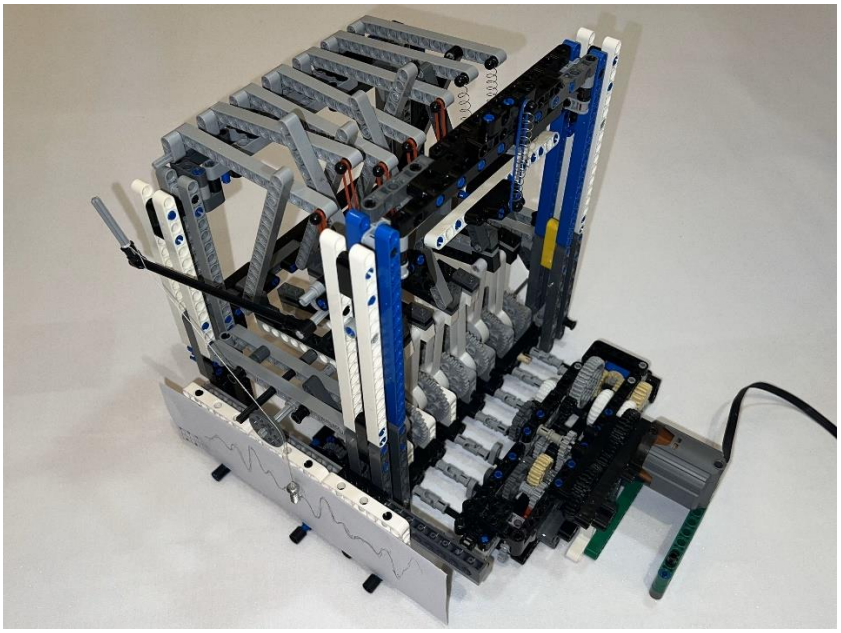
LEGO Fourier Analyzer

A LEGO machine that presents Fourier Analysis

GitHub Link: https://github.com/jadongao/fourier-analyzer

Video Link: https://www.youtube.com/watch?v=KJ-xnW8nUjk



1. What kind of thing is this work?

I made a LEGO machine that presents Fourier Analysis in a highly visual way. This machine replicates a Harmonic Analyzer designed in the late nineteenth century by the physicist Albert Michelson. I must clarify that I referred to the video of YouTuber @thoroughfareproduction, who created a 5-element analyzer. I redesigned the gearbox to increase the number of elements to 7 and made improvements in the design of the sinusoid generator gear set to make the operation smoother.

2. How to characterize/categorize this work?

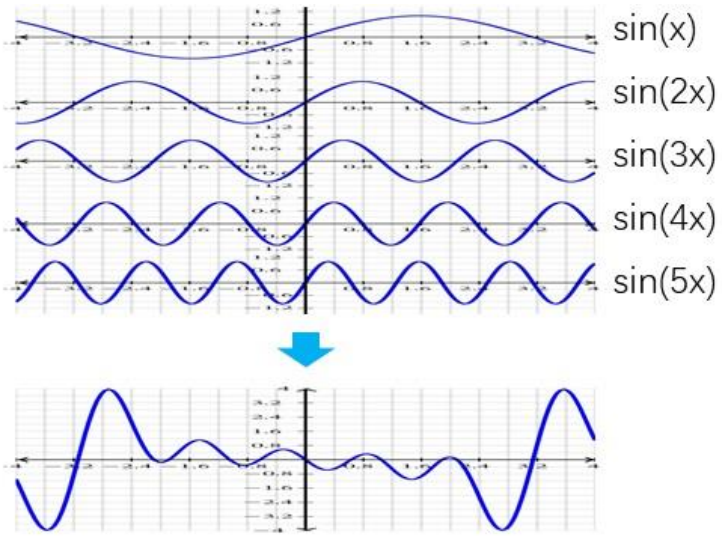
LEGO machine, Fourier Analysis, Sinusoid, 3D models, Visual Learning

3. Briefly description of this work

A Fourier analyzer can carry out two related tasks: add weighted sines or cosines to produce a function graph and perform the inverse operation of decomposing a given function graph into its constituent sinusoids. The addition of sinusoids is called Fourier synthesis, and the inverse operation is called Fourier Analysis. This type of machine is an analog computer that directly processes function graphs without the digital computer, which is almost considered essential by people today. It does not require electricity and can be operated by hand. The capability of the Fourier analyzer depends on the number of elements. I tried to increase the number of elements to 7, while Albert Michelson's copper analyzer once achieved 80 elements.

