Conditional autoregressive space-time models

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Abstract

This is a review of areal unit models of spatiotemporal variability in a fisheries oceanography context demonstrated by an application to Atlantic cod in the Maritimes Region of Atlantic Canada. In particular, we focus upon replicating historical methods and then advancing to more modern methods that operate in a model-based context. The utility of GLM, GAM and INLA-based approaches are demonstrated for both estimation of abundance via a Poisson assumption and estimation of viable habitat via a Binomial assumption. The addition of a CAR (Conditional Autoregressive) error assumption is found to significantly improve out understanding of both numerical abundance and viable habitat. These worked examples are provided to guide further studies of Atlantic cod and any other species that require stock assessments constrained by (adhoc) static areal management units that generally do not map onto dynamic environmental and biological space-time processes.

1 Introduction

Ecological and biological processes demonstrate variability in space and in time. Characterizing this variability is necessary to understand the potential generative processes. However, information is usually observed in designated areal units due to a number of factors: arbitrary administrative requirements; historical precedents; sampling design considerations due to the need to balance information obtained vs the practicalities of the costs of time and resources required for sampling. Strategies can range from completely random sampling in the absence of additional information, to some form of stratified random design. In the latter, samples are chosen randomly from strata that are characterised *a priori* by informative factor(s). Analysis of covariance (ANOVA) is a common application of such stratification, "blocking" variability of informative factor(s) that cannot be directly measured. In the marine context, factors such as depth, substrate type, temperature or some oceanic feature, are commonly used to define such strata, such that the variability within strata of such important factors will be smaller than that between strata. The lower the variability within strata (relative to between-strata variability), the more optimally stratification has controlled for these nuisance effects, as a sample within a stratum can be considered a representative sample.

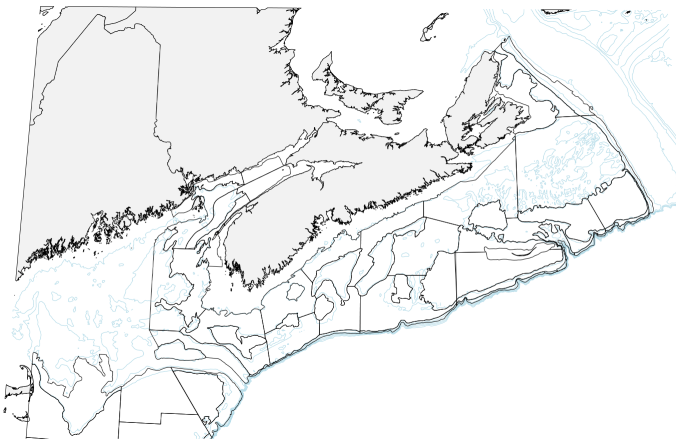

Figure 1.1. The area of interest in the Atlantic Maritimes Region of eastern Canada. Shown are the areal units (black lines) used for survey sampling for the purposes of stock abundance assessment. This area is at the confluence of the Gulf Stream from the south, Labrador Current and St. Lawrence outflow from the north. It is hydro-dynamically very complex. Overlaid are isobaths (light blue) that show that many of the areal units often follow bathymetric contours.

The problem of course is that for many of the processes of interest in ecological systems, the number of informative and interacting factors are large and the manner in which they interact are complex and not always the same in a given location. Ocean currents and water masses move, environmental state changes with invasive species or the collapse of predators. As the number of relevant factors increase, the number of strata required to adequately "block out" such factors statistically (i.e., in classical ANOVA) and the sampling required within each block increases exponentially, as more than one sample is required in each block, to non-viable levels in terms of required time and resources. When, the informative features are dynamic, their relevance in static strata can cause a mismatch with the presumed factors of importance. That is, there can be spatial and or temporal aliasing in that a sample is taken at a very different temperature (for example) or time of year than the overall average temperature of the strata that it is supposed to represent. Furthermore, this blocking or "factorial" approach, only crudely adjusts for the influence of these extraneous factors by "factoring them out" and otherwise ignoring them as nuisance factors. In reality, these factors are, by definition, actually very informative and can dominate the expression of the focal process(es) of interest. Ignoring them or at least trying to ignore can be detrimental, as will be shown below.

There exist two main approaches towards incorporating such additional information: (1) spatially continuous process and (2) spatially aggregated areal units. Both approaches decompose the spatial patterns into those that are associated with: informative factors; structured spatial autocorrelation patterns; and completely spatially unstructured errors. That is, to be explicit, the treatment of the units of study (e.g., strata) are no longer treated as a "fixed" factor but rather as being derived from some constrained, random process.

In another document, we develop methods to treat the continuous case, especially when the focal traits of interest are not spatially stationary ("**stmv**", an R-package found at https://github.com/jae0/stmv).

In the following, we will summarize the general background to the field, following closely Banerjee et al.' s (2004) exceptionally clear and thorough exposition of these ideas as they relate to this second class of spatially aggregated areal units. We then describe "**carstm**", an R-package (https://github.com/jae0/carstm) developed to leverage the speed and flexibility of "**INLA**" (http://www.r-inla.org/) in computing CAR models. It also leverages the data warehousing interfaces of the continuous biological and environmental data layers provided by **aegis** (https://github.com/jae0/aegis) and parenthetically, created by **stmv**.

2 Aggregated areal unit models

Background for Linear models, Generalized linear models (GLM) and Generalized additive models (GAM) are not provided as they are quite commonly used well understood. The CAR model is a "simple" extension of GLMs with a recently proposed parametrization (implemented by INLA as a "bym2" model) which we describe in the following.

2.1 Conditional Autoregressive (CAR) Models

Let the study region $S = S_1,...,S_K$ be a set of $k = 1,...,K$ non-overlapping areal units (**AU**s). Observations $\boldsymbol{Y} = (y_1,...,y_K)$ on such a set S, generally demonstrate spatial and temporal autocorrelation and can be modeled as a Generalized linear model:

$$Y|u \sim f(y|\mu,\nu^2)$$

$$g(\mu) = \boldsymbol{x}^T\boldsymbol{\beta} + \boldsymbol{O} + \boldsymbol{\psi},$$

with offsets $\boldsymbol{O} = (o_1,...,o_K)$, if any; a $K \times V$ matrix of covariates $\boldsymbol{x} = (x_{kv}) \in \Re^{K \times V}$; the V covariate parameters $\boldsymbol{\beta}$ with a multivariate normal prior with mean μ_β and diagonal variance matrix Σ_β; and $\boldsymbol{\psi} = (\psi_1,...,\psi_K)$ residual error structure. The $f(\cdot)$ indicates an exponential family (Binomial, Normal, Poisson) and $\mathrm{E}(Y) = \mu$ with an invertable link function $g(\cdot) = f^{-1}(\cdot)$. In the binomial case, $Y \sim \mathrm{Binomial}(\eta,\theta)$ and $\ln(\theta/(1-\theta)) = \boldsymbol{x}^T\boldsymbol{\beta} + \boldsymbol{O} + \boldsymbol{\psi}$, where η is the vector of number of trials and θ the vector of probabilities of success in each trial. In the Normal case, $Y \sim \mathrm{Normal}(\mu,\nu^2)$ and $\mu = \boldsymbol{x}^T\boldsymbol{\beta} + \boldsymbol{O} + \boldsymbol{\psi}$. In the Poisson case, $Y|\mu \sim \mathrm{Poisson}(\mu)$ and $\ln(\mu) = \boldsymbol{x}^T\boldsymbol{\beta} + \boldsymbol{O} + \boldsymbol{\psi}$.

