Sectoral Shocks and Labor Market Dynamics: A Sufficient Statistics Approach*

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Abstract

In this paper, we develop a sufficient statistics approach to evaluate the impact of sectoral shocks on labor market dynamics and welfare. Within a broad class of dynamic discrete choice models that allows for arbitrary persistent heterogeneity across workers, we show that knowledge of steady-state worker flows across sectors over different time horizons is sufficient to construct counterfactual predictions on labor reallocation and welfare changes, up to a first-order approximation. We also establish analytically that assuming away persistent worker heterogeneity, a common practice in existing literature, necessarily leads to overestimation of steady-state worker flows, resulting in systematic biases in counterfactual predictions. As an illustration of our sufficient statistics approach, we revisit the consequences of the rise of import competition from China. Using US panel data to measure steady-state worker flows prior to the shock, we conclude that the labor reallocation away from manufacturing is significantly slower, and the negative welfare effects on manufacturing workers are much more severe than those predicted by earlier models that abstract from persistent worker heterogeneity.

Keywords: Sectoral Shocks, Labor Market Dynamics, Sufficient Statistics, Self-Selection, Worker Heterogeneity, Dynamic Discrete Choice.

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1. Introduction

Labor markets in the United States, as well as in many other countries, have been subject to a variety of shocks, from globalization to the rise of automation, oil price shocks, and the Covid-19 pandemic. Although these shocks differ in many ways, they all have one thing in common: their effects tend to be highly asymmetric across sectors, potentially creating both winners and losers. How much do the winners gain and the losers lose? Can workers exposed to a negative shock in one sector avoid, or at least mitigate, its adverse consequences by moving to another sector? And if so, what determines the extent of this reallocation and the time it takes?

The goal of this paper is to shed light on these questions. The premise of our analysis is that both workers' exposure to shocks and their subsequent sectoral mobility depend on their comparative advantage across sectors. If comparative advantage is weak or highly transient, we expect frequent sector changes of workers, resulting in small welfare losses or even gains for negatively exposed workers. If instead comparative advantage is strong and persistent, we expect many workers to remain stuck in the negatively affected sector in the long run, leading to more severe negative welfare consequences. Intuitively, these differences could be revealed by panel data on worker flows across sectors, i.e., workers' propensity to move over different time horizons. If so, one might be able to use such data as sufficient statistics for evaluating the impact of sectoral shocks on labor market dynamics and welfare. In this paper, we formalize this general intuition and demonstrates, both theoretically and empirically, the importance of persistent comparative advantage.

We focus on a broad class of dynamic discrete choice models that allow for arbitrary persistent heterogeneity across workers. At each point in time, workers decide in which sector to work and are subject to sector-switching costs and transient idiosyncratic shocks drawn from an extreme value distribution, as in the canonical dynamic discrete choice framework (e.g., Artuç, Chaudhuri, and McLaren, 2010). However, workers may have time-invariant differences in productivity (or, more generally, in the utility they derive from being in a particular sector) and in the sector-switching costs. We impose no restriction on these persistent differences, in line with the general Roy model (e.g., Heckman and Honore, 1990). Within this general environment, we establish two theoretical results.

First, we provide a novel sufficient statistics approach that yields valid counterfactual predictions under arbitrary worker heterogeneity. This approach requires only two inputs: knowledge of steady-state sectoral worker flows over different time horizons and the dispersion of the idiosyncratic shocks. We show that this information is sufficient to construct counterfactual changes in welfare and sectoral employment in response to sectoral shocks, up to a first-order

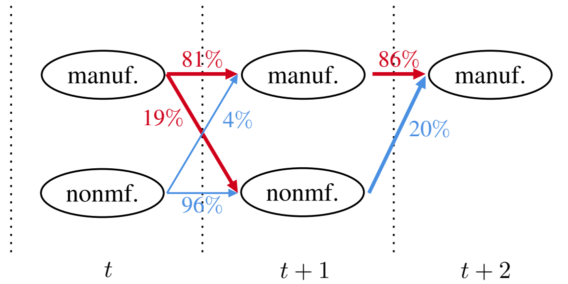


Figure 1. 1-Year and 2-Year Worker Flows in the Data

Notes: The arrows between years t and $t+1$ represent the steady-state 1-year worker flows between the manufacturing and non-manufacturing sectors. They represent $\Pr(s_{t+1} = \text{manuf.}|s_t = \text{manuf.}) = 1 - \Pr(s_{t+1} = \text{nonmf.}|s_t = \text{manuf.}) = 0.81$ and $\Pr(s_{t+1} = \text{nonmf.}|s_t = \text{nonmf.}) = 1 - \Pr(s_{t+1} = \text{manuf.}|s_t = \text{nonmf.}) = 0.96$. The arrows between years $t+1$ and $t+2$ provide additional information needed to compute the two-year staying probability for the manufacturing sector. It represents $\Pr(s_{t+2} = \text{manuf.}|s_{t+1} = s_t = \text{manuf.}) = 0.86$ and $\Pr(s_{t+2} = \text{manuf.}|s_{t+1} = \text{nonmf.}, s_t = \text{manuf.}) = 0.20$. Assuming that the economy was in a steady state between years 1980 and 2000, these worker flows are computed by pooling all observations of the NLSY79 data over this period.

approximation around a steady state. Steady-state worker flows contain information about the ease with which workers switch sectors over time, and this is precisely what we need to predict how their welfare and sector choices respond to sectoral shocks. We start by focusing on the labor supply side, examining the effect of exogenous changes in sectoral wages. However, we also demonstrate that when we augment the model with the labor demand side and endogenize wage changes, the same set of sufficient statistics, combined with knowledge of the labor demand side, can be used to perform counterfactual exercises.

Second, we show how persistent worker heterogeneity shapes our sufficient statistics and the consequences of sectoral shocks. We begin by characterizing the systematic bias in worker flow patterns implied by the canonical dynamic discrete choice model without persistent worker heterogeneity (hereafter, the canonical model). Ignoring heterogeneity and the resulting self-selection would lead to underestimation of the long-run probabilities of workers remaining in the same sector. Intuitively, workers who have self-selected into a sector are more likely to continue choosing the same sector in subsequent periods. More importantly, this bias, combined with the sufficient statistics result, implies systematic biases in the counterfactual predictions of the canonical model. In particular, we prove that the canonical model always underestimates the welfare losses of adversely affected workers and overestimates the speed of labor reallocation for given exogenous changes in wages. Interestingly, we show that when wages are endogenously determined, the underestimation of welfare losses is likely to be compounded by the overestimation of labor reallocation from given shocks to the labor market. These findings highlight the importance of incorporating persistent worker heterogeneity in evaluating the consequences of sectoral shocks, which is quantified in the remainder of the paper.

The next part of our paper provides empirical estimates of our sufficient statistics for the United States. We first use the longitudinal information in the National Longitudinal Survey of Youth 1979 (NLSY79) dataset to compute worker flows over different time horizons.[1] We find that the sectoral worker flows observed in the data are inconsistent with the canonical model without persistent worker heterogeneity. In line with our theoretical finding, the canonical model underestimates the probabilities of workers choosing the same sector by more than a factor of two. The same result holds if we break down workers by their demographic and socioeconomic characteristics, such as gender, education, race, and age. This suggests that the persistent heterogeneity driving this underestimation is mostly unobserved (to the econometrician).

To illustrate the inconsistency, we plot the 1-year and 2-year worker flows observed in the NLSY data in Figure 1. A detailed description of the data and calculations will be provided in Section 4, along with additional analysis. The figure shows, for example, that 81% of typical manufacturing workers remain in manufacturing after one year. The canonical model, which assumes away persistent heterogeneity, necessarily implies that workers who choose to stay in the manufacturing sector between years t and $t+1$ are as likely to stay in the following year as the typical manufacturing worker. However, the data reveals that workers who have self-selected to stay in manufacturing exhibit a higher probability of staying again in the following year ($86\% > 81\%$).[2] Similarly, workers who have self-selected into manufacturing in year t are more likely to choose it again in year $t+2$ even when they choose non-manufacturing sectors in year $t+1$ ($20\% > 4\%$). As a result of these discrepancies, the canonical model underestimates the probability of workers choosing the manufacturing sector again after 2 years.[3]

We then turn to estimation of the dispersion of the idiosyncratic shock. This estimation is based on the observation that the response of sectoral employment to wage shocks depends solely on the dispersion parameter and the sufficient statistics, independent of the specific details of worker heterogeneity. Thus, this parameter can be estimated by measuring the

[1] Given data constraints, we only compute worker flows over horizons of less than or equal to 18 years. To extrapolate longer-run worker flows from the available data, we leverage the structural model, which is calibrated by matching the observed worker flows.

[2] The data show considerable variation in the frequency of sector switching among workers. Some workers exhibit a high degree of sector mobility, transitioning frequently between different sectors. On the other hand, there are also workers who remain employed in a particular sector for most of their careers and rarely change sectors. This fact alone is difficult to reconcile with the canonical model.

[3] They contribute almost equally to this underestimation. The canonical model implies that the two-year staying probability is $81\% \times 81\% + 19\% \times 4\% = 66\%$, while the actual probability is $81\% \times 5\% + 19\% \times 16\% = 4\% + 3\%$ higher than this value.

response of sectoral employment, conditional on our sufficient statistics. We put this idea into practice by extending the standard Euler equation approach in the literature to allow for arbitrary worker heterogeneity.

In the final part of our paper, we combine the previous empirical estimates with the sufficient statistics result to revisit two applications in the literature. First, we apply our findings to a hypothetical trade liberalization exercise of Artuç, Chaudhuri, and McLaren (2010), in which manufacturing prices experience an unexpected permanent drop. This stylized exercise clearly illustrates how the failure to match worker flows across sectors at different horizons can lead to biases in counterfactual predictions. Second, as a more realistic application, we revisit an extensively studied topic: the impact of the rise in China's import competition on US labor markets. Following Caliendo, Dvorkin, and Parro (2019), we introduce a richer labor demand side that features trade, input-output linkages, and multiple production inputs. We extend the labor supply side of the model by allowing for arbitrary persistent worker heterogeneity.[4] Results demonstrate that when we correctly match the longer-run worker flow patterns with worker heterogeneity, labor reallocation following the China shock is significantly slower, and the negative welfare effects on manufacturing workers are more severe than previously suggested. Without persistent worker heterogeneity, workers initially employed in the manufacturing sector in all states appear to benefit, on average, from the China shock. However, when we do account for persistent worker heterogeneity, the welfare gains of manufacturing workers are close to zero in most states, and manufacturing workers experience welfare losses in five states.

Related Literature. This paper is related to several strands of literature. A large body of empirical literature studies the labor market impact of shocks that exhibit asymmetric effects across sectors, such as globalization (Goldberg and Pavcnik, 2007), automation (Acemoglu and Restrepo, 2020), the Covid-19 pandemic (Chetty et al., 2020), and oil price shocks (Keane and Prasad, 1996). Of particular relevance to our application is the literature that examines the impact of the rise in China import competition on US labor markets (e.g., Autor, Dorn, and Hanson, 2013, Autor et al., 2014, Acemoglu et al., 2016, Pierce and Schott, 2016). In this paper, we characterize welfare and labor reallocation in response to such sectoral shocks, taking into account the full general-equilibrium effect in a dynamic environment.

On the more structural side, recent papers have emphasized the importance of transitional dynamics in studying the effect of sectoral shocks, building models based on the dynamic discrete choice framework initially introduced in the IO literature (e.g., Rust, 1987). An important early contribution is Artuç, Chaudhuri, and McLaren (2010) who apply this framework to analyze the impact of a trade shock on labor market dynamics. Subsequent papers enrich this

[4] In contrast to their paper, we make the simplifying assumption that workers can switch sectors only within each state, thus abstracting from interstate migration.

framework by incorporating more realistic elements to investigate the effect of the China shock—including trade and input-output linkages (Caliendo, Dvorkin, and Parro, 2019); involuntary unemployment due to downward nominal wage rigidities or search frictions (Rodriguez-Clare, Ulate, and Vasquez, 2022); and endogenous trade imbalances (Dix-Carneiro et al., 2023). We extend the literature by introducing arbitrary persistent worker heterogeneity into the framework and show how this heterogeneity significantly affects the results of counterfactual exercises.

In allowing for persistent worker heterogeneity, we relate to a large, mostly static, literature emphasizing self-selection based on comparative advantage (see Borjas (1987), Heckman and Sedlacek (1985), and Ricardo's theory of comparative advantage for early contributions; and Lagakos and Waugh (2013), Burstein, Morales, and Vogel (2019), Hsieh et al. (2019), Porzio, Rossi, and Santangelo (2022), Grigsby (2022), and Adao, Beraja, and Pandalai-Nayar (2023) for recent applications and developments). The work most closely related to ours includes Costinot and Vogel (2010), Adão (2016), Lee (2020), and Galle, Rodríguez-Clare, and Yi (2023), who also study the distributional effects of trade shocks. We contribute to this literature by embedding this mechanism within a dynamic discrete choice framework, allowing us to take into account transitional dynamics and to highlight its importance in a dynamic context.

In terms of methodology, we follow the sufficient statistics tradition (e.g., Chetty, 2009; Hulten, 1978; Arkolakis, Costinot, and Rodríguez-Clare, 2012, Baqaee and Farhi, 2020, McKay and Wolf, 2022, Beraja, 2023). In the present context, the use of sufficient statistics offers multiple advantages. First, by eliminating the need to estimate numerous primitives, it reduces computational costs and identification requirements, while ensuring the transparency of the analysis. Second, our approach allows us to accommodate arbitrary worker heterogeneity without having to deal directly with the well-known identification challenges associated with unobserved heterogeneity (see, e.g., Heckman and Honore, 1990, Arellano and Bonhomme, 2017, Bonhomme, Lamadon, and Manresa, 2022).

Finally, our paper is also related to a large structural literature that incorporates rich heterogeneity—such as age, gender, education, nonpecuniary benefit, tenure, unobserved comparative advantage—in dynamic models. Prominent examples include Keane and Wolpin (1997) and Lee and Wolpin (2006) in the labor literature; and Dix-Carneiro (2014) and Traiberman (2019) in the trade literature. Our contribution to this body of literature is twofold. First, our results may serve to guide future structural analyses by suggesting that one can include worker flows over different time horizons as targeted moments in estimations to properly capture labor market dynamics. Second, our sufficient statistics provide a transparent way to assess which types of heterogeneity in the literature matter more for counterfactual predictions of the model.

Outline. The remainder of the paper is organized as follows. Section 2 presents a model of labor market dynamics with arbitrary persistent worker heterogeneity and derives our main sufficient statistics results. Section 3 analytically shows how ignoring persistent worker heterogeneity systematically affects the sufficient statistics and in turn biases counterfactual predictions about welfare and labor market dynamics. Section 4 measures our sufficient statistics using US panel data and estimates the bias in sufficient statistics due to ignoring persistent heterogeneity. Section 5 applies our sufficient statistics approach to study the impact of a hypothetical trade liberalization and the China shock. Section 6 concludes. The Appendix contains proofs omitted in the main text.

2. Dynamic Discrete Choice Model with Persistent Worker Heterogeneity

In this section, we present a model that extends the dynamic discrete choice model of Artuç, Chaudhuri, and McLaren (2010) (hereafter, ACM) by allowing arbitrary persistent (time-invariant) worker heterogeneity. We focus on the dynamic choice of workers over sectors, though the same framework can be applied to analyze their geographic location or occupation choices by a simple relabeling. We first describe the individual worker's dynamic discrete choice problem. The solution to this problem characterizes the dynamics of welfare and sectoral labor supply at individual level. We then show how we can aggregate the dynamics to macro level to derive equations that can be used to compute counterfactual changes in aggregate welfare and sectoral labor supply in response to sectoral shocks. We conclude by demonstrating how to combine this result with the labor demand side of the model to study the general equilibrium effects of sectoral shocks.

2.1 Workers' Dynamic Discrete Choice Problem

Time is discrete and indexed by t. There are S sectors indexed by $i, j \in \mathcal{S} = \{1, \ldots, S\}$. There is a continuum of infinitely lived heterogeneous workers. We allow for arbitrary persistent heterogeneity of workers by assigning each worker a type $\omega \in \Omega$, which is drawn from an unknown distribution W over Ω. Importantly, the type ω may capture not only observable demographic and socioeconomic characteristics, such as gender and education, but also unobserved differences in productivity and nonpecuniary preferences across sectors. At each point in time, workers decide in which sector to work.[5] A worker of type ω who is employed in sector i at period t chooses in which sector to work in period $t+1$ in order to maximize her continuation value. The value of this worker in period t can be written as[6]

$$V_{it}^{\omega} = w_{it}^{\omega} + \max_{j \in \mathcal{S}} \{\beta \mathbb{E}_t V_{jt+1}^{\omega} - C_{ij}^{\omega} + \rho^{\omega} \cdot \varepsilon_{jt}\}, \tag{1}$$

where the type-specific instantaneous utility, w_{it}^{ω}, can capture both the wage and nonpecuniary benefits from sector i in period t. For expositional purposes, we will refer to w_{it}^{ω} as sectoral wages (though in some of our applications, they will be the log of the real wages). The term $C_{ij}^{\omega} \geq 0$ captures the type-specific cost of switching from sector i to j. The idiosyncratic shock ε_{jt} is worker-specific and reflects nonpecuniary motives for workers to switch sectors.[7] The expectation operator, $\mathbb{E}_t$, is taken over realizations of future wages and future idiosyncratic shocks, conditional on the information available in period t. The parameter ρ^{ω} governs the relative importance of idiosyncratic shocks in sector choice decisions and hence determines the elasticity of sectoral employment with respect to sectoral wages. We allow the sectoral wages and switching costs to vary arbitrarily across different types of workers, as can be seen from the superscripts ω in equation (1).

When $|\Omega| = 1$—which means that there are no persistent differences across workers—our model reduces to the canonical homogeneous-worker sector choice model of ACM or, more broadly, to the standard dynamic discrete choice model (e.g., Rust, 1987). In these models, workers are ex ante homogeneous, and any ex post heterogeneity arising from different realizations of idiosyncratic shocks persists for only a single period. In the rest of the paper, we use the term *worker heterogeneity* exclusively to refer to persistent worker heterogeneity across different types of workers. When $C_{ij}^{\omega} = 0$ and $\rho^{\omega} = 0$, on the other hand, the worker problem boils down to choosing the sector that offers the highest wage, so this model reduces to the general Roy model of self-selection (e.g., Heckman and Honore, 1990).

As is standard in dynamic discrete choice models in trade, IO, and labor (e.g., Rust, 1987; Aguirregabiria and Mira, 2010), we assume that the idiosyncratic shocks, ε_{jt}, are drawn from a type I extreme-value distribution independently across workers, sectors, and time periods.

[5] Following the timeline of ACM, workers make their sector choice decision one period in advance. This means that the sector choice decision for period t is made in period $t-1$. Once period t arrives, the instantaneous utility w_{it} from their chosen sector is realized, and workers enjoy the realized utility. They then observe the realized value of period t idiosyncratic shocks, $\{\varepsilon_{jt}\}_j$, and subsequently make a sector choice decision for period $t+1$. The expectation operator $\mathbb{E}_t$ is defined with respect to workers' information set at the time of their sector choice decision for period $t+1$.

[6] Given the time series of instantaneous utility, $\{w_{it}^{\omega}\}_{i,t,\omega}$, there are no interactions between workers of different types, so each type of worker solves problem (1) independently.

[7] Idiosyncratic shocks result in gross flows that are an order of magnitude larger than net flows, a pattern that is consistent with the observed data.

Let v_{it}^{ω} be the expected value derived from choosing sector i in period t for workers of type ω, taking the average over the realizations of idiosyncratic shocks $\{\varepsilon_{jt}\}_j$, and let F_{ijt}^{ω} denote the probability that workers of type ω in sector i in period t choose sector j in period $t+1$. Standard extreme-value algebra gives an analytical characterization of the ex ante value and sector choice probabilities:

$$v_{it}^{\omega} \equiv \mathbb{E}_{\varepsilon} V_{it}^{\omega} = w_{it}^{\omega} + \rho^{\omega} \ln \sum_{j\in\mathcal{S}} \left(\exp(\beta \mathbb{E}_t v_{jt+1}^{\omega}) / \exp(C_{ij}^{\omega})\right)^{1/\rho}, \tag{2}$$

$$F_{ijt}^{\omega} \equiv \Pr_t(s_{t+1} = j | s_t = i, \omega) = \frac{\left(\exp(\beta \mathbb{E}_t v_{jt+1}^{\omega}) / \exp(C_{ij}^{\omega})\right)^{1/\rho^{\omega}}}{\sum_{k\in\mathcal{S}} \left(\exp(\beta \mathbb{E}_t v_{kt+1}^{\omega}) / \exp(C_{ik}^{\omega})\right)^{1/\rho^{\omega}}}, \tag{3}$$

where the expectation operator $\mathbb{E}_{\varepsilon}$ is taken over the realizations of $\{\varepsilon_{jt}\}_j$. Equation (2) expresses the value of being in sector i as the sum of the current period's instantaneous utility and a nonlinear aggregation of next-period values, net of switching costs. Equation (3) suggests that, all else being equal, workers are more likely to choose sectors with higher values net of switching costs. Because there is a continuum of workers for each type ω, we can characterize the law of motion of their sectoral employment share from their sector choice probabilities,

$$\ell_{jt+1}^{\omega} = \sum_{i\in\mathcal{S}} F_{ijt}^{\omega} \ell_{it}^{\omega}. \tag{4}$$

We also define the backward transition probability,

$$B_{jit}^{\omega} \equiv \Pr_t(s_t = i | s_{t+1} = j, \omega) = \frac{\ell_{it}^{\omega} F_{ijt}^{\omega}}{\ell_{jt+1}^{\omega}},$$

which is the probability that a type ω worker in sector j in period $t+1$ came from sector i in period t. We define $S \times S$ matrices F_t^{ω} and B_t^{ω}, whose (m,n)-element is F_{mnt}^{ω} and B_{mnt}^{ω}, respectively. We refer to them as the (forward) transition matrix and backward transition matrix, respectively. Note that the rows of these matrices sum to one.

2.2 Welfare and Labor Dynamics at the Micro Level

The system of equations (2)–(4) fully characterizes the labor supply side of the model. That is, given the series of sectoral wages, $\{w_{it}^{\omega}\}$, we can solve for the series of sectoral employment, $\{\ell_{it}^{\omega}\}$, and sectoral values, $\{v_{it}^{\omega}\}$, from this system of equations. These are the two variables of interest in our counterfactual analysis. As a first step toward deriving a sufficient statistics result, we consider infinitesimal sectoral shocks $\{\mathrm{d}w_{it}^{\omega}\}$ and take first-order approximations of equations (2) and (4) around a steady state.[8]

A *steady state* is associated with time-invariant sectoral wages ($w_{it}^{\omega} = w_i^{\omega}$ for all t), where the type-specific value, sector choice probabilities, and sectoral labor supply remain constant over time. In line with our previous notation, we denote the steady-state forward and backward transition matrices as F^{ω} and B^{ω}, respectively. The following equations summarize the responses of the endogenous variables in terms of deviations from steady state:

$$\mathrm{d}v_t^{\omega} = \mathrm{d}w_t^{\omega} + \beta F^{\omega}\,\mathbb{E}_t\,\mathrm{d}v_{t+1}^{\omega}, \tag{5}$$

$$\mathrm{d}\ln\ell_{t+1}^{\omega} = B^{\omega}\,\mathrm{d}\ln\ell_t^{\omega} + \tfrac{\beta}{\rho^{\omega}}(I - B^{\omega}F^{\omega})\mathbb{E}_t\,\mathrm{d}v_{t+1}^{\omega}, \tag{6}$$

where we use the vector notation,

$$\mathrm{d}v_t^{\omega} = \begin{pmatrix}\mathrm{d}v_{1t}^{\omega} & \cdots & \mathrm{d}v_{St}^{\omega}\end{pmatrix}^{\intercal},\ \mathrm{d}w_t^{\omega} = \begin{pmatrix}\mathrm{d}w_{1t}^{\omega} & \cdots & \mathrm{d}w_{St}^{\omega}\end{pmatrix}^{\intercal},\ \text{and}\ \mathrm{d}\ln\ell_t^{\omega} = \begin{pmatrix}\mathrm{d}\ln\ell_{1t}^{\omega} & \cdots & \mathrm{d}\ln\ell_{St}^{\omega}\end{pmatrix}^{\intercal}.$$

The algebra follows that of Kleinman, Liu, and Redding (2023), as described in Appendix D. It is worth highlighting the assumptions underlying these equations. Equation (5) is a direct application of the envelope theorem, often referred to the Williams-Daly-Zachary theorem in the discrete choice literature. Thus, it remains valid regardless of the distribution assumption on the idiosyncratic shock.[9] On the other hand, equation (6) is valid only under the assumption that idiosyncratic shock follows a type I extreme-value distribution. Whereas Taylor's theorem implies that it is always possible to express $\mathrm{d}\ln\ell_{t+1}^{\omega}$ as a linear function of $\mathrm{d}\ln\ell_t^{\omega}$ and $\mathbb{E}_t\,\mathrm{d}v_{t+1}^{\omega}$ up to the first order, the sufficient statistics results we derive below rely on the coefficients of this linear function's being polynomials of the transition matrices B^{ω} and F^{ω}, which extreme-value distribution assumption guarantees.[10] A more detailed intuition is provided in Appendix A.4.

It is important to note that equations (5) and (6) cannot be confronted directly with data if worker type ω is unobservable. The key idea of this paper is that despite this unobservability, these type-specific equations can be aggregated into equations that solely involve a few observable statistics, which can be used to construct counterfactuals. This idea allows us to bypass the well-known challenges associated with explicitly specifying and estimating the distribution of underlying unobserved heterogeneity (e.g., Heckman and Honore, 1990).

[8] We come back to the quality of this first order approximation in Section 5. For the shocks that we consider, the approximation is good.

[9] In fact, the same envelope-type result extends to a much wider class of models, including models with duration dependence mechanism.

[10] This assumption is sufficient but not necessary. In Appendix A.4, we demonstrate that this result is not specific to the extreme-value distribution. Specifically, we show that a version of equation (6) holds for any distribution of the idiosyncratic shock in a limit case of a perturbed economy, in which dispersion of the idiosyncratic shock converges to zero. Thus, all results in this paper apply to this limit case.

In order to prepare the aggregation at the macro level, we solve these equations forward and backward to write the responses of sectoral values and sectoral employment as a function of the expected past and future wage changes $(\cdots, \mathrm{d}w_{t-1}^{\omega}, \mathrm{d}w_t^{\omega}, \mathrm{d}w_{t+1}^{\omega}, \cdots)$. Lemma 1 summarizes the result.

Lemma 1. *For a given sequence of changes in sectoral wages $\{\mathrm{d}w_t^{\omega}\}$, the changes in type-specific sectoral values and sectoral employment $\{\mathrm{d}v_t^{\omega}, \mathrm{d}\ln \ell_t^{\omega}\}$ are given by:*

$$\mathrm{d}v_t^{\omega} = \sum_{k\geq 0}(\beta F^{\omega})^k \mathbb{E}_t\, \mathrm{d}w_{t+k}^{\omega}, \tag{7}$$

$$\mathrm{d}\ln \ell_t^{\omega} = \tfrac{\beta}{\rho^{\omega}} \sum_{s\geq 0}(B^{\omega})^s (I - B^{\omega}F^{\omega}) \left(\sum_{k\geq 0}(\beta F^{\omega})^k \mathbb{E}_{t-s-1}\, \mathrm{d}w_{t-s+k}^{\omega}\right). \tag{8}$$

Workers are forward-looking, so all future shocks affect the value of workers and sectoral employment, as can be seen from equations (7) and (8). Due to the presence of switching costs and idiosyncratic shocks, labor reallocation is sluggish, so past shocks also affect sectoral employment, as can be seen from equation (8).

2.3 Welfare and Labor Dynamics at the Macro Level

We focus on the effect of sectoral shocks on variables aggregated across workers of different types ω. In particular, we define total sectoral employment and average sectoral value as follows:

$$\ell_{it} = \int_{\Omega} \ell_{it}^{\omega}\, \mathrm{d}W(\omega) \quad \text{and} \quad v_{it} = \int_{\Omega} v_{it}^{\omega}\, \mathrm{d}W(\omega|s=i) \tag{9}$$

where $W(\cdot|s=i)$ is the steady-state type distribution of workers in sector i. The total employment of sector i is obtained by summing the employment of different types of workers. Likewise, the average value of workers in sector i is given by taking the weighted average across different types of workers, using the steady-state type distribution of that sector as weights.[11] In so doing, we implicitly assume a utilitarian social welfare function with equal weights across all workers. Next, we define worker flow matrices. As we will demonstrate, these matrices are sufficient statistics for characterizing the welfare and labor market consequences of sectoral shocks.

Definition 1. *For each $k \in \mathbb{N}_0$, the k-*period worker flow matrix *$\mathcal{F}_k$ is an $S \times S$ matrix whose (i,j)-element is given by the steady-state share of workers in sector i who switch to sector j*

[11] We therefore abstract from distributional consequences of sectoral shocks across unobservable types ω. Our interest here is in comparing the welfare of workers initially employed in different sectors.

after k periods:

$$(\mathcal{F}_k)_{i,j} = \Pr(s_{t+k} = j | s_t = i).$$

Unlike the type-specific transition matrices B^ω and F^ω in Lemma 1, these worker flow matrices can be computed directly from longitudinal information on workers' sector choices, as we will do using the NLSY data in Section 4.[12]

To derive an aggregation result, we make the following two assumptions.

Assumption 1. *Workers of different types share common sectoral shocks and common dispersion of idiosyncratic shocks:*

$$\mathrm{d}w_t^\omega = \mathrm{d}w_t \text{ and } \rho^\omega = \rho, \textit{ for all } t \textit{ and } \omega \in \Omega.$$

Assumption 2. *The bilateral switching costs between sectors satisfy either one of the following conditions:*

$$C_{ij}^\omega = C_{ji}^\omega \text{ or } C_{ij}^\omega = \begin{cases} C_i^\omega + \tilde{C}_j^\omega & \textit{if } i \neq j \\ 0 & \textit{if } i = j \end{cases},$$

for all $i, j \in \mathcal{S}$, and $\omega \in \Omega$.

It is worth emphasizing that the first part of Assumption 1 does not require that all workers have the same *level* of instantaneous utilities. For example, suppose workers have log utility and shocks are multiplicative to the wages of all workers. In this case, even if workers have different wages, the shocks manifest themselves as common additive shocks to instantaneous utilities for all workers.[13] Similarly, the second part of Assumption 1 does not necessarily imply that all workers have the same labor supply elasticity. Although the dispersion of idiosyncratic shocks governs the elasticity of sectoral labor supply with respect to sectoral shocks, the elasticity also depends on type-specific transition matrices, B^ω and F^ω, as can be seen in equation (8).[14] Although restrictive, this assumption is standard in the dynamic discrete choice literature, even in papers that incorporate rich heterogeneity of workers. Our approach can be applied to the models studied in those papers.

[12] In Appendix A.1, we formally show that if we have access to infinite-length longitudinal information on workers' sector choices, we can directly observe the full series of worker flow matrices. However, since panel data have finite time dimension in practice, we can only calculate worker flow matrices $\mathcal{F}_k$ for low enough k's. In Section 4, we discuss various ways of extrapolating worker flow matrices.

[13] In many applications, however, the shocks of interest are known to have heterogeneous effects across *observed* types—for example, between high-skilled and low-skilled workers or across geographic regions. In such cases, we can account for this possibility by conducting the same analysis separately for each observed type of workers. This is the approach we take when we study the effect of the China shock on US labor markets in Section 5. However, this approach is infeasible for *unobserved* types. Therefore, we cannot dispense with the assumption that shocks are common across unobserved types.

[14] What this implies is that all heterogeneity in the elasticity of sectoral labor supply with respect to sectoral shocks is revealed by shares. This observation will be useful in deriving our sufficient statistics result.

The conditions in Assumption 2 are often imposed in the literature for various purposes. For example, ACM (in Section IV.D) and Dix-Carneiro (2014) assume that the switching costs can be decomposed as in the second condition to reduce the number of parameters to be estimated. In a different context, Allen and Arkolakis (2014) and Desmet, Nagy, and Rossi-Hansberg (2018) assume the first condition for bilateral trade costs and bilateral switching costs, respectively, in order to simplify the equilibrium system into a single integral equation.

We are ready to state our main result.

Proposition 1. *Suppose that Assumptions 1 and 2 hold. For a given sequence of (common) changes in sectoral wages* $\{\mathrm{d}w_t\}$*, the changes in sectoral value and sectoral employment are given by:*

$$\mathrm{d}v_t = \sum_{k\geq 0} \beta^k \mathcal{F}_k \mathbb{E}_t \, \mathrm{d}w_{t+k}, \tag{10}$$

$$\mathrm{d}\ln \ell_t = \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho} (\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2}) \mathbb{E}_{t-s-1} \, \mathrm{d}w_{t-s+k}. \tag{11}$$

The logic behind this proposition is as follows. Starting from Lemma 1, we want to aggregate type-specific variables to the macro level. We first invoke Assumption 2, which simplifies the aggregation by giving the equality between the forward and backward transition matrices, $B^\omega = F^\omega$.[15] Although this equality is not strictly necessary for our purpose, it allows us to derive analytical results in Section 3 and reduces the data requirements needed to implement our sufficient statistics approach.[16] Figure A.1 shows that these two transition matrices are indeed very similar at the level of granularity at which both matrices can be computed from the data. After imposing this equality, we aggregate equations (7) and (8) to derive equations (10) and (11), respectively. In particular, the kth powers of the type-specific transition matrix $(F^\omega)^k$ are aggregated into the k-period worker flow matrix $\mathcal{F}_k$. Intuitively, since the (i, j)-element of the former is given by $\Pr(s_{t+k} = j | s_t = i, \omega)$, we can obtain the (i, j)-element of the latter, $\Pr(s_{t+k} = j | s_t = i)$, by taking an average over the type distribution of workers in sector i, $\Pr(\omega | s_t = i)$. In Appendix A.1, we prove Proposition 1 by introducing the population-average operator, which formalizes the idea of aggregation.

The role of Lemma 1 and Assumption 1 should also be clear at this point. We have just seen that products of transition matrices can be aggregated to worker flow matrices, but when

[15] See Appendix A.2 for a proof. Two matrices are equal if and only if the steady-state flow of type ω workers from sector i to j is equal to the flow from sector j to i for all sector pairs. However, the definition of the steady state does not necessarily imply this condition, since it only requires that the *total* outflow of type ω workers from a sector be equal to the *total* inflow into that sector. We further need Assumption 2 to guarantee that bilateral worker flows are balanced for all sector pairs.

[16] See Appendix A.2 for a general version of Proposition 1 without this assumption.

they are multiplied by another type-specific variable, such as $\mathrm{d}v_{t+1}^{\omega}$ in (5) and $\mathrm{d}w_{t+k}^{\omega}$ in (7), a complication arises because the aggregation then involves a covariance term that captures the extent to which two multiplicands comove across different worker types. Since the type index ω may include unobserved heterogeneity, it is not possible to characterize the covariance term without specifying the precise form of worker heterogeneity. Lemma 1 and Assumption 1 allow us to bypass this problem. Relatedly, given the structure in Proposition 1, the aggregate variables do not possess a recursive representation, hence going from equations (5) and (6) to (7) and (8) is key to our results. We will return to this issue in Section 4.

