

1. Second order $\rightarrow H(z) = \frac{z^2 + \dots}{z^2 + \dots}$

$$\textcircled{1} \quad h\left(\frac{z}{2}\right) = 0$$

$$\textcircled{2} \quad (-1)^n \rightarrow \frac{1}{3}(-1)^n \quad \cos(n\pi) = (-1)^n \rightarrow \pm \pi$$

$\textcircled{3}$ real

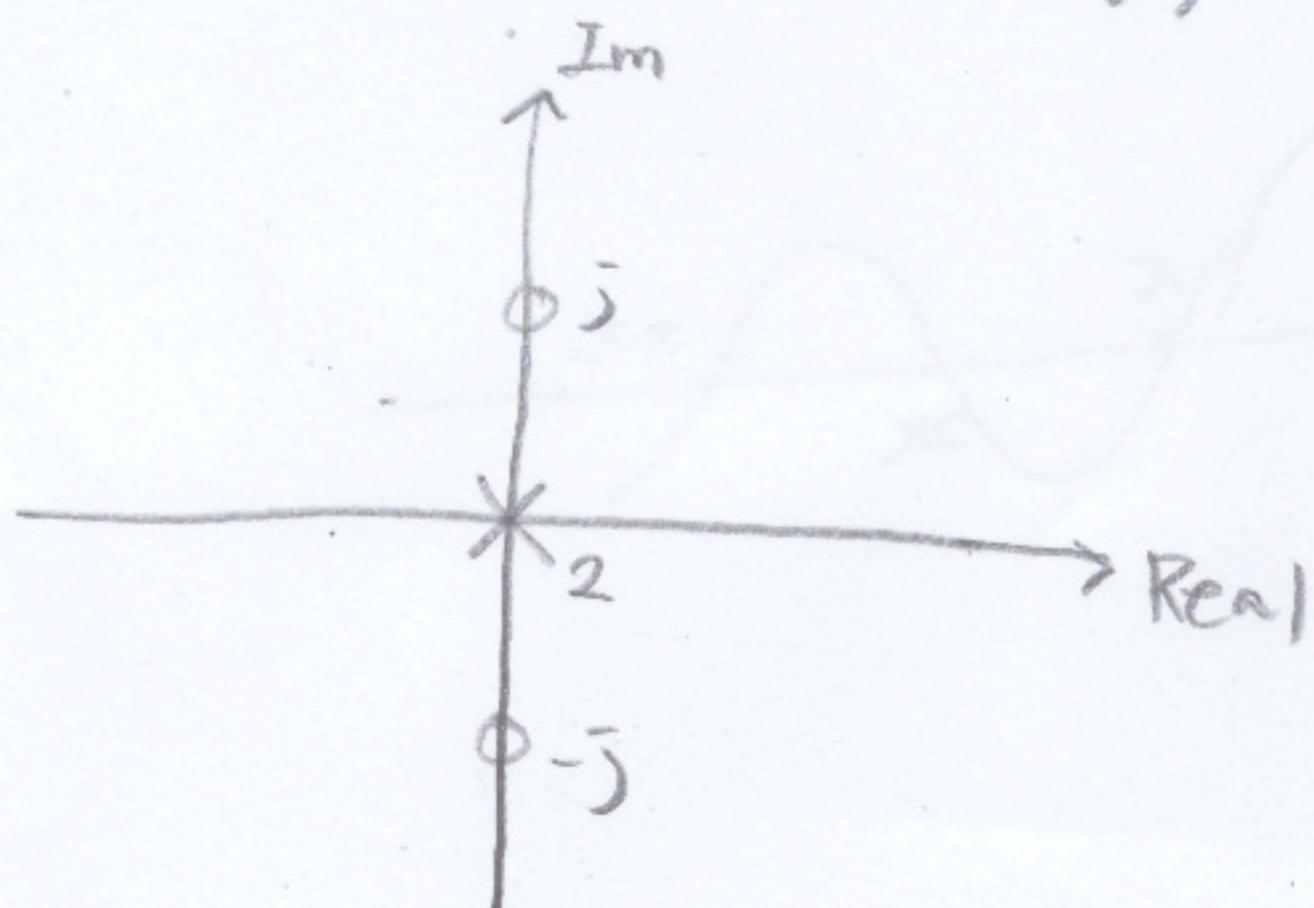
$$H(z) = \frac{z^2 + 1}{6z^2} = \frac{1 + z^{-2}}{6}$$

$$\frac{Y(z)}{X(z)} = \frac{1 + z^{-2}}{6} \quad \leftarrow 6Y(z) = (1 + z^{-2})X(z)$$