The above is basic statistics and more information can be found in any textbook. However, it is worthwhile emphasizing that in this family of models, there is a strong assumption that residuals are independent and identically distributed, that is: $\boldsymbol{\psi} \sim iid$. If any autocorrelation exists, such as when there is spatial or temporal, or spatio-temporal autocorrelation, then model parameter estimation becomes biased and uncertain if these correlations are unaccounted. The CAR-structured approach decomposes the residual error structure $\boldsymbol{\psi}$ into two additive random components rather than as a "fixed" factorial component. These components are a spatial, CAR-structured error ϕ and a non-spatial (unstructured) *iid* error ε:

$$\psi = \phi + \varepsilon.$$

The connectivity between AUs enters into the estimation of ϕ and is by convention, represented by the elements w_{ij} of an adjacency matrix $W_{K\times K}$ such that, connectivity is designated by $w_{ij} = 1$, non-connectivity is designated by $w_{ij} = 0$, and self-connectivity, i.e., diag(W), is designated as $w_{ii} = 0$. The residual spatial autocorrelation error ϕ, is treated as a random variable that is assumed to have a **local conditional** distribution between neighbouring lattices $\{p(\phi_i|\phi_j, j \neq i)\}$ which approximates the **global joint** distribution $p(\phi_1, \phi_2, \ldots, \phi_K)$, on the strength of the Hammersly-Clifford Theorem (Besag 1974, Besag et al. 1991, Leroux et al. 2000, Stern and Cressie 1999). The **local conditional** distribution (Banerjee et al. 2004) can be expressed as a weighted sum of the random quantity, where the weights are derived from the local neighbourhood connectivity in the adjacency matrix:

$$p(\phi_i|\phi_j, j \neq i, \tau_i) = N(\alpha \sum_{i\sim j}^{K} w_{ij}\phi_j, \tau_i^{-1}).$$

Here, τ_i is a spatially varying precision parameter; and $\alpha \in [0,1]$ controls the strength of spatial dependence/association such that when $\alpha = 0$ there is no spatial structure and $\alpha = 1$ indicates a fully intrinsic (conditional) autoregressive model, I(C)AR. The above assumes a single global spatial autocorrelation can be used to approximate the spatial structure of the area of interest. This assumption is unrealistic in many systems as covariates may not properly exist, are unknown, or be able to express these differences between subareas, especially when real discontinuities exist (but see **stmv**).

Under the assumption that ϕ is a Markov Random Field, the **joint distribution** of the random variable is, via Brook's Lemma:

$$\phi \sim N(\mathbf{0}, [\tau D(I - \alpha B)]^{-1}),$$

where, D is a diagonal matrix of the same shape as W with diag(D) = d_{ii} = no. of neighbours for each AU$_i$; and also a scaled adjacency matrix $B = D^{-1}W$, the relative strength/influence of the connectivity for each neighbour, where diag(B) = $b_{ii} = 0$. This simplifies to:

$$\phi \sim N(\mathbf{0}, [\tau(D - \alpha W)]^{-1}).$$

Further, the spatial dependence parameter is usually assumed to be $\alpha = 1$, which simplifies the prior further to:

$$\phi \sim N(\mathbf{0}, [\tau(D-W)]^{-1}).$$

This covariance matrix becomes singular as the ϕ are non-identifiable (adding a constant to ϕ leaves the joint distribution unchanged) and thus is an improper prior. Additional constraints are required to make it computationally tractable. By convention, this constraint is that they sum to zero ($\sum_k \phi_k = 0$). This model is a well known and frequently utilized simplification of the CAR model, and is known as an I(C)AR (Intrinsic conditional autoregressive) model. When applied to a Poisson process, it is also known in the epidemiological literature as a convolution model or a Besag-York-Mollie (BYM) model.

Setting sensible priors on the precision of the structured and unstructured random effects has been found to be challenging as they are dependent upon the connectivity of the area of interest. Simpson et al. (2017) and Riebler et al. (2016), following Leroux (1999), approach this issue by a clever parametrization the problem to what they call the "bym2" model in INLA:

$$\psi = \psi(\sqrt{1-\rho}\varepsilon^* + \sqrt{\rho}\phi^*)$$

where ψ is the overall (extra-Poisson) standard deviation; $\rho \in [0,1]$ models the relative balance of spatial and nonspatial heterogeneity; $\varepsilon^* \sim N(0,I)$ is the unstructured random error with fixed standard deviation of 1; and the ICAR random variable scaled so that $\text{Var}(\phi^* = \phi/\nu) \approx 1$. These assumptions ensure that $\text{Var}(\varepsilon) \approx \text{Var}(\phi) \approx 1$ which in turn ensures that ψ can be interpreted as an overall standard deviation. The priors can then be more conventionally understood and assigned as some prior of the standard deviation such as a half-normal or a half-t or exponential distribution and a Beta(0.5,0.5) for ρ. INLA's bym2 model by default uses the type 2 Gumbel distribution for ρ (Simpson et al. 2017).

2.2 Spatiotemporal CAR

Time variation of the focal process of interest can also be treated as an autoregressive process. The argument, similar to the spatial case, is that there is a continuity in time for many processes and so an inherent temporal autocorrelation. If covariates exist and they exhibit this temporal autocorrelation, then temporal autocorrelation enters mechanistically into the model. Any unaccounted or induced autocorrelation must, however, be explicitly accounted to ensure that $\boldsymbol{\psi} \sim iid$. In the timeseries literature, these are are known as the AR(I)MA type models (AutoRegressive Integrated Moving Average), often used when deterministic models do not exist or difficult to parameterize.

In a fisheries stock-assessment context, the stock assessment model is usually a constrained timeseries model, constrained by assumptions of biological rate processes such as mortality and growth. Current practice is to aggregate of all areal units and treat it as a single integral timeseries process. This is, of course, a stop-gap solution used to avoid the difficulties of model complexity and historical limitations of computational capacity. When all/most areal units act in an homogeneous manner, this approximation may be sufficient to express the focal process of interest. However, this is not always the case, especially in the oceanographic context as temporal and spatial variability is occurring at all scales.

Embedding such constrained temporal models into a CAR context is a viable way forward as the assumption is that each areal unit is indeed homogeneous (the correctness of this assumption of course is not always as clear nor verified). The variety of ways in which this can be done is expressed in the **stmv** document (https://github.com/jae0/stmv/blob/master/docs/stmvMethods.pdf). The mechanism would require model building using tools such as STAN or BUGS that permit more complex models than INLA currently permits. The primary limitation is computational load, which also depends upon the complexity of the constrained timeseries model; however, with advances in cloud computing this is becoming increasingly a viable option.

At present, in keeping with the focus upon autoregressive processes in CAR models and using computationally viable solutions, we extend the CAR model by the simple addition of a temporal AR(1) process. This amounts to what are currently known as space-time "separable" models, separable in that the spatial random error processes and temporal random processes are independent (See discussion in https://github.com/jae0/stmv/blob/master/docs/st

3 Application: Atlantic cod in Maritimes Region of Atlantic Canada (NAFO Div. 4VWX)

Here we present, a functional way forward in abundance index estimation that respects the spatial structure of biological data. For this example we focus upon the spatial and temporal variability of Atlantic cod (*Gadus morhua*) in the Maritimes Region of Atlantic Canada (NAFO Div. 4VWX), a frequently studied stock in a very environmentally and ecologically dynamic region (Figure 3.1). In particular, we focus upon a subset of these strata, collectively known as the "Summer Strata".

[Aside: "Summer Strata" are defined by the parameter list p$selection$survey$strata_toremove. Alternatively, one can define explicitly p$selection$survey$strata_tokeep or to the matching variable name, which in this case is also p$selection$survey$StrataID; see section 5].

The current standard is to compute areal unit (stratum)-specific averages that are utilized as inputs to a stock-assessment model that constrains the temporal dynamics based upon assumed growth and mortality rates to scale variations within biologically reasonable ranges. Here we will replicate this approach and then incrementally and didactically move to a model based approach that more flexibly describes these patterns and permits the addition of spatial and temporal autocorrelation patterns explicitly and implicitly via environmental covariates.

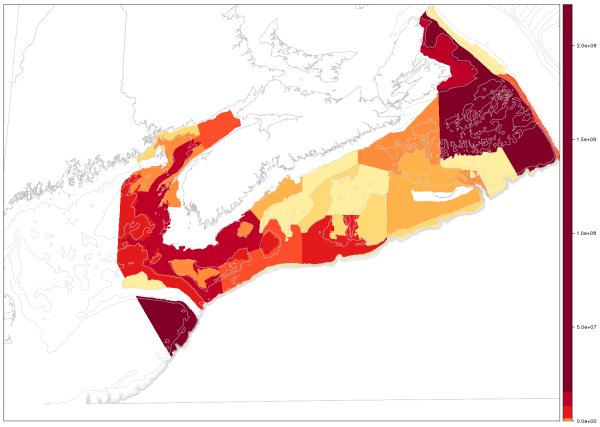

Figure 3.1. Arithmetic means densities per "standard tow" of Atlantic cod in the Maritimes Region and the strata boundaries used by groundfish surveys (red is high and yellow is low on a quantile scale of spatial densities in kg/km^2) for 2017 to show general spatial variability. In this document, we focus upon a subset of these strata, collectively called Summer Strata (see function aegis::strata_definitions() for specifics).

