Signals and Systems

Jaeha Huh

1.4.9

$$X(z) = \frac{2z+1}{z^2 - \frac{5}{6}z + \frac{1}{6}} = \frac{2z+1}{(z-\frac{1}{3})(z-\frac{1}{2})} = \frac{A}{(z-\frac{1}{3})} + \frac{B}{(z-\frac{1}{2})}$$

$$\Rightarrow (z-\tfrac{1}{2})A + (z-\tfrac{1}{3})B = 2z+1 = Az - \tfrac{1}{2}A + Bz - \tfrac{1}{3}B$$

$$\Rightarrow \begin{array}{l} A+B=2 \\ -\frac{1}{2}A - \frac{1}{3}B = 1 \end{array} \qquad \begin{array}{r} \cancel{A}+B=2 \\ -\cancel{A}-\frac{2}{3}B=2 \\ \hline \frac{1}{3}B=4 \end{array} \qquad B=12,\ A=-10$$

$$\Rightarrow \frac{-10}{z-\frac{1}{3}} + \frac{12}{z-\frac{1}{2}}$$

$$a^n u(n) \rightarrow \frac{z}{z-a}$$

$$a^{n-1} u(n-1) \rightarrow \frac{1}{z-a}$$

$$\therefore \boxed{x(n) = -10\left(\tfrac{1}{3}\right)^{n-1} u(n-1) + 12\left(\tfrac{1}{2}\right)^{n-1} u(n-1)}$$

1.5.2

$$h(n) = \delta(n) + 3.5\,\delta(n-1) + 1.5\,\delta(n-2)$$

(a) $\boxed{H(z) = 1 + 3.5z^{-1} + 1.5z^{-2}}$

(b) $$H^{-1}(z) = \frac{1}{1+3.5z^{-1}+1.5z^{-2}} = \frac{1}{(1+3z^{-1})(1+\frac{1}{2}z^{-1})}$$

$$= \frac{A}{(1+3z^{-1})} + \frac{B}{(1+\frac{1}{2}z^{-1})} \Rightarrow \begin{array}{l} A+B=1 \\ \frac{1}{2}A+3B=0 \end{array} \qquad \begin{array}{r} \cancel{A}+B=1 \\ -\cancel{A}-6B=0 \\ \hline -5B=1 \end{array}$$

$$B = -\frac{1}{5},\ A = \frac{6}{5} \Rightarrow \frac{\frac{6}{5}}{1+3z^{-1}} + \frac{-\frac{1}{5}}{1+\frac{1}{2}z^{-1}}$$

$$= \left(\frac{6}{5}\right)\frac{z}{z+3} + \left(-\frac{1}{5}\right)\frac{z}{z+\frac{1}{2}} \quad \boxed{\therefore h^{-1}(n) = \frac{6}{5}(-3)^n u(n) - \frac{1}{5}\left(-\frac{1}{2}\right)^n u(n)}$$

1.5.3 (a)

$h(n) = \delta(n+1) - \frac{10}{3}\delta(n) + \delta(n-1)$

$H(z) = z - \frac{10}{3} + z^{-1}$ $\quad G(z) = \frac{1}{H(z)} = \frac{1}{z - \frac{10}{3} + z^{-1}}$

$G(z) = \frac{z}{z^2 - \frac{10}{3}z + 1}$ $\quad \frac{G(z)}{z} = \frac{1}{z^2 - \frac{10}{3}z + 1} = \frac{A}{(z-\frac{1}{3})} + \frac{B}{(z-3)}$

$A + B = 0$
$-3A - \frac{1}{3}B = 1$

$A + B = 0$
$-9A - B = 3$
$-8A = 3$

$A = -\frac{3}{8}$ $\quad B = \frac{3}{8}$

$\Rightarrow \frac{G(z)}{z} = \frac{-\frac{3}{8}}{(z-\frac{1}{3})} + \frac{\frac{3}{8}}{(z-3)} \Rightarrow G(z) = \left(-\frac{3}{8}\right)\frac{z}{z-\frac{1}{3}} + \left(\frac{3}{8}\right)\frac{z}{z-3}$