$$(a) \quad 6y(n) = x(n) + x(n-2)$$

$$(b) \quad \text{Poles: } z^2 = 0 \quad z = 0, 0$$

$$\text{Zeros: } z^2 + 1 \quad z = j, -j$$



$$2. x(t) = \cos(\pi t) \sin(4\pi t)$$

$$x(t) = \frac{1}{2} [\sin(4\pi t + \pi t) + \sin(4\pi t - \pi t)]$$

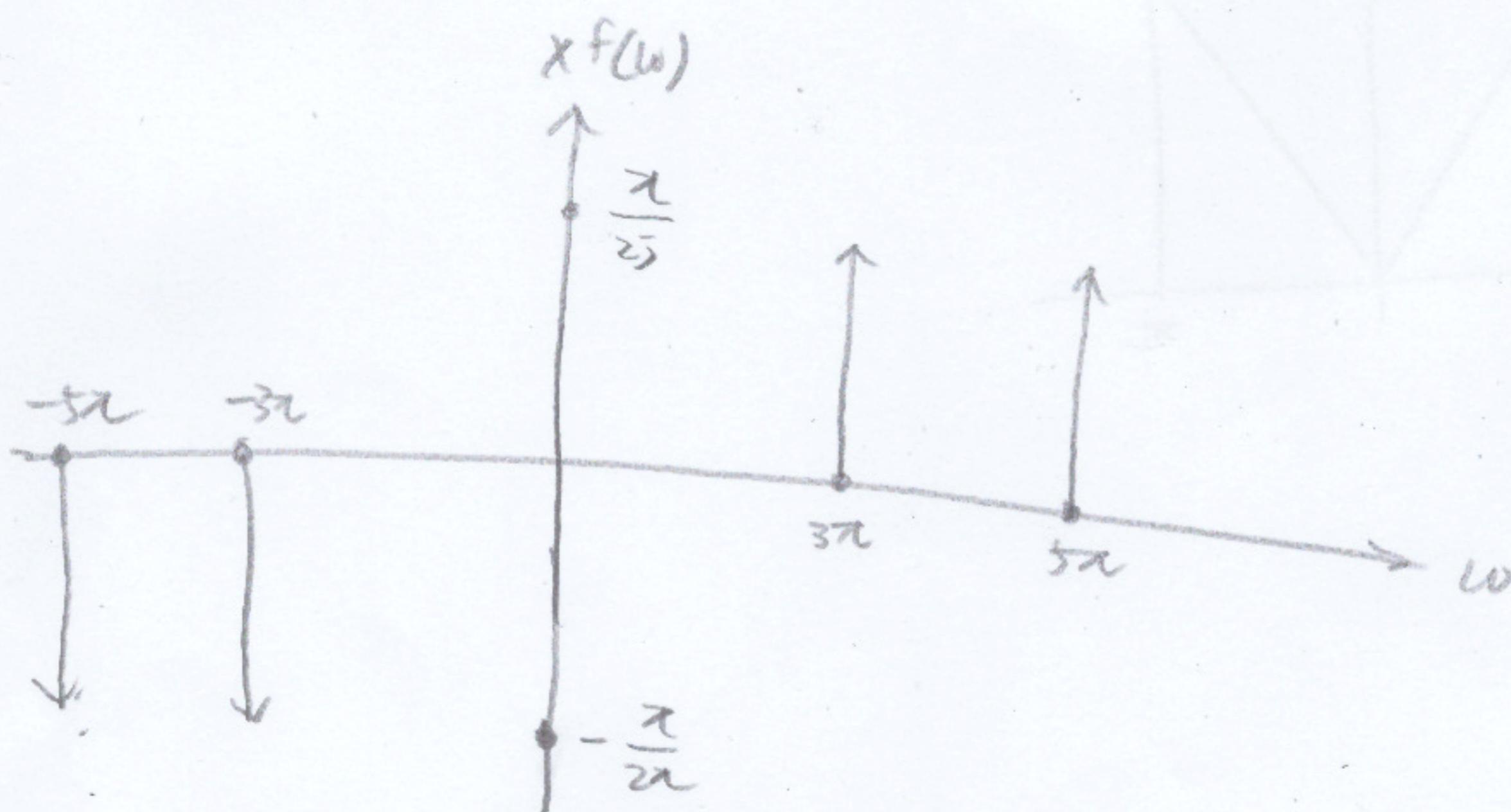
$$x(t) = \frac{1}{2} [\sin(5\pi t) + \sin(3\pi t)]$$

$$\sin(5\pi t) \xrightarrow{\text{F.T.}} \frac{\pi}{j} [\delta(\omega + 5\pi) - \delta(\omega - 5\pi)] = x^f(\omega),$$