3.1 Standard procedure: Stratanal

Stratanal is the name of the current standard in data treatment. Essentially, this procedure amounts to computing the arithmetic average of the abundance of a given species at each stratum. Afterwards a subset of the strata are chosen and the values are added together to provide an index of abundance that is then modeled via a biologically constrained timeseries model that usually attempts to account for latent observation and/or process errors. This procedure, in the context of Generalized linear models, is equivalent to computing the the fixed effects means of all strata-year combinations (aka, "interaction terms"). Seen in this light, the assumption is that the data (abundance of cod in a "standard tow"; kg) is normally distributed as there are no covariates, no main effects, no intercept terms and no variations in tow net configuration and tow length. The persistence of this procedure as a standard has prevented the development of other methods that might better incorporate information of environmental variability and spatial and/or temporal autocorrelations.

[Aside: This is straightforwardly implemented in the "fixed" factorial formula: totwgt ~ StrataID : yr_factor. Comparisons of the Stratanal solutions and the lm and glm derived solutions are found in the scripts directory of carstm, files 01* and 02*.]

Comparisons using this procedure for different swept area estimates (standard tow, tow length corrected, tow length and width corrected) suggest that individual tows can incorrectly have biases in abundance estimates (Choi et al. 2017). When aggregated, due to the law of large numbers, this effect is of course reduced, however, it is still evident in some years (Figures 3.1, 3.2, 3.3).

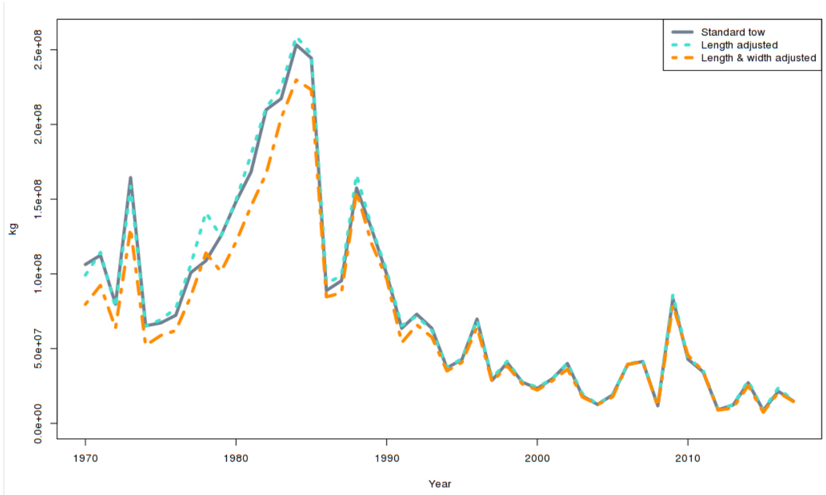

Figure 3.1. Comparison of abundance estimates based upon various swept area estimates.

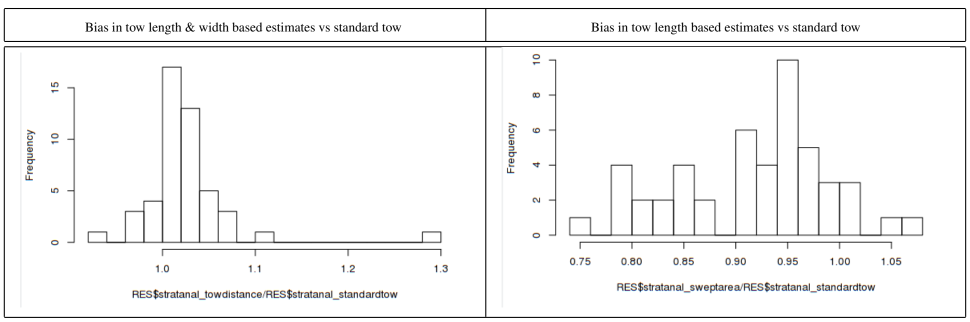

Figure 3.2. Left: Bias attributable to incorporation of tow length in swept area estimates vs "standard tow" assumptions. Generally, a positive bias is seen, with most variability in the range of +/- 10%, and an extreme positive bias of up to 30% in year 1978. See (Choi et al. 2017) for more details. Right: Bias attributable to incorporation of tow length and tow width in swept area estimates vs "standard tow" assumptions. Generally, a net negative bias is observed/estimated. Observations of net configuration are sparse and exist for limited number of years (1990-1992, 2004:2015). Other years were estimated from a series of empirical models of net behaviour with depth and substrate (Choi et al. 2017 for more details).

There is a significant in bias, depending upon what is assumed to be the best measure of swept area. The correlations are all >0.99 with most of the variability seen at high magnitudes of abundance. None of the methods are ideal: the length and width adjusted method is limited by incomplete and unreliable data in the post 2015 period; the tow length adjusted method is naive to width variations that are known to be large and so can result in over-estimation of abundance; and the standard tow method simply ignores important variations in tow length and width.

In what follows, we will use the tow length adjusted method for further comparisons with the caveat that it may be significantly over-estimating abundance in the pre-1984 period, relative to the sweptarea method ("tow length and width adjusted", Figure 3.1).

3.2 Model-based approach: Poisson and fixed effects

By adopting a model-based approach to the problem of abundance estimation, we can test the validity or utility of various data distribution models and covariates. Comparison of estimates of total biomass from a Gaussian model *vs* a Poisson model with subsequent conversion of numbers to biomass using mean weights shows that the latter generally results in a larger estimate of biomass than the former; the divergence being larger as the magnitude becomes larger (Figure 3.3). [Parenthetically, as stock assessment models usually use numerical abundance rather than biomass, this additional step and the associated compounding of the errors of mean weight can be avoided.]

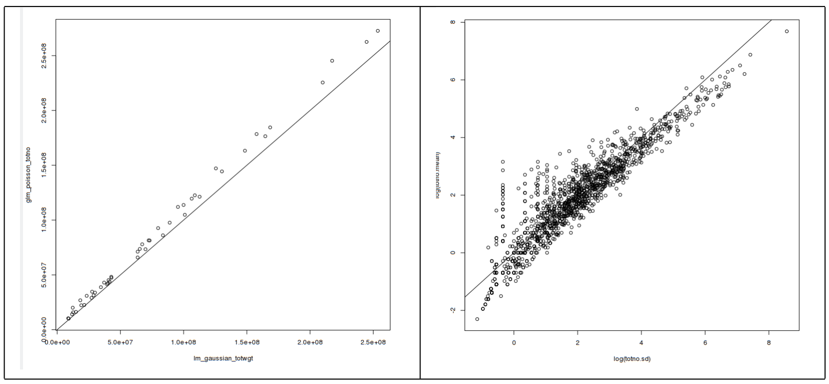

Figure 3.3. (Left panel) Comparison of estimates of total biomass from a Gaussian model (x-axis) vs a Poisson model with conversion of numbers to biomass using mean weights (y-axis). The 1:1 line is shown for comparison. (Right) The relationship between mean and variance (in each stratum and year). The 1:1 line is shown. The Poisson assumes that mean and variance are equal.

Distributionally, the data (total weights) are definitely non-Gaussian, being highly skewed and almost log-normal in shape (see Figure 3.3 right panel; 3.4). As a result the Gaussian assumption that is implicit in **Stratanal** is a concern as a magnitude and variance dependent bias is highly probable (underestimating the highs and overestimating the lows). The predictions based upon a Poisson model generally results in a larger estimate of biomass than that from a Gaussian model (Figure 3.3, left panel). Examination of other distributional models such as zero-inflated Poisson is would increase this divergence even more as a certain proportion of the zero-values are relegated to zero-values due to incorrect observations. A negative binomial model is also simple enough to implement, however, the interpretation of the process as a failure time is unclear.

