Spatial Job Misallocation

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Job Market Paper

November 12, 2025

Under active revision: Click here for latest version

Abstract

This paper studies the efficiency of job creation across space and job types. Because labor market dynamics differ systematically across locations and jobs, the magnitude of externalities associated with vacancy creation in a frictional labor market also varies, leading to misallocation in the spatial distribution and the composition of jobs. I quantify these inefficiencies using a frictional labor market model with multiple locations and ex ante job heterogeneity that is flexible enough to replicate observed heterogeneity in labor dynamics. Calibrating the model to match job-finding and job-to-job transition patterns in U.S. metropolitan areas, I find that a social planner would reallocate vacancy creation away from metropolitan areas with heated labor markets toward those with frozen ones, and, in particular, from moderately productive toward less productive jobs. Aggregate welfare rises by about 1.1%, providing an efficiency-based rationale for place-based policies that support weaker labor markets.

Keywords: Spatial inequality, Labor dynamics, frictional labor markets, job creation, externalities, job-to-job transitions, spatial misallocation, place-based policy.

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1. Introduction

Labor market conditions differ substantially across space: in some places, workers can easily find jobs, while in others, job opportunities are scarce. Such spatial disparities motivate many real-world place-based policies aimed at stimulating local job creation. Kline and Moretti (2013) theoretically illustrate how place-based policies can improve efficiency when unemployment rates differ across regions. However, little is known about the quantitative importance of such policies and how broader dimensions of heterogeneity in labor market dynamics—such as job-finding and job-to-job transition rates—affect policy implications. How does this multidimensional heterogeneity map into optimal place-based policy? Specifically, where should governments target job creation subsidies, and what kinds of jobs should these policies aim to create?

This paper addresses these questions by characterizing how externalities from vacancy creation in a frictional labor market vary across locations and job types, and by quantifying how such variation gives rise to systematic inefficiencies in the spatial distribution and composition of jobs. To this end, I develop a frictional labor market model with ex ante heterogeneous job creation—defined by region, sector, and productivity—that is flexible enough to replicate the observed heterogeneity in labor market dynamics. The estimated model implies that a social planner would implement a sizable reallocation of job creation, shifting jobs away from metropolitan areas (MSAs) with heated labor markets toward those with frozen ones, and, in particular, away from moderately to less productive jobs.

Using data from the U.S. Current Population Survey and the National Longitudinal Survey, I document two empirical patterns that discipline the model. First, labor market conditions vary widely across metropolitan areas, with job-finding and job-to-job transition rates strongly positively correlated: some local labor markets are ‘‘heated,” characterized by high job-finding rates, low unemployment, and frequent job-to-job transitions, while others are “frozen,” displaying the opposite conditions. Second, within a given labor market, low-wage jobs rely more heavily on hiring from unemployment and experience frequent poaching, whereas high-wage jobs hire mainly from other firms.

To translate these empirical patterns into implications for job misallocation, I build a continuous-time frictional labor market model with ex ante heterogeneous job types and on-the-job search. Within each local labor market, workers and firms engage in random matching under search frictions and bargain over the match surplus upon meeting. The model has two key features. First, it specifies origin-destination-specific matching functions whose matching efficiencies can differ freely across locations and job types, allowing the model to replicate the observed heterogeneity in unemployment-to-employment and job-to-job transition patterns. Second, vacancy posting decisions are made at the job-type level, so that free entry endogenously determines the spatial distribution and composition of posted vacancies. To isolate this compositional margin—rather than the aggregate level of vacancy creation—I assume that the economy is endowed with a fixed aggregate resource available for vacancy creation.

I then evaluate the efficiency of the decentralized equilibrium by comparing it with the socially optimal allocation of job creation. The magnitude of two externalities differs across locations and job types. The first is an opportunity cost externality, which arises because firms do not fully consider the fact that their workers could have been employed elsewhere had they not been hired, generating profits for other firms. When a firm hires an unemployed worker, it places insufficient weight on the possibility that the worker could have continued searching and potentially matched with another job. Similarly, when a firm poaches an employed worker, it neglects that the worker would have continued to work in her previous job had she not been hired. Thus, jobs that hire workers who would otherwise have quickly found employment elsewhere or remained employed tend to be created excessively, as their social value is low relative to their private return. The second is the stepping stone externality. It arises because a job that loses its worker to another job does not fully internalize the social gains generated by such job-to-job transitions, as it fails to take into account the gains accruing to the poaching job. Therefore, jobs that are more frequently poached are created less than is socially efficient.

To quantify the extent of misallocation, I calibrate the model to match labor market dynamics across heterogeneous jobs in U.S. metropolitan areas. To bring the model to the data, I extend the baseline model by introducing match-specific idiosyncratic shocks that generate bilateral job-to-job flows while preserving tractability and the key externalities. I then calibrate the model by inverting it to match a broad set of empirical moments: the direction of job-to-job transitions across job types, the MSA-level unemployment-to-employment (U2E) rates and employment-to-employment (E2E) transition rates, separation rates, and the wage distribution across jobs.

Using the calibrated model, I evaluate the magnitude and sources of misallocation in vacancy creation and decompose the relative contributions of the opportunity cost and stepping stone effects. Across metropolitan areas, the planner reallocates vacancy creation resources substantially, ranging from -27 percent to $+18$ percent. The reallocation pattern is tightly linked to local labor market conditions. In areas with low U2E rates and a high share of E2E transitions among new hires, many additional hires would have been employed even without additional vacancies, implying a lower social value of vacancy creation due to the opportunity cost effect. Although the stepping stone effect raises the social value of vacancies in markets with active E2E transitions, its quantitative contribution is smaller and dominated by the opportunity cost effect. Thus, the planner allocates more vacancy creation to metropolitan areas that exhibit either lower U2E rates or smaller E2E shares.

In practice, these two conditions often coincide, as U2E rates and E2E shares are positively correlated across MSAs. Therefore, these spatial disparities operate in the same direction, leading the planner to reallocate vacancy creation from heated labor markets toward frozen ones. The resulting optimal allocation thus effectively mitigates regional disparities even though the planner is indifferent to distributional concerns. This finding suggests that efficiency considerations alone can provide a rationale for the place-based policies observed in practice: reallocating vacancy creation toward less dynamic regions not only narrows spatial inequality but also raises aggregate efficiency. Although the model attributes cross-regional differences to exogenous parameter heterogeneity, such disparities can naturally arise from asymmetric shocks, such as trade liberalization or automation, that shift labor demand across regions. In such cases, the analysis helps rationalize why governments may wish to direct policy support toward adversely affected labor markets. Geographically, the planner indeed expands vacancy creation in structurally challenged labor markets—such as Rust Belt cities including Cleveland, Detroit, and Indianapolis—and reduces it in high-growth, innovation-driven areas like Minneapolis, Denver, Boston, and Austin.

Moreover, within a given MSA, the planner shifts vacancy creation away from moderately productive jobs (by up to about 10 percent) toward less productive ones (by up to about 20 percent), while leaving high-productivity jobs largely unchanged. This reallocation arises because low-productivity jobs primarily recruit from the unemployment pool and thus entail a low opportunity cost of hiring. More importantly, they serve as effective stepping stones that generate substantial dynamic gains through frequent job-to-job transitions to other jobs. In contrast, moderately productive jobs hire mainly from job-to-job transitions but yield relatively small improvements in match quality compared with high-productivity jobs. As a result, the decentralized equilibrium features excessive vacancy creation among these moderately productive jobs.

Thus, improving efficiency requires reallocating vacancy creation simultaneously across space and job types—expanding vacancy creation in frozen markets, particularly among low-productivity jobs, and reducing it in heated markets, especially among moderately productive ones. Welfare effects across MSAs range from about -20 percent to $+20$ percent, with larger gains in areas suffering from high unemployment. Although the distribution of welfare changes is slightly more concentrated on the left of zero, MSAs with positive gains tend to be larger, so aggregate welfare rises by roughly 1.1% under the planner's allocation.

Literature. This paper builds on the literature that studies the efficiency of frictional labor markets with random search. The seminal result by Hosios (1990) characterizes the inefficiency related to job search by unemployed workers, showing that the aggregate level of vacancy creation is inefficient unless worker's bargaining power equals the elasticity of the matching function, the well-known Hosios condition. A related strand of research characterizes inefficiencies associated with on-the-job search (e.g., Gautier et al., 2010; Coles and Mortensen, 2016; Fukui and Mukoyama, 2025). These papers show that on-the-job search introduces an additional externality through the worker stealing effect—i.e., poaching firms do not internalize the fact that they destroy existing matches—leading to excessive aggregate vacancy creation. This paper departs from the literature by allowing for multiple labor markets and ex ante job heterogeneity, making the spatial allocation and composition of jobs endogenous and enabling an analysis of inefficiencies in these dimensions beyond the aggregate level.

Search frictions have long been recognized as a fundamental feature of labor markets and have been extensively studied (e.g., Burdett and Mortensen, 1998; Postel-Vinay and Robin, 2002; Cahuc et al., 2006; Menzio and Shi, 2011; Dey and Flinn, 2005). A growing body of empirical research documents that worker employment dynamics are far richer than those implied by benchmark search models. Job-to-job transitions are not exclusively upward in terms of ex-ante surplus, but often bilateral due to idiosyncratic shocks; search intensity and mobility patterns vary systematically across employment status, types of jobs, and worker types (e.g., Faberman et al., 2017, 2022; Berger et al., 2024; Lamadon et al., 2024; Bagger and Lentz, 2019). Despite this mounting evidence on heterogeneous employment dynamics across jobs, the efficiencies of the spatial allocation and composition of jobs remain largely unexplored.

A growing strand of research examines frictional labor markets across space. Heterogeneity in labor market frictions—such as employment-to-employment wage growth or spatial search frictions—has been widely recognized as a key driver of spatial wage inequality (e.g., Baum-Snow and Pavan, 2012; Heise and Porzio, 2022). More recently, studies have emphasized the role of firm heterogeneity and local job creation in shaping these differences across local markets. In particular, these mechanisms account for substantial spatial variation in labor dynamics—such as unemployment rates, job finding rates, and vacancy filling rates (e.g., Kline and Moretti, 2013; Bilal, 2023; Kuhn et al., 2022)—as well as local wage ladders (e.g., Lindenlaub et al., 2024). This paper also focuses on the firm side but emphasizes spatial differences in labor market dynamics and their implications on misallocation.

Finally, this paper is related to the literature on optimal spatial policies. Much of the earlier work evaluates place-based policies in environments that abstract from search frictions. Depending on the framework, these studies either caution that spatial interventions can distort the spatial allocation of labor (e.g., Albouy, 2009; Hsieh and Moretti, 2019) or argue that they can improve welfare when local productivity is shaped by agglomeration externalities (e.g., Fajgelbaum and Gaubert, 2020; Rossi-Hansberg et al., 2019). Spatial models incorporating search frictions highlight that such frictions can themselves give rise to additional externalities. For example, several papers show that it may be optimal to subsidize job creation in low-income regions when hiring costs are large or when heterogeneous workers and firms are endogenously sorted across local labor markets with random search (e.g., Kline and Moretti, 2013; Bilal, 2023; Oh, 2025). This paper also characterizes the optimal spatial policies under a frictional labor market, but it focuses on a distinct source of inefficiency. Differences in local labor dynamics generate spatial variation in the magnitude of externalities, which in turn calls for place-based policies.

Outline. The remainder of the paper is organized as follows. Section 2 documents key empirical facts on heterogeneity in employment dynamics across job types and metropolitan areas. Section 3 develops a frictional labor market model with multiple locations and ex ante heterogeneous job creation, characterizes the decentralized equilibrium and the socially optimal allocations, and identifies two externalities. Section 4 describes the estimation strategy. Section 5 quantifies the extent of misallocation, decomposes it across job characteristics and locations, and discusses implications for place-based and job-targeted policies. Section 6 concludes. The Appendix provides proofs and additional empirical details.

2. Labor Dynamics in Data

This section documents the heterogeneity in labor market dynamics across regions, sectors, and productivity levels in the United States. These empirical patterns constitute the key input for the efficiency analysis that follows, linking observed labor market heterogeneity to the misallocation of job creation across space and job types.

Data. I use data from the Current Population Survey (CPS) from IPUMS (Flood et al., 2025), a monthly rotating panel. The CPS reports detailed information on each worker's employment status and history, including industry, wages, and job changes. Local labor markets are defined at the Metropolitan Statistical Area (MSA) level, and I pool data from 2013 to 2018 to ensure sufficient sample size for each MSA. I classify jobs into 30 job types by interacting three broad sectors—manufacturing, wholesale and retail, and services—with deciles of residualized wages. Sectors such as agriculture, mining, and construction are excluded because of their distinct labor market characteristics. Since productivity is not directly observed, I use residualized wages as a proxy, which in the model corresponds one-to-one with productivity. Including unemployment, this classification yields 31 distinct labor market states. For each job type, I compute the employment share, average wage, separation rate into unemployment, and the job-finding rate from unemployment. In addition, I compute the job-to-job transition rate across all pairs of job types.

To supplement the CPS, I use the National Longitudinal Survey of Youth 1979 (NLSY), a survey tracking a nationally representative sample of individuals who were between ages 14 and 21 as of December 1978. These individuals have been surveyed regularly since 1979. One limitation of the CPS is that it collects wages only in the fourth and eighth interview months. Thus, I use the NLSY primarily to measure job-to-job transitions across wage groups. Job types are defined analogously to the CPS, based on sector and residualized wage. See Appendix A for additional details.

2.1 Heterogeneity in Labor Dynamics Pattern

I first document heterogeneity in labor market dynamics across local labor markets (Fact 1) and then across job types (Fact 2).

In Figure 1, I plot cross-sectional patterns of monthly unemployment-to-employment (U2E) transition rates, monthly employment-to-employment (E2E) transition rates, and unemployment rates across Metropolitan Statistical Areas (MSA). The top panels show that U2E rates vary considerably, ranging from 0.2 to 0.4, while E2E rates, though less dispersed, range from 0.015 to 0.022. The unemployment rate also exhibits large spatial variation, ranging from 0.02 to 0.07, consistent with evidence from other countries (e.g. Bilal, 2023; Kuhn et al., 2022). The bottom panels plot E2E rates and unemployment rates against U2E rates across MSAs. The left panel shows that U2E and E2E rates are positively correlated.[1] This positive correlation is

[1] Kuhn et al. (2022) document that E2E rates are largely uncorrelated with unemployment rates in pooled CPS data (2000-2019). In contrast, I find greater cross-sectional variation in more recent years (2013-2018).

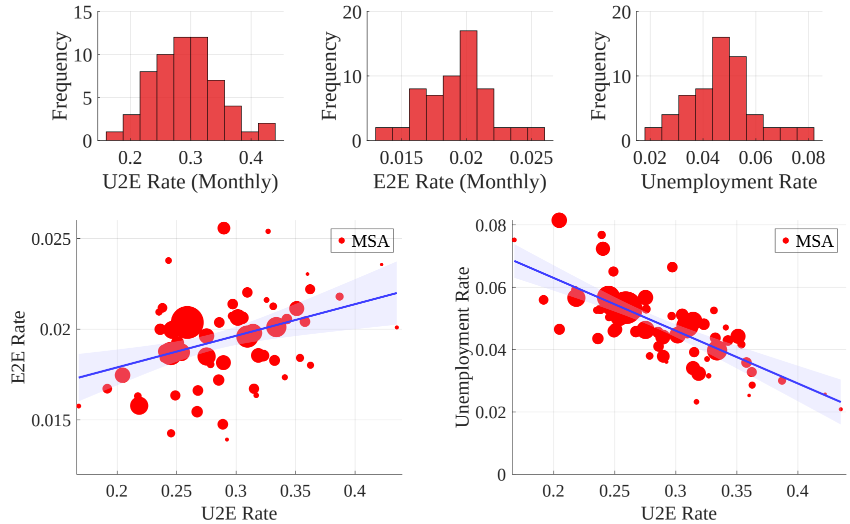


Figure 1. Employment Flows across MSAs

Notes: Data source: CPS (2013-2018). Each circle represents an MSA, with its size proportional to the size of the MSA's labor force. The blue line shows the fitted regression line, and the shaded area represents the 95 percent confidence band.

significant at the 1% confidence level, and a one-standard deviation increase in the U2E rate is associated with a 0.38-standard deviation increase in the E2E rate. The right panel shows that the unemployment rate is negatively correlated with the U2E rate, a relationship that is even more significant: a one-standard deviation increase in the U2E rate leads to a 0.73-standard deviation decrease in the unemployment rate.

Accordingly, local labor markets can be broadly classified into two groups based on these correlations: *heated* and *frozen* labor markets. Heated markets exhibit high U2E and E2E rates and low unemployment, whereas frozen markets feature low U2E and E2E rates alongside high unemployment. Compared to frozen markets, heated markets have up to 75 percent higher U2E rates and 40 percent higher E2E rates. Examples of heated markets include high-growth metropolitan areas such as Minneapolis, Denver, Boston, San Jose, Seattle, Salt Lake City, Dallas, and Austin, whereas frozen markets include many Rust Belt cities such as Cleveland, Detroit, Indianapolis. I summarize this observation in Fact 1 below.