Proposition 1 establishes that in order to calculate the counterfactual changes in aggregate welfare and sectoral employment for a known sequence of exogenous wage changes, $\{\mathrm{d}w_t\}$, we only require knowledge of the worker flow matrices, $\{\mathcal{F}_k\}$, and the shape parameter ρ. In particular, we do not need full knowledge of the detailed worker heterogeneity (i.e., the distribution of types, W) and resulting self-selection that generates these worker flow matrices. What matters is how frequently workers switch sectors over time, not the specific structural determinants of these patterns.[17] In Section 4, we estimate worker flow matrices and the value of the parameter ρ using panel data. The following corollary summarizes the discussion.[18]

Corollary 1. *Consider a sequence of exogenous changes in sectoral wages, $\{\mathrm{d}w_t\}$. Together with ρ, the worker flow matrices, $\{\mathcal{F}_k\}$, constitute sufficient statistics for changes in sectoral values, $\{\mathrm{d}v_t\}$, and sectoral employment, $\{\mathrm{d}\ln \ell_t\}$.*

We have so far focused on the labor supply side and considered exogenously given wage changes. In Section 2.4, we show that even when we endogenize the wage by augmenting the model with the labor demand side, the same set of sufficient statistics, combined with knowledge of the labor demand side, constitutes sufficient statistics for counterfactual changes in sectoral values and sectoral employment.

At this point, it is worth discussing how our sufficient statistics approach relates to structural work in this area. Our approach stands in stark contrast to the standard structural approach

[17] This result is surprising because, in principle, constructing the counterfactual in a dynamic context without a precisely specified model requires estimating all dynamic elasticities of sectoral values and sectoral employment with respect to shocks at all time horizons—that is, how past as well as future shocks affect these variables. McKay and Wolf (2022) propose a method to operationalize this approach in practice, but in general estimating all elasticities is challenging due to high information requirements and limited availability of data. Proposition 1 reveals that the dynamic discrete choice framework imposes a tight connection among these dynamic elasticities. This relationship enables us to parameterize these elasticities with a single parameter to be estimated, ρ, while the worker flow matrices contain all the remaining information needed to calculate the dynamic elasticities.

[18] In Arkolakis, Costinot, and Rodríguez-Clare (2012), a distinction is made between the ex ante sufficient statistics result and the ex post result. Proposition 1 provides the ex post sufficient statistics in the sense that this result is only useful if we can estimate or directly observe the change in wages resulting from the shock of interest. This is often feasible when examining the effects of shocks that occurred in the past. However, it becomes impossible when attempting to forecast the impact of hypothetical shocks.

to accounting for worker heterogeneity. First, our approach eliminates the need to estimate many primitives. This reduces computational costs and data requirements, while ensuring the transparency of the analysis. Like other studies of sufficient statistics (See, e.g., Chetty, 2009), our method yields counterfactual predictions that are immune to the Lucas critique, without requiring knowledge of the full structure of the model. Second, this approach effectively accommodates arbitrary worker heterogeneity without encountering the well-known challenges associated with estimating the distribution of unobserved heterogeneity from the data.[19] However, these advantages do not come without costs. First, we need to make restrictions on the set of shocks that can be studied and on the heterogeneity in the dispersion of idiosyncratic shocks (Assumption 1). Second, we rely on the first-order approximation around a steady state, the validity of which depends on the sectoral shock of interest. We will revisit this issue in Section 5.

2.4 Closing the Model: Labor Demand and Equilibrium Wages

We conclude this section by extending the sufficient statistics result to the case in which the wage is endogenously determined by the labor market equilibrium. For this purpose, we need to specify the labor demand side of the model. Although the main contribution of this paper centers on the labor supply side, our heterogeneous worker labor supply model can be integrated with any labor demand system. The specific nature of the labor demand system depends on preferences, technology, and good market structure. For expositional purposes, we specify it in a reduced-form manner in this section and return to its structural determinants in our applications in Section 5. Specifically, we assume that the wage of sector i is endogenously determined by the sectoral labor allocation $\{\ell_{jt}\}_j$ and exogenous shocks $\{\varepsilon_{jt}\}_j$ that affect the marginal productivity of labor. The variable ε_{jt} encompasses sector-specific factors, such as capital stock, technology shocks, policy variables, and the like. This relationship can be expressed as $w_{it}^{\omega} = f_i^{\omega}(\{\ell_{jt}\}_j, \{\varepsilon_{jt}\}_j)$.[20] In Appendix A.3, we show that under Assumption 1 we can write this relationship in terms of a first-order approximation as

$$\mathrm{d}w_t = D \cdot \mathrm{d}\ln \ell_t + E \cdot \mathrm{d}\varepsilon_t, \tag{12}$$

[19] Both Dix-Carneiro (2014) and Traiberman (2019) incorporate unobserved heterogeneity in their analyses. However, due to the identification challenge, they are constrained to use a limited number of unobserved types. While this could allow them to capture the absolute advantage of workers, it is difficult to capture the comparative advantage of workers and hence their self-selection into sectors.

[20] For simplicity, we assume that labor demand, unlike labor supply, is determined in a static manner. However, the results below could be extended to models with dynamic labor demand decisions, without affecting any of the main insights.

where there is no ω-index on matrices D and E.

Combining the labor demand curve represented by this equation with the labor supply curve characterized in the previous proposition, we can define the labor market equilibrium. It consists of paths of type-specific sectoral value, v_t^ω; type-specific labor allocation across sectors, ℓ_{t+1}^ω; type-specific sector choice probabilities, F_t^ω; aggregate sectoral value, v_t; and aggregate labor allocation, ℓ_{t+1}, that are measurable with respect to the period-t information set, and a path of sectoral wages, w_t, that are measurable with respect to the period-$(t-1)$ information set and the period-t shock such that: *(a)* type-specific variables $\{v_t^\omega, \ell_{t+1}^\omega, F_t^\omega\}$ solve problem (1) given the path of wages; *(b)* aggregate variables $\{v_t, \ell_t\}$ are consistent with the type-specific variables through equation (9); *(c)* wages are determined by the marginal productivity of labor, (12); and *(d)* the labor market clears.

The following proposition shows that the same set of worker flow matrices, combined with knowledge of the labor demand side, still constitutes sufficient statistics when wages are endogenously determined by the labor market equilibrium.

Proposition 2. *Suppose that Assumptions 1 and 2 hold. For a given sequence of labor demand shocks* $\{\mathrm{d}\varepsilon_t\}$*, the equilibrium values of* $\{\mathrm{d}v_t, \mathrm{d}\ln\ell_t, \mathrm{d}w_t\}_t$ *are given by solution of the following system of equations:*

$$
\begin{aligned}
\mathrm{d}v_t &= \sum_{k\geq 0} \beta^k \mathcal{F}_k \mathbb{E}_t\, \mathrm{d}w_{t+k}, \\
\mathrm{d}\ln\ell_t &= \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho} (\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2}) \mathbb{E}_{t-s-1}\, \mathrm{d}w_{t-s+k}, \\
\mathrm{d}w_t &= D \cdot \mathrm{d}\ln\ell_t + E \cdot \mathrm{d}\varepsilon_t.
\end{aligned}
$$

The intuition is simple. Conditional on a path of wage changes across time and across sectors, we can characterize the dynamic response of sectoral employment using Proposition 1. Conditional on the dynamics of sectoral employment, we can solve for prices and wages from the labor demand side to characterize the path of (real) wage changes. The equilibrium is determined as a fixed point of these relations.

Unlike Proposition 2, which requires knowledge of the path of wages, Proposition 1 requires the path of labor demand shocks $\{\varepsilon_i\}$. One can think of it as extending Proposition 2 to the case where labor demand are not perfectly elastic.

Given the shock path, $\{\mathrm{d}\varepsilon_t\}$, we can solve the system of equations to compute changes in values and sectoral employment. Conditional on the observed series of worker flow matrices, the value of ρ, and the reduced-form specification of the labor demand side, D and E, the responses of welfare, employment, and wages to labor demand shocks are identical.

3. Employment and Welfare Implications of Persistent Heterogeneity

The canonical dynamic discrete choice model commonly used in the trade, labor, and IO literature abstracts from persistent worker heterogeneity. The sufficient statistics results in the previous section provide a way to account for arbitrary persistent heterogeneity. In this section, we use these results to demonstrate why incorporating this consideration matters in evaluating the consequences of sectoral shocks on welfare and labor reallocation.

Our sufficient statistics result highlights that worker heterogeneity affects the results of counterfactual exercises only through its effect on the model's predictions for a particular set of moments of the data: the worker flow matrices $\{\mathcal{F}_k\}$.[21] In this section, we first theoretically characterize a systematic bias in worker flow matrices implied by the canonical model, which, due to the lack of persistent worker heterogeneity, imposes that the k-period worker flow matrix is equal to the one-period worker flow matrix to the kth power. In turn, the bias in worker flows leads to systematic biases in counterfactual predictions of welfare changes and labor reallocation.

3.1 Steady-state Worker Flow with and without Persistent Heterogeneity

Lemma 2 characterizes the restrictions that the absence of persistent worker heterogeneity imposes on our sufficient statistics; i.e., the worker flow matrices $\{\mathcal{F}_k\}$.

Lemma 2. *Without persistent worker heterogeneity ($|\Omega| = 1$), we have $\mathcal{F}_k = (\mathcal{F}_1)^k$. With (non-degenerate) persistent worker heterogeneity, we have $(\mathcal{F}_k)_{ii} > \left((\mathcal{F}_1)^k\right)_{ii}$ for all $i \in \mathcal{S}$ and $k > 1$.*

The first part of Lemma 2 shows that without persistent heterogeneity, the Markovian structure of the model implies that the same transition probabilities are applied to all workers, which enables us to compute the k-period worker flow matrix by multiplying the one-period matrix k times.[22] The second part of Lemma 2 illustrates how accommodating worker heterogeneity relaxes this restriction. With worker heterogeneity, the diagonal elements of the k-period worker flow matrices could be larger than they would be in the absence of worker heterogeneity.

[21] Another implication of the sufficient statistics result is that, conditional on the observed worker flow matrix series, the results of counterfactual exercises remain unchanged regardless of how we specify the underlying heterogeneity. However, in practice, not all worker flow matrices of the model match those observed in the data, and, as will be seen in this section, models with different worker heterogeneity yield different worker flow matrices.

[22] This is known as the *Chapman-Kolmogorov equation* in the theory of Markovian processes.

Accordingly, if we wrongly ignore persistent worker heterogeneity, we systematically *under-estimate* the probability of workers choosing the same sector after k periods, concluding that moving across sectors is more frequent than it actually is. To understand this underestimation, consider the case of $k = 2$, where we have

$$(\mathcal{F}_2)_{ii} = \sum_{j \in \mathcal{S}} \Pr(s_{t+1} = j | s_t = i) \Pr(s_{t+2} = i | \boldsymbol{s_t = i}, s_{t+1} = j), \tag{13}$$

$$\left((\mathcal{F}_1)^2\right)_{ii} = \sum_{j \in \mathcal{S}} \Pr(s_{t+1} = j | s_t = i) \Pr(s_{t+2} = i | s_{t+1} = j).$$

When workers are heterogeneous, the additional conditioning of $s_t = i$ in equation (13) increases the likelihood of choosing sector i again in period $t + 2$, since workers who have self-selected into sector i in period t are more likely to do so in subsequent periods (see, for example, Heckman, 1981).

Many widely used datasets only provide information on the short-run worker flows because they do not track individual workers, nor do they provide information on workers' past sector choice history or tenure.[23] In such situations, a common approach in the literature is to assume homogeneous workers and calibrate models by matching the one-period worker flow matrix. Lemma 2 shows how this calibration practice effectively extrapolates longer-run worker flows and why this extrapolation is necessarily biased. In Section 4, we indeed show that the canonical model performs poorly in matching the longer-run worker flow patterns observed in the data.

3.2 Counterfactual Predictions with and without Persistent Heterogeneity

Combining Lemma 2 with our sufficient statistics result, we can theoretically characterize the systematic biases in counterfactual predictions that arise from assuming away persistent heterogeneity. For the moment, we consider shocks to exogenously given wages, as in Proposition 1. For simplicity, we focus on a uniform permanent shock, either positive or negative, to a sector $s \in \mathcal{S}$ that is known to workers in period 1:

$$\mathrm{d}w_{st} = \Delta \in \mathbb{R}, \ \forall\, t \geq 1. \tag{14}$$

For more general shocks, Appendix A.6 characterizes the effect of one-time shocks, from which we can calculate the effect of any sequence of shocks.

[23] Even when researchers use panel data that contain the necessary information, it is unclear whether the model correctly matches longer-run worker flows unless they are directly targeted.

Counterfactual Welfare Changes. We begin with welfare changes. Compared to the predictions of the canonical model, workers who are initially employed in sector s are more likely to remain in the sector and to be affected by the wage change for a longer period of time. As a result, ignoring worker heterogeneity leads to underestimation of the welfare changes of these workers. This observation is formalized in Proposition 3.

Proposition 3. *Consider a uniform permanent shock of the form* (14) *known to workers in period* 1*. The canonical model, calibrated by matching the one-period worker flow matrix, underestimates the welfare effect on workers initially employed in sector* s*,* $|\,\mathrm{d}v_{s1}|$.

Counterfactual Employment Changes. We now turn to labor reallocation. A shock to sector s changes the employment share of that sector over time. This labor reallocation is characterized by equation (11) of Proposition 1, which involves terms of the form $\mathcal{F}_k - \mathcal{F}_{k+2}$. For ease of notation, we define b_k as the diagonal element of $\mathcal{F}_k - \mathcal{F}_{k+2}$ that corresponds to sector s:

$$b_k \equiv (\mathcal{F}_k - \mathcal{F}_{k+2})_{s,s}.$$

Roughly speaking, b_k measures the rate at which the probability of remaining in sector s decreases over time. To characterize the bias of the canonical model, we assume a single-crossing condition on b_k.

Assumption 3. *There exists* $\bar{k} \in \mathbb{N}$ *such that* b_k *is higher in the canonical model if and only if* $k \leq \bar{k}$.

This assumption requires that the probability of remaining in sector s initially decreases faster in the canonical model, but eventually decreases faster in the heterogeneous-worker model. Note that both models give the same value of the staying probabilities $(\mathcal{F}_k)_{s,s}$ for $k = 0, 1$, and the heterogeneous-worker model yields higher staying probabilities for all $k \geq 2$. Thus, staying probabilities must decrease faster in the canonical model for early periods. On the other hand, if staying probabilities converge to similar levels in both models as $k \to \infty$, then the decline should eventually become faster in the heterogeneous-worker model in order to compensate for the initial faster decline. Assumption 3 further requires the existence of a cutoff $\bar{k}$ at which the order of the speed of decline is reversed. In Appendix A.5, we show that this assumption indeed holds with $\bar{k} = 9$ (years) for the worker flow matrix series we observe in the data. Under this assumption, the following proposition shows that the canonical model initially overestimates the change in employment in sector s while underestimating the long-run labor reallocation.

Proposition 4. *Consider a uniform permanent shock of the form* (14) *known to agents in period* 1*. Under Assumption 3, there exists* $\bar{t} \in \mathbb{N} \cup \{\infty\} \backslash \{1\}$ *such that the canonical model, calibrated by matching the one-period worker flow matrix, overestimates the change in employment of sector* s *in period* t *if and only if* $1 < t \leq \bar{t}$.

The result implies that whether assuming away persistent heterogeneity leads to overestimation or underestimation of the labor reallocation depends on the time horizon. On the one hand, as discussed in Lemma 2, the canonical model overestimates the mobility of workers across sectors, leading to an overestimation of the change in employment of sector s. This intuition is what Proposition 4 describes when t is small. On the other hand, in the heterogeneous-worker model, once workers choose sector s, they have relatively higher probabilities of being stuck in that sector. Thus, in the face of a permanent negative (positive, respectively) shock, workers will dislike (like, respectively) sector s more relative to the canonical model. This aspect works in the opposite direction to our previous intuition and may become dominant when t is large enough.[24] In Appendix A, we show that the worker flow matrix series we observe in the data implies $\bar{t} = 11$ years. Thus, we can conclude that the canonical model overestimates the impact of shocks on sectoral employment within an 11-year horizon but underestimates their longer-run effects.

Until now, we have considered exogenous changes in sectoral wages. This scenario corresponds, for example, to a small open economy with linear technology affected by changes in world prices induced by trade liberalization. However, if wage changes are endogenously determined by labor market equilibrium, different models may also generate different predictions for wage changes in response to given exogenous shocks to the labor market. Interestingly, with endogenously determined wages, the underestimation of the welfare effect characterized in Proposition 3 is likely to be compounded by the overestimation of the speed of labor reallocation shown in Proposition 4. The intuition behind this is straightforward. Without loss of generality, consider a negative shock to a sector. Proposition 4 implies that in response to given negative wage changes, workers leave the sector more rapidly in the canonical model. The resulting decrease in labor supply raises the marginal productivity of labor in that sector, which partially offsets the initial decline in wages. Thus, the canonical model predicts a smaller decline in wages, at least in the short term. This, combined with discounting of the future, further contributes to underestimation of the welfare effect.

In sum, we show that the canonical model, without persistent worker heterogeneity, always underestimates the welfare losses of adversely affected workers and overestimates the speed of labor reallocation. In our counterfactual exercises in Section 5, we indeed document

[24] This is not always the case. In particular, there exists $\bar{\beta} \in (0, 1)$ such that when $\beta > \bar{\beta}$, the canonical model overestimates the change in employment in sector s in all periods t.

sizable differences in welfare effects and labor reallocation with and without persistent worker heterogeneity.

4. Sufficient Statistics in the Data

The sufficient statistics result in Proposition 2 requires three inputs to construct counterfactuals for a given shock of interest: the worker flow matrix series, the values of the parameters ρ and β, and the knowledge of the labor demand side. In this section, we first use longitudinal information in the NLSY data to compute the aggregate worker flow matrices and compare them with those implied by the canonical model without persistent worker heterogeneity. We then present a method for estimating the value of ρ without specifying worker heterogeneity, which extends the standard Euler-equation approach used in the literature. Finally, we impose $\beta = 0.96$ for the subsequent analysis. In Section 5, we close the model by specifying the details of the labor demand side.

4.1 Observed Worker Flow Matrices

We compute the worker flow matrices from the National Longitudinal Survey of Youth 1979, a rich dataset compiled by the US Bureau of Labor Statistics.[25] This survey follows a nationally representative sample of workers from 1979 onward, annually through 1994 and biennially thereafter. The sample consists of workers who were between 14 and 21 years old as of December 31, 1978, and entered the labor market in the 1980s. The NLSY79 provides detailed information on education, race, gender, age, and, importantly, the sector of employment. Specifically, we identify a worker's sector of employment in a year as the sector in which the worker was employed in the first week of that year.[26] We mainly follow the data-cleaning procedure of Lise and Postel-Vinay (2020).[27] We aggregate sectors into four broad sectors and consider worker flows across these sectors: (i) Agriculture and Construction; (ii) Manufacturing; (iii) Communication and Trade; and (iv) Services and Others.

Assuming that the economy was in a steady state between years 1980 and 2000, we calculate the series of worker flow matrices by pooling all observations in this period. Given data constraints, we only calculate the k-year worker flow matrices, $\mathcal{F}_k$, up to $k = 18$. Figure 2

[25] We also obtain quantitatively and qualitatively similar findings in the monthly Current Population Survey dataset; see Section 5.

[26] This ensures that we consistently measure mobility at a 1-year window.

[27] The survey comprises a cross-sectional subsample that is representative of young people living in the US and other subsamples that target ethnic minorities, people in the military, and the poor. We only use the representative subsample for our analysis. We also drop people seen in the military.

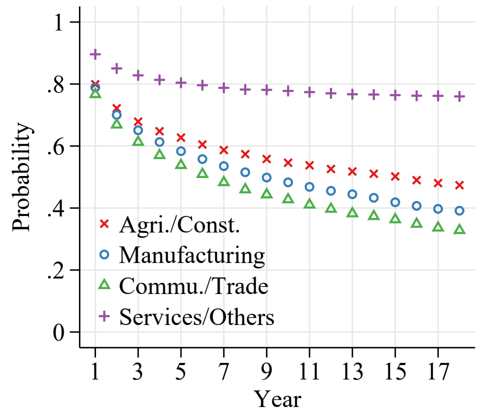


Figure 2. Worker Flow Matrix Series

Notes: Each marker in the figure represents the probability that workers choose the same sector after $k \in \{1, 2, \dots, 18\}$ years, $\Pr(s_{t+k} = s | s_t = s)$. There are four sectors: agriculture and construction, manufacturing, communication and trade, and services and others. Data source: NLSY79.

plots the diagonal elements of the obtained worker flow matrices. Each point represents the probability of workers choosing the same sector after k years (i.e., $\Pr(s_{t+k} = s | s_t = s)$; hereafter, *$k$-year staying probability*). 1-year staying probabilities are close to 80%, except in the services sector, in which it is just below 90%. The k-period staying probabilities decrease in k, reflecting the diminishing impact of being in a particular sector in the past over time.

4.2 Bias of the Canonical Model

To quantify the bias of the canonical model characterized in Lemma 2, we calculate the worker flow matrix series implied by the canonical model. Following Lemma 2, we compute the implied k-year worker flow matrix by multiplying the 1-year matrix k times. We first compare a specific diagonal element of the actual and the implied worker flow matrices: the k-year staying probability for the manufacturing sector. This is the element of primary interest because our main counterfactual exercise in Section 5 examines the impact of the China shock on US manufacturing sectors. Figure 3 plots both the actual staying probabilities (blue dots) and those implied by the canonical model (red line). It clearly shows that the canonical model significantly underestimates longer-run staying probabilities, which is in line with the prediction of Lemma 2 and becomes particularly pronounced at longer horizons.[28] Figure A.4 further

[28] In fact, the stationarity assumption is likely to lead to an *underestimation* of the gap between the actual and implied staying probabilities. To see this, suppose that worker flow matrices for two periods $t = 1, 2$ are F^1 and F^2, respectively. Under the stationarity assumption, we calculate the worker flow matrix $\bar{F}$, which is applied to both periods, by taking an average of F^1 and F^2. Suppose that we put equal weights on F_1 and F_2. Then, we

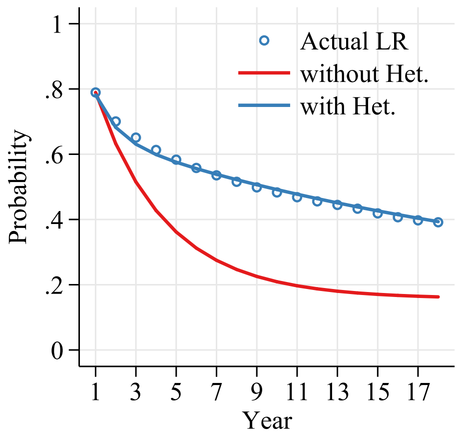


Figure 3. Actual and Model-Implied Worker Flow Matrices: Manufacturing Staying Probabilities

Notes: Blue dots in the figure represent the k-year manufacturing staying probabilities. The red solid line represents the staying probabilities implied by the canonical model. The blue solid line represents the fit of the estimated two-type worker model. Data source: NLSY79.

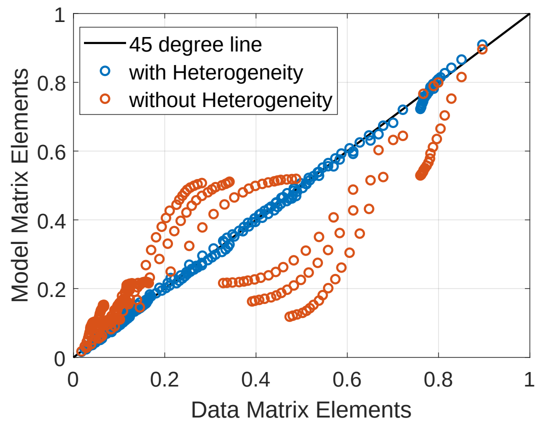


Figure 4. Actual and Model-Implied Worker Flow Matrices: All 4×4 Elements

Notes: Red dots in the figure plot elements of the worker flow matrix series implied by the canonical model against those in the data. Blue dots correspond to the estimated two-type worker model. Data source: NLSY79.

shows that this underestimation is not driven by the nonstationary nature of the data. The result remains qualitatively and quantitatively similar even when nonstationarity is taken into account.

As we have seen in Section 3, this discrepancy arises because the likelihood of choosing the manufacturing sector is higher for workers who have previously chosen the manufacturing sector. Workers who have self-selected to stay in manufacturing exhibit a higher probability of staying again in the following year, perhaps due to particularly high switching costs. Similarly, workers self-selected into manufacturing in the past are more likely to choose it again, possibly owing to their comparative advantage.

The canonical model also underestimates the diagonal elements of the worker flow matrices corresponding to the non-manufacturing sectors. In Figure 4, we plot all 4×4 elements of the worker flow matrix series implied by the canonical model against the actual values in the data, along with the 45-degree line. The points clustered below the 45 degree line correspond to the diagonal elements of the worker flow matrices that are underestimated by the canonical model. To compensate for this underestimation, the off-diagonal elements are overestimated, as seen

have

$$(F^1F^2)_{ii} \approx (F^1)_{ii}(F^2)_{ii} \leq \left(\frac{(F^1)_{ii} + (F^2)_{ii}}{2}\right)^2 = (\bar{F})^2_{ii} \approx (\bar{F}^2)_{ii}.$$

Thus, the implied two-period staying probability is overestimated under the stationarity assumption, leading to a *smaller* gap between the actual and implied staying probabilities.

in other points clustered above the 45-degree line. section quantifies how this inconsistency translates into systematic biases in counterfactual predictions of the effects of sectoral shocks.

4.3 Understanding the Bias of the Canonical Model

Demographic and Socioeconomic Characteristics. Where does the bias of the canonical model come from? One possibility is that worker heterogeneity in terms of observable characteristics could explain most of the bias. If so, we can easily correct for the bias by simply conditioning on these characteristics, obviating the need for our sufficient statistics approach. However, we will demonstrate that this is not at all the case. The literature has discussed various types of demographic and socioeconomic characteristics of workers. Here, we focus on three dimensions, gender, race, and education, which have been found to be important determinants of sectoral choices and welfare outcomes (e.g., Dix-Carneiro, 2014; Lee and Wolpin, 2006). We divided people into male and female, Hispanic/Black and non-Hispanic/Black, and low-skilled (less than high school and high school) and high-skilled (some college and college or more). Unique combinations of these three dimensions of heterogeneity define eight worker types. In Figure A.5, we plot the actual manufacturing staying probabilities separately for these eight worker types. Indeed, workers who differ along these characteristics exhibit highly distinct sectoral movement patterns. For example, low-skilled non-Hispanic/Black males are more than twice as likely to stay in the manufacturing sector in the longer-run than high-skilled Hispanic/Black females. However, Figure 5 shows that these characteristics do not explain the gap observed in Figure 3. We plot the manufacturing staying probabilities implied by the model that incorporates these three dimensions of observed characteristics. Specifically, we consider a model with the eight observed worker types. For each worker type ω^{obs}, we can calculate the 1-year transition matrix $(F^{\omega^{\text{obs}}})$ directly from the data. Since workers are assumed to be homogeneous within each of the eight types, their k-year transition matrix can then be computed by $(F^{\omega^{\text{obs}}})^k$. Thus, the model-implied aggregate k-year worker flow matrix is obtained by taking averages of these type-specific k-year transition matrices using the steady-state type composition as the weight. The green solid line shows the result and is almost indistinguishable from the red line, which plots the staying probabilities implied by the canonical model. These types of observed characteristics provide only a minor improvement in the model's ability to explain the actual pattern of worker flows. If we also incorporate age in the model, there is slight improvement in the fit, but the implied worker flow matrix

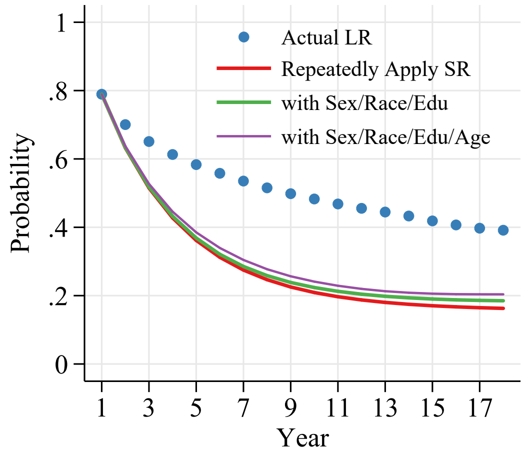


Figure 5. Predicted and Actual Staying Probabilities, with and without Socioeconomic Characteristics

Notes: The blue dots in the figure represent k-period manufacturing staying probabilities. The blue solid line represents the fit of the estimated two-type worker model. The red solid line represents the fit of the estimated canonical model. The green line incorporates three observed characteristics. The purple line additionally incorporates age. Data source: NLSY79.

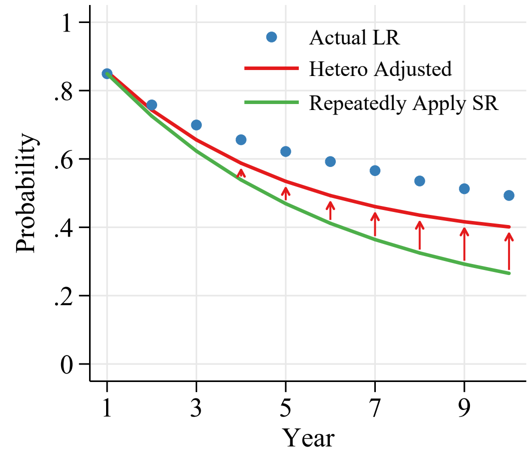


Figure 6. Predicted and Actual Staying Probabilities, with and without Socioeconomic Characteristics

Notes: The blue dots in the figure represent k-period manufacturing staying probabilities calculated using post-1990 data. The green and red lines represent the staying probabilities implied by the canonical model and the model with five worker types, respectively. Data source: NLSY79.

series still significantly differs from the actual one (purple solid line).[29] In sum, workers with different demographic and socioeconomic characteristics do indeed behave differently, but this fact barely changes *aggregate* labor market dynamics.[30] In turn, the sufficient statistics result implies that adding these worker characteristics into a model does not cause substantial changes to the results of the counterfactual analysis.[31]

Pure Duration Dependence. The limited role of demographic and socioeconomic characteristics implies two possibilities: either these characteristics are just poor proxies for underlying

[29] Age is a form of time-varying heterogeneity. To analyze this within the context of our persistent heterogeneity framework, we categorize age into two groups, "old" and "young," so that each represents about 50% of the total sample.

[30] Similar results can be found in various fields of economics. For example, Card, Rothstein, and Yi (2023) find that only a modest fraction of the variation in average wages across commuting zones is explained by differences in the observed characteristics of workers.

[31] This means that the discrepancy in Figure 3 is mostly driven by within-type heterogeneity instead of across-type heterogeneity. In ??, we demonstrate that we can observe similar gaps between actual and predicted longer-run staying probabilities for each of the eight types. In particular, the gaps are more pronounced for female Hispanic/Black workers.

worker heterogeneity, or the gap in Figure 3 arises from mechanisms other than worker heterogeneity, such as duration dependence. A notable example is the accumulation of sector-specific human capital, where otherwise identical workers who have spent more time in manufacturing may have accumulated more manufacturing-specific human capital, rendering them more likely to choose the sector again. Other examples include fixed adjustment costs (e.g., Stokey, 2008), learning about match productivity (e.g., Jovanovic, 1979), and psychological choice models (e.g., Cain, 1976).[32] This raises a concern, given that only the first equation of Proposition 1 extends to the case with a duration dependence mechanism.

In response to this issue, we provide suggestive evidence pointing to the importance of worker heterogeneity in generating the gap in Figure 3. Our strategy involves constructing an alternative proxy for worker heterogeneity. Instead of relying on demographic and socioeconomic characteristics, we leverage workers' sector choice histories prior to 1990 as a means to capture their heterogeneity. Differences among workers materialize as differences in their sector choice patterns, so their histories allow us to effectively control for their heterogeneity. Specifically, we use pre-1990 data to compute the 1-year manufacturing staying probability for each worker, then categorize workers into five types based on the quintiles of this probability. We then assess the extent to which accounting for these worker types narrows the gap between actual and model-implied staying probabilities. In Figure 6, the actual k-year staying probabilities calculated using post-1990 data are shown as blue dots, while the green and red lines represent the staying probabilities implied by the canonical model and the model with five worker types, respectively. Note that the red line is computed under the assumption that workers are homogeneous within each of the five worker types and that there is no duration dependence mechanism in play. The red line is close to blue dots, and the gap between the actual and implied staying probabilities is reduced by more than half. Given that worker heterogeneity is only partially controlled by sector choice history, this result suggests that at least half of the gap is due to worker heterogeneity, the mechanism emphasized in this paper.

4.4 Extrapolation Using the Structural Model

To apply sufficient statistics results, we need a full sequence of worker flow matrices from k equals one to infinity. Thus, we need a method to extrapolate longer-run worker flow matrices from the available finite-length data. Leveraging the structural model provides a natural method

[32] Distinguishing dynamic selection based on heterogeneity from duration dependence mechanisms is a recurring theme in various fields of economics. See Heckman (1981) for an important early contribution along these lines. Alvarez, Borovičková, and Shimer (2016) also study how to distinguish between the two in the context of unemployment duration.

Table 1: Estimation Results

$\lvert\Omega\rvert = 2$	Type ω_1 (31.1%)		Type ω_2 (68.9%)	
Sector	Wage	Switching Cost	Wage	Switching Cost
Agri/Const Manufacturing Commu/Trade Services/Others	$\begin{pmatrix} 0.58 \\ 0.60 \\ 0.66 \\ 0.77 \end{pmatrix}$	$\begin{pmatrix} 0.00 & 1.53 & 1.69 & 1.55 \\ 1.53 & 0.00 & 1.33 & 1.41 \\ 1.69 & 1.33 & 0.00 & 0.98 \\ 1.55 & 1.41 & 0.98 & 0.00 \end{pmatrix}$	$\begin{pmatrix} 1.02 \\ 1.02 \\ 1.00 \\ 1.07 \end{pmatrix}$	$\begin{pmatrix} 0.00 & 4.62 & 5.62 & 5.46 \\ 4.62 & 0.00 & 4.88 & 4.93 \\ 5.62 & 4.88 & 0.00 & 3.72 \\ 5.46 & 4.93 & 3.72 & 0.00 \end{pmatrix}$

for extrapolation.[33] For this purpose, we estimate the structural model by matching the worker flow matrices we computed directly from the NLSY data, $\{\mathcal{F}_k\}_{k=1}^{18}$. The estimated structural model generates the full set of worker flow matrices, which are used in Section 5 to perform counterfactual exercises.[34] Since the structural model reduces to the canonical model when there is only one worker type, this extrapolation is a strict generalization of the canonical model's extrapolation.

Specifically, we estimate the number of worker types along with their respective steady-state instantaneous utility vectors w_i^ω and switching costs C_{ij}^ω by matching 18 worker flow matrices (i.e., 216 moments). Note from the worker's sector-choice problem (1) that only the ratio between these values and the parameter ρ can be identified from the observed worker flow matrices. Thus, we only estimate these ratios until we estimate the value of ρ in Section 4.5. Following Assumption 2, we impose symmetry on the switching costs. The estimation process involves two steps: We first maximize the likelihood of observing $\{\mathcal{F}_k\}_{k=1}^{18}$ to estimate $\{\frac{1}{\rho}w_i^\omega, \frac{1}{\rho}C_{ij}^\omega\}$ for a given number of worker types, then use the Bayesian information criterion to determine the number of worker types; see Appendix B.1 for more details.

