Signals and System (HW 2) Jaeha Huh

1.2.1

(a) $y(n) = \cos(x(n))$.

→ the output depends on only the input at that same time. : memoryless.

→ it is memoryless system so it is causal system

→ $|y(n)| = |\cos(x(n))|$

if $|x(n)| < A$, then $|y(n)| < A$. : stable

$\boxed{\therefore \text{ memoryless, casual, and BIBO stable}}$

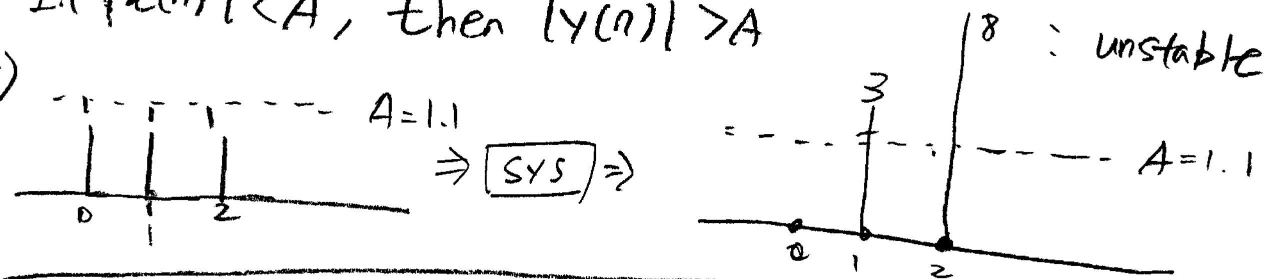
(b) $y(n) = 2n^2 x(n) + n x(n+1)$

→ the output does not depends on same time input signal : memory

→ the output depends on future value : Non causal

→ If $|x(n)| < A$, then $|y(n)| > A$: unstable.

ex)



$\boxed{\therefore \text{ with memory, non causal, and BIBO unstable}}$

(c) $y(n) = \max\{x(n), x(n+1)\}$

→ the output does not depends on same time input signal : memory

→ the output depends on future valuse : Non causal.

→ If $|x(n)| < A$, then $|y(n)| < A$: stable.

$\boxed{\therefore \text{ with memory, non causal, and BIBO stable.}}$

(d) $y(n) = \begin{cases} x(n) & \text{when } n \text{ is even} \\ x(n-1). & \text{when } n \text{ is odd} \end{cases}$

→ the output does not depends on same time input ∴ memory.

→ the output depends only on past and present input.
∴ causal.

→ 모든 $|x(n)| < A$, the $|y(n)| < A$ ∴ stable

$\boxed{\therefore \text{ with memory, causal, BIBO stable}}$

1.2.2

(c) i. $y(n) = 0.5\,x(2n) + 0.5\,x(2n-1)$.

The output depends on past and present inputs

$\boxed{\text{Thus, it is causal system}}$

1.2.4

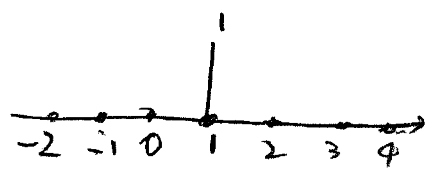
(d) $y(n) = x(n) + [x(n+1)]^2$

The output depends on future input..

$\boxed{\text{Thus, it is non causal system}}$

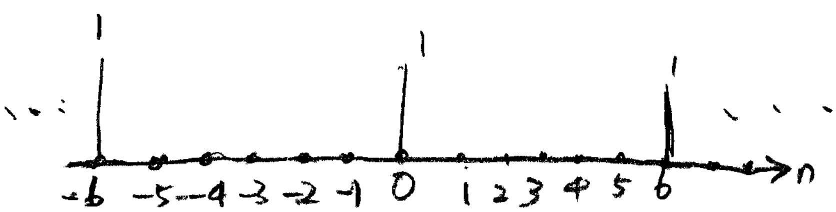
1.2.15

$h(n) = \delta(n-1)$

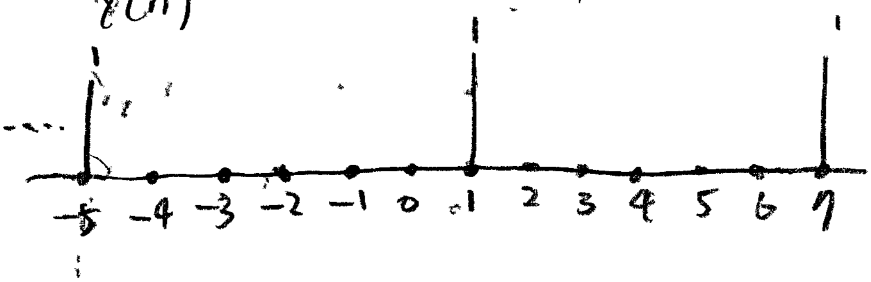


h(n).