Fact 1: *Labor markets exhibit substantial heterogeneity across MSAs. Heated markets display high U2E and E2E rates and low unemployment, while frozen markets exhibit the opposite pattern.*

Next, I turn to heterogeneity in labor market dynamics across job types. First, U2E transitions are disproportionately concentrated in low-wage jobs. In the left panel of Figure 2, I plot the distribution of wage

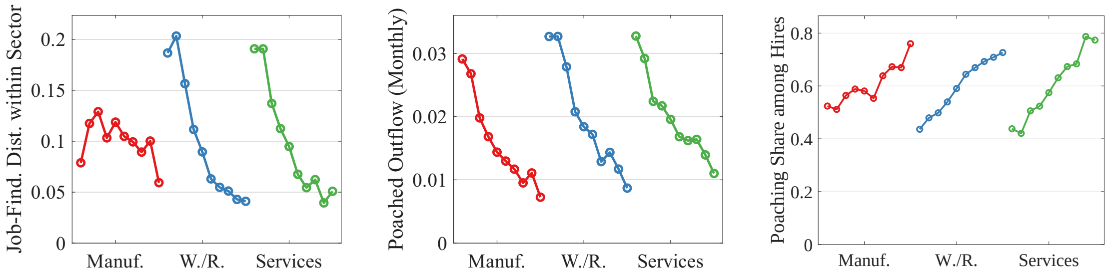


Figure 2. Employment Flows across Jobs

Notes: Data source: CPS and NLSY79. Jobs are classified into 30 groups by interacting three sectors—manufacturing, wholesale and retail, and services—with deciles of residualized wages. The left panel shows the distribution of job-finding probabilities across wage groups within each sector. The middle panel presents the outflow rates of employed workers who are poached by other firms, and the right panel depicts the share of new hires who were previously employed elsewhere.

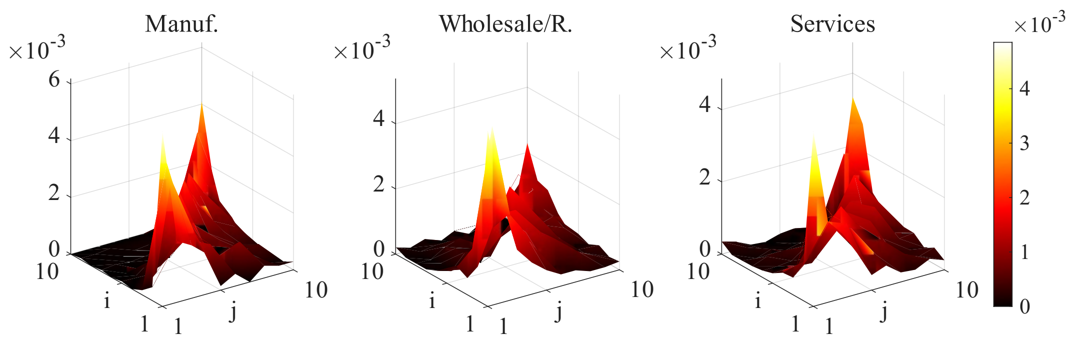


Figure 3. E2E Rate for Each Origin-Destination Pair

Notes: Data source: NLSY79. Each panel reports job-to-job transition rates from origin job type i to destination job type j, where jobs are classified into ten residualized wage groups within each sector.

deciles for jobs obtained by unemployed workers when they find employment, conditional on the sector. In both wholesale and retail and services, the probability of finding a bottom-decile job is roughly four times higher than that of getting a top-decile job. Although less pronounced, the manufacturing sector displays a similar qualitative pattern. In addition, low-wage jobs experience more frequent poaching. The middle panel of Figure 2 shows that, across all sectors, the probability that a bottom-decile job is poached is about three times higher than that for a top-decile job. Moreover, high-wage jobs rely more heavily on E2E transitions, that is, hiring directly from other firms. The right panel of Figure 2 plots poaching share, i.e., the share of hires drawn from the employed pool. For bottom-decile jobs, roughly 50-60 percent of hires come from unemployment. In contrast, for top-decile jobs, nearly 80 percent of new hires are recruited from other firms.

Taken together, these findings suggest that low-wage employment is characterized by strong inflows from unemployment and high turnover, whereas high-wage employment relies more on poaching workers from other firms. These patterns are qualitatively consistent with standard on-the-job search models, in which both unemployed and employed workers draw job offers from a common offer distribution and gradually move up the wage ladder, and are also in line with existing empirical evidence documenting similar wage-conditional mobility patterns. The data, however, reveal some quantitative departures from the benchmark model; for example, the poaching share for bottom-decile firms exceeds 40 percent, whereas the standard model would predict a much smaller value.

Having examined the intensity of job-to-job flows, I now turn to the directional pattern of job-to-job mobility, focusing on the destinations of workers who leave each job. Figure 3 plots the E2E transition rate for each origin-destination pair (i, j). The mass of transitions lies close to the 45-degree line, indicating that most E2E moves occur between jobs with similar wage levels. This pattern is consistent with recent evidence from Lentz et al. (2022), who document that employed workers tend to move to jobs with similar wages using Norwegian data. However, it contrasts with prediction of canonical frictional search models, which assume random job offers independent of a worker's current position.

The discussion above can be summarized in the following fact.

Fact 2: *Low-wage jobs rely more heavily on hiring unemployed workers and experience frequent poaching, whereas high-wage jobs hire workers mainly from other firms.*

3. Model

This section presents a model of frictional labor market with ex ante heterogeneous job creation, flexible enough to match labor dynamics pattern documented in Section 2. Jobs are freely created for each type, thereby endogenously determining the spatial distribution and the composition of jobs. I first present the model and derive the equilibrium conditions. I then compare it with the optimal allocation to characterize externalities in job creation.

3.1 A Model of Frictional Labor Market

Time t is continuous. Locations are indexed by $\ell \in \mathcal{L}$ and sectors are indexed by $s \in \mathcal{S}$. Within each location-sector pair (ℓ, s), jobs differ by their productivity. I discretize these productivity differences into a finite number of levels, indexed by $A \in \mathcal{A}(\ell, s)$. Thus, the type of job is determined by the combination of its location, sector and productivity level, which I index by $j \in \mathcal{J} \equiv \{1, \cdots, J\}$. A job of type j is therefore characterized by the triplet (ℓ_j, s_j, A_j), where ℓ_j denotes the location, s_j the sector, and A_j the productivity level.

Workers. In each location ℓ, there is a continuum of infinitely lived, homogeneous workers with measure L^ℓ. The measure of workers is exogenous and time invariant, and workers are immobile across locations. All workers and firms are risk-neutral and discount the future at rate r. Workers maximize the net present value of discounted utility. Let $\mathcal{J}^\ell \equiv \{j \in \mathcal{J} : \ell_j = \ell\}$ denote the set of job types operating in location ℓ. At each time t, a worker in location ℓ is either employed in a job of type $j \in \mathcal{J}^\ell$, or unemployed. Let M_{jt} denote the measure of workers employed in type-j jobs, and M_{ut}^ℓ denote the measure of unemployed workers in location ℓ. Thus, we have $\sum_{j \in \mathcal{J}^\ell} M_{jt} + M_{ut}^\ell = L^\ell$. Workers employed in type-j jobs earn a flow wage of w_{jt}, while unemployed workers receive a location-specific flow utility of b^ℓ from home production.

Jobs. Jobs produce a homogeneous numéraire good using only labor. When matched with a worker, a type-j job produces a flow output of A_j. Matches with type j jobs are exogenously destroyed at rate $s_j > 0$, sending the worker into unemployment. Posting and maintaining a type-j vacancy requires a type-specific vacancy-positing service, the price of which, P_{jt}, is determined in the vacancy-posting service market and increases with the total number of type-j vacancies posted at time t. Firms are atomistic and therefore take P_{jt} as given when posting vacancies. Vacancies are created under free entry, and thus they are posted until their net value is driven to zero.

Vacancy-posting service market. The economy is endowed with a fixed aggregate supply $\bar{R}$ of a generic resource that can be used for vacancy creation. This resource is competitively supplied by resource owners at price P_t^R.[2] Competitive intermediaries use this resource to provide vacancy-posting service for each job type. Posting V_j vacancies of type j requires $c_j(V_j)$ units of the resource. The cost function $c_j(\cdot)$ is strictly

[2] When comparing the decentralized equilibrium with the social planner's allocation, my focus is not on whether the aggregate level of vacancy creation is too high or too low, but rather on the allocation of vacancies across job types. The assumption of a fixed resource supply allows us to isolate this compositional margin.

increasing, convex, and continuously differentiable, with $c_j(0) = c_j'(0) = 0$. Workers hold diversified portfolios of claims on resource owners and intermediaries, so that all rents ultimately accrue to them.

Search. Search is random. Both unemployed and employed workers engage in job search. Matches are formed through constant-returns-to-scale matching functions with a constant matching elasticity η. Importantly, matching occurs separately for each origin-destination pair, where the origin refers to the type of job (or unemployment pool) a worker currently belongs to, and the destination refers to the type of vacancy.[3] Specifically, unemployed workers in location ℓ contact type-j vacancies according to a Poisson process with arrival rate $f_{jt} = \alpha_j (M_{ut}^{\ell})^{\eta-1} V_{jt}^{1-\eta}$. Workers employed in type-j jobs contact type-k vacancies at rate $f_{jkt} = \alpha_{jk} M_{jt}^{\eta-1} V_{kt}^{1-\eta}$. Type-$j$ vacancies contact unemployed workers at rate $q_{jt} = \alpha_j (M_{ut}^{\ell})^{\eta} V_{jt}^{-\eta}$ and contact workers employed by type-k firms at rate $q_{kjt} = \alpha_{kj} M_{kt}^{\eta} V_{jt}^{-\eta}$.

Technology and wage. When a worker and a type-j vacancy are matched, they produce flow output A_j. A matched worker and firm split the surplus of the match according to Nash bargaining, with worker's bargaining power $\beta \in (0,1)$. I assume that firms lack commitment and thus cannot match outside offers.[4]

3.2 Equilibrium

I assume that the economy is in steady state at $t = 0$. Thus, all equilibrium objects are time-invariant, and I will drop the time subscript in this section.

Values of workers and firms. I first characterize the value functions of workers and firms. Let U^{ℓ} denote the value of an unemployed worker in location ℓ, and let E_j denote the value of an employed worker matched to a type-j job. These values are given by

$$rU^{\ell} = b^{\ell} + \Pi + \sum_{j \in \mathcal{J}^{\ell}} f_j \max\{E_j - U^{\ell}, 0\},$$

$$rE_j = w_j + \Pi + \sum_{k \in \mathcal{J}^{\ell_j}} f_{jk} \max\{E_k - E_j, 0\} + s_j (U^{\ell_j} - E_j),$$

[3] Most of the literature assumes a single aggregate matching function that encompasses all vacancies and workers. However, aggregating the match technology mechanically introduces additional externalities when heterogeneous workers and firms search randomly (e.g., Acemoglu, 2001), which are not the focus of this paper. I therefore specify origin-and-destination-specific matching functions.

[4] This wage-setting protocol differs from the sequential bargaining framework in Cahuc et al. (2006). While assuming either full or no commitment represents an extreme case, the main insights of the paper remain valid for intermediate degrees of commitment. I discuss the implications of this assumption in greater detail in Section 3.4.

where Π denotes the profit from vacancy-posting service market. At rate f_j, an unemployed worker meets a type-j vacancy. If the offer is acceptable (i.e., $E_j > U^\ell$) her value jumps to E_j. An employed worker earns a flow wage w_j, which is endogenously determined through Nash bargaining, as will be discussed shortly. With arrival rate f_{jk}, this worker receives an offer from a type-k vacancy and switches if doing so raises her value. Matches are destroyed at rate s_j, in which case the worker becomes unemployed.

Similarly, let V_j^v be the value of a type-j vacancy, and let J_j denote the value of a type-j job matched with a worker. Then, the Bellman equations are

$$rV_j^v = -P_j + (q_j \mathbb{1}_{E_j > U^\ell} + \sum_{k \in \mathcal{J}^{\ell_j}} q_{kj} \cdot \mathbb{1}_{E_j > E_k}) \max\{J_j - V_j^v, 0\},$$

$$rJ_j = A_j - w_j + (s_j + \sum_{k \in \mathcal{J}^{\ell_j}} f_{jk} \cdot \mathbb{1}_{E_k > E_j})(V_j^v - J_j).$$

Vacancy pays a flow cost of P_j, meets an unemployed worker at rate q_j, and meets a worker employed by a type-k job at rate q_{kj}. When matched, its value increases by $J_j - V_j^v$. This gain is independent of the worker's previous employment status due to the lack of commitment. A matched job generates a flow profit of $A_j - w_j$ until the match ends, either through exogenous destruction at rate s_j or through the worker's transition to another job at rate $\sum_{k \in \mathcal{J}^{\ell_j}} f_{jk} \cdot \mathbb{1}_{E_k > E_j}$. Free entry implies that vacancies are posted until their value is driven to zero:

$$V_j^v = 0.$$

I define the joint surplus of a match as $S_j = J_j + E_j - U^\ell$. Then, the Nash bargaining has a well-known solution, in which a worker and a firm receive constant shares of the surplus:

$$E_j - U^\ell = \beta S_j \text{ and } J_j = (1-\beta) S_j.$$

Plugging these bargaining solutions into the definition of the surplus, I can characterize the type-specific match surplus:

$$rS_j = A_j - b^{\ell_j} - s_j S_j + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{jk}(\beta S_k - S_j) - \beta \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k S_k, \tag{1}$$

where $\tilde{f}_{jk} = f_{jk} \cdot \mathbb{1}_{S_k > S_j}$ and $\tilde{f}_k = f_k \cdot \mathbb{1}_{S_k > 0}$ denote the effective meeting rates of workers that result in matches with k-type jobs. The bargaining rule implies that wages are determined by

$$w_j = A_j - (1-\beta)(r + s_j + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{jk}) S_j.$$

Flow-balance condition. For each location ℓ, inflows into and outflows from unemployment must balance in steady state:

$$\sum_{j \in \mathcal{J}^\ell} s_j M_j = \left(\sum_{j \in \mathcal{J}^\ell} \tilde{f}_j \right) M_u^\ell$$

Similarly, for each job type j, the inflow and outflow of workers must be in balance:

$$\tilde{f}_j M_u^{\ell_j} + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kj} M_k = \left(s_j + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{jk} \right) M_j.$$

Type-j jobs hire workers both from unemployment, at rate $\tilde{f}_j M_u^{\ell_j}$, and from other jobs, at rate $\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kj} M_k$. They lose workers through exogenous separation $s_j M_j$ and job-to-job transitions $\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{jk} M_j$.

Vacancy-posting service market. Market clearing in the vacancy-posting service market determines the price of each type-specific vacancy-posting service:

$$P_j = P^R \cdot c_j'(V_j),$$

which is increasing in the number of type-j vacancies V_j. The resource price P^R is also pinned down by resource market clearing:

$$\sum_{j \in \mathcal{J}} c_j(V_j) = \bar{R}.$$

Therefore, the free-entry condition can be rewritten as

$$c_j'(V_j) = \frac{1-\beta}{P^R} \cdot \left(\tilde{q}_j + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{q}_{kj} \right) \cdot S_j. \tag{2}$$

where $\tilde{q}_j = q_j \cdot \mathbb{1}_{S_j > 0}$ and $\tilde{q}_{kj} = q_{kj} \cdot \mathbb{1}_{S_j > S_k}$ denote the effective meeting rates of firms that result in successful matches.

Equilibrium. A steady-state equilibrium consists of the measure of vacancies $\{V_j\}_{j\in\mathcal{J}}$, unemployment rates $\{M_u^\ell\}_{\ell\in\mathcal{L}}$, employment shares $\{M_j\}_{j\in\mathcal{J}}$, the resource price P^R, the prices of vacancy-posting service $\{P_j\}_{j\in\mathcal{J}}$, and wages $\{w_j\}_{j\in\mathcal{J}}$ such that (i) free entry condition holds, (ii) wages are determined by Nash bargaining, (iii) the markets for the resource and for vacancy-posting service clear, and (iv) employment shares and unemployment rates satisfy the flow-balance conditions.

3.3 Social Planner Problem

The social planner maximizes the discounted sum of output, taking the initial labor allocation at time $t = 0$ as given. The planner also discounts the future at rate r and is subject to search friction and resource constraint as in the decentralized equilibrium. The problem can be formulated as

$$\begin{aligned}
\max_{\{V_{jt},\mathcal{J}_{jt}^O,\mathcal{J}_{jt}^I\}_{j\in\mathcal{J},t\geq 0}} \quad & \int_0^\infty e^{-rt}\left\{\sum_{j\in\mathcal{J}} M_{jt}\cdot A_j + \sum_{\ell} M_{ut}^\ell \cdot b^\ell\right\}\mathrm{d}t \\
\text{s.t.} \quad & \dot{M}_{jt} = -s_j M_{jt} - \sum_{k\in\mathcal{J}_{jt}^I} \alpha_{jk} M_{jt}^\eta V_{kt}^{1-\eta} + \sum_{k\in\mathcal{J}_{jt}^O} \alpha_{kj} M_{kt}^\eta V_{jt}^{1-\eta} + \alpha_j \left(M_{ut}^\ell\right)^\eta V_{jt}^{1-\eta}, \\
& \sum_{j\in\mathcal{J}} c_j(V_{jt}) = \bar{R}.
\end{aligned}$$

where $M_{ut}^\ell = L^\ell - \sum_{j\in\mathcal{J}^\ell} M_{jt}$, $\mathcal{J}_{jt}^I \subset \mathcal{J}^{\ell_j}$ denotes the set of job types into which the planner reallocates workers away from j at time t, and $\mathcal{J}_{jt}^O \subset \mathcal{J}^{\ell_j}$ denotes the set of job types from which the planner reallocates workers toward j at time t whenever feasible. The first constraint describes the law of motion of the employment share of type-j jobs. The type-j matches are destroyed either exogenously, at rate s_j, or endogenously, when workers transition to other jobs. At the same time, the employment share of type-j jobs increases through poaching of workers from other jobs and through new hires from unemployment.

