1.7.3

Signals and Systems HW5

Jaeha Huh

$y(n) = x(n) - 4y(n-2)$

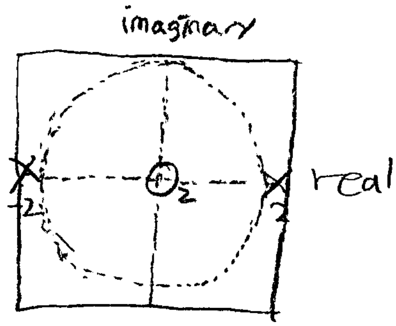
$Y(z) = X(z) - 4z^{-2}Y(z)$

$$\frac{Y(z)}{X(z)} = \frac{1}{1-4z^{-2}} = \frac{z^2}{z^2-4} = H(z)$$

(a) Zeros : $z^2 = 0$ $\quad z = 0$

Poles : $z^2 - 4 = 0$ $\quad (z+2)(z-2) = 0$

$z = -2,\ z = 2$



'o' : zeros

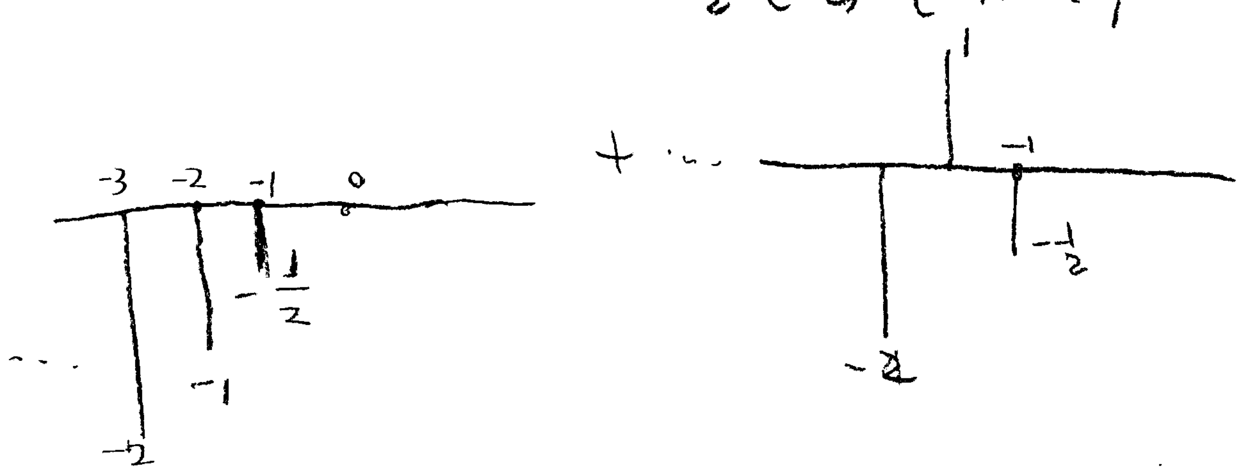
'x' : poles

(b) $H(z) = \dfrac{z^2}{z^2-4} = \dfrac{z^2}{(z+2)(z-2)}$ $\quad \dfrac{H(z)}{z} = \dfrac{z}{z^2-4} = \dfrac{A}{(z-2)} + \dfrac{B}{(z+2)}$

