HW 11 Jaeha Heh

3.1.3

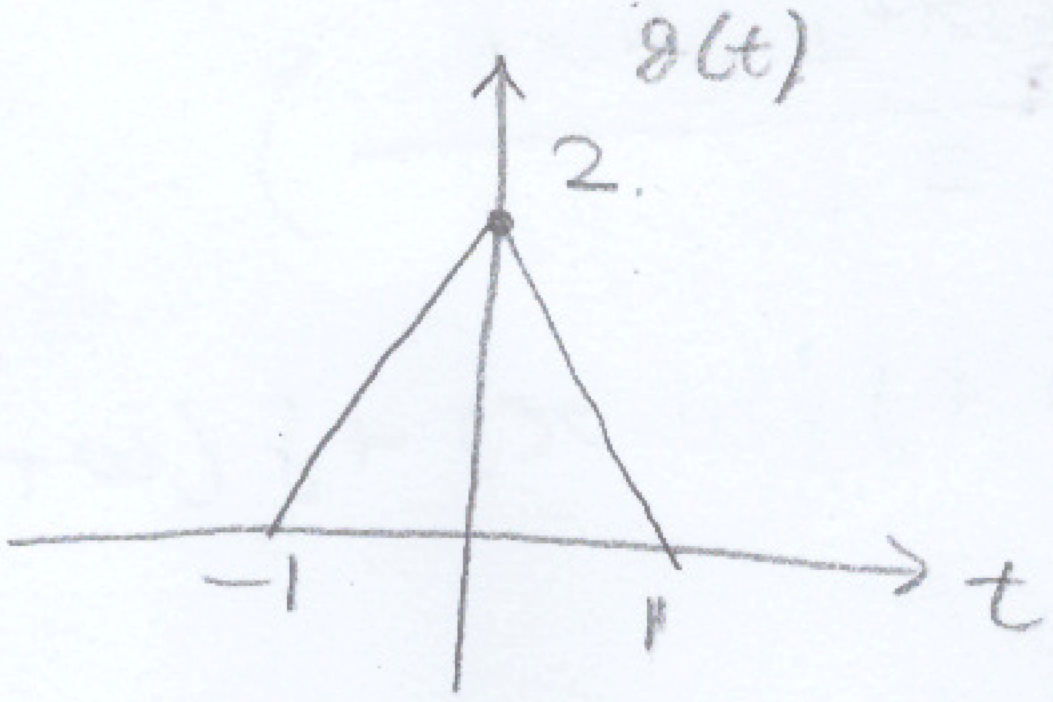
$$G^f(\omega) = X^f(0.5\omega) \qquad x(t) \xrightarrow{F.T} x^f(\omega) \qquad g(t) \xrightarrow{F.t} G^f(\omega)$$

$$\Rightarrow x(2t) \xrightarrow{F.T} \frac{1}{2} X^f(0.5\omega)$$

$$\Rightarrow \frac{G(\omega)}{2} = \frac{1}{2} X^f(0.5\omega) = F.T(x(2t))$$

$$\Rightarrow F.T^{-1}(G(\omega)) = 2x(2t)$$

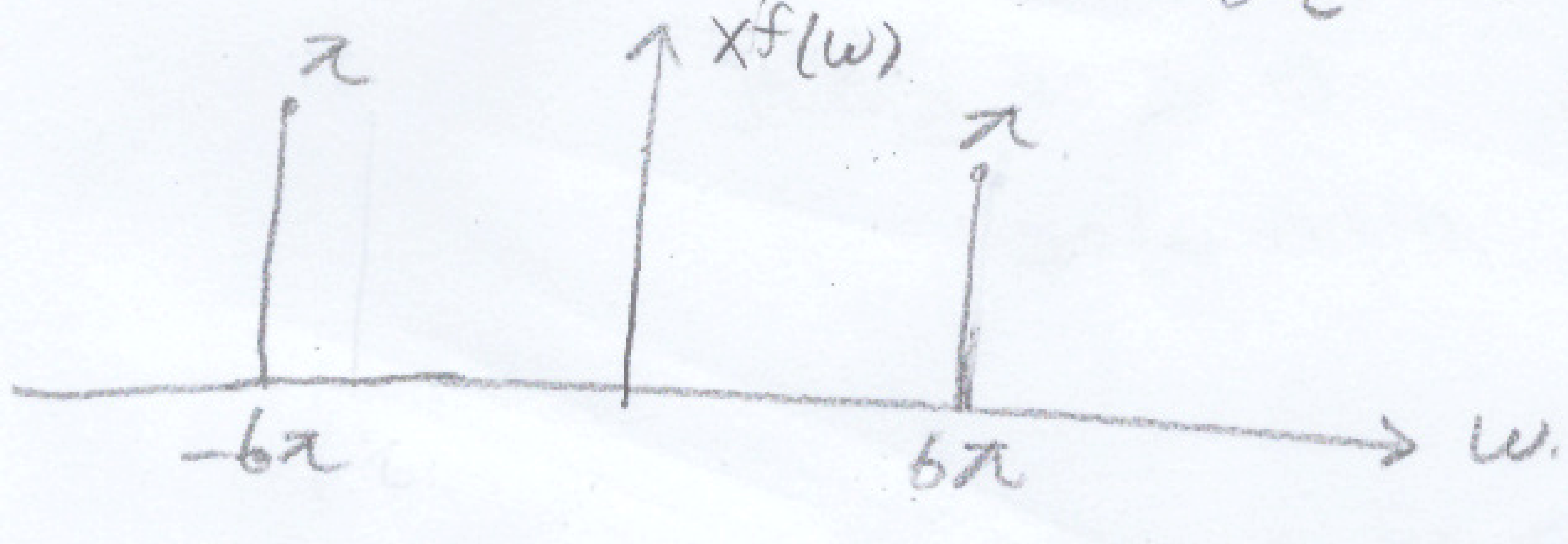
$$\Rightarrow g(t) = 2x(2t)$$



3.1.5

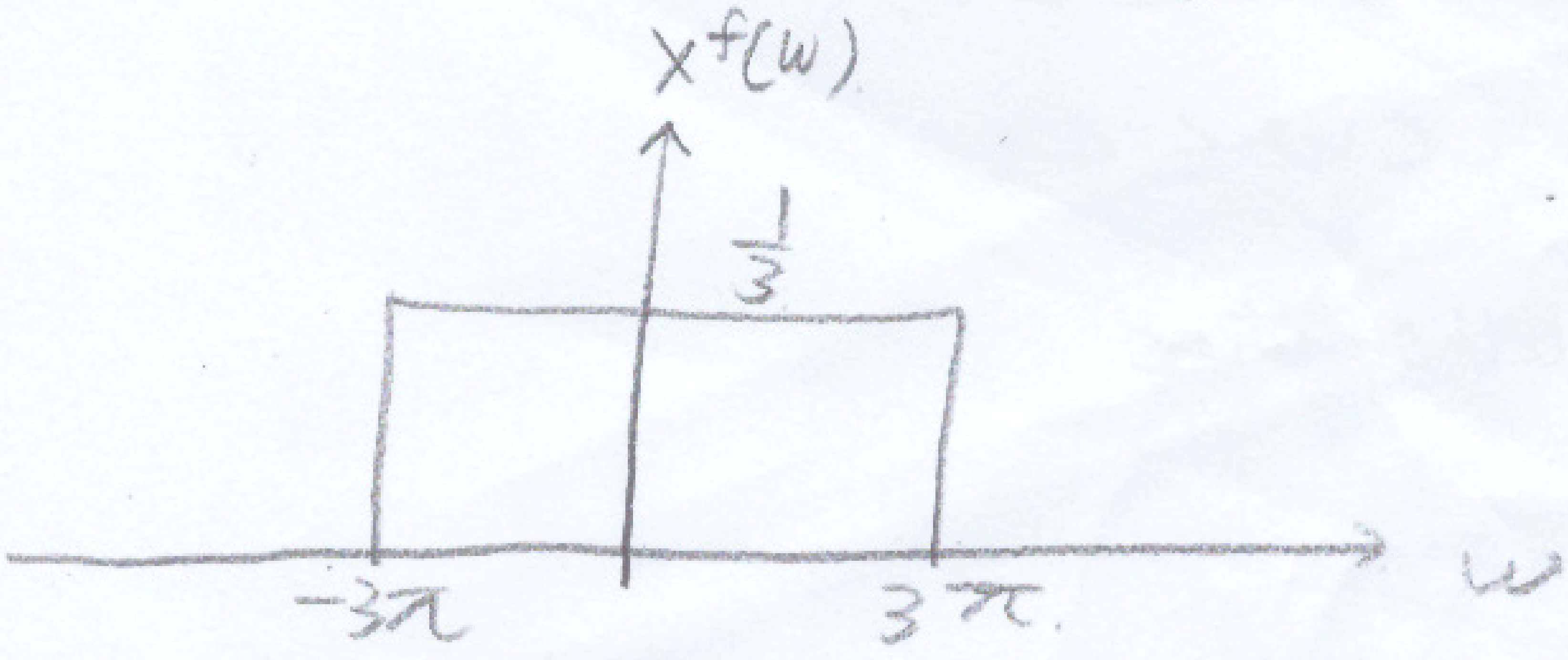
(a) $x(t) = \cos(6\pi t) = \dfrac{e^{j6\pi t} + e^{-j6\pi t}}{2}$

$$\Rightarrow x^f(\omega) = \pi[\delta(\omega - 6\pi) + \delta(\omega + 6\pi)]$$



(b) $x(t) = \mathrm{sinc}(3t) = \dfrac{\sin 3\pi t}{3\pi t} = \dfrac{1}{3} \cdot \dfrac{\sin \pi t}{\pi t}$