The planner's optimal vacancy creation is characterized by the following conditions. Following Fukui and Mukoyama (2025), I represent the planner's solution in terms of the valuation margin (3) and the investment margin (4).

$$rS_{jt}^{\mathrm{SP}} = A_j - b^{\ell_j} - s_j S_{jt}^{\mathrm{SP}} + \eta \sum_{k\in\mathcal{J}^{\ell_j}} \tilde{f}_{jkt}^{\mathrm{SP}}(S_{kt}^{\mathrm{SP}} - S_{jt}^{\mathrm{SP}}) - \eta \sum_{k\in\mathcal{J}^{\ell_j}} \tilde{f}_{kt}^{\mathrm{SP}} S_{kt}^{\mathrm{SP}} + \dot{S}_{jt}^{\mathrm{SP}}, \tag{3}$$

$$c_j'(V_{jt}^{\mathrm{SP}}) = \frac{1-\eta}{\tilde{P}_t^R}\cdot\left(\tilde{q}_{jt}^{\mathrm{SP}} S_{jt}^{\mathrm{SP}} + \sum_{k\in\mathcal{J}^{\ell_j}} \tilde{q}_{kjt}^{\mathrm{SP}}(S_{jt}^{\mathrm{SP}} - S_{kt}^{\mathrm{SP}})\right). \tag{4}$$

Here, S_{jt}^{SP} denotes the current-value costate variable, which measures the marginal value to the social planner of an additional worker employed in a type-j job at time t. These costate variables are the planner's analogue of match surpluses in the decentralized equilibrium. The variable $\tilde{P}_t^R$ is the Lagrange multiplier associated with the resource constraint, representing the shadow value of the resource. The effective meeting rates, $\tilde{f}^{\mathrm{SP}}$ and $\tilde{q}^{\mathrm{SP}}$, are defined analogously to those in the decentralized equilibrium. They are equal to underlying meeting rates, f and q, when forming a match is optimal for the social planner and are zero otherwise. See Appendix for the derivations.

Comparing the equilibrium conditions (1) and (2) with the planner's optimality conditions (3) and (4) reveals that they differ along several dimensions. Consequently, the decentralized equilibrium does not, in general, coincide with the optimal allocation, as summarized in the following proposition.

Proposition 1. *The decentralized equilibrium is inefficient.*

3.4 Externality in Vacancy Creation

In this section, I characterize two externalities and discuss how their magnitudes systematically vary across locations and job types. In the remainder of this section, I assume that the Hosios condition is violated in a manner consistent with empirical evidence in the literature. The opposite case would simply reverse the direction of the associated inefficiencies.

Assumption 1 (Violation of the Hosios Condition). *The bargaining power of workers is smaller than the elasticity of the matching function: $\beta < \eta$.*

Opportunity cost effect. The opportunity cost externality arises because firms do not *fully* consider the fact that their hired workers could have worked in other firms had they not been hired.

First, when a firm hires an unemployed worker, it places insufficient weight on the fact that the worker could have continued searching and potentially matched with another firm. This occurs because, unlike the social planner, firms do not take into account the profits that would have been generated by another firm that might have hired the worker. To compare the surplus and the marginal value of a match, I gather the equations (1) and (3) below:

$$rS_j = A_j - b^{\ell_j} - s_j S_j + \underbrace{\sum_k \tilde{f}_{jk}(\beta S_k - S_j)}_{\text{Stepping Stone}} - \underbrace{\beta \sum_k \tilde{f}_k S_k}_{\text{Opportunity Cost}}, \tag{1′}$$

$$rS_{jt}^{\mathrm{SP}} = A_j - b^{\ell_j} - s_j S_{jt}^{\mathrm{SP}} + \underbrace{\sum_k \tilde{f}_{jkt}^{\mathrm{SP}}(\eta S_{kt}^{\mathrm{SP}} - \eta S_{jt}^{\mathrm{SP}})}_{\text{Stepping Stone}} - \underbrace{\eta \sum_k \tilde{f}_{kt}^{\mathrm{SP}} S_{kt}^{\mathrm{SP}}}_{\text{Opportunity Cost}} + \dot{S}_{jt}^{\mathrm{SP}}. \tag{3′}$$

Unemployed workers find a type-k job at rate $\tilde{f}_k$ and thus generate an expected flow value of $\sum_k \tilde{f}_k S_k$ through their search activities. However, since the firm's share $(1-\beta)\sum_k \tilde{f}_k S_k$ is ignored in the joint surplus, the private opportunity cost is given by $\beta \sum_k \tilde{f}_k S_k$, as shown in the last term of equation (1′). From the social planner's perspective, increasing the measure of unemployed workers by one unit raises matches with type-k jobs by $\eta \tilde{f}_k^{\mathrm{SP}}$, implying a social (i.e., true) opportunity cost of $\eta \sum_k \tilde{f}_k^{\mathrm{SP}} S_k^{\mathrm{SP}}$, as shown in equation (3′). Therefore, under Assumption 1 ($\beta < \eta$), the opportunity cost term is smaller in the decentralized equilibrium, indicating that firms in the decentralized economy fail to fully account for the opportunity cost of hiring. This distortion is particularly severe in labor markets where the opportunity costs are high; for example, when $\tilde{f}_k$ is high. In such markets, even without the creation of additional jobs, unemployed workers would have been able to find high-quality jobs relatively easily. Consequently, the decentralized equilibrium features excessive job creation in tight labor markets and insufficient job creation in slack labor markets. The nature of this inefficiency is the same as the canonical inefficiency studied in the search literature (e.g., Hosios, 1990). However, that literature focuses on distortions in the *aggregate* level of vacancy creation. By contrast, this paper emphasizes *compositional* distortions in job creation across local labor markets arising from spatial heterogeneity in opportunity costs. Figure 1 suggests that there is indeed substantial heterogeneity in opportunity costs across local labor markets.

Second, when a job poaches a worker from another match, it neglects the fact that the worker would have continued to work in her previous job had the poaching not occurred. This again arises because the hiring firm does not internalize the profits that would have accrued to the worker's previous employer. Formally, from the social planner's perspective, the opportunity cost of such hiring corresponds to the value of the poached job, S_k^{SP}. Accordingly, in the last term of (4), the social gain from poaching a worker from a type-k job is given by $S_j^{\mathrm{SP}} - S_k^{\mathrm{SP}}$. In contrast, in the decentralized equilibrium, the poached firm's loss is not internalized. As shown in the free-entry condition in (2), the private value of a new match equals S_j, regardless of whether the worker is hired from unemployment or from another job. To further decompose the sources of inefficiency, I rewrite the planner's vacancy creation condition in (4) as

$$c_j'(V_{jt}^{\mathrm{SP}}) \propto \left(\tilde{q}_{jt}^{\mathrm{SP}} S_{jt}^{\mathrm{SP}} + \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{q}_{kjt}^{\mathrm{SP}} S_{jt}^{\mathrm{SP}} \right) \cdot \frac{1 + \left(\frac{\mathrm{EE}}{\mathrm{UE}}\right)_{jt} \cdot \Delta_{jt}}{1 + \left(\frac{\mathrm{EE}}{\mathrm{UE}}\right)_{jt}} \tag{4$'$}$$

where

$$\left(\frac{\mathrm{EE}}{\mathrm{UE}}\right)_{jt} = \frac{\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{q}_{kjt}^{\mathrm{SP}}}{\tilde{q}_{jt}^{\mathrm{SP}}} \quad \text{and} \quad \Delta_{jt} = \sum_{k \in \mathcal{J}^{\ell_j}} \frac{\tilde{q}_{kjt}^{\mathrm{SP}}}{\sum_{i \in \mathcal{J}^{\ell_j}} \tilde{q}_{ijt}^{\mathrm{SP}}} \left(1 - \frac{S_{kt}^{\mathrm{SP}}}{S_{jt}^{\mathrm{SP}}}\right).$$

It is without loss of generality to write this condition up to a constant, given the resource constraint.[5] The term inside the parentheses in (4′) mirrors the decentralized equilibrium condition (2), while the ratio term in (4′) captures the distortion. The extent of this distortion depends on two key factors. It decreases with $\left(\frac{\mathrm{EE}}{\mathrm{UE}}\right)_{jt}$—the relative reliance of hiring on job-to-job transitions—and increases with Δ_{jt}—the average surplus gain from job-to-job transitions, expressed as a share of the surplus of the destination job. For example, job types that hire primarily through job-to-job transitions with limited surplus gains are particularly excessively created relative to those that hire workers mainly from unemployment or from low-surplus jobs. This externality has also been studied in the literature and is often referred to as the *worker stealing externality* (e.g., Gautier et al., 2010; Coles and Mortensen, 2016; Fukui and Mukoyama, 2025). These studies show that such an externality leads to excessive *aggregate* vacancy creation in the decentralized equilibrium. In contrast, the focus of this paper is again on the *compositional* dimension of this inefficiency—i.e., how it distorts the allocation of job creation across locations and across job types with heterogeneous hiring patterns. Figures 1–3 suggest that there is indeed substantial heterogeneity in hiring patterns across both locations and jobs.

Stepping stone effect. The stepping stone externality arises because when a worker moves from one job to another, the firm that loses the worker does not fully internalize the social gain from this reallocation.

From the planner's perspective, job-to-job transitions improve the quality of matches by reallocating workers toward more productive jobs. An increase in employment at type-j jobs raises the number of transitions to type-k jobs by $\eta \tilde{f}_{jk}^{\mathrm{SP}}$, thereby generating an additional social surplus of $\eta \tilde{f}_{jk}^{\mathrm{SP}}(S_k^{\mathrm{SP}} - S_j^{\mathrm{SP}})$, as

[5] This is because the allocation of the fixed amount of resource $\bar{R}$ across job types depends solely on the relative size of marginal costs $c_j'(\cdot)$. For example, suppose there are two job types subject to a resource constraint $c_1(V_1) + c_2(V_2) = \bar{R}$. Suppose that $c_1'(V_1) : c_2'(V_2) = 1 : k$ for some $k > 0$. Because $c_2'(\cdot)$ is strictly increasing, I can define a function $\Phi(V_1) \equiv c_1(V_1) + c_2((c_2')^{-1}(kc_1'(V_1))) - \bar{R}$. $\Phi(\cdot)$ is continuous and strictly increasing, with $\Phi(0) = -\bar{R} < 0$ and $\Phi(V_1) \to \infty$ as $V_1 \to \infty$. Hence, there exists a unique V_1 solving $\Phi(V_1) = 0$, which in turn uniquely pins down V_2. Also, higher k implies higher V_2 and lower V_1.

shown in equation (3′). In this sense, jobs that are poached serve a stepping stone role: they temporarily employ workers who will later move on to more productive jobs. In the decentralized equilibrium, however, this dynamic contribution is insufficiently rewarded. Decomposing the stepping stone term in (1′) yields two components,

$$\beta \tilde{f}_{jk}(S_k - S_j) - (1-\beta)\tilde{f}_{jk}S_j.$$

The first term represents the gain from job-to-job transitions accruing to the worker. Under Assumption 1, it is smaller than the social gain represented by the stepping stone term in (3′). The second term captures the loss borne by the poached firm. As a result, job types that are frequently poached—those that serve as stepping stones—are created less than is socially efficient—mirroring, but in the opposite direction, the excessive job creation that mainly poach workers from others. Figure 2 documents significant variation across jobs in how frequently workers are poached by other firms.

The role of wage determination. The key source of inefficiencies in this paper lies in the limited sensitivity of wages. To illustrate, consider a labor market in weak conditions. The social value of creating an additional vacancy there exceeds that of creating an otherwise identical job elsewhere due to the opportunity cost effect. If wages in this location were sufficiently low to reflect the low opportunity cost, firms would post more vacancies, restoring efficiency. Similarly, when a firm mainly hires workers away from other jobs, efficiency requires that the wage of such a job be higher to discourage their creation. Conversely, for jobs that are frequently poached, efficiency requires wages to fall to reward their stepping stone role, providing incentives for their creation.

Because inefficiencies arise precisely from this limited wage sensitivity, the extent to which wages adjust is central for determining efficiency. To understand how much wage adjustment would be required for efficiency, it is useful to establish a theoretical benchmark. Following the approach of Fukui and Mukoyama (2025), I can show that the decentralized equilibrium of this model would be efficient if two conditions were satisfied: the Hosios condition and sequential-auction style wage bargaining (e.g., Dey and Flinn, 2005; Cahuc et al., 2006; Bagger et al., 2014). The Hosios condition ensures that wages adjust enough to internalize the opportunity cost, eliminating the first channel of the opportunity cost effect. Sequential-auction style wage bargaining, in turn, guarantees that the wage a poaching firm must offer is high enough relative to the wage it offers to unemployed workers, thereby penalizing poaching firms while rewarding poached firms by allowing them to offer lower wages. These two assumptions are frequently made in the literature, mainly for reasons such as tractability.

Empirical studies, however, consistently find that wage-setting behavior in practice falls far short of this benchmark. First, wages respond only weakly to workers' outside options, implying a bargaining power parameter, β, much smaller than the elasticity of matching function, η (e.g., Jäger et al., 2020; Lachowska et al., 2022; Caldwell and Harmon, 2019). Second, wage-setting protocols in practice deviate substantially from the sequential-auction style wage bargaining. Di Addario et al. (2023) shows that the contribution of the previous firm to wage variation is an order of magnitude smaller than that of the new firm, a finding inconsistent with the sequential-auction style wage bargaining.[6] Moreover, a considerable share of hiring occurs under protocols that restrict firms' ability to condition offers.[7] Evidence on counteroffers also suggests that they are infrequently used. Faberman et al. (2022) report that only about 12 percent of employed workers who received an outside offer were countered by their current employer—whether through a matching wage, promotion, or job-related benefits— a frequency far below that implied by sequential-auction style wage bargaining. In addition, See Kline (2024) for a more comprehensive review of the empirical evidence.

These empirical findings indicate that real-world wage setting is considerably less responsive than the benchmark efficient case would require, making the model's limited wage sensitivity—and the resulting inefficiency—a quantitatively relevant feature of actual labor markets.

3.5 Misallocation Across Locations and Jobs

In Section 3.4, I analyzed the nature of the externalities by comparing the conditions that characterize the decentralized equilibrium with those under the social planner's allocation. In this section, I explicitly solve these conditions to compare how resources are allocated across locations and across heterogeneous jobs using two simplified versions of the model. To obtain analytical results, I consider the limit case in which the matching elasticity η approaches one.[8] Throughout this section, I assume $s_j = s$ for all $j \in \mathcal{J}$. Detailed derivations of the results in this section are provided in Appendix C.

[6] While some studies find that wages partly depend on past jobs or outside option values (e.g., Bonhomme et al., 2019), wages generally do not adjust to the extent required for the sequential-auction mechanism to hold. Thus, the externalities highlighted in this paper remain relevant, although their quantitative magnitude may differ.

[7] For example, wage posting accounts for roughly one-third of hires, implying that a large fraction of jobs offer fixed wages with little or no scope for adjustment (e.g., Hall and Krueger, 2012).

[8] Without this assumption, changes in job creation by the social planner would affect job-finding and worker-finding rates, thereby altering employment shares and feeding back into the planner's job creation decisions. This interdependence makes the comparison of allocations analytically intractable.

Across Locations. I begin with a simplified version of the model that abstracts from job-to-job transitions by setting $\alpha_{jk} = 0$ for all $j, k \in \mathcal{J}$. This assumption shuts down all other externalities and allows me to focus on the opportunity cost effect associated with hiring unemployed workers. The following result shows that, in this environment, jobs are created insufficiently in weaker labor markets. In Proposition 2, let

$$(\overline{A-b})^{\ell} = \frac{\sum_{k \in \mathcal{J}^{\ell}} \tilde{f}_k \cdot (A_k - b^{\ell})}{\sum_{k \in \mathcal{J}^{\ell}} \tilde{f}_k}$$

denote the average net productivity of the jobs with which unemployed workers in location ℓ are matched, and let $\tilde{f}^{\ell} = \sum_{k \in \mathcal{J}^{\ell}} \tilde{f}_k$ be the total job-finding rate of unemployed workers in that location.