Table 1 shows the estimation result. The Bayesian information criterion supports the model with two worker types. Figure 7 plots the resulting transition matrix for each type of worker. The first type has a comparative advantage in non-manufacturing sectors and low switching costs. Thus, workers of this type switch sectors frequently, as indicated by the small diagonal elements of the transition matrix in Figure 7. In contrast, the second type has much higher switching costs, so workers of this type rarely move to other sectors. In Appendix B.4, we

[33] This approach has the advantage that extrapolation is disciplined by the model. However, the sufficient statistics approach does not, in principle, require that we estimate the details of the model. In Appendix B.3, we explore alternative extrapolation methods that do not rely on a structural model. In addition to extrapolation, there are two other benefits of the structural estimation. First, it serves as a proof of concept: By seeing whether our structural model can match the observed worker flow matrix series, we can test whether our framework is consistent with the data. Second, we can use the estimated structural model to evaluate the quality of the first-order approximation around a steady state. In Section 5, we compare the results of counterfactual exercises computed from the exact solution of the estimated model and those computed using the sufficient statistics result.

[34] In fact, we can simply treat the estimated model as if it were the true model and use it to perform counterfactual exercises directly. The sufficient statistics result guarantees that we will obtain correct counterfactual results regardless of whether this model is true.

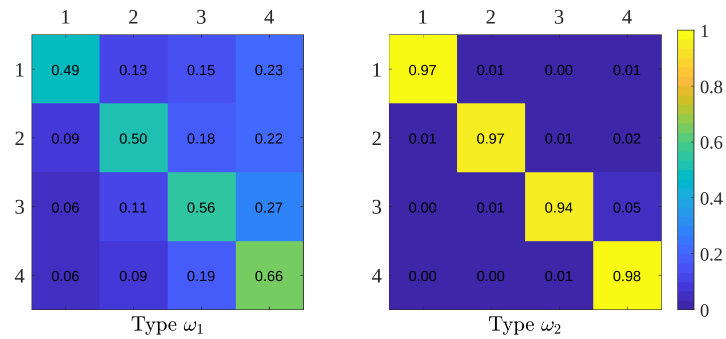


Figure 7. Type-Specific Transition Matrix

show how to interpret the figures in Table 1. In particular, paying one unit of switching costs means paying 3.25% of lifetime consumption. Thus, the switching costs in Table 1 are at most less than 20% of lifetime consumption. This is smaller than the estimates of ACM, who find that the *average* switching cost is at least 20% of lifetime consumption. Our estimates are close to those of Artuç and McLaren (2015), in which the switching costs are distributed around 12% of lifetime consumption.[35]

We also estimate primitives of the canonical by matching the one-period worker flow matrix, $\mathcal{F}_1$ (see Figure A.6 for the results). All parameters, including elements of the transition matrix, lie between the corresponding parameters for the model with two worker types.

Model Fit. Figures 3 and 4 document the fit of the estimated model. The blue solid line in Figure 3 represents the model-implied k-year staying probabilities for the manufacturing sector. The figure shows that the model with only two worker types closely matches actual short-run and longer-run staying probabilities. We obtain similar results for the other elements of the worker flow matrices. In Figure 4 we use blue dots to plot all 4×4 elements of the model-implied worker flow matrix series against the actual value in the data, along with the 45-degree line. Most of the dots lie roughly on the 45-degree line. The fit of the model is surprising for two reasons. First, the degrees of freedom (19 parameters) are much smaller than the number of moments we target (216 moments).[36] Second, the dynamic discrete choice

[35] As Artuç and McLaren (2015) argue, one reason for the smaller estimates is the inclusion of sector-specific nonpecuniary benefits in the model, which are absent in ACM's model.

[36] Suppose we want to match only the 1-year worker flow matrix. We can perfectly match this matrix with only one type of worker if we can choose an arbitrary transition matrix for this type. In terms of degrees of freedom, we match $N(N-1)$ values with $N(N-1)$ parameters, where $N = 4$ is the number of sectors. Suppose we also want to match one more worker flow matrix. This exercise can be seen as matching the level and slope of the dots in Figure 2. At least in terms of degrees of freedom, we can achieve this with only two types of workers: matching $2N(N-1)$ values with $2N(N-1)+1$ parameters ($N(N-1)$ parameters for each transition matrix, and 1 for the type share). However, we also want to match the overall shape of the dots in Figure 2, and confine ourselves to the case in which the type-specific transition matrices are generated by the structural primitives. Thus, we end up with 19 parameters that can be used to match 216 moments.

framework imposes systematic restrictions on the model-implied worker flow matrices, so we could not match any worker flow matrix series even with an arbitrary number of worker types.

The flexibility due to worker heterogeneity, characterized in Lemma 2, is necessary to match the observed worker flow matrices. As seen in Section 4.2, the canonical model with one type of workers fails to match the observed worker flow matrices. Table 1 clearly reveals why the canonical model significantly underestimates longer-run staying probabilities. Workers of the second type rarely change sectors and have comparative advantage in manufacturing. Thus, conditioning on the fact that workers have previously self-selected into the manufacturing sector greatly increases the probability that they are the second type, and thus increases the probability that they will stay or choose again the manufacturing sector in subsequent periods. While the fit of the model improves substantially with two types of workers, the additional increase in fit from adding more types of workers is negligible, causing the Bayesian information criterion to choose the two-type worker model. See Figure A.7 for comparison of the fits of the two-type model and the five-type model.

Identification. Comparing the fits of the models does not necessarily identify the true number of worker types, let alone the fact that two worker types cannot capture the multifaceted nature of real-world worker differences. However, a key feature of our approach is that it does not require identification of all elements of the true model. As long as the estimated model closely approximates the observed worker flow matrix series, our sufficient statistics result ensures that the model always provides valid counterfactuals for the outcome of interest—namely, aggregate welfare and employment. This feature distinguishes our approach from the latent variable approach in the literature, such as the finite mixture model and k-means clustering (e.g., Arcidiacono and Jones, 2003; Heckman and Singer, 1984; Bonhomme, Lamadon, and Manresa, 2022), in which the validity of counterfactual predictions requires a higher level of confidence in identification.

4.5 Estimation of ρ

In this section, we present a novel strategy for estimating the parameter ρ, which, in conjunction with the extrapolated worker flow matrix series, provides a complete description of the labor supply side of the model. Our goal is to propose an estimation method that does not require explicitly specifying the underlying heterogeneity.[37] The possibility of such an estimation

[37] In Propositions 1 and 2, we showed that the worker flow matrix series serves as sufficient statistics for counterfactual exercises *when we know the value of the parameter ρ.* However, if the estimation procedure for ρ depends on how we specify worker heterogeneity, the distribution of worker heterogeneity can affect the results of counterfactual exercises through its effect on estimation of ρ. In this sense, our estimation method is comparable to estimation of the trade elasticity using the gravity equation of Arkolakis, Costinot, and Rodríguez-Clare (2012).

is already suggested in the second equation of Proposition 1, which we restate here for convenience:

$$\mathrm{d}\ln \ell_t = \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k}. \tag{15}$$

This equation describes the response of sectoral employment to wage shocks, with the coefficients depending only on the value of ρ and the worker flow matrix series, independent of the specific details of worker heterogeneity. Thus, we can estimate ρ by measuring the responsiveness of sectoral employment to sectoral wage shocks, conditional on the observed worker flow matrix series.[38]

Implementation. In principle, equation (15) could be directly confronted with data to estimate ρ, but this requires that we fully specify how much information workers have about the future. The literature circumvents this demanding requirement by applying the Euler equation approach first used in ACM (e.g., Artuç and McLaren, 2015; Caliendo, Dvorkin, and Parro, 2019; Traiberman, 2019). We extend the Euler equation approach by allowing for arbitrary worker heterogeneity.[39]

The key idea of the Euler equation approach is to transform (15) into a recursive representation. For example, in the absence of persistent worker heterogeneity, we can use the restriction $\mathcal{F}_k = (\mathcal{F}_1)^k$ to rewrite equation (15) recursively as follows:[40]

$$\mathrm{d}\ln \ell_t = (\mathcal{F}_1^{-1} + \beta\mathcal{F}_1)\,\mathrm{d}\ln \ell_{t+1} - \beta\mathbb{E}_t\,\mathrm{d}\ln \ell_{t+2} - \tfrac{\beta}{\rho}(\mathcal{F}_1^{-1} - \mathcal{F}_1)\mathbb{E}_t\,\mathrm{d}w_{t+1}. \tag{16}$$

See Appendix B.2 for proofs of the results in this section. However, with arbitrary worker heterogeneity, this is impossible because the worker flow matrix series that determines the coefficients of equation (15) is based on empirical data and does not have a recursive structure. Nevertheless, we will demonstrate in two steps that it is still possible to derive an *approximate* recursive representation of equation (15). First, in Appendix B.2, we show that when there is a finite number of worker types, N, equation (15) always possesses an *exact* recursive

Just as their estimation based on the gravity equation provides a valid estimate within a class of "quantitative trade models," our method yields a valid estimate within a specific class of dynamic discrete choice models.

[38] The responsiveness is inversely related to the value of ρ. A higher value of ρ indicates that sector choice decisions are primarily driven by idiosyncratic shocks. As a consequence, the impact of wage shocks on sectoral employment is relatively diminished when ρ is high.

[39] ACM (p.1040) write that "Perhaps the biggest weakness in the Euler-equation approach … is that it assumes away workers with unobserved [heterogeneity] …. A full exploration of such effects probably requires a structural micro approach." However, the method described in this section demonstrates that up to the first-order approximation, this problem can be circumvented.

[40] Unlike equation (15), equation (16) contains only the term $\mathbb{E}_t\,\mathrm{d}w_{t+1}$ since the (expected) values of $\mathrm{d}\ln \ell_{t+1}$ and $\mathrm{d}\ln \ell_{t+2}$ summarize the effect of all other beliefs. Because of this advantage, the Euler equation approach has been widely used in the literature, although previous studies have used equations that involve migration probabilities rather than labor supply.

representation of the form[41]

$$\mathrm{d}\ln \ell_t = \sum_{k=1}^{4N-2} \Gamma_k \mathbb{E}_t \,\mathrm{d}\ln \ell_{t+k} + \tfrac{\beta}{\rho} \sum_{k=1}^{4N-3} \Lambda_k \mathbb{E}_t \,\mathrm{d}w_{t+k} \tag{17}$$

where Γ_k and Λ_k are functions of worker flow matrix series, $\{\mathcal{F}_k\}$. Second, recall from Section 4.4 that a model with two worker types provides a close approximation to the observed worker flow matrix series $\{\mathcal{F}_k\}$ in the data. Combining these two findings, we can conclude that equation (15), with $\{\mathcal{F}_k\}$ from the data, can be approximately represented in the recursive form (17) with $N = 2$. In Appendix B.2.1, we provide formulas for computing $\{\Gamma_k\}_{k=1,\ldots,6}$ and $\{\Lambda_k\}_{k=1,\ldots,5}$. In Figure A.8, we use simulation to demonstrate that the obtained approximate recursive representation provides a close fit to the actual dynamics of sectoral employment.

We further modify the obtained recursive representation in two ways:

$$\ln \ell_t - \sum_{k=1}^{6} \Gamma_k \ln \ell_{t+k} = \tfrac{\beta}{\rho} \sum_{k=1}^{5} \Lambda_k w_{t+k} + \mathsf{ExpectationError}_{t+1,t+6}\,. \tag{18}$$

First, instead of deviations from steady-state values, $\mathrm{d}\ln \ell$ and $\mathrm{d}w$, we use the actual values, $\ln \ell$ and w. This is possible because the recursive representation always satisfies $\sum_{k=1}^{6} \Gamma_k = I$ and $\sum_{k=1}^{5} \Lambda_k = O$, where I is the identity matrix and O is the zero matrix. Second, the expected values are substituted with the realized values plus an expectation error term that depends on the news revealed between time $t+1$ and $t+6$.

Equation (18) is our regression specification, where we regress the left-hand-side variable on the explanatory variable on the right-hand side, $\sum_{k=1}^{5} \Lambda_k \,\mathrm{d}w_{t+k}$, to estimate $\frac{\beta}{\rho}$. However, since both the explanatory variable and the expectation error term are affected by newly revealed information between period $t+1$ and $t+6$, they are likely to be correlated. To address this concern, we follow ACM and use past values of sectoral labor allocation and wages as instruments. Any variables included in the period t information set are theoretically valid instruments for the explanatory variable, providing a consistent estimate of ρ. In particular, we use the 1-year lag of sectoral wages and sectoral employment shares. For this approach to be valid, it is necessary to assume that workers have rational expectations and that the error term in equation (18) only reflects errors in workers' forecasts. If this term also incorporates shocks to the labor supply curve (e.g., unexpected aggregate shifts in preferences for particular sectors), we must assume an exclusion restriction whereby such shocks are uncorrelated with our instruments. For a discussion of the strengths and weaknesses of this approach in the case

[41] For further insights into how this result relates to the findings of Granger and Morris (1976), see Appendix B.2.

Table 2: Estimation of ρ

	(1)	(2)	(3)
β/ρ	-0.286	1.164**	0.877***
	(0.311)	(0.550)	(0.296)
Implied ρ	-3.358	0.825**	1.095***
	(3.652)	(0.390)	(0.369)
Method	OLS	IV	IV
Persistent Heterogeneity	✓	✓	
No Persistent Heterogeneity			✓
Observations	136	136	136
First-stage F	–	38.7	11.1

Notes: OLS and IV estimation results for specification (18) (heterogeneous workers: Columns (1) and (2)) and (16) (homogeneous workers: Column (3)). Standard errors robust to heteroskedasticity are reported in parentheses, with $^{***}: p < 0.01$. Data source is NLSY79 and BLS.

of homogeneous workers, refer to ACM and Traiberman (2019). Also, we include sector fixed effects in the regression to isolate within-sector variation from across-sector variation.

Our estimation method requires data on sectoral employment and sectoral wages. We use the Bureau of Labor Statistics' Current Employment Surveys (CES) to compute the time series of these variables. We use the share of workers in the dataset employed in each sector as our measure of sectoral labor allocation and average wages in each sector as our measure of sectoral wages. We again consider four broad sectors.[42] In Section 5, we will compare the counterfactual predictions of our model with those of Caliendo, Dvorkin, and Parro (2019), who analyze a homogeneous counterpart of our model. To facilitate clear comparison, we assume that the variable w represents the *log* sectoral wage.

Table 2 presents the estimation results. Column (1) estimates equation (18) by OLS. The estimated coefficient and the implied value of ρ are negative and insignificant. In Column (2), we estimate the same specification using IV. The implied value of ρ is 0.825. This means that a one standard deviation higher realization of the idiosyncratic shock is associated with 4.06% higher lifetime consumption (see Appendix B.4). Comparing the results in Columns (1) and (2), it appears that the estimated coefficient $\widehat{\beta/\rho}$ from OLS is biased downward due to the presence

[42] The assumption of a four-sector model raises an econometric issue similar to the one discussed by ACM. If the true model consists of more than four sectors with a dispersion parameter ρ, is it valid to approximate the model with a four-sector model and estimate the dispersion parameter based on this approximation? More importantly, can we use it to conduct counterfactual exercises? We can demonstrate that under certain assumptions on switching costs, the validity of using the approximated model is supported by the fact that the maximum of i.i.d. type I extreme-value distributed random variables follows another type I extreme-value distribution with the same scale parameter.

of expectation errors. This is consistent with the fact that expectation errors are likely to cause sectoral labor supply and sectoral wages to be negatively correlated.[43] The estimate in Column (2) implies that a temporary 1% decline in manufacturing wages leads to a 0.35% decrease in the manufacturing share, while a permanent 1% decline in manufacturing wages is associated with a 1.15% decrease in the manufacturing share. In column (3), we assume that workers are homogeneous and estimate equation (16) by IV with the same set of instrument variables. The implied value of ρ is slightly higher than our preferred estimate, but we cannot reject the null of equality between the two. Estimates in Columns (2) and (3) are broadly consistent with the estimates of ρ in the literature, which range from 0.5 to 2. The original ACM and subsequent papers estimate ρ to be around two. A more recent paper, Artuç and McLaren (2015), suggests a value of $\rho = 0.62$. Also, Rodriguez-Clare, Ulate, and Vasquez (2022) obtain a similar value of $\rho = 0.56$.[44] This estimate in Column (2) is our preferred specification, and we will use it in the subsequent analysis. However, our purpose is to see how the results of the counterfactual exercises would change if we accounted for worker heterogeneity—not to see how different values of ρ would change the results. Thus, we will also report the results of counterfactual exercises under $\rho = 0.5$ and $\rho = 2$ in the Appendix.

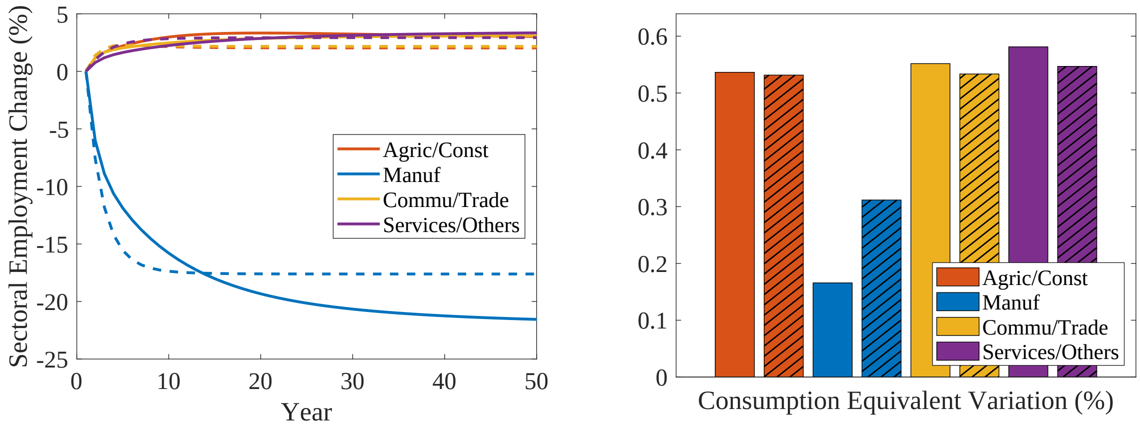
We now move on to the application of our model, in which we use the estimated model with worker heterogeneity to quantify the effect of sectoral shocks.

5. Applications

In this section, we consider two different sectoral shocks and quantify the implication of persistent worker heterogeneity for welfare and labor reallocation. We first apply our results to a stylized trade liberalization exercise of ACM, which illustrates how the framework of the literature can easily be extended to allow for worker heterogeneity. For a more realistic application, we then examine the dynamic effects of the rise in China's import competition on the US labor markets. We revisit this extensively studied topic using the sufficient statistics approach. Although we focus on these two exercises, our result can be applied more generally to other papers in the literature.

[43] If workers wrongly expect an increase in wages in a sector, they would supply more labor to that sector. This increased labor supply would lead to a decrease in wages in that sector.

[44] Note that ACM and other papers in the literature assume instantaneous utility linear in wage. This implies that the value of ρ governs semi-elasticity $\frac{\partial \ln \ell_{t+k}}{\partial \text{wage}_t}$ instead of elasticity $\frac{\partial \ln \ell_{t+k}}{\partial \ln \text{wage}_t}$. However, because they normalize sectoral wages so that the average annualized wage equals unity, both the semi-elasticity and elasticity can be interpreted as the percentage change in sectoral employment in response to a percentage change in sectoral wages.



(a) Changes in Sectoral Employment Share

(b) Changes in Sectoral Values

Figure 8. Counterfactual Changes in Sectoral Employment and Welfare: Trade Liberalization

Notes: This figure plots the transitional dynamics following an unexpected permanent drop in manufacturing prices. Solid lines correspond to the prediction from the sufficient statistics in the data, and dashed lines correspond to the prediction of the canonical model, without persistent worker heterogeneity.

5.1 Permanent Decline in Manufacturing Prices

The first counterfactual exercise closely follows ACM and considers an unanticipated permanent 10% drop in manufacturing prices for a small open economy—for example, due to trade liberalization. Following Section 2, we incorporate persistent worker heterogeneity in the labor supply side of the model, which is calibrated in Section 4. We specify the labor demand side of the model as in ACM. We assume log utility with Cobb-Douglas consumption aggregate and a sector-specific CES production function with fixed capital stock. All goods are traded and their prices are exogenously given at the world price level. Sectoral wages are competitively determined by the marginal productivity of labor. The labor demand side of the model is calibrated exactly as in ACM, see Appendix C.1 for details. Initially, the economy is in a steady state. Since wages are endogenously determined by the labor market equilibrium, we apply Proposition 2 to compute perfect-foresight transition path following announcement of the shock in year 1 until the economy reaches a new steady state.

Figure 8 shows results of the counterfactual exercise. In Figure 8a, we plot the dynamics of sectoral employment predicted by the models with and without worker heterogeneity. The manufacturing employment share drops sharply, by around 20%. Importantly, The canonical model overestimates the short-term labor reallocation, resulting in a much faster transition to the new steady state. The transition is completed within 10 years in the absence of worker heterogeneity, while it takes more than 50 years with worker heterogeneity. At the same time, the canonical model underestimates the magnitude of long-term reallocation. All of these results are consistent with the predictions of Proposition 4.[45]

In Figure 8b, we plot changes in welfare, measured in terms of consumption-equivalent variation; that is, the proportional change in lifetime consumption sequence that would have the same effect on household welfare as would the welfare effect of the China shock. In particular, plain bars represent the welfare changes of workers based on their initial sector of employment predicted by the model. Hatched bars represent the welfare changes implied by the canonical model. The result shows that even workers who were initially employed in manufacturing—the import-competing sector—benefit from trade liberalization. The increase in option value, driven by the increase in real wages in other sectors, more than compensates for the decline in manufacturing wages.[46] However, consistent with the prediction of Proposition 3, the canonical model overestimates the gains of manufacturing workers by nearly a factor of two. At the same time, it slightly underestimates the gains of non-manufacturing sector workers, resulting in a substantial underestimation of the distributional consequences of trade liberalization.

As discussed in Section 3, the welfare gains of manufacturing workers are overestimated in the canonical model for two reasons. Not only does it overestimate welfare gains for given wage changes, but it also predicts a smaller decline in manufacturing wages in the short term. To illustrate this, we show in Figure A.10 that if we used the changes in sectoral wages computed from the canonical model—instead of endogenizing them—and combined them with the worker flow matrix series as in Proposition 1, we would get a narrower gap between the welfare changes predicted by models with and without worker heterogeneity.

To gain a deeper understanding of the disparities between models with and without heterogeneity, we plot changes in sectoral values and employment shares separately for each of the two worker types in Figure A.11.[47] When the shock hits the economy, type 1 workers, who have

[45] In Appendix C.1, we compute the impulse responses of sectoral employment from the observed worker flow matrix series. As predicted by Proposition A.4, the canonical model tends to overestimate the short-term impact of shocks on sectoral employment but underestimates their long-term effects. Notably, within a 7-year time horizon, the canonical model consistently overestimates the effects of shocks on sectoral employment. This explains the difference between changes in sectoral employment in the models with and without worker heterogeneity documented in Figure 8a.

[46] Figure A.9 plots the time path of changes in sectoral wages. The real wage of the manufacturing sector initially overshoots because it takes time for labor to adjust. As the number of workers in manufacturing gradually declines, manufacturing wages rise and eventually exceed the preshock steady-state level. However, they increase less than those in other sectors. Thus, manufacturing employment share declines over time and in the new steady state.

[47] An important caveat to this interpretation pertains to the identification of the structural model. In estimating the structural model, many configurations of the model primitives are almost equally successful in matching the worker flow matrix series. This means that even small changes in the observed matrices can lead to significantly different estimation results. Thus, statements made at the unobserved type level should be viewed with caution, as they may be far from the true world.

lower switching costs and comparative advantages in non-manufacturing sectors, can easily move out of manufacturing and enjoy a higher welfare gain from the shock. Over time, as more type 2 workers leave the manufacturing sector, manufacturing wages begin to recover. Given that type 2 workers are more likely to be stuck in the manufacturing sector once they choose it, they dislike manufacturing more than type 1 workers, resulting in a higher proportion of type 1 workers in the manufacturing sector in the long run.

Finally, we assess the quality of the first-order approximation around a steady state. In Figure A.12, we compare the results of the counterfactual exercises obtained using sufficient statistics formulas with those calculated from the exact solution of the estimated structural model.[48] Despite the relatively large magnitude of the 10% shock considered in this section, the sufficient statistics results deliver a close approximation. In particular, the approximation error is an order of magnitude smaller than the difference between the counterfactual predictions of the models with and without worker heterogeneity. The approximation error becomes almost negligible for a shock size of 1%, but becomes more pronounced as the shock size increases to 30%.

5.2 The China Shock

As a second application, we apply our sufficient statistics result to a more realistic counterfactual exercise: the dynamic impact of the growth of China's manufacturing productivity and the resulting import competition on welfare of US workers and labor reallocation. We closely follow the dynamic quantitative trade model of Caliendo, Dvorkin, and Parro (2019) (hereafter, CDP) and extend the labor supply side by allowing for arbitrary persistent worker heterogeneity.[49] We make two simplifying assumptions relative to the original specification of CDP. First, we consider 4 broad sectors—Manufacturing, Wholesale/Retail, Construction, and Services—and another auxiliary sector representing nonemployment.[50] Second, for reasons discussed shortly, we abstract from interstate migration and assume that workers can switch sectors only within

[48] Figure A.12 also demonstrates that the exact value changes always exceed those calculated using a first-order approximation. We can show analytically that the second-order term is always positive due to the option value.

[49] A large body of subsequent literature studies additional elements that are missing in this framework: involuntary unemployment from downward nominal wage rigidities or search frictions (Kim and Vogel, 2021; Rodriguez-Clare, Ulate, and Vasquez, 2022); endogenous trade imbalances (Dix-Carneiro et al., 2023); occupation adjustment (Traiberman, 2019); and learning and expectations (Fan, Hong, and Parro, 2023). Incorporating worker heterogeneity in models with these additional features is an important direction for future research.

[50] In the CDP, there are 23 sectors: 12 from Manufacturing, 8 from Services, and 1 each for Wholesale/Retail, Construction, and nonemployment.

each state.[51,52] We first describe the labor supply side, then specify the labor demand side and the shock of interest to close the model.

Labor Supply Side. Two observations motivate us to conduct a separate analysis for each of the 50 US states. First, it is well known that there is considerable variation in the exposure to the China shock across different geographic regions in the US (see, for example, Autor, Dorn, and Hanson, 2013; Acemoglu et al., 2016). This suggests that Assumption 1 is better imposed within each state. Second, as shown in Figure A.15, worker flow matrices differ significantly across states; workers in states with a higher manufacturing employment share are more likely to remain in the manufacturing sector over time. By applying the sufficient statistics approach at the state level, we can account for such state-level heterogeneity. Under our simplifying assumptions, we can focus on 5-by-5 intersectoral worker flow matrices for each state.

State-level analysis requires computation of a worker flow matrix series for each state. However, the limited sample size of the NLSY data makes it difficult to estimate them accurately. Thus, we instead use the monthly Current Population Survey (CPS) dataset, which contains a substantial sample size for each state. This dataset tracks workers for 4 consecutive months, which allows us to compute worker flow matrices $\mathcal{F}_k^{\text{state}}$ for $k = 1, 2, 3$ months for each state. Specifically, we assume that the US was in a steady state before the China shock and compute worker flows by pooling transition observations between January 1995 and December 1999.[53,54] To compute worker flow matrices for other values of k, we

[51] CDP uses an 1150-by-1150 quarterly worker flow matrix between all US state-sector pairs (50 states, excluding the District of Columbia and the territories, and 22 sectors plus 1 additional sector representing non-employment). However, we need to estimate the longer-run worker flow matrix as well as the short-run one, and estimating them at this level of granularity is practically impossible.

[52] In principle, allowing for regional migration would dampen the welfare effects because it provides an additional margin of adjustment. However, US workers change sectors nearly 10 to 100 times more often than they change states, suggesting that the majority of the labor adjustment occurs at the sector change margin. In the same context of the China shock, Rodriguez-Clare, Ulate, and Vasquez (2022) also show that ignoring migration makes little difference to their model's prediction. Autor, Dorn, and Hanson (2013) and Autor et al. (2014) also demonstrate that regional migration is not an important mechanism through which the economy adjusts to the China shock.

[53] In contrast to the literature, which often uses only one worker flow matrix—either monthly ($k = 1$) or quarterly ($k = 3$)—we exploit the information contained in all worker flow matrices to identify underlying persistent worker heterogeneity. For any k, if we use the k-period worker flow matrix $\mathcal{F}_k^{\text{state}}$ and rely on the homogeneous-worker assumption to calculate the remaining worker flow matrices, we would underestimate the diagonal elements of k'-period worker flow matrices for k' greater than k.

[54] The monthly CPS dataset also tracks workers again for 4 additional consecutive months after 8 months after the first 4 consecutive months. Thus, in principle, we can observe $\mathcal{F}_k^{\text{state}}$ for $k = 1, 2, 3, 12, 13, 14, 15$. However, it is well known that the monthly CPS dataset suffers from underestimation of staying probabilities when comparing the first 4 months with the second 4 months because sectors are coded independently between these months (e.g., Kambourov and Manovskii, 2013). This can be clearly seen in Figure A.16, where we plot the manufacturing staying probabilities. This underestimation is critical to our analysis, so we focus only on the first three worker flow matrices. This also minimizes concerns from sample attrition and the resulting selection (e.g., Moscarini and Thomsson, 2007).

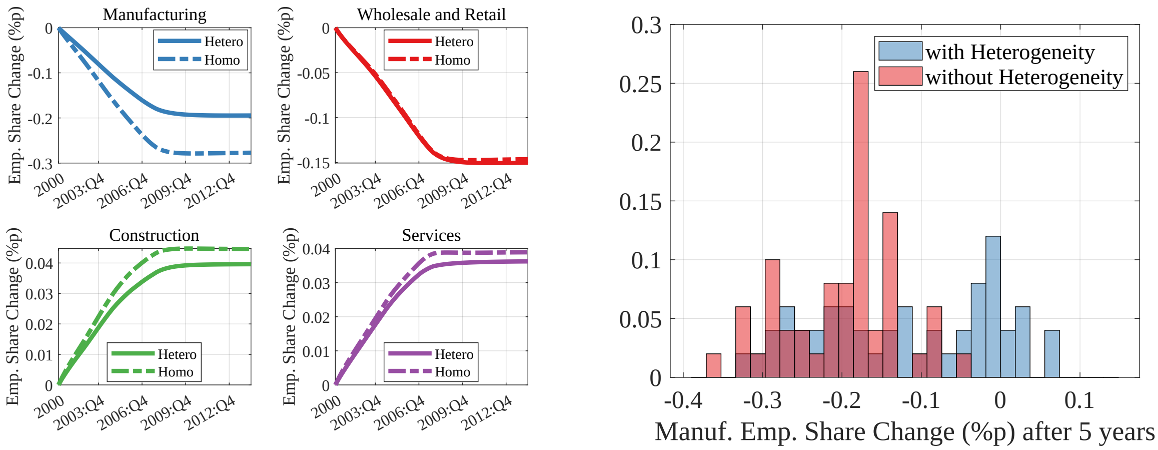
follow Section 4 and estimate the structural model with two types of workers that best match the observed worker flow matrices. We then use the estimated model to extrapolate longer-run worker flow matrix series.[55] In Figure A.17, we compare the fits of the models with and without worker heterogeneity to the observed worker flow matrix series. While the fits of the models vary across states, the canonical model consistently underestimates the staying probabilities. Finally, we use the value of the parameter ρ estimated in Section 4.[56]

Labor Demand Side and the China Shock. The labor demand side of the model is more complex than in Section 5.1: It features a large number of labor markets distinguished by sector and region, international trade, interregional trade within the US, input-output linkages, and multiple production inputs. Appendix C.2 describes the primitive assumptions of the model regarding households' consumption and sector choices; intermediate goods and final goods producing firms' profit maximization; and the definition of a sequential competitive equilibrium. We follow CDP in calibrating the structural parameters of the labor demand side; see Appendix C.2 for details of the calibration. The shock of interest is the growth of China's manufacturing productivity. Following CDP, we consider the China shock as a sequence of shocks to the growth rate of total factor productivity (TFP) of the Chinese manufacturing sector from 2000 to 2007, assuming a constant fundamental thereafter. We also assume that US agents anticipated the China shock in 2000 exactly as it occurred. We calibrate the manufacturing productivity growth such that the model's predicted increase in US imports from China exactly matches the predicted increase in imports, using the increase in imports from China of the other eight advanced economies as an instrument. See Appendix C.1 for the detailed calibration of the China shock.

Counterfactual Results. For exogenous changes in the manufacturing productivity of China, US sectoral wages are endogenously determined by the labor market equilibrium. Thus, we again use Proposition 2 to calculate counterfactual changes in sectoral welfare and sectoral employment. Given the rich structure of the model, it is computationally demanding to estimate all exogenous state variables of the model—including productivities, labor mobility costs, and trade costs—for every period. We reduce the computational burden by extending the CDP's dynamic hat algebra to models with arbitrary worker heterogeneity using our sufficient statistics

[55] One concern is that this requires too much extrapolation. In Figure A.19, we compare the extrapolated state-specific worker flow matrix series with the aggregate worker flow matrix series we computed in Section 4. Reassuringly, the average state-specific worker flow matrix series behaves very similarly to the aggregate series.

[56] The estimated ρ is at the opposite extreme of the CDP's estimate within the range of estimates in the literature. Thus, we also report results under the CDP's estimate in Section 4.5. Another issue is that we estimate the parameter ρ at an annual frequency. In Appendix C.2, we present a way to transform this to quarterly frequency. The resulting value is given by $\rho = 1.0011$.



(a) Manufacturing Employment: Aggregate Effect

(b) Manufacturing Employment: Individual States

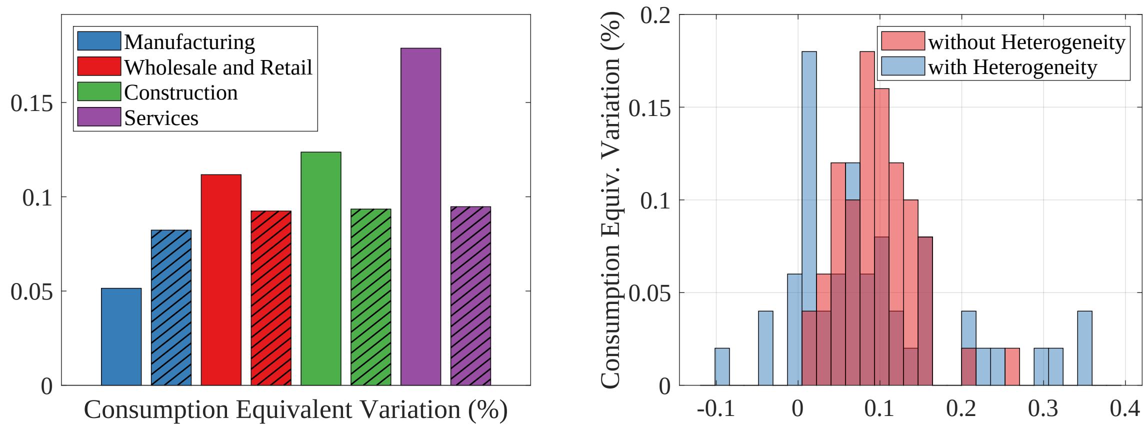
Figure 9. Effect of the China Shock on Welfare

result; see Appendix C.2 for details.[57] Figures 9 and 10 plot results of counterfactual exercises computed using the sufficient statistics approach.

Sectoral Employment Changes. Figure 9a plots the dynamic response of US sectoral employment to the China shock for models with and without worker heterogeneity. In both models, the increase in the manufacturing productivity of China shifts manufacturing production from the US to China, resulting in a decline in the share of US manufacturing employment (top left panel). With worker heterogeneity, the China shock reduces the share of manufacturing employment by around 0.2 percentage points (or, equivalently, 0.45 million manufacturing jobs) after 10 years. At the same time, the China shock increases employment in the construction and services sectors, as observed in the data. These sectors benefit from access to cheaper intermediate goods made available by the China shock. Dashed lines plot the changes in the sectoral employment share predicted by the canonical model. As expected from Proposition 4, the canonical model consistently overestimates the extent of labor reallocation by up to 50%. Figure 9b presents a histogram of state-level changes in manufacturing employment after 5 years. The impact of the China shock varies across states in both models, but in line with Figure 9a, the contraction of manufacturing is faster in the canonical model.

