

Syllabus

Lectures in Graphics

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This is the combined syllabus for several courses in Visual Computing offered at Georgia Tech.

The material posted here is organized into some 500 lecturettes (very short lectures), which are each assigned a category and a level of difficulty (which corresponds to a particular CS course).

The **categories** include:

ACC (acceleration): Culling, occlusion, levels-of-detail, panoramas, textures, and other acceleration techniques
ANI (animation): Represent and animate motions, morphs, and deformations of shapes
CUR (curved geometry): Represent, create, and process curves and surfaces (ellipse, Bezier, J-splines, NUBS...)
EXA (exams): Test samples, questions and past exams, quizzes, midterms
GEN (general): Course overview, policy, grading, software used, theory principles, project guidelines
GRA (graphics): Transforms, perspective, clipping, lighting, rasterization, and z-buffer in the graphics pipeline
GUI (graphic user interface): Design and implement user interaction (pick, drag, 3D and multi-touch input)
LIN (linear geometry): Create and process geometric primitives (points, vectors, frames, lines, planes, triangles).
MED (media): import, use, or export images, photos, textures, audio, and videos
MOD (modeling): Construct and process representations of shapes and scenes (polygons, triangle meshes, patterns).
PHO (photorealism): light, reflections, BRDF, shadows, highlights, ray tracing, radiosity, and other effects
PHY (physics): Equation of motion, collision prediction and reaction, flow simulation, dynamic systems
PRO (projects): Description of a project or project component, with deliverables, demos, reading material, code template
SOF (software): Tutorial of how to install and use the Processing language and its graphics API
VIS (visualization): Perception (color, resolution, illusions) and how to better visualize shapes, structures, and animations

Levels of difficulty are color coded as follows:

Clover green (apprentice): Every student should know this material and should be able to use it correctly.

Midnight blue (pro): Every graduate student and most developers in the field should understand this material and be able to reinvent it and explain it.

Cayenne red (guru): Advanced material for those interested in improving performance and scalability of graphic and animation applications, or interested in research and/or teaching.

Black (unassigned): Back-up material not included in any class, but of potential use in projects.

The short description of each lecturette summarizes what the students are expected to know

This syllabus and linked material are posted at <http://www.cc.gatech.edu/~jarek/LecturesInGraphics/>

The linked material includes:

- Videos explaining or demonstrating various concepts or projects: [videoXXX.m4v](#) or [videoXXX.mov](#)
- Processing sketches [sketchXXX.pde](#) or zipped folders (source code and data files): [sketchXXX.zip](#)
- Link to web pages with further details or different perspectives: [pageXXX.html](#),
- My notes and assigned reading (mandatory, unless in parentheses): [noteXXX.pdf](#)

This material is primarily designed to support the following courses at Georgia Tech,

but may be of value for other Visual Computing courses in rendering, animation, virtual and augmented reality, game design, and video effects:

- Undergraduate introduction to Computer Graphics (CS 3451)
- Graduate course in Computer Graphics (CS 6491)
- Advanced graduate course on 3D Complexity Techniques for Graphics, Modeling, and Animation (CS 7491)

Each course uses a different subset of lecturettes. Furthermore, depending on the instructor, students' skills and interests, the knowledge based needed for the projects, the progress made by the class that semester, and the evolution of the field, the particular subset and the order of the lecturettes may be different each time the course is taught.

But, unless specified otherwise, the following color coding applies to the different courses:

- **students of CS3451 are accountable for understanding and knowing the material in green,**
- **students of CS6491 are responsible for knowing the material in green and being able to explain and use the material in blue, and**
- **students of CS7491 are responsible for knowing the material in green and blue, and for understanding the research results and open questions in red.**

Overview

GEN	Scope of the course: What is covered in the course, target audience, resources. video001 ,
GEN	Grading: tests, projects, presentations, attendance, late policy (discussed in class)
GEN	Projects: Source (comments, header), demo (picture, name, GUI), write-up (header, format), videos, upload, team projects
GEN	Resources: Text book, examples & demo sketches, class notes, videos, slides, papers, sites

Intro to Processing and 2D graphics

Getting started and basics components of a Processing sketch

SOF	Install & run: Download Processing, run, edit, save, export. Processing site , video002 ,
SOF	Structure of a sketch: Global variables, initialization, display loop, user actions. sketch001 , video003 , tutorials
SOF	Software constructs: variables, methods, arrays, conditionals, loops, functions, classes, objects. reference , online Tutorials
SOF	Learning, writing, and debugging code: Load/edit examples, comments, references, coding and debugging techniques. ProcessingBooks.pdf

Graphics primitives and user interaction in 2D

GRA	Canvas: Coordinate system, canvas size, center, clear, background, double buffering. sketch002 ,
GUI	Mouse: Current and previous position, actions/modifiers (press, drag, release), rubber-band, sketch002 ,
GRA	Shapes: line, rectangle, disk, triangle, quad, polygon, weight. sketch004 ,
SOF	Colors: stroke & fill states, HLS color systems, stroke/fill, color picker, color ramp, opacity. sketch003 ,
GUI	Drag disk: display disk, drag with mouse, snap vs relative drag. sketch001 ,
GRA	Display disks: Retained model, array of disk objects, initialize, display, color, label. sketch003 ,
GUI	Pick & modify disk: pick center or border and drag to modify. sketch005 ,
GUI	Append disk: click&drag to append disks of different radii. sketch006
GUI	Insert or delete disk: click to delete disk, click & drag to append after and update array.

