Signals and system HW3 Jaeha Huh

1.4.7

$x(n) = 4\left(\frac{1}{3}\right)^n u(n) - \left(\frac{2}{3}\right)^n u(n)$

$x(n) = k a^n u(n) \Rightarrow X(z) = k\frac{z}{z-a}$, $|z|>|a|$

$X(z) = 4\left(\frac{z}{z-\frac{1}{3}}\right) - \left(\frac{z}{z-\frac{2}{3}}\right)$

$\therefore X(z) = 4\left(\frac{1}{1-\frac{1}{3}z^{-1}}\right) - \left(\frac{1}{1-\frac{2}{3}z^{-1}}\right)$

$\frac{1}{3}z^{-1}$

1.4.8

$x(n) = a^{|n|}$

① $|a| \geq 1$

i) $n \geq 0$: $x(n) = a^n u(n) \Rightarrow \frac{z}{z-a}$, $|z|>a \Rightarrow \underline{|z|>1}$

$n<0$: $x(n) = a^{-n} u(-1-n) \Rightarrow \left(\frac{1}{a}\right)^n u(-1-n)$

$\Rightarrow \frac{-z}{z-\frac{1}{a}}$, $|z| < \frac{1}{a}$ $\underline{|z|<1}$

$\therefore$ Thus, there is no ROC.

Z - Transform does not exist

② $|a|<1$

i) $n \geq 0$: $x(n) = a^n u(n) \Rightarrow \frac{z}{z-a}$, $|z|>a \Rightarrow -1<|z|<1$

$n<0$: $x(n) = a^{-n} u(n) \Rightarrow \frac{-z}{z-\frac{1}{a}}$, $|z| < \frac{1}{a}$

$\therefore$ ROC $= a<|z|<\frac{1}{a}$

$X(z) = \frac{z}{z-a} - \frac{z}{z-\frac{1}{a}} = \boxed{\frac{1}{1-az^{-1}} - \frac{1}{1-\frac{z^{-1}}{a}}}$

1.4.10

$h(n) = 3\left(\frac{2}{3}\right)^n u(n)$ $\quad H(z) = \frac{3z}{z-\frac{2}{3}} = \frac{3}{1-\frac{2}{3}z^{-1}}, \ |z| > \frac{2}{3}$

$x(n) = \left(\frac{1}{2}\right)^n u(n)$ $\quad X(z) = \frac{z}{z-\frac{1}{2}} = \frac{1}{1-\frac{1}{2}z^{-1}}, \ |z| > \frac{1}{2}$

$ROC = |z| > \frac{2}{3}$

$Y(n) = x(n) * h(n)$

$$Y(z) = X(z) \cdot H(z) = \frac{1}{1-\frac{1}{2}z^{-1}} \cdot \frac{3}{1-\frac{2}{3}z^{-1}}$$

$$= \frac{z}{z-\frac{1}{2}} \cdot \frac{3z}{z-\frac{2}{3}} = \frac{2z}{2z-1} \cdot \frac{9z}{3z-2}$$

$$= \frac{18z^2}{(2z-1)(3z-2)} = \left[\frac{A}{(2z-1)} + \frac{B}{(3z-2)}\right] z$$

ex) $(2z-1)B + (3z-2)A = 18z$ $\quad 3zA - 2A + 2zB - B = 18z.$

$$\begin{aligned} 3zA + 2zB &= 18z \\ -2A - B &= 0 \end{aligned} \Rightarrow A = -18, \ B = 36$$

$$= \left[\frac{-18}{2z-1} + \frac{36}{3z-2}\right] z = \frac{-18z}{2z-1} + \frac{36z}{3z-2}$$

$$= \frac{-18}{2-z^{-1}} + \frac{36}{3-2z^{-1}} = \frac{-9}{1-\frac{1}{2}z^{-1}} + \frac{12}{1-\left(\frac{2}{3}\right)z^{-1}}$$

$$\boxed{\therefore Y(n) = -9\left(\frac{1}{2}\right)^n u(n) + 12\left(\frac{2}{3}\right)^n u(n)}$$

