Group Theory

PHYS 7143 - spring 2008
Tu,Th 12:05-1:25 Howey S204

birdtracks.eu/~predrag/courses/PHYS-7143-08

Course Schedule

January 8 Predrag Cvitanović

1. History. Finite groups

Reading: Tinkham Chapter 2 - Abstract group theory

homework #1:Tinkham (2.1), (2.2); optional (2.6) - due Tue January 15

[solutions to exercises]

January 10

2. Finite groups

Cosets, classes, normal divisors and factor subgroups

January 15

3. Group representations

Matrix representations are unitary. Schur's lemma.

Reading: Tinkham Chapter 3 - Theory of group representations

homework #2: Tinkham (3.1), (3.3); optional (3,7), (3.8) - due Tue January 22

January 17

4. Characters

The great orthogonality theorem. Character orthogonality. Character tables.

January 21

MLK holiday

January 22

5. Characters

Hard work builds character.

January 24

6. Decomposition of reducible representations

Regular representation. Transformation operators. Representations.

Reading: Tinkham - sections 3.5-3.8

Reading: Harter Sect. 1.2Bd - Commuting matrices

homework #3: Harter (1.2.1), (1.2.6); optional (1.2.2) - due Tue January 29. [bra, ket refers to left/right eigenvectors. Sect. 1.2Bd is the same as my Appendix C, section 2.2]

January 29

7. Projection operators

All eigenvalues distinct. Complex eigenvalues in real representation. Degenerate eigenvalues: hermitian case, Jordan case.

January 31

8. Irreducible reps of abelian groups

Projection operators for abelian groups from character tables. D_2 example: Harter's propeller.

Reading: ChaosBook.org Appendix B - Linear stability (ver. Feb 1, 2008)

This appendix is continuously updated - wisest not to print it on paper yet.

homework #4: Appendix B exercise B.1, Appendix C exercise C.1 - due Tue January 29.

(You are in luck - class secretary is too exhausted to type yet another problem.)

February 5

9. Irreducible reps of abelian groups

Projection operators for abelian groups from character tables. C_2, C_3 coupled harmonic oscillators reduction to normal modes.

Reading: Lecture notes - Abelian groups reduction (ver. Feb 7 2008)

February 7

10. Irreducible reps of abelian groups

Irreps for C_n. Discrete Fourier transforms from character tables.

Reading: ChaosBook.org Appendix C - Discrete symmetries of dynamics (ver. Feb 8 2008)

Read sections C.3-C.5. This appendix is continuously updated - wisest not to print it on paper yet.

homework #5: Appendix C exercises C.2, C.3, C.5 - due Tue February 12.

February 12

11. Fourier transforms

If the symmetry group is the group of translations on a line of rotations/shifts on a circle, the reduction to 1-dimensional irreps is known as the Fourier transform. It trades in nonlocal operators, such as the Laplace operator for pure numbers, such as the momentum^2.

February 14

Valentine's day

February 14

12. Irreducible reps decomposition

Worked out problem C.2: 3 pendulums on a line, with mirror C2 symmetry. Reduce by symmetry first.

Reading: Harter - 3.2 Nonabelian symmetry analysis

Work through section 3.3.

Reading: Harter - Double group theory on the half-shell (1978)

Read appendices B and C on spectral decomposition and class algebras. Article works out some interesting examples.

homework #6: Appendix C exercise C.4 - due Tue February 19.

February 19

13. Irreducible reps of nonabelian groups

Projection operators for C_3v nonabelian group from character tables.

February 21

14. Continuous symmetries / back to triangulating C_3v

Rotations in a plane. Equilateral 3-mass spring system, not pinned down.

Reading: ChaosBook.org Chapter 4 - Local stability (ver. Feb 21, 2008)

Read sects. 4.2.2, 4.3.1 - how SO(2) Lie algebra generates rotations in a plane. This chapter is continuously updated - wisest not to print it on paper.

Reading: Frank Porter - CalTech Physics 129b

Read chapter "Representation theory," most of it for pleasure. Focus in particular on sect. 3.10.

homework #7, Problem 1, due February 26: Work through Porter sect. 3.10. (a) Derive (3.95), matrix U in terms of the 1/3 turn [2x2] rotation matrices (3.103), keep it in that format. (b) Verify that the matrix U is C_3v invariant. (c) evaluate (3.15), (3.16) using your invariant form of U (rather than the explicit [6x6] bunch of square roots of 3). (d) Compute explicitely \lambda_31=0 and its eigenvactors, show that they correspond to translations, rotations. (e) optional for everybody EXCEPT Jonathan and Vaggelis (for them it is required): quotient out T^2 and O(2), ie. rewrite dynamics so quotiented dynamics has no zero eigenvalues.

homework #7, Problem 2, due February 26: The relation of irreducible representations and the invariant subspaces of a vector space: Do problem 11 (click here). This problem takes some thought. Also, there many different, equally good ways to solve it.

[Porter solution to problem 11, now called 18]

February 26

15. Continuous groups

Lie groups defined. Examples. Lie algebras, first try.