Text, keys and files

SOF	Manipulate text: Char and string, operations and conversions, concatenation, formatting
SOF	Text: debugging print, write on canvas, load font, write coordinates next to mouse, label a point
GUI	Help: show and toggle help menu, assign actions to keys, state change vs while pressed
SOF	File Read/write: saving data to file, reading from file, managing file names
SOF	File selection: Define fixed, automatic, or user-specified path for loading or saving files
SOF	Keyboard input: read, parse

Images and videos

MED	Images: save snapshots of canvas as images, load and display images
MED	Pixels: read and change pixels in image and on screen, image processing
MED	Videos: make videos from saved snapshots, capture & process live video from camera
MED	Audio: load, play, record, save, analyze audio
MED	SVG: output vector graphics files
MED	PDF: output PDF files

Transformations

MOD	Transformations: translate, rotate, scale, order and non-commutativity.
MOD	Fixed point: Rotate or scale around a given fixed point.
MOD	Matrix stack: push/pop operations, sketch003 ,
MOD	Patterns: Translating, rotating, spiraling, patterns of patterns
MOD	Scene graph: Articulated shape, 2D human stick figure
MOD	Recursive patterns: Spirals, snowflakes, tree, P010.jpg , sketch010 ,

Animation

ANI	In-betweening: position and radius of disk controlled by mouse
ANI	Time: update time at each frame, loop / round trip, ease-in / ease-out, frame rate, speed control
VIS	Ghosts: elastic chain of previous positions while dragging or animating a disk
PHY	Free fall in box: Equation of free fall motion, collision with cube boundaries, reflected velocity

Projects on 2D graphics

PRO	Drag your picture and your name: Demo, file header, submission process
PRO	Rotate and scale: Relative to center of picture, use transforms and interrupts, nice GUI
PRO	Single click&drag: Specify translation, rotation, and scaling in a single gesture
PRO	Edit disks: Interactive editor for a set of disks (append, move, delete), display with IDs, archival
PRO	Disk chain creation: Attach tangentially next disk to previous
PRO	Prevent disk overlap: Limit disk creation and size to the empty space, combine with disk chain.
PRO	Disk chain editing: Grow disk as much as possible, Apollonius problem. Computational complexity.
PRO	Insert maximal disk: Grow (and adjust) disk as much as possible, Apollonius problem. Computational complexity.
PRO	Fill ring: Fill largest disk with all the other ones. Principles, strategy, complexity, heuristics.
PRO	Follow: Explain Examples>Topics>Interaction>Follow3
PRO	Reach: Fix Examples>Topics>Interaction>Reach3

Geometric primitives in 2D

Overview

GEN	Points & Vectors: Semantics, importance in graphics, representation, notation, usage, difference
GEN	Implicit, parametric, procedural: Definition, notation, representation, examples, advantages, conversion
GEN	algebraic sets, analytic sets, procedural sets, fractals

Points

LIN	Implementation: Class, methods, display, label, creation, assignment.
LIN	Distance: distance between points, pick closest point, two closest neighbors and tracing
LIN	Affine combinations: LERP, Different formulations, more points, constraint, implementation
LIN	Bi-linear interpolation: (s,t) mapping, mouse control, parametric expression, inverse map

Vectors

LIN	Implementation: Class, methods, display as line/arrow at point, creation from points,
LIN	Comparing: testing vector equality, number of constraints
LIN	Special alignment: testing whether two vectors are parallel or orthogonal
LIN	Linear combinations: scaling a vector, add/subtract, weighted combination
LIN	Magnitude and direction: formula, norm vs normalization, angle vs direction, unit vectors
LIN	Angles: computing the angle between two vectors, normalizing angle between -180 and +180
LIN	Dot-product: definition, properties, computation, use as test, use as constraint
MOD	Reflected vector: from a line
LIN	Cross-product (2D version): definition, properties, computation, use as test, use as constraint
LIN	Other vector products: geometric, outer
MOD	Rotated vector: 90 degree rotation formula and justification, arbitrary angle rotation
MOD	Polar coordinates: cartesian vs polar, conversions
LIN	Dot-product proof: representing vectors as complex numbers, Euler's formula, proof of dot product formula
MOD	In-betweening: Linear versus steady interpolation between vectors, de Moivre's formula
EXA	Review: Linear algebra for graphics, PDF ,

