

Course information for Math 1564

Linear Algebra with Abstract Vector Spaces

Fall 2015

This is an intensive course on linear algebra, taught at a sophisticated and abstract level. We will work mostly with abstract vector spaces. Many of the basic theorems of the subject will be proven in class, and some proofs will be assigned for homework.

Instructor:

- George Burdell, School of Mathematics
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- Office hours: Mondays 2:00 - 3:00, Thursdays 3:00 - 4:00, or by appointment. Check my web site for changes.

Teaching assistants:

- Section K1: Philip Bengé
 - Office: Skiles 149
 - E-mail: pbenge3@math.gatech.edu
 - Office hours: Tuesdays and Thursdays 11:00 - 12:00, and in the Math lab (Clough 278) on Tuesdays from 12:00-1:00
- Section K2: James Scurry
 - Office: Skiles 147
 - E-mail: jscurry3@math.gatech.edu
 - Office hours: Mondays 12:00 - 1:00 and Wednesdays 12:00 - 1:00

Course info:

- Title: Linear Algebra with Abstract Vector Spaces
- Number: 1564-K1 (82100) and 1564-K2 (81874)
- Lectures: TTh 1:35 - 2:55 in Skiles 202
- Recitations: MW 11:05 - 11:55 in Skiles 271 (K1) or Skiles 268 (K2)
- Pace: Topics will be covered more quickly and at a higher level of sophistication than Math 1604: Linear Algebra.
- Audience: Students with strong ability, interest, and motivation to master university-level calculus and linear algebra.
- Topics: See the [syllabus](#) below for a detailed list of topics.

Learning Objectives:

- Students will learn the definition of a vector space, and will learn to convert between this abstract point of view and the concrete representation (after choosing a basis) of a finite-dimensional real vector space as \mathbb{R}^n .
- Similarly, students will learn the definition of a linear transformation, and will learn to convert between this abstract point of view and the concrete representation (after choosing bases for the

source and target spaces) of a linear transformation by a matrix.

- Students will learn how to deduce the foundational results of linear algebra from the axioms for vector spaces and linear transformations.
- Students will master the concepts of eigenvalues, eigenvectors, and diagonalizability, and will be introduced to the theory of canonical forms.
- Students will learn to present complex, multi-step solutions clearly and succinctly.
- Students will learn important algorithmic and computational techniques in linear algebra such as Gaussian elimination, Gram-Schmidt orthogonalization, and Laplace's expansion for determinants.
- Students will learn some of the important real-world applications of linear algebra, including the method of least squares (linear regression), the theory of Markov chains, and Google's PageRank algorithm.

Requirements and grades:

- **Homework** will be assigned regularly, collected, and graded. Normally it will be assigned on Thursday in lecture and will be due at the beginning of lecture the following Tuesday. Assignments will be posted on t-square. Part of each assignment will be done on-line (and graded instantly), part will be traditional, written homework
- There will be frequent short **quizzes** using clickers at the beginning of class. Quiz points will be part of the homework grade
- There will be two **in-class exams: on Tuesday, September 25 and Tuesday, November 6.**
- The **final exam will Tuesday, December 11, 2:50 - 5:40, in Skiles 202.** It is a cumulative exam, covering the whole course.
- Grades will be based on the percentage of possible points earned. The in class exams will each count for 20% of the grade, the final will count for 40%, and homework and quizzes will count for 20%.
- Cutoff percentages for A, B, C, D are 90%, 80%, 70%, and 60% respectively.

Resources:

- Text: "Linear Algebra" by Jim Hefferon. This book is available for free online.
- Clickers: You will need a ResponseCard NXT clicker from TurningPoint Technologies. See the [CETL FAQ](#) for more information about clickers.
- Most homework assignments will include an on-line component. You will have unlimited attempts to solve the problem, with immediate feedback. This routine practice will be an important way to build computational skills. The login page for on-line homework is <https://courses.webwork.maa.org/webwork2/ft-georgiainst-math1512/>.

Other important policies:

- In an emergency situation, I may allow a make-up quiz or test if I am notified prior to the exam and provided with a reasonable, **written** confirmation of your absence. Any make-ups must be completed before the corresponding quiz or test has been graded and returned to other students. If you will miss a test due to a university-sponsored event or athletics, please provide me with the official documentation in advance.
- Arrangements for extensive absences (e.g., for extracurricular activities) must be made by the end of the first week of classes. Students requiring arrangements from the ADAPTS-Disability Services Program should consult their [information page](#) and make arrangements as soon as possible.