Welfare Changes. In terms of welfare, the heterogeneous-worker model predicts a 0.09% increase (in terms of consumption-equivalent variation), which is similar to the 0.11% welfare

[57] Dynamic hat algebra solves the equilibrium of the model in terms of time differences and differences between the actual and counterfactual economies. This method allows one to perform counterfactual exercises without the need to estimate the level of exogenous state variables of the model. For this advantage, it is widely used in the literature (e.g., Rodriguez-Clare, Ulate, and Vasquez, 2022; Caliendo et al., 2021; Balboni, 2019; Kleinman, Liu, and Redding, 2023).



(a) Sectoral Welfare Changes: Aggregate Effect

(b) Manufacturing Welfare Changes: Individual States

Figure 10. Effect of the China Shock on Sectoral Employment

change predicted by the canonical model. Despite this similarity, the two models differ in their predictions regarding the distributional conflicts—how much the winners win and the losers lose. In Figure 10a, we plot sectoral welfare changes measured at the time the China shock was known to the US labor market. Again, plain bars represent the sectoral welfare changes predicted using the sufficient statistics in the data, and hatched bars represent those implied by the canonical model. As in Section 5.1, both models predict that even workers who were initially employed in the manufacturing sector benefit from the China shock. However, as expected from the results of Proposition 3, the canonical model significantly overestimates the welfare gain of manufacturing workers, but underestimates the welfare gain of workers in non-manufacturing sectors. Thus, the model significantly underestimates the distributional impact of the China shock. In particular, in the absence of worker heterogeneity, workers who were initially employed in the manufacturing sector enjoy about the same welfare gains as non-manufacturing workers. However, once we account for worker heterogeneity and correctly match longer-run worker flow patterns, their average gain becomes less than half of the gains of workers in the other sectors.

Similar to employment effects, the welfare impact of China's import competition varies substantially across regions. In Figure 10b, we present a histogram of state-level changes in the welfare of manufacturing workers. In the canonical model, manufacturing workers in all states benefit from the China shock. In contrast, the heterogeneous-worker model predicts that the welfare gain of manufacturing workers is close to zero in most states, and manufacturing workers from 5 states—Alabama, Illinois, Louisiana, Michigan, and Ohio—

experience a decline in welfare.[58] These states have the highest manufacturing employment shares, experiencing higher reallocation from manufacturing to nonemployment. The figure also shows that worker heterogeneity not only amplifies the negative welfare effects of the China shock but also increases regional disparities in the welfare effects on manufacturing workers.

Figure A.21 presents the percentage changes in sectoral wages averaged across states induced by the China shock. The average effect is positive for all sectors, although some states experience declines in the manufacturing real wage. In contrast, non-manufacturing sectors experience wage increases in all states. More importantly, the heterogeneous-worker model predicts slower labor reallocation from manufacturing to non-manufacturing, resulting in a larger decline in manufacturing wages. Motivated by this observation, in Figure A.22 we plot sectoral welfare changes calculated by combining the worker flow matrix series implied by the heterogeneous-worker model and sectoral real wage changes implied by the model without worker heterogeneity. The result implies that around a quarter of the welfare change gap between the two models arises from this difference in real wage changes, and the remaining three-quarters result from differences in the worker flow matrix series. This highlights the importance of endogenizing wage changes when studying the role of worker heterogeneity.

In sum, results of the counterfactual exercises demonstrate the quantitative importance of accounting for worker heterogeneity.

6. Concluding Remarks

Large economic shocks often have asymmetric effects across different sectors. Such shocks can necessitate a substantial labor reallocation across sectors and may have significant distributional consequences for workers employed in different sectors, raising a paramount concern for policymakers. The key determinant of the dynamic effects of sectoral shocks is the ease with which workers can switch sectors over time. In this paper, we develop a dynamic sector choice model that incorporates a self-selection mechanism based on persistent worker heterogeneity in an otherwise standard dynamic discrete choice framework. Our sufficient statistics approach, which relies on the information contained in steady-state worker flows over various horizons, highlights in a transparent way the critical role of this self-selection mechanism in shaping the dynamic effects of sectoral shocks. Assuming away persistent worker heterogeneity results

[58] Workers from these five states continue to experience negative welfare effects even in the long run, which is in stark contrast to the findings in the literature.

in overestimation of steady-state worker flows, which in turn leads to underestimation of the distributional consequences and overestimation of the speed of labor reallocation.

By revisiting the two applications in the literature with empirical estimates of the sufficient statistics, we illustrate the applicability of our approach and quantify the importance of the added flexibility from worker heterogeneity. Our results present a more pessimistic view of the consequences of sectoral shocks: The reallocation of workers is significantly slower, and the welfare losses of adversely affected workers are more severe than previously suggested. Although we have focused on specific applications in this paper, the general insights we have developed could be applied more broadly—not only to other sectoral shocks but also to models incorporating richer mechanisms. For example, if involuntary unemployment is considered, our finding that workers are more likely to be stuck in a negatively affected sector will materialize as a higher unemployment rate in that sector, leading to even greater welfare losses. Applying our approach in the context of different shocks and to models with additional structures is an important direction for future research.

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Appendix

A. Theoretical Appendix

A.1 Aggregation Result: Proposition 1

In this section, we first prove a general result that does not rely on Assumption 2. In the next section, we show how Assumption 2 simplifies this result. To state the general result, we define another type of worker flow matrices. As we will demonstrate, two types of worker flow matrices are sufficient statistics for characterizing the welfare and labor market consequences of sectoral shocks.

Definition A.1. *For each $m, k \in \mathbb{N}$, the (m, k)-period* worker flow matrix*, is an $S \times S$ matrix denoted as $\mathcal{F}_{m,k}$ whose (i, j)-element is given by*

$$(\mathcal{F}_{m,k})_{i,j} = \Pr(s_{\tau(t,m)+k} = j | s_t = i),$$

where the random variable $\tau(t, m) \equiv \min\{\tau \geq t : s_\tau = s_{t-m}\}$ denotes the first period in which a worker returns to the sector she chose m periods ago.

The (i, j)-element of the (m, k)-period worker flow matrix equals the steady-state probability that a randomly selected worker from sector i will move to sector j after k periods after returning to the sector she chose m periods ago.

To formalize the idea of aggregation, we define the population-average operator.

Definition A.2. *The* population-average operator $\bar{\mathbb{E}}_\omega$ *is an operator that can be applied to $S \times N$ matrices for any $N \in \mathbb{N}$. It maps a type-specific matrix to an aggregate matrix,*

$$\begin{pmatrix} a_{11}^\omega & \cdots & a_{1N}^\omega \\ \vdots & \ddots & \vdots \\ a_{S1}^\omega & \cdots & a_{SN}^\omega \end{pmatrix} \mapsto \sum_\omega \begin{pmatrix} \tilde{\ell}_1^\omega a_{11}^\omega & \cdots & \tilde{\ell}_1^\omega a_{1N}^\omega \\ \vdots & \ddots & \vdots \\ \tilde{\ell}_S^\omega a_{S1}^\omega & \cdots & \tilde{\ell}_S^\omega a_{SN}^\omega \end{pmatrix}$$

where $\tilde{\ell}_i^\omega = \frac{\ell_i^\omega}{\sum_{\omega'} \ell_i^{\omega'}} = \Pr(\omega | s_t = i)$ is the steady-state proportion of type ω in sector i.

If the i-th row of a matrix contains variables related to sector i, then the steady-state type distribution of sector i gives the appropriate weights for computing the average across different types. This is precisely how the population-average operator is defined. The following lemma shows that certain type-specific variables can be converted to their aggregate equivalents through the application of the population-average operator.[59]

[59] The population-average operator does not transform all type-specific variables into their aggregate counterparts. For example, nonlinear functions of $\mathrm{d}\ln \ell_{t+1}^\omega$, $\mathrm{d}v_t^\omega$, or $(F^\omega)^k$ do not possess this property (e.g., $\bar{\mathbb{E}}_w \, \mathrm{d}\ell_{t+1}^\omega \neq \mathrm{d}\ell_{t+1}$).

Lemma A.1. *If we apply the population-average operator to* $\mathrm{d}\ln \ell_{t+1}^{\omega}$, $\mathrm{d}v_t^{\omega}$, $(F^{\omega})^k$, *or* $(B^{\omega})^m(F^{\omega})^k$, *we obtain aggregate variables:*

$$\begin{aligned}
\bar{\mathbb{E}}_{\omega}\,\mathrm{d}\ln \ell_{t+1}^{\omega} &= \mathrm{d}\ln \ell_{t+1} \\
\bar{\mathbb{E}}_{\omega}\,\mathrm{d}v_t^{\omega} &= \mathrm{d}v_t \\
\bar{\mathbb{E}}_{\omega}\big[(F^{\omega})^k\big] &= \mathcal{F}_k \\
\bar{\mathbb{E}}_{\omega}\big[(B^{\omega})^m(F^{\omega})^k\big] &= \mathcal{F}_{m,k}.
\end{aligned}$$

In particular, with infinite-length longitudinal information on workers' sector choices, we can observe $\bar{\mathbb{E}}_{\omega}[(F^{\omega})^k]$ and $\bar{\mathbb{E}}_{\omega}[(B^{\omega})^m(F^{\omega})^k]$ for all $k, m \in \mathbb{N}_0$.[60] We can now apply the population-average operator to the left- and right-hand sides of equations (7) and (8) to derive a general version of the sufficient statistics result. First, we have

$$\begin{aligned}
\mathrm{d}v_t = \bar{\mathbb{E}}_{\omega}\,\mathrm{d}v_t^{\omega} &= \bar{\mathbb{E}}_{\omega}\Big[\sum_{k\geq 0}(\beta F^{\omega})^k \mathbb{E}_t\,\mathrm{d}w_{t+k}\Big] \\
&= \sum_{k\geq 0}\beta^k \bar{\mathbb{E}}_{\omega}\Big[(F^{\omega})^k\Big]\mathbb{E}_t\,\mathrm{d}w_{t+k} \\
&= \sum_{k\geq 0}\beta^k \mathcal{F}_k \mathbb{E}_t\,\mathrm{d}w_{t+k}.
\end{aligned}$$

Second, we have

$$\begin{aligned}
\mathrm{d}\ln \ell_t &= \bar{\mathbb{E}}_{\omega}\,\mathrm{d}\ln \ell_t^{\omega} \\
&= \bar{\mathbb{E}}_{\omega}\left[\tfrac{\beta}{\rho}\sum_{s\geq 0}(B^{\omega})^s(I - B^{\omega}F^{\omega})\left(\sum_{k\geq 0}(\beta F^{\omega})^k \mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k}^{\omega}\right)\right] \\
&= \bar{\mathbb{E}}_{\omega}\left[\sum_{s\geq 0, k\geq 0}\tfrac{\beta^{k+1}}{\rho}\big((B^{\omega})^s(F^{\omega})^k - (B^{\omega})^{s+1}(F^{\omega})^{k+1}\big)\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k}\right] \\
&= \sum_{s\geq 0, k\geq 0}\tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s,k} - \mathcal{F}_{s+1,k+1})\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k}.
\end{aligned}$$

The result is summarized in the following proposition.

Proposition A.1 (Sufficient Statistics Result without Assumption 2). *Suppose that Assumption 1 holds. For a given sequence of (common) changes in sectoral wages* $\{\mathrm{d}w_t\}$, *the changes in sectoral value and*

[60] For the second type of worker flow matrices, this requires an additional assumption that $(\mathcal{F}_k)_{i,j}$ is strictly positive for all $i, j \in \mathcal{S}$ and $k \in \mathbb{N}$.

sectoral employment are given by:

$$\begin{aligned} \mathrm{d}v_t &= \sum_{k\geq 0} \beta^k \mathcal{F}_k \mathbb{E}_t \,\mathrm{d}w_{t+k}, \\ \mathrm{d}\ln \ell_t &= \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho} (\mathcal{F}_{s,k} - \mathcal{F}_{s+1,k+1}) \mathbb{E}_{t-s-1} \,\mathrm{d}w_{t-s+k}. \end{aligned}$$

Proposition A.1 establishes that in order to construct the counterfactual changes in welfare and sectoral employment for a given sequence of sectoral wage changes, we only require knowledge of the two types of worker flow matrices, $\{\mathcal{F}_k\}$ and $\{\mathcal{F}_{s,k}\}$, and the parameter ρ that governs the dispersion of the idiosyncratic shocks. Proposition 2 can be generalized in a similar manner.

A.2 Simplified Sufficient Statistics Result under Assumption 2

Lemma A.1 demonstrates that with infinite-length longitudinal information on workers' sector choices, we can observe two types of worker flow matrices, $\{\mathcal{F}_k\}$ and $\{\mathcal{F}_{m,k}\}$, for all k and m. In practice, however, we can only observe them for small enough k and m due to the finite nature of the real-world datasets. The following lemma characterizes a certain symmetry property satisfied by the second type of worker flow matrices that can be used to reduce the data requirements.

Lemma A.2. *We have*

$$\begin{pmatrix} \ell_1 & & 0 \\ & \ddots & \\ 0 & & \ell_N \end{pmatrix} \mathcal{F}_{m,k} = (\mathcal{F}_{k,m})^{\intercal} \begin{pmatrix} \ell_1 & & 0 \\ & \ddots & \\ 0 & & \ell_N \end{pmatrix}.$$

However, the data requirements for estimating the second type of worker flow matrices are still very demanding. In this section, we show how these data requirements can be significantly reduced under an additional assumption.

Lemma A.3. *Under Assumption 2, we have $B^{\omega} = F^{\omega}$ for all $\omega \in \Omega$. In this case, we have $\mathcal{F}_{m,k} = \mathcal{F}_{m+k}$.*

Assumption 2 imposes a certain structure on the sector-switching costs. Lemma A.3 shows that this assumption implies the equality between the backward transition matrix and the forward transition matrix, which in turn implies that the second type of worker flow matrices reduces to the first type of worker flow matrices.[61] Thus, we can focus on the first type of worker flow matrices, and Proposition A.1 is simplified to Proposition 1 in the main text.

In the left panel of Figure A.1, we compute the transition matrices between the four sectors considered in the main text and plot the sixteen elements of the backward transition matrices against the corresponding elements of the forward transition matrices. All elements lie closely on the 45 degree

[61] Assumption 2 is almost necessary and sufficient in the sense that when the number of sectors is three, the necessary and sufficient condition is $C_{12} + C_{23} + C_{31} = C_{13} + C_{32} + C_{21}$. When the number of sectors is four, the necessary and sufficient condition is $C_{ij} + C_{jk} + C_{ki} = C_{ik} + C_{kj} + C_{ji}$ for all distinct $i, j, k \in \{1, 2, 3, 4\}$; and for all distinct i, j (and remaining k, ℓ), we have either $C_{ij} = C_{ji}$ or $C_{ik} + C_{j\ell} = C_{i\ell} + C_{jk}$.

line. It is not possible to test this implication empirically at the unobserved type level. Instead, we compute the backward and forward transition matrices for observed types. Specifically, we consider four dimensions of observed heterogeneity as in Section 4: gender, race, education, and age, resulting in sixteen groups. In the right panel of Figure A.1, we compare the matrices for all sixteen groups. Again, there is a striking similarity between the backward and forward transition matrices.

A.3 First-Order Approximation of Labor Demand Side

Suppose that type-specific sectoral wages are determined by the labor allocation and exogenous shocks:

$$w_{it}^{\omega} = f_i^{\omega}\left(\{\ell_{jt}\}_j, \{\varepsilon_{jt}\}_j\right)$$

for $i \in \mathcal{S}$ and $\omega \in \Omega$. Up to a first-order approximation, we can write

$$\mathrm{d}w_{it}^{\omega} = \sum_j \frac{\partial f_i^{\omega}}{\partial \ln \ell_{jt}} \cdot \mathrm{d}\ln \ell_{jt} + \sum_j \frac{\partial f_i^{\omega}}{\partial \varepsilon_{jt}} \mathrm{d}\varepsilon_{jt}.$$

Assumption 1 requires that $\mathrm{d}w_{it}^{\omega} = \mathrm{d}w_{it}$ for all $\omega \in \Omega$ for any realizations of $\{\ell_{jt}\}_j$ and $\{\varepsilon_{jt}\}_j$. This in turn requires that

$$\mathrm{d}w_{it}^{\omega} = \mathrm{d}w_{it} \equiv \sum_j \frac{\partial f_i^{\omega_1}}{\partial \ln \ell_{jt}} \cdot \mathrm{d}\ln \ell_{jt} + \sum_j \frac{\partial f_i^{\omega_1}}{\partial \varepsilon_{jt}} \mathrm{d}\varepsilon_{jt}$$

for all $\omega \in \Omega$ for a given $\omega_1 \in \Omega$.[62] Thus, we can write

$$\begin{aligned}
\mathrm{d}w_t &= \begin{pmatrix} \frac{\partial f_1^{\omega_1}}{\partial \ln \ell_{1t}} & \frac{\partial f_1^{\omega_1}}{\partial \ln \ell_{2t}} & \cdots & \frac{\partial f_1^{\omega_1}}{\partial \ln \ell_{St}} \\ \frac{\partial f_2^{\omega_1}}{\partial \ln \ell_{1t}} & \frac{\partial f_2^{\omega_1}}{\partial \ln \ell_{2t}} & \cdots & \frac{\partial f_2^{\omega_1}}{\partial \ln \ell_{St}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_S^{\omega_1}}{\partial \ln \ell_{1t}} & \frac{\partial f_S^{\omega_1}}{\partial \ln \ell_{2t}} & \cdots & \frac{\partial f_S^{\omega_1}}{\partial \ln \ell_{St}} \end{pmatrix} \mathrm{d}\ln \ell_t + \begin{pmatrix} \frac{\partial f_1^{\omega_1}}{\partial \varepsilon_{1t}} & \frac{\partial f_1^{\omega_1}}{\partial \varepsilon_{2t}} & \cdots & \frac{\partial f_1^{\omega_1}}{\partial \varepsilon_{St}} \\ \frac{\partial f_2^{\omega_1}}{\partial \varepsilon_{1t}} & \frac{\partial f_2^{\omega_1}}{\partial \varepsilon_{2t}} & \cdots & \frac{\partial f_2^{\omega_1}}{\partial \varepsilon_{St}} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_S^{\omega_1}}{\partial \varepsilon_{1t}} & \frac{\partial f_S^{\omega_1}}{\partial \varepsilon_{2t}} & \cdots & \frac{\partial f_S^{\omega_1}}{\partial \varepsilon_{st}} \end{pmatrix} \mathrm{d}\varepsilon_t. \\
&= D \cdot \mathrm{d}\ln \ell_t + E \cdot \mathrm{d}\varepsilon_t.
\end{aligned}$$

A.4 Intuition for the Sufficient Statistics Result

We begin by providing intuition at the micro (i.e., type) level, then show how this intuition is preserved at the macro level, when type-specific equations are aggregated.

Intuition at the Micro Level. The key equations at the micro level are (5) and (6). As discussed in the main text, equation (5) is just an application of the envelope theorem (e.g., Milgrom and Segal, 2002). The envelope theorem implies that changes in future optimal sector choices do not contribute to the change in current welfare. Thus, we can evaluate the effect of changes in future sectoral wages using workers' sector-choice probabilities as weights. For example, equation (7) shows that the effect of a unit

[62] Note that this does not necessarily imply that $w_{it}^{\omega} = w_{it}$ for all $\omega \in \Omega$.

change in the period-$(t+k)$ wage of sector j on the period-t welfare of workers in sector i is given by the probability that these workers transition to sector j in period $t+k$ (after discounting the future):

$$\frac{\partial v_{it}^{\omega}}{\partial \mathbb{E}_t w_{j,t+k}^{\omega}} = \beta^k \Pr(s_{t+k} = j | s_t = i, \omega) \equiv \beta^k (F^{\omega})^k.$$

Thus, the matrix $(F^{\omega})^k$ is—by providing information about workers' k-period sector choices—informative about the effect of future sectoral shocks on current welfare.

Equation (6) summarizes the labor reallocation in response to sectoral shocks, which is rewritten here for convenience:

$$\mathrm{d}\ln \ell_{t+1}^{\omega} = B^{\omega}\,\mathrm{d}\ln \ell_t^{\omega} + \tfrac{\beta}{\rho^{\omega}}(I - B^{\omega}F^{\omega})\mathbb{E}_t\,\mathrm{d}v_{t+1}^{\omega}. \tag{6}$$

The first term captures the mechanical effect of changes in labor allocation in the previous period, which holds under any assumption about the distribution of the idiosyncratic shocks. On the other hand, the second term captures the response of workers' sector choices to changes in sectoral values in period $t+1$. In particular, it implies

$$\left.\frac{\partial \ln \ell_{it+1}^{\omega}}{\partial \mathbb{E}_t v_{jt+1}^{\omega}}\right|_{\text{fix } \ell_t} = \tfrac{\beta}{\rho}\Big(\mathbb{1}_{i=j} - \big(B^{\omega}F^{\omega}\big)_{ij}\Big).$$

To understand this equation, suppose first $i \neq j$. Then, for a given value of $\frac{\beta}{\rho}$, this means that the semi-elasticity of employment in sector i with respect to the expected value of sector j is proportional to the (i,j) element of $B^{\omega}F^{\omega}$. Note that we have

$$\begin{aligned}(B^{\omega}F^{\omega})_{ij} &= \sum_{k\in\mathcal{S}} B_{ik}^{\omega}F_{kj}^{\omega} \\ &= \sum_{k\in\mathcal{S}} \Pr(s_t = k | s_{t+1} = i, \omega)\Pr(s_{t+1} = j | s_t = k, \omega).\end{aligned}$$

Thus, this element can be interpreted as the probability that workers who choose $s_{t+1} = i$ will switch to $s_{t+1} = j$ if they are allowed to choose period-$(t+1)$ sector again after redrawing period-$(t+1)$ idiosyncratic shocks. Since their sector choice depends on the realization of idiosyncratic shocks, this probability is non-zero. If this probability is high for a given sector pair (i,j), it means that there are many workers who are at the margin between sectors i and j. In this case, a small change in the value of sector j leads to a relatively larger decline in employment in sector i.

Second, consider $i = j$. Then for a similar reason, an increase in the value of sector i leads to a relatively larger increase in sector i employment when many workers are at the margin between sector i and other sectors, which is measured by $1 - (B^{\omega}F^{\omega})_{ii}$.

Moving to the longer-run, consider changes in the values in period $t+1$ that are known to workers in period $t+1-\tau$, $\tau \in \mathbb{N}$. To make the intuition clear, we assume that $\beta = 1$, but the intuition remains the same for $\beta < 1$. Then, the effect of the shock on period t labor allocation can be written as follows:

$$\begin{aligned}\frac{\partial \ln \ell^{\omega}_{it+1}}{\partial \mathbb{E}_{t+1-\tau} v^{\omega}_{jt+1}} &= \frac{1}{\rho}\Big((I - B^{\omega}F^{\omega}) + (B^{\omega}F^{\omega} - (B^{\omega})^2(F^{\omega})^2) + \cdots + ((B^{\omega})^{k-1}(F^{\omega})^{k-1} - (B^{\omega})^k(F^{\omega})^k)\Big)_{ij} \\ &= \frac{1}{\rho}\Big(\mathbb{1}_{i=j} - \big((B^{\omega})^k(F^{\omega})^k\big)_{ij}\Big).\end{aligned}$$

Again, the (i,j)-element of the matrix $(B^{\omega})^k(F^{\omega})^k$ can be interpreted as the probability that workers who choose sector i in period $t+1$ will switch to sector j if they are allowed to choose sectors again for periods $t+2-\tau, t+3-\tau, \cdots, t+1$ after redrawing idiosyncratic shocks for these periods. This again measures the k-period indifference of workers between sectors i and j, and thus provides information about the responsiveness of workers to shocks known k-periods ahead of time.

In this sense, the steady-state transition matrices and their kth powers are—by providing information about the indifference of workers between sectors–informative about the response of sectoral employment to sectoral shocks. The role of the extreme-value assumption is to make this qualitative relationship exact. As an alternative justification, we show in Appendix A.4.1 that we have the same result up to a multiplicative constant in the limit of idiosyncratic volatility converging to zero.

Intuition at the Macro Level. At the micro level, the intuition can be formulated in terms of products of transition matrices. Appendix A.1 demonstrates that we can aggregate type-specific equations to derive macro-level equations, establishing the sufficient statistics result. In particular, Lemma A.3 shows that products of transition matrices can be aggregated to two types of worker flow matrices. This preserves the same intuition to the macro level: the matrix $\mathcal{F}_k$ is—by providing information about the *average* workers' k-period sector choices—informative about the effect of future sectoral shocks on current welfare; and the matrix $\mathcal{F}_{s,k}$ is—by providing information about the *average* workers' indifference between sectors—informative about the response of sectoral employment to sectoral shocks.

A.4.1 Small Idiosyncratic Shock Limit

Our goal is to show that a version of equation (6) is valid in the limit as ρ approaches zero, without any distribution assumption on the idiosyncratic shocks. Without loss of generality, consider a two-period version of the model in Section 2, where two periods are denoted by t and $t+1$ for notation consistent with equation (6). To make the limit non-trivial, we modify the canonical model in two ways. First, we perturb the sectoral values of workers in period $t+1$, which are written as $\Delta_s = \beta \mathbb{E}_t V_{st+1}$ for notational simplicity. Fix a positive number ε. For each pair of sectors $i \neq j \in \mathcal{S}$, consider a set of workers, $\Omega_{ij}(\varepsilon)$, who have $|\Delta_i - \Delta_j| < \varepsilon$ and $\Delta_s < \min\{\Delta_i, \Delta_j\} - \varepsilon$ for all $s \in \mathcal{S}\backslash\{i,j\}$. Since we will take a limit along which the importance of idiosyncratic shocks converges to zero, we *assume* that workers in the set $\Omega_{ij}(\varepsilon)$ never choose a sector other than i and j. For workers in $\Omega_{ij}(\varepsilon)$, the relative value of sector i to j is distributed as follows:[63]

$$\Delta_{ij} \equiv \Delta_i - \Delta_j \sim g_{ij}(\cdot), G_{ij}(\cdot).$$

[63] We consider workers with $|\Delta_{ij}| < \bar{\varepsilon}$, so this is a local distribution around 0.

Second, switching costs are given by $C_{ij} = \rho \cdot \tilde{C} \cdot \mathbb{1}_{i \neq j}$.[64] Finally, we dispense with the distributional assumption on the idiosyncratic shock: Idiosyncratic utility from sector i relative to sector j follows an unknown distribution:

$$\rho \cdot \varepsilon_{it} - \rho \cdot \varepsilon_{jt} \equiv \rho \cdot \varepsilon_{ij} \quad \text{where } \varepsilon_{ij} \sim f(\cdot), F(\cdot).$$

Within this environment, the following proposition establishes that equation (6) holds up to $o(\rho)$ and up to a multiplicative constant, which depends only on the shape of the distribution of idiosyncratic shocks and the switching costs.

Proposition A.2. *When $g_{ij}(\cdot)$ is continuous around 0 and ε_{ij} has a finite first moment for all $i, j \in \mathcal{S}$, we have*

$$\frac{\partial \ln \ell_{t+1}}{\partial \mathbb{E}_t v_{t+1}} = \textit{const.} \cdot \tfrac{\beta}{\rho} \cdot (I - BF) + o(\rho)$$

where the constant term only depends on the shape of $F(\cdot)$ and $\tilde{C}$.

A.5 Single Crossing Condition

We know from Lemma 2 that, compared to the canonical model calibrated by matching the one-period worker flow matrix, the model with worker heterogeneity implies lower values of b_k at least for $k = 0$ and $k = 1$:

$$b_0 = \big(\mathcal{F}_0 - \mathcal{F}_2\big)_{ss} = 1 - (\mathcal{F}_2)_{ss}$$
$$b_1 = \big(\mathcal{F}_1 - \mathcal{F}_3\big)_{ss} = (\mathcal{F}_1)_{ss} - (\mathcal{F}_3)_{ss}.$$

Thus, $\bar{k} \geq 1$. In Figure A.2, we plot the difference between $\{b_k\}$ implied by the canonical model and those observed in the data (extrapolated using the method described in the main text). Initially, the canonical model yields larger values of b_k, but eventually it leads to smaller values compared to those implied by the data (although not plotted, this is true for all values of k greater than 9; i.e., there is no more crossing).

A.6 Response to One-Time Shock

Consider a one-time negative shock to a sector $s \in \mathcal{S}$ that is known to agents in period 1:

$$\mathrm{d}w_{s\tau} = -\Delta < 0 \tag{A.1}$$

for given $\tau > 1$. The effect of any series of negative shocks to sector s can be calculated as the sum of the effects of such one-time negative shocks.

[64] If the switching costs are fixed along the limit of $\rho \to 0$, no workers would change the sectors over time.

Effects on Sectoral Welfare. With worker heterogeneity, workers initially employed in sector s are more likely to stay in sector s when the shock hits the sector. Thus, they suffer more from the one-time shock.

Proposition A.3. *Consider a one-time negative shock to sector s of the form* (A.1) *known to agents in period* 1. *The canonical model, calibrated by matching the one-period worker flow matrix, underestimates the negative welfare effect on workers initially employed in sector s,* $\mathrm{d}v_{s1}$.

This result in turn implies that for any series of negative shocks to sector s, the canonical model underestimates the negative welfare effect on workers initially employed in sector s, proving Proposition 3.

Effects on Sectoral Employment. The following proposition characterizes the condition under which the canonical model overestimates the decline in employment in sector s in period $t > 1$.

Proposition A.4. *Consider a one-time negative shock to sector s of the form* (A.1) *known to agents in period* 1. *Under Assumption 3, there exists a decreasing function $B : \mathbb{N} \to \mathbb{N}$ such that the canonical model overestimates the decline in employment of sector s in period t in response to the shock if and only if $|t - \tau| \leq B(t \wedge \tau)$ where $t \wedge \tau$ denotes the minimum of t and τ.*

When $|t - \tau|$ and/or $t \wedge \tau$ are small, the canonical model calibrated by matching the one-period worker flow matrix overestimates the decline in employment in sector s in period t in response to the shock, $\frac{\partial \ln \ell_{s,t}}{\partial w_{s,\tau}}$.[65]

The result implies that whether the models without worker heterogeneity overestimate or underestimate labor reallocation depends on the time horizon. On the one hand, as discussed in Lemma 2, the canonical model overestimates the mobility of workers across sectors, leading to an overestimation of the decline in employment in a negatively affected sector. This intuition is what Corollary 4 describes when $|t - \tau|$ and/or $t \wedge \tau$ are small; i.e., when the shock is recently known or when the period affected by the shock is close to the period of interest. On the other hand, in the canonical model, workers have relatively lower probabilities of remaining in a sector, implying that their sector choices in a given period do not have long-lasting impacts on their future sector choices. This aspect works in the opposite direction to our previous intuition and can become dominant if $|t - \tau|$ and/or $t \wedge \tau$ are large enough. For example, suppose that t is much larger than τ. A negative shock to a sector's wage in period τ reduces employment in the sector around that period. However, this reduced employment has only a limited impact on the employment of the sector in the distant period t in the canonical model. Figure A.3 plots the relative size of the decline in employment of sector s in period t in response to a shock to the period τ wage predicted by the canonical model and that implied by the data. As expected, the decline is overestimated in the canonical model for small t or τ or small $|t - \tau|$ (red cells). In short, the canonical model tends to overestimate the short-term impact of shocks on sectoral employment but underestimates their long-term effects. In particular, within a seven-year time horizon, the canonical model consistently overestimates the impact of shocks on sectoral employment.

[65] If w is log wage, then this measures the elasticity of sectoral employment with respect to sectoral wages.

A.7 Two-Sector Model

In this section we consider a special case of our model with two sectors, $\mathcal{S} = \{1, 2\}$. This special case serves two purposes. First, it highlights the restriction on the worker flow matrix series imposed by the class of dynamic discrete choice models studied in this paper. Second, the analytical results derived in this section will be used for proofs in Appendix D.

For simplicity, we assume that there are a finite number of worker types. The types are indexed by $i = 1, \cdots, I$, and the population share of type i is $\theta_i \in (0, 1)$. We denote the transition matrix of type-i workers by

$$F_i = \begin{pmatrix} \bar{\alpha}_i & \alpha_i \\ \beta_i & \bar{\beta}_i \end{pmatrix}$$

where $\bar{\alpha}_i = 1 - \alpha_i$ and $\bar{\beta}_i = 1 - \beta_i$. We also write $F_i^k \equiv \begin{pmatrix} \bar{\alpha}_{i,k} & \alpha_{i,k} \\ \beta_{i,k} & \bar{\beta}_{i,k} \end{pmatrix}$. For example, we have

$$F_i^2 = \begin{pmatrix} \bar{\alpha}_i^2 + \alpha_i \beta_i & \alpha_i(\bar{\alpha}_i + \bar{\beta}_i) \\ \beta_i(\bar{\alpha}_i + \bar{\beta}_i) & \bar{\beta}_i^2 + \alpha_i \beta_i \end{pmatrix} \text{ and } F_i^3 = \begin{pmatrix} \cdot & \alpha_i(\bar{\alpha}_i^2 + \bar{\alpha}_i \bar{\beta}_i + \bar{\beta}_i^2 + \alpha_i \beta_i) \\ \beta_i(\bar{\alpha}_i^2 + \bar{\alpha}_i \bar{\beta}_i + \bar{\beta}_i^2 + \alpha_i \beta_i) & \cdot \end{pmatrix}.$$

By induction, we can obtain a general formula for the off-diagonal elements of the matrix F_i^k

Lemma A.4. *F_i^k has the following form:*

$$F_i^k = \begin{pmatrix} 1 - \alpha_i f^k(\bar{\alpha}_i + \bar{\beta}_i) & \alpha_i f^k(\bar{\alpha}_i + \bar{\beta}_i) \\ \beta_i f^k(\bar{\alpha}_i + \bar{\beta}_i) & 1 - \beta_i f^k(\bar{\alpha}_i + \bar{\beta}_i) \end{pmatrix}$$

where $f^k(x) = \frac{1-(x-1)^k}{2-x}$.

The steady-state sectoral employment share of type i workers are given by

$$\begin{pmatrix} \Pr(\text{sector } 1|\text{type } i) \\ \Pr(\text{sector } 2|\text{type } i) \end{pmatrix} = \begin{pmatrix} \frac{\beta_i}{\alpha_i + \beta_i} \\ \frac{\alpha_i}{\alpha_i + \beta_i} \end{pmatrix} \equiv \begin{pmatrix} \tilde{\beta}_i \\ \tilde{\alpha}_i \end{pmatrix},$$

which gives

$$\tilde{\ell}_1^i = \frac{\tilde{\beta}_i \theta_i}{\sum_j \tilde{\beta}_j \theta_j} \text{ and } \tilde{\ell}_2^i = \frac{\tilde{\alpha}_i \theta_i}{\sum_j \tilde{\alpha}_j \theta_j}.$$

Thus, the k-period worker flow matrix is given by

$$\mathcal{F}_k = \begin{pmatrix} 1 - \dfrac{\sum_i \tilde{\beta}_i \theta_i \alpha_{i,k}}{\sum_i \tilde{\beta}_i \theta_i} & \dfrac{\sum_i \tilde{\beta}_i \theta_i \alpha_{i,k}}{\sum_i \tilde{\beta}_i \theta_i} \\ \dfrac{\sum_i \tilde{\alpha}_i \theta_i \beta_{i,k}}{\sum_i \tilde{\alpha}_i \theta_i} & 1 - \dfrac{\sum_i \tilde{\alpha}_i \theta_i \beta_{i,k}}{\sum_i \tilde{\alpha}_i \theta_i} \end{pmatrix} \equiv \begin{pmatrix} 1 - \sum_i x_i \alpha_{i,k} & \sum_i x_i \alpha_{i,k} \\ \sum_i y_i \beta_{i,k} & 1 - \sum_i y_i \beta_{i,k} \end{pmatrix}$$

where $x_i = \frac{\tilde{\beta}_i \theta_i}{\sum_j \tilde{\beta}_j \theta_j}$ and $y_i = \frac{\tilde{\alpha}_i \theta_i}{\sum_j \tilde{\alpha}_j \theta_j}$.