Lines

MOD	Implicit line and half-space: defined by point and normal, defined by 2 points.
MOD	Parametric line and ray: defined by point and tangent, by two points
LIN	Special configuration of two lines: orthogonal, parallel, coincident
LIN	Point projection: compute normal projection of point on line
LIN	Point distance: compute point/line distance
MOD	Numeric rounding: floating point round-off, representable points and lines

Linear halfspaces

MOD	Representation of a half-space: point&normal, point&tangent, two-points, implicit equation
GUI	Display a half-space: line through screen, sidewalk, tangent/normal arrow, label inside
MOD	Point-in-halfspace inclusion test: Implicit, parametric, round-off errors, cost
MOD	Numeric rounding: point inclusion tolerance

Rays

LIN	Applications: photon, collisions, visibility/shadows
LIN	Representation: point and (unit) tangent direction
LIN	Reflection: principle, formula of reflected ray, its derivation

Edges

MOD	Motivation: borders, trajectories, skeletons
MOD	Parametric representation: representation by end-points, map from [0,1], evaluation
MOD	Implicit formulation: Semi-algebraic representation, point-inclusion test
MOD	Point-in-slab test: test whether a point projects inside the edge
MOD	Point-in-edge test: test whether a point lies in an edge, distinguish inside and endpoint
MOD	Point-edge distance: formulation, justification, computation
MOD	Closest point projection: on edge
MOD	Edge/edge distance: Definition, computation, applications
MOD	Singular cases: Vertex contacts, parallel, overlaps

Triangle

GRA	Rendering: Closed shape, smooth fill, linear border
MOD	Construction: from 3 sides
MOD	Centroid: definition, intersection of medians, average of points
LIN	Barycentric coordinates: formulation, computation
LIN	Local vs barycentric coordinates: conversion
MOD	Measures and properties: angles, distances, perimeter, signed area, circumradius
MOD	Point-in-triangle inclusion test: areas signs, barycentric coordinates
MOD	Triangle centers: barycenter, orthocenter, circumcenter, Euler line, other centers
MOD	Triangle subdivision: in 2, 3, 4

Circle and arc

CUR	Implicit circle/disk: equation, inclusion test.
CUR	Parametric circle: equations, inclusion test
CUR	Arc: display arc, fit arc through 3 ordered points.
CUR	Fit circle to 2 points and a tangent: compute center and radius of circumcircle
CUR	Fit circle to 3 points: compute center and radius of circumcircle
CUR	Closest point projection: on circle, on arc
CUR	Fast display of circle: avoid trig at each step, use GPU
CUR	Circle inversion: Formula, properties, and applications
CUR	Conformal maps: Definition, applications, case of circle inversion
CUR	Appolonian gasket: Definition, generation
CUR	Appolonian circles: Definition, generation, applications

Intersections

LIN	Line/line intersection computation: special cases, solving using implicit forms, parametric forms, mixed forms
MOD	Edge/edge intersection test: using parameter, direct test, overlap, contact
MOD	Edge/edge intersection test -- proof: prove that the direct intersection test is necessary and sufficient
MOD	Edge/edge intersection acceleration: bounding boxes, cells, trees
LIN	Ray/edge intersection: mixed form, proper solution, and application to visible edge in 2D
CUR	Ray/circle bounce: intersection, reflection

MOD	Edge/triangle intersection test: formulation, computation
MOD	Triangle/triangle intersection test in 2D: half-space intersection
MOD	Acceleration of triangle/triangle intersection tests in 2D: disks, boxes,
CUR	Circle/circle intersection: Tes, contact plane
CUR	Ray/circle intersection: Mixed form, proper solutions
CUR	Ray/arc intersection: Mixed form, proper solutions
CUR	Arc/arc intersection: Mixed form, proper solutions

Frames and local coordinate systems

LIN	Local frame: basis, origin, local coordinates interpretation
LIN	Computing local coordinates: relative vector, coordinates
LIN	Change of coordinate systems: relative vector, coordinates
LIN	Affine transforms: {I,J,O} representation, 2x3 matrix form
LIN	Homogeneous coordinates: matrix, points, vectors, multiplication
LIN	Affine transforms: {I,J,O} representation, 2x3 matrix form
LIN	Cascades of affine transforms: associativity, non-commutativity
LIN	Inverse of an affine transform: Definition, calculation, inverse of a product
LIN	Rigid transform: definition, properties, rotation angle, translation, calculation, inverse, applications
LIN	Similarity transforms: definition, properties, applications, computing the scale, inverse

Quad

MOD	Quad: definition, interpolation, application to hight field
GRA	Rendering: Closed shape, linear border, smooth fill
MOD	Bilinear interpolation : Parametric form, calculation, implicit form
MOD	Inverse of a bilinear interpolation : Formulation, implementation, applications
MOD	Triangulation of a quad : diagonal, ambiguity, fan, error