(a) $x(n) = \sum_{k=-\infty}^{\infty} \delta(n-6k)$



y(n)



≡ output signal

1.2.15

(b) $|x(n)| \leq A$, $A = 1$. $|y(n)| \leq 1$

$\boxed{\text{Thus, The system is BIBO stable}}$

1.3.1

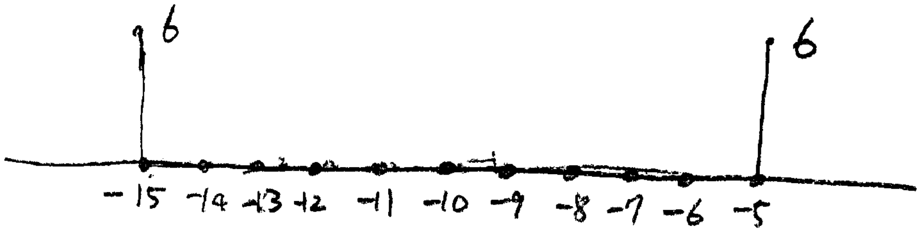
(a) $f(n) = 2\delta(n+10) + 2\delta(n-10)$

$g(n) = 3\delta(n+5) + 3\delta(n-5)$

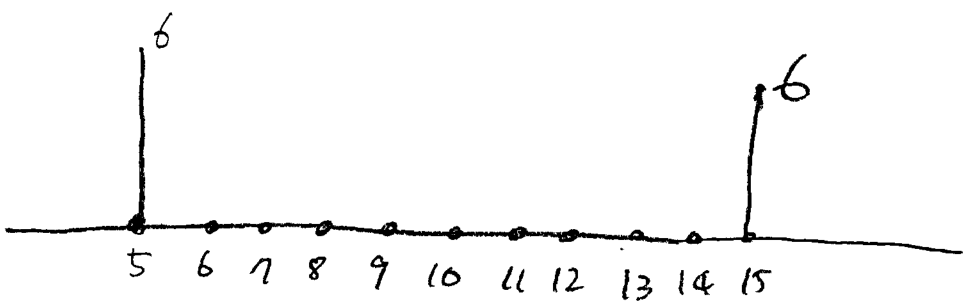
$(f*g)(n) = \sum_{k=-\infty}^{\infty} f(k)\, g(n-k)$

$= \cdots + f(-10)\, g(n+10) + \cdots + f(10)\, g(n-10) + \cdots$

$x(-10) = f(-10)\, g(n+10) = 2\, g(n+10)$

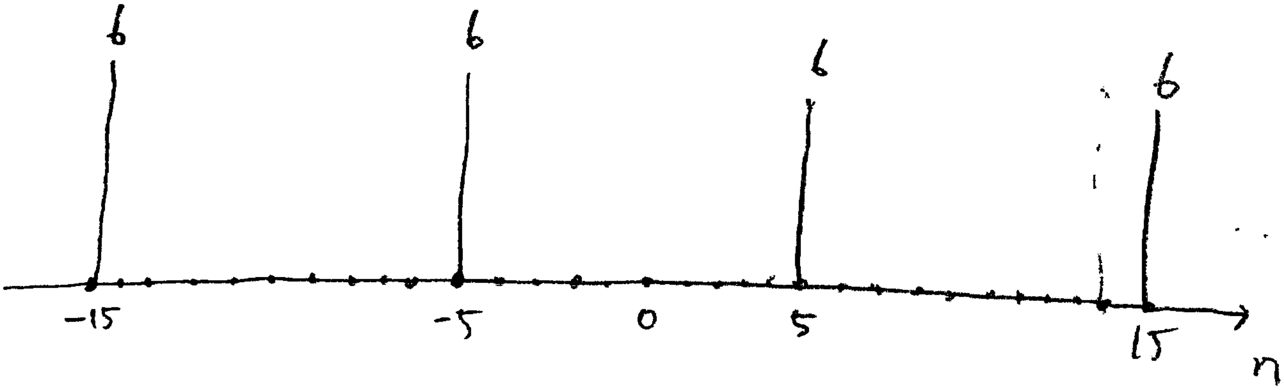


$x(10) = f(10)\, g(n-10) = 2\, g(n-10)$



$x(n) = x(-10) + x(10)$.