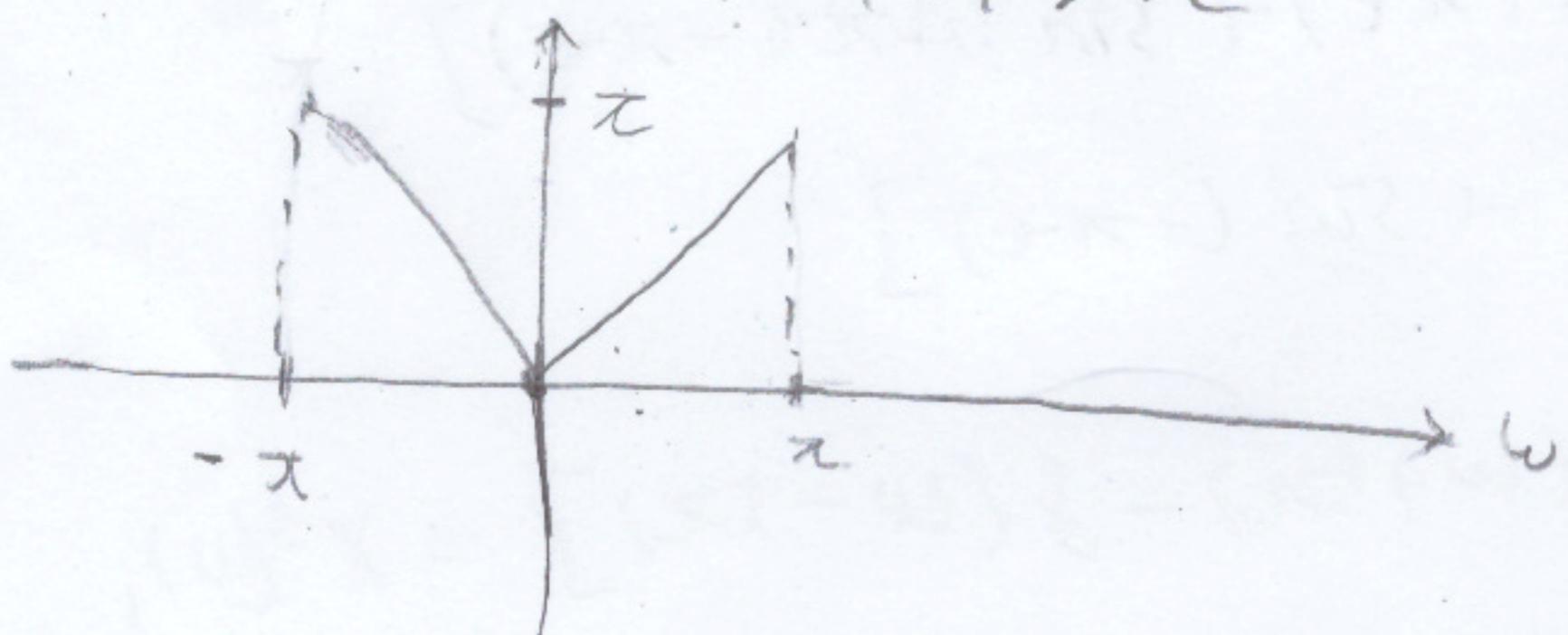
$$\Rightarrow x^f(\omega) = x^f(\omega)_1 \cdot x^f(\omega)_2$$

$$x^f(\omega) = \frac{\pi}{2j} \delta(\omega + 5\pi) - \frac{\pi}{2j} \delta(\omega - 5\pi)$$