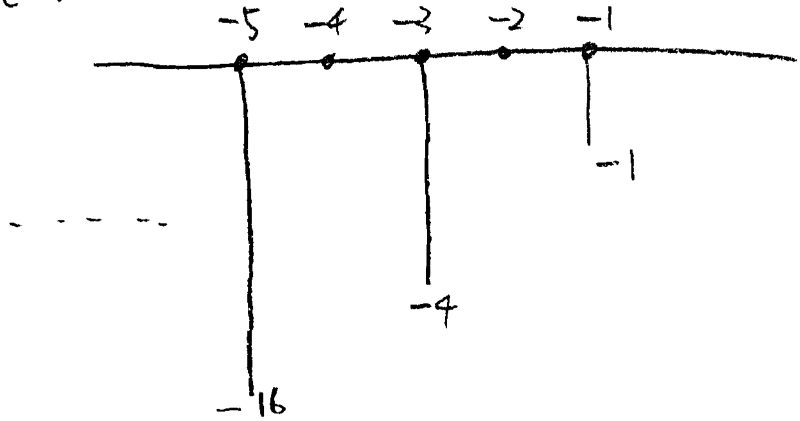
$A = \dfrac{z}{z-2}\Big|_{z=-2} = \dfrac{-2}{-4} = \dfrac{1}{2}$ $\quad B = \dfrac{z}{z+2}\Big|_{z=2} = \dfrac{2}{4} = \dfrac{1}{2}$

$\dfrac{H(z)}{z} = \dfrac{\frac{1}{2}}{(z-2)} + \dfrac{\frac{1}{2}}{(z+2)}$ $\quad H(z) = \frac{1}{2}\left(\dfrac{z}{z-2}\right) + \frac{1}{2}\left(\dfrac{z}{z+2}\right)$

$H(n) = -\frac{1}{2}(2)^n(-n-1) - \frac{1}{2}(-2)^n(-n-1)$



h(n).



(c) the system is unstable.

(d) $x(n) = 2\left(\frac{1}{3}\right)^n u(n) \qquad X(z) = \frac{2z}{z-\frac{1}{3}}$

$$H(z) = \frac{1}{2}\left(\frac{z}{z-2}\right) + \frac{1}{2}\left(\frac{z}{z+2}\right)$$

$$Y(z) = X(z)\,H(z) = 2\left(\frac{z}{z-\frac{1}{3}}\right)\left[\frac{1}{2}\left(\frac{z}{z-2}\right) + \frac{1}{2}\left(\frac{z}{z+2}\right)\right]$$

$$= \frac{z^2}{\left(z-\frac{1}{3}\right)(z-2)} + \frac{z^2}{\left(z-\frac{1}{3}\right)(z+2)} = \frac{z^2(z+2) + z^2(z-2)}{\left(z-\frac{1}{3}\right)(z-2)(z+2)}$$

$$\frac{Y(z)}{z} = \frac{z(z+2) + z(z-2)}{\left(z-\frac{1}{3}\right)(z-2)(z+2)} = \frac{A}{\left(z-\frac{1}{3}\right)} + \frac{B}{(z-2)} + \frac{C}{(z+2)}$$

$$A = \left.\frac{z(z+2)+z(z-2)}{(z-2)(z+2)}\right|_{z=\frac{1}{3}} = \frac{\frac{1}{3}\left(\frac{7}{3}\right) + \frac{1}{3}\left(-\frac{5}{3}\right)}{\left(-\frac{5}{3}\right)\left(\frac{7}{3}\right)} = \frac{\frac{7}{9} - \frac{5}{9}}{-\frac{35}{9}} = -\frac{\frac{2}{9}}{\frac{35}{9}}$$

$$A = -\frac{2}{35}$$

$$B = \left.\frac{z(z+2)+z(z-2)}{\left(z-\frac{1}{3}\right)(z+2)}\right|_{z=2} = \frac{8+0}{\left(\frac{5}{3}\right)(4)} = \frac{8}{\frac{20}{3}} = \frac{24}{20} = \frac{6}{5} \qquad B = \frac{6}{5}$$

$$C = \left.\frac{z(z+2)+z(z-2)}{\left(z-\frac{1}{3}\right)(z-2)}\right|_{z=-2} = \frac{0+8}{\left(-\frac{7}{3}\right)(-4)} = \frac{8}{+\frac{28}{3}} = \frac{24}{28} = \frac{6}{7}$$