$\therefore x(n)$



(b) $f(n) = \delta(n-4) - \delta(n-1)$

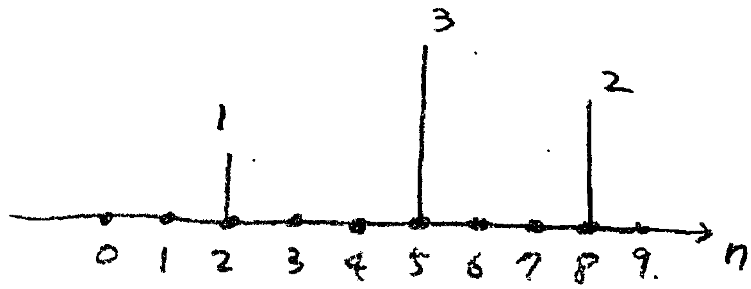
$g(n) = 2\delta(n-4) - \delta(n-1)$

$x(n) = (f * g)(n)$.

$f(n) = [\underset{n=0}{\underline{0}}\ 1\ 0\ 0\ 1]$ $\quad g(n) = [\underset{n=0}{\underline{0}}\ 1\ 0\ 0\ 2]$

$$\begin{array}{r}
0\,1\,0\,0\,1 \\
\times\ 0\,1\,0\,0\,2 \\
\hline
0\,0\,0\,0\,0 \\
0\,1\,0\,0\,1 \\
0\,0\,0\,0\,0 \\
0\,0\,0\,0\,0 \\
+\ 0\,2\,0\,0\,2 \\
\hline
0\,0\,1\,0\,0\,3\,0\,0\,2
\end{array}$$

$\therefore$ $x(n)$

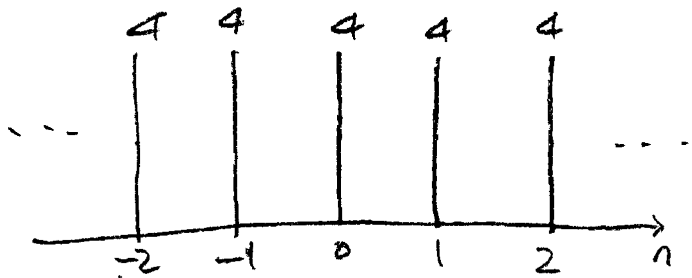


(d) $f(n) = 4$

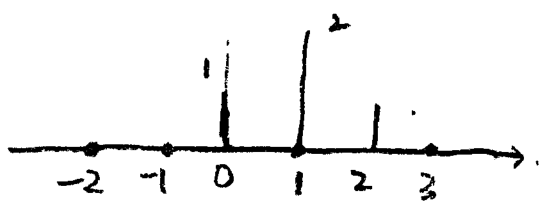
$g(n) = \delta(n) + 2\delta(n-1) + \delta(n-2)$

