

**UPDATED** December 5, 2010  
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<http://www.cns.gatech.edu/~roman/phys4267>

# Introduction to Nonlinear Dynamics and Chaos

## Instructor

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Office hours: Tuesday 1-2pm

## Place and Times

Tuesdays and Thursdays, 9:35-10:55am  
Room S107, Howey Physics Building

## Course Description

The material covered includes differential equations, their stability and bifurcations, iterated maps, deterministic chaos, fractals, and strange attractors with applications to physical, chemical, and biological systems.

## Textbook

Steven Strogatz: *Nonlinear Dynamics and Chaos* (Perseus Books, 1998). All chapter and exercise numbers refer to this book, unless stated otherwise.

Other books you might find useful:

- Maps:
  - H. Schuster, *Deterministic Chaos*, VCH, Weinheim.
  - K. Aligood, T. Sauer, J. Yorke, *Chaos: an Introduction to Dynamical Systems*, Springer-Verlag, New York.
- Hamiltonian Systems:
  - A. Lichtenberg, M. Lieberman, *Regular and Stochastic Motion*, Springer-Verlag, New York.
- Perturbation Theory:
  - C. M. Bender and S. A. Orszag, *Advanced Mathematical Methods for Scientists and Engineers: Asymptotic Methods and Perturbation Theory*, Springer, New York.

## Homeworks

Homework assignments will be posted on the web every Thursday and will be

due next Thursday **in class**. You can discuss problems with each other, but the solutions have to be executed and submitted **individually**. Some assignments will include problems (**in blue font**), which are mandatory for graduates and optional (for extra credit) for undergraduates. In general you are expected to comply with the [academic honor code](#). Grades will be based on the results of the homework assignments (50%), midterm (20%), and final exam (30%).

## Course Schedule

August 24

### 1. Introduction

Reading: Chapter 1, [lecture notes](#)

August 26

### 2. Flows on the line

Reading: Chapter 2, [lecture notes](#)

Fun stuff: [snowflakes](#) and [synchronization](#)

Problem set #1: 2.1.5, 2.2.10, 2.3.3, 2.4.2, 2.4.9 ([solutions](#))

August 31

### 3. Lyapunov function

Reading: Chapter 2, [lecture notes](#)

September 2

### 4. Numerical solution of nonlinear ODEs

Reading: Chapter 2, [lecture notes](#), Sections 16.0-16.3 of *Numerical Recipes* by Press *et al.*

Problem set #2: [2.5.3](#), 2.7.6, 2.8.3 ([solutions](#))

**Note:** There is a typo in 2.8.3 part (c): you are to plot  $\ln(E)$  vs.  $\ln(\Delta t)$ . Explain how the slope is related to the order of the method.

September 7

### 5. Bifurcations in one-dimensional systems

Reading: Chapter 3, [lecture notes](#)

September 9

### 6. Bifurcations in one-dimensional systems (continued)

Reading: Chapter 3, [lecture notes](#)

Problem set 3: 3.1.3, 3.2.6, 3.3.1, 3.4.8, 3.5.7 ([solutions](#))

September 14

### 7. Flows on a circle

Reading: Chapter 4, [lecture notes](#)

September 16

### 8. Two-dimensional systems

Reading: Chapter 5, [lecture notes](#)

Problem set 4: 3.6.2, [3.7.6](#), 4.1.8, 4.3.7, 4.6.3 ([solutions](#))

[September 21](#)

## **9. Phase plane analysis**

Reading: Chapter 6, [lecture notes](#)

[September 23](#)

## **10. Conservative systems**

Reading: Chapter 6, [lecture notes](#)

Problem set 5: 5.1.10, 5.2.13, 6.1.3, 6.2.1, 6.3.1, 6.4.7 ([solutions](#))

[September 28](#)

## **11. Limit Cycles**

Reading: Chapter 7, [lecture notes](#)

[September 30](#)

## **12. Perturbation theory**

Reading: Chapter 7, [lecture notes](#)

Problem set 6: [6.5.10](#), 6.6.3, 6.7.2, 6.8.5, 6.8.7 ([solutions](#))

[October 5](#)

## **13. Nonlinear oscillators and averaging**

Reading: Chapter 7, [lecture notes](#), Bender and Orszag

[October 7](#)

## **14. Bifurcations in two dimensions**

Reading: Chapter 8, [lecture notes](#)

Problem set 7: 7.1.6, 7.2.9, [7.3.9](#), 7.4.1 ([solutions](#))

[October 12](#)

## **Mid-term exam**

[October 14](#)

## **15. Hopf bifurcation**

Reading: Chapter 8, [lecture notes](#)

[October 21](#)

## **16. Global bifurcations of cycles**

Reading: Chapter 8, [lecture notes](#)

Problem set 8: 7.5.4, 7.6.3, 7.6.6, [7.6.17](#) ([solutions](#))

[October 26](#)

## **17. Quasiperiodicity and Poincare maps**

Reading: Chapter 8, [lecture notes](#)

October 28

## 18. Floquet Theory

Reading: Chapter 8, [lecture notes](#)

Problem set 9: 8.1.8, 8.1.11, 8.2.1, 8.2.9, 8.3.1 ([solutions](#))

November 2

## 19. Lorenz equations

Reading: Chapter 9, [lecture notes](#)

Numerical exploration of different [dynamical regimes](#) (Maple)

November 4

## 20. Lorenz equations

Reading: Chapter 9, [lecture notes](#)

Problem set 10: 8.4.2, [8.4.12](#), 8.5.2, 8.6.1, 8.7.5 ([solutions](#))

November 9

## 21. One-dimensional maps

Reading: Chapter 10, [lecture notes](#)

November 11

## 22. Universality

Reading: Chapter 10, [lecture notes](#)

Matlab simulations of the Rossler system: [reduction to 2D and 1D maps](#) and [stretching of phase space volumes](#)

Problem set 11: [assignment](#) ([solutions](#))

November 16

## 23. Statistical characterization of chaotic motion

Reading: Chapter 10, [lecture notes](#), Schuster

November 18

## 24. Fractals

Reading: Chapter 11, [lecture notes](#), Schuster

Fractals in [nature](#), [biology](#), and [mathematics](#).

Problem set 12: 10.1.11, 10.2.6 (see comments for 10.2.3), [10.3.11](#), 10.4.1 ([solutions](#))

November 23

**No class**

November 25

**Holiday**

November 30

## 25. Symbolic Dynamics

Reading: [lecture notes](#), Aligood, Sauer, and Yorke

Problem set 13: [assignment](#) ([solutions](#))

[December 2](#)

## **26. Strange Attractors**

Reading: Chapter 12, [lecture notes](#), Schuster

[December 7](#)

## **27. Hamiltonian chaos**

Reading: [lecture notes](#), Lichtenberg and Liebermann

[December 9](#)

## **28. Hamiltonian chaos**

Reading: [lecture notes](#), Lichtenberg and Liebermann

## **Final exam**

Take home, see [instructions](#)

## **Course Instructor Opinion Survey**

Please fill out the online [Course Survey](#). This is your real opportunity to provide feedback regarding the contents of the course, the style and quality of the presentation, or any other subject related to the course. Tell us what you liked and what you did not like. Your input is very valuable and will benefit students taking this course in subsequent years.