Example of Fourier analysis

Taking Fourier synthesis (5 sinusoids element) as an example,

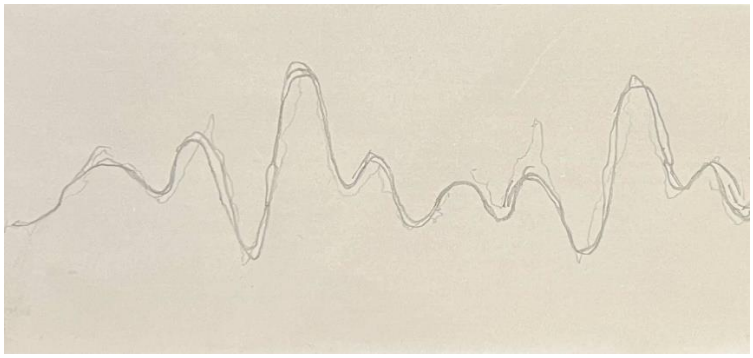


$$f(x) = \sum_{n=1}^{5} a_n * \sin(nx)$$

where a_n is -1, +1, -1, +1, -1

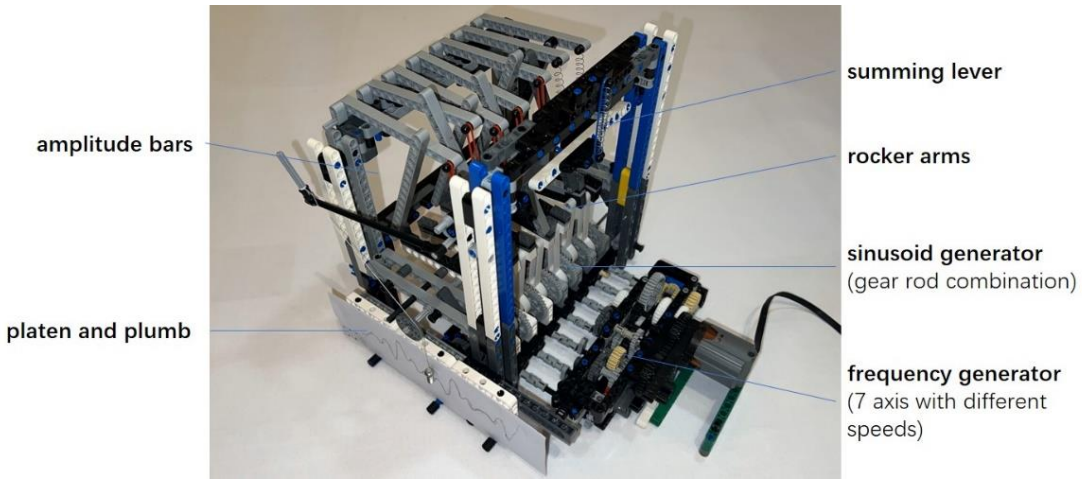
F (x)= -sin(x)+sin(2x)-sin(3x)+sin(4x)-sin(5x)

Using this LEGO Fourier Analyzer, the actual output function graph is as follows, which is very close to the graph drawn by the computer.



Parts of analyzer

This LEGO machine consists of the following parts:



3.1 Frequency generator

The frequency generator is a gearbox with seven output shafts of different frequencies. The rotational speeds on the seven axes are 1x, 2x, 3x, 4x, 5x, 6x, and 7x, respectively.

3.2 Sinusoid generator

The sinusoid generator has seven gears to move Rocker arms, converting the circular rotation generated by the frequency generator into sinusoidal motion. This combination of mechanical elements produces the seven different sinusoidal waves: sin(1x), sin(2x), sin(3x), sin(4x), sin(5x), sin(6x), and sin(7x).

3.3 Ample bars, weight the sinusoids

There are 7 sinusoidal motions weighted:
a_1*sin(1x), a_2*sin(2x), a_3*sin(3x), a_4*sin(4x), a_5*sin(5x), a_6*sin(6x), and a_7*sin(7x).

3.4 Summing lever

The summing lever at the top of the machine adds together the weighted sinusoids.

3.5 Platen and Plumb

The platen moves horizontally, and the plumb moves vertically. The pen tied to the plumb draws a function graph of the calculation result on the recording paper.