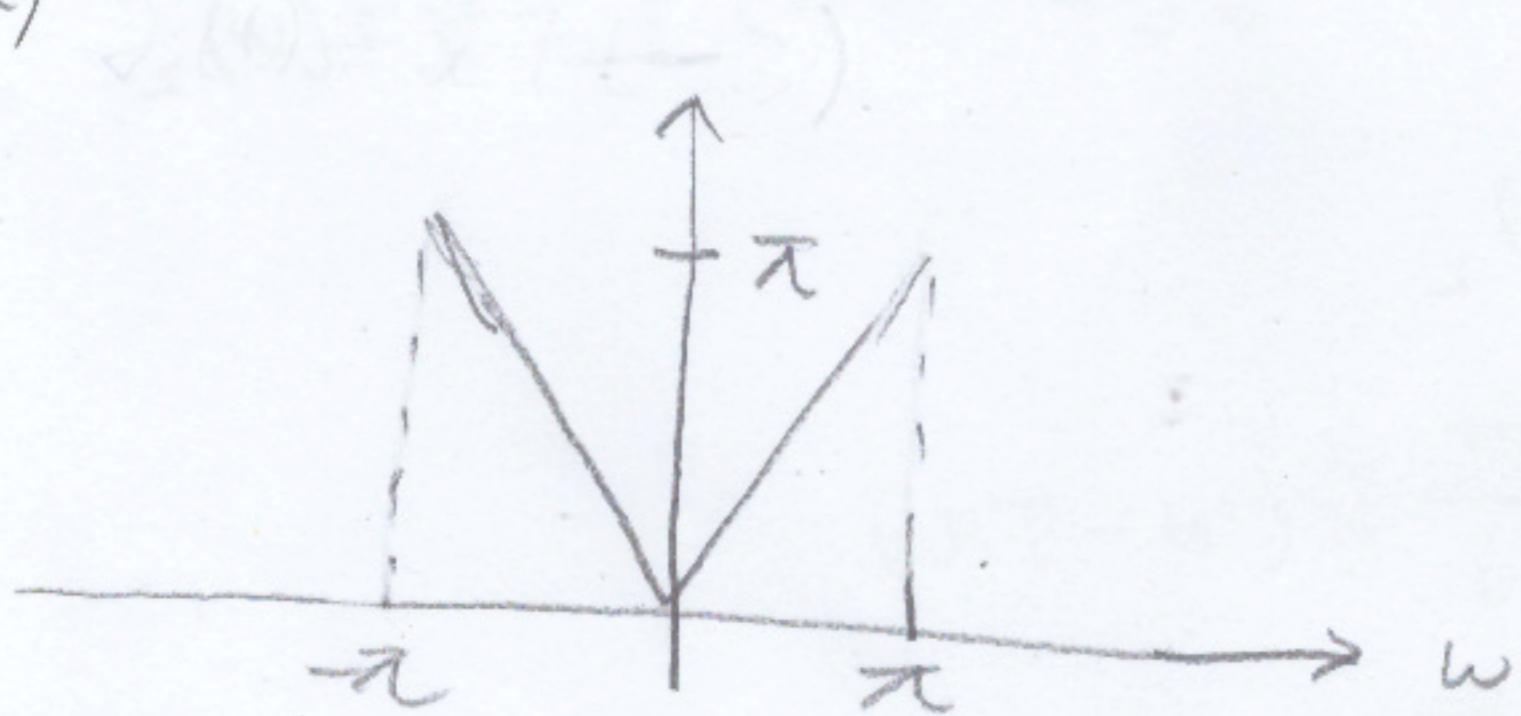
$$+ \frac{\pi}{2j} \delta(\omega + 3\pi) - \frac{\pi}{2j} \delta(\omega - 3\pi).$$



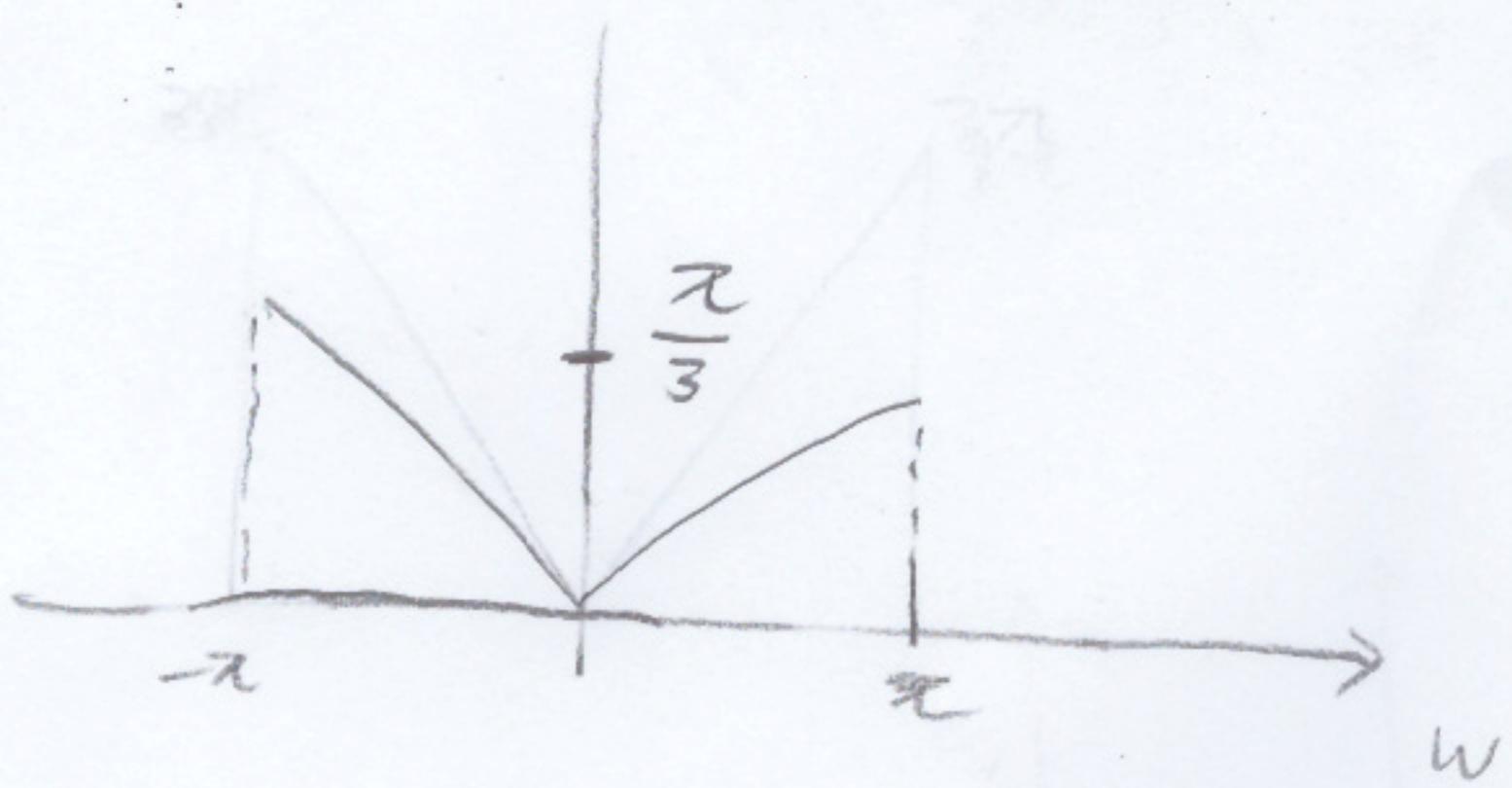
$$3. \quad X^f(\omega) = \begin{cases} 1|\omega|, & |\omega| < \pi \\ 0, & |\omega| > \pi \end{cases}$$