$x(n) = (f * g)(n) = \sum_{k=-\infty}^{\infty} f(k)\, g(n-k)$

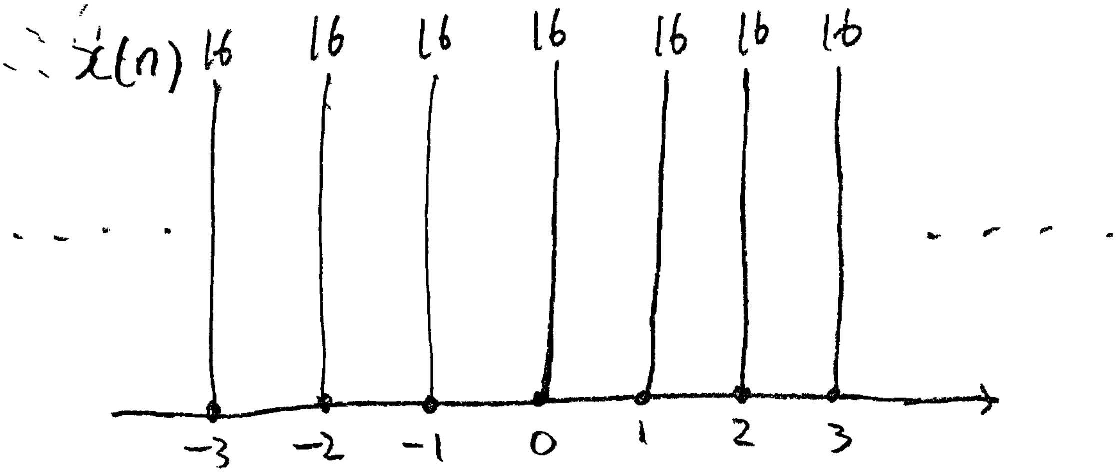
$f(n) = 4$



$g(n)$



$\therefore x(n)$

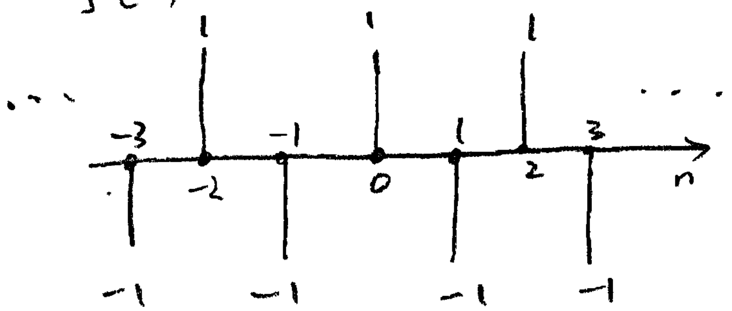


(f) $f(n) = -1^{(n)}$

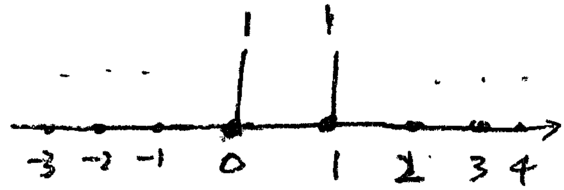
$g(n) = \delta(n) + \delta(n-1)$

$x(n) = (f * g)(n) = \sum_{k=-\infty}^{\infty} f(k)\, g(n-k)$.

