UPDATED *April 21, 2011* http://cns.physics.gatech.edu/~roman/phys7268/index.html

Spatiotemporal Dynamics and Pattern Formation

with Examples from Physics, Chemistry, and Biology

Instructor

Roman Grigoriev
Office: Howey W304 (office hours: Tuesday 1:30pm-2:30pm)
Phone: (404) 385-1130
E-mail: roman.grigoriev@physics.gatech.edu

Place and Times

Tusday, Thursday, 12pm-1:30pm
Room N210, Howey Physics Building

Course Description

Most macroscopic structures or patterns we see around us (clouds, ocean waves, sand dunes, zebra stripes, contraction waves of the heart muscle) arise as a result of competition between external driving and internal dissipation. Although the nature of pattern forming systems can be very different, essentially all of them possess a number of universal traits, which allows a unified treatment of their dynamics. This course will develop a general theory of pattern formation using stability theory, bifurcations and symmetry analysis using a variety of physical, chemical, and biological systems as examples.

This is a graduate level course intended for math, science and engineering students. Good knowledge of partial and ordinary differential equations and linear algebra is the main prerequisite.

Textbook

The main textbook is

- Michael Cross and Henry Greenside, *Pattern Formation and Dynamics in Nonequilibrium Systems* (Cambridge University Press, 2009)

There are a few other good books that provide better coverage of some topics:

- Paul Manneville, *Dissipative structures and weak turbulence* (Academic Press, 1990)
- Daniel Walgraef, *Spatio-Temporal Pattern Formation: With Examples from Physics, Chemistry, and Materials Science* (Springer-Verlag, 1997).
- James Murray, *Mathematical Biology*, Vol. 2 (Springer-Verlag, 2002).
- Philip Ball, *The Self-made Tapestry: Pattern Formation in Nature* (available electonically at

NetLibrary within GT)

There is also a collection of review articles on this subject:

- Cross and Hohenberg, *Pattern formation outside of equilibrium*, Reviews of Modern Physics **65**, 851 (1993).
- Koch and Meinhardt, *Biological pattern formation: from basic mechanisms to complex structures*, Reviews of Modern Physics **66**, 1481 (1994).
- Merzhanov and Rumanov, *Physics of reaction waves*, Reviews of Modern Physics **71**, 1173 (1999).
- Aranson and Kramer, *The world of the complex Ginzburg-Landau equation*, Reviews of Modern Physics **74**, 99 (2002).

Homeworks

There will be two exams (a mid-term and a final). The grades will be based on the homeworks (33%) and the exams (midterm 33%, final 34%). Homework assignments will be posted on the web every Thursday and will be due next Thursday in class. You can discuss problems with each other, but the solutions have to be executed and submitted individually. In general you are expected to comply with the academic honor code.

Course Schedule

January 13

1. Patterns in nature

Reading: Chapter 1, lecture notes
Related website: snowflakes
Movies: solar granular convection, slime mold evolution, vene formation in a dragonfly wing, dendritic solidification, cracking soil

January 18

2. Patterns in the lab

Reading: Chapter 1, lecture notes
Related websites: Cardiac Dynamics
Movies: onset of Taylor vortices and wavy vortex flow in a Taylor-Couette system; spiral defect chaos in a convection cell; Faraday waves on the surface of water; hexagonal pattern and localized structures (oscillons) in oscillated granular layers.

January 20

3. Linear stability analysis

Reading: Chapter 2, lecture notes

January 25

4. Stability of uniform states

Reading: Chapter 2, lecture notes

January 27
5. The emergence of a pattern
Reading: Chapter 2, lecture notes
Assignment #1: problems, solutions

February 1
6. Types of instabilities and boundary conditions
Reading: Chapter 2, lecture notes

February 3
7. Reaction-diffusion systems
Reading: Chapter 3, lecture notes
Assignment #2: problems, solutions

February 8
8. Turing instability
Reading: Chapter 3, lecture notes, Biological Pattern Formation by Koch and Meinhardt
Matlab simulation: Brusselator

February 10
9. Railegh-Benard convection
Reading: lecture notes, another source
Assignment #3: problems, solutions

February 15
10. Railegh-Benard convection
Reading: lecture notes, another source
Matlab simulation: Boussinesq equations

February 17
11. D'Arcy convection
Reading: lecture notes
Rayleigh-Taylor instability in statics, and again in the presence of vertical oscillation, salt fingers
Assignment #4: problems, solutions

February 22
12. Patterns and symmetry
Reading: Chapter 4, lecture notes

February 24
13. Nonlinear saturation (Swift-Hohenberg equation)
Reading: Chapter 4, lecture notes

Assignment #5: problems, solutions

March 1

14. Nonlinear saturation (d'Arcy convection)

Reading: Chapter 4, lecture notes

March 3

15. Stability of patterns

Reading: Chapter 4, lecture notes
Matlab simulations: zigzag, Eckhaus, cross-roll, skew-varicose instability
Assignment #6: problems, solutions (Maple) for problems 1, 2 and 3

Midterm (due 3/10/11)

Click here to download the assignment and solutions

March 8

16. Secondary instabilities

Reading: Chapter 4, lecture notes
Nonlinear saturation and secondary instabilities in the Swift-Hohenberg equation: Maple code

March 10

17. Amplitude equations for stripe states

Reading: Chapter 6, lecture notes
Assignment #7: problems, solutions, Maple code

March 15

18. Amplitude equations for stripe states

Reading: Chapter 6, lecture notes

March 17

19. Applications of amplitude equations

Reading: Chapter 6, lecture notes
Assignment #8: problems, solutions (Maple code)

March 29

20. Amplitude equations for lattice states

Reading: Chapter 7 and 8, lecture notes

March 31

21. Moving fronts

Reading: Chapter 8, lecture notes
Assignment #9: problems, solutions, Maple code

April 5

22. Pattern selection by moving fronts

Reading: Chapter 8, lecture notes

April 7

23. Dynamics of excitable media

Reading: Chapter 11, lecture notes

Assignment #10: problems, solutions (incomplete)

April 12

24. Waves in excitable media

Reading: Chapter 11, lecture notes

Matlab simulation: Spiral waves in the 2D FitzHugh-Nagumo model

Java simulations: Wave propagation in models of cardiac tissue

April 14

25. Stability of traveling waves

Reading: Chapter 11, lecture notes

Example: Stability of a spreading liquid film

Assignment #11: problems

April 19

26. Oscillatory patterns

Reading: Chapter 10, lecture notes

Matlab simulation: Spiral waves in the 2D complex Ginzburg-Landau equation

April 21

27. Amplitude and phase equations

Reading: Chapter 10, lecture notes

April 26

28. Stability baloon and defects

Reading: Chapter 10, lecture notes

April 28

No class

Course Instructor Opinion Survey

Please fill out the online Course Survey. This is your real opportunity to provide feedback regarding the contents of the course, the style and quality of the presentation, or any other subject related to the course. Tell us what you liked and what you did not like. Your input is very valuable and will benefit students taking this course in subsequent years.