Proposition 2. *Consider two job types j and j' located in different locations ℓ and ℓ', respectively. Suppose that $c_j(\cdot)$ and $c_{j'}(\cdot)$ are constant-elasticity functions with the same elasticity. If (i) the relative productivity of other job offers is lower in location ℓ, i.e., $\frac{(\overline{A-b})^{\ell}}{A_j - b^{\ell}} < \frac{(\overline{A-b})^{\ell'}}{A_{j'} - b^{\ell'}}$, while $\tilde{f}^{\ell} = \tilde{f}^{\ell'}$; or if (ii) $\frac{(\overline{A-b})^{\ell}}{A_j - b^{\ell}} = \frac{(\overline{A-b})^{\ell'}}{A_{j'} - b^{\ell'}} = 1$, but job-finding rates are lower in ℓ (due to smaller α_j's or higher c_j's), i.e., $\tilde{f}^{\ell} < \tilde{f}^{\ell'}$, then the social planner creates relatively more vacancies of type j than of type j' compared with the decentralized equilibrium, i.e.,*

$$\frac{V_{jt}^{SP}}{V_{j't}^{SP}} > \frac{V_j}{V_{j'}}.$$

If a job type j is less productive than the average job in its location, or operates in a labor market with a higher job-finding rate, it carries a higher opportunity cost and is therefore socially less valuable. However, because the decentralized equilibrium does not fully consider this cost, such jobs tend to be created excessively.

Within Location. I now consider another simplified version of the model that reintroduces job-to-job transitions. The goal is to isolate the opportunity cost effect and the stepping stone effect associated with job-to-job transitions. Accordingly, the next proposition imposes the Hosios condition and examines how the social planner reallocates job creation across different job types within a location. To derive analytical results, I assume that there are three job types in location ℓ, and that job-to-job transitions occur only from type-1 to type-3 jobs.

Proposition 3. *Assume that the Hosios condition holds, $\beta = \eta$. Consider three job types 1, 2, and 3, and suppose that $\alpha_{jk} > 0$ only when $(j, k) = (1, 3)$. Suppose that $c_1(\cdot)$, $c_2(\cdot)$, and $c_3(\cdot)$ are constant-elasticity* *functions with the same elasticity. Then, the social planner creates relatively more vacancies of type-*1 *than of type-*2*, and fewer vacancies of type-*3 *than of type-*2*, compared with the decentralized equilibrium, i.e.,*

$$\frac{V_{1t}^{SP}}{V_1} > \frac{V_{2t}^{SP}}{V_2} > \frac{V_{3t}^{SP}}{V_3}.$$

Type-3 jobs entail a high opportunity cost because they destroy type-1 jobs when workers transition from type-1 to type-3. Thus, the social planner view their creation as excessive relative to type-2 jobs. At the same time, type-1 jobs generate a stepping stone value by facilitating job-to-job transitions toward more productive type-3 jobs, leading the social planner to create more type-1 jobs than type-2 jobs.

3.6 Preference Heterogeneity

In this section, I extend the baseline model by introducing idiosyncratic mobility shocks. This extension is motivated by the need to better align the model with empirical patterns of job-to-job transitions. As shown in Figure 3, these transitions exhibit substantial bilateral flows in the data. In contrast, the baseline model in Section 3 allows job-to-job transitions only from lower- to higher-surplus jobs. If we were to assume that workers are occasionally forced to move from higher- to lower-suplus jobs, such transitions would reduce total match surplus and thus be costly from the perspective of the social planner. Under such an assumption, the model would overstate the extent of excessive job creation among job types that primarily rely on poaching.

To address this issue, I assume that when a worker contacts a vacancy, a match-specific idiosyncratic shock ε is realized. After observing the realization of this shock, the worker decides optimally whether to switch jobs, ensuring that all job-to-job transitions are surplus-increasing, while still allowing for positive bilateral flows between any pair of job types. Idiosyncratic taste or amenity shocks are standard in quantitative search models, and their quantitative importance in explaining employment flows has been emphasized in recent studies (e.g., Faberman et al., 2022; Berger et al., 2024). For tractability, I impose two simplifying assumptions. First, both the worker and the firm derive flow utility that is proportional to the realized shock, with exogenous scaling factors. Specifically, a worker obtains flow utility $g_j^w \varepsilon$, while a firm obtains flow utility $g_j^f \varepsilon$, where

$$g_j^w = \beta \cdot (r + s_j), \quad g_j^f = (1 - \beta)(r + s_j + \tilde{f}_{j\cdot}), \tag{5}$$

and $\tilde{f}_{j\cdot}$ denotes the rate at which workers in type-j jobs move to other jobs in the decentralized equilibrium.[9] Although it is typically assumed that only the worker directly values the idiosyncratic shock, even in that case, the firm also benefits indirectly because a higher realization of the shock leads to a lower wage through Nash bargaining. This assumption, however, weakens the link between wages and productivity compared with Lemma 1. Second, I assume that the idiosyncratic shock accumulates over successive job-to-job transitions until the worker is exogenously separated. Specifically, when an employed worker with an existing shock realization ε meets a new vacancy, she draws an additional random component $\tilde{\varepsilon}$ from the distribution $\varphi_\varepsilon(\cdot)$, and, upon moving, her new match-specific shock becomes $\varepsilon' = \varepsilon + \tilde{\varepsilon}$. This assumption eliminates the need to track the distribution of shocks within each job type when characterizing job-to-job transitions (see Lemma 1), similar in spirit to Desmet et al. (2018) and Lamadon et al. (2024).

Let $E_j(\varepsilon)$ and $J_j(\varepsilon)$ denote, respectively, the values of a worker and a type-j matched job with a shock ε. Likewise, let $S_j(\varepsilon)$ and $w_j(\varepsilon)$ denote the match surplus and the wage. With idiosyncratic shocks, workers move probabilistically upon meeting a new job. A worker matched with a type-j job meets a type-k job at rate f_{jk}, but the effective job-to-job transition rate is given by

$$\tilde{f}_{jk}(\varepsilon) \equiv f_{jk} \cdot \mathrm{Pr}_{\tilde{\varepsilon}}[E_k(\varepsilon + \tilde{\varepsilon}) > E_j(\varepsilon)].$$

If the random component $\tilde{\varepsilon}$ is always zero, this expression collapses to the baseline case, i.e., $\tilde{f}_{jk} = f_{jk}\mathbb{1}_{E_k > E_j}$. Finally, let $\tilde{f}_{j\cdot}(\varepsilon) = \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{jk}(\varepsilon)$ denote the total rate at which the worker transitions to other jobs.

The following result characterizes the decentralized equilibrium of the extended model. The structure of the equilibrium remains closely aligned with that of the baseline model. Detailed derivations are provided in Appendix B.

Lemma 1. *The wage w_j and the job-to-job transition rate $\tilde{f}_{jk}$ are independent of ε. Moreover, the value functions and the match surplus are additively separable in ε:*

$$E_j(\varepsilon) = E_j + \beta\varepsilon, \;\; J_j(\varepsilon) = J_j + (1-\beta)\varepsilon, \;\; \text{and} \;\; S_j(\varepsilon) = S_j + \varepsilon,$$

[9] Importantly, the firm factor g_j^f is an exogenous parameter chosen for tractability. For example, in counterfactuals where vacancy creation differs from the baseline, it is held fixed rather than recomputed

where the match surplus satisfies

$$rS_j = A_j - b^{\ell_j} - s_j S_j + \sum_{k \in \mathcal{J}^{\ell_j}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}}\left[\{\beta(S_k + \tilde{\varepsilon}) - S_j\} \cdot \mathbb{1}_{S_k + \tilde{\varepsilon} > S_j}\right] - \beta \sum_{k \in \mathcal{J}^{\ell_j}} f_k\, \mathbb{E}_{\tilde{\varepsilon}}\left[(S_k + \tilde{\varepsilon})^+\right]. \quad (6)$$

Under the sharing rule (5), a unit increase in the realized ε raises the worker's and the firm's values by exactly β and $1-\beta$, respectively. This ensures that the shock affects only workers' mobility decisions, without influencing wages. The assumption that shocks accumulate over transitions further guarantees that, conditional on the current job type, the idiosyncratic shock does not introduce additional state dependence into future mobility decisions, making the job-to-job transition rate independent of the current shock ε. Without this assumption—i.e., if ε' were independently drawn from $\varphi_\varepsilon(\cdot)$—the model would feature richer dynamics, such as a declining hazard of job-to-job transitions with job tenure, but at the cost of substantially reduced tractability. The absence of such state dependence implies additive separability of the value functions and the match surplus.

The free-entry condition determines the equilibrium number of vacancies V_j:

$$c_j'(V_j) = \frac{1-\beta}{P^R}\left(q_j\, \mathbb{E}_{\tilde{\varepsilon}}\left[(S_j + \tilde{\varepsilon})^+\right] + \sum_{k \in \mathcal{J}^{\ell_j}} q_{kj}\, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_k}\left[(S_j + \varepsilon_k + \tilde{\varepsilon}) \cdot \mathbb{1}_{S_j + \tilde{\varepsilon} > S_k}\right]\right), \quad (7)$$

where ε_k follows the steady-state distribution of idiosyncratic shocks among type-k matches. The complete set of equilibrium conditions for E_j, J_j, $\tilde{f}_{jk}$, and w_j is provided in Appendix B.

Social Planner Problem. The social planner maximizes the discounted sum of output and flow utility from idiosyncratic shocks that workers and firms enjoy, taking the initial ($t = 0$) labor allocation and idiosyncratic shock distribution as given. In Appendix B, I formulate the social planner problem and derive the first-order conditions for optimality. The result gives

$$
\begin{aligned}
rS^{\mathrm{SP}}_{jt}(\varepsilon) &= A_j - b^{\ell_j} + (g^w_j + g^f_j)\cdot\varepsilon - s_j S^{\mathrm{SP}}_{jt}(\varepsilon) + \eta \sum_{k\,\in\,\mathcal{J}^{\ell_j}} f_{jkt}\,\mathbb{E}_{\tilde{\varepsilon}}[(S^{\mathrm{SP}}_{kt}(\varepsilon+\tilde{\varepsilon}) - S^{\mathrm{SP}}_{jt}(\varepsilon))^+] \\
&\quad - \eta \sum_{k\,\in\,\mathcal{J}^{\ell_j}} f_{kt}\,\mathbb{E}_{\tilde{\varepsilon}}[(S^{\mathrm{SP}}_{kt}(\tilde{\varepsilon}))^+] + \dot{S}^{\mathrm{SP}}_{jt}(\varepsilon) \\
&\quad + (1-\eta)\Bigg(\sum_{k\,\in\,\mathcal{J}^{\ell_j}} f_{jkt}\,\mathbb{E}_{\tilde{\varepsilon}}[(S^{\mathrm{SP}}_{kt}(\varepsilon+\tilde{\varepsilon}) - S^{\mathrm{SP}}_{jt}(\varepsilon))^+] - \sum_{k\,\in\,\mathcal{J}^{\ell_j}} f_{jkt}\,\mathbb{E}_{\tilde{\varepsilon},\varepsilon_{jt}}[(S^{\mathrm{SP}}_{kt}(\varepsilon_{jt}+\tilde{\varepsilon}) - S^{\mathrm{SP}}_{jt}(\varepsilon_{jt}))^+]\Bigg), \\
c'_j(V^{\mathrm{SP}}_{jt}) &\propto q_{jt}\,\mathbb{E}_{\tilde{\varepsilon}}[(S^{\mathrm{SP}}_{jt}(\tilde{\varepsilon}))^+] + \sum_{k\,\in\,\mathcal{J}^{\ell_j}} q_{kjt}\,\mathbb{E}_{\tilde{\varepsilon},\varepsilon_{kt}}[(S^{\mathrm{SP}}_{jt}(\varepsilon_{kt}+\tilde{\varepsilon}) - S^{\mathrm{SP}}_{kt}(\varepsilon_{kt}))^+].
\end{aligned}
$$

where ε_{kt} follows the distribution of idiosyncratic shocks among workers employed in type-k jobs at time t.

Compared to the equation (3), the planner's condition includes additional terms capturing the idiosyncratic utility shocks enjoyed by both workers and firms, while the overall structure in the first two lines remains nearly identical. The new terms only appear in the third line. These arise because matches with type-j jobs that differ in their realizations of ε share a common matching function (e.g., Acemoglu, 2001; Fukui and Mukoyama, 2025). Quantitatively, however, the magnitude of this effect is small and does not affect any of the subsequent results.[10]

4. Structural Estimation

4.1 Estimation Strategy

Functional forms. For each metropolitan statistical area (MSA), I define 15 job types by interacting three broad sectors with five productivity groups. To construct these productivity groups, I partition residualized wages within each sector into quintiles, such that $|\mathcal{A}(\ell, s)| = 5$. The analysis focuses on the 60 largest MSAs, as smaller regions are more prone to small-sample bias. The analysis is restricted to employment flows within each MSA.[11] Idiosyncratic shocks $\tilde{\varepsilon}$ are assumed to follow a logistic distribution with scale parameter ν; see Appendix D for details on how it simplifies the equilibrium expressions. The vacancy creation cost function

[10] This is because the entire term in the third line of the expression vanishes when S^{SP} is additively separable. It is approximately additive for the estimated model.

[11] The framework could be extended to incorporate interregional mobility if reliable cross-MSA migration data were available.

takes the constant-elasticity form

$$c_j(V_j) = \frac{\bar{c}_j}{1+\xi} \cdot V_j^{1+\xi},$$

where $\bar{c}_j$ captures job-type-specific cost shifters, and ξ denotes the elasticity of vacancy creation costs.

Externally set or calibrated parameters. I first discuss the parameters that are externally set: the discount rate ρ, the elasticity of the matching functions η, worker's bargaining power β, the vacancy creation cost elasticity ξ, separation rates s_j, the matching efficiencies of off-the-job search α_j, and region-specific unemployment benefit b^ℓ. The discount rate is set to $\rho = 0.004$, corresponding to an annual real interest rate of 5%. The matching elasticity takes the standard value in the literature, $\eta = 0.4$ (e.g., Petrongolo and Pissarides, 2001). The bargaining power of workers, β, is calibrated using empirical estimates of the elasticity of wages with respect to firm rents. These estimates vary across studies, ranging from nearly zero to about one-half, depending on data sources, measurement of rents, and empirical strategies (e.g., Lamadon et al., 2022; Card et al., 2014; Hagedorn and Manovskii, 2008; Kline et al., 2019). Comprehensive reviews are provided by Card et al. (2018) and Alan (2011). I set $\beta = 0.1$, which implies an elasticity of wages of about 0.15, a moderate value within the empirical range and consistent with Assumption 1.[12] The vacancy creation cost elasticity, ξ, is set to 2, which lies near the midpoint of the range of estimates reported in the literature (e.g., Kaas and Kircher, 2015; Bilal et al., 2022; Berger et al., 2024).[13] The type-specific separation rate s_j is calibrated to match the average monthly transition probability from employment to unemployment. The matching efficiency of off-the-job search α_j is normalized to one, consistent with the random search assumption commonly adopted in search models. Unlike in those models, however, the vacancy contact rate here varies with job-type-specific market tightness, which is endogenously determined in equilibrium. These externally set parameters are summarized in the top panel of Table 1.

Internally estimated parameters. In the second step, I estimate the remaining parameters internally using structural estimation combined with model inversion. These parameters include the dispersion of idiosyncratic shocks ν, the matching efficiencies of on-the-job search α_{kj}, the type-specific productivity levels A_j, and the vacancy creation cost shifters c_j. For a given value of ν, the remaining parameters are obtained by inverting the model. Although all parameters are jointly determined in equilibrium, it is useful to describe the moments

[12] This value is also close to recent estimates that exploit firm-level (rather than market-level) and permanent (rather than transitory) shocks, the source of variation most relevant for identifying this parameter in this paper.

[13] These studies estimate the elasticity at the firm level rather than the market level. In this paper, however, a firm corresponds to a job, so the parameter captures the elasticity of creating a particular type of job.

Table 1: Parameter Values

Parameter	Value	Target
Discount rate ρ	0.00096	Interest rate
Worker's bargaining power β	0.1	Literature
Matching elasticity η	0.4	Literature
Vacancy creation cost elasticity ξ	2	Literature
Type-specific separation rate $\{s_j\}_j$	Figure A.1a	U2E rates
Matching efficiency (Off-the-Job) $\{\alpha_j\}_j$	1	Normalization
Scale parameter of idiosyncratic shock ν	0.04	Share of downward transitions
Matching efficiency (On-the-Job) $\{\alpha_{jk}\}_{j,k}$	Figure 4	E2E rates (Figure 3)
Type-specific productivity $\{A_j\}_j$	Figure 5a	Wages (Figure A.1c)
Vacancy creation cost $\{c_j\}_j$	Figure 5b	Free-entry condition

Notes: The top panel shows parameters that are externally calibrated, and the bottom panel shows parameters that are internally calibrated.

that primarily identify each parameter. The matching efficiencies of on-the-job search are identified from observed job-to-job transition rates. Type-specific productivity levels are inferred from wage data, and the free-entry condition pins down the vacancy creation cost shifters. Finally, the dispersion parameter ν is calibrated so that the average matching efficiency for upward transitions equals that for downward transitions across adjacent job types. Intuitively, frequent transitions of workers toward higher- to lower-productivity jobs can result either from high volatility of idiosyncratic shocks or from high contact rates. Because these two forces are not separately identified, I impose symmetry in matching efficiencies between upward and downward transitions and choose ν to match the observed share of downward job-to-job transitions. This approach is analogous to targeting the share of downward transitions in a random-search model (e.g., Berger et al., 2024). The internally estimated parameters are summarized in the bottom panel of Table 1.