1.4.15

$$x(n) \longrightarrow \underset{h_1(n)}{\boxed{\text{sys 1}}} \longrightarrow \underset{h_2(n)}{\boxed{\text{sys 2}}} \longrightarrow y(n)$$

$$h_{total}(n) = h_1(n) * h_2(n) \Rightarrow H(z) = H_1(z) \cdot H_2(z)$$

$$h_1(n) = \delta(n) + 0.5\,\delta(n-1) - 0.5\,\delta(n-2)$$

$$H_1(z) = 1 + 0.5z^{-1} - 0.5z^{-2}$$

$$H_2(z) = z^{-1} + 2z^{-2} + 2z^{-3}$$

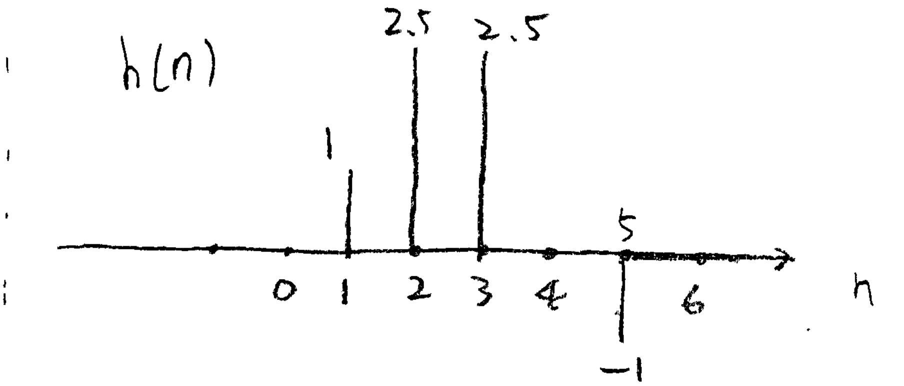
$$H(z) = H_1(z) \cdot H_2(z) = (1 + 0.5z^{-1} - 0.5z^{-2})(z^{-1} + 2z^{-2} + 2z^{-3})$$

$$= z^{-1} + 2z^{-2} + 2z^{-3} + 0.5z^{-2} + z^{-3} + z^{-4}$$

$$- 0.5z^{-3} - z^{-4} - z^{-5}$$

$$= z^{-1} + 2.5z^{-2} + 2.5z^{-3} - z^{-5}$$

a) $h(n) = \delta(n-1) + 2.5\,\delta(n-2) + 2.5\,\delta(n-3) - \delta(n-5)$



b) $H(z) = z^{-1} + 2.5z^{-2} + 2.5z^{-3} - z^{-5}$

1.5.1 (a)

$h(n) = -\delta(n) + 2\left(\frac{1}{2}\right)^n u(n)$

$H(z) = -1 + 2\frac{z}{z-\frac{1}{2}} = -1 + 2\frac{1}{1-\frac{1}{2}z^{-1}}$

$= \frac{-1+\frac{1}{2}z^{-1}+2}{1-\frac{1}{2}z^{-1}} = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}}$

Inverse $H(z) = \frac{1}{H(z)} = \frac{1-\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}}$

$= \frac{1}{1+\frac{1}{2}z^{-1}} - \frac{\frac{1}{2}z^{-1}}{1+\frac{1}{2}z^{-1}} = \frac{1}{1-(-\frac{1}{2})z^{-1}} - \frac{\frac{1}{2}}{1-(-\frac{1}{2})z^{-1}}z^{-1}$

$\therefore$ The inverse of the system

$\boxed{\text{Inverse } h(n) = \left(-\frac{1}{2}\right)^n u(n) - \frac{1}{2}\left(-\frac{1}{2}\right)^{n-1} u(n-1)}$

1.6.1

$y(n) = x(n) + 3x(n-1) + 2x(n-4)$

a): $Y(z) = X(z) + 3z^{-1}X(z) + 2z^{-4}X(z)$.