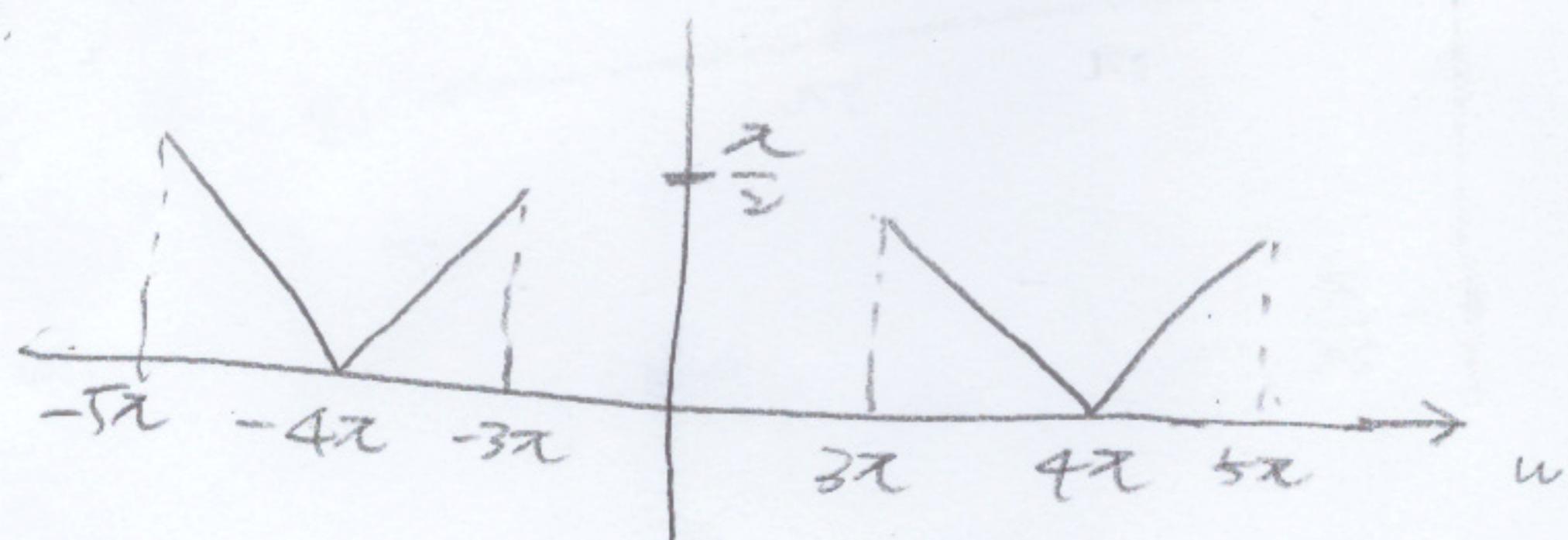
(a)