$\therefore \boxed{g(n) = -\frac{3}{8}\left(\frac{1}{3}\right)^n u(n) + \frac{3}{8}(3)^n u(n)}$

1.6.4

(a) $h(n) = \left(\frac{1}{3}\right)^n u(n)$ $\quad H(z) = \frac{Y(z)}{X(z)} = \frac{z}{z-\frac{1}{3}} = \frac{1}{1-\frac{1}{3}z^{-1}}$

$\left(1 - \frac{1}{3}z^{-1}\right) Y(z) = X(z)$ $\quad Y(z) - \frac{1}{3}z^{-1}Y(z) = X(z)$

$y(n) - \frac{1}{3}y(n-1) = x(n)$

$\therefore \boxed{y(n) = x(n) + \frac{1}{3}y(n-1)}$

(b) $x(n) = \left(\frac{1}{2}\right)^n u(n)$

$\therefore \boxed{y(n) = \left(\frac{1}{2}\right)^n u(n) + \frac{1}{3}y(n-1)}$

1.6.7

(a) $y(n) - \frac{5}{6}y(n-1) + \frac{1}{6}y(n-2) = 2x(n) + \frac{2}{3}x(n-1)$

$$Y(z) - \frac{5}{6}z^{-1}Y(z) + \frac{1}{6}z^{-2}Y(z) = 2X(z) + \frac{2}{3}z^{-1}X(z)$$

$$\left(1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}\right)Y(z) = \left(2 + \frac{2}{3}z^{-1}\right)X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{2 + \frac{2}{3}z^{-1}}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}} = \frac{2z^2 + \frac{2}{3}z}{z^2 + \frac{5}{6}z + \frac{1}{6}}$$

$$\therefore H(z) = \frac{2z^2 + \frac{2}{3}z}{z^2 + \frac{5}{6}z + \frac{1}{6}}$$

1.6.9

(a) $r(n) = x(n) + \frac{2}{3}x(n-10)$. $\quad r(n) \to \boxed{g(n)} \to x(n)$

$$r(n) * g(n) = x(n) \Rightarrow R(z)G(z) = X(z) \Rightarrow G(z) = \frac{X(z)}{R(z)}$$

$$R(z) = X(z) + \frac{2}{3}z^{-10}X(z) = X(z)\left(1 + \frac{2}{3}z^{-10}\right)$$

$$G(z) = \frac{X(z)}{R(z)} = \frac{1}{1 + \frac{2}{3}z^{-10}} = \frac{3z^{10}}{3z^{10} + 2}$$

$$\boxed{\therefore g(n) = \left(-\frac{2}{3}\right)^{\frac{n}{10}} u\left(\frac{n}{10}\right)}$$

(b) $\frac{X(z)}{R(z)} = \frac{3z^{10}}{3z^{10} + 2}$ $\quad (3z^{10} + 2)X(z) = 3z^{10}R(z)$

$$3z^{10}X(z) + 2X(z) = 3z^{10}R(z)$$

$$3z^{10}X(z) + 2X(z) - 3z^{10}R(z) = 0$$

$$\boxed{\therefore 3x(n+10) + 2x(n) - 3r(n+10) = 0}$$

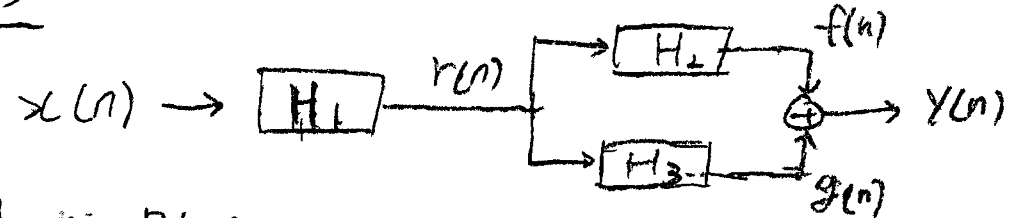
(c) $g(n) = \left(-\frac{2}{3}\right)^{\frac{n}{10}} u\left(\frac{n}{10}\right)$

$\sum_{n=0}^{\infty} |g(n)| < \infty$, stable.