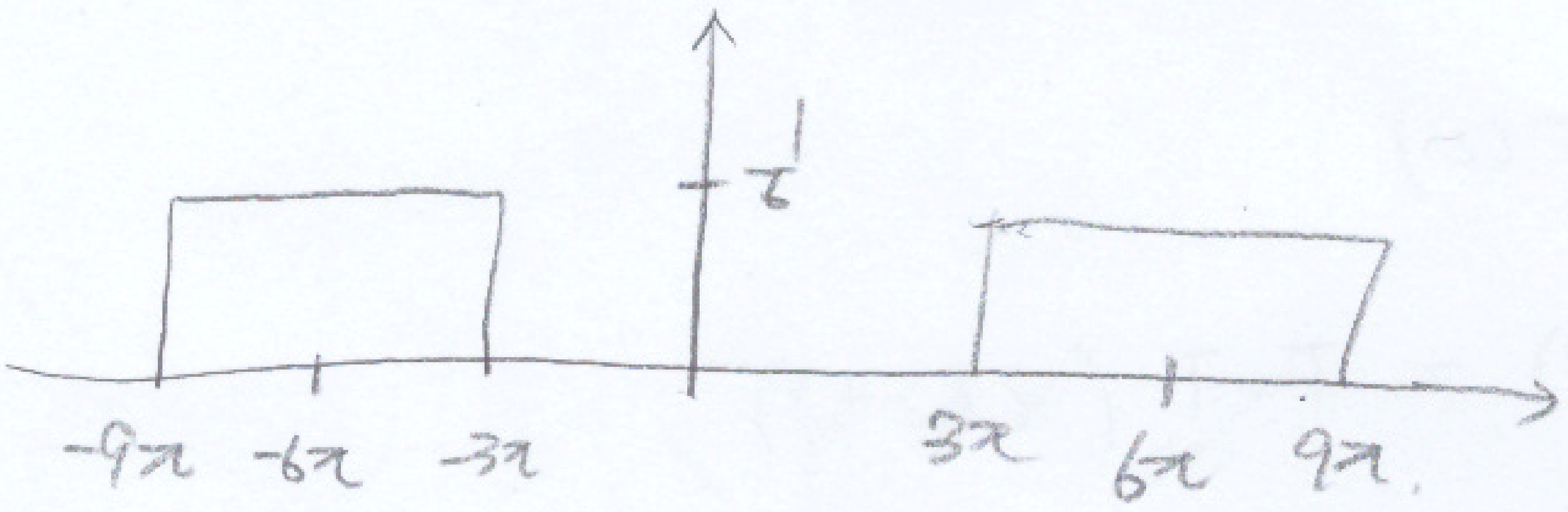
$$= \frac{1}{3}\,\mathrm{rect}\left(\frac{\omega}{6\pi}\right)$$



(c) $x(t) = \cos(6\pi t)\ \mathrm{sinc}(3t)$

$$x^f(\omega) = \frac{1}{2} x_0^f(\omega - 6\pi) + \frac{1}{2} x_0^f(\omega - 6\pi)$$

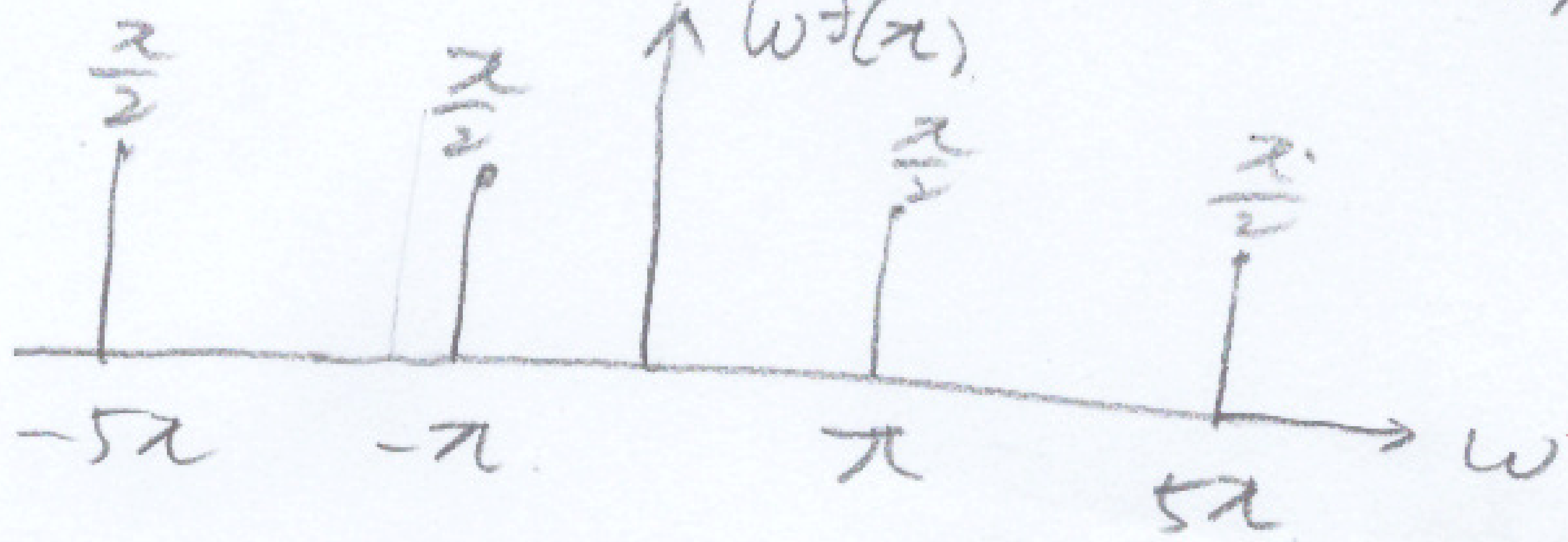
$$x_0^f(\omega) = \frac{1}{3}\,\mathrm{rect}\left(\frac{\omega}{6\pi}\right)$$



(d) $x(t) = \cos 3\pi t \cdot \cos 2\pi t = \frac{1}{2}\left[\cos(5\pi t) + \cos(\pi t)\right]$

$$= \frac{1}{2}\left(\frac{e^{j5\pi t} + e^{-j5\pi t}}{2} + \frac{e^{j\pi t} + e^{-j\pi t}}{2}\right)$$

$$\xrightarrow{F.T} \frac{\pi}{2}\left[\delta(\omega - 5\pi) + \delta(\omega + 5\pi) + \delta(\omega - \pi) + \delta(\omega + \pi)\right]$$