Reading: Chen, Ping and Wang - Group Representation Theory for Physicists Sect 5.2

Definition of a Lie group, with examples

February 28

16. Lie algebras

Groups, vector spaces, tensors, invariant tensors, invariance groups.

Reading: birdtracks.eu Chapter 3 - Invariants and reducibility

Reading: C K Wong - 1-D continuos groups (power point notes)

Wong is entirely optional, not covered in the lectures, but completes discussion of Fourier analysis as continuom limit of cyclic groups C_n: Read chapter 6 on representations of SO(2), O(2) and translational group.

homework #8, due March 4: Same as homework #7 - sing it until you get it right.

March 4

17. So many indices, so little time

Indices. Tensors. Invariant tensors. Indices.

March 6

18. Birdtracks

Goodbye to indices. Clebsch-Gordan coefficients. Infinitesimal transformations. Lie algebras.

Reading: birdtracks.eu Chapter 4 - Diagrammatic notation

homework #9: Derive the Lie algebra commutator and the Jacobi identity as particular examples of the invariance condition on invariant tensor, using both index and birdtracks notations.

March 11 John Wood

19. The nature and use of dynamical groups

March 13 Evangelos Siminos and Jonathan Halcrow

20. Trading in a dogeared Lorenz for a cute Van Gogh

Quotienting symmetries of nonlinear dynamical systems, or: How Lorenz lost one ear.

Reading: ChaosBook.org limbo - Desymmetrization of the Lorenz flow (rev. 459 03/27/2008)

Reading: Golubitsky and Stewart Chapter 1 - Steady-state bifurcation

[optional: this chapter was not used in the course]

Reading: ChaosBook.org Chapter 9 - World in a mirror

[optional: this chapter was not used in the course]

March 18

spring break

Alex has read no Dyson, so here is a fun sample:

Reading: Freeman J. Dyson in NYRB - The World on a String

March 20

spring break

A fun read on group theory we definitely will not cover:

Reading: Marcus du Sautoy - Finding Moonshine: A Mathematician's Journey Through Symmetry

March 25

21. Birdtracks refresher

Reading: ChaosBook.org limbo - Desymmetrization of the Lorenz flow (rev. 459 03/27/2008)

homework #10, due April 1: Exercise 5.1 in "Desymmetrization of the Lorenz flow"

March 27

22. Mutiny in the class

Reading: birdtracks.eu Chapter 5 - Recouplings

Reading: Abel Prize - J. G. Thompson and J. Tits

You doubt group theory is good for anything? How does $1.2 million sound to you?

April 1

23. Symmetrizations. Antisymmetrizations

Reading: birdtracks.eu Chapter 6 - Permutations

April 3

24. Unitary representations, Young tableaux

Reading: birdtracks.eu Chapter 9 - Unitary groups

Read sects. 9.1, 9.2, 9.11 and 9.12. Optional: sects. 9.3, 9.4.

homework #11, due April 8: Derive projection operators and dimensions listed in Table 9.3. (Ignore "indices," we have not defined them).

April 8

25. Orthogonal groups

Reading: birdtracks.eu Chapter 10 - Orthogonal groups

Read sects. 10.1, 10.2, 10.4 and 10.5

homework #12, due April 15: Decompose the Riemann-Christoffel curvature tensor into its SO(n) irreducible tensors: curvature scalar, traceless Ricci tensor and Weyl tensor, equations (10.57) to (10.59). How many components does each irreducible tensor have in n=4 dimensions? You do not need to know general relativity or worry about SO(1,3) Lorenz group for this exercise - this is a question only of the reduction of V^4 tensor representations of SO(n).

April 10

26. Symplectic groups. SU(2), SU(3) as invariance groups

Reading: birdtracks.eu Chapter 12 - Symplectic groups

Reading: birdtracks.eu Chapter 15 - SU(n) family of invariance groups

Read sects. 15.1 and 15.2.

April 15

27. Invariance group of a cubic invariant

A quick overview of the construction of exceptional Lie algebras.

Reading: birdtracks.eu lite - the webbook in 20 minutes

April 16

Fall registration starts

April 17 Jogia Bandyopadhyay

28. Group theory made coherent

Representation of SU(1,1) and the construction of coherent states.

Reading: J. Bandyopadhyay - Optimal Concentration for SU(1; 1) Coherent State Transforms and An Analogue of the Lieb-Wehrl Conjecture for SU(1; 1)

April 22

29. Exceptional group E_6

E_6 family of invariance groups of a symmetric cubic invariant.

April 24

30. Exceptional magic

A summary of the continous Lie groups part of the course.

Reading: P Cvitanovic - The webbook at a cyclist pace, in 50 overheads

takehome final: Do any part of problems 1, 2 and 5 in the order you find most convenient. 5 is straightforward, for 1 and/or 2 a partial solution is good enough. Sorry for few illegible lines of problem 1, I am learning how to use a CyberPad.

April 25

GT classes end

May 1

10:50 take-home final exam due, Predrag's office

notes

solutions to the final exam

to May 2

Course opinion survey

CETL web link

May 5

GT grades due at noon

May 6