$g(n) = 0$ for $n < 0$, causal

$\therefore$ $\boxed{\text{The system is causal and stable.}}$

1.6.13



H_1 : $R(Z) = 2X(z) - \frac{1}{2} z^{-1} R(z)$ $\qquad (1+\frac{1}{2}z^{-1}) R(z) = 2X(z)$

H_2 : $F(Z) = R(z) - \frac{1}{3} z^{-1} F(z)$ $\qquad (1+\frac{1}{3}z^{-1}) F(z) = R(z)$

H_3 : $G(Z) = R(Z) - \frac{1}{4} z^{-1} R(Z)$ $\qquad G(z) = (1-\frac{1}{4}z^{-1}) R(z)$

$H_1(z) = \frac{R(z)}{X(z)} = \frac{2}{1+\frac{1}{2}z^{-1}}$ $\qquad H_3(z) = \frac{G(z)}{R(z)} = 1 - \frac{1}{4} z^{-1}$

$H_2(z) = \frac{F(z)}{R(z)} = \frac{1}{1+\frac{1}{3}z^{-1}}$ $\qquad H_{Total} = H_1(z) \cdot H_2(z) + H_1(z) H_3(z)$

$H_1(z) \cdot H_2(z) = \frac{F(z)}{X(z)} = \frac{2}{1+\frac{1}{2}z^{-1}} \cdot \frac{1}{1+\frac{1}{3}z^{-1}} = \frac{2}{1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}}$

$H_1(z) \cdot H_3(z) = \frac{G(z)}{X(z)} = \frac{2}{1+\frac{1}{2}z^{-1}} \cdot (1-\frac{1}{4}z^{-1}) = \frac{2-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$

$H_{Total} = \frac{2}{1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}} + \frac{2-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}} = \frac{2(1+\frac{1}{2}z^{-1}) + (2-\frac{1}{2}z^{-1})(1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2})}{(1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2})(1+\frac{1}{2}z^{-1})}$

$\Rightarrow$ Calculations :

$\Rightarrow (1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2})(1+\frac{1}{2}z^{-1}) = 1 + \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2} + \frac{1}{2}z^{-1} + \frac{5}{12}z^{-2} + \frac{1}{12}z^{-3} = 1 + \frac{8}{6}z^{-1} + \frac{7}{12}z^{-2} + \frac{1}{12}z^{-3}$

$\Rightarrow$

$$\Rightarrow \left(2-\frac{1}{2}z^{-1}\right)\left(1+\frac{5}{6}z^{-1}+\frac{1}{6}z^{-2}\right) = 2+\frac{5}{3}z^{-1}+\frac{1}{3}z^{-2}$$

$$-\frac{1}{2}z^{-1}-\frac{5}{12}z^{-2}-\frac{1}{12}z^{-3} = 2+\frac{7}{6}z^{-1}-\frac{1}{12}z^{-2}-\frac{1}{12}z^{-3}$$

$$H_{total}(z) = \frac{2+z^{-1}+2+\frac{7}{6}z^{-1}-\frac{1}{12}z^{-2}-\frac{1}{12}z^{-3}}{1+\frac{8}{6}z^{-1}+\frac{7}{12}z^{-2}+\frac{1}{12}z^{-3}}$$

$$= \frac{4+\frac{13}{6}z^{-1}-\frac{1}{12}z^{-2}-\frac{1}{12}z^{-3}}{1+\frac{8}{6}z^{-1}+\frac{7}{12}z^{-2}+\frac{1}{12}z^{-3}} \cdot \frac{12z^{3}}{12z^{3}}$$

$$= \frac{48z^{3}+26z^{2}-z-1}{12z^{3}+16z^{2}+7z+1}$$

$$\therefore \boxed{H_{total}(z) = \frac{48z^{3}+26z^{2}-z-1}{12z^{3}+16z^{2}+7z+1}}$$