4.2 Estimation Results

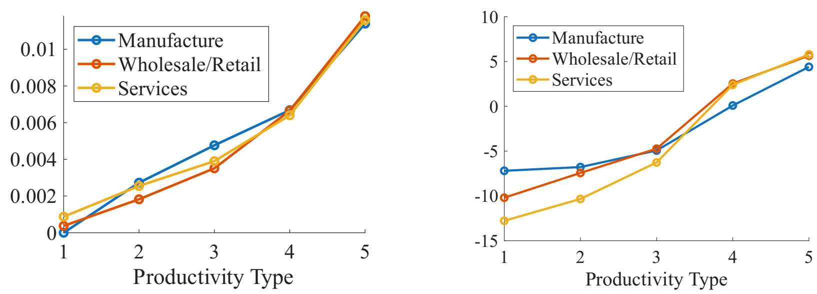
In Figure 4, I plot the estimated matching efficiencies for on-the-job search, α_{jk}. Since these parameters are estimated separately for each MSA, I report their averages across MSAs. The estimated matching efficiencies exhibit substantial heterogeneity across job types. The ratio of the third to the first quartile is about 12. Efficiencies tend to be higher around the diagonal, indicating that workers are more likely to move to jobs with similar productivity. Notably, the estimated efficiencies differ from the observed job-to-job transition probabilities. For example, the relative efficiency of moving to lower-productivity jobs is much higher than the corresponding observed transition probability. This difference arises because workers endogenously decide whether to accept an offer: they accept jobs with lower productivity only when idiosyncratic shocks are sufficiently favorable. In particular, the value of jobs with the highest productivity is much higher than other jobs, and hence the estimated matching efficiencies out of the top productivity jobs are substantially higher than average. Although we report average matching efficiencies across job types, we also see substantial spatial variation in matching efficiencies. The ratio of the third to the first quartile of the average MSA-level matching efficiencies is about 6.

The estimated scale parameter of idiosyncratic shock is 0.04. The standard deviation of job-specific amenity shocks is 1.1 times that of wages offered to unemployed workers, after scaling by expected job duration for comparability. Although direct comparison is difficult due to model differences, this magnitude is broadly consistent with the literature. For instance, Hall and Mueller (2018) report that the ratio of the standard deviation of the non-wage component of job values (0.34) to that of offered wages (0.24), conditional on worker productivity, is about 1.4. Thus, the relative importance of amenities in my estimated model is slightly smaller but of comparable order.

Figure 5a displays the estimated productivities across the 15 job types, averaged across MSAs. As expected, productivity increases monotonically within each sector. The dispersion in productivity is much larger than that of wages, as only part of productivity differences are passed through to wages. Given the relatively small worker bargaining power, productivity variation is an order of magnitude greater than wage variation. Finally, Figure 5b displays the log of estimated vacancy creation cost parameters, averaged across MSAs. The estimated costs rise steeply with job productivity, indicating that creating high-productivity jobs is considerably more expensive. This pattern reflects the fact that such jobs generate greater surplus yet are created less frequently than lower-productivity jobs, as reflected in the small share of hires into high-productivity positions from unemployment. Average vacancy creation cost shifters are broadly comparable across sectors.

Figure 4. Estimation Results: Matching Efficiency of On-the-Job Search

Notes: This figure plots the estimated matching efficiencies of on-the-job search α_{jk} within each sector, averaged across MSAs.



(a) Productivity (b) Vacancy Creation Cost Shifters

Figure 5. Estimation Results

Notes: The left panel plots the estimated productivity for each type of job, averaged across MSAs. I normalize the minimum value of A_j to zero since it is not separately identified from the mean of idiosyncratic shocks. The right panel plots the log of estimated vacancy creation parameters $\log c_j$.

5. Misallocation of Job Creation

In this section, I use the calibrated model to quantify the extent of misallocation in vacancy creation by comparing the decentralized equilibrium allocation with the optimal allocation. I then decompose the relative contribution of the opportunity cost effect and the stepping stone effect to this misallocation.

I present the results along two dimensions to highlight clear and systematic patterns of misallocation. First, by aggregating across sectors and productivity levels within each metropolitan statistical area (MSA), I examine spatial patterns of misallocation. Second, by aggregating across MSAs within each sector and productivity quintile, I study misallocation across different jobs. The pattern of misallocation across different

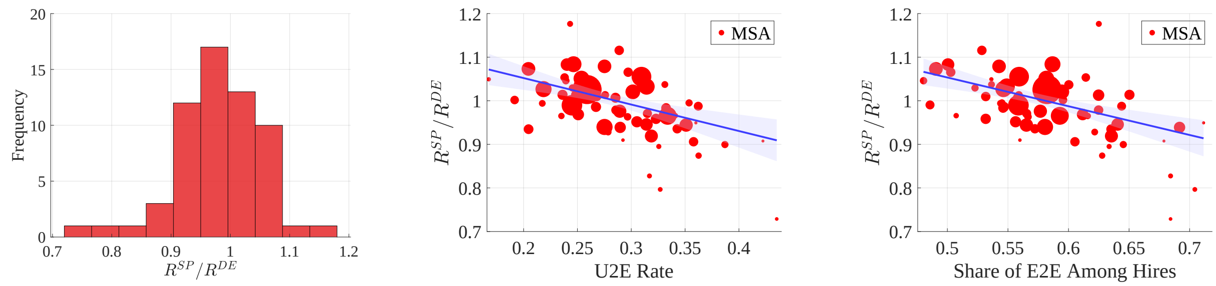


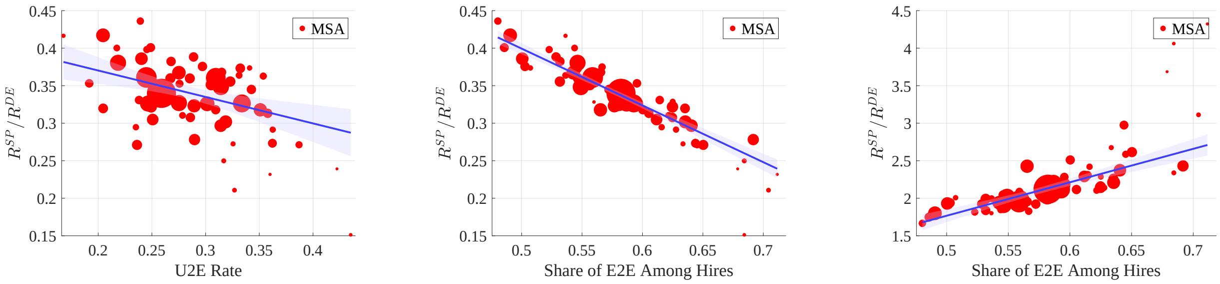
Figure 6. Misallocation of Job Creation: MSA

Notes: The left panel shows the histogram of the ratio of vacancy-creation resources used in the planner's problem to those in the decentralized equilibrium across MSAs. The middle and right panels show scatter plots of this ratio against the U2E rate and the share of E2E transitions among hires, respectively. Each circle represents an MSA, with its size proportional to the size of the MSA's labor force. The blue line shows the fitted regression line, and the shaded area represents the 95 percent confidence band.

jobs within MSAs closely mirrors the aggregate pattern, and similarly, misallocation across MSAs for a given sector and productivity quintile parallels the aggregate pattern across different jobs.

Across MSAs. The histogram in Panel (a) of Figure 6 shows the extent of misallocation in vacancy creation across MSAs, aggregated over jobs within each MSAs. For each MSA, I compute the ratio of vacancy creation resources that the social planner would allocate compared to the decentralized equilibrium $\frac{R^{SP}}{R^{DE}}$. Values above one indicate the planner would expand vacancy creation relative to the decentralized equilibrium of a certain MSA, while values below one denote MSAs where vacancy creation is excessive. Note that the total supply of resources is fixed, so the weighted average of these ratios must equal one. The social planner substantially reallocates vacancy creation resources across local labor markets, with values ranging from approximately -27 percent to $+18$ percent relative to the decentralized equilibrium. To examine which MSA characteristics drive this reallocation, Panels (b) and (c) plot the ratio $\frac{R^{SP}}{R^{DE}}$ against the unemployment-to-employment (U2E) rate and the share of employment-to-employment (E2E) transitions among hires (hereafter, the E2E share), respectively. These figures imply that the planner allocates more vacancy-creation resources to labor markets with lower U2E rates and lower E2E shares. When the ratio $\frac{R^{SP}}{R^{DE}}$ is regressed jointly on the U2E rate and the E2E share, both coefficients are highly significant. Controlling for the other variable, a one-standard-deviation increase in the E2E share (in the U2E rate, respectively) is associated with a 0.42 standard-deviation (0.42 standard-deviation, respectively) decrease in $\frac{R^{SP}}{R^{DE}}$.

To better understand the sources of this misallocation, I decompose the contributions of the opportunity cost and stepping stone effects. Panels (a) and (b) of Figure 7 show the results from a modified planner



(a) Opportunity Cost Effect (b) Opportunity Cost Effect (c) Stepping Stone Effect

Figure 7. Misallocation of Job Creation: Decomposition

Notes: Panels (a) and (b) show the results of the modified planner problem that corrects opportunity cost effect, while Panel (c) show the results of the modified planner problem correcting stepping stone effect. Each panel shows a scatter plot of the ratio of vacancy-creation resources used in the planner's problem to those in the decentralized equilibrium against either U2E rate or the share of E2E transitions among hires. Each circle represents an MSA, with its size proportional to the size of the MSA's labor force. The blue line shows the fitted regression line, and the shaded area represents the 95 percent confidence band.

problem that corrects only for the externality associated with the opportunity cost effect. In this modified problem, I relax the aggregate resource constraint and instead allow the planner to purchase any amount of the vacancy-posting resource at the decentralized equilibrium price, P^R. This assumption enables the planner to adjust total resource usage endogenously—spending less when vacancy creation is, on average, socially less valuable and more when it is more valuable. The resulting level effect is negative, which is expected to be stronger in MSAs with high U2E rates and larger E2E shares. In such areas, many of the additional hires would have been employed elsewhere even without new vacancies, implying a low social value of vacancy creation. Consistent with this interpretation, the graphs show that both the U2E rate and the E2E share have negative effects on the ratio $\frac{R^{SP}}{R^{DE}}$. Panel (c) of Figure 7 focuses on the stepping stone effect by solving a modified planner problem that corrects only this externality. This effect leads to insufficient vacancy creation in MSAs with a high share of E2E transitions. While the opportunity cost and stepping stone effects work in opposite directions with respect to the E2E shares, the right panel of Figure 6 indicates that the opportunity cost effect dominates quantitatively. As a result, the planner would create more vacancies in local labor markets with either lower U2E rates or lower E2E shares.

Implications. As documented in Section 2, the U2E rate and the E2E share are positively correlated across MSAs: heated markets exhibit both high U2E and E2E rates, whereas frozen markets display low values of

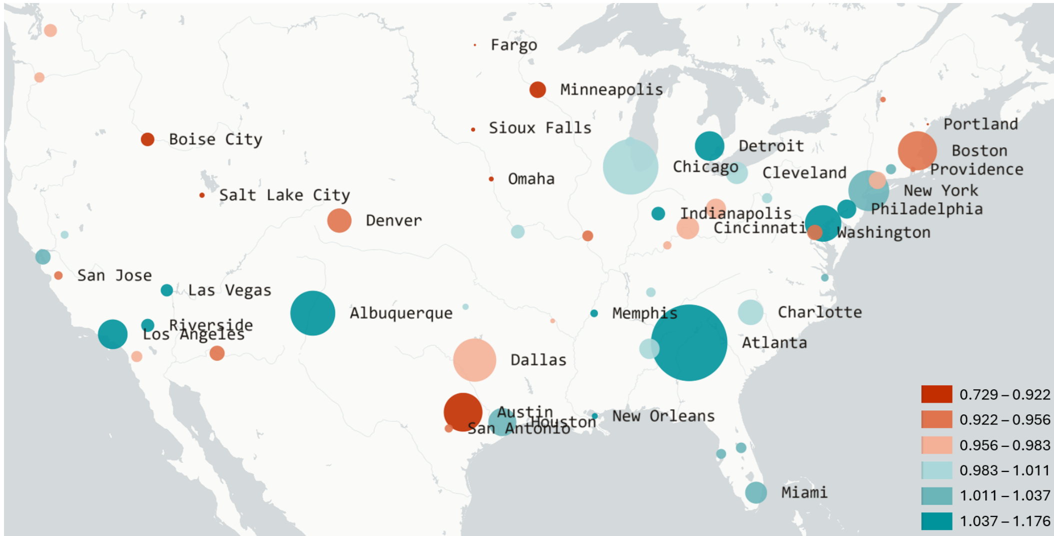


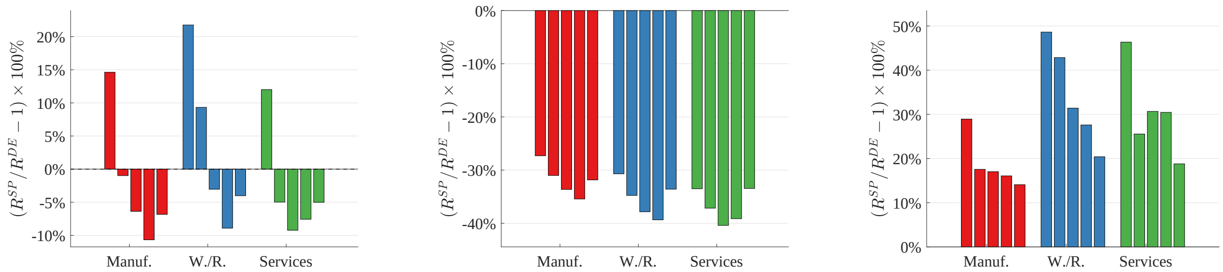
Figure 8. Misallocation of Job Creation: MSA

Notes: The map shows the ratio of vacancy-creation resources used in the planner's problem to those in the decentralized equilibrium across MSAs. Each circle represents an MSA, with its size proportional to the size of the MSA's labor force.

both. Therefore, these spatial disparities operate in the same direction, jointly shaping the planner's spatial reallocation of vacancies.

Interestingly, although the social planner is assumed to have a linear utility function that assigns equal weight to all MSAs—thus being entirely indifferent to distributional concerns—the optimal allocation effectively acts to reduce disparities across local labor markets. This result suggests that efficiency considerations alone can provide a rationale for the place-based policies observed in practice: reallocating vacancy creation toward less dynamic regions not only narrows spatial inequality but also raises aggregate efficiency.

Figure 8 illustrates how the social planner would reallocate vacancy creation resources across major U.S. metropolitan areas. The pattern reveals a clear regional pattern. Many Rust Belt cities—such as Cleveland, Detroit, and Indianapolis—feature frozen markets and display ratios above one, indicating that the planner would expand vacancy creation relative to the decentralized equilibrium. These areas experienced slower recoveries following the Great Recession and continued to face slack labor markets, where additional vacancies would generate substantial social gains. In contrast, high-growth metropolitan areas—including Minneapolis, Denver, Boston, San Jose, Seattle, Salt Lake City, Dallas, and Austin—exhibit ratios well below one, implying the planner would like to reallocate vacancies away from these areas. These cities maintained strong labor markets between 2013 and 2018, characterized by rapid job-to-job transitions,



(a) Total Effect (b) Opportunity Cost Effect (c) Stepping Stone Effect

Figure 9. Misallocation of Job Creation: Sector and Productivity Level

Notes: Panel (a) plots the ratio of vacancy-creation resources used in the planner's problem to those in the decentralized equilibrium across job characteristics (sector, productivity). Panels (b) and (c) decompose the total externality into the opportunity cost effect and the stepping stone effect, respectively.

low unemployment, and diversified, innovation-driven industries. From the planner's perspective, vacancy creation in such dynamic labor markets is relatively excessive.

Within MSA. Panel (a) of Figure 9 summarizes the extent of misallocation in vacancy creation across sectors and productivity levels, averaged over MSAs. Misallocation is measured by the ratio of vacancy-creation resources used by the planner to those in the decentralized equilibrium, $R^{\text{SP}}/R^{\text{DE}}$. The figure shows that the planner shifts vacancy creation away from moderately productive jobs toward less productive ones, leaving high-productivity jobs largely unaffected. To interpret this pattern, Panels (b) and (c) decompose the total effect in panel (a) into the opportunity cost effect and the stepping stone effect, respectively.

In Panel (b), I plot the reallocation implied by the modified social planner problem that corrects only for the externality arising from the opportunity cost effect. The level effect is negative: because the opportunity cost effect arises mainly from firms' failure to internalize (potential) loss of other firms, the decentralized equilibrium exhibits excessive vacancy creation across all job types. This distortion is particularly severe among moderately productive jobs, whose hires largely come from job-to-job transitions as shown in the right panel of Figure 2. In contrast, low-productivity jobs mainly hire from the unemployment pool, while high-productivity jobs—although relying even more on poaching—generate large increases in surplus relative to the surplus of the poached jobs. Thus, excessive vacancy creation is less pronounced for both low- and high-productivity jobs.

