**Finite Element Method for Coupled Processes in Elastic Porous Media – Spring 2018**

**Meeting Times:** Tuesday & Thursday, 9.30am-10.45am

**Classroom:** ES&T L1118

**Office hours:** Monday & Wednesday, 2pm-3pm (or by appointment)

**Prerequisites:** COE 3001 Mechanics of Deformable Bodies (or equivalent)

**Scope of the course:** The course covers basic knowledge of poroelasticity, variational methods, space and time discretization. It explains Finite Elements Methods (FEM) for elastic solids and elastic porous media, with a special emphasis on coupled thermo-mechanical and hydro-mechanical processes. Applications include problems of solid mechanics, heat and mass transfer, consolidation, seepage, precipitation and stress/pressure/temperature variations around cavities. Topics covered include the derivation of coupled thermo-hydro-mechanical FEM equations for elastic problems, the resolution of the FEM equations, the typical convergence issues encountered in coupled problems, the basic modeling strategies used in numerical poro-mechanics and the interpretation of coupled FEM simulations for engineering design.

**Learning Objectives:**

1. Write the set of equations and constraints necessary to solve a poro-mechanical problem.
2. Approximate the solution of Partial Differential Equations (PDEs) by using the weighted-residual formulation and variational methods.
3. Build a Finite Element Model (weak form, interpolation functions, element governing equation, global stiffness matrix) for singe-variable problems in 1D - including eigenvalue and time-dependent problems.
4. Solve axis-symmetric, plane strain and plane stress poro-mechanical problems with the Finite Element Method, by using analytical and numerical computations.
5. Interpret numerical errors and convergence problems.
6. Design an engineering prototype; optimize the performance of a porous medium under thermo-hydro-mechanical constraints with the FEM.
7. Recommend FEM strategies (e.g., meshing, interpolation order, time discretization) for the modeling of coupled processes in porous media.

**Course Assessment:** There will not be any final examination but there will be two tests in the course of the semester. Sample problems similar to the ones assigned in the tests will be solved in class beforehand. Homework will include linear algebra and analysis (by hand) and symbolic calculus (mostly in MATLAB). The projectis an individual assignmentaimed to design an engineering structure or system that requires the use of porous media subjected to thermo-hydro-mechanical constraints. Students will be required to do two presentations and write a report. A dedicated project syllabus will be posted later, in the course of the semester. Students will propose a project topic and, after the instructor’s approval, will complete their project with the computational software of their choice (e.g. MATLAB, ABAQUS, ANSYS, PLAXIS).

*Example projects from past semesters: Tunnel of the port of Miami; MARTA Peachtree Center Station in Atlanta; Compressed Air Energy Storage in a salt cavern; nuclear waste disposal; heat exchanger piles; oil extraction and CO2 injection in shale; construction of flow networks in competitive environments; decontamination of cementitious materials with wet poultices; design of composite materials.*

**Recommended References:**

* Reddy, J.N. *An Introduction to the Finite Element Method,* 3rd edition (2006), McGraw-Hill
* Lewis, R. W., & Schrefler, B. A. (1998). *The finite element method in the static and dynamic deformation and consolidation of porous media*. John Wiley, second edition
* Dormieux, L., Kondo, D., & Ulm, F. J. (2006). *Microporomechanics*. John Wiley & Sons.

For further reading:

* Reddy, J.N. *An Introduction to the Nonlinear Finite Element Analysis,* (2004), Oxford University Press
* Zienkiewicz, O. C., Chan, A. H. C., Pastor, M., Schrefler, B. A., & Shiomi, T. (1999). *Computational geomechanics*. John Wiley.
* Coussy, O. (2011). *Mechanics and physics of porous solids*. John Wiley & Sons.

**Outline & Schedule:** Lecture topics, test dates and deadlines will be adjusted as needed in the course of the semester. *Project 1: mid-term presentation. Project 2: final presentation and report.*

|  |  |  |
| --- | --- | --- |
| **Week** | **Tests & Deadlines** | **Topics** |
| **01/09** |  | *FEM and poromechanics.* Introduction to poromechanics and governing equations of common thermal, hydraulic and mechanical problems. Concept of mathematical approximation. Weighted Integral Method. Variational Formulation. |
| **01/16** |  |
| **01/23** | HW 1 | *Finite Element Method in 1D problems with one dependent variable.* Integral formulation, approximation functions, stiffness matrix assembly, boundary conditions, resolution methods and post-processing techniques. Applications: solid mechanics, 1D heat transfer and 1D fluid flow. Introduction to beam elements. |
| **01/30** |  |
| **02/06** | HW 2 |
| **02/13** | Test 1 | *Eigenvalue and transient problems.* Time discretization for parabolic and hyperbolic equations. Applications: 1D heat and mass transfer, beam vibration. |
| **02/20** |  |
| **02/27** | HW 3 | *2D-Finite Element models.* Triangular and rectangular linear elements, higher order elements, serendipity elements, master elements, coordinate transformation. Numerical integration in 1D and 2D: Newton Cotes quadrature, Gauss quadrature. Stiffness matrix in plane elasticity. Applications: elastic stress and strain around cavities, irrotational flow around impermeable solids, seepage problems. |
| **03/06** | Project 1 |
| **03/13** |  |
| **03/27** | HW 4 |
| **04/03** | Test 2 | *Coupled poro-elastic processes with the FEM.* FEM model for the two-phase isothermal poro-elastic medium. Applications: drained and undrained oedometer tests, drained and undrained triaxial tests, fields of displacements and pore pressure around a well bore. Thermo-poro-elastic equations for unsaturated media. Applications: precipitation problems with conditional boundary conditions (pressure vs. flow), long-term geological storage of nuclear waste. Reduction of errors in the FEM. |
| **04/10** |  |
| **04/17** | HW 5 |
| **04/24** | Project 2 |

**Grading:** *Final grad*e: F<60%≤D<70%≤C<80%≤B<90%≤A≤100%

*Score:* 2 tests @ 20% each: 40%. Project: 35%. 5 HW @ 5% each = 25%.

**Academic Integrity:** Working in group on homework is allowed (and encouraged). However, each student must write up and turn in his/her own solutions. In-class exams are strictly individual. Any student suspected of cheating or plagiarizing on a quiz, exam, or assignment, will be reported to the Office of Student Integrity, who will investigate the incident and identify the appropriate penalty for violations. Georgia Tech aims to cultivate a community based on trust, academic integrity, and honor. Students are expected to act according to the highest ethical standards.  For information on Georgia Tech's Academic Honor Code, please visit <http://www.catalog.gatech.edu/policies/honor-code/> or <http://www.catalog.gatech.edu/rules/18/>.

**Accommodations for Students with Disabilities:** If you are a student with learning needs that require special accommodation, contact the Office of Disability Services at (404)894-2563 or <http://disabilityservices.gatech.edu/>, as soon as possible, to make an appointment to discuss your special needs and to obtain an accommodations letter.  Please also e-mail me as soon as possible in order to set up a time to discuss your learning needs.