1.6.14

$$y(n) = 2x(n)+x(n-2)-0.2y(n-1)$$

(a) $Y(z) = 2X(z)+z^{-2}X(z)-0.2z^{-1}Y(z)$

$$Y(z)+0.2z^{-1}Y(z) = 2X(z)+z^{-2}X(z)$$

$$(1+0.2z^{-1})Y(z) = (1+z^{-2})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1+z^{-2}}{1+0.2z^{-1}} \cdot \frac{5z^{2}}{5z^{2}}$$

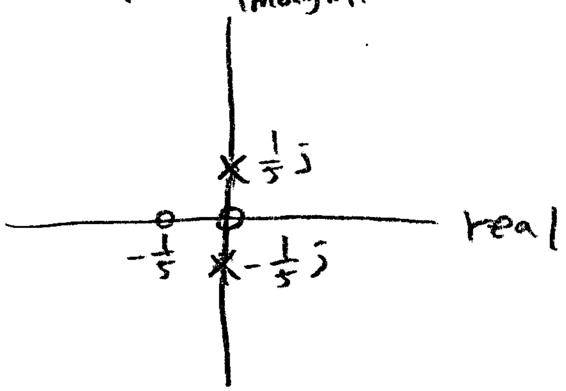
$$\therefore \boxed{H(z) = \frac{5z^{2}+5}{5z^{2}+z}}$$

(b) $H(z) = \dfrac{5z^2+1}{5z^2+z}$

Zeros : $5z^2+1=0$ $\quad z^2 = -\frac{1}{5}$ $\quad \boxed{z = \pm\frac{1}{5}j}$

poles : $5z^2+z=0$ $\quad z(5z+1)=0$

$\boxed{\text{poles} = 0,\ -\frac{1}{5}}$



(c) $H(z) = \dfrac{5z^2+1}{5z^2+z}$

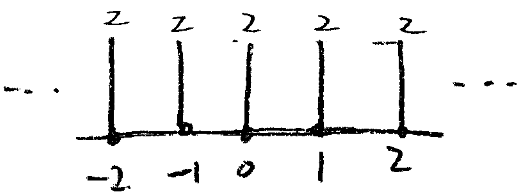
$\lim_{t\to\infty} y(t) = \lim_{z\to 0} \dfrac{5z^2+1}{z(5z+1)} = \dfrac{1}{1} = 1$

$\therefore$ $\boxed{\text{DC gain} = 1}$

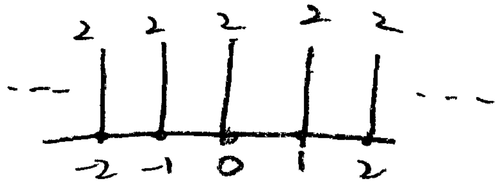
(d). input $x(n) = 2$ $\qquad$ DC gain $= 1 = \dfrac{y(n)}{x(n)}$

$\therefore$ $\boxed{\text{output } y(n) = 2}$

input $x(n)$



output $y(n)$



1.7.1

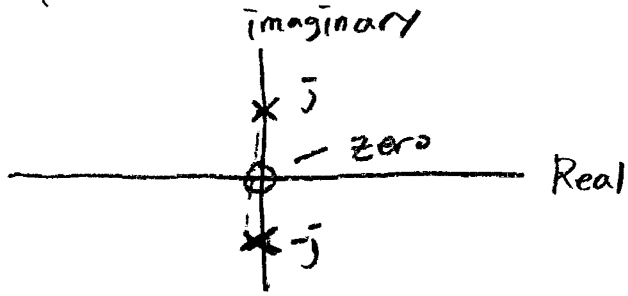
$y(n) = x(n) - y(n-2)$

(a) $Y(z) = X(z) - z^{-2}Y(z)$

$Y(z) + z^{-2}Y(z) = X(z) \qquad H(z) = \dfrac{1}{1+z^{-2}} = \dfrac{z^2}{z^2+1}$