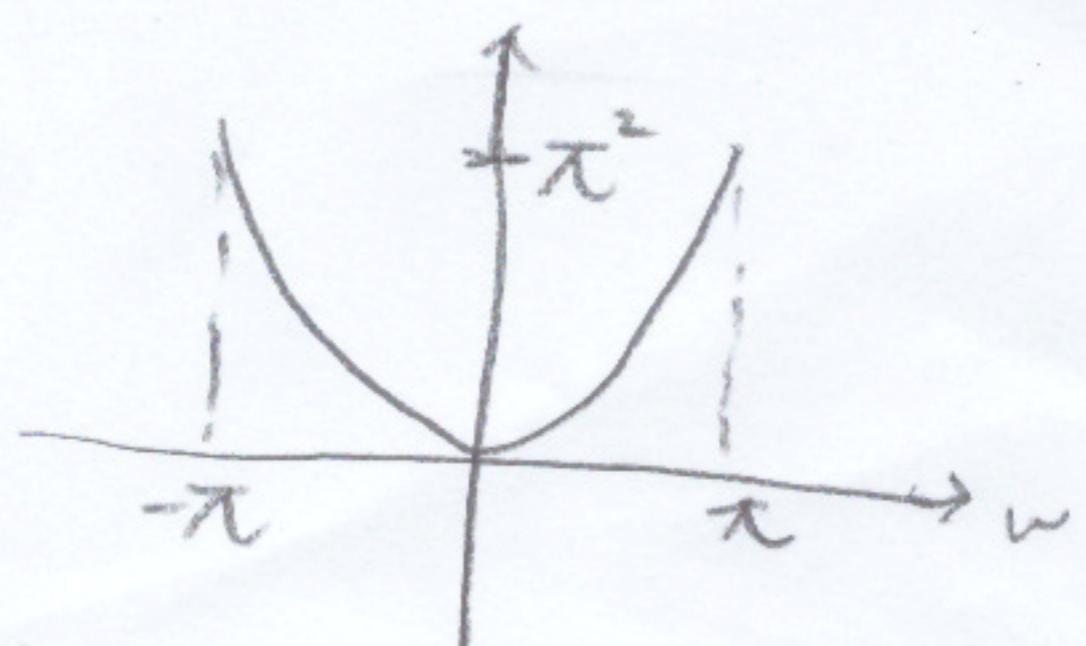
(b)



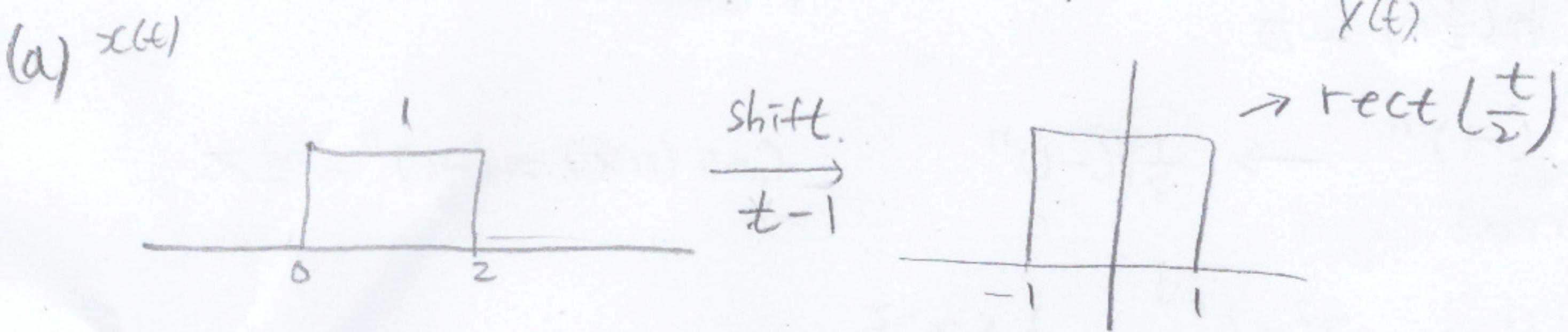
(c)



(d)



$$4. x(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \leq t \leq 2 \\ 0 & t > 2 \end{cases}$$