Our preference, in this document, is to use the Poisson distributional model to estimate numerical abundance due to the simplicity of the model as a random counting process, reserving complexity to covariates and autocorrelated variations and the justification based upon improved DIC (see Table 3.1). This numerical abundance can be converted to biomass using mean weight information for each stratum-year as done in this document. Or alternatively, if a fishery model is being used that models numerical abundance, this conversion would not be necessary and used directly by the model.

Further, in the context of model design, **Stratanal**'s use of only the crossed effects (StrataID X yr_factor) explicitly shows that the assumptions are simplistic and limiting. The influence of main effects of year and strata are ignored, even if some strata are consistently higher or lower, or if a given year had consistently higher or lower overall abundance. These main effects variations can be informative and can be used to help impute or stabilize expectations when some strata are not visited or with fewer survey sets in a given year due to logistical reasons ("unbalanced designs") and help understand the sources of variability in a more systematic manner. Note also that these are all factorial models and conceptually treating each year and stratum as a "fixed" or static entity.

Total weight	Total number

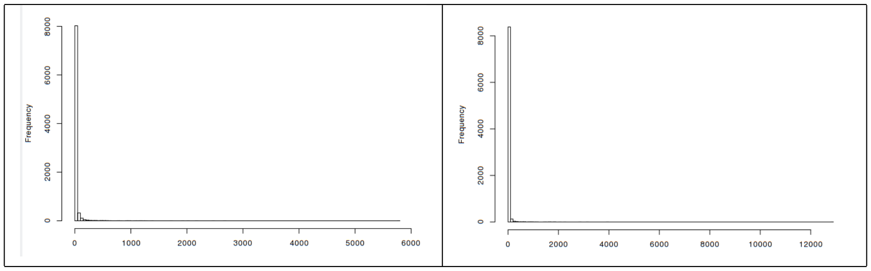

Figure 3.4. Histograms of total weight and total numbers of Atlantic cod caught in survey gear. They are not Gaussian distributions.

3.3 Environmental covariates

Environmental covariates can also be easily added to the model-based inference and prediction. Leveraging the **aegis** data warehousing of environmental data for the area of interest, we show worked examples of how this is done in the supplemental Section 5 and in the associated code (https://github.com/jae0/carstm/blob/master/inst/scripts/ 03_cod_comparisons_environment_abundance.R). At present a limited number of temperature and depth related covariates are available, but this will eventually expand to biochemical measures such as dissolved oxygen, pH, conductivity, chlorophyll-a, zooplankton abundance, etc., once these other databases are more fully assimilated into **aegis**.

For demonstration purposes, we focus upon two important environmental variables, temperature and depth. This is accomplished by adding to the full factorial of strata and year model the two factors as simple linear effects. This has the effect of reducing the AIC from 359785 to 348154 (Table 3.1). In terms of the time series of aggregate abundance in the "summer strata", there is some divergence observed between these two models though the overall pattern remains similar (Figure 3.5).

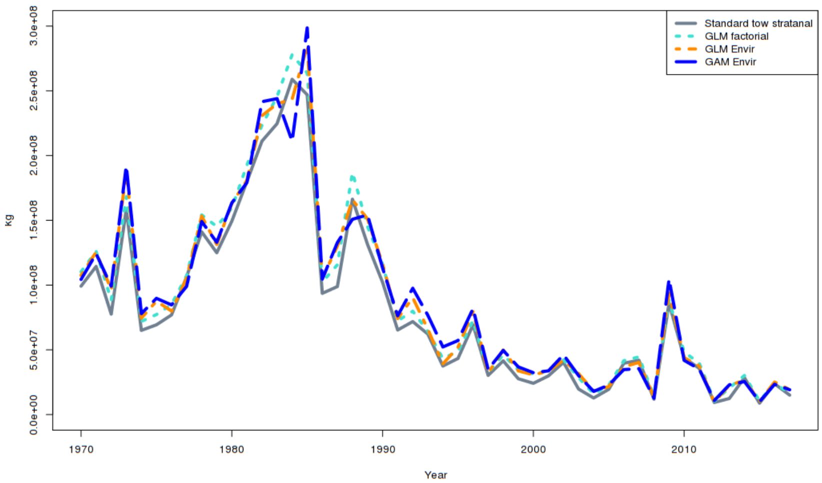

Figure 3.5. Timeseries of abundance in "summer strata" based upon factorial (fixed effects) models. Note that these models use tow-length adjusted estimates unlike the "stratanal" which uses standard tow estimates.

Table 3.1. Summary of the performance factorial models. A full factorial model with environmental covariates performs the best in this class of models. However, most are rank-deficient due to insufficient data in some year-stratum combinations. There are for example, 65 stratum-year combinations where there has been only one sampling station (event), rendering variance estimation difficult and model matrices highly collinear. Indeed some 378 strata-years were not visited and in 2018, the survey was not undertaken.

Model name	Strata	Time	Environment	Family	No Parameters	AIC	Rank AIC
GLM totwgt standard tow	fixed	fixed	NA	Gaussian	2203	-	-
GLM totno standard tow	fixed	fixed	NA	Poisson	2203	-	-
GLM totno tow length	fixed	fixed	NA	Poisson	2203	359785	4
GLM totno towlength full factorial	fixed	fixed	NA	Poisson	2202	359785 (rank deficient)	3
GLM totno towlength full factorial Envir	fixed	fixed	linear: temp + depth	Poisson	2204	348154 (rank deficient)	2
GAM	fixed	fixed	thin plate spline, 3 knots: temp + depth	Poisson	2207	340300 (rank deficient)	1

The expectation of a linear relationship between numerical density and temperature or depth is, however, naive. Animal abundance usually has a nonlinear, modal relationship with environmental factors. To alleviate this problem, a penalized smooth can be used with a limited number of nodes in a Generalized additive model (GAM). This model, however, is unstable. Convergence occurs occasionally upon a solution where model AIC improves to 340300, even with the additional parameters (Table 3.1). The presence of single sampling locations to represent a stratum without error results in rank-deficient and unstable solutions. This instability of the factorial models suggests that the model form may be inappropriate for the kind of data being collected and the constraints faced by the survey. An attempt to solve the same factorial model using INLA resulted in similar difficulties.

Looking at the environmental relationships more carefully, there are indeed modal relationships: a maximal abundance in the 4-5 Celsius range and in depths less than 60 m (Figure 3.6). The aggregate timeseries again shows some minor divergence relative to stratanal-based solutions (Figure 3.5) with the spatial distribution of densities as depicted in Figure 3.7. The problem is that these solutions are rank deficient and unstable (Table 3.1).

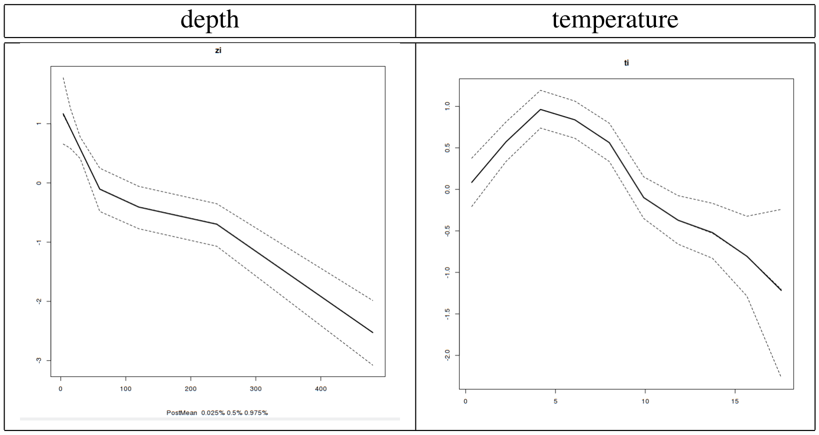

Figure 3.6. The relationship of numerical density of Atlantic cod with depth and temperature, based upon "INLA Envir 1". Similar patterns are also found in the GAM parameterizations.