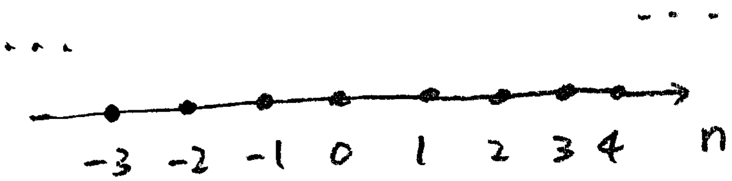
$f(n)$



$g(n)$

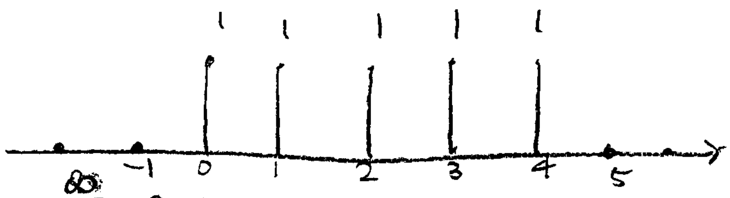


$\therefore x(n)$

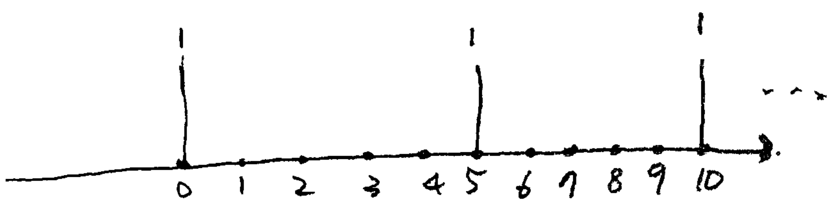


1.3.2

$h(n) = u(n) - u(n-5)$

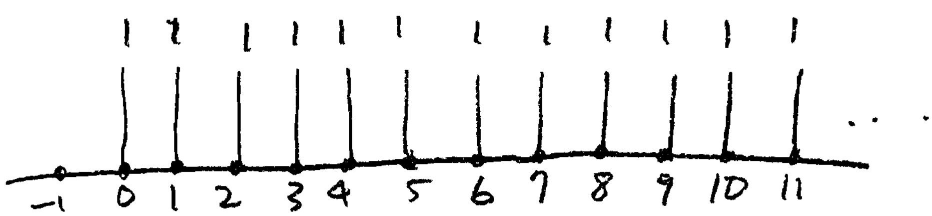


$x(n) = \sum_{k=0}^{\infty} \delta(n-5k)$

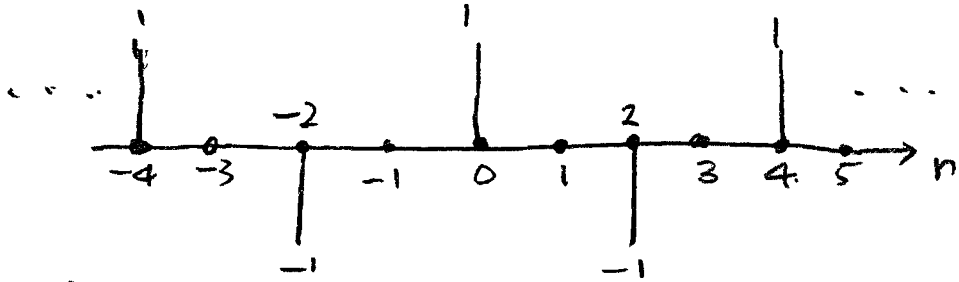


$Y(n) = x(0)h(n) + x(5)\, h(n-5) + x(10)\, h(n-10) + \cdots$

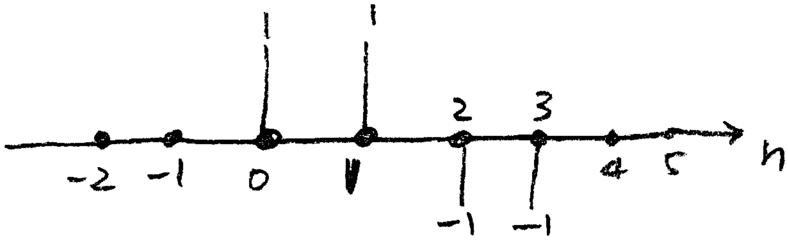
$\therefore Y(n)$



1.3.3 $f(n) = \cos\left(\frac{\pi}{2} n\right)$



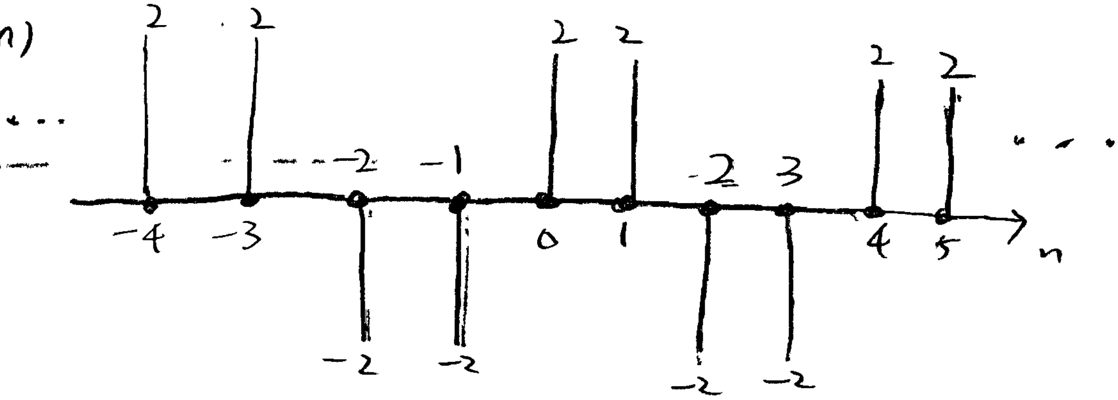
$g(n)$



$$x(n) = (f * g)(n) = \sum_{k=-\infty}^{\infty} f(k)\, g(n-k).$$

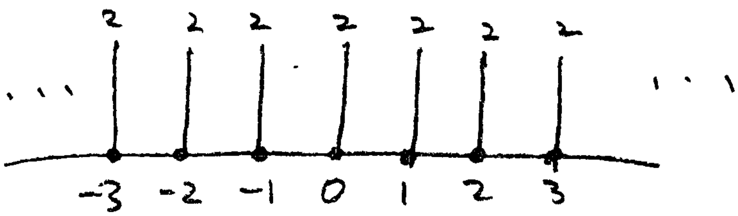
$$x(n) = \cdots + f(-4)\, g(n+4) + f(-2)\, g(n+2) + f(0)\, g(n) + \cdots$$