$$\frac{Y(z)}{z} = -\frac{2}{35}\frac{1}{\left(z-\frac{1}{3}\right)} + \frac{6}{5}\frac{1}{(z-2)} + \frac{6}{7}\frac{1}{(z+2)}$$

$$Y(z) = -\frac{2}{35}\frac{z}{\left(z-\frac{1}{3}\right)} + \frac{5}{6}\frac{z}{(z-2)} + \frac{6}{7}\frac{z}{(z+2)}$$

$$\boxed{\therefore y(n) = -\frac{2}{35}\left(\frac{1}{3}\right)^n u(n) - \frac{5}{6}(2)^n u(-1-n) + \frac{6}{7}(-2)^n u(-1-n)}$$

1.7.4 $y(n) = x(n) - \frac{1}{2}x(n-1) + \frac{1}{2}y(n-1) - \frac{5}{8}y(n-2)$

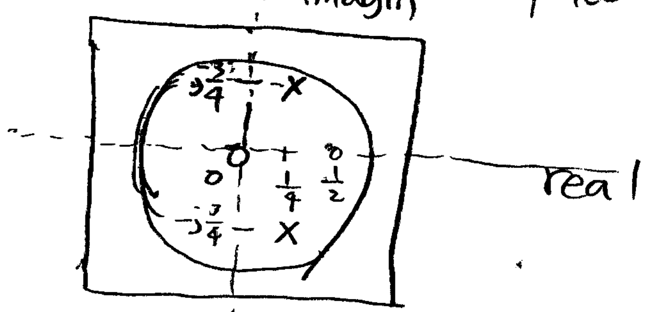
(a) $Y(z) = X(z) - \frac{1}{2}z^{-1}X(z) + \frac{1}{2}Y(z) - \frac{5}{8}z^{-2}Y(z)$

$$Y(z)\left(1-\frac{1}{2}z^{-1}-\frac{5}{8}z^{-2}\right) = 1+\frac{1}{2}z^{-1}X(z)$$

$$\frac{Y(z)}{X(z)} = \frac{1+\frac{1}{2}z^{-1}}{1-\frac{1}{2}z^{-1}-\frac{5}{8}z^{-2}} = \frac{z^2-\frac{1}{2}z^1}{z^2-\frac{1}{2}z^1-\frac{5}{8}}$$

$$H(z) = \frac{z\left(z-\frac{1}{2}\right)}{\left(z-\frac{1}{4}+j\frac{3}{4}\right)\left(z-\frac{1}{4}-j\frac{3}{4}\right)}$$

zeros: $z=0$, imagin poles: $\frac{1}{4} \pm j\frac{3}{4}$



(b) $$\frac{H(z)}{z} = \frac{z-\frac{1}{2}}{\left(z-\frac{1}{4}+j\frac{3}{4}\right)\left(z-\frac{1}{4}-j\frac{3}{4}\right)} = \frac{A}{\left(z-\frac{1}{4}+j\frac{3}{4}\right)} + \frac{B}{\left(z-\frac{1}{4}-j\frac{3}{4}\right)}$$

$A = \frac{1}{2} - j/6 \qquad B = \frac{1}{2} + \frac{j}{6}$

$$H(z) = \left(\frac{1}{2} - \frac{j}{6}\right)\frac{z}{z-\frac{1}{4}+j\frac{3}{4}} + \left(\frac{1}{2}+\frac{j}{6}\right)\frac{z}{z+\frac{1}{4}-j\frac{3}{4}}$$

$$h(n) = \left(\frac{1}{2} - \frac{j}{6}\right)\left(\frac{1}{4}-j\frac{3}{4}\right)^n u(n) + \left(\frac{1}{2}+\frac{j}{6}\right)\left(\frac{1}{4}+j\frac{3}{4}\right)^n u(n)$$

$$= \frac{1}{2}\left(\frac{1}{4}-j\frac{3}{4}\right)^n - \frac{j}{6}\left(\frac{1}{4}-j\frac{3}{4}\right)^n u(n) + \frac{1}{2}\left(\frac{1}{4}+j\frac{3}{4}\right)^n u(n)$$