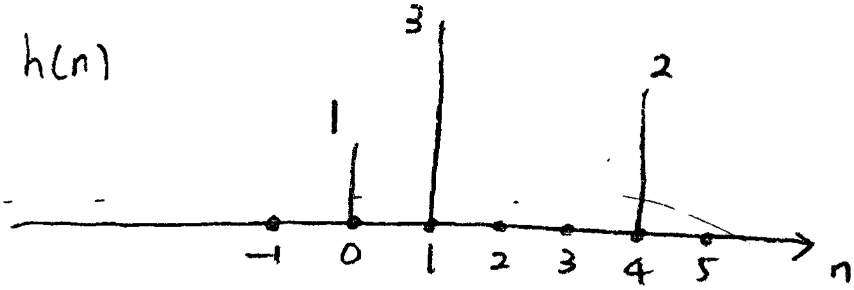
$Y(z) = (1+3z^{-1}+2z^{-4})X(z)$

$\therefore H(z) = \frac{Y(z)}{X(z)} = 1+3z^{-1}+2z^{-4}$

1.6.1 (b)

$$H(z) = 1 + 3z^{-1} + 2z^{-4}$$

$$\Rightarrow h(n) = \delta(n) + 3\delta(n-1) + 2\delta(n-4)$$



1.6.2

$$h(n) = 2\delta(n) + \delta(n-1) + \delta(n-3)$$

a) $\boxed{H(z) = 2 + z^{-1} + z^{-3}}$

b) $H(z) = \dfrac{Y(z)}{X(z)} = 2 + z^{-1} + z^{-3}$

$$Y(z) = 2X(z) + z^{-1}X(z) + z^{-3}X(z)$$

$$\therefore \boxed{y(n) = 2x(n) + x(n-1) + x(n-3)}$$

1.6.3

$$y(n) = x(n) + x(n-1) + 0.5\,y(n-1)$$

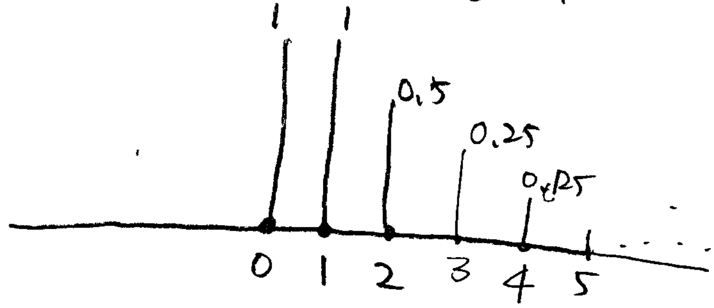
$$Y(z) = X(z) + z^{-1}X(z) + 0.5z^{-1}Y(z)$$

$$(1 - 0.5z^{-1})\,Y(z) = (1 + z^{-1})\,X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-0.5z^{-1}} = \frac{1}{1-0.5z^{-1}} + z^{-1}\frac{1}{1-0.5z^{-1}}$$

$$\boxed{\therefore H(z) = \frac{Y(z)}{X(z)} = \frac{1}{1-0.5z^{-1}} + \frac{1}{1-0.5z^{-1}} z^{-1}}$$

$$h(n) = (0.5)^n u(n) + (0.5)^n u(n-1)$$



1.6.5

$$H_1(z):\ R(z) + \frac{1}{3} z^{-1} R(z) = X(z) + 2z^{-1} X(z)$$

$$R(z)\left(1 + \frac{1}{3} z^{-1}\right) = X(z)\left(1 + 2z^{-1}\right)$$

$$H_1(z) = \frac{R(z)}{X(z)} = \frac{1+2z^{-1}}{1+\frac{1}{3}z^{-1}}$$

$$H_2(z):\ Y(z) + \frac{1}{3} z^{-1} Y(z) = R(z) - 2z^{-1} R(z)$$

$$Y(z)\left(1 + \frac{1}{3} z^{-1}\right) = R(z)\left(1 - 2z^{-1}\right)$$

$$H_2(z) = \frac{Y(z)}{R(z)} = \frac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}}$$