Projects on 2D primitives

PRO	Edge/edge intersection: Implementation, visualization and testing
PRO	Edge soup editor: Implement display, select, add, edit, delete, save, load edge soup
PRO	Ray path in triangle: Trace finite length ray in triangle, interactive ray dragging
PRO	Draw a spiral: Using recursive pattern, iterative pattern and trigonometry
PRO	Arc/arc intersection: implementation
PRO	Ricochet: Compute direction of ray from A to reach B while bouncing off edge soup

Curves in 2D

General

CUR	Representation: Parametric/implicit, algebraic/procedural, interpolating/approximating
CUR	Derivatives: Tangent, curvature, from continuous function, from samples
CUR	Continuity: C^k vs G^k , measure, approximation
CUR	Limitations: Domain, accuracy, scalability

Parabola

CUR	Parabola equation: constant acceleration, integration, parametric form
ANI	Free-fall simulation: update P and V, correction, 3-point prediction
ANI	Free-fall shooting: Compute angle to hit target, maximum reach
CUR	Bézier quadric from bi-linear diagonal: demonstration, justification, implementation, sketch011.zip
CUR	Parabolic interpolant from bi-linear diagonal: demonstration, justification, implementation
CUR	Non-uniform parabolic interpolant: formulation, advantages, implementation

Cubic polynomial

CUR	Cubic Bézier : geometric construction, implementation, parametric form
CUR	Cubic Bézier convex hull : justification, union of triangles, applications
CUR	Cubic Bézier subdivision : geometric construction, justification, implementation, applications
CUR	Cubic Bézier derivatives : parametric form, at ends
CUR	Hermite interpolation : formulation, conversion to Bézier, reverse conversion
CUR	Higher order interpolation : interpolate end points, velocities, and curvatures
CUR	Interpolating cubic : general solution
CUR	Neville's algorithm : geometric construction, justification, implementation

Bézier splines and Catmull Rom splines

CUR	Bézier splines : Hermite spans, C1 continuity
CUR	Specify velocity and curvature : 3 points per sample
CUR	Catmull-Rom splines : estimating derivatives, formulation as Hermite spans

B-splines

CUR	Uniform B-spline : spans, control polygon of span, evaluation
CUR	Uniform B-spline conversion to Bézier : Image of control point, conversion
CUR	Non-uniform B-spline : formulation, intuition, sevaluation

Polyloop smoothing

CUR	Polyloop : definition, representation, class, n(), p(), display, edit points, edit curve, archive
CUR	Polyloop operations : split, dual, tuck(s), implementation
CUR	Smoothing : tuck($\frac{2}{3}$)*, shrinking problem, (tuck($\frac{2}{3}$),tuck($-\frac{2}{3}$))*

Piecewise circular curves (PCCs)

CUR	Bi-arc : definition, degree-of-freedom, computation
CUR	Bi-arcs : inscribed, interpolating

Polyloop subdivision

CUR	Quadratic B-spline subdivision : (split, tuck($\frac{1}{2}$))*
CUR	Cubic B-spline subdivision : (split, tuck($\frac{1}{2}$), tuck($\frac{1}{2}$))*
CUR	Non-uniform cubic B-spline subdivision : formulation, intuition, application, benefits
CUR	Quintic B-spline subdivision : (split, tuck($\frac{1}{2}$), tuck($\frac{1}{2}$), tuck($\frac{1}{2}$), tuck($\frac{1}{2}$))*
CUR	FourPoint subdivision : (split, tuck($\frac{1}{2}$),tuck(-1))*
CUR	FourPoint subdivision using parabola average : formulation, left
CUR	J-splines: definition J_s=(split, tuck($\frac{1}{2}$), tuck(s/2-1))*
CUR	J-splines properties : J_0=FourPoint, J_1=Cubic B-spline, J_(3/2)=Quintic B-spline
CUR	Jarek's curve : J_ $\frac{1}{2}$,motivation, popping minimization, area preservation
CUR	Variable width : normal, radial, ball offsets, definitions, computation
CUR	Ringin : Footprint, rings, algorithm, further improvements

Polyline subdivision

CUR	FourPoint subdivision using parabola average : no average at the end
CUR	End-extension heuristics : parabolic prediction

Polyloop resampling

MOD	Arc-length measure : definition, implementation, total length
MOD	Uniform arc-length resampling : algorithm
MOD	Equidistant resampling : Definition (constant edge length), challenge

Curvature

CUR	Curvature estimators : circle, parabola
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CUR **Curvature sensitive resampling:** benefits, strategies, implementation

Animation

ANI **Animate a point:** parametric forms, subdivision form, resampling
ANI **Ease-in and -out:** Bezier splines, B-splines, time warp
ANI **Animate a curve:** animate its control points, show/edit keyframes or trajectories, [sketch012.zip](#)
ANI **Compare curve animations:** cyclic keyframed motion