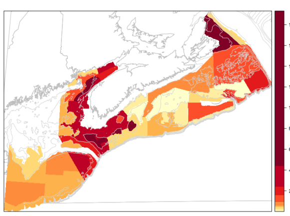

Figure 3.7. A map depicting the predicted biomass densities (kg/km^2) for 2017. Basis is a smoothed model of temperature and depth ("rw2"; see model "INLA Envir 1" for more details). Spatial autocorrelation is not a part of the model, however, as temperature and depth are autocorrelated factors, they indirectly add spatial autocorrelation to the model. Compare with Figure 3.1 (a stratanal solution for the same year).

If survey stations were perfectly unbiased with regards to, in this simple case of depth and temperature, then the difference between those factors and the stratum averages should be zero. As can be seen in Figure 3.8, there is important variability and potential for bias due to improper accounting for these factors.

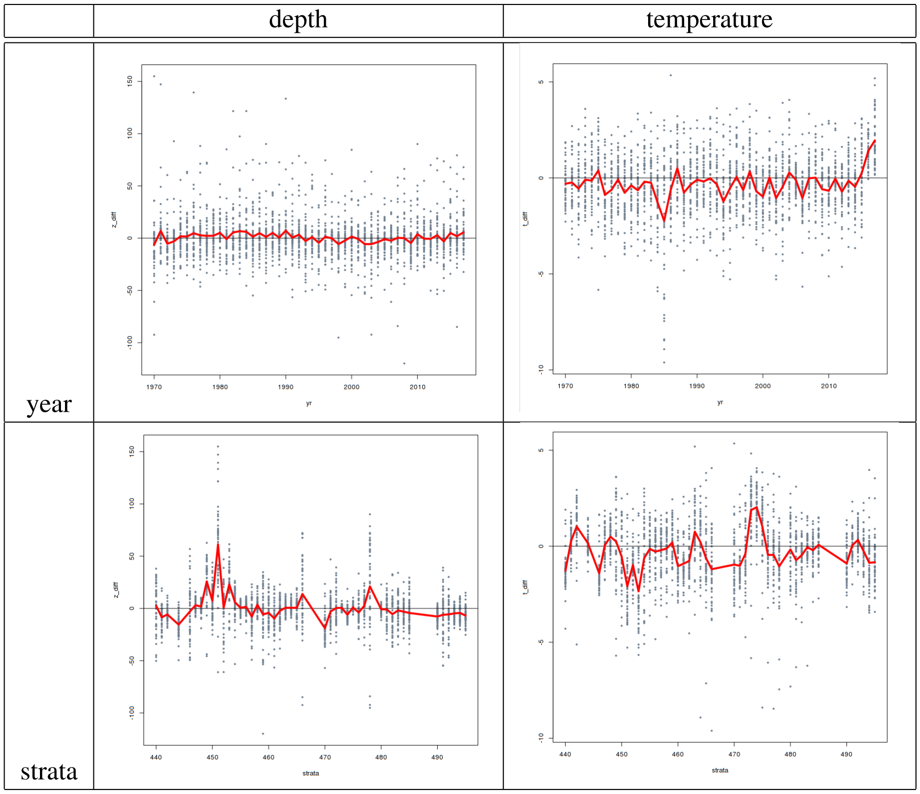

Figure 3.8. Bias in depth and temperature of each survey location relative to strata and year averages. Bias and large variability in this bias exists which indicates that the sampling strategy is creating a bias in estimates. The use of environmental covariates can reduce the magnitude of some of these biases.

3.4 Random-effects

The usual solution entertained when a fixed effects model fails to produce stable results is to consider a random effects model; with in this case, some marginally informative priors in a Bayesian context. Use of a fixed effects parameterization is called for when a factor being studied (stratum-year) is truly unique. If one accepts that a stratum-year represents an abstraction of a number of different interacting factors (e.g., Figure 3.6), including depth, temperature and a multitude of other interacting factors, a stratum-year represents not a fixed category but rather one that is defined contextually and so perhaps better treated as a random entity following some statistical distributional model that depend upon these characteristics.

Philosophical basis aside, such random effects parameterizations, though conceptually more complex in some ways, represents a powerful simplification in providing a way forward in situations where data are poorly behaved and do not conform to the matrix rank/positive definiteness requirement of fixed effects estimation. Their assumption results in a very different interpretation of how abundance in the strata and year vary. The net result is a timeseries that is significantly divergent relative to the factorial analysis (Figure 3.11).

3.5 Autocorrelation

Autocorrelation of the spatial process enters indirectly into the estimation process via the mechanistic autocorrelation that exists naturally in bathymetry and bottom temperatures and any other covariates that might enter into the estimation process. However, to gauge the importance of other unaccounted for spatial correlation it is necessary to model it explicitly. In CAR-models, this spatial autocorrelation is treated as an extra-Poisson source of variability (Section 2). Various model formulations were examined and their performance compared in Table 3.2.

Table 3.2. Summary of model performance. The '|*' notation is used to represent "grouped by *", in the same manner as in lme and lattice.

Model name	Time	Strata	Environment	Family	Eff. no. params	DIC	DIC Rank
INLA full factorial	fixed	fixed	-	Poisson	6136	unstable	-
INLA Envir	iid	iid|year	rw2: temp+depth	Poisson	5960	33652	5
INLA Envir AR1	ar1	iid|year	rw2: temp+depth	Poisson	5966	33681	6
INLA Envir CAR	iid	bym2	rw2: temp+depth	Poisson	6056	33748	7
INLA Envir AR1 CAR	ar1	bym2	rw2: temp+depth	Poisson	5976	33810	8
INLA Envir AR1 CAR|year	ar1	bym2|year	rw2: temp+depth	Poisson	5945	33655	3
INLA Envir AR1|strata CAR	ar1|strata	bym2	rw2: temp+depth	Poisson	5903	33552	2
INLA Envir AR1|strata CAR|year	ar1|strata	bym2|year	rw2: temp+depth	Poisson	5905	33554	1
INLA Envir CAR|year	iid	bym2|year	rw2: temp+depth	Poisson	5947	33652	4

The importance of the spatial vs the unstructured extra-Poisson variability is expressed by the the parameter $\rho \in [0,1]$. In the case of a simple CAR random effects model ("INLA Envir CAR"), the extra-Poisson variability is large (Table 3.3) with a mode of approximately 80% of the extra-Poisson variance being attributable to spatial autocorrelation (Figure 3.9). The distribution is, however, quite wide suggesting a single CAR effect across all years may not be appropriate.

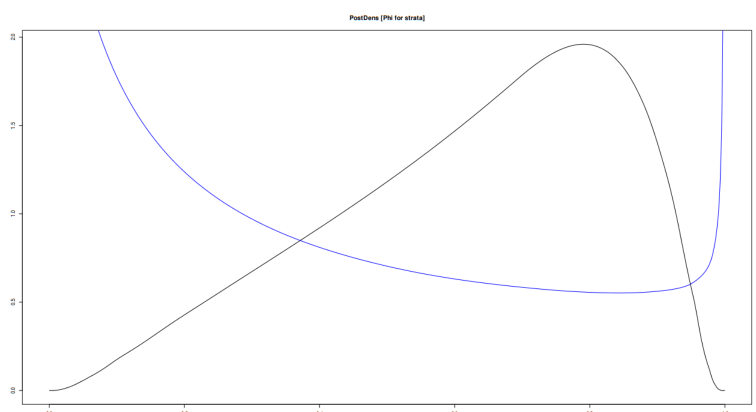

Figure 3.9. Posterior distribution of ρ in the "INLA Envir CAR" model with PC prior in blue. The wide distribution of the posterior suggests that a single CAR effect across all years might not be appropriate.

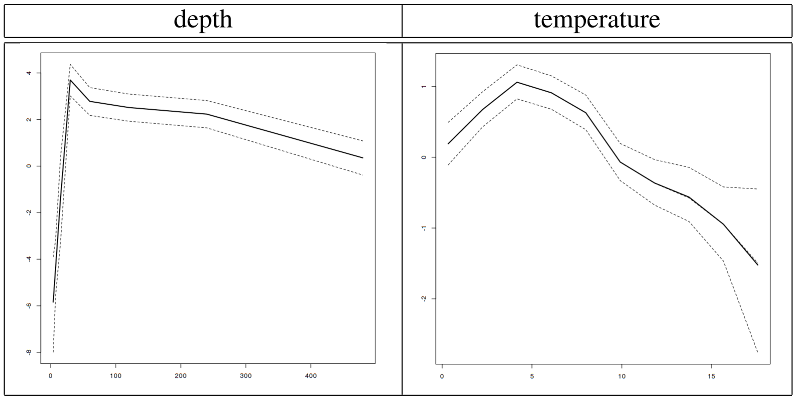

Figure 3.10. Relationship of abundance with environmental covariates in the "INLA Envir CAR" model.