In Panel (c), I solve the modified social planner problem that corrects only for the externality from the stepping stone effect. The level effect is positive: because poaching jobs not being fully rewarded for the

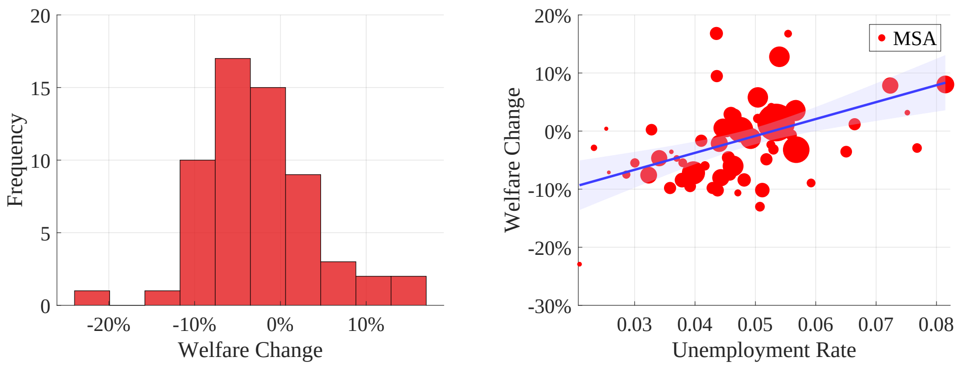


Figure 10. Welfare Change: MSA

Notes: The left panel plots the distribution of welfare changes across MSAs under the optimal vacancy creation. The right panel relates these welfare changes to the unemployment rate. Each circle represents an MSA, with its size proportional to the size of the MSA's labor force. The blue line shows the fitted regression line, and the shaded area represents the 95 percent confidence band.

social gain generated by job-to-job transitions, the decentralized equilibrium features insufficient vacancy creation for all job types. The pattern of undercreation closely mirrors the pattern of poached outflow shown in the middle panel of Figure 2, as the higher the rate at which a job loses workers to poaching, the stronger its stepping stone effect. Therefore, the social planner that corrects this externality creates disproportionately more vacancies among low-productivity jobs.

Panel (a) reflects the combined impact of these two forces. The planner increases the vacancy creation of low-productivity jobs, mainly due to the stepping stone effect, reinforced by the opportunity cost effect, while reducing vacancies for moderately productive jobs, primarily driven by the opportunity cost effect.

Summing up the results so far, improving efficiency requires reallocating vacancy creation simultaneously across space and job types—expanding vacancy creation in frozen markets, particularly among low-productivigy jobs, and reducing it in heated markets, especially among moderately productive ones.

Effects on welfare. Under the planner's optimal allocation of vacancies across MSAs, aggregate welfare—defined as the discounted sum of aggregate output and the utility derived from idiosyncratic shocks—increases by 1.06%. This aggregate gain, however, masks substantial spatial heterogeneity in welfare changes. As shown in the histogram in Figure 10, welfare changes across MSAs range from about -20% to $+20\%$. Although more than half of the MSAs experience welfare losses, those with positive welfare gains tend to be larger, so that when aggregating across MSAs, the overall effect on welfare is positive. These welfare changes systematically vary across MSAs. as illustrated in the right panel of Figure 10, MSAs with higher unemployment rates benefit more, as the planner reallocates job creation toward these weak labor markets.

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Appendix

A. Data

CPS. I restrict the sample to labor force participants aged 21 to 63 and exclude workers who do not earn salaries, e.g., self-employed or unpaid family workers. I limit the analysis to spells in manufacturing, wholesale/retail, and services. Transitions involving other sectors, such as EE flows from agriculture to manufacturing or unemployment spells terminating in non-target sectors, are dropped. If an unemployed individual subsequently reports employment in a non-target sector within the four-month CPS window, I exclude that observation from the analysis. I prepare wage data by first dropping the top and bottom 5% of outliers and residualizing log wages on year, age, gender, race, and education. I then construct ten wage-based groups. Combined with three sectors, this yields 30 job types and an unemployment state. In Section 5, I construct five wage-based groups, yielding 15 job types.

The CPS rotates respondents over four consecutive months, followed by an eight-month gap, then reinterviews for four months, corresponding to the basic monthly CPS. Wage data (usual weekly earnings) are reported only during the fourth and eighth interviews, corresponding to the outgoing rotation groups. Since the analysis requires detailed MSA-sector-wage group statistics, I pool data from 2013 to 2018 to ensure sufficient sample size. I compute employment shares, average wages, and job-finding probabilities at the MSA-sector-wage group level. The latter is constructed by combining sectoral transition rates from the basic monthly CPS with relative transition probabilities across wage groups from the outgoing rotation sample. For job-finding probabilities across wage groups, I pool the outgoing ration sample from 2001 onward to mitigate potential small-sample bias. To construct job-to-job transitions, I first compute the overall MSA-level E2E rate. Job-to-job transitions across wage groups cannot be directly computed because the CPS does not contain consistent wage information across consecutive months. To address this limitation, I use the NLSY, which I convert into a pseudo–monthly panel by selecting observations from the first week of each month. Specifically, I scale the national E2E rates across to account for differences in employment shares across MSAs, ensuring consistency with the aggregate matching technology and matching the overall MSA-level E2E rate. Similarly, since the CPS does not allow identification of separation rates by wage group, I estimate MSA-sector–level separations from CPS and apply relative within-sector separation rates across wage groups derived from the NLSY. Finally, each MSA is scaled by the size of its labor force.

NLSY. I focus on male respondents, as female labor market behavior differs substantially for this cohort, with notably lower labor force participation rates. I primarily follow the data-cleaning procedures of Lise and Postel-Vinay (2020), organizing the data into a weekly panel. As in the CPS, I focus on observations in manufacturing, wholesale/retail, services, or unemployment. To align with the infinite-horizon framework of the model, sample exits, presumably driven by retirement, are coded as transitions into unemployment. Before grouping jobs based on wages, I drop the top and bottom 1% outliers and then residualize wages on gender, race, education, month, the interaction of year and education, the interaction of race and year. I group jobs into wage deciles based on these residualized wages, which yields 30 job types and an unemployment state. For regional analysis in Section 5, I divide jobs into wage quintiles.

B. Generalized Model

When an unemployed worker meets a vacancy of type j, the pair draws a match-specific idiosyncratic shock ε. The worker derives flow utility $g_j^w \cdot \varepsilon$, while the firm derives flow utility $g_j^f \cdot \varepsilon$, where

$$g_j^w = \beta \cdot (r + s_j), \quad g_j^f = (1-\beta)(r + s_j + \tilde{f}_{j\cdot}),$$

and $\tilde{f}_{j\cdot}$ denotes the rate at which workers in type-j jobs move to other jobs. I conjecture that this rate does not depend on ε, and verify this later in Lemma A.1.

If the match is formed, the worker's value is $E_{jt}(\varepsilon)$ and the firm's value is $J_{jt}(\varepsilon)$. Nash bargaining implies

$$E_{jt}(\varepsilon) - U_t^{\ell(j)} = \beta S_{jt}(\varepsilon) \text{ and } J_{jt}(\varepsilon) = (1-\beta) S_{jt}(\varepsilon),$$

where $S_{jt}(\varepsilon) \equiv E_{jt}(\varepsilon) + J_{jt}(\varepsilon) - U_t^{\ell(j)}$ is the joint match surplus. A match is formed if and only if $S_{jt}(\varepsilon) > 0$.

Consider on-the-job search where a worker evaluates offers from two job types, j and k, with match-specific idiosyncratic shocks ε_j and ε_k, respectively, where one firm is the current employer and the other provides a new offer.

Without loss of generality, suppose that $S_{jt}(\varepsilon_j) > S_{kt}(\varepsilon_k)$. Let $\omega \in [0,1]$ denote the degree of offer matching, which determines how the value of the outside option enters the bargaining outcome.

If the worker forms a match with firm j, her value relative to unemployment is

$$E_{kjt}(\varepsilon_k,\varepsilon_j) - U_t^{\ell(j)} \;=\; \omega S_{kt}(\varepsilon_k) + \beta\big(S_{jt}(\varepsilon_j) - \omega S_{kt}(\varepsilon_k)\big),$$

while firm j obtains

$$J_{kjt}(\varepsilon_k,\varepsilon_j) \;=\; (1-\beta)\big(S_{jt}(\varepsilon_j) - \omega S_{kt}(\varepsilon_k)\big).$$

Alternatively, if the worker were to choose firm k, the hypothetical continuation values would be

$$\begin{aligned}\tilde{E}_{jkt}(\varepsilon_j,\varepsilon_k) - U_t^{\ell(k)} &= \beta S_{kt}(\varepsilon_k) + \omega(1-\beta)S_{kt}(\varepsilon_k),\\ \tilde{J}_{jkt}(\varepsilon_j,\varepsilon_k) &= (1-\beta)S_{kt}(\varepsilon_k) - \omega(1-\beta)S_{kt}(\varepsilon_k).\end{aligned}$$

Since $S_{jt}(\varepsilon_j) > S_{kt}(\varepsilon_k)$, these allocations are never observed in equilibrium.

In this setup, the value functions for the worker and the firm can be expressed as:

$$\begin{aligned}
rU_t^{\ell} &= b^{\ell} + \sum_{j\in\mathcal{J}^{\ell}} f_{jt}\,\mathbb{E}_{\tilde{\varepsilon}}[(E_{jt}(\tilde{\varepsilon}) - U_t^{\ell})^{+}] + \dot{U}_t^{\ell}\\
rE_{jt}(\varepsilon) &= w_{jt}(\varepsilon) + g_j^{\omega}\cdot\varepsilon + s_j(U_t^{\ell(j)} - E_{jt}(\varepsilon))\\
&\quad + \sum_{k\in\mathcal{J}^{\ell(j)}} f_{jkt}\,\mathbb{E}_{\tilde{\varepsilon}}\left[\begin{array}{c}(E_{jkt}(\varepsilon,\varepsilon+\tilde{\varepsilon}) - E_{jt}(\varepsilon))\cdot\mathbb{1}_{S_{kt}(\varepsilon+\tilde{\varepsilon})>S_{jt}(\varepsilon)}\\ +(E_{kjt}(\varepsilon+\tilde{\varepsilon},\varepsilon) - E_{jt}(\varepsilon))\cdot\mathbb{1}_{S_{kt}(\varepsilon+\tilde{\varepsilon})<S_{jt}(\varepsilon)}\end{array}\right] + \dot{E}_{jt}(\varepsilon)\\
rJ_{jt}(\varepsilon) &= A_j - w_{jt}(\varepsilon) + g_j^{f}\cdot\varepsilon - s_j J_{jt}(\varepsilon)\\
&\quad + \sum_{k\in\mathcal{J}^{\ell(j)}} f_{jkt}\,\mathbb{E}_{\varepsilon'}\left[\begin{array}{c}-J_{jt}(\varepsilon)\cdot\mathbb{1}_{S_{kt}(\varepsilon')>S_{jt}(\varepsilon)}\\ +(J_{kjt}(\varepsilon+\varepsilon',\varepsilon) - J_{jt}(\varepsilon))\cdot\mathbb{1}_{S_{kt}(\varepsilon+\varepsilon')<S_{jt}(\varepsilon)}\end{array}\right] + \dot{J}_{jt}(\varepsilon)\\
rV_{jt}^{v} &= -P_j + q_{jt}\,\mathbb{E}_{\tilde{\varepsilon}}[(J_{jt}(\tilde{\varepsilon}))^{+}] + \sum_{k\in\mathcal{J}^{\ell(j)}} q_{kjt}\,\mathbb{E}_{\tilde{\varepsilon},\varepsilon_k}[J_{kjt}(\varepsilon_k,\varepsilon_k+\tilde{\varepsilon})\cdot\mathbb{1}_{S_{kt}(\varepsilon_k)<S_{jt}(\varepsilon_k+\tilde{\varepsilon})}]
\end{aligned} \tag{A.1}$$

$$rE_{kjt}(\varepsilon',\varepsilon) = w_{kjt}(\varepsilon',\varepsilon) + g_j^{\omega}\cdot\varepsilon + s_j(U_t^{\ell(j)} - E_{kjt}(\varepsilon',\varepsilon))$$

$$+ \sum_{i \in \mathcal{J}^{\ell(j)}} f_{jit}\, \mathbb{E}_{\tilde{\varepsilon}} \begin{bmatrix} (E_{ji}(\varepsilon, \varepsilon + \tilde{\varepsilon}) - E_{kjt}(\varepsilon', \varepsilon)) \cdot \mathbb{1}_{S_{it}(\varepsilon+\tilde{\varepsilon}) > S_{jt}(\varepsilon)} \\ +(E_{ijt}(\varepsilon + \tilde{\varepsilon}, \varepsilon) - E_{kjt}(\varepsilon', \varepsilon)) \cdot \mathbb{1}_{S_{it}(\varepsilon+\tilde{\varepsilon}) < S_{jt}(\varepsilon)} \end{bmatrix} + \dot{E}_{kjt}(\varepsilon', \varepsilon)$$

$$rJ_{kjt}(\varepsilon', \varepsilon) = A_j - w_{kjt}(\varepsilon', \varepsilon) + g_j^f \cdot \varepsilon - s_j J_{kjt}(\varepsilon', \varepsilon)$$

$$+ \sum_{i \in \mathcal{J}^{\ell(j)}} f_{jit}\, \mathbb{E}_{\tilde{\varepsilon}} \begin{bmatrix} -J_{kjt}(\varepsilon', \varepsilon) \cdot \mathbb{1}_{S_{it}(\varepsilon+\tilde{\varepsilon}) > S_{jt}(\varepsilon)} \\ +(J_{ijt}(\varepsilon + \tilde{\varepsilon}, \varepsilon) - J_{kjt}(\varepsilon', \varepsilon)) \cdot \mathbb{1}_{S_{it}(\varepsilon+\tilde{\varepsilon}) < S_{jt}(\varepsilon)} \end{bmatrix} + \dot{J}_{kjt}(\varepsilon', \varepsilon)$$

Thus, the joint surplus satisfies

$$\begin{aligned} rS_{jt}(\varepsilon) &:= r(E_{kjt}(\varepsilon', \varepsilon) - U_t^{\ell(j)} + J_{kjt}(\varepsilon', \varepsilon)) = r(E_{jt}(\varepsilon) - U_t^{\ell(j)} + J_{jt}(\varepsilon)) \\ &= A_j - b^{\ell(j)} + g_j \cdot \varepsilon - s_j S_{jt}(\varepsilon) \\ &\quad + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jkt}\, \mathbb{E}_{\tilde{\varepsilon}} \left[\{(\omega - 1) S_{jt}(\varepsilon) + \beta(S_{kt}(\varepsilon + \tilde{\varepsilon}) - \omega S_{jt}(\varepsilon))\} \cdot \mathbb{1}_{S_{kt}(\varepsilon+\tilde{\varepsilon}) > S_{jt}(\varepsilon)} \right] \\ &\quad - \beta \sum_{k \in \mathcal{J}^{\ell(j)}} f_{kt}\, \mathbb{E}_{\tilde{\varepsilon}}[(S_{kt}(\tilde{\varepsilon}))^+] + \dot{S}_{jt}(\varepsilon), \end{aligned}$$

where $g_j = g_j^w + g_j^f$, and the free-entry condition can be rewritten as

$$c_j'(V_{jt}) = \frac{1-\beta}{P^R} \cdot \left(q_{jt}\, \mathbb{E}_{\tilde{\varepsilon}}[(S_{jt}(\tilde{\varepsilon}))^+] + \sum_{k \in \mathcal{J}^{\ell(j)}} q_{kjt}\, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_k}[(S_{jt}(\varepsilon_k + \tilde{\varepsilon}) - \omega S_{kt}(\varepsilon_k)) \mathbb{1}_{S_{kt}(\varepsilon_k) < S_{jt}(\varepsilon_k + \tilde{\varepsilon})}] \right).$$

B.1 Special Case: the Steady-State Decentralized Equilibrium

The model nests the decentralized economy by setting $\omega = 0$. Together with the steady-state assumption, the previous equations simplify to

$$\begin{aligned} rU^\ell &= b^\ell + \beta \sum_{j \in \mathcal{J}^\ell} f_j\, \mathbb{E}_{\tilde{\varepsilon}}[(E_j(\tilde{\varepsilon}) - U^\ell)^+] \\ rE_j(\varepsilon) &= w_j(\varepsilon) + g_j^w \cdot \varepsilon + s_j(U_t^{\ell(j)} - E_j(\varepsilon)) + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[(E_k(\varepsilon + \tilde{\varepsilon}) - E_j(\varepsilon))^+ \right] \\ rJ_j(\varepsilon) &= A_j - w_j(\varepsilon) + g_j^f \cdot \varepsilon - s_j J_j(\varepsilon) - \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[J_j(\varepsilon) \cdot \mathbb{1}_{S_k(\varepsilon+\tilde{\varepsilon}) > S_j(\varepsilon)} \right] \\ &= A_j - w_j(\varepsilon) + g_j^f(\varepsilon) - (s_j + \tilde{f}_{j\cdot}(\varepsilon)) J_j(\varepsilon) \end{aligned}$$

$$
\begin{aligned}
rS_j(\varepsilon) &= A_j - b^{\ell(j)} + g_j \cdot \varepsilon - s_j S_j(\varepsilon) \\
&\quad + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[\{\beta S_k(\varepsilon + \tilde{\varepsilon}) - S_j(\varepsilon))\} \cdot \mathbb{1}_{S_k(\varepsilon+\tilde{\varepsilon}) > S_j(\varepsilon)} \right] - \beta \sum_{k \in \mathcal{J}^{\ell(j)}} f_k\, \mathbb{E}_{\tilde{\varepsilon}}[(S_k(\tilde{\varepsilon}))^+] \\
&= A_j - b^{\ell(j)} + g_j \cdot \varepsilon - (s_j + \tilde{f}_{j\cdot}(\varepsilon)) S_j(\varepsilon) \\
&\quad + \beta \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[S_k(\varepsilon + \tilde{\varepsilon}) \cdot \mathbb{1}_{S_k(\varepsilon+\tilde{\varepsilon}) > S_j(\varepsilon)} \right] - \beta \sum_{k \in \mathcal{J}^{\ell(j)}} f_k\, \mathbb{E}_{\tilde{\varepsilon}}[(S_k(\tilde{\varepsilon}))^+] \\
c_j'(V_j) &= (1-\beta) \left(q_j\, \mathbb{E}_{\tilde{\varepsilon}}[(S_j(\tilde{\varepsilon}))^+] + \sum_{k \in \mathcal{J}^{\ell(j)}} q_{kj}\, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_k}[S_j(\varepsilon_k + \tilde{\varepsilon}) \cdot \mathbb{1}_{S_k(\varepsilon_k) < S_j(\varepsilon_k + \tilde{\varepsilon})}] \right),
\end{aligned}
$$

where $\tilde{f}_{j\cdot}(\varepsilon) \equiv \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk} \Pr_{\tilde{\varepsilon}} \left(S_k(\varepsilon + \tilde{\varepsilon}) > S_j(\varepsilon) \right)$.