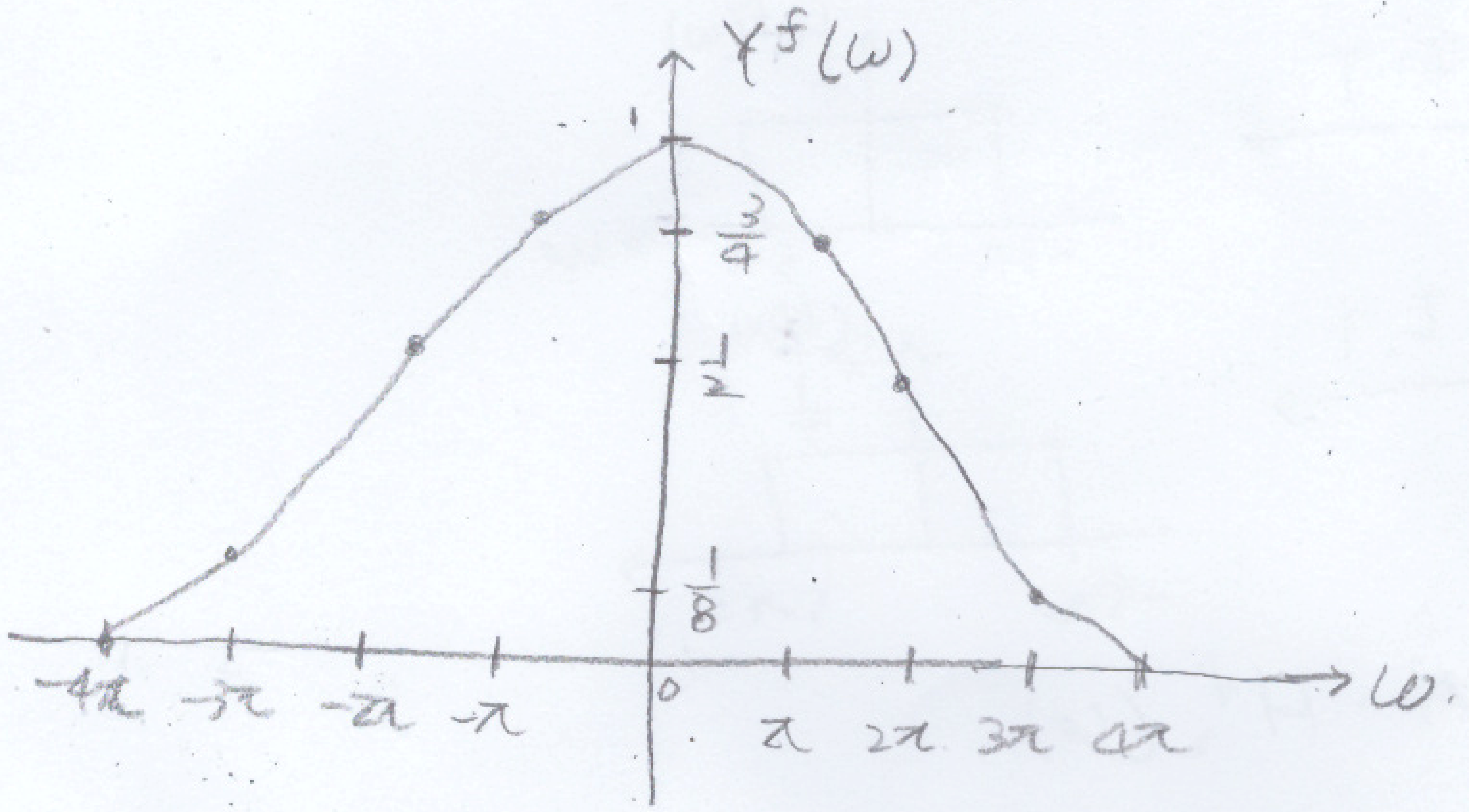
3.1.12

$$x^f(\omega) = \begin{cases} 1 + \frac{\omega}{4\pi}, & -4\pi < \omega \le 0 \\ 1 - \frac{\omega}{4\pi}, & 0 < \omega \le 4\pi \\ 0, & \text{otherwise} \end{cases}$$

$$H^f(\omega) \begin{cases} 2 + \frac{\omega}{2\pi}, & -4\pi < \omega \le -2\pi \\ 1, & -2\pi < \omega \le 2\pi \\ 2 - \frac{\omega}{2\pi}, & 2\pi < \omega \le 4\pi \\ 0, & \text{otherwise} \end{cases}$$

$$Y^f(\omega) = X^f(\omega) \cdot H^f(\omega)$$

$$Y^f(\omega) = \begin{cases} \left(1+\frac{\omega}{4\pi}\right)\left(2+\frac{\omega}{2\pi}\right), & -4\pi < \omega \le -2\pi \\ \left(1+\frac{\omega}{4\pi}\right), & -2\pi < \omega \le 0 \\ \left(1-\frac{\omega}{4\pi}\right), & 0 < \omega \le 2\pi \\ \left(1-\frac{\omega}{2\pi}\right)\left(2-\frac{\omega}{2\pi}\right), & 2\pi < \omega \le 4\pi \\ 0, & \text{otherwise} \end{cases}$$



3.1.14

$$x(t) = \cos\left(5\pi t + \frac{\pi}{4}\right)$$

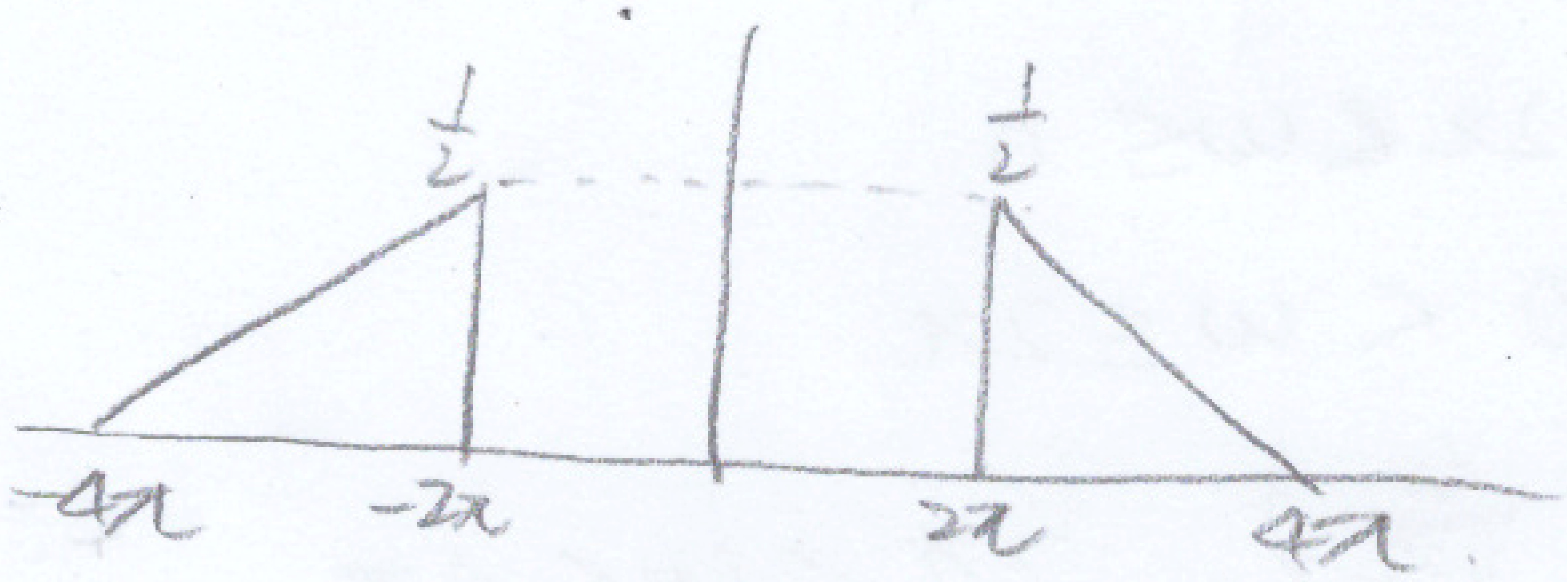
$$x(t) = \frac{e^{j\left(5\pi t + \frac{\pi}{4}\right)} + e^{-j\left(5\pi t + \frac{\pi}{4}\right)}}{2}$$

$$= \frac{(1+j)\, e^{j5\pi t} + (1-j)\, e^{-j5\pi t}}{2\sqrt{2}}$$

$$\xrightarrow{F.T} X^f(\omega) = \frac{1}{2\sqrt{2}}\left[(1+j)\,\delta(\omega - 5\omega) + (1-j)\,\delta(\omega + 5\pi)\right]$$