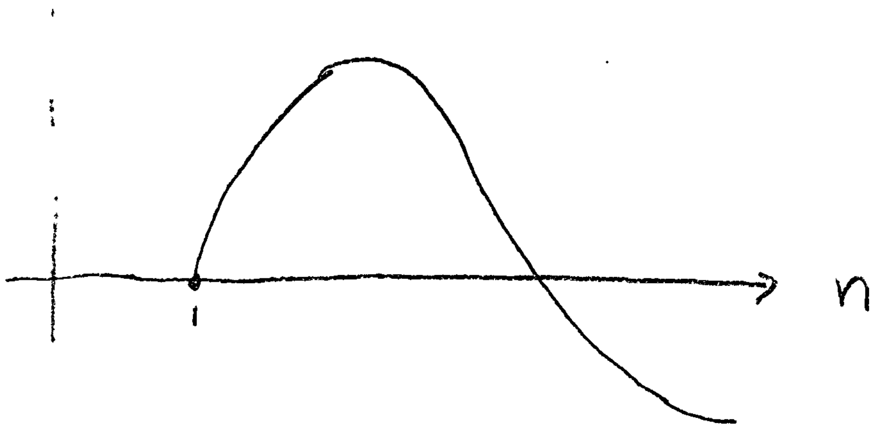
$$+ \frac{j}{6}\left(\frac{1}{4}+j\frac{3}{4}\right)^n u(n)$$

$$= \frac{1}{2}\left(\frac{1}{4}\right)^n u(n) + \frac{j}{6}\left(j\frac{3}{4}\right)^n u(n) + \frac{1}{2}\left(\frac{1}{4}\right)^n u(n)$$

$$+ \frac{j}{6}\left(j\frac{3}{4}\right)^n u(n)$$

$$h(n) = \left(\frac{1}{4}\right)^n u(n) + \frac{j}{3}\left(j\frac{3}{4}\right)^n u(n)$$

$$h(n) = \begin{cases} \left(\frac{1}{4}\right)^n u(n) - \frac{1}{3}\left(3/4\right)^n u(n) & \text{for } n \text{ is odd} \\ \left(\frac{1}{4}\right)^n u(n) - \frac{j}{3}\left(3/4\right)^n u(n) & \text{for } n \text{ is even.} \end{cases}$$



1.7.8

Impulse Resonse	Pole-Zero Diagram
1	3
2	6
3	7
4	1
5	4
6	2
7	5
8	8

1.7.10

Pole-Zero Diagram	Impulse Response
1	1
2	7
3	4
4	6
5	5
6	3
7	8
8	2

1.7.12

$h(n) = A(0.7)^n u(n).$

$H(z) = A \frac{z}{z-0.7}$ $\quad H(e^{j\omega_0}) = A \frac{e^{j\omega_0}}{[e^{j\omega_0} - 0.7]}$

$x(n) = B \cos[0.2\pi n]$ $\quad \omega_0 = 0.2\pi.$

$$\boxed{Y(n) = k(0.7)^n \cos(0.2\pi n + \theta)\, u(n) \quad (a)}$$

1.7.13

$Y(n) = b_0 x(n) - a_1 y(n-1) - a_2 y(n-2)$

$Y(z) = b_0 X(z) - a_1 z^{-1} Y(z) - a_2 z^{-2} Y(z)$

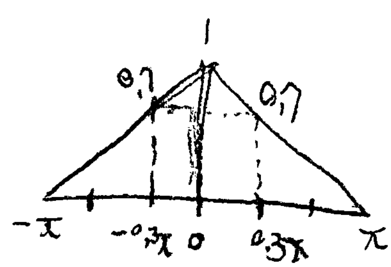
$\frac{Y(z)}{X(z)} = H(z) = \frac{b_0}{1 - a_1 z^{-1} - a_2 z^{-2}} = \ldots$