Projects

PRO **Visualize acceleration and jerk:** vectors, dynamic, 3D

Animations in 2D

Physics

PHY **Free fall:** parabolic path, advection/correction, prediction, aim,
PHY **Dynamic:** translation, forces, rotation, torques
PHY **Inertia:** formulae of inertia moments, computation for polygons
PHY **Drag:** physical drag with constraint or using a spring
PHY **Collision test:** detection/prediction (continuous) for disks and polygons
PHY **Collision response:** contact plane, forces, friction
PHY **Complex collisions:** strategy, examples
PHY **Particles:** simulate motion, density, pressure, viscosity
PHY **Fluid:** Eulerian/Lagrangian formulations, Navier-Stokes equation, advection, area preservation, interaction with air and solids

Point and curve animation

ANI **Animate a point:** parametric forms, subdivision form, resampling
ANI **Ease-in and -out:** Bezier splines, B-splines, time warp
ANI **Animate a curve:** animate its control points, show/edit keyframes or trajectories
ANI **Curvature morph:** compute curvature, morph, reconstruct, register
ANI **Compare curve animations:** cyclic keyframed motion

Primitive motions

ANI **Rigid motions:** Definition of congruency, rotation/translation, formulation, computation of morph, comparison with dynamics
ANI **Spiral motions:** Definition of similarity, spiral, formulation, computation of morph, comparison with dynamics
ANI **As-Rigid-As-Possible:** Definition, computation, properties
ANI **Steady motions:** Definition of affinity, steady, formulation, computation of morph, solvability, properties, comparison with
ANI **3 frame interpolant:** Definition, solutions, bi-SAM, benefits, UAM, comparison

Composite motions

ANI **Subdivision of key framed polySAMs:** Definition, computation, examples, special cases

Cloud motions

MOD **Registration:** Rigid, similar, affine
ANI **Moving Least Square Blending:** Definition, computation, examples, special cases
ANI **SAMBA:** Problem statement, formulation, computation, examples, special cases

Projects

PRO **Cyclic keyframe animation editor:** design, archive, play smooth animations

Geometric arrangements in 2D

Point cloud

LIN	Transformation: translation, rotation, rigid motion, scaling, similarity, shear/squash, affine
LIN	Centroid: formula, rotate / scale around centroid
MOD	Point cloud registration: rigid, similar, affine
MOD	Delaunay triangulation: definition, computation, applications (spanning tree, triangulation)
LIN	Convex hull: Intuition, definition, computation
MOD	Voronoi regions: definition, point-in-cell test, Voronoi vertices, duality with Delaunay
MOD	Alpha hull, shape, and complex: definition, properties, computation, applications
MOD	Gabriel graph: definition, computation, properties, applications
MOD	Computational geometry: computational complexity
MOD	Fast convex hull: Jarvis march, Graham scan, Aki-Toussaint acceleration
MOD	Fast Delaunay: Insertion (edge test and flip, propagation), sweep line

Half-space and line arrangements

MOD	General configurations: definition, importance, realization
MOD	Cells: Definition, properties, point inclusion test, identification
MOD	Counting cells: count edges and faces in general configuration of lines
MOD	BSP: definition, representation, construction, point classification
MOD	CSG: definition, representation, construction, point classification, stack
MOD	Active Zone in CSG: definition, properties, use, applications
MOD	BSP/CSG comparison and optimization: classification cost, footprint, optimization
MOD	Blist, definition, CSG-to-Blist conversion, footprint, OBF

Polygons

MOD	Simple polygon: Definition, interior, representation, display, editing
MOD	Face: disjoint, holes, non-manifold vertices, represent with simple polygon
MOD	Self-crossing polygons: winding number, alternating parity
MOD	Point-in-polygon: ray-intersection parity
MOD	Point-in-polygon: point-in-triangle parity
MOD	Point-in-polygon: fast and robust implementation

Edge soup

MOD	Edge soup: definition, interactive editing
MOD	Intersections: testing, computing, ordering
MOD	Sidewalk loops: Right-most turn, follow loop
MOD	Point-in-loop test: ray intersection parity
MOD	Faces: Loop containment graph

Triangle meshes in 2D

Planar graphs

MOD	Planar graph definition: Vertices, reactively open edges, open faces, disjoint, valence
MOD	Connected components of a planar graph: Definition, computation (union find)
MOD	Euler-Poincaré formula: formula, examples, importance
MOD	Euler-Poincaré formula proof: derivation/proof

Triangle mesh representation and traversal

MOD	Maximal planar graph: planar triangulation, $T=2v-4$, average valence
MOD	Meshes with borders: definition, counting, tracing borders

