Math 7018 (Probability Methods in Combinatorics) Spring'13

- **Instructor:** Prasad Tetali, tetali-at-math.gatech.edu; 404-894-9238 (o)
- **Time/Location:** Skiles 168, MW 11:00am–Noon
- **Office hours:** Skiles 234, Wed, Fri 1:00-2:00pm, Thurs 2:00-3:00pm
- **Prerequisites:** Undergrad Probability and Combinatorics or Consent of Instructor

Objective: Two-fold: To cover a few topics from classical probability theory and to develop an appreciation for the strength and beauty of the probabilistic techniques in combinatorics.

List of Topics:

* Introduction: Sigma algebras, discrete and absolutely continuous distributions, Univariate r.v.s
* Expectation, Moment Generating Function and Properties of Characteristic Function
* Multivariate r.v.s: Joint Normal Distribution, Conditional Expectation, Chain rule for Variance, Entropy
* Convergence of Random Variables : Convergence in Probability, Almost Sure etc.
* Limit Laws: Weak and Strong Law of Large Numbers, The CLT, Berry-Esseen Theorem (Proofs are self-contained with the exception of assuming Dominated Convergence Theorem and Fubini's Theorem; treatment as in Marc Berger's book)

- Introduction to the probabilistic method: First moment method and variations
- The Second Moment Method: Examples from random graphs
- Lovasz Local Lemma : Applications, variants and the algorithmic version
- Applications of Conditioning: Independence number of triangle-free graphs, Radhakrishan-Srinivasan lower bound for Property B
- Random Graphs: Connectivity, Threshold for appearance of balanced subgraphs, The Chromatic number, Existence of limits (for independence number, maxcut, max-coloring) in sparse random graphs
- Correlation Inequalities: FKG and the Ahlswede-Daykin Theorem, Janson's inequality
- Entropy Techniques : Fractional subadditivity and submodularity, Shearer Lemma and applications to enumerative/additive combinatorics
- Dependent Random Choice : Extremal numbers for bipartite graphs, The Balog-Szemerédi-Gowers theorem
- Miscellaneous: a subset of the following topics from Alon-Spencer:
  The semi-random method: Proof sketch of R(3,k) lower bound
  Geometric Applications: Sign Matrices, epsilon-nets, and VC-dimension
  Discrepancy Theory: Discrepancy of set systems

Suggested Textbooks:

* *Probability and Random Processes*, by Grimmett and Stirzaker, (3rd ed.) Oxford University Press, 2001.

* *Introduction to Probability and Stochastic Processes*, by Marc Berger, Springer-Verlag, 1992.

- *The Probabilistic Method*, by N. Alon and J. Spencer, Wiley (Third Edition, 2008).

additional references:

! *Random Graphs*, S. Janson, T. Luczak, and A. Rucinski (Wiley-Interscience Series, 2000).

! *Concentration of Measure for the Analysis of Randomized Algorithms*, by D. P. Dubhashi and A. Panconesi (Cambridge 2009).

General grading policy : HWs 40% ; Two Tests 30% ; Final 30%

Homeworks will be assigned, collected and graded on a regular basis. You are strongly advised to (attempt to) solve all the homework problems. You are allowed to discuss your homework assignments with other students, but are required to write the solutions on your own. In other words, you are **not** allowed to copy another student's solution.

Late submission of HWs is discouraged with a penalty of 20%.

Academic Dishonesty: All students are expected to comply with the Georgia Tech Honor Code. Any evidence of cheating or other violations of the Georgia Tech Honor Code will be submitted directly to the Dean of Students. The institute honor code is available at:
http://www.deanofstudents.gatech.edu/
http://osi.gatech.edu/plugins/content/index.php?id=46