$\therefore x(n)$

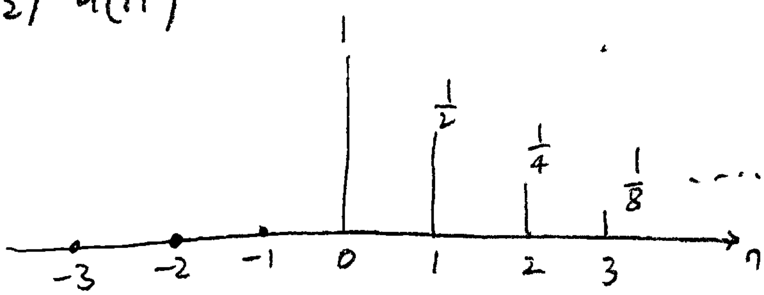


1.3.4

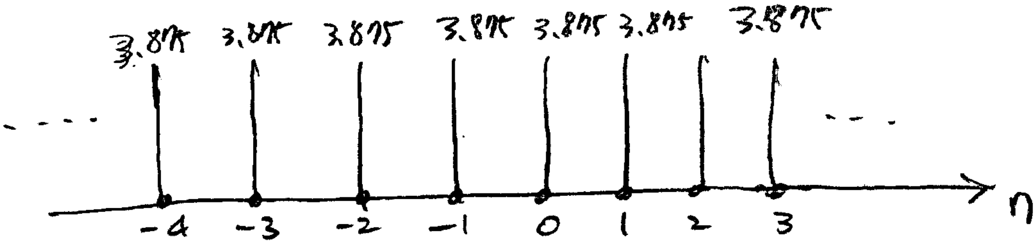
$f(n) = 2$



$g(n) = \left(\frac{1}{2}\right)^n u(n)$



$x(n) = (f * g)(n)$



1.3.5

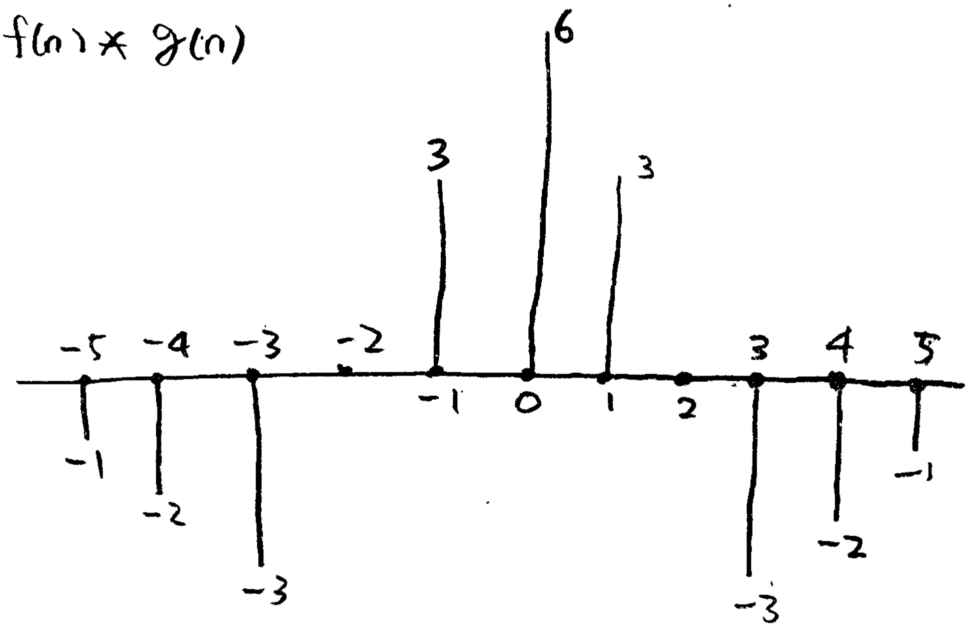
$f(n) = \delta(n+2) + 2\delta(n+1) + 3\delta(n) + 2\delta(n-1) + \delta(n-2)$

$g(n) = -\delta(n+3) + 2\delta(n) - \delta(n-3)$

$[1 \quad 2 \quad \underset{n=0}{\underline{3}} \quad 2 \quad 1] * [-1 \quad 0 \quad 0 \quad \underset{n=0}{\underline{2}} \quad 0 \quad 0 \quad -1]$

-1	-2	-3	-2	-1						
	0	0	0	0	0					
		0	0	0	0	0				
			2	4	6	4	2			
				0	0	0	0	0		
					0	0	0	0	0	
						-1	-2	-3	-2	-1
-1	-2	-3	0	3	6	3	0	-3	-2	-1

$x(n) = f(n) * g(n)$



1.3.9

x=[1 4 2 5]; h= [1 3 -1 2];

According to the results, two Matlab codes, conumtx(h',4) * X' and conv(h,x)', execute Convolution.

The form of the convolution matrix is

```
>> conumtx(h',4)
ans =
   1  0  0  0
   3  1  0  0
  -1  3  1  0
   2 -1  3  1
   0  2 -1  3
   0  0  2 -1
   0  0  0  2
```

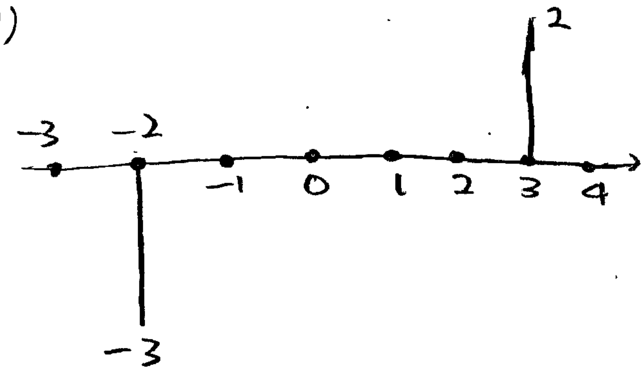
So, conumtx(h',4) * X

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -1 & 3 & 1 & 0 \\ 2 & -1 & 3 & 1 \\ 0 & 2 & -1 & 3 \\ 0 & 0 & 2 & -1 \\ 0 & 0 & 0 & 2 \end{bmatrix} \times \begin{bmatrix} 1 \\ 4 \\ 2 \\ 5 \end{bmatrix} = \begin{matrix} 1 \\ 7 \\ 13 \\ 9 \\ 21 \\ -1 \\ 10 \end{matrix}$$