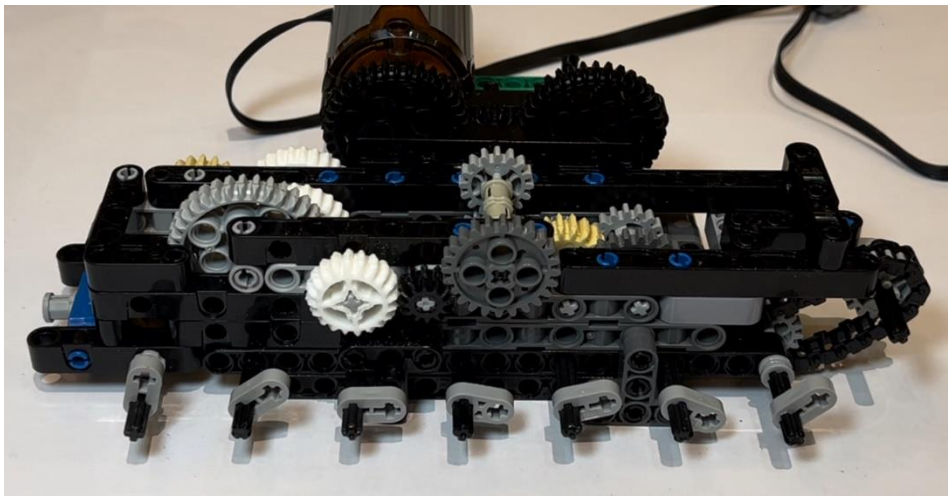
4. How do I make it?

Since March 2022, I have been in contact with the Harmonic Analyzer, designed by physicist Albert Michelson for studying calculus and Fourier transform, and its clever design has deeply impressed me. So I wanted to make one myself at home. The entire process took considerable time and effort to complete each component.

The sinusoid generator and frequency generator are two essential parts:

Frequency generator

The frequency generator is a gearbox, and I am amazed by the clever design of Youtuber @thoroughfareproduction. However, because this YouTuber only provided videos without detailed installation drawings, I couldn't see his internal structure and had to explore it myself. I first completed a 5-axis gearbox like a YouTuber's machine, and after finishing it, my confidence doubled. I challenged and assembled a 7-axis gearbox.