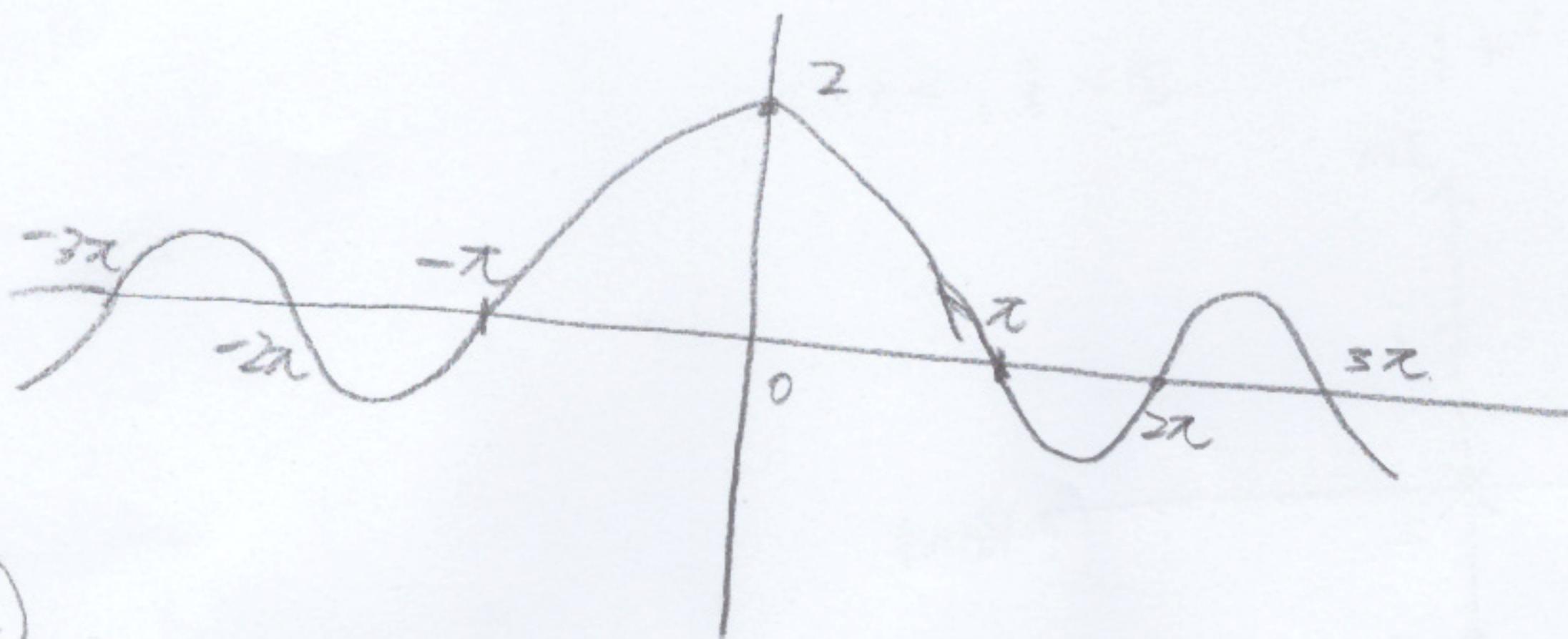
$$\text{rect}\left(\frac{t}{2}\right) \xrightarrow{\text{FT}} 2 \sin\left(\frac{\omega}{\pi}\right)$$

$$X^f(\omega) = \frac{2 \sin \omega}{\omega}$$

$$x(t-1) \rightarrow e^{-j\omega}$$

$$X^f(\omega) = e^{-j\omega} \left[\frac{2 \sin \omega}{\omega} \right]$$

④ $|X^f(\omega)| = \frac{2 \sin \omega}{\omega}$



⑤

$$\frac{2 \sin \omega}{\omega} = 0$$

$$\Rightarrow \dots, -2\pi, -\pi, \pi, 2\pi, \dots$$

$$5. x(t) = 3 \cos(15\pi t) + 4 \cos(20\pi t + 0.5\pi)$$

$$x(t) = \sum c(k) e^{jk\omega_0 t}$$

$$c(k) = \frac{1}{T} \int_{-\infty}^{\infty} x(t) e^{-jk\omega_0 t} dt$$

$$\omega_1 = 15\pi \quad \omega_2 = 20\pi$$

$$\omega_0 = \boxed{5\pi} \quad \omega_0 = \frac{2\pi}{T_0} \Rightarrow T_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$f_0 = \frac{1}{T_0} = \frac{5}{2} \text{ Hz}$$

$$(a) 3 \left[\frac{e^{15\pi t j} + e^{-15\pi t j}}{2} \right] + \frac{4}{25} \left[e^{20\pi j t + 0.5\pi j} - e^{-(20\pi j t + 0.5\pi j)} \right]$$

$$= -\frac{3}{2} e^{15\pi t j} + \frac{3}{2} e^{-15\pi t j} + \frac{4}{25} [e^{20\pi t j} + e^{-20\pi t j}]$$

$$= \frac{3}{2} e^{15\pi t j} + \frac{3}{2} e^{-15\pi t j} + 2 e^{20\pi t j} + 2 e^{-20\pi t j}$$

