1:35 pm - 2:55 pm TR Engr Science & Mech 210

Reference books:

J. W. Thomas, *Introduction to Numerical Methods for Partial Differential Equations*, Springer, ISBN 0-387-97999-9.

L. N. Trefethen, *Spectral Methods in Matlab*, SIAM.

Claes Johnson, *Numerical solution of partial differential equations by the finite element method*. ISBN 0521 345 146.

S. C. Brenner and R. Scott, *The Mathematical Theory of Finite Element Methods*, Second Edition.

R. J. LeVeque, *Numerical Methods for Conservation Laws*, Birkhauser Verlag, 1992.

S. V. Patankar, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing Corp., 1980.

C.-W. Shu, *Essentially non-oscillatory and weighted essentially non-oscillatory schemes for hyperbolic conservation laws*, in *Advanced Numerical Approximation of Nonlinear Hyperbolic Equations*, B. Cockburn, C. Johnson, C.-W. Shu and E. Tadmor (Editor: A. Quarteroni), *Lecture Notes in Mathematics*, volume 1697, Springer, 1998, pp.325-432.

#### Syllabus

Instructor: Dr. Yingjie Liu  
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Office Hours: 3pm-4pm TR, Skiles 134

Homework: Homeworks will be assigned once every couple of weeks and they must be turned in on time.

Grading: Homeworks 50%, final exam 50%.

Topic Outline: Finite difference methods for parabolic equations, including heat conduction, forward and backward Euler schemes, Crank-Nicolson scheme, L infinity stability and L2 stability analysis including Fourier analysis, boundary condition treatment, Peaceman-Rachford scheme and ADI schemes in 2D, line-by-line methods etc.  
Finite difference and finite volume schemes for hyperbolic equations and conservation laws, including upwind schemes, Lax-Friedrich scheme, characteristic method, Lax-Wendroff scheme, MacCormack scheme, back and forth error compensation and correction (BFEC), time splitting and Strang splitting, Godunov scheme for conservation laws (such as the Euler equation for gas

dynamics), monotone fluxes, MUSCL scheme, ENO scheme, flux corrected transport etc. Finite element methods for elliptic equations, including variational formulations, conforming Galerkin methods, Dirichlet boundary condition treatment and Neumann boundary condition treatment (nature boundary condition), formation of the stiff matrix, error analysis, steepest decent and conjugate gradient methods, multigrid method, construction of Lagrangian nodal basis functions etc.

## Introduction to Numerical Methods for Partial Differential Equations

**Department:** MATH

**Course Number:** 6640

**Hours - Lecture:** 3

**Hours - Lab:** 0

**Hours - Recitation:** 0

**Hours - Total Credit:** 3

**Typical Scheduling:** Every fall semester

**Description:**

Introduction to the implementation and analysis of numerical algorithms for the numerical solution of the classic partial differential equations in science and engineering

**Prerequisites:**

[Math 4347](#) and [Math 4640](#)

**Course Text:**

No text

**Topic Outline:**

Quick Review of Basic Models - Elliptic, parabolic, and hyperbolic problems, classification and behavior of solutions, numerical methods

Elliptic Problems and the Finite Element Method - Two-point boundary value problems, Laplace and Poisson equations, variational formulation, finite element methods, interpolation theory, quadrature, energy norm, a priori convergence, order of convergence

Brief Review of Numerical Linear Algebra - Specialized to systems arising from discretization of differential equations: sparse matrices, direct methods, basic iterative methods

Parabolic Problems and the Method of Lines - Explicit and implicit discretization schemes, numerical stability, stiffness and convergence

Hyperbolic Problems - Transport equation, characteristics, finite difference schemes, stability, dissipativity, dispersion, the wave equation

Miscellaneous Topics as Time Permits - Error estimation, adaptive schemes, conservation laws