3.1.15

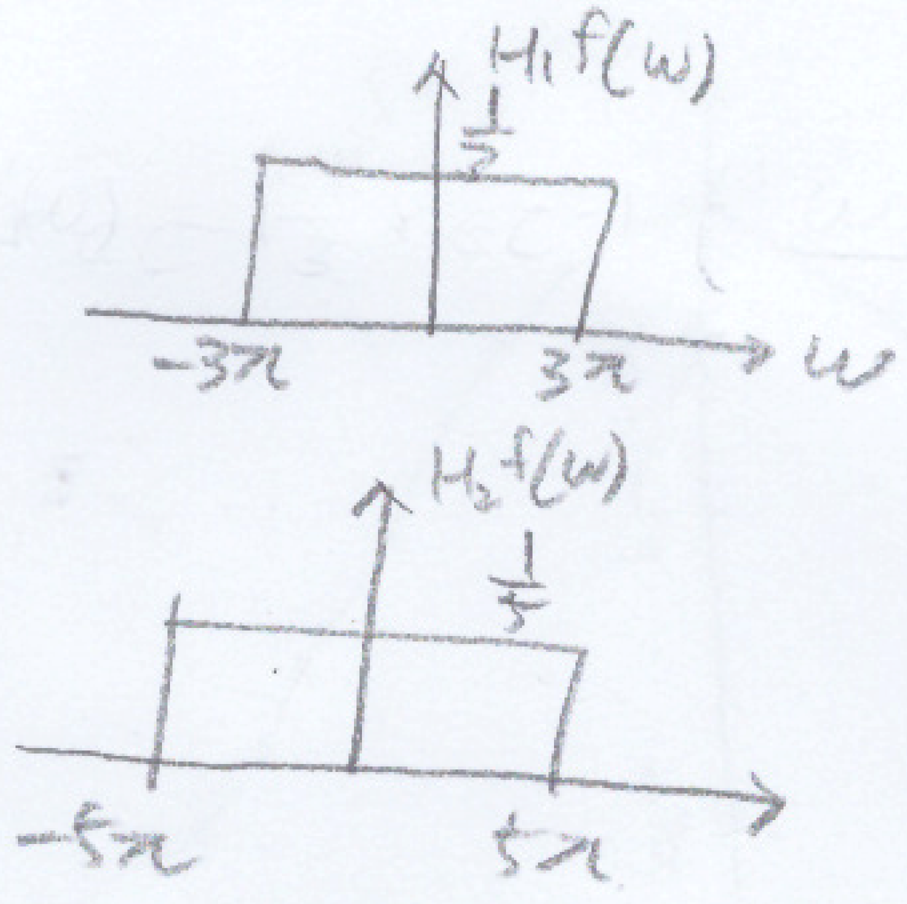
$$Y^f(\omega) = X^f(\omega) \cdot H^f(\omega)$$



3.1.16

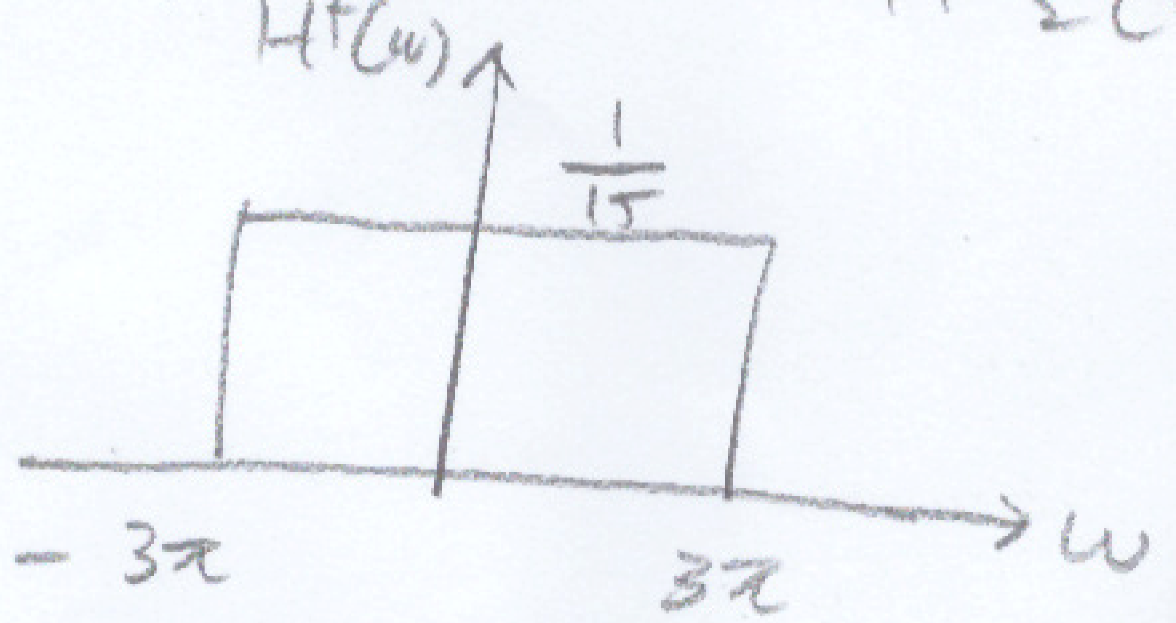
$h_1(t) = \text{sinc}(3t) \xrightarrow{F.T}$