Table 3.3. Hyperparameters for "INLA Envir CAR". Note that "Precision for strata" is the precision of the extra-Poisson variability, ψ) and is relatively important. Note that ρ (INLA calls it, "Phi for strata"), the proportion of spatial variability in the extra-Poisson is about 62%. The variable "zi" is depth and "ti" is temperature. Precision = 1/variance. Here the marginal variance associated depth dominates.

	mean	sd	0.025quant	0.5quant	0.975quant	mode
Precision for iid_error	0.3526	0.0083	0.3378	0.3521	0.3703	0.3501
Precision for ti	4.4226	3.4519	0.7379	3.5242	13.4633	1.9830
Precision for zi	0.0029	0.0024	0.0003	0.0022	0.0093	0.0010
Precision for year	1.8576	1.2141	0.7927	1.4857	5.0897	0.9917
Precision for strata	0.4162	0.1082	0.2571	0.3978	0.6763	0.3620
Phi for strata	0.6236	0.2104	0.1684	0.6587	0.9309	0.7902

The addition of a CAR error modifies the relationship of abundance with environmental characteristics. As CAR variability absorbs some of the overall variability in the data, on average, it acts to reduce the magnitude and variability associated the other processes; in this case, the relative influence of environmental and interannual variability is tempered. Indeed, when modeled with a single CAR effect across all years, the interannual variability is dampened relative to the non-CAR case (compare "INLA Envir" vs "INLA Envir CAR" or "INLA Envir AR1" in Figure 3.11).

The addition of a separable, AR(1) random effect to the CAR model dampens and smooths the timeseries ("INLA Envir AR1 CAR"): the temporal autocorrelation is >99% but the spatially structured component drops to 61%.

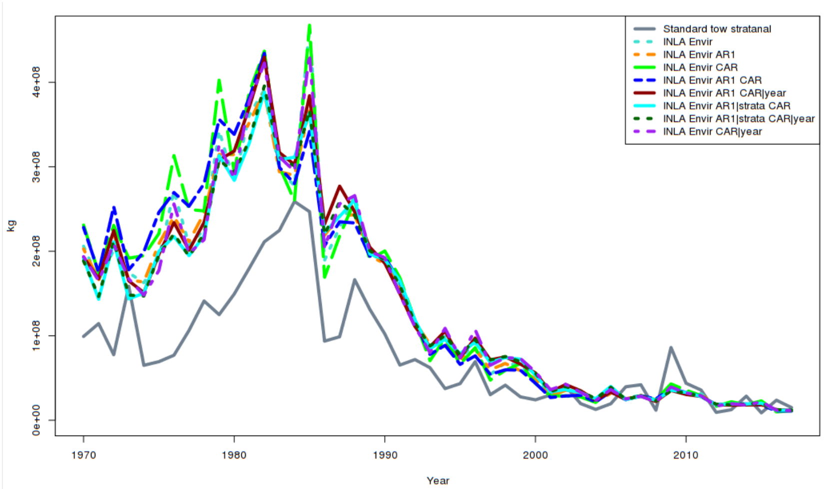

Figure 3.11. Comparison of annual timeseries of Atlantic cod biomass of summer strata under various model assumptions. See Table 3.2 for details.

The "INLA Envir AR1 CAR|year" model treats each year as a separate CAR process and performs optimally in terms of DIC (Table 3.2) and helps to refine our understanding of the spatial process. The results are very interesting (Table 3.4). First, the temporal autocorrelation (lag 1) is still about the same at 91%. The extra-Poisson variability (ψ) still dominates. However, the spatial component has increased to 54%. The overall CAR structure is also stable across the years with a lag 1 correlation of 82%. Further addition and testing of covariates should most likely be done using this model. It is, however, slow to complete (days).

Table 3.4. Hyperparameter estimates from "INLA Envir AR1 CAR|year".

	mean	sd	0.025quant	0.5quant	0.975quant	mode
Precision for iid_error	0.4034	0.01140	0.3840	0.4022	0.4284	0.3982
Precision for ti	6.7918	3.55378	2.3474	5.9976	15.8801	4.7169
Precision for zi	11.2908	22.23496	0.6430	5.2435	59.4956	1.5948
Precision for year	8.3944	1.79123	5.4799	8.1840	12.4807	7.7694
Rho for year	0.9116	0.02558	0.8545	0.9140	0.9541	0.9187
Precision for strata	0.4627	0.09394	0.3187	0.4483	0.6848	0.4175
Phi for strata	0.5433	0.09985	0.3540	0.5410	0.7402	0.5283
GroupRho for strata	0.8233	0.04275	0.7209	0.8307	0.8863	0.8481

Though the "INLA Envir AR1 CAR|year" model seems to work well, we examine an additional refinement where the temporal autocorrelation (AR1) is also broken down by each stratum ("INLA Envir AR1|strata CAR|year"). This more complex model, on the basis of DIC seems to perform best. However, it also takes a longer time to complete. Annual autocorrelation is still high at 98% and spatial autocorrelation is at 78% of the extra-Poisson variation, though the latter precision increases significantly (Table 3.5). However, irrespective of the model formulation, most suggest similar time trajectories of Atlantic cod abundance (Figure 3.7).

Table 3.5. Hyperparameter estimates from "INLA Envir AR1|strata CAR|year". Of note, GroupRho (the correlation of CAR errors over time) for strata declines as the autocorrelation gets absorbed by strata-specific autocorrelations in AR1|strata.

```
                            mean     sd 0.025quant 0.5quant 0.975quant    mode
Precision for iid_error   0.4050 0.0107     0.3866   0.4041     0.4282  0.4008
Precision for ti          3.7876 3.7486     0.5902   2.6850    13.6721  1.4429
Precision for zi          1.3279 1.9961     0.0507   0.7144     6.3839  0.1206
Precision for year        0.2728 0.0443     0.1913   0.2714     0.3645  0.2700
Rho for year              0.9840 0.0036     0.9766   0.9841     0.9906  0.9843
GroupRho for year         0.4248 0.0701     0.3039   0.4183     0.5768  0.3961
Precision for strata     12.5043 5.9177     3.4042  11.8055    25.6231  9.2699
Phi for strata            0.7764 0.1605     0.3994   0.8101     0.9824  0.9477
GroupRho for strata       0.0355 0.1002    -0.0210  -0.0030     0.3449 -0.0211
```

4 Habitat

To this point, we have focused upon abundance estimation via a Poisson error assumption and conversion of numerical abundance to biomass using mean weights at each stratum-year, under various model formulations. Utility was evident and so represents a good way forward.

It is extremely informative is to also examine the presence or absence of an organism under the same context. This permits a rapid quantitative assessment of viable habitat space. This can of course be done in the continuous case, an example of which is found in the snow crab assessment which uses **stmv** to accomplish this. However, it is also possible to do this in the areal-unit case, as was identified in the introduction, when data density limitations and management constraints exist.

Indeed, the only difference is assumption of the the error distribution which in the case of presence-absence data becomes a Bernoulli binomial process with a logit link function:

$$Y \sim \text{Binomial}(\eta, \theta)$$

and

$$\ln(\theta/(1-\theta)) = \boldsymbol{x}^T \boldsymbol{\beta} + \boldsymbol{\psi},$$

where, η is the vector of number of trials and θ the vector of probabilities of success in each trial; a $K \times V$ matrix of covariates $\boldsymbol{x} = (x_{kv}) \in \Re^{K \times V}$; the V covariate parameters $\boldsymbol{\beta}$ with a multivariate normal prior with mean μ_β and diagonal variance matrix Σ_β; and $\boldsymbol{\psi} = (\psi_1, ..., \psi_K)$ residual error structure which again follows the "bym2" error parametrization of Simpson et al (2017).