That is why conumtx(h',4) and conv(h,x') do same thing

1.4.1 $X(z) = -3z^2 + 2z^{-3}$

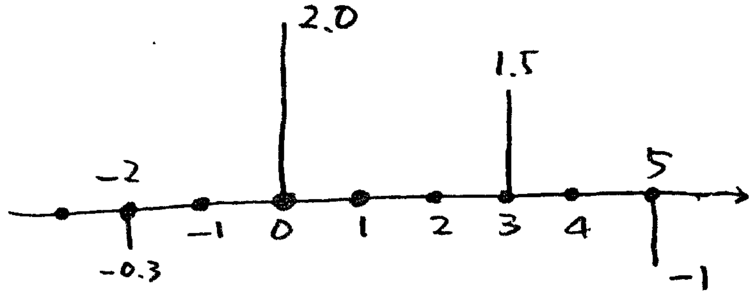
$x(n)$



1.4.2

$x(n) = -0.3\,\delta(n+2) + 2.0\,\delta(n) + 1.5\,\delta(n-3) - \delta(n-5)$

(a) $x(n)$

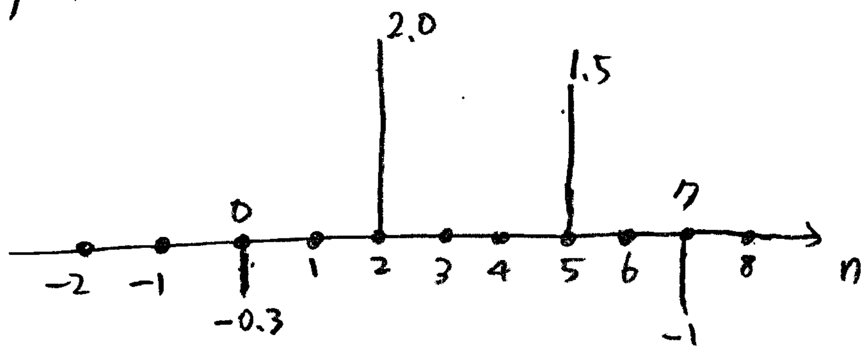


(b) $X(z) = -0.3z^2 + 2.0 + 1.5z^{-3} - z^{-5}$

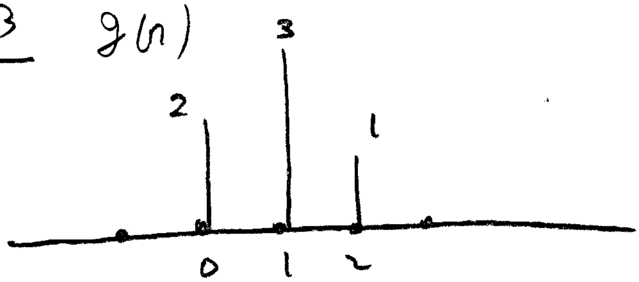
(c) $G(z) = z^{-2}X(z)$.

$G(z) = -0.3 + 2.0z^{-2} + 1.5z^{-5} - z^{-7}$

$\therefore g(n)$



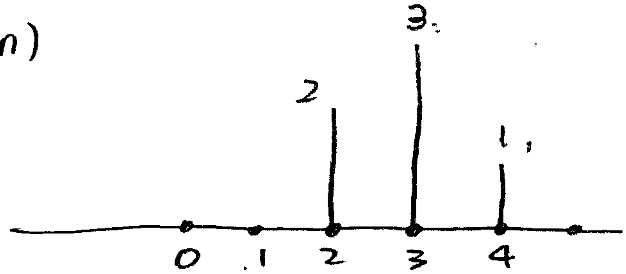
1.4.3 $g(n)$



$$G(z) = 2 + 3z^{-1} + z^{-2}$$
$$= 2 + \frac{3}{z} + \frac{1}{z^2}$$

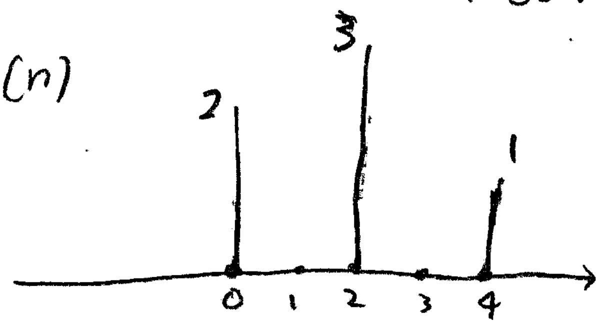
a) $X(z) = z^{-2}G(z)$ $\quad X(z) = 2z^{-2} + 3z^{-3} + z^{-4}$