Under equation (5), we can prove the following result:

Lemma A.1. *The wage $w_j(\varepsilon)$ and the rate $\tilde{f}_{j\cdot}(\varepsilon)$ are independent of ε, (hence, I omit the argument and write them simply as w_j and $\tilde{f}_{j\cdot}$) and the value functions are additively separable in the realization of idiosyncratic shocks,*

$$
E_j(\varepsilon) = E_j + \beta\varepsilon, \;\; J_j(\varepsilon) = J_j + (1-\beta)\varepsilon, \;\; \text{and} \;\; S_j(\varepsilon) = S_j + \varepsilon,
$$

where

$$
\begin{aligned}
w_j &= A_j - (1-\beta)(r + s_j + \tilde{f}_{j\cdot}) S_j, && \text{(A.2)} \\
\tilde{f}_{j\cdot} &= \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk} \Pr_{\tilde{\varepsilon}} \left(S_k + \tilde{\varepsilon} > S_j \right), \\
rU^\ell &= b^\ell + \sum_{j \in \mathcal{J}^\ell} f_j\, \mathbb{E}_{\tilde{\varepsilon}}[(E_j + \beta\tilde{\varepsilon} - U^\ell)^+] \\
rE_j &= w_j + s_j(U^{\ell(j)} - E_j) + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[(E_k + \beta\tilde{\varepsilon} - E_j)^+ \right] \\
rJ_j &= A_j - w_j - (s_j + \tilde{f}_{j\cdot}) J_j \\
rS_j &= A_j - b^{\ell(j)} - s_j S_j + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jk}\, \mathbb{E}_{\tilde{\varepsilon}} \left[\{\beta(S_k + \tilde{\varepsilon}) - S_j\} \cdot \mathbb{1}_{S_k + \tilde{\varepsilon} > S_j} \right] - \beta \sum_{k \in \mathcal{J}^{\ell(j)}} f_k\, \mathbb{E}_{\tilde{\varepsilon}}[(S_k + \tilde{\varepsilon})^+]. && \text{(A.3)}
\end{aligned}
$$

Finally, the free-entry condition can be rewritten as

$$c_j'(V_j) = \frac{1-\beta}{P^R} \cdot \left(q_j \, \mathbb{E}_{\tilde{\varepsilon}}[(S_j + \tilde{\varepsilon})^+] + \sum_{k \in \mathcal{J}^{\ell(j)}} q_{kj} \, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_k}[(S_j + \varepsilon_k + \tilde{\varepsilon}) \cdot \mathbb{1}_{S_k < S_j + \tilde{\varepsilon}}] \right). \tag{A.4}$$

B.2 Social Planner Problem

This section derives the optimality conditions for the social planner's problem in the extended model with idiosyncratic shocks. The planner maximizes the discounted sum of aggregate output and the flow utilities obtained by workers and firms from idiosyncratic match-specific shocks. Formally, the planner solves:

$$\begin{aligned}
\max_{\{V_{jt}\}_{j \in \mathcal{J}, t \geq 0}} \quad & \int_0^\infty e^{-rt} \sum_{j \in \mathcal{J}, \varepsilon} M_{jt}(\varepsilon) \cdot (A_j - b^{\ell(j)} + (g_j^w + g_j^f) \cdot \varepsilon) \, \mathrm{d}t \\
\text{s.t.} \quad & \dot{M}_{jt}(\varepsilon) = -s_j M_{jt}(\varepsilon) - \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{jk} M_{jt}^{\eta-1} V_{kt}^{1-\eta} M_{jt}(\varepsilon) \Pr_{\tilde{\varepsilon}} \left(S_{kt}(\varepsilon + \tilde{\varepsilon}) > S_{jt}(\varepsilon) \right) \\
& + \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{kj} M_{kt}^{\eta-1} V_{jt}^{1-\eta} \sum_{\varepsilon'} M_{kt}(\varepsilon') f_\varepsilon(\varepsilon - \varepsilon') \cdot \mathbb{1}_{S_{jt}(\varepsilon) > S_{kt}(\varepsilon')} \\
& + \alpha_j \left(L^{\ell(j)} - \sum_{i \in \mathcal{J}^{\ell(j)}} M_{it} \right)^\eta V_{jt}^{1-\eta} f_\varepsilon(\varepsilon) \cdot \mathbb{1}_{S_{jt}(\varepsilon) > 0}, \\
& \sum_{j \in \mathcal{J}} c_j(V_{jt}) = \sum_{j \in \mathcal{J}} c_j(V_{jt}^{\text{DE}}) \\
\text{where} \quad & M_{jt} = \sum_{\varepsilon} M_{jt}(\varepsilon), \ \tilde{\varepsilon} \sim f_\varepsilon(\cdot).
\end{aligned}$$

To solve this problem, we set up the current-value Hamiltonian and derive the associated first-order conditions.

$$\begin{aligned}
rS_{jt}(\varepsilon) = \; & A_j - b^{\ell(j)} + (g_j^w + g_j^f) \cdot \varepsilon - s_j S_{jt}(\varepsilon) \\
& - \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{jk} \sum_{\varepsilon'} (\eta - 1) M_{jt}^{\eta-2} V_{kt}^{1-\eta} \sum_{\varepsilon'} M_{jt}(\varepsilon') S_{jt}(\varepsilon') \Pr_{\tilde{\varepsilon}}(S_{kt}(\varepsilon' + \tilde{\varepsilon}) > S_{jt}(\varepsilon')) \\
& - \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{jk} M_{jt}^{\eta-1} V_{kt}^{1-\eta} S_{jt}(\varepsilon) \Pr_{\tilde{\varepsilon}}(S_{kt}(\varepsilon + \tilde{\varepsilon}) > S_{jt}(\varepsilon)) \\
& + \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{jk} (\eta - 1) M_{jt}^{\eta-2} V_{kt}^{1-\eta} \sum_{\varepsilon'} M_{jt}(\varepsilon') \sum_{\varepsilon''} f_\varepsilon(\varepsilon'') S_{kt}(\varepsilon + \varepsilon'') \cdot \mathbb{1}_{S_{kt}(\varepsilon' + \varepsilon'') > S_{jt}(\varepsilon')} \\
& + \sum_{k \in \mathcal{J}^{\ell(j)}} \alpha_{jk} M_{jt}^{\eta-1} V_{kt}^{1-\eta} \sum_{\varepsilon'} f_\varepsilon(\varepsilon') S_{kt}(\varepsilon + \varepsilon') \mathbb{1}_{S_{kt}(\varepsilon + \varepsilon') > S_{jt}(\varepsilon)}
\end{aligned}$$

$$
\begin{aligned}
&\quad - \eta \sum_{k \in \mathcal{J}^{\ell(j)}} f_{kt} \sum_{\varepsilon'} f_\varepsilon(\varepsilon') S_{kt}(\varepsilon') \cdot \mathbb{1}_{S_{kt}(\varepsilon')>0} + \dot{S}_{jt}(\varepsilon) \\
&= A_j - b^{\ell(j)} + (g_j^w + g_j^f) \cdot \varepsilon - s_j S_{jt}(\varepsilon) \\
&\quad - (1-\eta) \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jkt}\, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_{jt}}[(S_{kt}(\varepsilon_{jt} + \tilde{\varepsilon}) - S_{jt}(\varepsilon_{jt}))^+] \\
&\quad + \sum_{k \in \mathcal{J}^{\ell(j)}} f_{jkt}\, \mathbb{E}_{\tilde{\varepsilon}}[(S_{kt}(\varepsilon + \tilde{\varepsilon}) - S_{jt}(\varepsilon))^+] \\
&\quad - \eta \sum_{k \in \mathcal{J}^{\ell(j)}} f_{kt}\, \mathbb{E}_{\tilde{\varepsilon}}[(S_{kt}(\tilde{\varepsilon}))^+] + \dot{S}_{jt}(\varepsilon),
\end{aligned}
$$

$$
c_j'(V_{jt}) \propto q_{jt}\, \mathbb{E}_{\tilde{\varepsilon}}[(S_{jt}(\tilde{\varepsilon}))^+] + \sum_{k \in \mathcal{J}^{\ell(j)}} q_{kjt}\, \mathbb{E}_{\tilde{\varepsilon}, \varepsilon_{kt}}[(S_{jt}(\varepsilon_{kt} + \tilde{\varepsilon}) - S_{kt}(\varepsilon_{kt}))^+].
$$

where ε_{kt} is drawn from the distribution of idiosyncratic shocks among workers employed in type-k jobs at time t.

C. Limit Case with $\eta \to 1$

C.1 Without job-to-job transitions

Suppose that job-to-job transitions are not possible and $s_j = s$ for all $j \in \mathcal{J}$. Then, equations (1′) and (3′) become

$$
\begin{aligned}
RS_j &= \hat{A}_j - \beta \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k S_k, \\
RS_j^{\mathrm{SP}} &= \hat{A}_j - \eta \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k S_k^{\mathrm{SP}},
\end{aligned}
\tag{A.5}
$$

where $R = r + s$ and $\hat{A}_j = A_j - b^{\ell_j}$. This can be solved in terms of $\hat{A}$'s as in the following lemma:

Lemma A.2. *S_j and S_j^{SP} are given by*

$$
\begin{aligned}
RS_j &= \hat{A}_j - \frac{\beta \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k \hat{A}_k}{R + \beta \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k}, \\
RS_j^{SP} &= \hat{A}_j - \frac{\eta \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k \hat{A}_k}{R + \eta \sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_k}.
\end{aligned}
$$

Proof. We first multiply both sides of (A.5) by $\tilde{f}_j$:

$$R\tilde{f}_j S_j = \tilde{f}_j \hat{A}_j - \beta \tilde{f}_j \sum_{k\in\mathcal{J}^{\ell_j}} \tilde{f}_k S_k.$$

Next, summing over all j gives:

$$R\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k S_k = \sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k \hat{A}_k - \beta \sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k \cdot \sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k S_k.$$

Rearranging the term, we can isolate $\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k S_k$:

$$\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k S_k = \frac{1}{R+\beta\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k} \sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k \hat{A}_k.$$

Finally, substituting this result back into the original equation yields:

$$RS_j = \hat{A}_j - \frac{\beta\sum_{k\in\mathcal{J}^{\ell_j}} \tilde{f}_k \hat{A}_k}{R+\beta\sum_{k\in\mathcal{J}^{\ell_j}} \tilde{f}_k}.$$

Similarly, by following the same steps and replacing β with η, we obtain the expression for S_j^{SP}. □

Define $\tilde{f}^{\ell} = \sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k$, and $(\overline{A-b})^{\ell} = \frac{\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k\cdot(A_k-b^{\ell})}{\sum_{k\in\mathcal{J}^{\ell}} \tilde{f}_k}$. Then,

$$S_j = \mathcal{S}(\beta, R, \tilde{f}^{\ell_j}, \hat{A}_j, (\overline{A-b})^{\ell_j}) \;\text{ and }\; S_j^{\mathrm{SP}} = \mathcal{S}(\eta, R, \tilde{f}^{\ell_j}, \hat{A}_j, (\overline{A-b})^{\ell_j}),$$

where

$$S(\kappa, R, \tilde{f}^{\ell}, \hat{A}, (\overline{A-b})^{\ell}) = \frac{1}{R}\left(\hat{A} - \frac{\kappa f^{\ell}\cdot(\overline{A-b})^{\ell}}{R+\kappa\cdot f^{\ell}}\right) = \frac{1}{R}\frac{R\hat{A}+\kappa\cdot f^{\ell}\cdot(\hat{A}-(\overline{A-b})^{\ell})}{R+\kappa\cdot f^{\ell}}.$$

Holding fixed $R, \tilde{f}^{\ell}, \hat{A}$, we can show that the mapping $(\kappa, \frac{(\overline{A-b})^{\ell}}{\hat{A}}) \mapsto S(\kappa, R, \tilde{f}^{\ell}, \hat{A}, (\overline{A-b})^{\ell})$, is log-submodular. This means that, if $\frac{(\overline{A-b})^{\ell}}{A_j - b^{\ell}} < \frac{(\overline{A-b})^{\ell'}}{A_{j'} - b^{\ell'}}$, we have

$$\frac{\mathcal{S}(\eta, R, \tilde{f}^{\ell}, \hat{A}_j, (\overline{A-b})^{\ell})}{\mathcal{S}(\eta, R, \tilde{f}^{\ell'}, \hat{A}_{j'}, (\overline{A-b})^{\ell'})} > \frac{\mathcal{S}(\beta, R, \tilde{f}^{\ell}, \hat{A}_j, (\overline{A-b})^{\ell})}{\mathcal{S}(\beta, R, \tilde{f}^{\ell'}, \hat{A}_{j'}, (\overline{A-b})^{\ell'})},$$

or equivalently,

$$\frac{S_j^{\mathrm{SP}}}{S_{j'}^{\mathrm{SP}}} > \frac{S_j}{S_{j'}}. \tag{A.6}$$

Under the constant-elasticity assumption, vacancy creation cost functions are of the form $c_j(V_j) = \frac{c_j}{\xi+1} \cdot V_j^{\xi+1}$. Thus, we have

$$c_j V_j^{\xi+1} \propto \alpha_j U^{\ell_j} S_j,$$

implying

$$\frac{V_j^{\mathrm{SP}}}{V_{j'}^{\mathrm{SP}}} = \left(\frac{S_j^{\mathrm{SP}}}{S_{j'}^{\mathrm{SP}}}\right)^{\frac{1}{\xi}} \quad \text{and} \quad \frac{V_j}{V_{j'}} = \left(\frac{S_j}{S_{j'}}\right)^{\frac{1}{\xi}}. \tag{A.7}$$

Combining (A.6) and (A.7), we immediately have $\frac{V_j^{\mathrm{SP}}}{V_{j'}^{\mathrm{SP}}} > \frac{V_j}{V_{j'}}$.

Similarly, when $\frac{(\overline{A-b})^{\ell}}{A_j - b^{\ell}} = \frac{(\overline{A-b})^{\ell'}}{A_{j'} - b^{\ell'}} = 1$, the mapping $(\kappa, \tilde{f}^{\ell}) \mapsto S(\kappa, R, \tilde{f}^{\ell}, \hat{A}, (\overline{A-b})^{\ell})$, is log-submodular. This means that, if $\tilde{f}^{\ell} < \tilde{f}^{\ell'}$, we have

$$\frac{\mathcal{S}(\eta, R, \tilde{f}^{\ell}, \hat{A}_j, (\overline{A-b})^{\ell})}{\mathcal{S}(\eta, R, \tilde{f}^{\ell'}, \hat{A}_{j'}, (\overline{A-b})^{\ell'})} > \frac{\mathcal{S}(\beta, R, \tilde{f}^{\ell}, \hat{A}_j, (\overline{A-b})^{\ell})}{\mathcal{S}(\beta, R, \tilde{f}^{\ell'}, \hat{A}_{j'}, (\overline{A-b})^{\ell'})}.$$

This again leads to $\frac{V_j^{\mathrm{SP}}}{V_{j'}^{\mathrm{SP}}} > \frac{V_j}{V_{j'}}$. This proves Proposition 2.