Although many more should be ideally tested and added, here we add one more covariates relative to section 4 (depth, z and temperature, t) for the purposes of demonstration: the number of degree days which is well known to be influential for most poikilothermic organisms (integral of days * temperature; degreedays).

The simplest possible formulation is the GLM form with only the covariates which provides a rapid assessment of these factors (Figure 4.1) and a prediction of the associated habitat space (Figure 4.2).

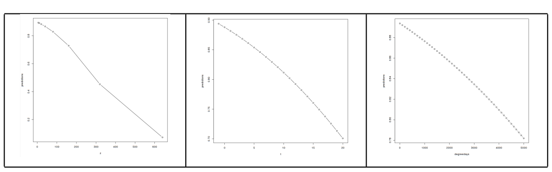

Figure 4.1. Relationship of probability of observing Atlantic cod as a function of depth (left; z), temperature (middle; t) and degree days (right; degreeday) derived from a GLM fit (DIC=10774). The observed curvature is derived from the use of the logit transform.

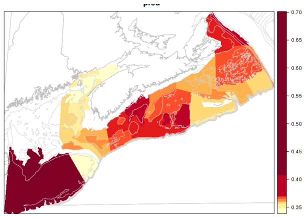

Figure 4.2. A spatial representation of the probability of observing Atlantic cod in the year 2017, based upon a GLM fit.

One can improve upon the GLM representation by using smooths from a Generalized Additive Model (GAM). This results in an improved DIC and a modal relationship can be seen in the probability of observing Atlantic cod with temperature-related covariates (Figure 4.3). A very different expectation of viable habitat can be seen as well (Figure 4.4).

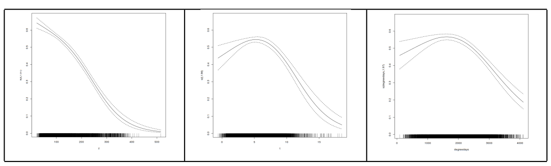

Figure 4.3. Relationship of probability of observing Atlantic cod as a function of depth (left; z), temperature (middle; t) and degree days (right; degreeday) derived from a GAM fit (DIC=10579). Note the modal nature of temperature and degree days.

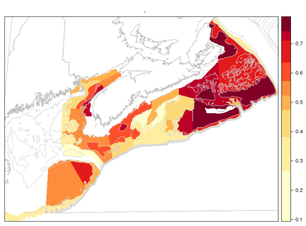

Figure 4.4. A spatial representation of the probability of observing Atlantic cod in the year 2017, based upon a GAM fit.

Further improvement though the use of a random effects approach in a Bayesian context (via INLA's rw2 model) provides a mechanism to describe the modality of the temperature related effects a little more carefully. However, the DIC is marginally worse than that of the GAM model as more parameters are used.

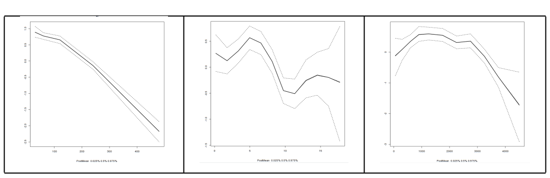

Figure 4.5. Relationship of probability of observing Atlantic cod as a function of depth (left; z), temperature (middle; t) and degree days (right; degreeday) derived from an INLA rw2 fit (DIC=10585). Note the modal nature of temperature and degree days.

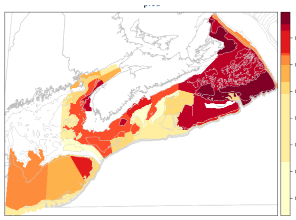

Figure 4.6. A spatial representation of the probability of observing Atlantic cod in the year 2017, based upon an INLA fit. Overall features are similar to those of the GAM solution. However, of note is the ability of this approach to impute the missing information in the westernmost regions due to the adoption of a random effects model.

Finally, the addition of a simple spatially autocorrelated error provides a solution that notably improves the model in terms of DIC. The definition of an appropriate spatial error also permits a more reliable estimation of the influence of the environmental covariates (Figure 4.7). The spatial representation of this habitat is also distinct, showing separation between a western and an eastern area (Figure 4.8). This CAR model is a simple one as only a single global CAR effect was parameterized. The addition of an annual CAR effect improves DIC (8730) with minor differences in covariate relationships and predictions relative to the single global CAR effect (Figures 4.8, 4.9).

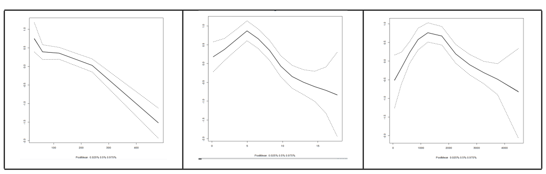

Figure 4.7. Relationship of probability of observing Atlantic cod as a function of depth (left; z), temperature (middle; t) and degree days (right; degreeday) derived from an INLA rw2 with a bym2 spatial error fit (DIC=8885). The same patterns were observed with an annually varying CAR effect (DIC=8730).

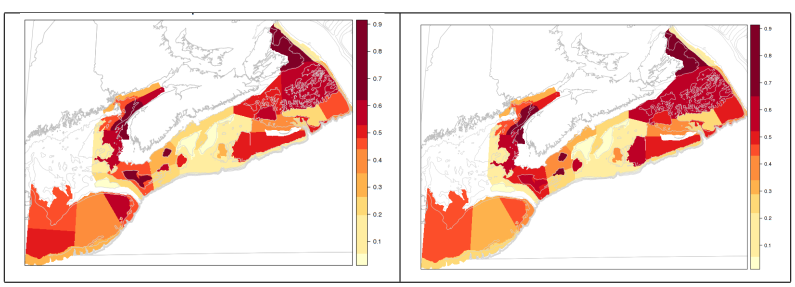

Figure 4.8. A spatial representation of the probability of observing Atlantic cod in the year 2017, based upon an INLA-based single CAR fit (left) and separate CAR effects for each year (right). Overall features are

similar to the INLA base model. However, significant differences in local distributions are seen, especially in the separation of western and eastern areas, relative to the base model (Figure 4.6).

Finally, summarizing some of the above results, we can look to the surface area-weighted average probability of observing Atlantic cod for each of the model forms (Figure 4.9). What we observe immediately is that most the models that do not account for spatial autocorrelation suggest minimal variations in viable habitat over time, except perhaps in the post-2010 period where some large scale warming of bottom temperatures have been observed. The addition of an extremely simple CAR error suggest that, in fact, this degradation of viable habitat is more nuanced with a significant erosion that started in 1991 continued to degrade for another decade with some variations thereafter.

Of course more exploration with more refined spatio-temporal autocorrelations are justified and will be the subject to be examined next. But for now, it seems sufficient to suggest that ignoring spatial (and temporal) autocorrelation in the context of viable habitat studies is unwise.

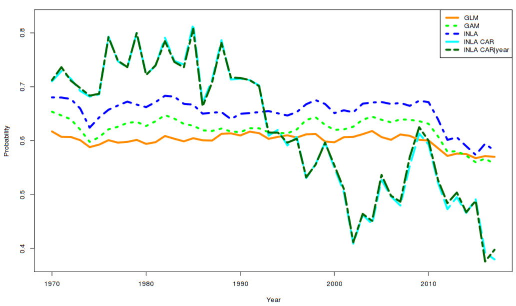

Figure 4.9. The temporal variations of area-weighted average probability of observing Atlantic cod. Significant declines in habitat viability occurred in the early 1990s.

5 Concluding thoughts

It is likely that there has been a severe underestimation of biomass for the early part of the timeseries, making the decline in Atlantic cod even more extreme than previously thought by "stratanal". The cause is due to an incorrect assumption of distributional error causing a magnitude-dependent bias; assumptions of survey samples being representative of a homogeneous strata that are, in fact, environmentally quite dynamic; and most importantly, an assumption of fixed effects vs random effects in terms of how we treat the strata and year and whether they express autocorrelation in time and space. A way forward is to adjust for these issues and examples of this approach have been provided in this document.