MOD	Corner operators: n, p, s, u, o, l, r, t, v, c
MOD	Representations: Corner Table (CT), Swing Table (ST), Winged Edge (WE), others
MOD	Construction: Quadratic algorithm
MOD	Fast construction: Swing lists, sorted swing lists, hash table
MOD	Streaming: Problem statement, motivation, approaches
MOD	Graph drawing: Integer coordinates, Schnyder woods
MOD	Compact representations: SOT, Squad, Zipper, Meshlets
MOD	Non-manifold border vertices: definition, counting, tracing borders, extending CT and ST

Mesh processing

MOD	Valence: local computation, global computation
MOD	Border: identify border edges
MOD	Components: identify connected components
MOD	Border loops: count, trace border loops, trace border between two parts
MOD	Smoothing: border, interior
MOD	Regular subdivision: definition, implementation, point estimation
MOD	Adaptive subdivision: definition, criteria, implementation

Connectivity compression

MOD	Swirl traversal: definition and C, L, E, R, S triangle labels, shallow stack implementation
MOD	Vertex spanning edge tree (VSET): definition, computation, encoding
MOD	Vertex spanning triangle tree (VSTT): definition, encoding, border
MOD	Topological surgery: definition, encoding
MOD	Compression with offsets: definition and reconstruction algorithm
MOD	EdgeBreaker: formula for implicit offsets, decompression (build and zip)

Projects

PRO	Point cloud editor: Append, pick, move, delete, translate, transform all (rotate, scale, around centroid)
PRO	Graph editor: Add/delete edge; move add, pick, delete, move vertices; transform all
PRO	Ray path in edge soup: Trace finite length ray in edge soup, interactive ray dragging
PRO	Edge soup connectivity: Edit, save, read, edges. Detect all edge/edge intersections. Compute connected sets.
PRO	Graph processing: Test if connected, compute minimum spanning tree
PRO	Draw fat stroke: Capture speed, variable weight, draw as 2 rows of quads, smooth shading with flat normals
PRO	Edge soup walks: Trace and count sidewalk loops for crossing edge soup

Projects

PRO	Delaunay triangulation: Show Delaunay triangles and closest neighbor oriented edges
PRO	Constrained Delaunay: Delaunay triangulation that respects non-crossing edge soup

Morphology

Distance

MOD	Distance measures: Minimum, Hausdorff, Frechet, Ball
MOD	Voronoi: Definition, computation

Correspondence

MOD	Correspondence formulations: parametric, arc-length, closest, normal, ball
MOD	Compatibility: normal/ball, definition, properties

Offset and blends

MOD	Blending functions: Ricci's blends, RBF, r-sets, issues.
MOD	Grow & shrink: Definition, computation (discrete, continuous), applications
MOD	Area compensation: offset distance formula, curvature-dependent offset, motivation, formula

MOD **Fiil & round:** Definition, applications
MOD **Tightening:** Definition, computation, applications
MOD **Relative rounding:** Motivation, definition, computation, applications

Medial Axis

MOD **MAT:** Definition, computation (discrete, continuous), applications
MOD **Envelop:** Normal/ball,radial, definition, computation
MOD **Curve average:** Definition, computation (discrete, continuous), applications

Minkowski sums and morphs

MOD **Minkowski sum:** Definition, computation, implementation for convex polygons
MOD **Minkowski morphs:** Definition, computation, properties, applications to animations
MOD **Minkowski morphs of non-convex polygons:** Self-crossing, index, selection

Ball morphs

MOD **Ball morph of PCCs:** Definition, computation, implementation
MOD **Ball morph of polygons:** Definition, computation, implementation

Inflation

MOD **Distance transform:** Definition, computation, implementation, application
MOD **Regularity transform:** Definition, computation, implementation, application
MOD **Inflation:** From medial axis transform

Topology

Mathematical concepts

MOD **Point set definition:** Concept, notation, motivation
MOD **Booleans:** Union, intersection, difference, XOR, properties
MOD **Interior, boundary, exterior:** Definition, examples, closure, open, closed, regularized (open/closed)

Polygons

MOD **What is a polygon:** Examples, definition, options, loops, non-manifold
MOD **Point-in-polygon test:** Ray stab parity, triangle inclusion parity, fast implementation, alternating difference, XOR
MOD **Representation:** Loops, loop, edge soup, constructive, axis-aligned
MOD **Self-crossing loop:** Examples, index, semantics, applications to Minkowski sums, offsets, drawings, animation
MOD **Triangulation:** Motivation, definition, algorithms

Simplicial complexes

MOD **Simplicial complexes:** Definition of simplices and complexes, operators, applications

3D geometry

Points and vectors in 3D

LIN **Cross-product (3D):** definition, notation, computation, properties, use in constructions
MOD **Angles:** computing the angle between two vectors, rotating a vector
MOD **Special alignment:** testing whether two vectors are parallel or orthogonal
LIN **Decomposition:** tangent/normal decomposition of a vector and applications to reflections
LIN **Reflection:** Formula, derivation, implementation, applications
MOD **Mixed product:** Formula, intuition, properties, applications to signed volume, flatness

