

Calculus of Variations

Department: MATH

Course Number: 7581

Hours - Lecture: 3

Hours - Lab: 0

Hours - Recitation: 0

Hours - Total Credit: 3

Typical Scheduling: Every even spring

Description:

Minimization of functionals, Euler Lagrange equations, sufficient conditions for a minimum, geodesic, isoperimetric and time of transit problem
variational principles of mechanics, applications to control theory

Prerequisites:

[Math 4317](#) or equivalent

Course Text:

No text

Topic Outline:

The basic setup: Bernoulli and the Brachistochrone. The general setup: functionals and boundary conditions; isoperimetric problems, geodesic problems

Minimizing in a linear space; directional derivatives; convex functions

Convex functionals and calculus of variations; variations; sufficient conditions for minimum of convex functional -- the Euler Lagrange equation; applications in mechanics and minimum area problems

The lemmas of DuBois-Raymond

Minimizing without prior assumptions of smoothness, the Euler-Lagrange equations again

Optimizing with respect to piecewise smooth functions; general linear space background, norms; the Weierstrass corner conditions

Applications to mechanics, Lagrangians, Hamiltonians, the 2-body problem and generalizations; Hamilton-Jacobi equations

Necessary conditions for minimization

An introduction to control theory in the context of Calculus of Variations; examples; rocket problems