$x(n)$



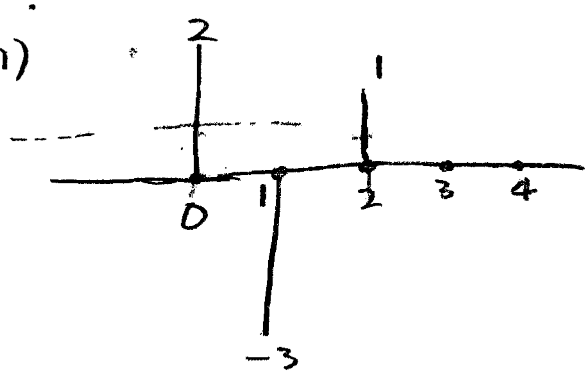
b) $F(z) = G(z^2)$ $\quad F(z) = 2 + \frac{3}{(z)^2} + \frac{1}{(z^2)^2} = 2 + 3z^{-2} + z^{-4}$

$f(n)$



c) $V(z) = G(-z)$ $\quad V(-z) = 2 + \frac{3}{(-z)} + \frac{1}{(-z)^2} = 2 - 3z^{-1} + z^{-2}$

$v(n)$



1.4.5

$$X(z) = (1+2z)(1+3z^{-1})(1-z^{-1})$$

$$= (1+2z)(1-z^{-1}+3z^{-1}-3z^{-2})$$

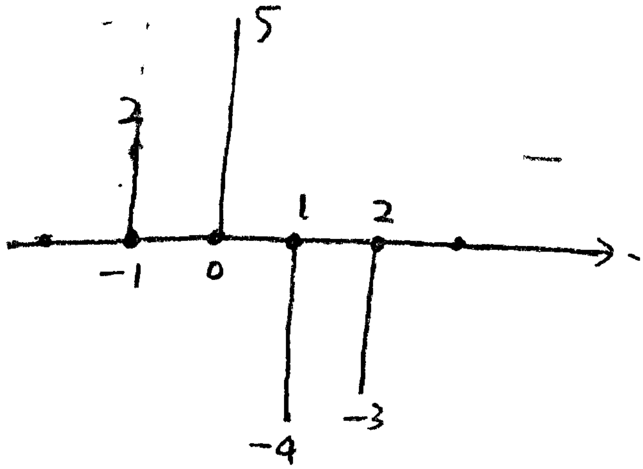
$$= (1+2z)(1+2z^{-1}-3z^{-2})$$

$$= 1+2z^{-1}-3z^{-2}+2z+4-6z^{-1})$$

$$= 2z+5-4z^{-1}-3z^{-2}$$

$$X(z) = 2z+5-4z^{-1}-3z^{-2}$$

$\therefore\ x(n)$



1.4.12

$$H_1(z) = 1+2z^{-1}+z^{-2}$$

$$H_2(z) = 1+z^{-1}+z^{-2}$$

Let $x(n) = \delta(n)$, so $x(z) = 1$

a)

$$x(z) \longrightarrow \boxed{H_1(z)} \longrightarrow \boxed{H_2(z)} \longrightarrow Y(z).$$

$$\uparrow$$
$$r(z)$$

$\Rightarrow r(z) = x(z) \times H_1(z)$

$Y(z) = r(z) \times H_2(z) = X(z) \times \boxed{H_1(z) \times H_2(z)}$

$\Rightarrow \boxed{H_{Tot}(z) = H_1(z) \times H_2(z).}$

$H_{Tot}(z) = (1 + 2z^{-1} + z^{-2})(1 + z^{-1} + z^{-2})$

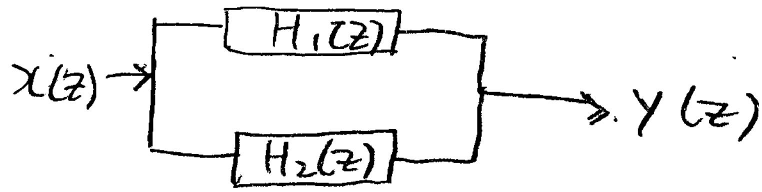
$= 1 + z^{-1} + z^{-2} + 2z^{-1} + 2z^{-2} + 2z^{-3} + z^{-2} + z^{-3} + z^{-4}$

$= 1 + 3z^{-1} + 4z^{-2} + 3z^{-3} + z^{-4}$

$\therefore H_{tot}(z) = 1 + 3z^{-1} + 4z^{-2} + 3z^{-3} + z^{-4}$

$H_{Tot}(n) = \delta(n) + 3\delta(n-1) + 4\delta(n-2) + 3\delta(n-3) + \delta(n-4)$

(b)



$Y(z) = X(z)(H_1(z) + H_2(z))$

$\Rightarrow \boxed{H_{Tot}(z) = H_1(z) + H_2(z)}$

$H_{Tot}(z) = (1 + 2z^{-1} + z^{-2}) + (1 + z^{-1} + z^{-2})$

$= 2 + 3z^{-1} + 2z^{-2}$

$\therefore H_{Tot}(z) = 2 + 3z^{-1} + 2z^{-2}$

$H_{Tot}(n) = 2\delta(n) + 3\delta(n-1) + 2\delta(n-2)$