Finally, we use the results to characterize a restriction imposed on the off-diagonal elements of the worker flow matrix series.

Proposition A.5. *We have*

$$\frac{(\mathcal{F}_k)_{1,2}}{(\mathcal{F}_k)_{2,1}} = \frac{\alpha_1 x_1}{\beta_1 y_1} \quad \textit{for all } k \geq 1.$$

This proposition shows that the ratio between the two off-diagonal elements is identical for all worker flow matrix series. Thus, if these rations are far from constant in the data, we cannot match the worker flow matrix series observed in the data with the class of models we consider in this paper. The result in Section 4.4 demonstrates that this is not the case.

B. Structural Estimation Appendix

B.1 Details of Structural Estimation

To be written.

B.2 Estimation of ρ

Section 4.5 proposes a method to estimate the parameter ρ, which is based on the second equation of Proposition 1:

$$\mathrm{d}\ln \ell_t = \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k}.$$

We refer to this relation as a forward-looking infinite-order MA process, because it resembles infinite-order MA processes, but involving forward-looking variables.

We first use equations (2) and (3) to prove equation (16) under the homogeneous worker assumption. Imposing Assumption 1 and assumptions in Lemma A.3, these equations become

$$\begin{aligned} \mathrm{d}v_t^\omega &= \mathrm{d}w_t + \beta F^\omega \mathbb{E}_t\,\mathrm{d}v_{t+1}^\omega \\ \mathrm{d}\ln \ell_{t+1}^\omega &= F^\omega\,\mathrm{d}\ln \ell_t^\omega + \tfrac{\beta}{\rho}(I - (F^\omega)^2)\mathbb{E}_t\,\mathrm{d}v_{t+1}^\omega \end{aligned} \tag{A.2}$$

Pre-multiplying one-period forwarded version of equation (A.2) with βF^ω and taking expectation operator $\mathbb{E}_t$, we have

$$\beta F^\omega \mathbb{E}_t\,\mathrm{d}\ln \ell_{t+2}^\omega = \beta (F^\omega)^2\,\mathrm{d}\ln \ell_{t+1}^\omega + \tfrac{\beta}{\rho}(I - (F^\omega)^2)\beta F^\omega \mathbb{E}_t\,\mathrm{d}v_{t+2}^\omega \tag{A.3}$$

Subtracting equation (A.3) from equation (A.2), we have

$$\mathrm{d}\ln \ell_{t+1}^\omega = F^\omega\,\mathrm{d}\ln \ell_t^\omega + \beta F^\omega(\mathbb{E}_t\,\mathrm{d}\ln \ell_{t+2}^\omega - F^\omega\,\mathrm{d}\ln \ell_{t+1}^\omega) + \tfrac{\beta}{\rho}(I - (F^\omega)^2)\mathbb{E}_t\,\mathrm{d}w_{t+1}.$$

Rearranging it, we obtain equation (16):

$$\mathrm{d}\ln \ell_t = (\mathcal{F}_1^{-1} + \beta\mathcal{F}_1)\,\mathrm{d}\ln \ell_{t+1} - \beta\mathbb{E}_t\,\mathrm{d}\ln \ell_{t+2} - \tfrac{\beta}{\rho}(\mathcal{F}_1^{-1} - \mathcal{F}_1)\mathbb{E}_t\,\mathrm{d}w_{t+1}.$$

We refer to this equation as a *forward-looking process with order 2* because we write a period t variable as a function of period $t+1$ and $t+2$ variables and a shock.

Now, suppose that there are N number of worker types, $\omega \in \{1, 2, \ldots, N\}$. We derive a recursive representation of the form

$$\mathrm{d}\ln \ell_t = \sum_{k=1}^{K} \mathbf{\Gamma}_k \mathbb{E}_t\,\mathrm{d}\ln \ell_{t+k} + \tfrac{\beta}{\rho}\sum_{k=1}^{K'+1} \mathbf{\Lambda}_k \mathbb{E}_t\,\mathrm{d}w_{t+k}.$$

We refer to this equation as a forward-looking process with order(K, K'). Note that the change in aggregate labor supply can be written as

$$\mathrm{d}\ln \ell_{t+1} = \bar{\mathbb{E}}_\omega[\mathrm{d}\ln \ell^\omega_{t+1}] = L_1 \,\mathrm{d}\ln \ell^1_{t+1} + L_2 \,\mathrm{d}\ln \ell^2_{t+1} + \cdots + L_N \,\mathrm{d}\ln \ell^N_{t+1}$$

where each $\mathrm{d}\ln \ell^\omega_{t+1}$ follows a forward-looking process with order w, and L_ω is a diagonal matrix, whose i-th diagonal element is given by the stationary proportion of type ω in sector i. Granger and Morris (1976) show that the scalar-weighted sum of N number of autoregressive processes of order 2 follows an autoregressive process of order at most $2N$. Here, we instead have diagonal-matrix-weighted sum of N number of forward-looking process of order 2, but we can apply a modified version of their proof to show proposition. Due to an invertibility issue, we need forward-looking process of order $(4N-2, 4N-4)$ instead of order $2N$. The next subsection is devoted to the proof of Proposition A.6.

Proposition A.6. *If the number of types is N, $\mathrm{d}\ln \ell_t$ has a recursive representation of the form*

$$\mathrm{d}\ln \ell_t = \sum_{k=1}^{4N-2} \mathbf{\Gamma}_k \mathbb{E}_t \,\mathrm{d}\ln \ell_{t+k} + \frac{\beta}{\rho} \sum_{k=1}^{4N-3} \mathbf{\Lambda}_k \mathbb{E}_t \,\mathrm{d}w_{t+k}.$$

B.2.1 Proof of Proposition A.6

We prove this proposition for the case with $N = 2$. The same proof can be inductively applied to show the case with $N > 2$. For the case with two worker types, aggregate labor supply is given by

$$\mathrm{d}\ln \ell_t = L_1 \,\mathrm{d}\ln \ell^1_t + L_2 \,\mathrm{d}\ln \ell^2_t \tag{A.4}$$

where

$$\mathrm{d}\ln \ell^1_{t+1} - \beta F^1 \mathbb{E}_t \,\mathrm{d}\ln \ell^1_{t+2} = F^1 \,\mathrm{d}\ln \ell^1_t - \beta (F^1)^2 \,\mathrm{d}\ln \ell^1_{t+1} + \frac{\beta}{\rho}(I - (F^1)^2)\mathbb{E}_t \,\mathrm{d}w_{t+1} \tag{A.5}$$

$$\mathrm{d}\ln \ell^2_{t+1} - \beta F^2 \mathbb{E}_t \,\mathrm{d}\ln \ell^2_{t+2} = F^2 \,\mathrm{d}\ln \ell^2_t - \beta (F^2)^2 \,\mathrm{d}\ln \ell^2_{t+1} + \frac{\beta}{\rho}(I - (F^2)^2)\mathbb{E}_t \,\mathrm{d}w_{t+1}. \tag{A.6}$$

Using equation (A.5) to cancel out $\mathrm{d}\ln \ell^1_t$ from equation (A.4), we have

$$\begin{aligned}
&L_1((F^1)^{-1} + \beta F^1)L_1^{-1} \,\mathrm{d}\ln \ell_{t+1} - \beta \mathbb{E}_t \,\mathrm{d}\ln \ell_{t+2} - \mathrm{d}\ln \ell_t \\
&= L_1((F^1)^{-1} + \beta F^1)(\mathrm{d}\ln \ell^1_{t+1} + L_1^{-1} L_2 \,\mathrm{d}\ln \ell^2_{t+1}) - \beta \mathbb{E}_t(L_1 \,\mathrm{d}\ln \ell^1_{t+2} + L_2 \,\mathrm{d}\ln \ell^2_{t+2}) - (L_1 \,\mathrm{d}\ln \ell^1_t + L_2 \,\mathrm{d}\ln \ell^2_t) \\
&= L_1 (F^1)^{-1}\big((I + \beta (F^1)^2)\,\mathrm{d}\ln \ell^1_{t+1} - \beta F^1 \mathbb{E}_t \,\mathrm{d}\ln \ell^1_{t+2} - F^1 \,\mathrm{d}\ln \ell^1_t\big) + \mathbb{E}_t \Xi_t \\
&= L_1 (F^1)^{-1} \frac{\beta}{\rho}(I - (F^1)^2)\mathbb{E}_t \,\mathrm{d}w_{t+1} + \mathbb{E}_t \Xi_t
\end{aligned}$$

where

$$
\begin{aligned}
\Xi_t &= L_1((F^1)^{-1} + \beta F^1)L_1^{-1}L_2 \,\mathrm{d}\ln \ell_{t+1}^2 - \beta L_2 \,\mathrm{d}\ln \ell_{t+2}^2 - L_2 \,\mathrm{d}\ln \ell_t^2 \\
&= L_1((F^1)^{-1} + \beta F^1)L_1^{-1}L_2 \,\mathrm{d}\ln \ell_{t+1}^2 - \beta L_2 \,\mathrm{d}\ln \ell_{t+2}^2 \\
&\quad - L_2(F^2)^{-1}\Big(((I + \beta(F^2)^2) \,\mathrm{d}\ln \ell_{t+1}^2 - \beta F^2 \,\mathrm{d}\ln \ell_{t+2}^2 - \tfrac{\beta}{\rho}(I - (F^2)^2)\mathbb{E}_t \,\mathrm{d}w_{t+1}\Big) \\
&= \Big(L_1((F^1)^{-1} + \beta F^1)L_1^{-1}L_2 - L_2((F^2)^{-1} + \beta F^2)\Big) \,\mathrm{d}\ln \ell_{t+1}^2 + \tfrac{\beta}{\rho} L_2((F^2)^{-1} - F^2)\mathbb{E}_t \,\mathrm{d}w_{t+1}.
\end{aligned}
$$

This can be rearranged to

$$
\begin{aligned}
&L_1((F^1)^{-1} + \beta F^1)L_1^{-1} \,\mathrm{d}\ln \ell_{t+1} - \beta \mathbb{E}_t \,\mathrm{d}\ln \ell_{t+2} - \mathrm{d}\ln \ell_t \\
&= \tfrac{\beta}{\rho}\big(L_1((F^1)^{-1} - F^1) + L_2((F^2)^{-1} - F^2)\big)\mathbb{E}_t \,\mathrm{d}w_{t+1} + \big(L_1((F^1)^{-1} + \beta F^1)L_1^{-1} - L_2((F^2)^{-1} + \beta F^2)L_2^{-1}\big)L_2 \,\mathrm{d}\ln \ell_{t+1}^2.
\end{aligned}
$$

or equivalently

$$
y_t = \Psi x_t
$$

where

$$
\begin{aligned}
y_t &= \boldsymbol{X} \,\mathrm{d}\ln \ell_{t+1} - \beta \mathbb{E}_t \,\mathrm{d}\ln \ell_{t+2} - \mathrm{d}\ln \ell_t - \tfrac{\beta}{\rho} \boldsymbol{Y} \mathbb{E}_t \,\mathrm{d}w_{t+1} \\
x_t &= \mathrm{d}\ln \ell_{t+1}^2 \\
\boldsymbol{X} &= L_1((F^1)^{-1} + \beta F^1)L_1^{-1} \\
\boldsymbol{Y} &= L_1((F^1)^{-1} - F^1) + L_2((F^2)^{-1} - F^2) \\
\Psi &= \big(L_1((F^1)^{-1} + \beta F^1)L_1^{-1} - L_2((F^2)^{-1} + \beta F^2)L_2^{-1}\big)L_2.
\end{aligned}
$$

From equation (A.6), the law of motion of x_t is given by

$$
x_t = \boldsymbol{A} x_{t+1} + \boldsymbol{B} \mathbb{E}_{t+1} x_{t+2} + \varepsilon_{t+1}
$$

where $\boldsymbol{A} = (F^2)^{-1} + \beta F^2$, $\boldsymbol{B} = -\beta I$, and $\varepsilon_t = \frac{\beta}{\rho} \boldsymbol{C} \mathbb{E}_t \,\mathrm{d}w_{t+1}$ where $\boldsymbol{C} = -((F^2)^{-1} - F^2)$.

Lemma A.5. *Suppose $x_t = \boldsymbol{A} x_{t+1} + \boldsymbol{B} x_{t+2} + \varepsilon_{t+1} \in \mathbb{R}^S$ and $y_t = \boldsymbol{Z} x_t$ where $\boldsymbol{A}, \boldsymbol{B} \in \mathbb{R}^{S\times S}$ are invertible and $\boldsymbol{Z} \in \mathbb{R}^{S\times S}$ is of rank $S-1$. Then, we can always write*

$$
y_t = \Theta_1 y_{t+1} + \Theta_2 y_{t+2} + \Theta_3 y_{t+3} + \Theta_4 y_{t+4} + \Omega_1 \varepsilon_{t+1} + \Omega_2 \varepsilon_{t+2} + \Omega_3 \varepsilon_{t+3}.
$$

for some matrices $\Theta_i = \Theta_i(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{Z})$ and $\Omega_i = \Omega_i(\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{Z})$.

Proof. Write $\boldsymbol{x} = \begin{pmatrix} x_{t+3} \\ x_{t+4} \end{pmatrix}$, then we have

$$\begin{aligned}
y_{t+4} &= \begin{pmatrix} \boldsymbol{O} & \boldsymbol{Z} \end{pmatrix} \boldsymbol{x} \equiv \boldsymbol{M}_1 \boldsymbol{x} \\
y_{t+3} &= \begin{pmatrix} \boldsymbol{Z} & \boldsymbol{O} \end{pmatrix} \boldsymbol{x} \equiv \boldsymbol{M}_2 \boldsymbol{x} \\
y_{t+2} &= \boldsymbol{Z}(\boldsymbol{A} x_{t+3} + \boldsymbol{B} x_{t+4} + \varepsilon_{t+3}) \\
&= \begin{pmatrix} \boldsymbol{ZA} & \boldsymbol{ZB} \end{pmatrix} \boldsymbol{x} + \boldsymbol{Z} \varepsilon_{t+3} \\
&\equiv \boldsymbol{M}_3 \boldsymbol{x} + \boldsymbol{Z} \varepsilon_{t+3} \\
y_{t+1} &= \boldsymbol{Z}(\boldsymbol{A} x_{t+2} + \boldsymbol{B} x_{t+3} + \varepsilon_{t+2}) \\
&= \boldsymbol{Z}(\boldsymbol{A}(\boldsymbol{A} x_{t+3} + \boldsymbol{B} x_{t+4} + \varepsilon_{t+3}) + \boldsymbol{B} x_{t+3} + \varepsilon_{t+2}) \\
&= \begin{pmatrix} \boldsymbol{Z}(\boldsymbol{A}^2 + \boldsymbol{B}) & \boldsymbol{ZAB} \end{pmatrix} \boldsymbol{x} + \boldsymbol{ZA} \varepsilon_{t+3} + \boldsymbol{Z} \varepsilon_{t+2} \\
&\equiv \boldsymbol{M}_4 \boldsymbol{x} + \boldsymbol{ZA} \varepsilon_{t+3} + \boldsymbol{Z} \varepsilon_{t+2} \\
y_t &= \boldsymbol{Z}(\boldsymbol{A} x_{t+1} + \boldsymbol{B} x_{t+2} + \varepsilon_{t+1}) \\
&= \boldsymbol{Z}(\boldsymbol{A}(\boldsymbol{A}(\boldsymbol{A} x_{t+3} + \boldsymbol{B} x_{t+4} + \varepsilon_{t+3}) + \boldsymbol{B} x_{t+3} + \varepsilon_{t+2}) + \boldsymbol{B}(\boldsymbol{A} x_{t+3} + \boldsymbol{B} x_{t+4} + \varepsilon_{t+3}) + \varepsilon_{t+1}) \\
&= \begin{pmatrix} \boldsymbol{Z}(\boldsymbol{A}^3 + \boldsymbol{AB} + \boldsymbol{BA}) & \boldsymbol{Z}(\boldsymbol{A}^2 \boldsymbol{B} + \boldsymbol{B}^2) \end{pmatrix} \boldsymbol{x} + \boldsymbol{Z}(\boldsymbol{A}^2 + \boldsymbol{B}) \varepsilon_{t+3} + \boldsymbol{ZA} \varepsilon_{t+2} + \boldsymbol{Z} \varepsilon_{t+1} \\
&\equiv \boldsymbol{M}_5 \boldsymbol{x} + \boldsymbol{Z}(\boldsymbol{A}^2 + \boldsymbol{B}) \varepsilon_{t+3} + \boldsymbol{ZA} \varepsilon_{t+2} + \boldsymbol{Z} \varepsilon_{t+1}.
\end{aligned}$$

Note that $\boldsymbol{N}_1 \equiv \boldsymbol{M}_1$ is of rank $S-1$, $\boldsymbol{N}_2 \equiv \begin{pmatrix} \boldsymbol{M}_1 \\ \boldsymbol{M}_2 \end{pmatrix}$ is of rank $2(S-1)$. If all rows of $\boldsymbol{M}_3$ can be written as a linear combination of rows of $\boldsymbol{N}_2$, then we can write $y_t = \Theta_1 y_{t+1} + \Theta_2 y_{t+2}$. If not, $\boldsymbol{N}_3 \equiv \begin{pmatrix} \boldsymbol{N}_2 \\ \boldsymbol{M}_3 \end{pmatrix}$ is of rank at least $2(S-1)+1$.[66] By the same logic, we either prove the result, or $\boldsymbol{N}_4 \equiv \begin{pmatrix} \boldsymbol{N}_3 \\ \boldsymbol{M}_4 \end{pmatrix}$ is of rank (at least) $2S$. Thus, all rows of $\boldsymbol{M}_5$ can be written as a linear combination of rows of $\boldsymbol{N}_4$. This implies that we can find matrices $\Theta_1, \cdots, \Theta_4$ such that

$$\boldsymbol{M}_5 = \Theta_1 \boldsymbol{M}_4 + \Theta_2 \boldsymbol{M}_3 + \Theta_3 \boldsymbol{M}_2 + \Theta_4 \boldsymbol{M}_1.$$

Thus, we only need

$$\boldsymbol{Z}(\boldsymbol{A}^2 + \boldsymbol{B}) \varepsilon_{t+3} + \boldsymbol{ZA} \varepsilon_{t+2} + \boldsymbol{Z} \varepsilon_{t+1} = \Theta_1(\boldsymbol{ZA} \varepsilon_{t+3} + \boldsymbol{Z} \varepsilon_{t+2}) + \Theta_2(\boldsymbol{Z} \varepsilon_{t+3}) + \Omega_1 \varepsilon_{t+1} + \Omega_2 \varepsilon_{t+2} + \Omega_3 \varepsilon_{t+3}$$

or

$$\Omega_1 = \boldsymbol{Z}, \;\; \Omega_2 = \boldsymbol{ZA} - \Theta_1 \boldsymbol{Z}, \;\; \text{and} \;\; \Omega_3 = \boldsymbol{Z}(\boldsymbol{A}^2 + \boldsymbol{B}) - \Theta_1 \boldsymbol{ZA} - \Theta_2 \boldsymbol{Z}. \qquad \square$$

Lemma A.6. Ψ *is non-invertible.*

[66] With numerical simulation, we can see that generically $\boldsymbol{N}_3$ is of rank $2(S-1)+1$.

Proof. Denote population share of type ω as θ^ω, transition matrix as F^ω, and stationary distribution over sectors as π^ω. Define $\pi \equiv \sum_\omega \theta^\omega \pi^\omega$, then we have

$$\pi^\intercal L_\omega = \theta^\omega (\pi^\omega)^\intercal \text{ and } (\pi^\omega)^\intercal F^\omega = (\pi^\omega)^\intercal$$

Thus,

$$\pi^\intercal \Psi = \pi^\intercal \Big(L_1((F^1)^{-1} + \beta F^1)L_1^{-1} - L_2((F^2)^{-1} + \beta F^2)L_2^{-1}\Big) L_2 = (\pi^\intercal + \beta\pi^\intercal - \pi^\intercal - \beta\pi^\intercal)L_2 = 0. \quad \square$$

By combining the previous two lemmas, we can write

$$y_t = \Theta_1 y_{t+1} + \Theta_2 \mathbb{E}_{t+1} y_{t+2} + \Theta_3 \mathbb{E}_{t+1} y_{t+3} + \Theta_4 \mathbb{E}_{t+1} y_{t+4} + \Omega_1 \varepsilon_{t+1} + \Omega_2 \mathbb{E}_{t+1} \varepsilon_{t+2} + \Omega_3 \mathbb{E}_{t+1} \varepsilon_{t+3}.$$

where $\Theta_i = \Theta_i(\boldsymbol{A}, \boldsymbol{B}, \Psi)$ and $\Omega_i = \Omega_i(\boldsymbol{A}, \boldsymbol{B}, \Psi)$. Plugging in the definitions of y_t and ε_t, we have

$$\begin{aligned}
-\mathrm{d}\ln \ell_t + \boldsymbol{X}\mathrm{d}\ln \ell_{t+1} - \beta\mathbb{E}_t \mathrm{d}\ln \ell_{t+2} - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+1} = {} & \Theta_1(-\mathrm{d}\ln \ell_{t+1} + \boldsymbol{X}\mathbb{E}_t \mathrm{d}\ln \ell_{t+2} - \beta\mathbb{E}_t \mathrm{d}\ln \ell_{t+3} - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+2}) \\
& + \Theta_2(-\mathbb{E}_t \mathrm{d}\ln \ell_{t+2} + \boldsymbol{X}\mathbb{E}_t \mathrm{d}\ln \ell_{t+3} - \beta\mathbb{E}_t \mathrm{d}\ln \ell_{t+4} - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+3}) \\
& + \Theta_3(-\mathbb{E}_t \mathrm{d}\ln \ell_{t+3} + \boldsymbol{X}\mathbb{E}_t \mathrm{d}\ln \ell_{t+4} - \beta\mathbb{E}_t \mathrm{d}\ln \ell_{t+5} - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+4}) \\
& + \Theta_4(-\mathbb{E}_t \mathrm{d}\ln \ell_{t+4} + \boldsymbol{X}\mathbb{E}_t \mathrm{d}\ln \ell_{t+5} - \beta\mathbb{E}_t \mathrm{d}\ln \ell_{t+6} - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+5}) \\
& + \tfrac{\beta}{\rho}\Omega_1 \boldsymbol{C}\mathbb{E}_t \mathrm{d}w_{t+2} + \tfrac{\beta}{\rho}\Omega_2 \boldsymbol{C}\mathbb{E}_t \mathrm{d}w_{t+3} + \tfrac{\beta}{\rho}\Omega_3 \boldsymbol{C}\mathbb{E}_t \mathrm{d}w_{t+4}
\end{aligned}$$

or equivalently

$$\begin{aligned}
\mathrm{d}\ln \ell_t = {} & (\Theta_1 + \boldsymbol{X})\mathrm{d}\ln \ell_{t+1} + (-\Theta_1\boldsymbol{X} + \Theta_2 - \beta I)\mathbb{E}_t \mathrm{d}\ln \ell_{t+2} + (\beta\Theta_1 - \Theta_2\boldsymbol{X} + \Theta_3)\mathbb{E}_t \mathrm{d}\ln \ell_{t+3} \\
& + (\beta\Theta_2 - \Theta_3\boldsymbol{X} + \Theta_4)\mathbb{E}_t \mathrm{d}\ln \ell_{t+4} + (\beta\Theta_3 - \Theta_4\boldsymbol{X})\mathbb{E}_t \mathrm{d}\ln \ell_{t+5} + (\beta\Theta_4)\mathbb{E}_t \mathrm{d}\ln \ell_{t+6} \\
& - \tfrac{\beta}{\rho}\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+1} + (\tfrac{\beta}{\rho}\Theta_1\boldsymbol{Y} - \tfrac{\beta}{\rho}\Omega_1\boldsymbol{C})\mathbb{E}_t \mathrm{d}w_{t+2} + (\tfrac{\beta}{\rho}\Theta_2\boldsymbol{Y} - \tfrac{\beta}{\rho}\Omega_2\boldsymbol{C})\mathbb{E}_t \mathrm{d}w_{t+3} + (\tfrac{\beta}{\rho}\Theta_3\boldsymbol{Y} - \tfrac{\beta}{\rho}\Omega_3\boldsymbol{C})\mathbb{E}_t \mathrm{d}w_{t+4} + \tfrac{\beta}{\rho}\Theta_4\boldsymbol{Y}\mathbb{E}_t \mathrm{d}w_{t+5}.
\end{aligned}$$

This can be rearranged to obatin Proposition A.6.[67]

$$\begin{aligned}
\mathrm{d}\ln \ell_t = {} & \boldsymbol{\Gamma}_1 \mathrm{d}\ln \ell_{t+1} + \boldsymbol{\Gamma}_2\mathbb{E}_t \mathrm{d}\ln \ell_{t+2} + \boldsymbol{\Gamma}_3\mathbb{E}_t \mathrm{d}\ln \ell_{t+3} + \boldsymbol{\Gamma}_4\mathbb{E}_t \mathrm{d}\ln \ell_{t+4} + \boldsymbol{\Gamma}_5\mathbb{E}_t \mathrm{d}\ln \ell_{t+5} + \boldsymbol{\Gamma}_6\mathbb{E}_t \mathrm{d}\ln \ell_{t+6} \\
& + \tfrac{\beta}{\rho}(\boldsymbol{\Lambda}_1\mathbb{E}_t \mathrm{d}w_{t+1} + \boldsymbol{\Lambda}_2\mathbb{E}_t \mathrm{d}w_{t+2} + \boldsymbol{\Lambda}_3\mathbb{E}_t \mathrm{d}w_{t+3} + \boldsymbol{\Lambda}_4\mathbb{E}_t \mathrm{d}w_{t+4} + \boldsymbol{\Lambda}_5\mathbb{E}_t \mathrm{d}w_{t+5}).
\end{aligned}$$

$\square$

B.3 Alternative Methods of Extrapolation

[67] If the number of types is 3, then we need $(\mathrm{d}\ln \ell_{t+1}, \cdots, \mathrm{d}\ln \ell_{t+10})$, and with the number of types W, we need $(\mathrm{d}\ln \ell_{t+1}, \cdots, \mathrm{d}\ln \ell_{t+4W-2})$.

B.3.1 Pure Extrapolation

Suppose we have panel data of length $K < \infty$. From the data, we can observe the conditional probability

$$\Pr\left(s_{t+k} = s_k | s_t = s_0, s_{t+1} = s_1, \cdots, s_{t+k-1} = s_{k-1}\right),$$

for all $s_0, s_1, \ldots, s_k \in \mathcal{S}$ and k less than K. To extrapolate the probabilities with k greater than or equal to K, we truncate the history and assume the following:

$$\begin{aligned}
&\Pr\left(s_{t+k} = s_k \middle| s_t = s_0, s_{t+1} = s_1, \cdots, s_{t+k-1} = s_{k-1}\right) \\
=& \Pr\left(s_{t+k} = s_k | s_{t+k-K+1} = s_{k-K+1}, \cdots, s_{t+k-1} = s_{k-1}\right).
\end{aligned}$$

In short, we assume a $(K-1)$-th order Markov process and calculate the probabilities accordingly. In this sense, this extrapolation is a strict generalization of the canonical model's extrapolation, which is based on the assumption that the sector choice follow a first-order Markov process.

B.3.2 Extrapolation Using Retention Model

The retention model developed in Henry (1971) is based on the idea that Markov chains can be viewed as the conjunction of two processes: One determines whether workers change sectors or not, and the other governs which sector they choose, conditional on sector switching. Any transition matrix F can be decomposed as follows:

$$F = F^{\text{diag}} + (I - F^{\text{diag}})F^{\text{off-diag}}$$

where

$$\begin{aligned}
(F^{\text{diag}})_{i,j} &= F_{i,j} \cdot \mathbb{1}_{i=j} \\
(F^{\text{off-diag}})_{i,j} &= \Pr(s_{t+1} = j | s_t = i, s_{t+1} \neq i) = \frac{F_{i,j}}{1 - F_{i,i}}.
\end{aligned}$$

The idea of extrapolation is to treat the two parts of the process separately. For example, we can extrapolate each element of the first part of the worker flow matrices to compute the full series of $\{(\mathcal{F}_k)^{\text{diag}}\}$. Then, we can assume that the second part remains constant for all k: $(\mathcal{F}_k)^{\text{off-diag}} = (\mathcal{F}_1)^{\text{off-diag}}$ for all k.

B.4 Interpretation of the Estimation Results

Denote the log wages and switching costs estimated under the normalization $\rho = 1$ by $\ln w^0$ and C^0. Then, the true log wages and switching costs are given by

$$\ln w = \rho \cdot \ln w^0 \text{ and } C = \rho \cdot C^0.$$

Thus, when $C^0 = 1$, the consumption equivalent variation of paying switching cost is implicitly given by

$$\frac{\ln w(1 - \texttt{CEV})}{1-\beta} = \frac{\ln w}{1-\beta} - \rho,$$

which gives

$$\texttt{CEV} = 1 - \exp(-\rho(1-\beta)) = 3.25\%.$$

Likewise, our estimate of Fréchet parameter is 0.825, which means that the consumption equivalent variation corresponding to one standard deviation lower realization of the idiosyncratic shock is given by

$$\frac{\ln w(1 - \texttt{CEV})}{1-\beta} = \frac{\ln w}{1-\beta} - 2 \cdot \frac{\pi\rho}{\sqrt{6}}$$

where we multiply two because it is the difference between two realizations of idiosyncratic shocks. This gives

$$\texttt{CEV} = 1 - \exp(-\frac{2\pi\rho(1-\beta)}{\sqrt{6}}) = 8.12\%.$$

In contrast, ACM assume linear utility function, so the consumption equivalent variation can be computed as

$$\frac{w(1 - \texttt{CEV}_{\text{ACM}})}{1-\beta} = \frac{w}{1-\beta} - C_{\text{ACM}}$$

Thus, we have (given their normalization of average wages to one)

$$\texttt{CEV}_{\text{ACM}} = \frac{(1-\beta)C_{\text{ACM}}}{w} = 19.7\%$$

where we use the number in Panel IV of Table 3, which is used in their counterfactual exercises. Likewise, their estimate of Fréchet parameter is 1.884, which means that the consumption equivalent variation corresponding to one standard deviation lower realization of the idiosyncratic shock is given by

$$\frac{w(1 - \texttt{CEV}_{\text{ACM}})}{1-\beta} = \frac{w}{1-\beta} - 2 \cdot \frac{\pi\rho_{\text{ACM}}}{\sqrt{6}}$$

which gives

$$\texttt{CEV}_{\text{ACM}} = \frac{2\pi\rho_{\text{ACM}}(1-\beta)}{w\sqrt{6}} = 14.5\%.$$

C. Empirical Applications Appendix

C.1 Application 1: Hypothetical Trade Liberalization

Closing the Model. We consider the first version of the model in ACM, in which all sectors produce tradable output, and world prices are exogenously determined. Each sector s has a constant elasticity of substitution (CES) production function

$$y_t^s = \psi^s(\alpha^s(L_t^s)^{\rho^s} + (1-\alpha^s)(K_t^s)^{\rho^s})^{\frac{1}{\rho^s}}$$

with fixed sector-specific capital $K_t^s = 1$ normalized to one. The parameters should satisfy $\alpha^s \in (0,1)$, $\rho^s < 1$, and $\psi^s > 0$. Then, the sectoral wage is given by the marginal productivity of labor

$$w_t^s = p_t^s \alpha^s \psi^s (L_t^s)^{\rho^s - 1}(\alpha^s(L_t^s)^{\rho^s} + (1-\alpha^s))^{\frac{1-\rho^s}{\rho^s}}$$

where p_t^s is the domestic price of the sector-s good. Without loss of generality, we normalize the domestic prices to one in the initial steady state prior to the shock. Finally, workers have identical Cobb-Douglas with shares μ^s for sector s.

We follow the calibration strategy of ACM (except for the number of sectors). We set the values of α^s, ρ^s, and ψ^s to minimize the Euclidean distance between the model-implied values of sectoral wages, sectoral labor shares, and sector share of GDP; and the values computed from the data. The values of μ^s are calibrated from consumption shares from the Bureau of Labor Statistics (BLS).

C.2 Application 2: The China Shock

C.2.1 CDP's Model Extended with Worker Heterogeneity

In this section and the next, we closely follow the modeling decisions and notation of CDP. See CDP for more details and equilibrium characterization. CDP consider a world with N locations (indexed by n or i) and J sectors (indexed by j or k), where sector $j = 0$ represents non-employment. Time is discrete and indexed by $t \in \mathbb{N}_0$. In each location-sector combination, (n, j), there is a competitive local labor market.

Heterogeneous Workers. In each location n, there is a continuum of forward-looking workers who optimally decide which sector to supply their labor for each period. The heterogeneity of workers are indexed by ω. Similar to equation (1), the value of a worker of type ω who is employed in sector j at period t is given by

$$V_{jt}^{n,\omega} = U(c_{jt}^{n,\omega}) + \max_k\{\beta \mathbb{E}_t V_{kt+1}^{n,\omega} - C_{jk}^{n,\omega} + \rho \cdot \varepsilon_{kt}\}$$

where $c_{jt}^{n,\omega} = \prod_k (c_{jt}^{k,n,\omega})^{\alpha^k}$ is a Cobb-Douglas aggregator across local sectoral goods, with the corresponding price index $P_t^n = \prod_j (P_t^{nk}/\alpha^k)^{\alpha^k}$. Following CDP, we assume $U(c) = \log c$.

Households employed in a local labor market (n,j) earn a nominal wage of w_t^{nj}, consuming $c_{jt}^{n,\omega} = \tilde{c}^{nj,\omega} \cdot w_t^{nj}/P_t^n$ units of consumption aggregate where $\tilde{c}^{nj,\omega}$ is a type-specific shifter representing non-pecuniary sectoral preferences. Non-employed household (who chooses sector $j = 0$) consume $c_{0t}^{n,\omega} = b^{n,\omega}$ units of consumption aggregate in terms of home production. The ex-ante value and sector choice probabilities are characterized as in equations (2) and (3):

$$v_{jt}^{n,\omega} = U(c_{jt}^{n,\omega}) + \rho \ln \sum_k (\exp(\beta \mathbb{E}_t v_{kt+1}^{n,\omega}) / \exp(C_{jk}^{n,\omega}))^{1/\rho} \tag{A.7}$$

$$F_{jkt}^{n,\omega} = \frac{(\exp(\beta \mathbb{E}_t v_{kt+1}^{n,\omega}) / \exp(C_{jk}^{n,\omega}))^{1/\rho}}{\sum_{k'} (\exp(\beta \mathbb{E}_t v_{k't+1}^{n,\omega}) / \exp(C_{jk'}^{n,\omega}))^{1/\rho}}. \tag{A.8}$$

Thus, the law of motion of sectoral labor supply of type ω workers in region n is

$$\ell_{kt+1}^{n,\omega} = \sum_j F_{jkt}^{n,\omega} \ell_{jt}^{n,\omega}. \tag{A.9}$$

Finally, let $L_t^{nj} = \sum_\omega \ell_{jt+1}^{n,\omega}$ be the total labor supply to local labor market (n,j). In this environment, Assumption 1 holds, and we implicitly maintain Assumption 2 (with n superscript) as well.