$h_2(t) = \text{sinc}(5t) \xrightarrow{F.T}$



$$H^f(\omega) = H^f_1(\omega) \cdot H^f_2(\omega)$$

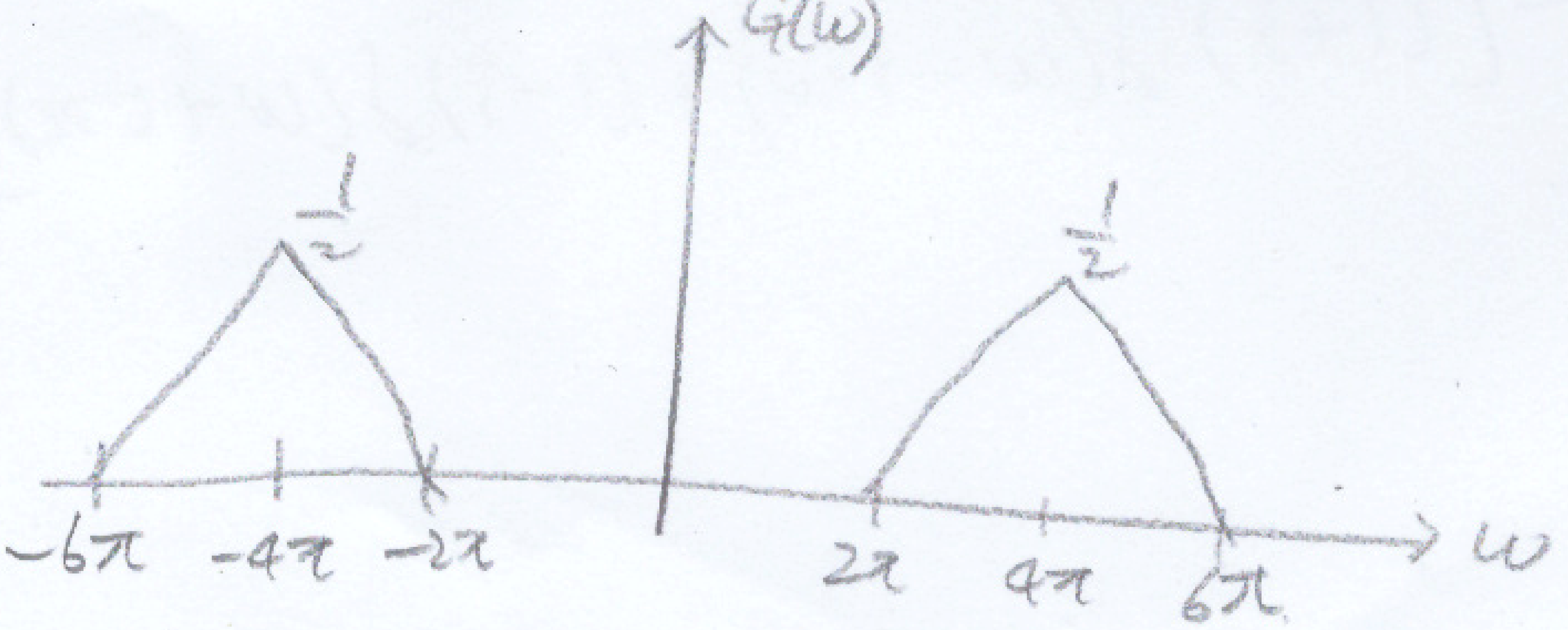
$\Rightarrow$



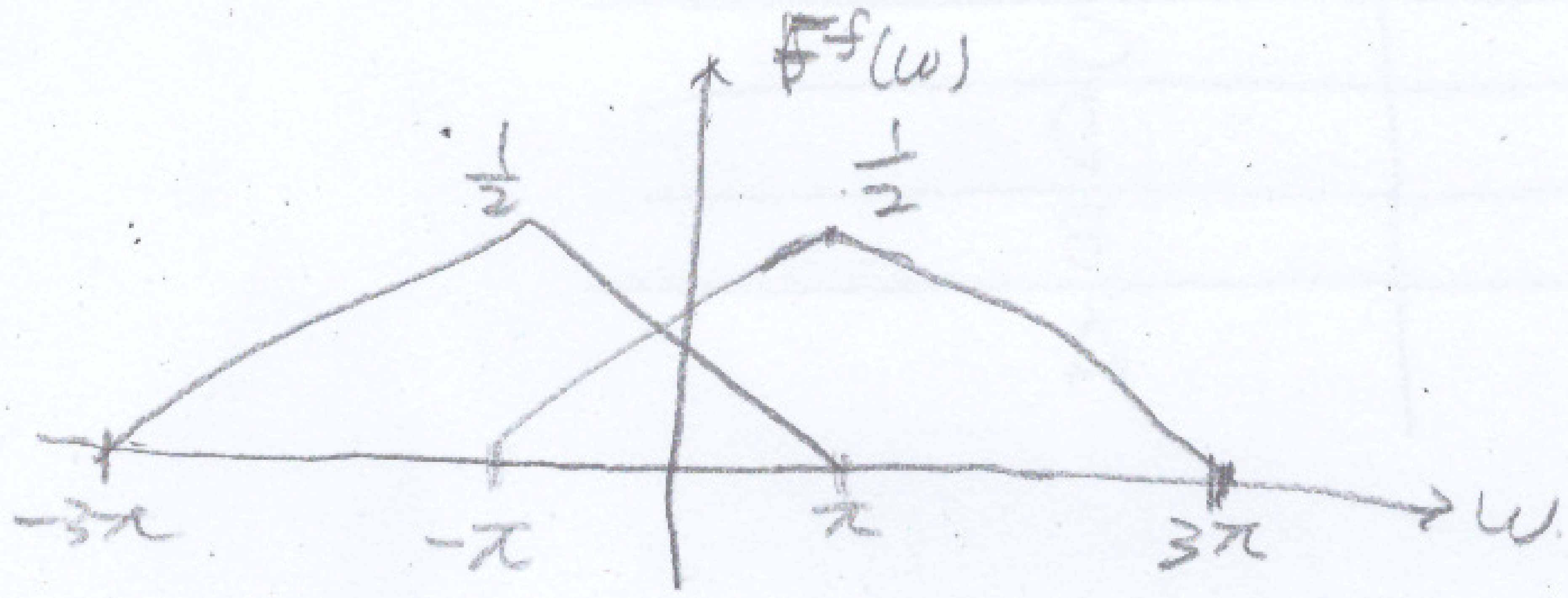
$\therefore \boxed{h(t) = \frac{1}{5}\,\text{sinc}(3t)}$

3.1.19

(a) $g(t) = x(t)\cos(4\pi t) \xrightarrow{F.T} G^f(\omega) = \frac{1}{2}X(\omega - 4\pi) + \frac{1}{2}X(\omega + 4\pi)$



(b) $f(t) = x(t)\cos(\pi t) \xrightarrow{F.T} F^f(\omega) = \frac{1}{2}x(\omega-\pi) + \frac{1}{2}x(\omega+\pi)$



3.1.20

$G^f(\omega) = X^f(\omega - 2\pi) + X^f(\omega + 2\pi)$

$\xrightarrow{F.T^{-1}} g(t) = x(t)\left[e^{j2\pi t} + e^{-j2\pi t}\right]$

$\therefore g(t) = 2\cos(2\pi t)\, x(t)$

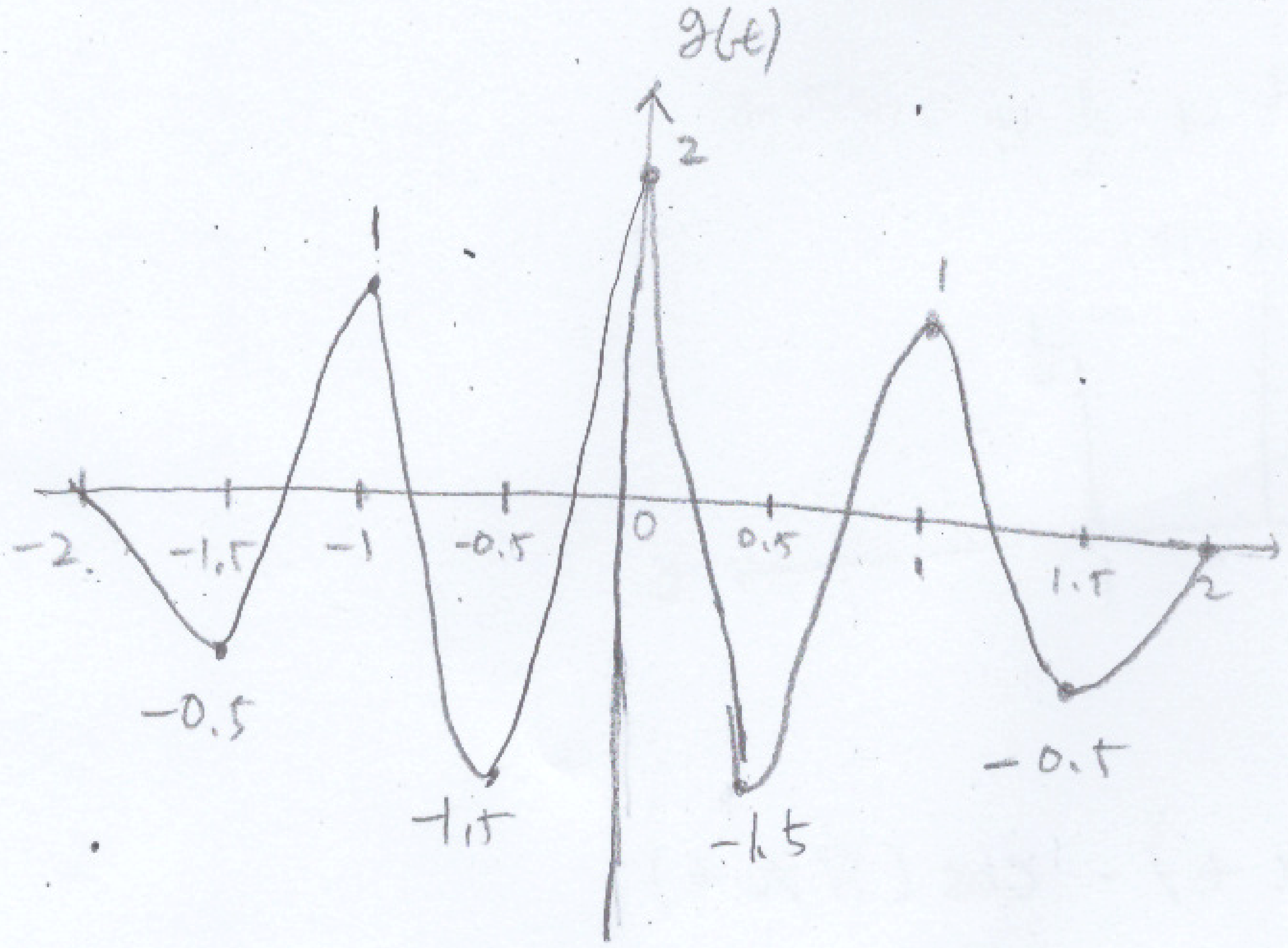
$\Rightarrow t=0,\ g(0) = 2$

$t=1,\ g(1) = 2\cdot\frac{1}{2} = 1$

$t=2,\ g(2) = 2\cdot 0 = 0$

$t=0.5,\ g(0.5) = 2\cdot 1.5(-1) = -1.5$

$t=1.5,\ g(1.5) = 2(0.25)(-1) = -0.5$



3.1.21

Signal	Fourier transform
1	C
2	D
3	B
4	A

3.1.22

$H^f(\omega) = \begin{cases} 0, & |\omega| \le \omega_c \\ 1, & |\omega| > \omega_c \end{cases}$