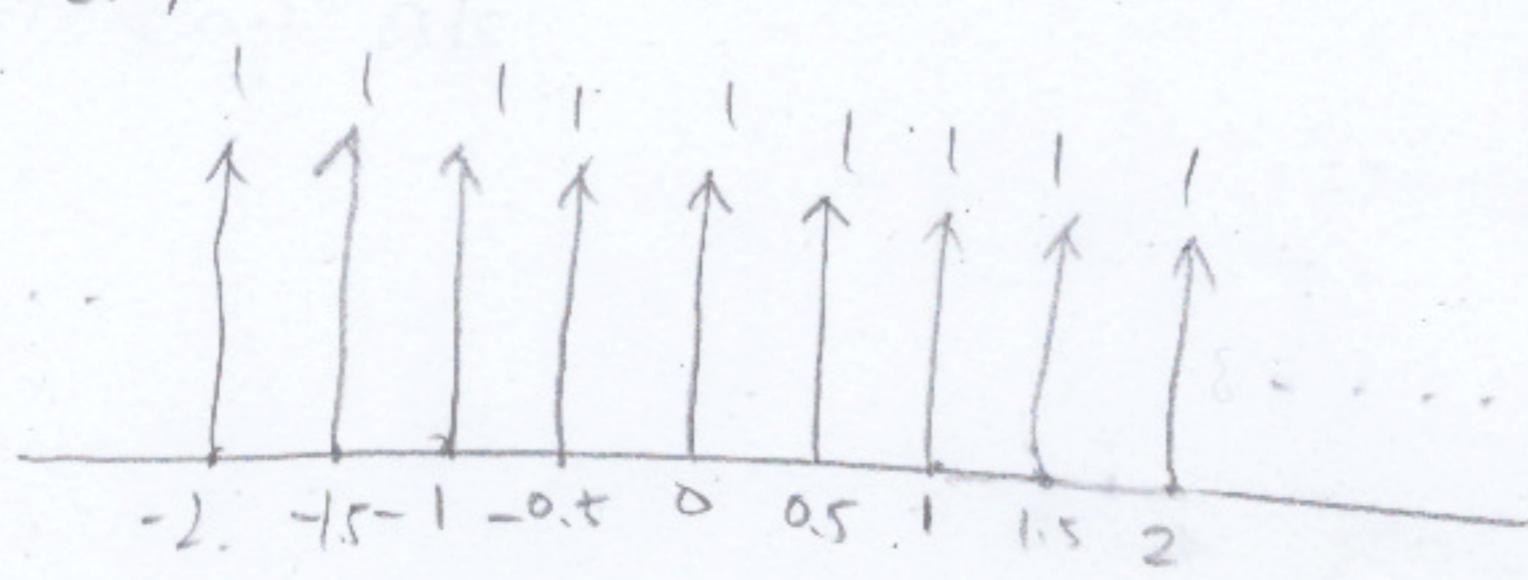
$$\therefore C_1 = \frac{3}{2} \quad C_2 = \frac{3}{2} \quad C_3 = 2 \quad C_4 = 2$$

$$(b) f_0 = \frac{1}{T_0} = \frac{5}{2} \text{ Hz} \quad T_0 = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$\Rightarrow f = 2.5 \text{ Hz}, \quad T = 0.4 \text{ s.}$$

$$6. x(t) = \sum_{k=-\infty}^{\infty} \delta(t - 0.5k)$$

(a) $x(t)$.

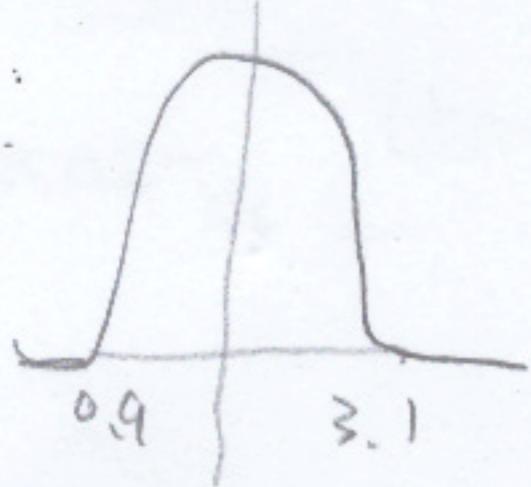


$$(b) T_0 = 0.5k \quad f_0 = \frac{1}{T_0} = \frac{1}{0.5k}$$

Band pass filter $0.9 \text{ Hz} \sim 3.1 \text{ Hz}$.

$$T_1 = \frac{1}{f_L} = \frac{1}{0.9} = 1.1111 \quad T_2 = \frac{1}{f_H} = \frac{1}{3.1} = 0.32258$$

(Hz)



Band pass filter

$\Rightarrow Y(t)$