Intersections and containment tests in 3D

MOD	Ray/triangle and edge/triangle intersection tests: Formula, intuition, implementation
MOD	Point-in-tetrahedron test: Formula, intuition, implementation
MOD	Tet-in-tet test: Formula, intuition, implementation
MOD	Tet/tet intersection test: Formula, intuition, justification, implementation
MOD	Tet/sphere & sphere/tet containment test: Formula, intuition, justification, implementation

Frames and transforms in 3D

LIN	Orthonormal frame: definition, orientation, matrix, construction from l, from z, , composition, inverse
LIN	Quaternions: formulation, rotation, composition, implementation
LIN	Affinity: definition, matrix, computation from 4 point displacements, properties

Motions in 3D

MOD	Direction interpolation: linear, minimal angle rotation
MOD	Rotation: around given axis
MOD	Screw: definition, , computation, applications
MOD	SAM in 3D: definition, computation, solvability
MOD	Patterns: definition, patterns of patterns, animation
MOD	bi-SAM: parabolic interpolant, Bezier evaluation, implementation, results
MOD	Poly-SAM subdivision: definition, computation, application

Curves and surfaces

MOD	Curves in 3D: lifting definitions and construction from 2D schemes, topology (tubes, tori)
MOD	Tensor-product surfaces: Bezier, Neville, NUBS, mixed,
MOD	Surface tangent & normals: computing parametric tangents, computing surface normals
MOD	Normals under non-similar transforms: problem statement, motivation, solution

3D graphics basics

Basics graphics and user interaction in 3D

GRA	Renderer: invoke P3D, coordinates, transforms, primitives
GRA	Primitives: sphere, triangle, quad (bilinear and triangulated), block, cone, cylinder
GRA	Light and surface properties: position, color, ambient, Lambertian, specular

Basics graphics and user interaction in 3D

GUI	Pick surface point: 3D pick, feedback, applications
GUI	Screen coordinates: screen aligned coordinates, obtaining them
GUI	Drag point in 3D: 3D pick, feedback, screen coordinates
GRA	Rotate view: set view, rotate view around fixed point
GRA	Rotate view around picked point: formulation, implementation

3D triangle mesh processing

Topology

MOD	Water tight meshes: definition, properties, detection
MOD	Manifold: non-manifold edges and vertices, detection
MOD	Genus and holes: definitions, Euler-Poincaré equation for manifold shell
MOD	Orientation: definition, encoding in order of vertex references, computation
MOD	Shells: definition, identification (swirl), topological classification, containment tree, solid parts

Normals and shading

MOD	Normals: triangle normals, area weighted, vertex normals
MOD	Normals at border and sharp edges: border vertices, sharp edges, per-corner normals
MOD	Gouraud shading: Definition, implementation, benefits, limitation
MOD	Phong shading: Definition, implementation, limitations

Traversal

MOD	Silhouettes: Definition, proof of loop, tracing, subdivision for rendering
MOD	Geodesic walk: Definition, exit edge, new direction
MOD	Geodesic walk: Definition, exit edge, new direction
MOD	Geodesic path: Definition, computation, acceleration

Smoothing and subdivision

MOD	Smoothing: Dual, tuck, tuck-untuck, Beltrami-Laplace
MOD	Filling holes: Build fan, construct center
MOD	Subdivision: 1-to-4 split, implementation, butterfly, Loop, adaptive subdivision

Simplification and Levels-of-Detail

MOD	Vertex clustering: Quantization, degenerate triangles, degenerate edges, implementation, properties
MOD	Edge collapse: Implementation, error measures: normal displacement, minimal, quadric
MOD	Simplification strategies: greedy, optimal vertex placement, memory-less
MOD	Levels of Detail: Static vs progressive, stream of v-splits/e-col, adaptive
MOD	Multi-resolution representations: simplicial complex, Quad-tree

Geometry/texture compression

MOD	Coordinate quantization: Application requirements, non-uniform quantization,
MOD	Geometry prediction: Quad predictor, more advanced predictors
MOD	Correction encoding: Quantized correction, variable length encoding, Huffman tree, entropy
MOD	Normal compression: Normal representation, quantization, prediction
MOD	Texture compression: Texture representation, quantization, prediction

Offsetting

MOD	Normal offsetting: Vertex displacement, self-crossing, chamfering
MOD	Volume compensation: Adjust displacement, curvature computation, applications
MOD	Constant flux: local formula

Intersections and Booleans

MOD	Point-inclusion test: ray-shooting, tet-inclusion parity, robustness, speed
MOD	Interference test: edge-triangle pairs, co-planarity
MOD	Intersection loops: labels, connection, tracing, splitting triangles, approximation
MOD	Booleans: propagation vs testing, incremental, regularization
MOD	Numeric accuracy: tolerance, extended precision rationals, determinant tests, SoS, general position
MOD	Self-crossing test: edge-triangle pairs, why they are insufficient