$z^2+1=0 \qquad z^2=-1 \qquad \boxed{z=\pm j}$: poles

$z^2 = 0 \qquad \boxed{z=0}$: zero

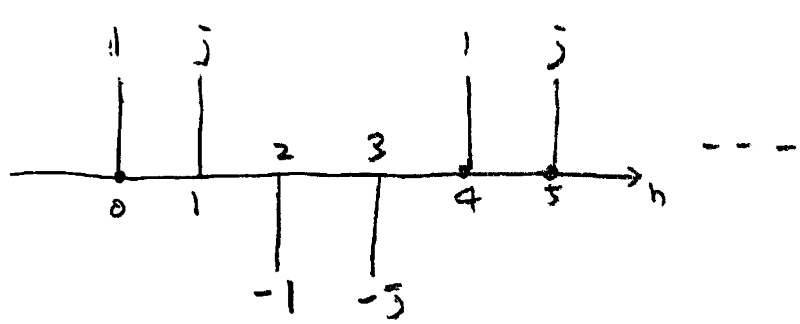


(b) $H(z) = \dfrac{z^2}{z^2+1} = \dfrac{z^2}{(z+j)(z-j)}$

$\dfrac{H(z)}{z} = \dfrac{z}{(z+j)(z-j)} = \dfrac{A}{(z+j)} + \dfrac{B}{(z-j)} \qquad A = \frac{1}{2} \quad B = \frac{1}{2}$

$\dfrac{H(z)}{z} = \dfrac{\frac{1}{2}}{z+j} + \dfrac{\frac{1}{2}}{z-j} \qquad H(z) = \frac{1}{2}\left[\dfrac{z}{z+j} + \dfrac{z}{z-j}\right]$

$\therefore h(n) = \frac{1}{2}\left[(j)^n + (-j)^n\right] u(n)$



(c) $\sum_{n=-\infty}^{\infty} h(n) = 0$ So it is stable.

1.7.2

$$h(n) = \left(\frac{1}{2}\right)^n \cos\left(\frac{2\pi}{3} n\right) u(n).$$

$$a^n \cos(\omega_0 n) u(n) \xrightarrow{Z.T} \frac{1 - a z^{-1} \cos(\omega_0)}{1 - 2a z^{-1} \cos(\omega_0) + a^2 z^{-2}}$$

$$H(z) = \frac{1 - \cos\left(\frac{2\pi}{3}\right) z^{-1}}{1 - 2\cos\left(\frac{2\pi}{3}\right) z^{-1} + z^{-2}} = \frac{1 + \frac{1}{4} z^{-1}}{1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}}$$

$$\frac{Y(z)}{X(z)} = \frac{1 + \frac{1}{4} z^{-1}}{1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}}$$

$$\left(1 + \frac{1}{2} z^{-1} + \frac{1}{4} z^{-2}\right) Y(z) = \left(1 + \frac{1}{4} z^{-1}\right) X(z).$$

$$Y(z) + \frac{1}{2} z^{-1} Y(z) + \frac{1}{4} z^{-2} Y(z) = X(z) + \frac{1}{4} z^{-1} X(z)$$