- **Collaboration and plagiarism:** Discussing ideas and homework exercises with your peers is not only acceptable, it is a good idea. However, you must write your own solutions in your own words. Copying another's words or otherwise passing off someone else's work as your own is plagiarism and will result in a score of 0 on the entire assignment in question. Egregious cases will be dealt with more harshly. **ASK** if you have **any questions whatsoever about this**
- Homework is due at the beginning of lecture, usually on Tuesdays. Late homework is strongly discouraged. It will be accepted up to 24 hours after the due time and will be assessed a 20% penalty.
- Read assignments before class and come prepared with questions.
- The [Georgia Tech Honor Code](#) applies 100% without exception. Know it and live it.
- Show some respect for your professor and your classmates---save Facebook, YouTube, and Technique for other times.

Tips for success:

- The pace will be very fast and if you fall behind it will be very difficult to catch up. **Keep up!**
- **Read** the book. The high school technique of flipping through the section looking for an example like the problem will not work in college. The key is to read **actively**. This means constantly asking yourself questions like “What is the point of this example? What idea does it illustrate?” In a theorem, “why are these assumptions being made? What happens if we relax one of them?”
- It is *very* beneficial to read the relevant parts of the text **before class**. The discussion will be much more meaningful if you already know what the key issues are.
- Your goal should be **active understanding**. This means being able to apply concepts in a new setting. The best way to develop active understanding is to do lots of problems. The texts have far more good problems than can be graded in this class. Doing them is an excellent way to study.
- **On writing:** The solution to an exercise or the proof of a theorem is a form of communication---it's a conversation between the reader and the author. The goal is to convince the reader that the solution or proof is correct. Doing this well involves all the same skills as writing an essay, in particular, knowing something about the audience (What can be assumed? What is “trivial” and what requires explanation?) and structuring the discussion so that it is easy to follow. Learning to write mathematics well is an important goal for this course.
- Good solutions and proofs usually involve more words than formulas. The author should **explain** what he or she is doing. I'll give some examples in class.
- Working in small groups is a great idea. If you can explain the ideas in this course to your friends, then you really know them.
- The TAs and I are *resources*. We can't learn the material for you, but we can help a lot, by explaining the concepts and clarifying subtle or tricky points. Take advantage, after all, we're why you are going to Tech, not that other, lesser place down the road.
- **Ask questions**, in class and in office hours. This is how we know what you need help with, and helping you is what we are trying to do.

Topics:

Solving Linear Systems and Gaussian Elimination, Sections One.I.1-3, 3 Lectures

Reduced Echelon Form, Sections One.III.1-2, 2 Lectures

Geometry of vectors, Sections One.II.1-2, 1 Lecture

Abstract vector spaces, Sections Two.I.1, 1 Lecture

Subspaces and spanning sets, Sections Two.I.2, 2 Lectures

Linear independence, Sections Two.II.1, 1 Lecture

Bases, Sections Two.III.1, 2 Lectures

Dimension, Sections Two.III.2, 2 Lectures

Vector spaces and linear systems, Sections Two.III.3, 1 Lecture

Isomorphisms, Sections Three.I.1-2, 2 Lectures

Linear transformations, Sections Three.II.1-2, 3 Lectures

Computing linear maps, Sections Three.III.1-2, 2 Lectures
Matrix operations, Sections Three.IV.1-3, 2 Lectures
Inverses, Sections Three.IV.4, 1 Lecture
Markov chains, Sections Three.Topic, 1 Lecture
Change of basis, Sections Three.V.1-2, 2 Lectures
Orthogonal projection, Sections Three.VI.1,3, 1 Lecture
Gram--Schmidt orthogonalization, Sections Three.VI.2, 1 Lecture
Line of best fit, Sections Three.Topic, 1 Lecture
Determinants, Sections Four.I.1-2, 2 Lectures
Laplace's expansion, Sections Four.III.1, 1 Lecture
Complex vector spaces, Sections Five.I.1-2, 1 Lecture
Similarity, Sections Five.II.1, 1 Lecture
Diagonalizability, Sections Five.II.2, 1 Lecture
Eigenvalues and eigenvectors, Sections Five.II.3, 2 Lectures
Method of Powers and Page Rank, Sections Five.Topics, 2 Lectures
Jordan canonical form, Notes, 1 Lecture