3D model creation

Acquisition

MOD	3D scans: Depth scanners (time of flight, triangulation), z-buffers
MOD	Surface meshing: Triangulation vs z-buffer merging

Drawing contours

MOD	Contour inflation: MAT, elevation model, Teddy, Teddy ,
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MOD **Redraw silhouette:** Project, deform, RoI, smoothing

Constructive models

MOD **Extrusions:** contours (polygon, PCC), linear, circular, morph
MOD **Booleans:** primitive, transformations, CSG, parameterization, constraints

Skinning

MOD **Canal surface:** Definition, 4D curve subdivision
MOD **Spine:** Frénét frame, twist propagation, local coordinates, perfect loop
MOD **Pipe skinning:** Quad-rings, checkerboard pattern
MOD **Blending spheres:** contact rings, Dupin cyclide, interpolation, [Basti11](#),

Interpolating points and curves

MOD **Radial Basis Function (RBF):** Definition, computation, example
MOD **Curve networks:** Normals, patches, interpolation

3D deformations and morphs

Triangle mesh deformations

MOD **Vertex displacement:** Pick, drag in 3D, along normal
MOD **Region of influence:** Decay function,
MOD **Twister:** Frame grab and move, 3D GUI, screw, adaptive tessellation
MOD **Twister two hands:** Overlap RoI, squashing, cos and sin squares,
MOD **Bender:** Ribbon, PCC, twist distribution closest projection
MOD **Silhouette:** Select, drag, apply

Triangle mesh morphing

MOD **Correspondence:** One-to-one, partial, closest projection
MOD **Registration:** Rigid, similar, affine, non-rigid
ANI **Linear:** Two key-frames, more key-frames
ANI **Ball morph:** Ball correspondence, circular trajectories, synchronization, incompatible areas
ANI **Minkowski morph of convex shapes:** Normal correspondence, T-V and E-E pairs, algorithm, representation, multiple keyframes
ANI **Minkowski morph of general shapes:** Self-crossing, point containment

3D rendering

Perception

PSY **Color:** Tristimulus, RGB, HLS
PSY **Perception:** Eye, cones/rods, fovea/periphery, acuity, motion, color separation/recognition
PSY **Illusions:** Shape, color, size, motion,
PSY **Aesthetics:** Gestalt, Prägnanz, complexity, steadiness, grace

Light and photorealistic rendering

PHY **Light:** Photons, wave, speed, linear propagation, measure, dimension
PHY **Photon paths:** Snell's law of refraction, Fermat's principle of reflection, [wiki](#),
PHY **Optics:** Pinhole camera, spherical lenses, focus
PHY **Mirrors:** virtual images, multiple reflections, retroreflector, [wiki](#),
PHY **Specular reflection:** Highlight, spread, models (Phong, Cook-Torrance), dependence on distance, [wiki](#),
PHY **Diffuse reflection:** Lambertian surface, independence of viewer location
GRA **BRDF:** general formulation, special cases

GRA	Shadows: Filler rays, shadow buffer
GRA	Ray tracing: from light source, from eye, primary highlights, secondary highlights
GRA	Area light sources: umbra/penumbra, Minkowski sums, sampling, curved boundaries
GRA	Radiosity: Justification, mutual transfer between two patches, solvers

Non-Photorealistic Rendering (NPR)

PSY	Silhouettes: Importance, definition, classification, rendering
PSY	Characteristic lines: Definitions, computation
PSY	Hatching: Models, computation, coherence during motion and deformation

Volumes

Voxels

MOD	Voxel models: binary (in, out, partial)
MOD	Rasterize triangle meshes: regular sampling, robust point-in-solid, speed-ups
MOD	Compact representations: run-length encoding, compression

Distance fields

MOD	Definition: point-mesh-distance, sign, point-surface distance, sharp features
MOD	Computation: exact, propagation, sweeps

Iso-surfaces

MOD	Binary: face cutting, consistency (the Marching Cube mistake), cube splitting
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GPU

Graphics pipeline

GRA	Transforms: model space, screen space
GRA	Lighting: ambient, Lambertian, specular, finite/infinite light
GRA	Clipping: viewing frustum, polygon clipping
GRA	Perspective: projection, properties
GRA	Rasterization: drawing lines, filling triangles
GRA	Texture: texture coordinates, blending, assignments, procedural, distortion, correction
GRA	Z-buffer: principle, implementation, linear interpolation, accuracy issues, turning off
GRA	Visibility: depth order, correct perspective transform, homogeneous coordinates, inverse perspective

Shaders

GRA	Programmable shaders: Architecture, shaders, parallelization, languages
GRA	Vertex shader: function, input/output, coordinates, matrices, normals, effects
GRA	Fragment shader: function, input/output, coordinates, texture mapping, normals, Phong shading, bump mapping
GRA	Image processing: filters, blending images
GRA	Geometry & tessellation shaders: Bezier curves, subdivision surface, normals, silhouettes