Production. For each sector j, there is a continuum of different varieties of intermediate goods. Each region-sector combination draws a variety-specific productivity z^{nj}, which follows a Fréchet distribution with the dispersion parameter θ^j. Without loss, each variety is indexed by $z^j = (z^{1j}, z^{2j}, \ldots, z^{Nj})$. In each local labor market, (n,j), there is a continuum of perfectly competitive firms producing variety z^j. They have a Cobb-Douglas technology combining labor (l), structures (h), and local sectoral goods from all sectors (M):

$$q_t^{nj} = z^{nj} (A_t^{nj} (h_t^{nj})^{\xi^n} (l_t^{nj})^{1-\xi^n})^{\gamma^{nj}} \prod_k (M_t^{nj,nk})^{\gamma^{nj,nk}}$$

where A_t^{nj} is a sector-region specific productivity. Thus, the unit cost of producing this intermediate good is

$$\frac{x_t^{nj}}{z^{nj} (A_t^{nj})^{\gamma^{nj}}} \quad \text{where} \;\; x_t^{nj} = B^{nj} ((r_t^{nj})^{\xi^n} (w_t^{nj})^{1-\xi^n})^{\gamma^{nj}} \prod_k (P_t^{nk})^{\gamma^{nj,nk}} \tag{A.10}$$

where B^{nj} is a constant, r_t^{nj} is the rental price of structures, and P_t^{nk} is the price of the local sector-k goods.

Local sectoral goods are produced from intermediate goods in a competitive way, which are then used as final consumption and as materials for the production of intermediate varieties. The technology is given by:

$$Q_t^{nj} = \left(\int (\tilde{q}_t^{nj}(z^j))^{1-1/\eta^{nj}} \, \mathrm{d}\phi^j(z^j) \right)^{\eta^{nj}/(\eta^{nj}-1)}$$

where $\tilde{q}_t^{nj}(z^j)$ is the quantity of variety z^j used in the production, and $\phi^j(\cdot)$ is the joint distribution of the vector z^j. The intermediate good of variety z^j is sourced from a country with the minimum price, taking into account bilateral iceberg-type trade costs (κ). The minimized price is then given by

$$p_t^{nj}(z^j) = \min_i \left\{ \frac{\kappa_t^{nj,ij} x_t^{ij}}{z^{ij}(A_t^{ij})^{\gamma^{ij}}} \right\}.$$

Thus, the price of the local sectoral good is

$$P_t^{nj} = \Gamma\left(\frac{1+\theta^j - \eta^{nj}}{\theta^j}\right) \cdot \left(\sum_i (x_t^{ij}\kappa_t^{nj,ij})^{-\theta^j}(A_t^{ij})^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j} \tag{A.11}$$

Finally, the share of total expenditure in local market (n,j) on goods from market (i,j) is given by

$$\pi_t^{nj,ij} = \frac{(x_t^{ij}\kappa_t^{nj,ij})^{-\theta^j}(A_t^{ij})^{\theta^j\gamma^{ij}}}{\sum_{i'}(x_t^{i'j}\kappa_t^{nj,i'j})^{-\theta^j}(A_t^{i'j})^{\theta^j\gamma^{i'j}}} \tag{A.12}$$

Structure Rentier. There is a continuum of structure rentiers in each region n. The own the local structures of fixed amount $\{H^{nj}\}_j$ and rent them to local firms. The received rents are aggregated at the global-level, and rentiers in each region n receive a constant share ι^n of the total global revenue:

$$\iota^n \chi_t \quad \text{where } \chi_t = \sum_i \sum_k r_t^{ik} H^{ik}.$$

Market Clearing. Market clearing for goods market, labor market, and structure market is given by

$$X_t^{nj} = \sum_k \gamma^{nk,nj} \sum_i \pi_t^{ik,nk} X_t^{ik} + \alpha^j \left(\sum_k w_t^{nk} L_t^{nk} + \iota^n \chi_t\right) \tag{A.13}$$

$$w_t^{nj} L_t^{nj} = \gamma^{nj}(1-\xi^n)\sum_i \pi_t^{ij,nj} X_t^{ij} \tag{A.14}$$

$$r_t^{nj} H^{nj} = \gamma^{nj}\xi^n \sum_i \pi_t^{ij,nj} X_t^{ij} \tag{A.15}$$

where X_t^{nj} is the total expenditure on sector j good in region n.

Equilibrium. Following CDP, we group exogenous state variables of the economy into time-varying ones and time-invariant ones:

$$\Theta_t \equiv (\{A_t^{nj}\}_{n,j}, \{\kappa_t^{nj,ij}\}_{n,i,j}) \text{ and } \bar{\Theta} \equiv (\{C_{jk}^{n,\omega}\}_{j,k,n,\omega}, \{H^{nj}\}_{n,j}, \{\tilde{c}^{nj,\omega}\}_{n,j,\omega}, \{b^{n,\omega}\}_{n,\omega}).$$

Given the initial distribution of labor and the path of exogenous state variables $(\{\ell_{j0}^{n,\omega}\}_{j,n,\omega}, \{\Theta_t\}_{t=0}^{\infty}, \bar{\Theta})$, a *sequential competitive equilibrium* corresponds to a sequence of $\{L_t, F_t, v_t, x_t, P_t, \pi_t, X_t, w_t, r_t\}_{t=0}^{\infty}$, where $L_t = \{\ell_{jt}^{n,\omega}\}_{j,n,\omega}$, $F_t = \{F_{jkt}^{n,\omega}\}_{j,k,n,\omega}$, $v_t = \{v_{jt}^{n,\omega}\}_{j,n,\omega}$, $x_t = \{x_t^{nj}\}_{n,j}$, $P_t = \{P_t^{n,j}\}_{n,j}$, $\pi_t = \{\pi_t^{ij,nj}\}_{i,j,n}$, $X_t = \{X_t^{nj}\}_{n,j}$, $w_t = \{w_t^{nj}\}_{n,j}$, and $r_t = \{r_t^{nj}\}_{n,j}$, such that households optimally make sector choice decisions, as described in (A.7)–(A.9); firms maximize their profits, as described in (A.10)–(A.12); all markets clear, as described in (A.13)–(A.15). A *stationary equilibrium* is a sequential competitive equilibrium such that $\{L_t, F_t, v_t, x_t, P_t, \pi_t, X_t, w_t, r_t\}$ is time-invariant.

C.2.2 Dynamic Hat Algebra with Worker Heterogeneity

Following CDP, we solve for the equilibrium in time differences. We denote by $\dot{y}_{t+1} \equiv (y_{t+1}^1/y_t^1, y_{t+1}^2/y_t^2, \cdots)$ the proportional change in any scalar or vector. The following proposition corresponds to Propositions 1 and 2 of CDP, but allowing for worker heterogeneity.

Proposition A.7 (Solving the model). *Suppose that the economy is initially starting from a stationary equilibrium at period $t = 0$. Up to the first-order approximation around a stationary equilibrium, given a sequence of changes in exogenous state variables, $\{\dot{\Theta}_t\}_{t=1}^{\infty}$ satisfying $\lim_{t\to\infty} \dot{\Theta}_t = 1$, known to agents in period $t = 1$, the solution to the sequential equilibrium in time differences does not require information on the level of the exogenous state variables $\{\Theta_t\}_{t=0}^{\infty}$ or $\bar{\Theta}$, and solves the following system of equations:*

$$
\begin{aligned}
v_t^{nj} &= v_0^{nj} + \sum_k \beta^k \mathcal{F}_k \ln\left(\frac{\dot{w}_{t+k}^n}{\dot{P}_{t+k}^n} \cdot \frac{\dot{w}_{t+k-1}^n}{\dot{P}_{t+k-1}^n} \cdot \cdots \cdot \frac{\dot{w}_1^n}{\dot{P}_1^n}\right) \\
L_t^{nj} &= L_0^{nj} \cdot \exp\left(\left(\sum_{s=0}^{t-2}\sum_{k=0}^{\infty} \frac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k}^n - \mathcal{F}_{s+k+2}^n) \ln\left(\frac{\dot{w}_{t-s+k}^n}{\dot{P}_{t-s+k}^n} \cdot \frac{\dot{w}_{t-s+k-1}^n}{\dot{P}_{t-s+k-1}^n} \cdot \cdots \cdot \frac{\dot{w}_1^n}{\dot{P}_1^n}\right)\right)_j\right) \\
\dot{x}_{t+1}^{nj} &= (\dot{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n} (\dot{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_k (\dot{P}_{t+1}^{nk})^{\gamma^{nj,nk}} \\
\dot{P}_{t+1}^{nj} &= \left(\sum_i \pi_t^{nj,ij} (\dot{x}_{t+1}^{ij}\dot{\kappa}_{t+1}^{nj,ij})^{-\theta^j} (\dot{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j} \\
\pi_{t+1}^{nj,ij} &= \pi_t^{nj,ij} \left(\frac{\dot{x}_{t+1}^{ij}\dot{\kappa}_{t+1}^{nj,ij}}{\dot{P}_{t+1}^{nj}}\right)^{-\theta^j} (\dot{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}} \\
X_{t+1}^{nj} &= \sum_k \gamma^{nk,nj} \sum_i \pi_{t+1}^{ik,nk} X_{t+1}^{ik} + \alpha^j \left(\sum_k \dot{w}_{t+1}^{nk}\dot{L}_{t+1}^{nk} w_t^{nk} L_t^{nk} + \iota^n \chi_{t+1}\right) \\
\dot{w}_{t+1}^{nj}\dot{L}_{t+1}^{nj} w_t^{nj} L_t^{nj} &= \gamma^{nj}(1-\xi^n) \sum_i \pi_{t+1}^{ij,nj} X_{t+1}^{ij}
\end{aligned}
$$

where $\chi_{t+1} = \sum_i \sum_k \frac{\xi^i}{1-\xi^i} \dot{w}_{t+1}^{ik} \dot{L}_{t+1}^{ik} w_t^{ik} L_t^{ik}$ and $\dot{w}_t^n$ is a vector whose jth element is $\dot{w}_t^{nj}$.

Proof. The last five equations write the equilibrium conditions for the static multicountry interregional trade model in time differences. See CDP for a proof of this representation. Note that the real wage in period t can be written as

$$\frac{w_t^{nj}}{P_t^n} = \frac{\dot{w}_t^{nj}}{\dot{P}_t^n} \cdot \frac{\dot{w}_{t-1}^{nj}}{\dot{P}_{t-1}^n} \cdot \cdots \cdot \frac{\dot{w}_1^{nj}}{\dot{P}_1^n} \frac{w_0^{nj}}{P_0^n}.$$

Since the economy is initially starting from a stationary equilibrium at period $t = 0$, we have

$$\mathrm{d}\ln\left(\frac{w_t^{nj}}{P_t^n}\right) = \ln\left(\frac{\dot{w}_t^{nj}}{\dot{P}_t^n} \cdot \frac{\dot{w}_{t-1}^{nj}}{\dot{P}_{t-1}^n} \cdot \cdots \cdot \frac{\dot{w}_1^{nj}}{\dot{P}_1^n}\right). \tag{A.16}$$

Note that with shocks known to agent at period $t = 1$, the sufficient statistics results of Proposition 1 can be simplified to

$$\begin{aligned}
\mathrm{d}v_t &= \sum_{k=0}^{\infty} \beta^k \mathcal{F}_k \,\mathrm{d}w_{t+k} \\
\mathrm{d}\ln \ell_t &= \sum_{s=0}^{t-2} \sum_{k=0}^{\infty} \tfrac{\beta^{k+1}}{\rho} (\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2}) \,\mathrm{d}w_{t-s+k}.
\end{aligned}$$

Plugging expression (A.16) into these equations gives the first two equations. □

In the baseline economy, the path of exogenous state variables are given by $\{\Theta_t\}_{t=0}^{\infty}$ and $\bar{\Theta}$. In the counterfactual economy, we consider changes in exogenous state variables. We denote the new path by $\{\Theta'_t\}_{t=0}^{\infty}$. We assume that agents learn about these changes at period $t = 1$. This proposition corresponds to Proposition 3 of CDP, but allowing for worker heterogeneity. It shows how to solve for the counterfactual changes in endogenous variables in time differences and relative to a baseline economy without the need to estimate the level of the exogenous state variables. We denote by $\hat{y}_{t+1} \equiv \dot{y}'_{t+1}/\dot{y}_{t+1}$ the ratio of time differences between the counterfactual equilibrium and the baseline equilibrium.

Proposition A.8 (Solving for Counterfactuals). *Suppose that the economy is initially starting from a stationary equilibrium at period $t = 0$. Up to the first-order approximation around a stationary equilibrium, given a baseline equilibrium, $\{L_t, \pi_t, X_t\}_{t=0}^{\infty}$, and a counterfactual sequence of changes in exogenous state variables, $\{\hat{\Theta}_t\}_{t=1}^{\infty}$ satisfying $\lim_{t\to\infty} \hat{\Theta}_t = 1$, known to agents in period $t = 1$, the solution to the counterfactual sequential equilibrium in time differences does not require information on* *the level of the exogenous state variables* $\{\Theta_t\}_{t=0}^{\infty}$ *or* $\bar{\Theta}$*, and solves the following system of equations:*

$$
\begin{aligned}
v_t^{\prime nj} &= v_t^{nj} + \sum_k \beta^k \mathcal{F}_k \ln\left(\frac{\hat{w}_{t+k}^n}{\hat{P}_{t+k}^n} \cdot \frac{\hat{w}_{t+k-1}^n}{\hat{P}_{t+k-1}^n} \cdot \dots \cdot \frac{\hat{w}_1^n}{\hat{P}_1^n}\right) \\
L_t^{\prime nj} &= L_t^{nj} \cdot \exp\left(\left(\sum_{s=0}^{t-2}\sum_{k=0}^{\infty} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k}^n - \mathcal{F}_{s+k+2}^n)\ln\left(\frac{\hat{w}_{t-s+k}^n}{\hat{P}_{t-s+k}^n} \cdot \frac{\hat{w}_{t-s+k-1}^n}{\hat{P}_{t-s+k-1}^n} \cdot \dots \cdot \frac{\hat{w}_1^n}{\hat{P}_1^n}\right)\right)_j\right) \\
\hat{x}_{t+1}^{nj} &= (\hat{L}_{t+1}^{nj})^{\gamma^{nj}\xi^n} (\hat{w}_{t+1}^{nj})^{\gamma^{nj}} \prod_k (\hat{P}_{t+1}^{nk})^{\gamma^{nj,nk}} \\
\hat{P}_{t+1}^{nj} &= \left(\sum_i \pi_t^{\prime nj,ij} \dot{\pi}_{t+1}^{nj,ij} (\hat{x}_{t+1}^{ij}\hat{\kappa}_{t+1}^{nj,ij})^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}}\right)^{-1/\theta^j} \\
\pi_{t+1}^{\prime nj,ij} &= \pi_t^{\prime nj,ij} \dot{\pi}_{t+1}^{\prime nj,ij} \left(\frac{\hat{x}_{t+1}^{ij}\hat{\kappa}_{t+1}^{nj,ij}}{\hat{P}_{t+1}^{nj}}\right)^{-\theta^j} (\hat{A}_{t+1}^{ij})^{\theta^j\gamma^{ij}} \\
X_{t+1}^{\prime nj} &= \sum_k \gamma^{nk,nj} \sum_i \pi_{t+1}^{\prime ik,nk} X_{t+1}^{\prime ik} + \alpha^j \left(\sum_k \hat{w}_{t+1}^{nk}\hat{L}_{t+1}^{nk} w_t^{\prime nk} L_t^{\prime nk} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk} + \iota^n \chi_{t+1}^{\prime}\right) \\
\hat{w}_{t+1}^{nj}\hat{L}_{t+1}^{nj} &= \frac{\gamma^{nj}(1-\xi^n)}{w_t^{\prime nk} L_t^{\prime nk} \dot{w}_{t+1}^{nk} \dot{L}_{t+1}^{nk}} \sum_i \pi_{t+1}^{\prime ij,nj} X_{t+1}^{\prime ij}
\end{aligned}
$$

where $\chi_{t+1}^{\prime} = \sum_i \sum_k \frac{\xi^i}{1-\xi^i} \hat{w}_{t+1}^{ik} \hat{L}_{t+1}^{ik} w_t^{\prime ik} L_t^{\prime ik} \dot{w}_{t+1}^{ik} \dot{L}_{t+1}^{ik}$ *and* $\hat{w}_t^n$ *is a vector whose* j*th element is* $\hat{w}_t^{nj}$*.*

C.2.3 Calibration of the Model

There are 87 regions, 50 US states and 37 other countries, and 4 sectors. The model has the following parameters: value added shares ($\{\gamma^{nj}\}_{n,j}$), the share of structures in value added ($\{\xi^n\}_n$), the input-output coefficients ($\{\gamma^{nk,nj}\}_{n,k,j}$), rentier shares ($\{\iota^n\}_n$), consumption Cobb-Douglas shares ($\{\alpha^j\}_j$), the discount factor (β), the sectoral trade elasticities ($\{\theta^j\}_j$), and the inverse sector-choice elasticity (ρ).[68] The year 2000 corresponds to the period $t = 0$ of the model. To apply dynamic hat algebra, we use data on bilateral trade flows π_t and value added $\{w_t^{nj} L_t^{nj} + r_t^{nj} H^{nj}\}_{n,j}$ from year 2000 to 2007. The data comes from the World Input-Output Database (WIOD), the 2002 Commodity Flow Survey (CFS), and regional employment data from the Bureau of Economic Analysis (BEA). See CDP for more details. Finally, we need to identify the magnitude of the China shock.

Parameters. Following CDP, value added shares ($\{\gamma^{nj}\}_{n,j}$), the share of structures in value added ($\{\xi^n\}_n$), and the input-output coefficients ($\{\gamma^{nk,nj}\}_{n,k,j}$) are constructed from the BEA and the WIOD data. Rentier shares ($\{\iota^n\}_n$), consumption Cobb-Douglas shares ($\{\alpha^j\}_j$) are calculated from the constructed trade and production data. We set the quarterly discount factor to $\beta = 0.99$. We use the sectoral trade elasticities from Dix-Carneiro et al. (2023), $\theta^j = 4$. Finally, we obtain the inverse migration elasticity at a quarterly frequency from the estimate in Section 4.5. In particular, the value of

[68] Without loss of generality we can ignore $\{\eta^{nj}\}_{n,j}$ as they only appear in the constant term of the price index.

rho at a quarterly frequency is calibrated such that both the yearly and quarterly analysis deliver the same elasticity of labor with respect to the wage changes. Up to a first-order approximation, the response of labor to a permanent change in wage $w_t = w$ known to households at $t = 1$ is given by

$$\mathrm{d}\ln \ell_t{}^{\text{quarterly}} = \sum_{s=0}^{t-2} \sum_{k=0}^{\infty} \frac{(\beta^{\text{quarterly}})^{k+1}}{\rho^{\text{quarterly}}} (\mathcal{F}_{s+k}^{\text{quarterly}} - \mathcal{F}_{s+k+2}^{\text{quarterly}})\, \mathrm{d}w$$
$$\mathrm{d}\ln \ell_t{}^{\text{yearly}} = \sum_{s=0}^{t-2} \sum_{k=0}^{\infty} \frac{(\beta^{\text{yearly}})^{k+1}}{\rho^{\text{yearly}}} (\mathcal{F}_{s+k}^{\text{yearly}} - \mathcal{F}_{s+k+2}^{\text{yearly}})\, \mathrm{d}w.$$

We calculate the value of $\rho^{\text{quarterly}}$ that minimizes the difference between $\frac{\mathrm{d}\ln \ell_2^{\text{yearly}}}{\mathrm{d}w}$ and $\frac{\mathrm{d}\ln \ell_2^{\text{quarterly}}}{\mathrm{d}w}$:

$$\left\| \sum_{k=0}^{\infty} \frac{(\beta^{\text{quarterly}})^{k+1}}{\rho^{\text{quarterly}}} (\mathcal{F}_k^{\text{quarterly}} - \mathcal{F}_{k+2}^{\text{quarterly}}) - \sum_{k=0}^{\infty} \frac{(\beta^{\text{yearly}})^{k+1}}{\rho^{\text{yearly}}} (\mathcal{F}_k^{\text{yearly}} - \mathcal{F}_{k+2}^{\text{yearly}}) \right\|_2 .$$

The resulting value is $\rho = 1.0011$. Figure A.14a plots the elements of $\frac{\mathrm{d}\ln \ell_2^{\text{yearly}}}{\mathrm{d}w}$ against the correponding elements of $\frac{\mathrm{d}\ln \ell_2^{\text{quarterly}}}{\mathrm{d}w}$. In Figure A.14b, we also compare the (normalized) sectoral value changes at the yearly and quarterly frequency:

$$(1 - \beta^{\text{quarterly}})\, \mathrm{d}v_0 = (1 - \beta^{\text{quarterly}}) \sum_{k=0}^{\infty} (\beta^{\text{quarterly}})^k \mathcal{F}_k^{\text{quarterly}}\, \mathrm{d}w$$
$$(1 - \beta^{\text{yearly}})\, \mathrm{d}v_0 = (1 - \beta^{\text{yearly}}) \sum_{k=0}^{\infty} (\beta^{\text{yearly}})^k \mathcal{F}_k^{\text{yearly}}\, \mathrm{d}w.$$

China Shock. Following CDP, we first compute the predicted increases in US imports from China between 2000 and 2007 using the increases in imports from China of other eight advanced economies during the same period as an instrument. Given this plausibly China-driven increase in imports, we calibrate the increase in China's manufacturing TFP, $\hat{A}_t^{\text{China,manufacturing}}$, from 2000 to 2007 such that the structural model the increase in imports from China that exactly matches the predicted imports increase.

D. Omitted Proofs

Proof of Equations (5) and (6). Taking the first-order approximations to equations (2) and (4) and the definition of F_{ijt}, we have

$$
\begin{aligned}
\mathrm{d}v_{it} &= \mathrm{d}w_{it} + \rho \sum_j \left. \frac{\Big(\exp(\beta \mathbb{E}_t v_{jt+1}) / \exp(C_{ij})\Big)^{1/\rho}}{\sum_k \Big(\exp(\beta \mathbb{E}_t v_{kt+1}) / \exp(C_{ik})\Big)^{1/\rho}} \right|_{\text{steady state}} \cdot \tfrac{\beta}{\rho} \mathbb{E}_t \, \mathrm{d}v_{jt+1} \\
&= \mathrm{d}w_{it} + \beta \sum_j F_{ij} \mathbb{E}_t \, \mathrm{d}v_{jt+1} \qquad \text{(A.17)} \\
\mathrm{d}\ln \ell_{jt+1} &= \sum_i \left. \frac{F_{ijt}\ell_{it}}{\ell_{jt+1}} \right|_{\text{steady state}} \cdot \left(\mathrm{d}\ln \ell_{it} + \mathrm{d}\ln F_{ijt}\right) \\
&= \sum_i B_{ji} \cdot \left(\mathrm{d}\ln \ell_{it} + \mathrm{d}\ln F_{ijt}\right) \qquad \text{(A.18)} \\
\mathrm{d}\ln F_{ijt} &= \tfrac{\beta}{\rho} \left(\mathbb{E}_t \, \mathrm{d}v_{jt+1} - \sum_k F_{ik} \mathbb{E}_t \, \mathrm{d}v_{kt+1} \right). \qquad \text{(A.19)}
\end{aligned}
$$

Plugging equation (A.19) into equation (A.18), we have

$$
\begin{aligned}
\mathrm{d}\ln \ell_{jt+1} &= \sum_i B_{ji} \, \mathrm{d}\ln \ell_{it} + \tfrac{\beta}{\rho} \sum_i B_{ji} \cdot \left(\mathbb{E}_t \, \mathrm{d}v_{jt+1} - \sum_k F_{ik} \mathbb{E}_t \, \mathrm{d}v_{kt+1} \right) \\
&= \sum_i B_{ji} \, \mathrm{d}\ln \ell_{it} + \tfrac{\beta}{\rho} \cdot \left(\mathbb{E}_t \, \mathrm{d}v_{jt+1} - \sum_i B_{ji} \sum_k F_{ik} \mathbb{E}_t \, \mathrm{d}v_{kt+1} \right). \qquad \text{(A.20)}
\end{aligned}
$$

With vector notation, equations (A.17) and (A.20) become equations (5) and (6). □

Proof of Lemma 1. Equation (5) relates $\mathrm{d}v_t^\omega$ with $\mathrm{d}v_{t+1}^\omega$. Solving this equation forward, we can write $\mathrm{d}v_t^\omega$ as a linear combination of the expected value of a sequence $(\mathrm{d}w_t^\omega, \mathrm{d}w_{t+1}^\omega, \mathrm{d}w_{t+2}^\omega, \cdots)$,

$$\mathrm{d}v_t^\omega = \sum_{k \geq 0} (\beta F^\omega)^k \mathbb{E}_t \, \mathrm{d}w_{t+k}^\omega.$$

Plugging this result into equation (6), we obtain a formula that relates $\mathrm{d}\ln \ell_{t+1}^\omega$ with $\mathrm{d}\ln \ell_t^\omega$,

$$\mathrm{d}\ln \ell_{t+1}^\omega = B^\omega \, \mathrm{d}\ln \ell_t^\omega + \tfrac{\beta}{\rho}(I - B^\omega F^\omega) \sum_{k \geq 0} (\beta F^\omega)^k \mathbb{E}_t \, \mathrm{d}w_{t+k+1}^\omega.$$

Solving this equation backward, we can write $\mathrm{d}\ln \ell_t^\omega$ as a linear combination of the expected value of a two-sided sequence $(\cdots, \mathrm{d}w_{t-1}^\omega, \mathrm{d}w_t^\omega, \mathrm{d}w_{t+1}^\omega, \cdots)$,

$$\mathrm{d}\ln \ell_t^\omega = \tfrac{\beta}{\rho} \sum_{s \geq 0} (B^\omega)^s (I - B^\omega F^\omega) \left(\sum_{k \geq 0} (\beta F^\omega)^k \mathbb{E}_{t-s-1} \, \mathrm{d}w_{t-s+k}^\omega \right).$$ □

Proof of Lemma A.3. Write $\theta_j = \exp(\frac{\beta}{\rho}\mathbb{E}v'_j)$ and $k_{ij} = \exp(\frac{1}{\rho}C_{ij})$, then

$$F_{ij} = \frac{\theta_j/k_{ij}}{\varphi_i} \quad \text{where } \varphi_i = \sum_k \theta_k/k_{ik}.$$

First, assume

$$k_{ij} = \begin{cases} k_i \cdot \tilde{k}_j & \text{if } i \neq j \\ 1 & \text{if } i = j \end{cases}.$$

Then, we have

$$\varphi_i = \sum_k \theta_j/k_{ik} = \theta_i - \frac{\theta_i}{k_i\tilde{k}_i} + \frac{1}{k_i}\sum_k \frac{\theta_k}{\tilde{k}_k} \tag{A.21}$$

and

$$\begin{aligned} \ell_i &= \sum_k \ell_k F_{ki} = \sum_k \ell_k \frac{\theta_i/k_{ki}}{\varphi_k} \\ &= \ell_i \frac{\theta_i}{\varphi_i} + \ell_i \frac{\theta_i}{\varphi_i}\frac{1}{k_i\tilde{k}_i} + \frac{\theta_i}{\tilde{k}_i}\sum_k \frac{\ell_k}{\varphi_k k_k} \end{aligned}$$

hence, rearranging,

$$\begin{aligned} \ell_i &= \left(1 - \frac{\theta_i}{\varphi_i}\left(1 - \frac{1}{k_i\tilde{k}_i}\right)\right)^{-1} \frac{\theta_i}{\tilde{k}_i}\sum_k \frac{\ell_k}{\varphi_k k_k} \\ &\overset{\text{(A.21)}}{=} \frac{\varphi_i\theta_i k_i}{\tilde{k}_i} \frac{\sum_k \frac{\ell_k}{\varphi_k k_k}}{\sum_k \frac{\theta_k}{\tilde{k}_k}}. \end{aligned}$$

Thus, we can show that $\ell_i F_{ij} = \ell_j F_{ji}$ from

$$\frac{\ell_i F_{ij}}{\ell_j F_{ji}} = \frac{\frac{\varphi_i\theta_i k_i}{\tilde{k}_i} \cdot \frac{\theta_j/k_{ij}}{\varphi_i}}{\frac{\varphi_j\theta_j k_j}{\tilde{k}_j} \cdot \frac{\theta_i/k_{ji}}{\varphi_j}} = 1.$$

second, assume instead $k_{ij} = k_{ji}$ for all j, i. Under this assumption, we can easily check that

$$\ell_i = \frac{\theta_i\varphi_i}{\sum_k \theta_k\varphi_k}$$

solves the equation

$$\ell_i = \sum_k \ell_k F_{ki}, \ \forall i.$$

Thus, we again have

$$\frac{\ell_i F_{ij}}{\ell_j F_{ji}} = \frac{\theta_i\varphi_i \frac{\theta_j/k_{ij}}{\varphi_i}}{\theta_j\varphi_j \frac{\theta_i/k_{ji}}{\varphi_j}} = 1.$$

□

Proof of Lemma 2. It is without loss to assume that there are two sectors. From Lemma A.4, we have

$$(\mathcal{F}_k)_{11} = 1 - \frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega f^k(2-\alpha_\omega-\beta_\omega)}{\sum_\omega \tilde{\beta}_\omega \theta_\omega}$$

$$\left((\mathcal{F}_1)^k\right)_{11} = 1 - \frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} f^k\left(2 - \frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} - \frac{\sum_\omega \tilde{\alpha}_\omega \theta_\omega \beta_\omega}{\sum_\omega \tilde{\alpha}_\omega \theta_\omega}\right).$$

Thus, we need to show that

$$\frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega \tilde{f}^k(\alpha_\omega+\beta_\omega)}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} < \frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} \tilde{f}^k\left(\frac{\sum_\omega \tilde{\beta}_\omega \theta_\omega \alpha_\omega}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} + \frac{\sum_\omega \tilde{\alpha}_\omega \theta_\omega \beta_\omega}{\sum_\omega \tilde{\alpha}_\omega \theta_\omega}\right)$$

where $\tilde{f}^k(x) = f^k(2-x)$, or equivalently

$$\frac{\sum_\omega \frac{\alpha_\omega \beta_\omega}{\alpha_\omega+\beta_\omega} \theta_\omega \tilde{f}^k(\alpha_\omega+\beta_\omega)}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} < \frac{\sum_\omega \frac{\alpha_\omega \beta_\omega}{\alpha_\omega+\beta_\omega} \theta_\omega}{\sum_\omega \tilde{\beta}_\omega \theta_\omega} \tilde{f}^k\left(\frac{\sum_\omega \frac{\alpha_\omega \beta_\omega}{\alpha_\omega+\beta_\omega} \theta_\omega}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)}\right)$$

Define $g^k(x) = x \cdot \tilde{f}^k(x)$, then this becomes

$$\frac{\sum_\omega \tilde{\alpha}_w \tilde{\beta}_w \theta_w g^k(\alpha_w+\beta_w)}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)} < g^k\left(\frac{\sum_\omega \tilde{\alpha}_w \tilde{\beta}_w \theta_\omega (\alpha_w+\beta_w)}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)}\right).$$

Define, $\tau_\omega = \frac{\tilde{\alpha}_\omega \tilde{\beta}_\omega \theta_\omega}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)}$, then $\kappa \equiv \sum_\omega \tau_w \in (0,1)$. We can inductively show that $g^k(x) \in [0,1]$ is weakly increasing and weakly concave for $x \in [0,1]$. Thus,

$$\begin{aligned}
g^k\left(\frac{\sum_\omega \tilde{\alpha}_w \tilde{\beta}_w \theta_\omega (\alpha_w+\beta_w)}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)}\right) &= g^k\left(\sum_\omega \tau_w(\alpha_\omega+\beta_\omega)\right) \\
&= g^k\left(\kappa \sum_\omega \frac{\tau_\omega}{\kappa}(\alpha_w+\beta_w)\right) \\
&\geq \kappa g^k\left(\sum_\omega \frac{\tau_\omega}{\kappa}(\alpha_w+\beta_w)\right) \\
&> \kappa \sum_\omega \frac{\tau_\omega}{\kappa} g^k(\alpha_w+\beta_w) \\
&= \frac{\sum_\omega \tilde{\alpha}_w \tilde{\beta}_w \theta_w g^k(\alpha_w+\beta_w)}{\left(\sum_\omega \tilde{\beta}_\omega \theta_\omega\right)\cdot\left(\sum_\omega \tilde{\alpha}_\omega \theta_\omega\right)}.
\end{aligned}$$

□

Proof of Proposition 3. We prove a more general result in Proposition A.3. □

Proof of Proposition 4. For $t \geq 2$,

$$\begin{aligned}
\mathrm{d}\ln \ell_t &= \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k} \\
&= \sum_{s=0}^{t-2}\sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\,\mathrm{d}w_{t-s+k} \\
&= \sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{k} + \mathcal{F}_{k+1} - \mathcal{F}_{t+k-1} - \mathcal{F}_{t+k})\,\mathrm{d}w
\end{aligned}$$

where $\mathrm{d}w = (0, \cdots, 0, \overbrace{-\Delta}^{s\text{th}}, 0, \cdots, 0)$. Thus, we can write

$$|\,\mathrm{d}\ln \ell_{st}| = \Delta \cdot \sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho}\left(\sum_{s=0}^{t-2} b_{s+k}\right).$$

This gives

$$|\,\mathrm{d}\ln \ell_{st+1}| - |\,\mathrm{d}\ln \ell_{st}| = \Delta \cdot \sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho} b_{k+t-1}$$

Under the single-crossing condition of Assumption 3, if this term is higher in the heterogeneous-worker model, then $|\,\mathrm{d}\ln \ell_{st+2}| - |\,\mathrm{d}\ln \ell_{st+1}|$ is also higher in the heterogeneous-worker model. This means that there exists a cutoff $\bar{t} \in \mathbb{N} \cup \{\infty\}$ such that the canonical model calibrated by matching the one-period worker flow matrix overestimates the decline in employment of sector s in period t if and only if $1 < t \leq \bar{t}$. Note that

$$b_0 + \cdots + b_{t-2} = (I + \mathcal{F}_1 - \mathcal{F}_{t-1} - \mathcal{F}_t)_{ss}$$

is always higher in the canonical model. Thus, $|\,\mathrm{d}\ln \ell_{st}|$ is always higher in the canonical model if β is sufficiently close to zero. In that case, $\bar{t} = \infty$ is possible. On the other hand, for $t = 2$, we have

$$\begin{aligned}
|\,\mathrm{d}\ln \ell_{s2}| &= \Delta \cdot \sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_k - \mathcal{F}_{k+2})_{s,s} \\
&= \Delta \cdot \tfrac{\beta}{\rho}\big((I - \mathcal{F}_2) + \beta(\mathcal{F}_1 - \mathcal{F}_3) + \beta^2(\mathcal{F}_2 - \mathcal{F}_4) + \cdots\big)_{s,s} \\
&= \Delta \cdot \tfrac{\beta}{\rho}\big(I + \beta\mathcal{F}_1 - (1-\beta^2)\mathcal{F}_2 - \beta(1-\beta^2)\mathcal{F}_3 - \cdots\big)_{s,s},
\end{aligned}$$

which is always higher in the canonical model. Thus, $\bar{t} \neq 1$. $\square$

Proof of Proposition A.3. We can write the absolute changes in the value as

$$|\,\mathrm{d}v_{s1}| = \beta^{\tau-1}(\mathcal{F}_{\tau-1})_{s,s}\Delta \geq \beta^{\tau-1}\big((\mathcal{F}_1)^{\tau-1}\big)_{s,s}\Delta = \Big|\,\mathrm{d}v_{s1}|_{\text{canonical}}\Big|. \qquad \square$$