$H(\omega) = H_2(\omega) - H_1(\omega) \Rightarrow h(t) = h_2(t) - h_1(t)$

$h_1(t) = \dfrac{\sin \omega_c t}{\pi t}$ $\quad h_2 = \delta(t)$

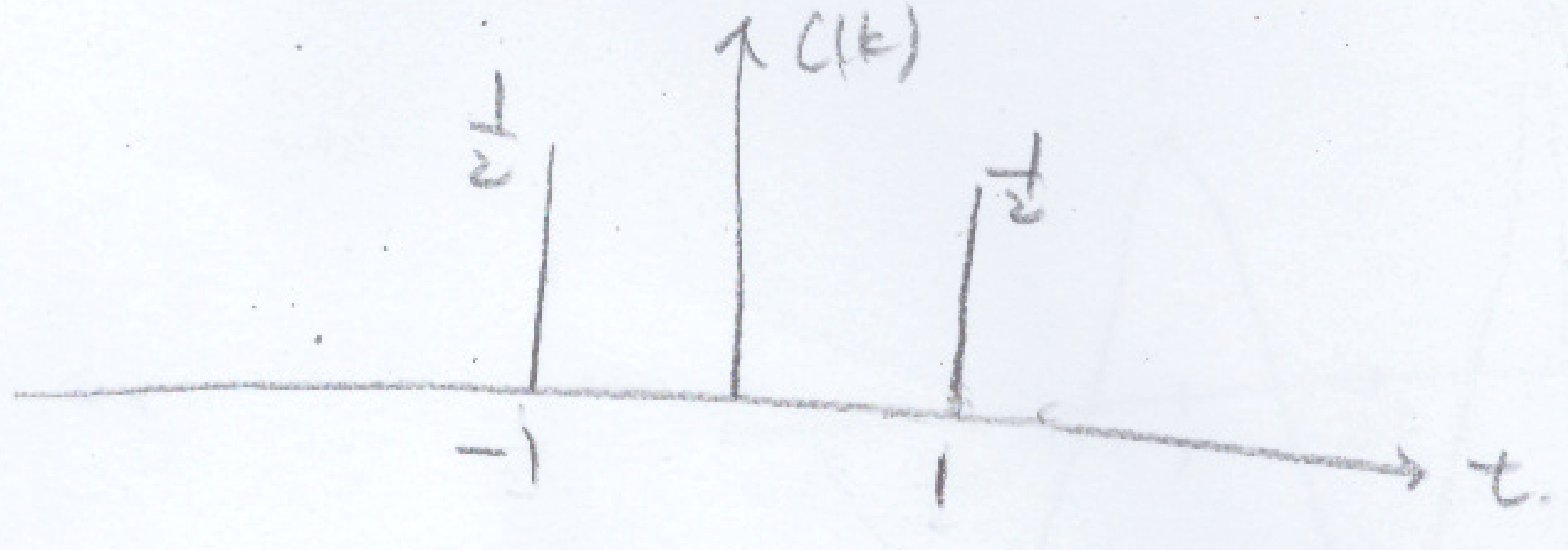
$h(t) = \delta(t) - \dfrac{\sin \omega_c t}{\pi t}$ $\quad \therefore h(t) = \delta(t) - \dfrac{\omega_c}{\pi} \operatorname{sinc}\left(\dfrac{\omega_c}{\pi} t\right)$

3.2.1

$x(t) = \cos(10\pi t)$

$x(t) = \sum_{k=-\infty}^{\infty} C(k) \cdot e^{jk\omega_0 t}$

$\Rightarrow x(t) = \frac{1}{2} e^{j10\pi t} + \frac{1}{2} e^{-j10\pi t}$

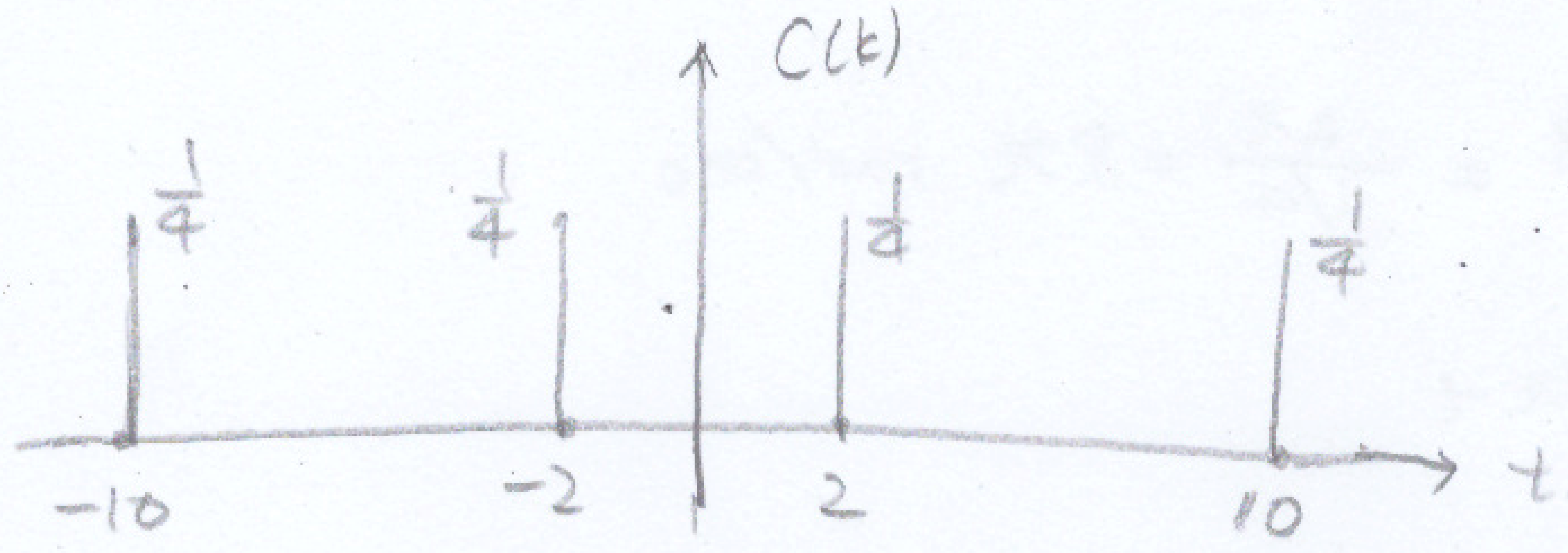


3.2.2

$x(t) = \cos(12\pi t) \cdot \cos(8\pi t)$

$x(t) = \frac{1}{2}[\cos(20\pi t) + \cos(4\pi t)]$

$\Rightarrow x(t) = \frac{1}{4} e^{j20\pi t} + \frac{1}{4} e^{-j20\pi t} + \frac{1}{4} e^{j4\pi t} + \frac{1}{4} e^{-j4\pi t}$

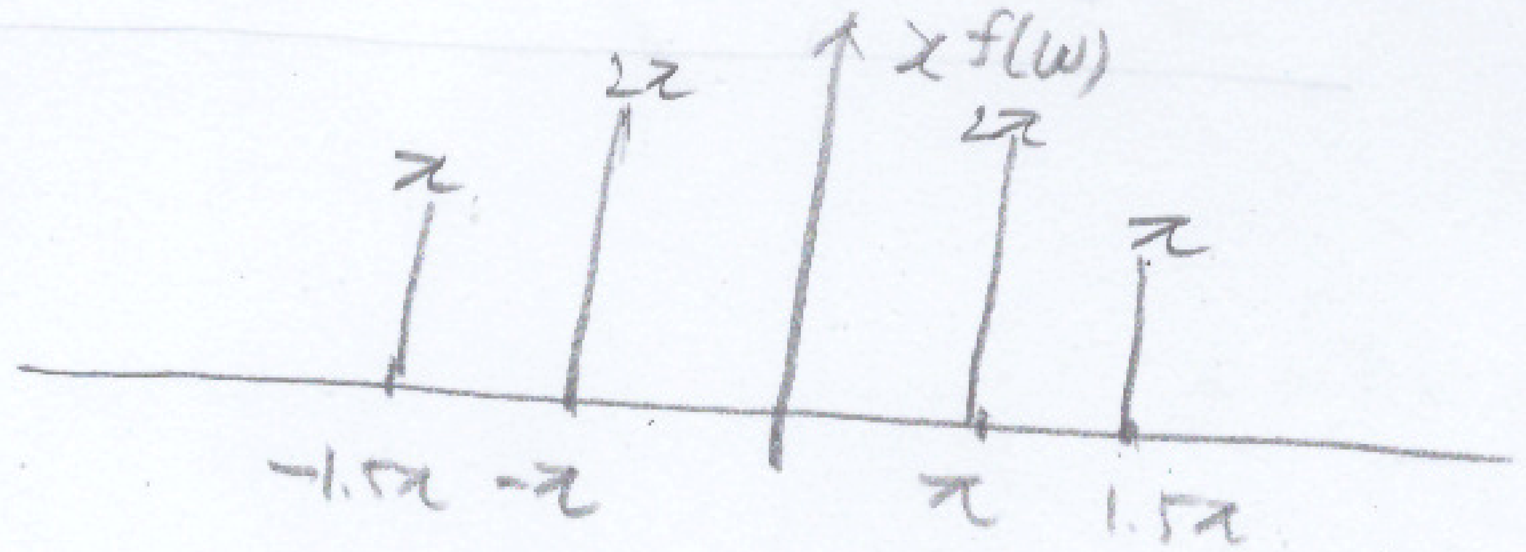


3.2.6

$x(t) = 2\cos(\pi t) + \cos(1.5\pi t)$

a) $x(t) = \left(e^{j\pi t} + e^{-j\pi t}\right) + \frac{1}{2}\left(e^{j1.5\pi t} + e^{-j1.5\pi t}\right)$