$$Y(n) + \frac{1}{2} Y(n-1) + \frac{1}{4} Y(n-2) = x(n) + \frac{1}{4} x(n-1)$$

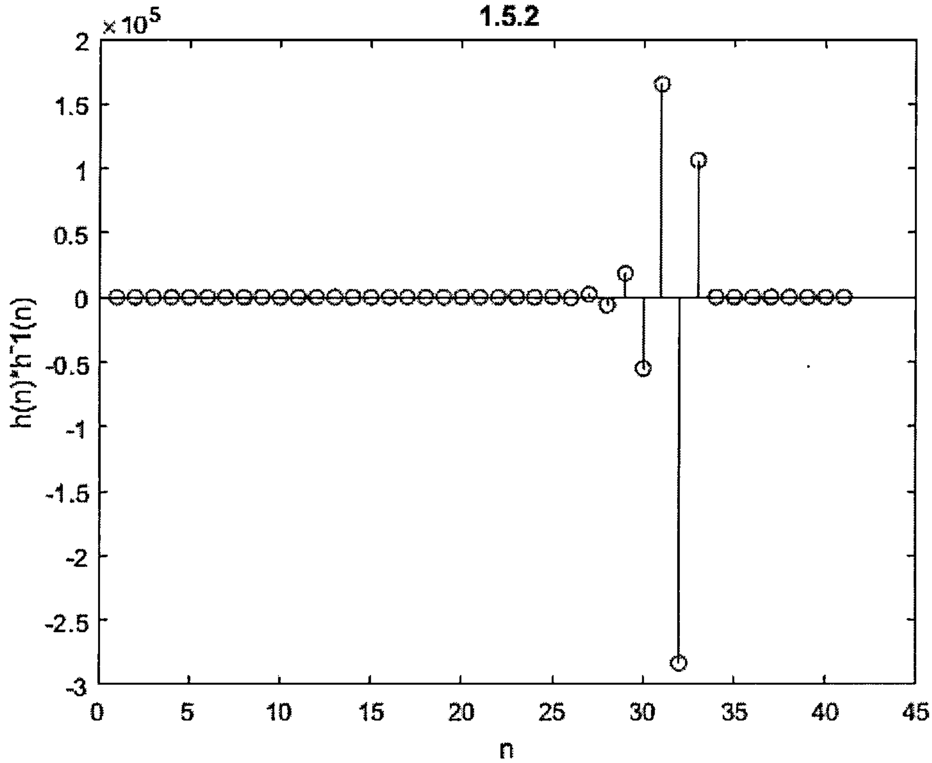
$$\therefore Y(n) = X(n) + \frac{1}{4} x(n-1) - \frac{1}{2} Y(n-1) - \frac{1}{4}(n-2)$$

Homework 4

Contents

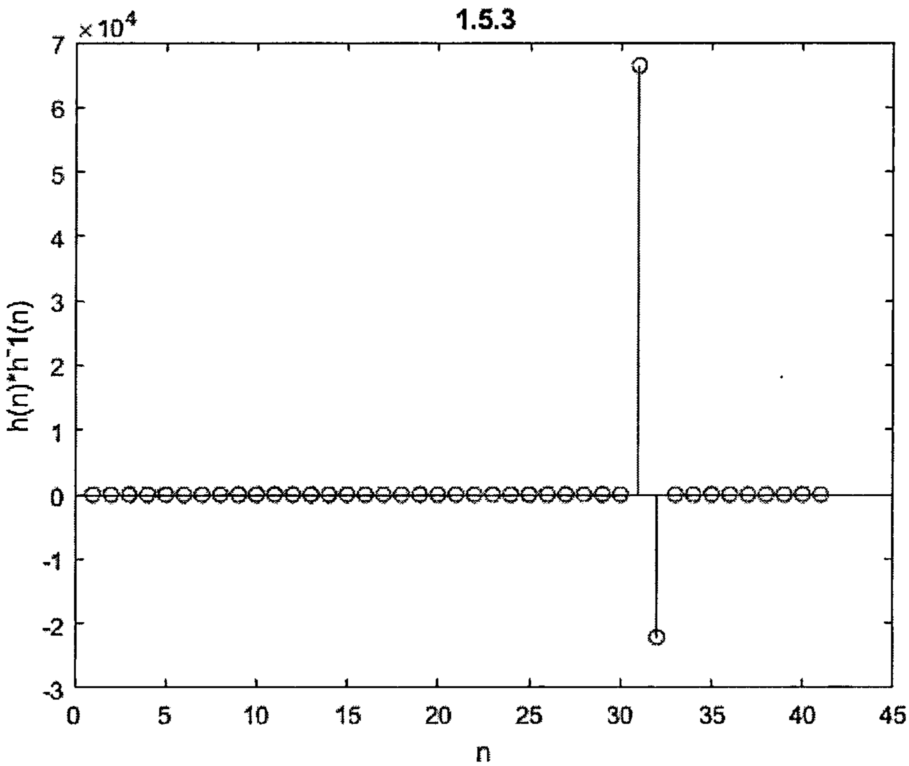
1.5.2

```
n = -10:10;

h = (1.*(n==0))-(3.5.*(n==1))+(1.5.*(n==2));

h_inv = ((6/5).*(-3).^n.*(n>=0))-((1/5).*(-1/2).^n.*(n>=0));



x = conv(h,h_inv);

figure (1);

stem(x)

title('1.5.2')

ylabel('h(n)*h^-1(n)')

xlabel('n')
```