Proof of Proposition A.4. For $t \geq 2$,

$$
\begin{aligned}
\mathrm{d}\ln \ell_t &= \sum_{s\geq 0, k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\mathbb{E}_{t-s-1}\,\mathrm{d}w_{t-s+k} \\
&= \sum_{s=0}^{t-2}\sum_{k\geq 0} \tfrac{\beta^{k+1}}{\rho}(\mathcal{F}_{s+k} - \mathcal{F}_{s+k+2})\,\mathrm{d}w_{t-s+k} \\
&= \tfrac{\beta}{\rho}(\mathcal{F}_{t-2} - \mathcal{F}_t)\,\mathrm{d}w_2 \\
&+ \left(\tfrac{\beta}{\rho}(\mathcal{F}_{t-3} - \mathcal{F}_{t-1}) + \tfrac{\beta^2}{\rho}(\mathcal{F}_{t-1} - \mathcal{F}_{t+1})\right)\mathrm{d}w_3 \\
&+ \left(\tfrac{\beta}{\rho}(\mathcal{F}_{t-4} - \mathcal{F}_{t-2}) + \tfrac{\beta^2}{\rho}(\mathcal{F}_{t-2} - \mathcal{F}_{t}) + \tfrac{\beta^3}{\rho}(\mathcal{F}_{t} - \mathcal{F}_{t+2})\right)\mathrm{d}w_4 \\
&+ \cdots \\
&+ \left(\tfrac{\beta}{\rho}(\mathcal{F}_{0} - \mathcal{F}_{2}) + \cdots + \tfrac{\beta^{t-1}}{\rho}(\mathcal{F}_{2t-4} - \mathcal{F}_{2t-2})\right)\mathrm{d}w_t \\
&+ \left(\tfrac{\beta^2}{\rho}(\mathcal{F}_{1} - \mathcal{F}_{3}) + \cdots + \tfrac{\beta^{t}}{\rho}(\mathcal{F}_{2t-3} - \mathcal{F}_{2t-1})\right)\mathrm{d}w_{t+1} \\
&+ \cdots \\
&\equiv \sum_{\tau=2}^{\infty} \boldsymbol{A}_{\tau,t}\,\mathrm{d}w_\tau
\end{aligned}
$$

where the impulse response functions are given by

$$
\boldsymbol{A}_{\tau,t} = \begin{cases} \sum_{s=0}^{t-2} \tfrac{\beta^{s+\tau-t+1}}{\rho}(\mathcal{F}_{2s+\tau-t} - \mathcal{F}_{2s+\tau-t+2}) & \text{if } \tau \geq t \\ \sum_{s=0}^{\tau-2} \tfrac{\beta^{s+1}}{\rho}(\mathcal{F}_{2s+t-\tau} - \mathcal{F}_{2s+t-\tau+2}) & \text{if } t > \tau. \end{cases}
$$

Thus, the s-th diagonal element of $\boldsymbol{A}_{\tau,t}$ is a weighted sum of $\{b_{2s+|t-\tau|}\}_{s=0,\ldots,t\wedge\tau-2}$ where more weights are given to those with small s. Thus, under Assumption 3, the s-th diagonal element of $\boldsymbol{A}_{\tau,t}$ is higher in the canonical model when $|t-\tau|$ and/or $t\wedge\tau$ are small. In particular, for given $t\wedge\tau$, we can find $B\in\mathbb{N}$ such that the s-th diagonal element of $\boldsymbol{A}_{\tau,t}$ is higher in the canonical model when $|t-\tau|\leq B$. We can show that $B\geq 1$:

$$
\begin{aligned}
(\boldsymbol{A}_{t,t})_{ss} &= \sum_{s=0}^{t-2}\tfrac{\beta^{s+1}}{\rho}b_{2s} \\
&= \tfrac{\beta}{\rho} - \tfrac{\beta(1-\beta)}{\rho}(\mathcal{F}_2)_{ss} - \tfrac{\beta^2(1-\beta)}{\rho}(\mathcal{F}_4)_{ss} - \cdots - \tfrac{\beta^{t-2}(1-\beta)}{\rho}(\mathcal{F}_{2t-4})_{ss} - \tfrac{\beta^m}{\rho}(\mathcal{F}_{2t-2})_{ss},
\end{aligned}
$$

$$
\begin{aligned}
(\boldsymbol{A}_{t,t+1})_{ss} &= \sum_{s=0}^{t-2}\tfrac{\beta^{s+2}}{\rho}b_{2s+1} \\
&= \tfrac{\beta^2}{\rho}(\mathcal{F}_1)_{ss} - \tfrac{\beta^2(1-\beta)}{\rho}(\mathcal{F}_3)_{ss} - \tfrac{\beta^3(1-\beta)}{\rho}(\mathcal{F}_5)_{ss} - \cdots - \tfrac{\beta^{t-1}(1-\beta)}{\rho}(\mathcal{F}_{2t-3})_{ss} - \tfrac{\beta^t}{\rho}(\mathcal{F}_{2t-1})_{ss},
\end{aligned}
$$

and

$$
\begin{aligned}
(\boldsymbol{A}_{t+1,t})_{ss} &= \sum_{s=0}^{t-2} \tfrac{\beta^{s+1}}{\rho} b_{2s+1} \\
&= \tfrac{\beta}{\rho}(\mathcal{F}_1)_{ss} - \tfrac{\beta(1-\beta)}{\rho}(\mathcal{F}_3)_{ss} - \tfrac{\beta^2(1-\beta)}{\rho}(\mathcal{F}_5)_{ss} - \cdots - \tfrac{\beta^{t-2}(1-\beta)}{\rho}(\mathcal{F}_{2t-3})_{ss} - \tfrac{\beta^m}{\rho}(\mathcal{F}_{2t-1})_{ss}.
\end{aligned}
$$

Thus, by Lemma 2, we can see that $(\boldsymbol{A}_{\tau,t})_{ss}$ is higher in the canonical model if $|t-\tau| \leq 1$. This proves that B maps $\mathbb{N}$ into itself. □

Proof of Lemma A.1. The second equation holds by construction. For the first equation, we have

$$
[\bar{\mathbb{E}}_\omega \,\mathrm{d}\ln \ell^\omega_{t+1}]_i = \sum_\omega \tilde{\ell}^\omega_i \,\mathrm{d}\ln \ell^\omega_{it+1} = \frac{\sum_\omega \ell^\omega_i \,\mathrm{d}\ln \ell^\omega_{it+1}}{\ell_i} = \frac{\sum_\omega \mathrm{d}\ell^\omega_{it+1}}{\ell_i} = \frac{\mathrm{d}\ell_{it+1}}{\ell_i} = \mathrm{d}\ln \ell_{it+1}. \qquad \square
$$

For the remaining equations, fix time t, we then have

$$
\begin{aligned}
\left[\bar{\mathbb{E}}_\omega[(F^\omega)^k]\right]_{ij} &= \sum_\omega \tilde{\ell}^\omega_i \left[(F^\omega)^k\right]_{ij} \\
&= \sum_\omega \Pr(\omega | s_t = i) \cdot \Pr(s_t = i, s_{t+k} = j | s_t = i, \omega) \\
&= \sum_\omega \Pr(s_t = i, s_{t+k} = j, \omega | s_t = i) \\
&= \Pr(s_t = i, s_{t+k} = j | s_t = i) \\
&= (\mathcal{F}_k)_{ij}.
\end{aligned}
$$

Again fix time t, and define a random variable $\tau(t,m)$ as follows:

$$
\tau(t,m) \equiv \min\{\tau > t : s_\tau = s_{t-m}\}
$$

which is well-defined based on two assumptions we begin with.

$$
\begin{aligned}
\left[\bar{\mathbb{E}}_\omega[(B^\omega)^m (F^\omega)^k]\right]_{ij} &= \sum_\omega \tilde{\ell}^\omega_i \sum_h [(B^\omega)^m]_{ih} \cdot \left[(F^\omega)^k\right]_{hj} \\
&= \sum_\omega \Pr(\omega | s_t = i) \sum_h \Pr(s_{t-m} = h, s_t = i | s_t = i, \omega) \cdot \Pr(s_{\tau(t,m)+k} = j | s_{t-m} = h, s_t = i, \omega) \\
&= \sum_\omega \sum_h \Pr(s_{t-m} = h, s_t = i, s_{\tau(t,m)+k} = j, \omega | s_t = i) \\
&= \sum_\omega \Pr(s_t = i, s_{\tau(t,m)+k} = j, \omega | s_t = i) \\
&= \Pr(s_t = i, s_{\tau(t,m)+k} = j | s_t = i)
\end{aligned}
$$

Proof of Lemma A.2. We have

$$\begin{aligned}
\ell_i \cdot \left[\bar{\mathbb{E}}_\omega\left[(B^\omega)^m (F^\omega)^k\right]\right]_{ij} &= \sum_\omega \ell_i^\omega \sum_h [(B^\omega)^m]_{ih} \cdot \left[(F^\omega)^k\right]_{hj} \\
&= \sum_\omega \sum_h \ell_h^\omega [(F^\omega)^m]_{hi} \cdot \left[(F^\omega)^k\right]_{hj} \\
&= \sum_\omega \sum_h \ell_j^\omega [(F^\omega)^m]_{hi} \cdot \left[(B^\omega)^k\right]_{jh} \\
&= \ell_j \left[\bar{\mathbb{E}}_\omega\left(B^\omega)^k (F^\omega)^m\right]\right]_{ji}.
\end{aligned}$$

Thus,

$$(LHS)_{ij} = \ell_i \cdot \left[\bar{\mathbb{E}}_\omega\left[(B^\omega)^m (F^\omega)^k\right]\right]_{ij} = \ell_j \left[\bar{\mathbb{E}}_\omega\left(B^\omega)^k (F^\omega)^m\right]\right]_{ji} = (RHS)_{ij} \qquad \square$$

Proof of Proposition A.2. Consider shifts in the distributions of Δ_i and Δ_j by $\rho \cdot \mathrm{d}\Delta_i$ and $\rho \cdot \mathrm{d}\Delta_j$ units, respectively. Denote the share of workers working in sector s as L_s. The following lemma represents levels and changes of sector choice probabilities in terms of $\mathrm{d}\Delta_i$ and $\mathrm{d}\Delta_j$.

Lemma A.7. *When $g_{ij}(\cdot)$ is continuous around 0 and ε_{ij} has a finite first moment for all $i, j \in \mathcal{S}$, we have*

$$\begin{aligned}
F_{ij} &= \frac{\rho g_{ij}(0)\mathcal{F}_1}{L_i} + o(\rho) \\
\mathrm{d}F_{ijt} &= \left(\frac{\rho g_{ij}(0)\mathcal{F}_2}{L_i} + o(\rho)\right) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) \\
\mathrm{d}\ln F_{ijt} &= (\mathcal{F} + o(1)) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) \\
F_{ii} &= 1 + o(1) \\
\mathrm{d}F_{iit} &= \mathcal{F} \cdot \left(\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j\right) + o(\rho) \\
\mathrm{d}\ln F_{iit} &= \mathcal{F} \cdot \left(\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j\right) + o(1)
\end{aligned}$$

where the constants $\mathcal{F}_1, \mathcal{F}_2$ and $\mathcal{F}$ only depend on $F(\cdot)$ and $\tilde{C}$.

Plugging the final equation into equation (A.18), we have

$$\begin{aligned}
\mathrm{d}\ln \ell_i &= \sum_j B_{ij}\,\mathrm{d}\ln F_{jit} \\
&= F_{ii}\,\mathrm{d}\ln F_{iit} + \sum_{j\neq i} B_{ij}\,\mathrm{d}\ln F_{jit} \\
&= \mathrm{d}F_{iit} + \sum_{j\neq i} B_{ij}\,\mathrm{d}\ln F_{jit} \\
&= \mathcal{F}\cdot\left(\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j\right) + \sum_{j\neq i} T_{ij}\cdot\mathcal{F}\cdot(\mathrm{d}\Delta_i - \mathrm{d}\Delta_j) + o(\rho) \\
&= \mathcal{F}\cdot\left(2\,\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j - \sum_j T_{ij}\,\mathrm{d}\Delta_j\right) + o(\rho)
\end{aligned}$$

Lemma A.8. *We have* $2I - F - B = I - BF + o(\rho)$.

Proof. This directly follows from $(I-B)(I-F) = o(\rho)$, which in turn comes from the fact that all the elements of both matrices $I - F$ and $I - B$ are proportional to ρ. □

Thus, we obtain

$$\begin{aligned}
\mathrm{d}\ln \ell &= \mathcal{F}\cdot(2I - F - B)\,\mathrm{d}\Delta + o(\rho) \\
&= \mathcal{F}\cdot\tfrac{\beta}{\rho}(I - BF)\mathbb{E}_t\,\mathrm{d}v_{t+1} + o(\rho)
\end{aligned}$$ □

Proof of Lemma A.7. We start with the following lemma.

Lemma A.9 (Conditional Distribution). *Suppose that in the steady state, shares* ℓ_i *and* ℓ_j *of workers in* $\Omega_{ij}(\varepsilon)$ *are working in sectors* i *and* j*, respectively, and that they have*

$$\Delta_{ij}|_{\textit{in sector } i,\,\Omega_{ij}(\varepsilon)} \sim g^i_{ij}(\cdot), G^i_{ij}(\cdot) \;\text{ and }\; \Delta_{ij}|_{\textit{in sector } j,\Omega_{ij}(\varepsilon)} \sim g^j_{ij}(\cdot), G^j_{ij}(\cdot).$$

Then, these conditional distributions satisfy

$$g^i_{ij}(x)\ell_i = \frac{F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right)}{F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right) + F\left(-\frac{x}{\rho} - \tilde{C}_{ij}\right)} g_{ij}(x).$$

Without a shock, the share of workers in $\Omega_{ij}(\varepsilon)$ who move from i to j is given by

$$\ell_{ij} = \ell_i \Pr(\Delta_{ij} + \rho\varepsilon_{ij} < -\rho\tilde{C}_{ij}|\text{in } i) = \begin{cases} \ell_i \cdot \displaystyle\int_{-\infty}^{\infty} G^i_{ij}(-\rho\varepsilon_{ij} - \rho\tilde{C}_{ij})\,\mathrm{d}F(\varepsilon_{ij}) \\ \ell_i \cdot \displaystyle\int_{-\infty}^{\infty} F\left(\frac{-\Delta_{ij} - \rho\tilde{C}_{ij}}{\rho}\right)\mathrm{d}G^i_{ij}(\Delta_{ij}) \end{cases}$$

$$
\begin{aligned}
&= \int_{-\infty}^{\infty} \frac{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) \cdot F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)}{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) + F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)} g_{ij}(x)\,\mathrm{d}x \\
&= \rho \int_{-\infty}^{\infty} \frac{F(t - \tilde{C}_{ji}) \cdot F(-t - \tilde{C}_{ij})}{F(t - \tilde{C}_{ji}) + F(-t - \tilde{C}_{ij})} g_{ij}(\rho t)\,\mathrm{d}t
\end{aligned}
\tag{A.22}
$$

Suppose that the distributions of Δ_i and Δ_j shift by $\rho \cdot \mathrm{d}\Delta_i$ and $\rho \cdot \mathrm{d}\Delta_j$, respectively. Then, new share is given by

$$
\begin{aligned}
\ell_{ij} + \mathrm{d}\ell_{ij} &= \ell_i \cdot \Pr(\Delta_{ij} + \rho(\mathrm{d}\Delta_i - \mathrm{d}\Delta_j) + \rho\varepsilon_{ij} < -\rho\tilde{C}_{ij} | \text{in } i) \\
&= \ell_i \cdot \int_{-\infty}^{\infty} F\big(-\tfrac{\Delta_{ij}}{\rho} - \tilde{C}_{ij} - (\mathrm{d}\Delta_i - \mathrm{d}\Delta_j)\big)\,\mathrm{d}G_{ij}^i(\Delta_{ij}) \\
&= \int_{-\infty}^{\infty} \frac{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) \cdot F\Big(-\frac{x}{\rho} - \tilde{C}_{ij} - (\mathrm{d}\Delta_i - \mathrm{d}\Delta_j)\Big)}{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) + F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)} g_{ij}(x)\,\mathrm{d}x \\
&= \int_{-\infty}^{\infty} \frac{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) \cdot \Big(F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big) - f\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big) \cdot (\mathrm{d}\Delta_i - \mathrm{d}\Delta_j)\Big)}{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) + F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)} g_{ij}(x)\,\mathrm{d}x
\end{aligned}
$$

Thus,

$$
\begin{aligned}
\mathrm{d}\ell_{ij} &= \int_{-\infty}^{\infty} \frac{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) \cdot f\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)}{F\Big(\frac{x}{\rho} - \tilde{C}_{ji}\Big) + F\Big(-\frac{x}{\rho} - \tilde{C}_{ij}\Big)} g_{ij}(x)\,\mathrm{d}x \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) \\
&= \rho \int_{-\infty}^{\infty} \frac{F\big(t - \tilde{C}_{ji}\big) \cdot f\big(-t - \tilde{C}_{ij}\big)}{F\big(t - \tilde{C}_{ji}\big) + F\big(-t - \tilde{C}_{ij}\big)} g_{ij}(\rho t)\,\mathrm{d}t \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i)
\end{aligned}
\tag{A.23}
$$

We will take a limit $\rho \to 0$ to equations (A.22) and (A.23). Note that we have

$$
\begin{aligned}
\left| \frac{F(t - \tilde{C}_{ji}) \cdot F(-t - \tilde{C}_{ij})}{F(t - \tilde{C}_{ji}) + F(-t - \tilde{C}_{ij})} g_{ij}(\rho t) \right| &\leq \big(F(-t - \tilde{C}_{ij}) \mathbb{1}_{t \geq 0} + F(t - \tilde{C}_{ji}) \mathbb{1}_{t < 0}\big) \cdot \sup_x \{g_{ij}(x)\} \\
\left| \frac{F\big(t - \tilde{C}_{ji}\big) \cdot f\big(-t - \tilde{C}_{ij}\big)}{F\big(t - \tilde{C}_{ji}\big) + F\big(-t - \tilde{C}_{ij}\big)} g_{ij}(\rho t) \right| &\leq f(-t - \tilde{C}_{ij}) \cdot \sup_x \{g_{ij}(x)\}
\end{aligned}
$$

with

$$
\begin{aligned}
\int_0^{\infty} F(-t - \tilde{C}_{ij})\,\mathrm{d}t + \int_{-\infty}^{0} F(t - \tilde{C}_{ji})\,\mathrm{d}t &= \int_0^{\infty} \int_{-\infty}^{-t-\tilde{C}_{ij}} f(x)\,\mathrm{d}x\,\mathrm{d}t + \int_{-\infty}^{0} \int_{-\infty}^{t-\tilde{C}_{ji}} f(x)\,\mathrm{d}x\,\mathrm{d}t \\
&= \int_{-\infty}^{-\tilde{C}_{ij}} \int_0^{-x-\tilde{C}_{ij}} f(x)\,\mathrm{d}t\,\mathrm{d}x + \int_{-\infty}^{-\tilde{C}_{ji}} \int_{x+\tilde{C}_{ji}}^{0} f(x)\,\mathrm{d}t\,\mathrm{d}x \\
&= \int_{-\infty}^{-\tilde{C}_{ij}} (-x - \tilde{C}_{ij}) f(x)\,\mathrm{d}x + \int_{-\infty}^{-\tilde{C}_{ji}} (-x - \tilde{C}_{ji}) f(x)\,\mathrm{d}x,
\end{aligned}
$$

which is finite because $\mathbb{E}[\varepsilon_{ij}]$ is well-defined, and

$$\int_{-\infty}^{\infty} f(-t-\tilde{C}_{ij})\,\mathrm{d}t = 1,$$

which is also finite. Thus, we can apply dominated convergence theorem to conclude

$$\ell_{ij} = \rho \cdot \underbrace{\int_{-\infty}^{\infty} \frac{F(t-\tilde{C}_{ji}) \cdot F(-t-\tilde{C}_{ij})}{F(t-\tilde{C}_{ji}) + F(-t-\tilde{C}_{ij})}\,\mathrm{d}t}_{\mathcal{F}_1^{ij}} \cdot g_{ij}(0) + o(\rho)$$

$$\mathrm{d}\ell_{ij} = \left(\rho \cdot \underbrace{\int_{-\infty}^{\infty} \frac{F(t-\tilde{C}_{ji}) \cdot f(-t-\tilde{C}_{ij})}{F(t-\tilde{C}_{ji}) + F(-t-\tilde{C}_{ij})}\,\mathrm{d}t}_{\mathcal{F}_2^{ij}} \cdot g_{ij}(0) + o(\rho)\right) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i)$$

Assume $\tilde{C}_{ji} = \tilde{C}_{ij} = \tilde{C}$ for all $i \neq j$ and write $\mathcal{F}_1 = \mathcal{F}_1^{ij}$ and $\mathcal{F}_2 = \mathcal{F}_2^{ij}$. Then, we have

$$\begin{aligned}
F_{ij} &= \frac{\ell_{ij}}{L_i} = \frac{\rho g_{ij}(0)\mathcal{F}_1}{L_i} + o(\rho) \\
\mathrm{d}F_{ij} &= \frac{\mathrm{d}\ell_{ij}}{L_i} = \left(\frac{\rho g_{ij}(0)\mathcal{F}_2}{L_i} + o(\rho)\right) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) \\
\implies \mathrm{d}\ln F_{ij} &= \left(\frac{\mathcal{F}_2}{\mathcal{F}_1} + o(1)\right) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) \equiv (\mathcal{F} + o(1)) \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i)
\end{aligned}$$

for all $i \neq j$, Thus,

$$\begin{aligned}
F_{ii} &= 1 - \sum_{j \neq i} F_{ij} = 1 + o(1) \\
\mathrm{d}F_{ii} &= -\sum_{j \neq i} \mathrm{d}F_{ij} = -\sum_{j \neq i} F_{ij}\,\mathrm{d}\ln F_{ij} = -\sum_{j \neq i} F_{ij}\mathcal{F} \cdot (\mathrm{d}\Delta_j - \mathrm{d}\Delta_i) + o(\rho) \\
&= \mathcal{F} \cdot \left(\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j\right) + o(\rho) \\
\implies \mathrm{d}\ln F_{ii} &= \mathcal{F} \cdot \left(\mathrm{d}\Delta_i - \sum_j F_{ij}\,\mathrm{d}\Delta_j\right) + o(1). \qquad \square
\end{aligned}$$

Proof of Lemma A.9. In the steady state, we have

$$\begin{aligned}
\ell_i G_{ij}^i(x) &= \ell_i \Pr(\Delta_{ij} \leq x | \text{in } i) \\
&= \ell_i \Pr(\Delta_{ij} \leq x | \text{in } i) \Pr(\text{stay in } i | \Delta_{ij} \leq x, \text{in } i) + \ell_j \Pr(\Delta_{ij} \leq x | \text{in } j) \Pr(\text{leave } j | \Delta_{ij} \leq x, \text{in } j) \\
&= \ell_i \int_{-\infty}^{x} \underbrace{\Pr(\Delta_{ij} + \rho\varepsilon_{ij} \geq -\rho\tilde{C}_{ij})}_{=\bar{F}(-\frac{\Delta_{ij}}{\rho} - \tilde{C}_{ij})} \mathrm{d}G_{ij}^i(\Delta_{ij}) + \ell_j \int_{-x}^{\infty} \underbrace{\Pr(\Delta_{ji} + \rho\varepsilon_{ji} < -\rho\tilde{C}_{ji})}_{=F(-\frac{\Delta_{ji}}{\rho} - \tilde{C}_{ji})} \mathrm{d}G_{ji}^j(\Delta_{ji})
\end{aligned}$$

Differentiating with respect to x, we have

$$\ell_i g_{ij}^i(x) = \ell_i \bar{F}\left(-\frac{x}{\rho} - \tilde{C}_{ij}\right) g_{ij}^i(x) + \ell_j F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right) g_{ji}^j(-x)$$

or

$$F\left(-\frac{x}{\rho} - \tilde{C}_{ij}\right) g_{ij}^i(x)\ell_i = F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right) g_{ij}^j(x)\ell_j.$$

We also have

$$g_{ij}^i(x)\ell_i + g_{ij}^j(x)\ell_j = g_{ij}(x),$$

so we can conclude that

$$g_{ij}^i(x)\ell_i = \frac{F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right)}{F\left(\frac{x}{\rho} - \tilde{C}_{ji}\right) + F\left(-\frac{x}{\rho} - \tilde{C}_{ij}\right)} g_{ij}(x).$$

□

Proof of Lemma A.4.

$$\begin{aligned}
F_i^{k+1} &= \begin{pmatrix} 1 - \alpha_i f^k & \alpha_i f^k \\ \beta_i f^k & 1 - \beta_i f^k \end{pmatrix} \begin{pmatrix} \bar{\alpha}_i & \alpha_i \\ \beta_i & \bar{\beta}_i \end{pmatrix} \\
&= \begin{pmatrix} \bar{\alpha}_i - \alpha_i \bar{\alpha}_i f^k + \alpha_i \beta_i f^k & \alpha_i(1 - \alpha_i f^k + \bar{\beta}_i f^k) \\ \beta_i(\bar{\alpha}_i f^k + 1 - \beta_i f^k) & \alpha_i \beta_i f^k + \bar{\beta}_i - \beta_i \bar{\beta}_i f^k \end{pmatrix} \\
&= \begin{pmatrix} 1 - \alpha_i f^{k+1} & \alpha_i f^{k+1} \\ \beta_i f^{k+1} & 1 - \beta_i f^{k+1} \end{pmatrix}.
\end{aligned}$$

Thus, we have $f^{k+1}(x) = (x-1) \cdot f^k(x) + 1$. Define $g^k(x) = x \cdot f^k(2-x) - 1$, then we have

$$\begin{aligned}
g^{k+1}(x) &= x \cdot f^{k+1}(2-x) - 1 \\
&= x \cdot \big((1-x) \cdot f^k(2-x) + 1\big) - 1 \\
&= (1-x) g^k(x).
\end{aligned}$$

and $g^1(x) = -(1-x)$. Thus, $g^k(x) = -(1-x)^k$ and hence $f^k(x) = \frac{1+g^k(2-x)}{2-x} = \frac{1-(x-1)^k}{2-x}$. □

Proof of Proposition A.5. Note from the definitions of x_i and y_i that we have

$$\frac{\alpha_i x_i}{\alpha_1 x_1} = \frac{\beta_i y_i}{\beta_1 y_1}.$$

Let λ_i denote this ratio. We then have

$$\frac{(\mathcal{F}_k)_{1,2}}{(\mathcal{F}_k)_{2,1}} = \frac{\sum_i x_i \alpha_{i,k}}{\sum_i y_i \beta_{i,k}} = \frac{\sum_i \alpha_i x_i \left(\frac{\alpha_{i,k}}{\alpha_i}\right)}{\sum_i \beta_i y_i \left(\frac{\beta_{i,k}}{\beta_i}\right)} = \frac{\alpha_1 x_1 \sum_i \lambda_i \left(\frac{\alpha_{i,k}}{\alpha_i}\right)}{\beta_1 y_1 \sum_i \lambda_i \left(\frac{\beta_{i,k}}{\beta_i}\right)} = \frac{\alpha_1 x_1}{\beta_1 y_1}$$

where the last equality uses $\frac{\alpha_{i,k}}{\alpha_i} = \frac{\beta_{i,k}}{\beta_i} = f^k(\bar{\alpha}_i + \bar{\beta}_i)$. □

E. Additional Figures

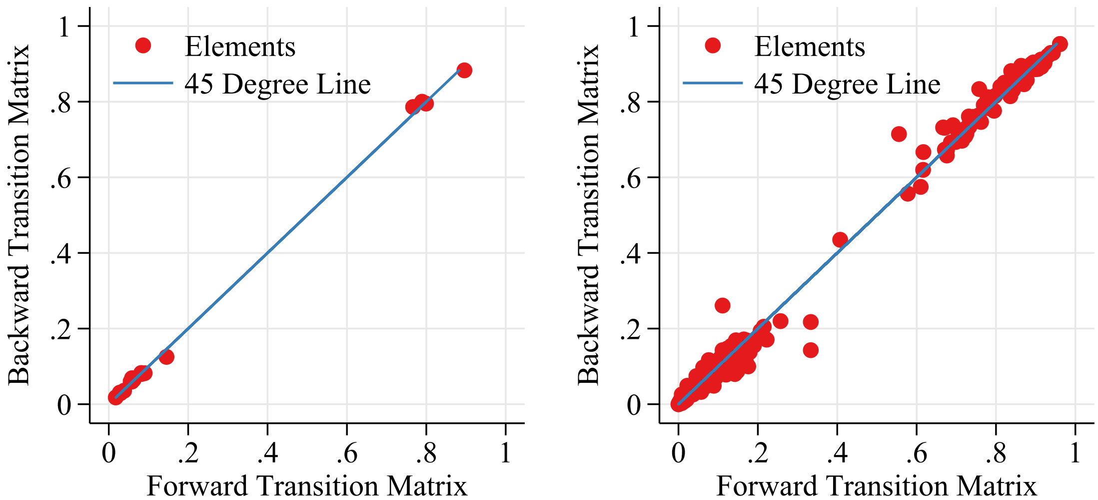


Figure A.1. The Backward and Forward Transition Matrices: NLSY

Notes: Assuming that the economy was in a steady state between years 1980 and 2000, the backward and forward transition matrices are computed by pooling all observations of the NLSY79 data over this period. In the left panel, we plot the elements of the aggregate forward transition matrix against those of the aggregate backward transition matrix. With four sectors, the backward and forward transition matrices are four-by-four matrices with sixteen elements. In the right panel, we consider four dimensions of observed heterogeneity—sex, race, education, and age, leading to sixteen groups—and compare the matrices for all sixteen groups.

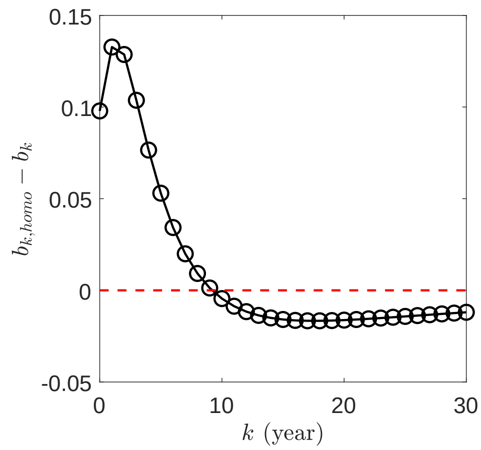


Figure A.2. Differences in b_k Series: Manufacturing Sector

Notes: For each k, this figure plots the difference between the diagonal elements of $(\mathcal{F}_k - \mathcal{F}_{k+2})$ corresponding to the manufacturing sector implied by the canonical model and those observed in the data. For the canonical model, we compute $\mathcal{F}_k$ by multiplying $\mathcal{F}_1$ k times. Since we only observe a finite number of worker flow matrices in the data, we extrapolate it using the estimated structural model. The same pattern is observed for the other sectors. Data source: NLSY79.

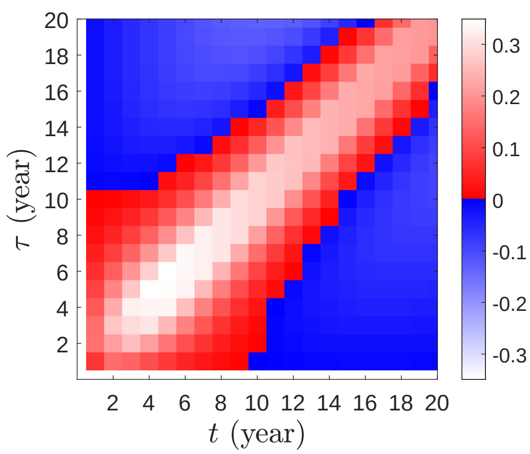


Figure A.3. Differences in the Response of Sectoral Employment

Notes: This figure plots the values of $\left.\frac{\partial \ln \ell_{s,t}}{\partial w_{s,\tau}}\right|_{\text{canonical}} - \left.\frac{\partial \ln \ell_{s,t}}{\partial w_{s,\tau}}\right|_{\text{data}}$ for $1 \leq t, \tau \leq 20$. These derivatives are calculated from equation (11) using the worker flow matrices $\mathcal{F}_k$ from the NLSY79 data for $\left.\frac{\partial \ln \ell_{s,t}}{\partial w_{s,\tau}}\right|_{\text{data}}$ and using the worker flow matrices implied by the canonical model for $\left.\frac{\partial \ln \ell_{s,t}}{\partial w_{s,\tau}}\right|_{\text{canonical}}$.

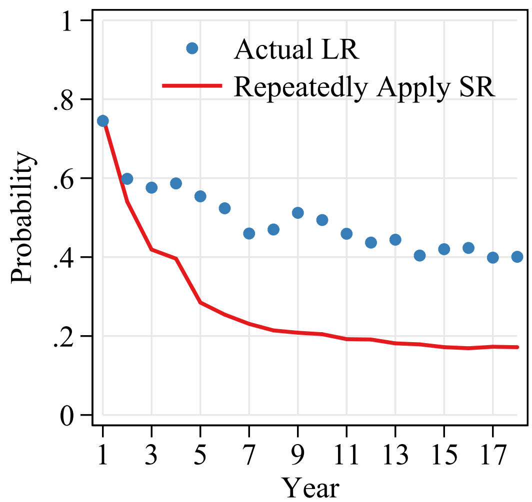


Figure A.4. Actual and Model-Implied Staying Probabilities: Non-Stationarity

Notes: For each $k = 1, \ldots, 18$ (year), this figure plots the actual probability of workers who choose manufacturing in 1980 choosing manufacturing again in year 1980+k (blue dots) and the probability computed by repeatedly multiplying time-varying one-year worker flow matrices (red line); i.e., the diagonal element of the matrix $\prod_{\kappa=1}^{k-1} \mathcal{F}_1^\kappa$ corresponding to the manufacturing sector. $\mathcal{F}_1^\kappa$ is the aggregate worker flow matrix computed using transition observations between years $1980 + \kappa - 1$ and $1980 + \kappa$ from the NLSY data.

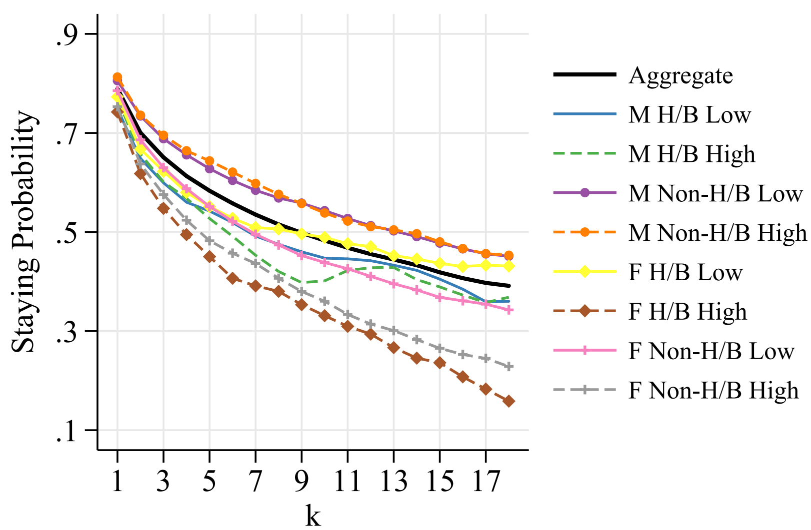
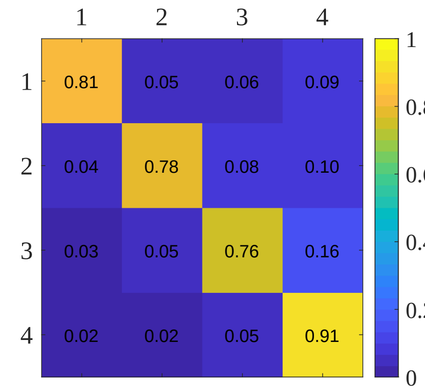


Figure A.5. Actual Staying Probabilities for Different Worker Groups

Notes: For each unique combination of (male, female), (Hispanic/Black, non-Hispanic/Black), and (low-skilled, high-skilled), this figure plots the steady-state k-year manufacturing staying probabilities, $\Pr(s_{t+k} = \text{manufacturing} | s_t = \text{manufacturing})$. Data source: NLSY79.