$\xrightarrow{F.T} X^f(\omega) = 2\pi\left[\delta(\omega-\pi) + \delta(\omega+\pi)\right] + \pi\left[\delta(\omega-1.5\pi) + \delta(\omega+1.5\pi)\right]$



b) $2\cos(\pi t) \Rightarrow \omega_1 = \pi \qquad T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{\pi} = 2$

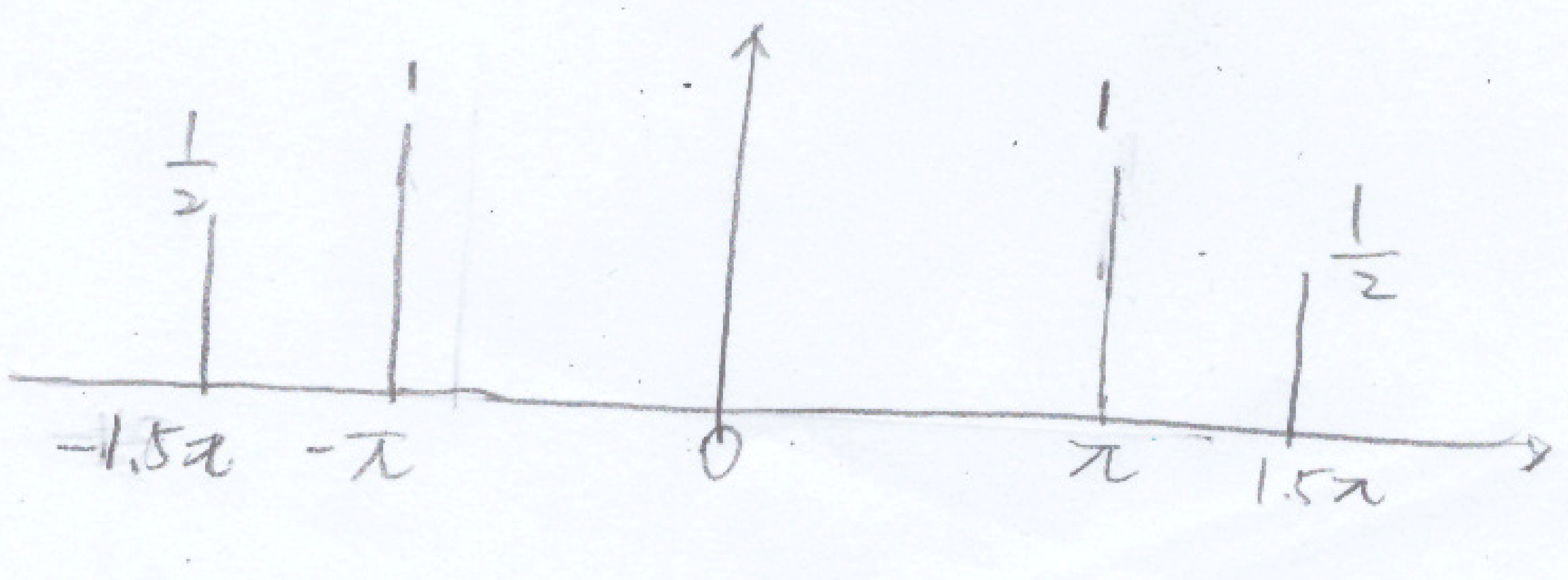
$\cos(1.5\pi t) \Rightarrow \omega_2 = 1.5\pi \qquad T_2 = \frac{2\pi}{1.5\pi} = \frac{4}{3}$

$T_0 = 2 \cdot T_1 = 4$

$\Rightarrow \omega_0 = \frac{2\pi}{T_0} = \frac{2\pi}{4} = \frac{\pi}{2}$

$\therefore C_1 = 1, \ C_2 = 1, \ C_3 = \frac{1}{2}, \ C_4 = \frac{1}{2}, \ \omega_0 = \frac{\pi}{2}$ rad/sec

c)



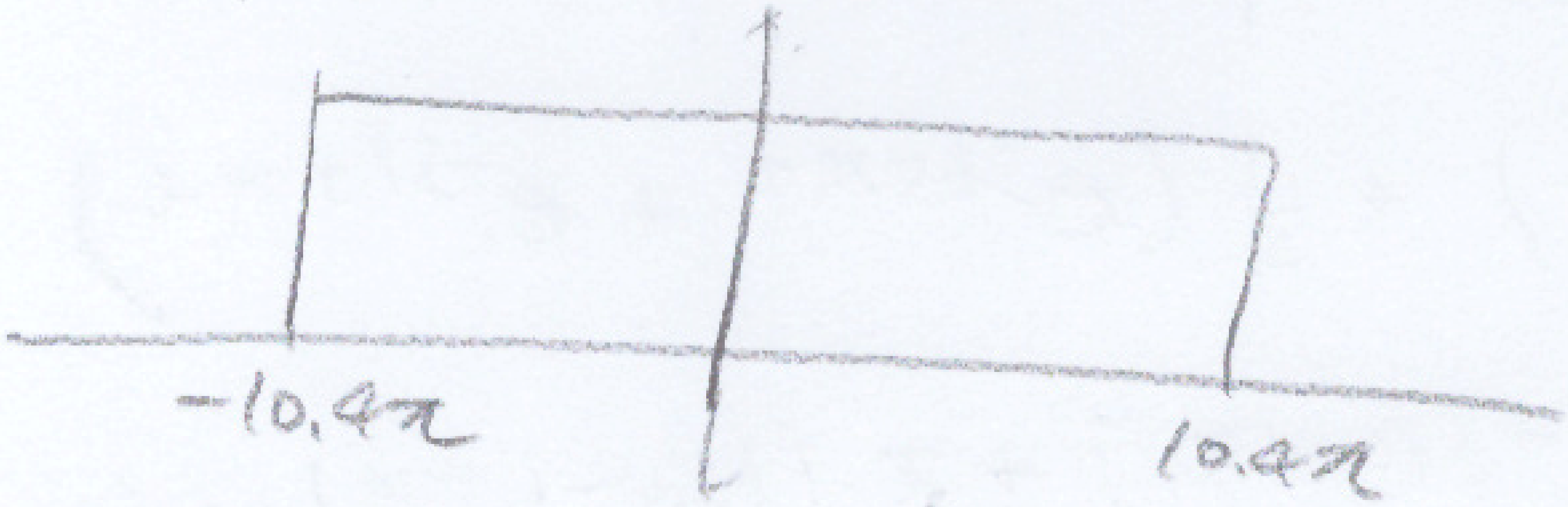
3.2.8

$$C_k = \frac{1}{1+k^2} \qquad w_0 = \frac{2\pi}{T} = \frac{2\pi}{1/4} = 8\pi \text{ rad/sec}$$

$$x(t) = \sum_{k=-\infty}^{\infty} \left(\frac{1}{1+k^2}\right) e^{-jk8\pi t}$$

$f_c = 5.2$ Hz (low - pass filter)