C.2 With job-to-job transitions

Assume that $\alpha_{jk} > 0$ only for $(j,k) = (1,3)$ and $\beta = \eta$. Write

$$\begin{aligned}
RS_1(\omega) &= \hat{A}_1 + f_{13}(\eta(S_3(\omega) - S_1(\omega)) - (1-\eta)(1-\omega)S_1(\omega)) - \eta(f_1 S_1(\omega) + f_2 S_2(\omega) + f_3 S_3(\omega)) \\
RS_2(\omega) &= \hat{A}_2 - \eta(f_1 S_1(\omega) + f_2 S_2(\omega) + f_3 S_3(\omega)) \\
RS_3(\omega) &= \hat{A}_3 - \eta(f_1 S_1(\omega) + f_2 S_2(\omega) + f_3 S_3(\omega))
\end{aligned}$$

$$c_1 V_1(\omega)^{\xi+1} \propto \alpha_1 U S_1(\omega)$$

$$c_2 V_2(\omega)^{\xi+1} \propto \alpha_2 U S_2(\omega)$$

$$c_3 V_3(\omega)^{\xi+1} \propto \alpha_3 U S_3(\omega) + \alpha_{13} M_1(S_3(\omega) - \omega S_1(\omega)).$$

Then, the decentralized equilibrium is characterized by $S_j = S_j(0)$ and $V_j = V_j(0)$ for $j = 1,2,3$, and the socially optimal allocation is characterized by $S_j^{\mathrm{SP}} = S_j(1)$ and $V_j^{\mathrm{SP}} = V_j(1)$ for $j = 1,2,3$. Consider an infinitesimal increase in ω, $\mathrm{d}\omega$, then

$$
\begin{aligned}
(1+\xi)\frac{\mathrm{d}V_1(\omega)}{V_1(\omega)} &= \frac{\mathrm{d}S_1(\omega)}{S_1(\omega)} + \Omega \\
(1+\xi)\frac{\mathrm{d}V_2(\omega)}{V_2(\omega)} &= \frac{\mathrm{d}S_2(\omega)}{S_2(\omega)} + \Omega \\
(1+\xi)\frac{\mathrm{d}V_3(\omega)}{V_3(\omega)} &= (1-\pi(\omega))\cdot\frac{\mathrm{d}S_3(\omega)}{S_3(\omega)} + \pi(\omega)\cdot\frac{\mathrm{d}(S_3(\omega)-\omega S_1(\omega))}{S_3(\omega)-\omega S_1(\omega)} + \Omega,
\end{aligned}
$$

for some constant Ω, where $\pi(\omega) = \frac{\alpha_{13}M_1(S_3(\omega)-\omega S_1(\omega))}{\alpha_3 U S_3(\omega)+\alpha_{13}M_1(S_3(\omega)-\omega S_1(\omega))}$. We have

$$
\underbrace{\begin{pmatrix} R + f_{13}(1-\eta)(1-\omega) + \eta(f_1+f_{13}) & \eta f_2 & \eta(f_3 - f_{13}) \\ \eta f_1 & \eta f_2 + R & \eta f_3 \\ \eta f_1 & \eta f_2 & \eta f_3 + R \end{pmatrix}}_{\equiv Y(\omega)} \begin{pmatrix} S_1(\omega) \\ S_2(\omega) \\ S_3(\omega) \end{pmatrix} = \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2 \\ \hat{A}_3 \end{pmatrix},
$$

which can be inverted to be solved for the surplus:

$$
\begin{pmatrix} S_1(\omega) \\ S_2(\omega) \\ S_3(\omega) \end{pmatrix} = \frac{1}{\det Y(\omega)} \underbrace{\begin{pmatrix} X_{11} & X_{12} & X_{13} \\ X_{21} & X_{22} & X_{23} \\ X_{31} & X_{32} & X_{33} \end{pmatrix}}_{\equiv X(\omega)} \begin{pmatrix} \hat{A}_1 \\ \hat{A}_2 \\ \hat{A}_3 \end{pmatrix},
$$

where $X(\omega)$ is the transpose of the matrix of cofactors of $Y(\omega)$:

$$
X = \begin{pmatrix} \eta R(f_2+f_3) + R^2 & -\eta f_2(R+\eta f_{13}) & \eta(Rf_{13} - Rf_3 + \eta f_{13} f_2) \\ -\eta R f_1 & \begin{pmatrix} R(R+\eta(f_1+f_{13}+f_3)) + \eta^2 f_{13}(f_1+f_3) \\ +f_{13}(R+\eta f_3)(1-\eta)(1-\omega) \end{pmatrix} & -\eta(f_3(R+\eta f_{13}) + \eta f_1 f_{13} + f_{13} f_3 (1-\eta)(1-\omega)) \\ -\eta R f_1 & -\eta f_2(R+\eta f_{13} + f_{13}(1-\eta)(1-\omega)) & \begin{pmatrix} R(R+\eta(f_1+f_2+f_{13})) + \eta^2 f_{13} f_2 \\ +(R+\eta f_2) f_{13}(1-\eta)(1-\omega) \end{pmatrix} \end{pmatrix}.
$$

Thus,

$$
\begin{aligned}
\frac{\mathrm{d}V_1(\omega)}{V_1(\omega)} - \frac{\mathrm{d}V_2(\omega)}{V_2(\omega)} &\overset{\mathrm{sgn}}{=} \frac{\mathrm{d}S_1(\omega)}{S_1(\omega)} - \frac{\mathrm{d}S_2(\omega)}{S_2(\omega)} \\
&\overset{\mathrm{sgn}}{=} (R+\eta f_3)\hat{A}_2 - \eta f_3 \hat{A}_3,
\end{aligned}
$$

which is positive by the following lemma:

Lemma A.3. *If $S_2 \geq 0$, then We have $(R + \eta f_3)\hat{A}_2 > \eta f_3 \hat{A}_3$.*

Proof. We have

$$
\begin{aligned}
(R + \eta f_3)\hat{A}_2 &\geq (R + \eta f_3)\eta(f_1 S_1(0) + f_2 S_2(0) + f_3 S_3(0)) \\
&> \eta f_3(R S_3(0) + \eta(f_1 S_1(0) + f_2 S_2(0) + f_3 S_3(0))) \\
&= \eta f_3 \hat{A}_3.
\end{aligned}
$$

The first inequality follows from $S_2(0) = S_2 \geq 0$, and the second inequality is obtained by expanding and comparing the terms. $\square$

Therefore, we have $\frac{\mathrm{d}\log(V_1(\omega)/V_2(\omega))}{\mathrm{d}\omega} > 0$. Integrating this from $\omega = 0$ to $\omega = 1$, we obtain

$$
\frac{V_1^{\mathrm{SP}}}{V_2^{\mathrm{SP}}} = \frac{V_1(1)}{V_2(1)} > \frac{V_1(0)}{V_2(0)} = \frac{V_1}{V_2}.
$$

We also have

$$
\begin{aligned}
\frac{\mathrm{d}V_2(\omega)}{V_2(\omega)} - \frac{\mathrm{d}V_3(\omega)}{V_3(\omega)} &\overset{\mathrm{sgn}}{=} \frac{\mathrm{d}S_2(\omega)}{S_2(\omega)} - (1 - \pi(\omega)) \cdot \frac{\mathrm{d}S_3(\omega)}{S_3(\omega)} - \pi(\omega) \cdot \frac{\mathrm{d}(S_3(\omega) - \omega S_1(\omega))}{S_3(\omega) - \omega S_1(\omega)} \\
&= \frac{\mathrm{d}S_2(\omega)}{S_2(\omega)} - (1 - \pi(\omega)) \cdot \frac{\mathrm{d}S_3(\omega)}{S_3(\omega)} - \pi(\omega) \cdot \frac{S_3(\omega)}{S_3(\omega) - \omega S_1(\omega)} \frac{\mathrm{d}S_3(\omega)}{S_3(\omega)} \\
&\quad + \pi(\omega) \frac{\omega S_1(\omega)}{S_3(\omega) - \omega S_1(\omega)} \left(\frac{\mathrm{d}S_1(\omega)}{S_1(\omega)} + \frac{\mathrm{d}\omega}{\omega} \right)
\end{aligned}
$$

As $\eta \to 1$, we have $\mathrm{d}S_j(\omega) \to 0$; hence, the effect of $\mathrm{d}\omega$ dominates, and this value is positive. Accordingly, by the same line of reasoning as before, we can show that:

$$
\frac{V_2^{\mathrm{SP}}}{V_3^{\mathrm{SP}}} > \frac{V_2}{V_3}.
$$

D. Computation

D.1 Equilibrium Conditions under the Distributional Assumption

In this section, we impose the distributional assumption that

$$\tilde{\varepsilon} \sim \text{Logistic}(0, \nu)$$

whose cumulative distribution function is given by $\Pr(\tilde{\varepsilon} \le x) = \frac{1}{1+e^{-x/\nu}}$, or equivalently,

$$\tilde{\varepsilon} \overset{d}{=} \mu_k - \mu_j$$

where $\mu_k, \mu_j \overset{iid}{\sim} \text{Gumbel}(0, \nu)$, whose cumulative distribution function is given by $\Pr(\mu \le x) = \exp(-e^{-x/\nu})$.

Under this assumption, the following results can be derived

$$\begin{aligned}
\pi_{jkt} \equiv \Pr_{\tilde{\varepsilon}}(S_{kt} + \tilde{\varepsilon} > S_{jt}) &= \frac{e^{\frac{1}{\nu}S_{kt}}}{e^{\frac{1}{\nu}S_{kt}} + e^{\frac{1}{\nu}S_{jt}}} = 1 - \pi_{kjt}, \\
\pi_{ujt} \equiv \Pr_{\tilde{\varepsilon}}(S_{jt} + \tilde{\varepsilon} > 0) &= \frac{e^{\frac{1}{\nu}S_{jt}}}{e^{\frac{1}{\nu}S_{jt}} + 1}, \\
\mathbb{E}_{\tilde{\varepsilon}}\left[(S_{kt} + \tilde{\varepsilon} - S_{jt})^+\right] &= \nu \log\left(e^{\frac{1}{\nu}(S_{kt} - S_{jt})} + 1\right), \\
\mathbb{E}_{\tilde{\varepsilon}}[\tilde{\varepsilon} \cdot \mathbb{1}_{S_{kt} + \tilde{\varepsilon} > S_{jt}}] &= \nu \log\left(e^{\frac{1}{\nu}S_{kt}} + e^{\frac{1}{\nu}S_{jt}}\right) - \pi_{jkt}S_{kt} - \pi_{kjt}S_{jt}.
\end{aligned}$$

Proof.

$$\begin{aligned}
\mathbb{E}_{\tilde{\varepsilon}}\left[(S_{kt} + \tilde{\varepsilon} - S_{jt})^+\right] &= \mathbb{E}\left[(S_{kt} + \mu_k - S_{jt} - \mu_j)^+\right] = \mathbb{E}\left[\max\{S_{kt} + \mu_k, S_{jt} + \mu_j\} - (S_{jt} + \mu_j)\right] \\
&= \nu \log\left(e^{\frac{1}{\nu}S_{kt}} + e^{\frac{1}{\nu}S_{jt}}\right) - S_{jt} = \nu \log\left(e^{\frac{1}{\nu}(S_{kt} - S_{jt})} + 1\right) \\
\mathbb{E}_{\tilde{\varepsilon}}[\tilde{\varepsilon} \cdot \mathbb{1}_{S_{kt} + \tilde{\varepsilon} > S_{jt}}] &= \nu \log\left(e^{\frac{1}{\nu}(S_{kt} - S_{jt})} + 1\right) - \pi_{jkt} \cdot (S_{kt} - S_{jt}) \\
&= \nu \log\left(e^{\frac{1}{\nu}S_{kt}} + e^{\frac{1}{\nu}S_{jt}}\right) - \pi_{jkt}S_{kt} - \pi_{kjt}S_{jt}.
\end{aligned}$$

□

Using these results, the equilibrium conditions (A.3)–(A.4) simplify to

$$rS_{jt} = A_j - b^{\ell_j} - s_jS_{jt} + \sum_{k \in \mathcal{J}^{\ell_j}} f_{jkt}\left(\beta\nu \log\left(e^{\frac{1}{\nu}(S_{kt} - S_{jt})} + 1\right) - (1-\beta)S_{jt}\pi_{jkt}\right)$$

$$
\begin{aligned}
&- \beta \sum_{k \in \mathcal{J}^{\ell_j}} f_{kt} \nu \log\Big(e^{\frac{1}{\nu} S_{kt}} + 1\Big) + \dot{S}_{jt}, \\
w_{jt} &= A_j - (1-\beta)(r + s_j + \tilde{f}_{j \cdot t}) S_{jt} + (1-\beta)\dot{S}_{jt}, \\
c_j'(V_{jt}) &= (1-\beta)\left(q_{jt} \nu \log\Big(e^{\frac{1}{\nu} S_{jt}} + 1\Big) + \sum_{k \in \mathcal{J}^{\ell_j}} q_{kjt} \Big\{ (\mathbb{E}_{\varepsilon_{kt}}[\varepsilon_{kt}] + S_{kt}) \cdot \pi_{kj} + \nu \log\Big(e^{\frac{1}{\nu}(S_{jt} - S_{kt})} + 1\Big) \Big\} \right).
\end{aligned}
$$

Substituting the second equation into the first, we obtain

$$
(r + s_j) S_{jt} = \beta^{-1}(w_{jt} - b^{\ell_j}) + \sum_{k \in \mathcal{J}^{\ell_j}} f_{jkt} \nu \log\Big(e^{\frac{1}{\nu}(S_{kt} - S_{jt})} + 1\Big) - \sum_{k \in \mathcal{J}^{\ell_j}} f_{kt} \nu \log\Big(e^{\frac{1}{\nu} S_{kt}} + 1\Big) + \dot{S}_{jt}.
$$

We can characterize the dynamics of M_{jt} and $\bar{\varepsilon}_{jt} \equiv \mathbb{E}_{\varepsilon_{jt}}[\varepsilon_{jt}]$:

$$
\begin{aligned}
M_{j,t+\mathrm{d}t} &= U_t^{\ell_j} \tilde{f}_{jt}\, \mathrm{d}t + (1 - (s_j + \tilde{f}_{j \cdot t})\, \mathrm{d}t) M_{jt} + \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kjt}\, \mathrm{d}t M_{kt}, \\
M_{j,t+\mathrm{d}t} \cdot \bar{\varepsilon}_{j,t+\mathrm{d}t} &= U_t^{\ell_j} \tilde{f}_{jt}\, \mathrm{d}t \cdot \bar{\varepsilon}_{ujt} + (1 - (s_j + \tilde{f}_{j \cdot t})\, \mathrm{d}t) M_{jt} \cdot \bar{\varepsilon}_{jt} + \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kjt}\, \mathrm{d}t M_{kt} \cdot (\bar{\varepsilon}_{kt} + \bar{\varepsilon}_{kjt}),
\end{aligned}
$$

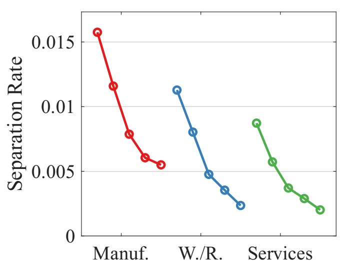
where

$$
\begin{aligned}
\bar{\varepsilon}_{ujt} &\equiv \mathbb{E}[\tilde{\varepsilon} | S_{jt} + \tilde{\varepsilon} > 0] = \Big(\nu \log\Big(e^{\frac{1}{\nu} S_{jt}} + 1\Big) - \pi_{ujt} S_{jt}\Big) / \pi_{ujt} \\
\bar{\varepsilon}_{kjt} &\equiv \mathbb{E}[\tilde{\varepsilon} | S_{jt} + \tilde{\varepsilon} > S_{kt}] = \Big(\nu \log\Big(e^{\frac{1}{\nu} S_{jt}} + e^{\frac{1}{\nu} S_{kt}}\Big) - \pi_{kjt} S_{jt} - \pi_{jkt} S_{kt}\Big) / \pi_{kjt}.
\end{aligned}
$$

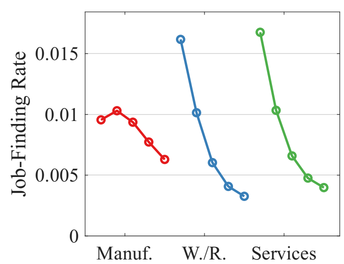
Steady State. Steady state requires

$$
\begin{aligned}
0 &= U^{\ell_j} \tilde{f}_j - (s_j + \tilde{f}_{j\cdot}) M_j + \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kj} M_k \\
0 &= U^{\ell_j} \tilde{f}_j \bar{\varepsilon}_{uj} - (s_j + \tilde{f}_{j\cdot}) M_j \bar{\varepsilon}_j + \textstyle\sum_{k \in \mathcal{J}^{\ell_j}} \tilde{f}_{kj} M_k \cdot (\bar{\varepsilon}_k + \bar{\varepsilon}_{kj}).
\end{aligned}
$$

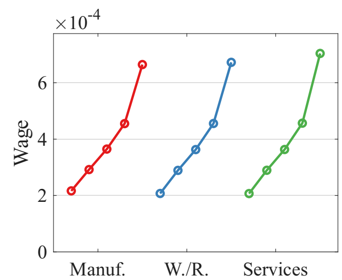
E. Additional Figures



(a) Separation Rate



(b) Job Finding Rates



(c) Wages

Figure A.1. Targeted Moments

Data source: CPS, NLSY. I use the CPS (2013–2018) to compute separation and job-finding moments at the sector–MSA level. To obtain within–sector–MSA heterogeneity, I supplement these estimates with the NLSY for relative separation rates and with the pooled CPS (2001–2018) for relative job-finding rates across jobs.