Written part HW1

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1\. (a)

```
total = 0                  - θ(1)
for i in range(n):
    total += i             - θ(1)   ] θ(n)   ]  θ(2n)
for j in range(1, n+1):                           ||
    total += j             - θ(1)   ] θ(n)   ]  θ(n)
```

$\boxed{\therefore T_{worst}(n) = \Theta(n)}$

(b)

```
total = 0                  - θ(1)
for i in range(n):
    total += i             - θ(1)
    for j in range(i, n):               ]  θ(n^2)
        total += j         - θ(1)  ] θ(n)
```

$\boxed{\therefore T_{worst}(n) = \Theta(n^2)}$

(c)

```
total = 0                  - θ(1)
while n > 1:               - θ(n)
    total += n % 2         - θ(1)   ]  θ(log_2 n)
    n = n // 2
```

$n \to \frac{n}{2} \to \frac{n}{4} \to \frac{n}{8} \to \cdots$: size

$\boxed{\therefore T_{worst}(n) = \Theta(\log_2 n)}$

2(a) $3n^4+8n^3-3n = \theta(n^4)$

$c_1 n^4 \le 3n^4+8n^3-3n \le c_2 n^4$

upper bound:

$3n^4+8n^3-3n \le 3n^4+8n^4+3n^4$

$3n^4+8n^3-3n \le 14n^4$ for $n \ge 0$

lower bound:

$3n^4 \le 3n^4+8n^3-3n$ for $n \ge 0$

$3n^4 \le 3n^4+8n^3-3n \le 14n^4$ $(n \ge 0)$

$\boxed{\therefore\ c_1 = 3,\ c_2 = 14,\ n_0 = 0}$

(b) $\sqrt{17n^2+4n-7} = \theta(n)$

$c_1 n \le \sqrt{17n^2+4n-7} \le c_2 n$

$c_1^2 n^2 \le 17n^2+4n-7 \le c_2^2 n^2$

upper bound:

$17n^2+4n-7 \le 17n^2+4n^2+7n^2 = 28n^2$ for $n \ge 0$

lower bound:

$17n^2-4n^2-7n^2 \le 17n^2+4n-7$

$6n^2 \le 17n^2+4n-7n$ for $n \ge 1$

$6n^2 \le 17n^2+4n-7 \le 28n^2$ $\quad \underline{c_1^2 = 6}$, $\underline{c_2^2 = 28}$

$c_1 = \sqrt{6}$ $\quad c_2 = \sqrt{28}$

$\boxed{\therefore\ c_1 = \sqrt{6},\ c_2 = \sqrt{28},\ n_0 = 1}$

(c) $f(n) = O(g(n)) \Rightarrow f(n)$ is upper bound.

$f(n) = 4n \quad g(n) = 2n$

$g(n) = O(h(n)) \Rightarrow g(n)$ is upper bound.

$g(n) = 2n \quad h(n) = n$

$\therefore n \le 4n \rightarrow h(n) \le f(n)$ for $n \ge 0$

$f(n) = O(h(n))$

3.

```
def find_primes(n):
    prime_lst = []                     - θ(1)
    for i in range(1, n+1):            - θ(n)      ] θ(n√n)
        if is_prime(i):                - θ(√n)
            prime_lst.append(i)        - θ(1) Amortized
    return prime_lst                   - θ(1)

def is_prime(n):
    if n == 1:                         - θ(1)
        return False
    for k in range(2, int(math.sqrt(n)+1)):   ] θ(√n).
        if n % k == 0:                 - θ(1)
            return False
    return True
```

$$\boxed{\therefore\ T(n) = \theta(n\sqrt{n})}$$

4. (a)

```
def reverse1(lst):
    rev_lst = []                       - θ(1)
    for i in range(len(lst)):          → θ(n).    ] θ(n²).
        rev_lst.insert(0, lst[i])      → θ(n)
    return rev_lst                     - θ(1)
```

$$1+2+3+4+5+\cdots\cdots+n = \frac{n(n+1)}{2} = \theta(n^2).$$

$$\boxed{\therefore\ T_{worst}(n) = \theta(n^2)}$$

4. (b)

```
def reverse2 (lst):
    rev_lst = []  - θ(1)
    for i in range (len(lst)-1, -1, -1):              ⎤ θ(n).
        rev_lst.append (lst[i])  - θ(1) Amortized    ⎦
    return rev_lst  - θ(1)
```

$1 + 1 + 1 + 1 + 1 + \cdots + 1 = \Theta(n)$.

$\boxed{\therefore\ T_{worst}(n) = \Theta(n)}$