How these influences will alter fishing mortality estimates and the interpretation of overfishing vs environmental change vs a state change in natural mortality vs intrinsic population limitation, needs to be re-examined in this new context. Peak abundance seems to have occurred early in 1980 and not the mid-1980's; and severe declines in abundance had already occurred by 1989, well in advance of the moratorium in 1992. Significant viable habitat degradation seems to have occurred in the post 1990 period and in combination with density dependent factors exacerbated this reduction in viable habitat may have caused the precipitous decline their abundance in the 1990s. Currently, spatial constriction of Atlantic cod into core areas is more apparent with the "INLA Envir AR1 CAR|year" modeled results (Figure 3.7) relative to the "stratanal" representation (Figure 3.1).

As environmental variability has been significant in the area/time, it invalidates the random stratified design's assumption of unbiased sampling within an areal unit, variations that need to be accounted. Further, spatial and temporal autocorrelations exist and need to be parameterized and incorporated into models; they represent important sources of variation that provide a more reasonable estimate of abundance while properly weighting the influence of these environmental covariates. The models presented here represent a first step in this direction and a means forward towards better understanding the dynamics of Atlantic cod and many other species.

Furthermore and perhaps much more important: these methods represent a unified, viable and coherent solution for modeling the abundance and ecological niche of many species that urgently require assessment in the Maritimes Region; urgent due to rapid climate change and rapid changes in human exploitation patterns and disruption of their ecosystem.

6 References and supplemental readings

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Dean, C. B., Ugarte, M. D. and Militino, A. F. 2001. Detecting interaction between random region and fixed age effects in disease mapping. Biometrics 57: 197–202.

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Simpson, Daniel; Rue, Håvard; Riebler, Andrea; Martins, Thiago G.; Sørbye, Sigrunn H. 2017. Penalising Model Component Complexity: A Principled, Practical Approach to Constructing Priors. Statist. Sci. 32(1): 1-28.

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7 Using carstm

Carstm and associated libraries use a parameter list of settings and options to control the data environment and analyses. The data object "p" is used by default to do this message passing and so is reserved. Here we explain the data environment and data selection mechanisms.

The data environment is initiated with a call to carstm::carstm_parameters() which loads the required libraries and any additional parameters expected by the data warehousing interface **aegis** which are required for the analysis of environmental covariates. The result is a list of default settings relevant to the subsequent analyses. In this case, the function accepts a named list of various parameters that will be added into "p" and help control choices that relate to defining the spatial extent and cartographic projections used for plotting and surface area calculations. It also defines the the polygons to be used, (by default, pre-2014 definitions) as well as ensuring that the variable "trawlable_units" is defined.

```
p = carstm::carstm_parameters(
    id ="Atlantic cod summer standardtow", # identifier
    speciesname = "Atlantic_cod",
    groundfish_species_code = 10, # 10= cod for data selection later on
    yrs = yrs, # study years for data selection, later on
    trawlable_units = "standardtow" # choices are "standardtow", "towlength", "sweptarea"
)
```

After setting up the data environment, additions to the parameter list are made that control data extraction from the groundfish databases. The expectation is that the aegis databases are all set up. This data can be downloaded upon request, or, alternatively set up on a server and code run remotely.

```
p$selection=list(
    biologicals=list(
        spec_bio = bio.taxonomy::taxonomy.recode(
            from="spec", to="parsimonious", tolookup=p$groundfish_species_code
        ) # aegis uses an internal code
    ),
    survey=list(
```

```
                data.source = "groundfish",
                yr = p$yrs, # years are already specified above
                months=6:8, # which months to use
                settype = 1, # set quality indicator used in groundfish surveys .. 1=good
                gear = c("Western IIA trawl", "Yankee #36 otter trawl"), # choice of gear types
                strata_toremove=c("Gulf", "Georges_Bank", "Spring", "Deep_Water"), # for stratanal
                polygon_enforce=TRUE # make sure data positions are inside the polygons: incorrect strata or posi-
            tions
        )
)
    set = survey.db( p=p, DS="filter" add_groundfish_strata=TRUE ) # the call that returns the filtered data
```

The selection mechanism is quite general. If a survey variable is passed with an explicit list, then only data matching that list will be returned {e.g. p$selection$survey$yr = c(1990:2000) will return only those years where yr=(1990,...,2000)}. If a data variable is ranged then a range can be passed {p$selection$survey$yr = c(1990, 2000); p$selection$ranged_data="yr"} which will return the same years. This operation is conducted through {aegis::filter_data()} which can be easily extended for alternate methods.

Also of more general interest are the following functions:

```
    weight_year = meanweights_by_strata(
        set=set,
        StrataID=as.character( sppoly$StrataID ),
        yrs=p$yrs,
        fillall=TRUE,
        annual_breakdown=TRUE
)
```

which generates a matrix of mean weights broken down by strata and year. This is used to convert numbers to weights.

Environmental covariates interpolated via **stmv** can be extracted from the **aegis** data tables through a simple call:

```
    covars = c("t", "tmin", "tmax", "degreedays", "z", "dZ", "ddZ" )
    res = aegis_db_extract_by_polygon(
        sppoly=sppoly, # polygons to overlay onto covariate fields
        vars=covars,
        yrs=p$yrs,
        dyear=0.6 # 0.6*12 months = 7.2 = early July
)
```

When modeling, the input data likely will not have measured the appropriate variables of interest at the time of sampling. These can be filled via a fast lookup by location (longitude, latitude) and possibly a (POSIXct) timestamp . This can be accomplished via the following call:

```
    covars = c("t", "tmin", "tmax", "degreedays", "z", "dZ", "ddZ" )
    set = aegis_db_lookup( X=set, lookupvars=covars, xy_vars=c("lon", "lat"), time_var="timestamp" )
```

where, set$timestamp is a POSIXct variable, and set$lon is longitude and set$lat is latitude.

The list of variable names are defined by the user and so is project driven and growing. The generic environmental variables that might be of interest to most are:

```
z = depth (m)
dZ = bottom slope (m/km)
ddZ = bottom curvature (m/km^2)
substrate.grainsize = mean grain size of bottom substrate (mm)
t = temperature (C) – subannual
tlb = temperature lower 95% bound (C) –subannual
tub = temperature upper 95% bound (C) –subannual
tmean = mean annual temperature
tsd = standard deviation of the mean annual temperature
tmin = minimum value of temperature in a given year – annual
tmax= maximum value of temperature in a given year – annual
tamplitude = amplitude of temperature swings in a year (tmax-tmin) – annual
degreedays = number of degree days in a given year – annual
```

Finally, the output from `res = aegis_db_extract_by_polygon(...)`, above, can be reformatted into a simple tabular form that facilitates plotting and prediction:

```
APS = aegis_prediction_surface( aegis_data=res$means )
predictions = predict( model_fit, newdata=APS, type="response" )
```

8 Using INLA

INLA provides an interface to approximate Bayesian modeling that is extremely fast, making previously inaccessible problems accessible. The difficulty is that, as it is an approximation, there is less control over the specification of priors. The latter are also notoriously difficult to specify sensibly due to the possibility of unwittingly forcing an unwanted pattern in the joint distribution. Nonetheless, an important advance in prior specification has been made via "PC priors" (Penalized Complexity; Simpson et al. 2017). Fortunately, the clever folks developing INLA also are the proponents of PC priors. The perspective is that this prior is the base form towards which a null model might be expected to "shrink" (i.e., penalized by the prior). Any deviations in the posterior identifies the information expressed in the model of interest and the likelihood of the data.

In a number of important problems, one can state the prior in terms of a requirement that the parameter being estimated be less than some upper limit at some probability level. Syntactically, one provides parameters in the format c(upper bound, probability associated with the upper bound). Expressing things in terms of the upper tail probability makes excellent sense. However, to make things even more intuitive, in our examples, we express them in terms of medians, that is, with a probability of 0.5. This facilitates comprehension of priors as a scaling of the problem that shrinks towards the defined null/base model (prior). In the case of variance parameters, this scaling would be ideally be independently obtained and potentially informed; here we cheat and use the standard deviation of the data to scale these PC priors (see https://github.com/jae0/carstm /blob/master/inst/scripts/04_cod_comparisons_car.R) using the default type 2 Gumbel distribution that shrinks the distribution towards zero. For the AR1 process we use the the "cor0"

PC prior which as has a base model of 0 correlation, that is, no correlation. For the spatial process ("bym2") we use the recommended "pc" prior (rho=0.5, alpha=0.5).

END