Wage	Switching Cost
$\begin{pmatrix} 0.87 \\ 0.85 \\ 0.84 \\ 1.00 \end{pmatrix}$	$\begin{pmatrix} 0.00 & 2.86 & 2.95 & 3.15 \\ 2.86 & 0.00 & 2.46 & 2.88 \\ 2.95 & 2.46 & 0.00 & 2.22 \\ 3.15 & 2.88 & 2.22 & 0.00 \end{pmatrix}$

(a) Primitives



(b) Transition Matrix

Figure A.6. Estimation Result: Canonical Model

Notes: Panel (a) shows the estimated values of the primitives of the canonical model. The four sectors are Agriculture and Construction; Manufacturing; Communications and Trade; and Services and Others. Panel (b) shows the resulting transition matrix (or, equivalently, one-year worker flow matrix).

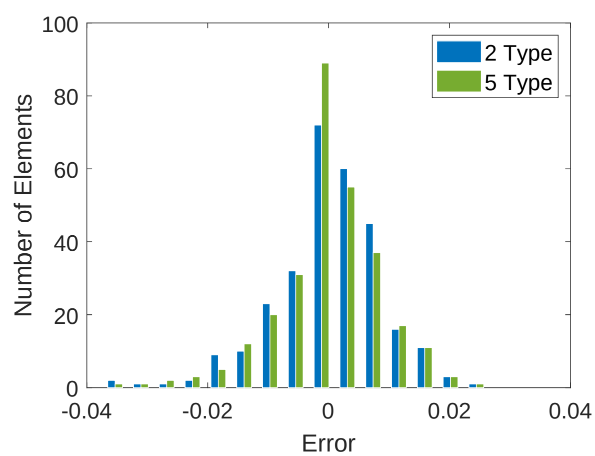


Figure A.7. Fits of Two-type Model and Five-type Model

Notes: For each of the two-type and five-type models, we compute the model-implied worker-flow matrix series, $\{\mathcal{F}_k\}_{k=1,\ldots,18}$. There are a total of 288 elements. This figure plots the differences between these elements and the actual values observed in the data for each of the two models. The five-type model provides a slightly better fit, but the difference is not significant.

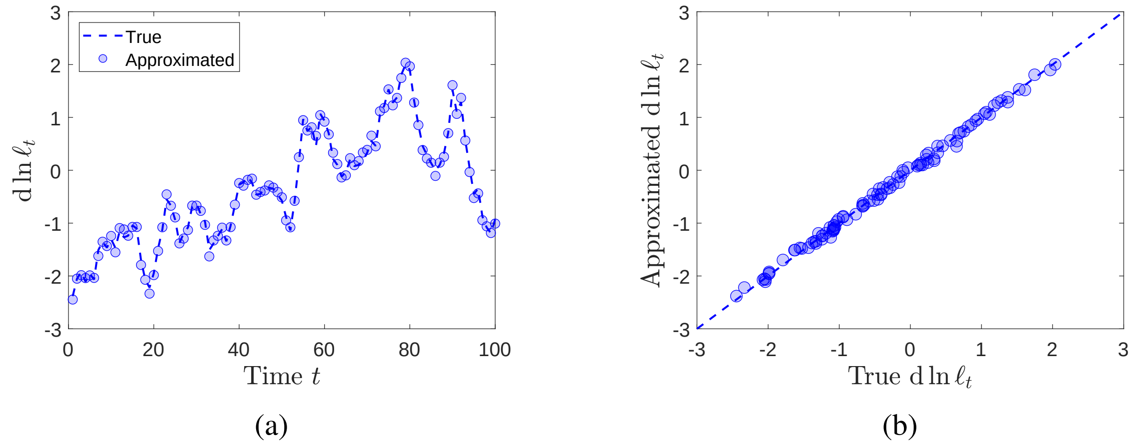


Figure A.8. Fit of Recursive Representation

Notes: Plugging randomly generated values of $\{\mathrm{d}w_t\}$ (from standard normal distribution) and worker flow matrices computed from the NLSY data (extrapolated using the structural model) into equation (15), we can generate a sequence of changes in sectoral employment $\{\mathrm{d}\ln \ell_t\}$. Using the computed values of $\{\Gamma_k\}_{k=1,\ldots 6}$ and $\{\Lambda_k\}_{k=1,\ldots 5}$ and equation (17) instead, we can also compute an approximated sequence of changes in sectoral employment. These figures compare the actual sequence with the approximated sequence.

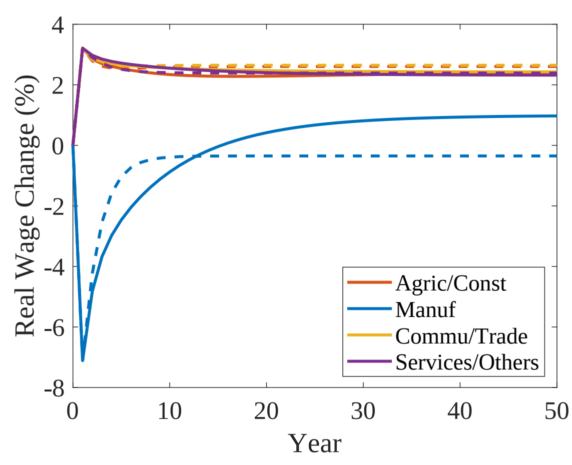


Figure A.9. Changes in Sectoral Real Wages

Notes: This figure plots changes in sectoral real wages over time following an unexpected permanent drop in manufacturing prices. Solid lines correspond to the prediction from the sufficient statistics in the data, and dashed lines correspond to the prediction of the canonical model, without persistent worker heterogeneity.

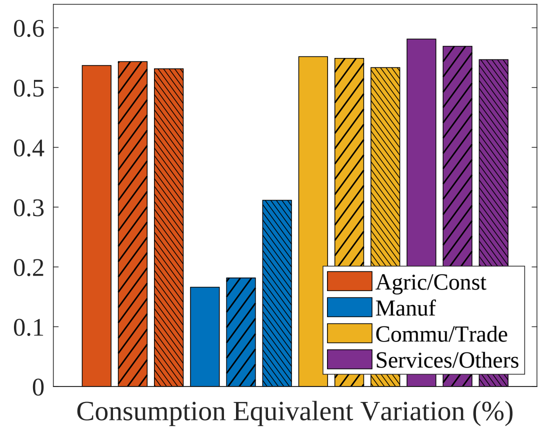
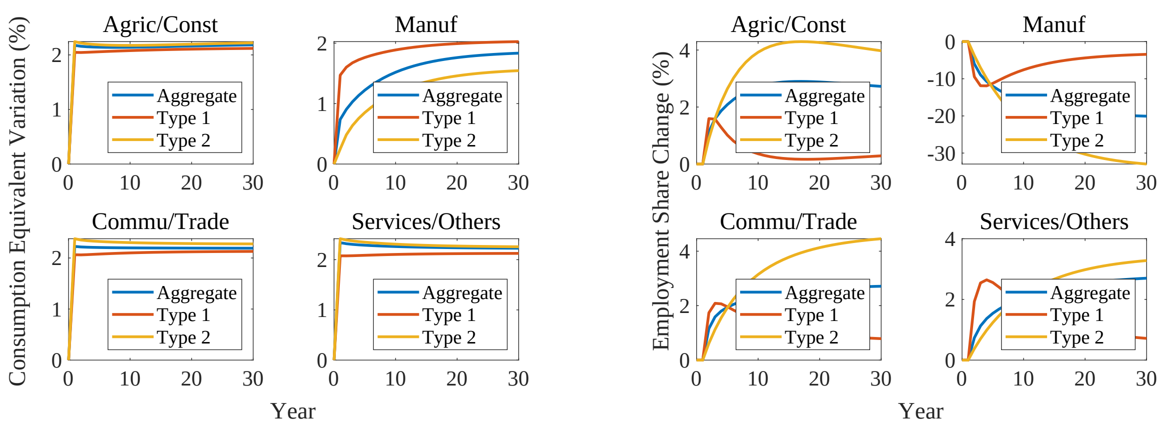


Figure A.10. Changes in Sectoral Values: Exogenous and Endogenous Wage Changes

Notes: This figure plots changes in sectoral values in terms of consumption-equivalent variation for workers initially employed in different sectors. The plain bars correspond to the prediction from sufficient statistics in the data. The rightmost hatched bars (Northwest to Southeast) correspond to the prediction of the canonical model. The hatched bars in the middle (Northeast to Southwest) correspond to the prediction made by combining the wage changes of the canonical model and the sufficient statistics.



(a) Changes in Sectoral Values

(b) Change in Sectoral Employment Share

Figure A.11. Counterfactual Changes in Employment Share and Welfare: Type-specific Changes

Notes: This figure plots the transitional dynamics following an unexpected permanent drop in manufacturing prices for each type of worker. The orange line corresponds to type-1 workers (frequent movers), and the yellow line corresponds to type-2 workers (infrequent movers).

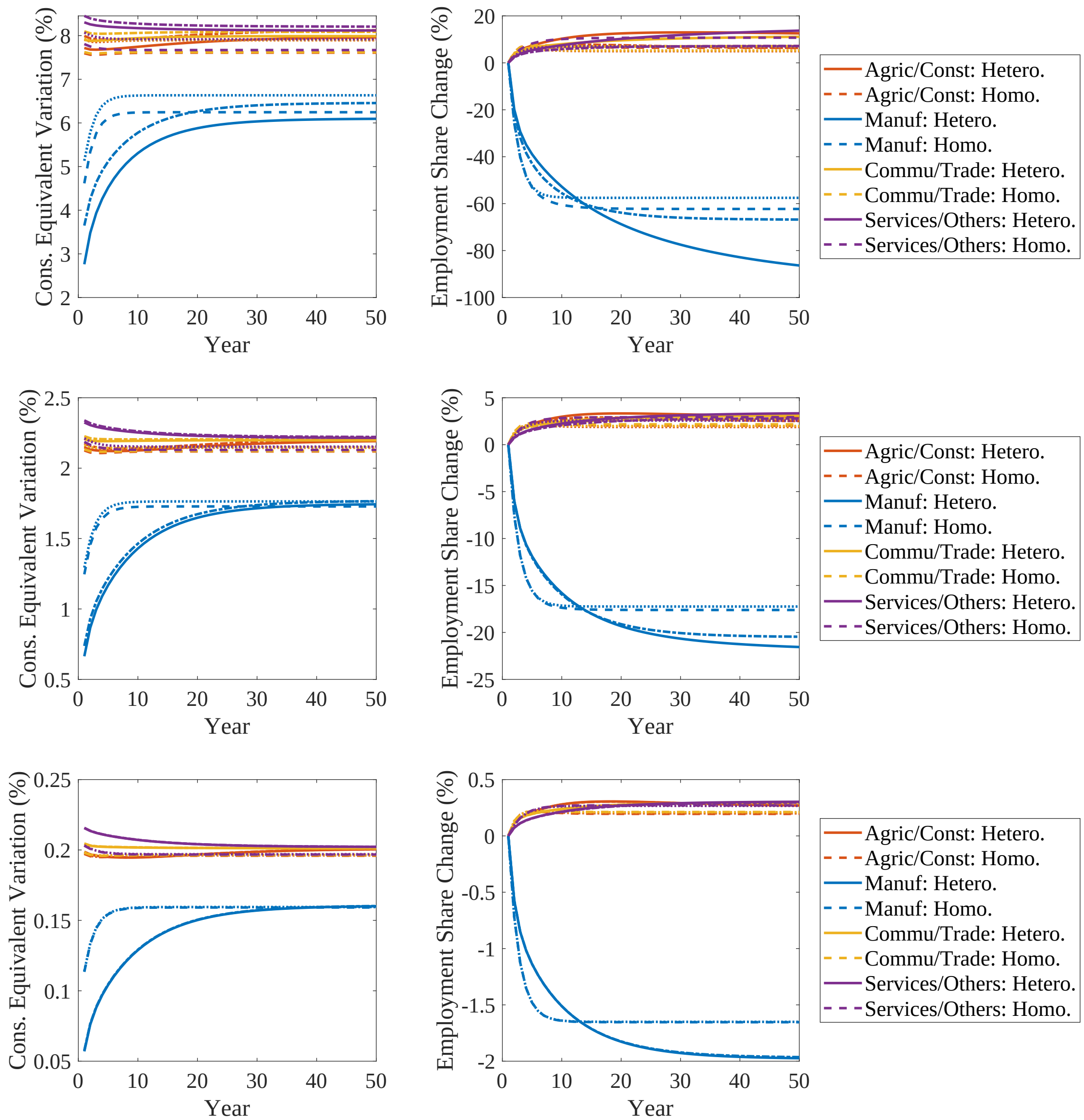


Figure A.12. The Quality of First-Order Approximation

Notes: This figure compares the transitional dynamics of welfare (left column) and employment share (right column) obtained using sufficient statistics formula with those calculated from the exact solution of the estimated structural model. The top row corresponds to 30% drop in manufacturing prices, the middle row to 10%, and the bottom row to 1%.

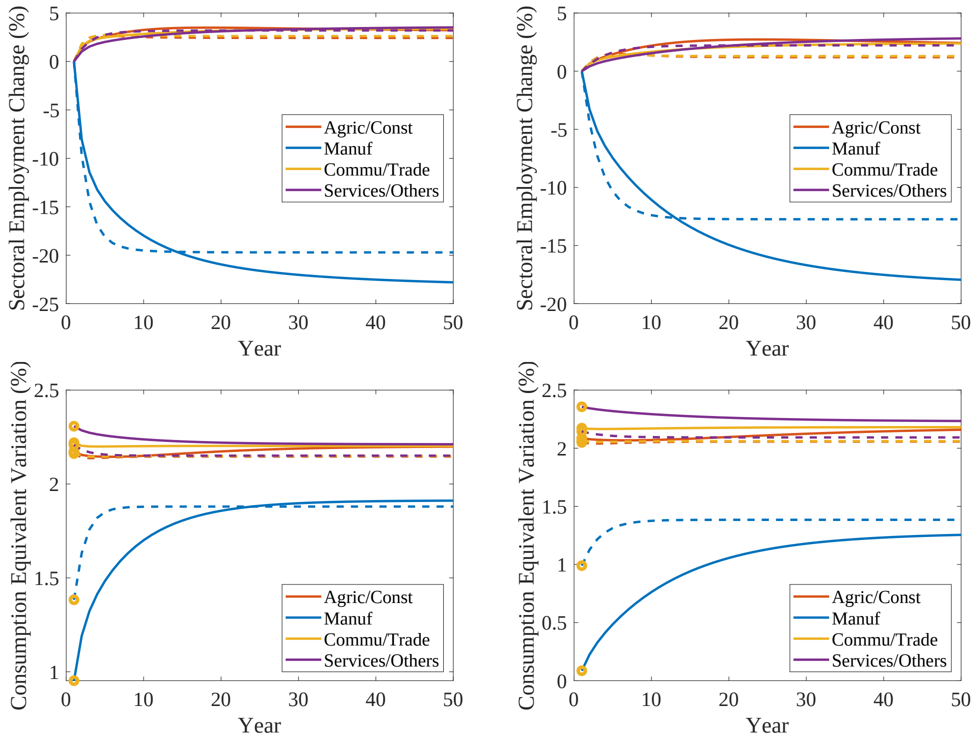
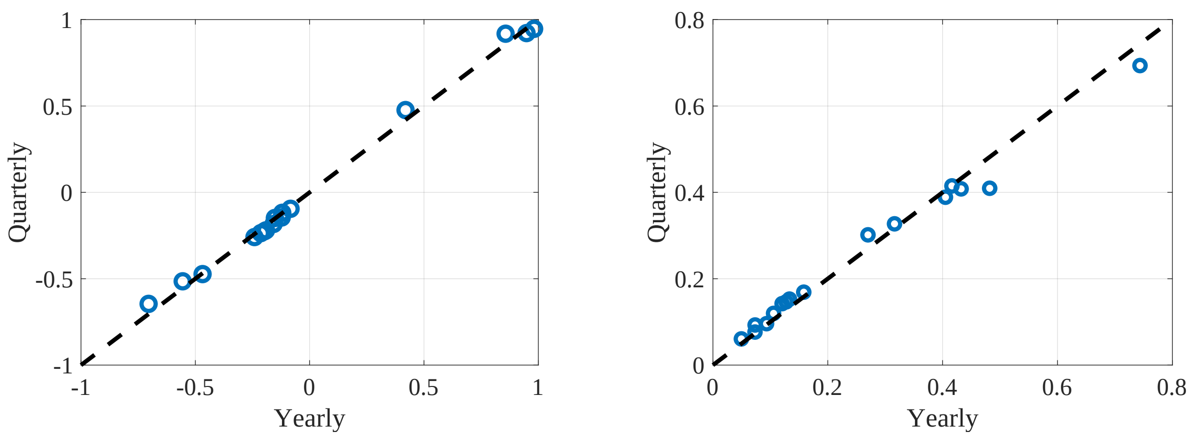


Figure A.13. Counterfactual Exercises with Different Values of ρ

Notes:



(a) Coefficients on Sectoral Employment Changes (b) Coefficients on Sectoral Welfare Changes

Figure A.14. Yearly to Quarterly

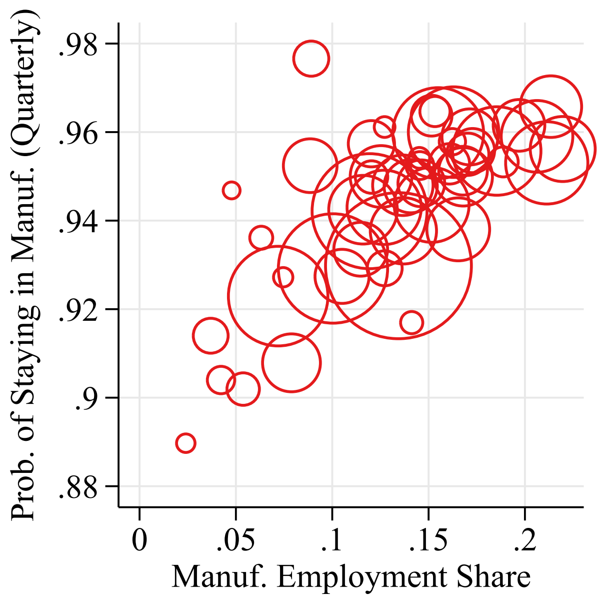


Figure A.15. Manufacturing Shares and Staying Probabilities Across States

Notes: This figure plots the probability of working in manufacturing sector again after one year against the manufacturing employment share. The size of the circle represents the relative size of the manufacturing sector. It is the largest in California, followed by Texas, Ohio, and Michigan.

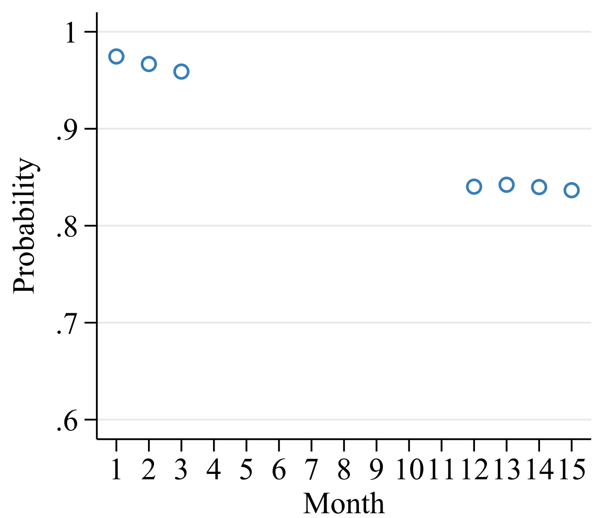


Figure A.16. Aggregate Worker Flow Matrix Series: Monthly CPS

Notes: This figure clearly shows that the initial three circles are not in line with the remaining four circles. In particular, the last four circles should be shifted upwards. This is a well-known problem of the CPS dataset.

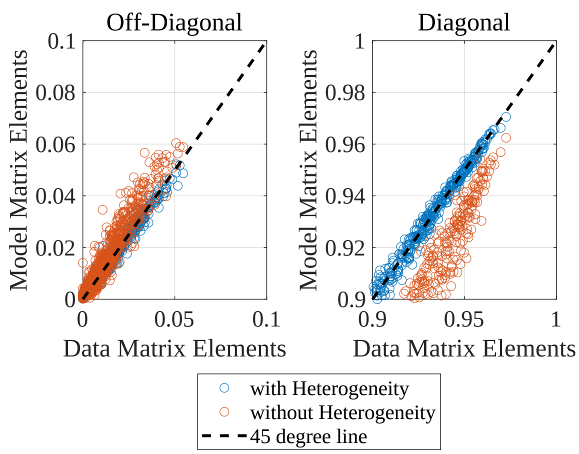


Figure A.17. Fit of the Models with and without Heterogeneity: State-Level Worker Flow Matrices

Notes: In this figure, we plot the model-implied state-level worker flow matrix series for $k = 2, 3$ against that in the data. Blue circles represent the results of the heterogeneous-worker model and orange circles represent those of the canonical model, which exactly matches the 1-month worker flow matrix. Due to the short time horizon, worker flow matrices have diagonal elements close to one and off-diagonal elements close to zero. We separately plot them in the left and right panels.

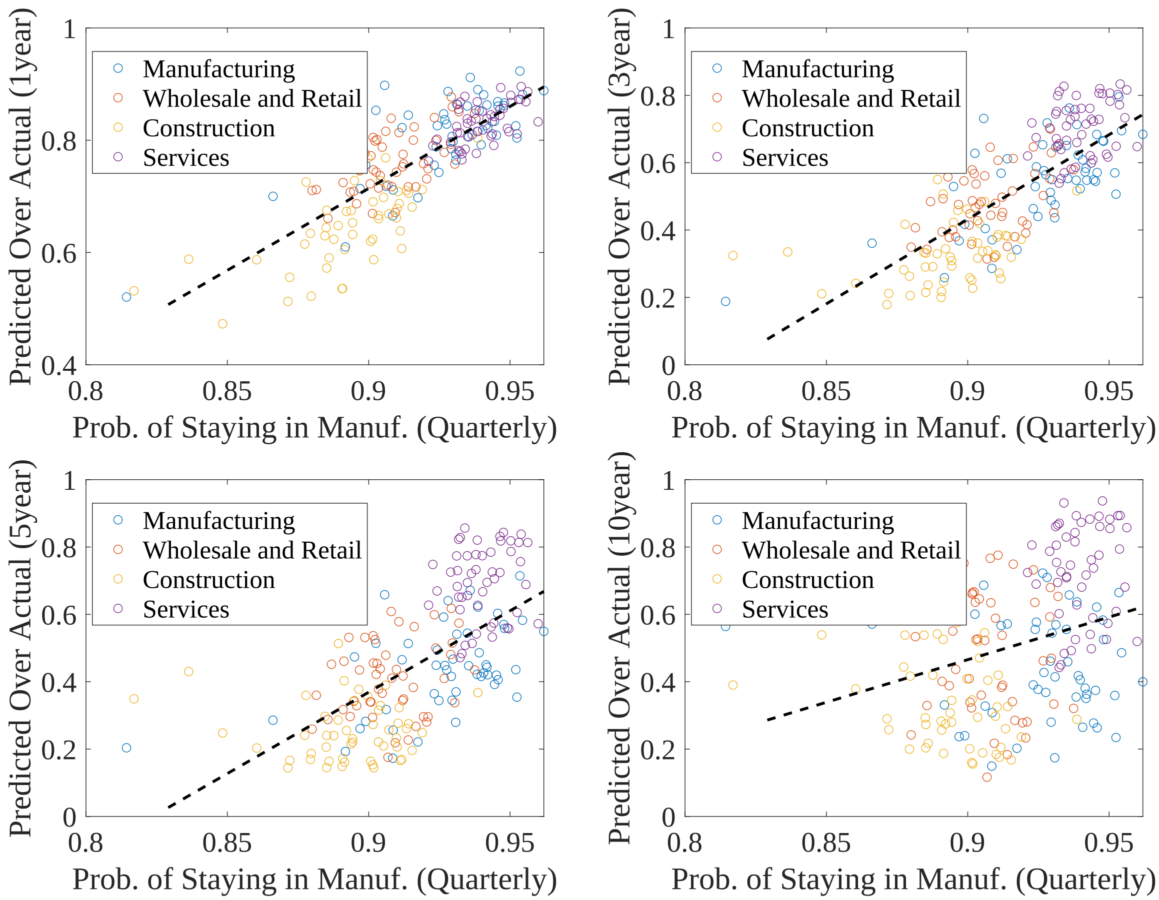
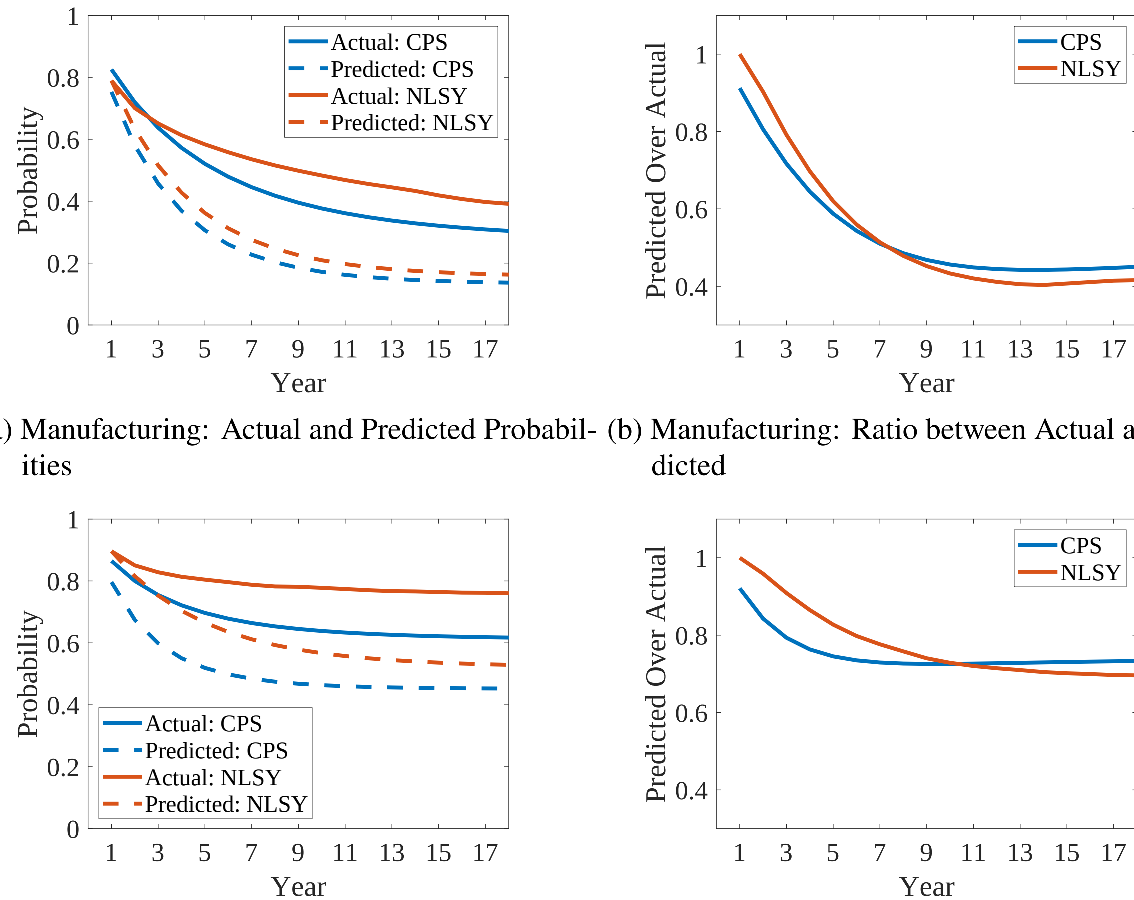


Figure A.18. Predicted and Actual Staying Probabilities: State-Level



(a) Manufacturing: Actual and Predicted Probabilities (b) Manufacturing: Ratio between Actual and Predicted

(c) Services: Actual and Predicted Probabilities (d) Services: Ratio between Actual and Predicted

Figure A.19. Comparison: CPS and NLSY

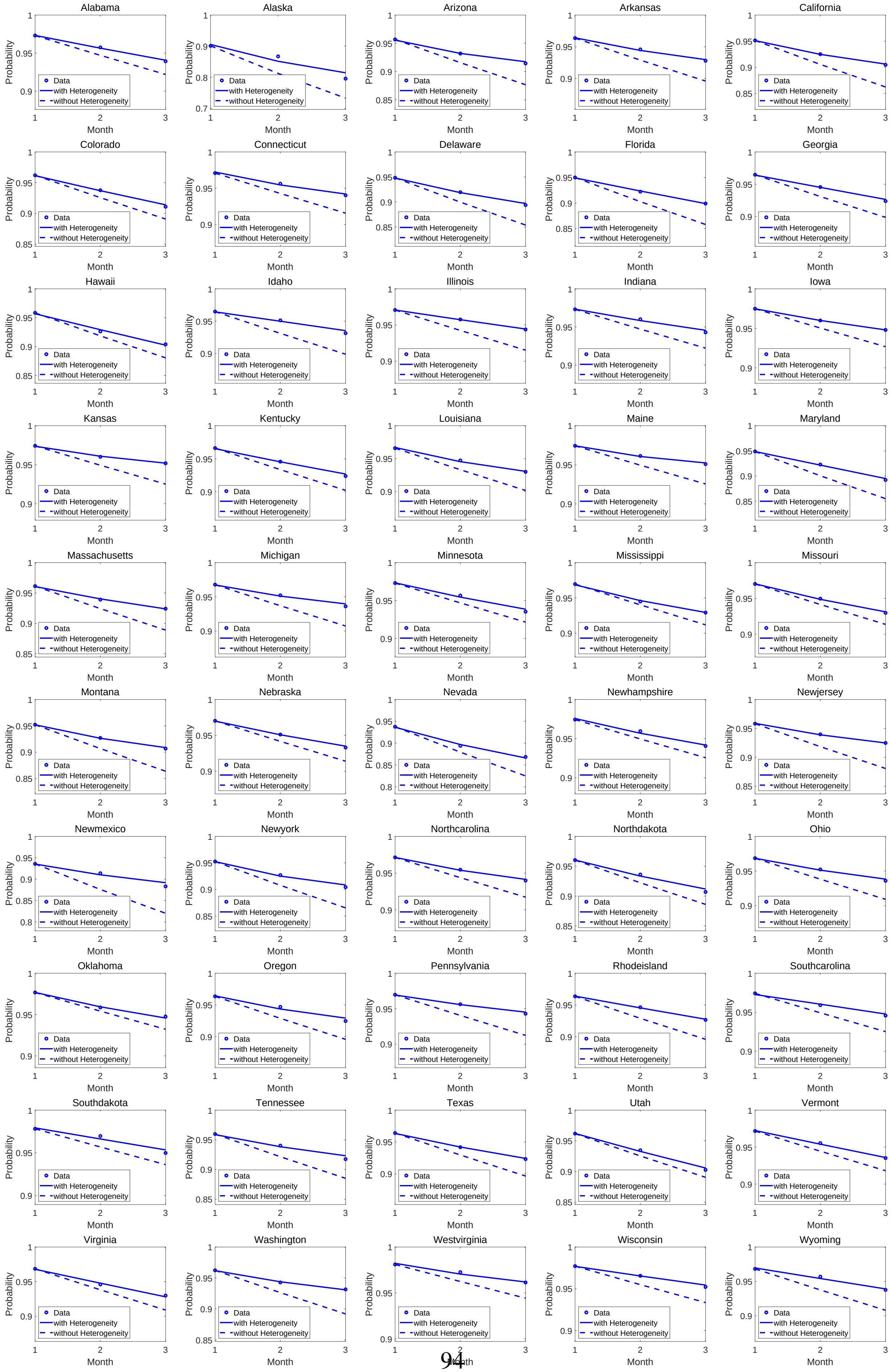
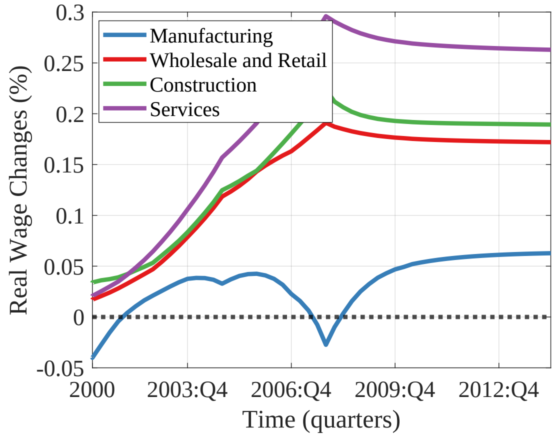
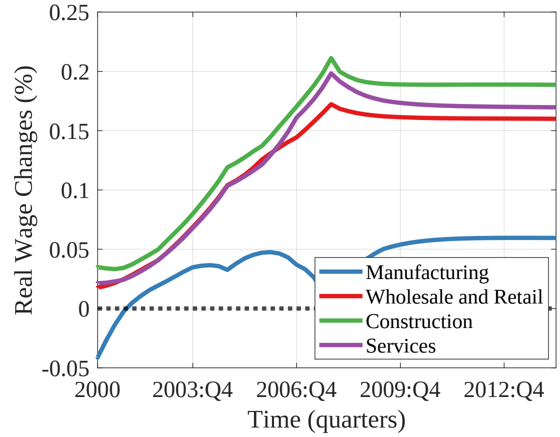


Figure A.20. Model Fit to State-Level Worker Flow Matrix Series: Manufacturing Sector



(a) With Worker Heterogeneity



(b) Without Worker Heterogeneity

Figure A.21. Changes in Real Wages

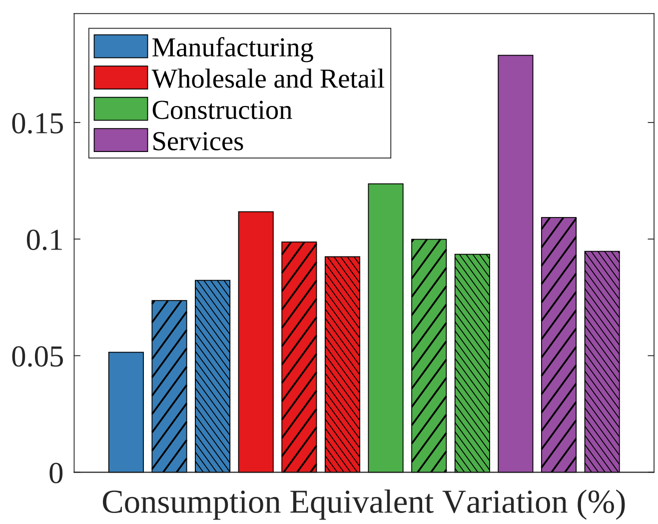


Figure A.22. Exogenous Wage