$w_c = 2\pi f_c = 2\pi \times 5.2\text{Hz} = 10.4\text{ Hz}$



$\Rightarrow k = 0, 1, -1$

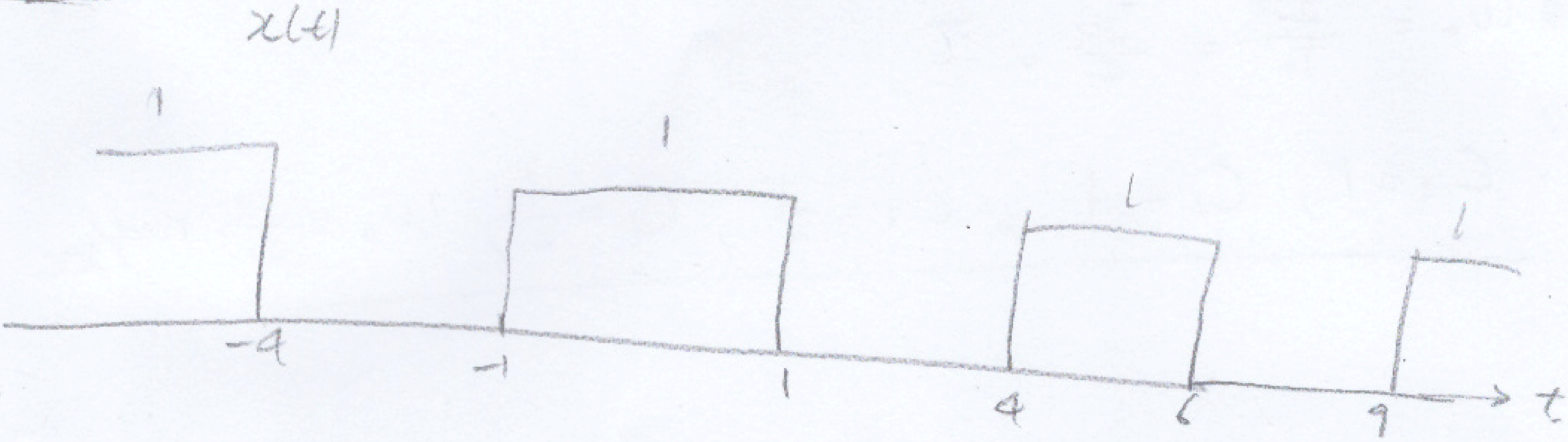
$\Rightarrow$ Output of the filter

$$Y(t) = 1 + \frac{1}{2}e^{j8\pi t} + \frac{1}{2}e^{-j8\pi t}$$

$$= 1 + \cos 8\pi t.$$

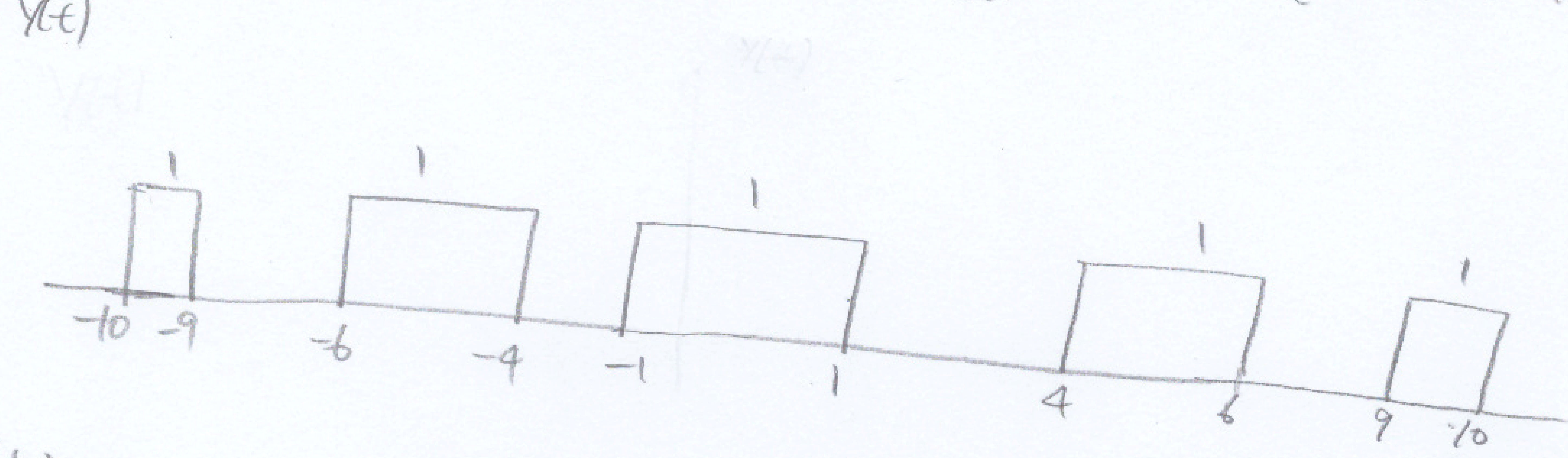
$\therefore Y(t) = 1 + \cos 8\pi t$

3.2.11



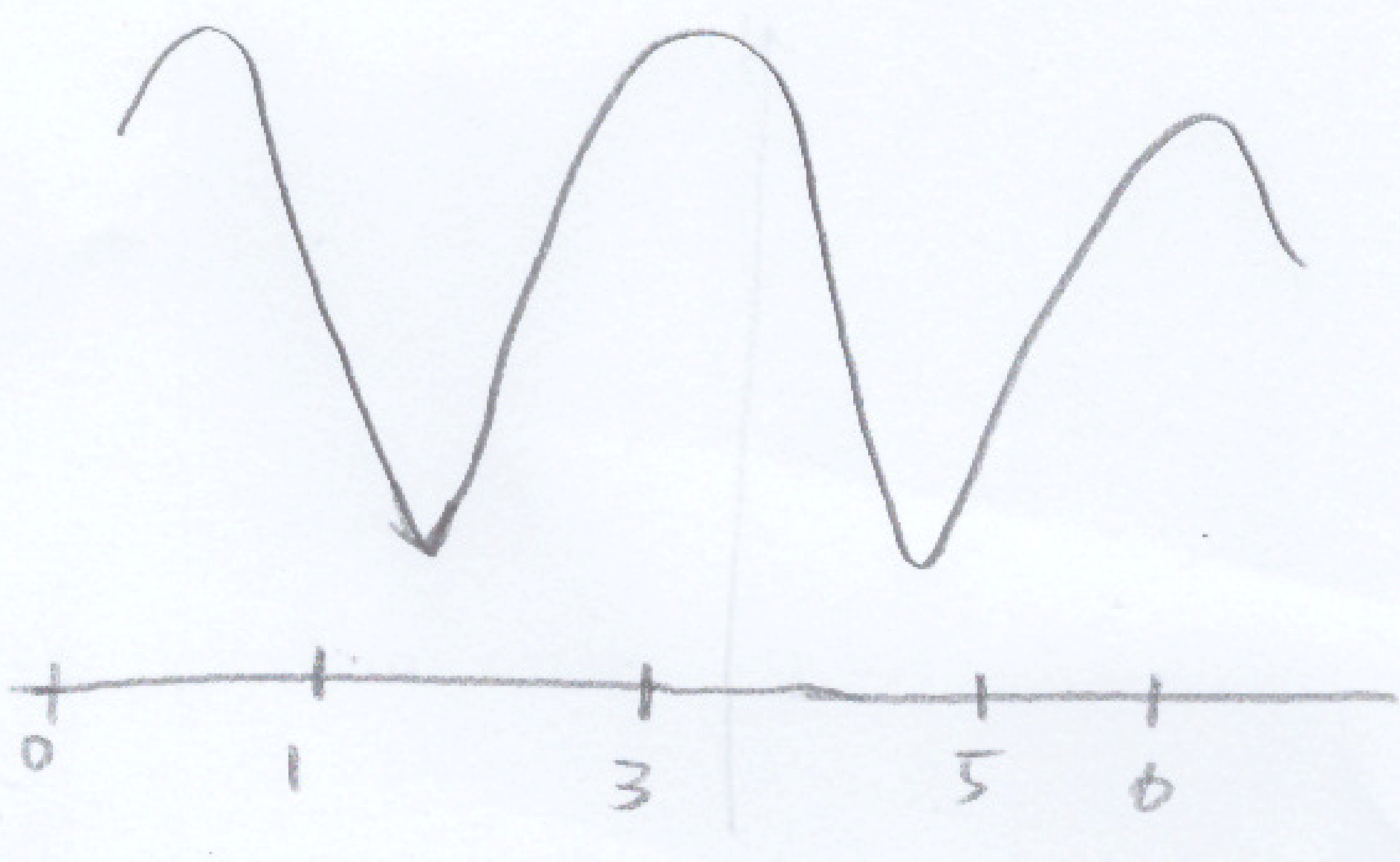
3.2.11

(a) $T = \frac{1}{f}$, $f = 0.1$. $T = \frac{1}{0.1} = 10\,s$ (low-pass filter)



(b) $T = \frac{1}{f}$, $f = 0.1$ $T = 10s$ (high pass filter)

3.2.3



Yes, Fourier series for $N=10$ graph does resemble the signal $x(t)$.