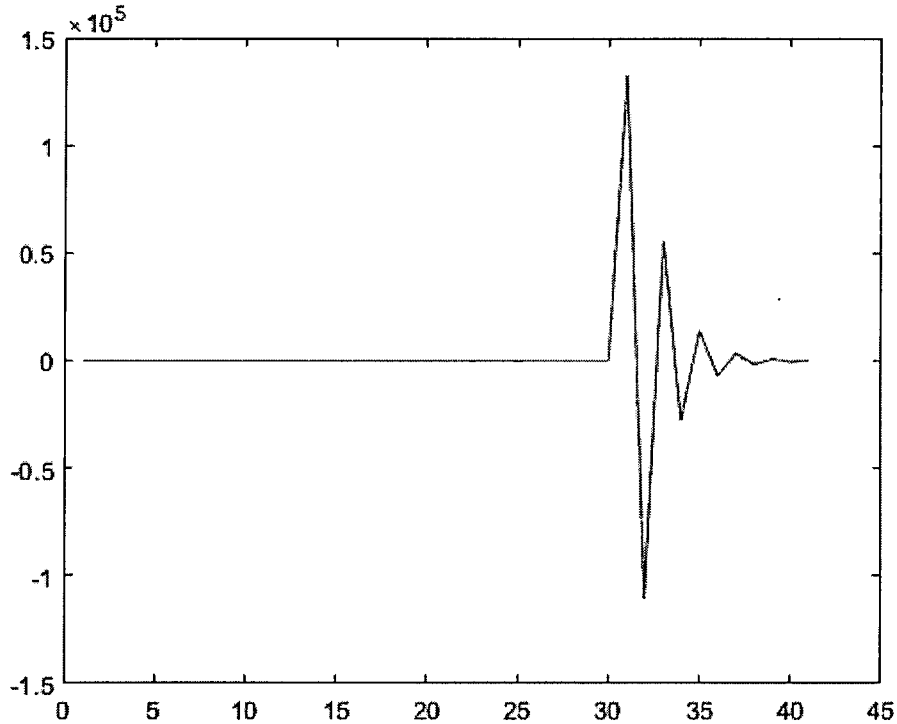
1.5.3

```
n = -10:10;

h = (1.*(n==-1))-((10/3).*(n==0))+(1.*(n==1));

h_inv = ((-3/8).*(1/3).^n.*(n>=0))-((3/8).*(3).^n.*(n>=0));



x = conv(h,h_inv);

figure (2);

stem(x)

title('1.5.3')

ylabel('h(n)*h^-1(n)')

xlabel('n')
```



1.6.7 (b)

```
[r,p,k] = residue ([2, 2/3], [1, 5/6, 1/6]);

h1 = filter([2, 2/3], [1, 5/6, 1/6],x);

figure(3);

plot(h1)
```



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