$$H(z) = H_1(z) \cdot H_2(z) = \frac{\cancel{R(z)}}{X(z)} \cdot \frac{Y(z)}{\cancel{R(z)}} = \frac{Y(z)}{X(z)}$$

$$\frac{Y(z)}{X(z)} = \frac{1+2z^{-1}}{1+\frac{1}{3}z^{-1}} \cdot \frac{1-2z^{-1}}{1+\frac{1}{3}z^{-1}} = \frac{1-2z^{-2}}{1+\frac{2}{3}z^{-1}+\frac{1}{9}z^{-2}}$$

$\therefore H(z)$ is stable and causal

1.6.6

H_1: $f(n) = x(n) + x(n-2) + 0.1 f(n-1)$

$$F(z) = X(z) + z^{-2} X(z) + 0.1 z^{-1} F(z)$$

$$F(z)(1 - 0.1 z^{-1}) = (1 + z^{-2}) X(z)$$

$$H_1(z) = \frac{F(z)}{X(z)} = \frac{1+z^{-2}}{1-0.1z^{-1}}$$

H_2: $g(n) = x(n) + x(n-1) + 0.1 g(n-1)$

$$G(z) = X(z) + z^{-1} X(z) + 0.1 z^{-1} G(z)$$

$$G(z)(1 - 0.1 z^{-1}) = (1 + z^{-1}) X(z)$$

$$H_2(z) = \frac{G(z)}{X(z)} = \frac{1+z^{-1}}{1-0.1z^{-1}}$$

$$H(z) = H_1(z) + H_2(z) = \frac{1+z^{-2}}{1-0.1z^{-1}} + \frac{1+z^{-1}}{1-0.1z^{-1}}$$

$$= \frac{2+z^{-1}+z^{-2}}{1-0.1z^{-1}}$$

$$\frac{Y(z)}{X(z)} = \frac{2+z^{-1}+z^{-2}}{1-0.1z^{-1}} \Rightarrow (1-0.1z^{-1}) X(z) = (2+z^{-1}+z^{-2}) Y(z)$$

$$X(z) - 0.1z^{-1}X(z) = 2Y(z) + z^{-1}Y(z) + z^{-2}Y(z)$$

$$x(n) - 0.1x(n-1) = 2y(n) + y(n-1) + y(n-2)$$

$$2y(n) = x(n) - 0.1x(n-1) - y(n-1) - y(n-2)$$

$$\therefore y(n) = \underline{\frac{x(n) - 0.1x(n-1) - y(n-1) - y(n-2)}{2}}$$

1.6.8

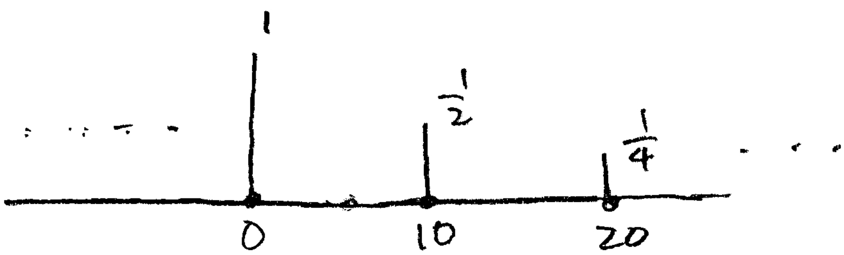
$$y(n) = \sum_{k=0}^{\infty} 2^{-k} x(n-10k)$$

$$Y(z) = \sum_{k=0}^{\infty} 2^{-k} z^{-10k} X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{\infty} 2^{-k} z^{-10k}$$

$$h(n) = \sum_{k=0}^{\infty} 2^{-k} \delta(n-10k).$$

(a)



(b)

$$H(z) = \sum_{k=0}^{\infty} 2^{-k} z^{-10k}$$

(c)

$$y(n) = 1 + 2^{-1} x(n-10) + 2^{-2} x(n-20) + \cdots$$