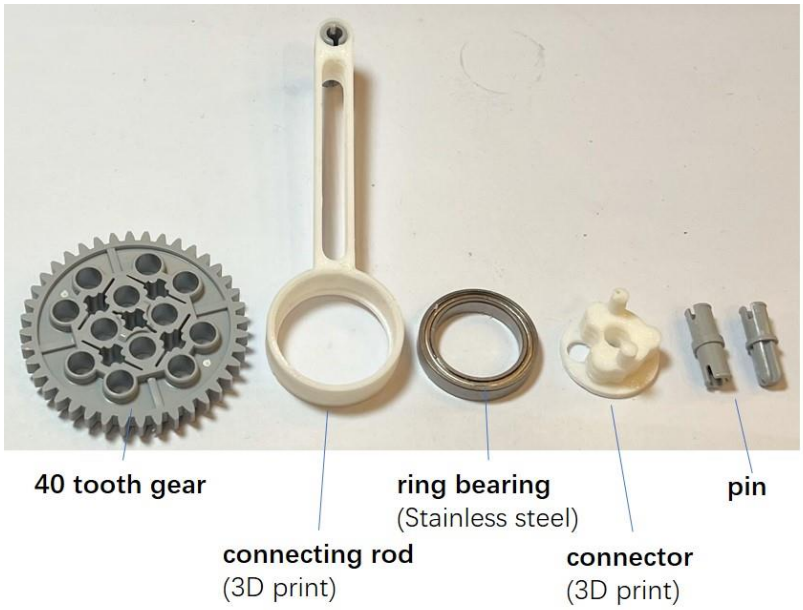


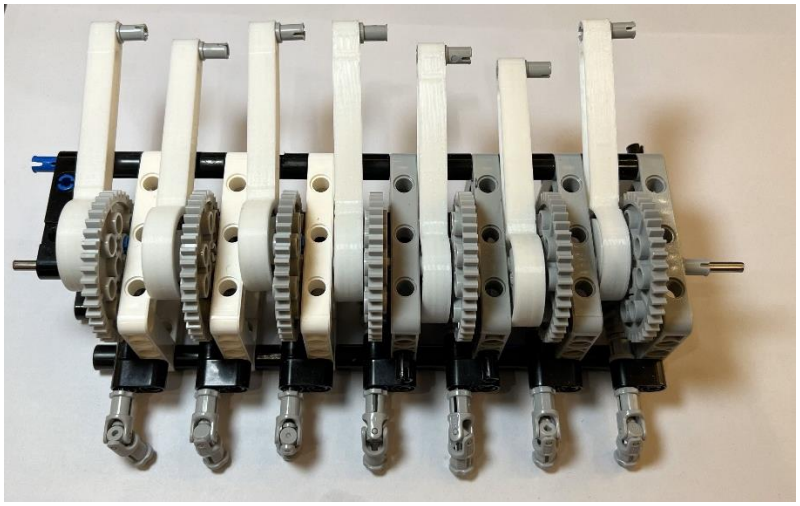
This machine uses almost all LEGO gears: 8, 12, 16, 20, 24, 28, and 36 teeth gear. Since I don't have all gear types at home, I temporarily used a 3D printer to print several types. Later, I replaced the 3D-printed parts with purchased LEGO parts, which made the operation more stable. This method of testing before formal procurement dramatically improves production efficiency, eliminating the need to wait several days to receive LEGO parts before testing.

Sinusoid generator

The sinusoid generator uses LEGO worm gear to transmit circumferential rotation, but during the manufacturing process, there was a problem. Due to the fixed position of the gears and worm, the worm would get stuck during operation, and the entire device could not operate then. I had to take a break until a few months later when I had time to conduct many repeated

experiments and finally found a solution. The final method found is straightforward: to make the worm position lose to avoid jamming. But to see this method, I should do many repeated disassembly and assembly. The device that drives the LEGO beam to form sinusoidal motion is crucial. Youtuber @thoroughfareproduction used a simple bolt connection, but the bolt connection was loose due to the need for rotation, which caused the gears and beam to shake often. I made improvements using a 3D-printed model and bearings, and it is no longer a classic LEGO work.





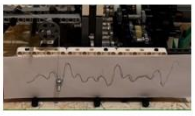
5. Why do I make it?

When learning sound waves, I came across the idea that chords are composed of different combinations of sounds, and using Fourier Analysis, chords can be decomposed into the frequencies of their constituent sounds. I am very interested in Fourier Analysis, and I saw videos online from Professor Bill Hammack and YouTuber @thoroughfareproduction, both dedicated to visualizing complex mathematical concepts in Fourier Analysis. So, I decided to learn from them and make a LEGO machine myself. When making this machine, I also traced the history of Fourier Analysis. I had a chance to communicate with these masters during this production process, and I felt their imaginative ideas in the era without electronic computers.

Time	Name	Invention
1807	French mathematician and physicist Fourier	He published a paper that used sine curves to describe temperature distribution: periodic functions can be represented by a series of cosines and sines.
1879	Sir William Thomson (Lord Kelvin)	He designed a 15 constituent sinusoids tide predicting machine (TPM).
1898	Physicist Albert Michelson (Nobel Prize Winner)	He designed the Harmonic Analyzer that calculates with 20 sinusoids.
November 12, 2014	Professor Bill Hammack	He uses videos and PDF books to introduce Albert Michelson's Harmonic Analyzer.
August 19, 2021	Youtuber @thoroughfareproduction	They designed a wondrous LEGO contraption of gears, springs, levers, and wheels to perform Fourier analysis.

6. What's the most meaningful thing you've made?

This LEGO machine can correspond one-to-one with mathematical formulas, which visualizes mathematical concepts and helps me understand them.

x		The variable x is proportional to the rotation of the LEGO motor.
n		The first axis (far right in the figure) has the slowest speed. The nth axis in gear set spins at a rate n times as fast as the first gear.
$\sin(nx)$		Each connecting rod offset-mounted on the large gear with produce near-sinusoidal motion at the tips of the rocker arms.
a_n		The positions of amplitude bars along the rocker arms set the values of the coefficients a_n that weight the sinusoids.
$\sum_{n=1}^{7}$		The springs and summing lever add together the weighted sinusoids.
$f(x)$		The pen tied to plumb draws continuous function graphs on recording paper.

The production process of this machine was a perfect visual learning. It's very consistent with my experience as a tutor in a schoolhouse. The visual teaching method enables students to see and touch the abstract concepts they have learned, bringing students (including myself) closer to the abstract concepts.