Derivation of Online Weighted Upper Variance Bound

Jadon Wagstaff

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Trying to find:

$$\sigma' = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{m} w_i w_j \sigma_i \sigma_j}{\left(\sum_{i=1}^{m} w_i\right)^2}}$$

Keep track of sum of weights $\sum_{i=1}^{k} w_i$, sum of weighted standard error $\sum_{i=1}^{k} w_i \sigma_i$, and U_k where

$$
\begin{aligned}
U_k &= \sum_{i=1}^{k}\sum_{j=1}^{k} w_i w_j \sigma_i \sigma_j \\
&= \sum_{i=1}^{k-1}\sum_{j=1}^{k-1} w_i w_j \sigma_i \sigma_j + w_k^2 \sigma_k^2 + 2\sum_{i=1}^{k-1} w_k w_i \sigma_k \sigma_i \\
&= U_{k-1} + w_k \sigma_k \left(w_k \sigma_k + 2\sum_{i=1}^{k-1} w_i \sigma_i \right)
\end{aligned}
$$

So,

$$\sigma' = \frac{\sqrt{U_m}}{\sum_{i=1}^{m} w_i}$$