$\boxed{(a), (c)}$

1.8.7

$x(n) = 1 + \cos(0.3\pi n)$

$x(n) = 1 + \frac{e^{j0.3\pi n} + e^{-j0.3\pi n}}{2}$



$x(n) = Ae^{j\omega_0 n} \longrightarrow \boxed{H(\omega)} \longrightarrow Y_1(n)$

$Y_1(n) = H(\omega)|_{\omega=\omega_0} X_1(n - \angle H(\omega)|_{\omega=\omega_0})$

$e^{j0N} \Rightarrow H(\omega)|_{\omega_0=0}\, x_1(n) \Rightarrow |X| = 1$

$e^{j0.3\pi n} \Rightarrow H(\omega)|_{\omega_0=0.3\pi}\, x_1(n - \angle H(\omega)|_{\omega_0=0.3\pi}) \Rightarrow 0.7e^{j0.3\pi n}$

$e^{-j0.3\pi n} \Rightarrow H(\omega)|_{\omega_0=-0.3\pi}\, x_1(n - \angle H)|_{\omega_0=-0.3\pi}) \Rightarrow 0.7e^{-j0.3\pi n}$

$$x(n) = 1 + \frac{e^{j0.3\pi n} + e^{-j0.3\pi n}}{2}$$

$$\therefore \boxed{y(n) = 1 + 0.1 \cos(0.3\pi n)}$$

1.8.3

$$H(z) = \frac{z(z+2)}{(z-\frac{1}{2})(z+4)}$$

(a) $z = e^{j\omega}$

$$H^f(\omega) = \frac{e^{j\omega}(e^{j\omega}+2)}{(e^{j\omega}-\frac{1}{2})(e^{j\omega}+4)}$$

$$\therefore H^f(\omega) = \frac{1+2e^{-j\omega}}{(1-\frac{1}{2}e^{-j\omega})(1+4e^{-j\omega})}$$

(b) $\omega = 0.2\pi$

$$H^f(\omega)\Big|_{\omega=0.2\pi} = \frac{1+2e^{-j0.2\pi}}{(1-\frac{1}{2}e^{-j0.2\pi})(1+4e^{-j0.2\pi})}$$

$$= \boxed{0.892 \angle -21.42^\circ}$$

(c) $x(n) = \cos(0.2\pi n) \rightarrow y(n) = |H^f(0.2\pi)| \cos(0.2\pi n + \angle H^f(0.2\pi))$

$$H^f(0.2\pi n) = 0.892 \angle -21.42^\circ$$

$$\therefore \boxed{y(n) = 0.892 \cos(0.2\pi n - 21.42^\circ)}$$

1.8.4

$$y(n) = 0.5x(n) + 0.2x(n-1) + 0.5y(n-1) - 0.1y(n-2)$$

$$Y(z) = 0.5X(z) + 0.2z^{-1}X(z) + 0.5z^{-1}Y(z) - 0.1z^{-2}Y(z)$$

$$(1+0.5z^{-1} - 0.1z^{-2})Y(z) = (0.5+0.2z^{-1})X(z)$$

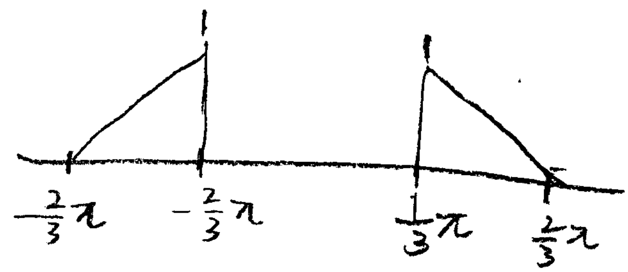
$$H(z) = \frac{Y(z)}{X(z)} = \frac{0.5 + 0.2z^{-1}}{(1 + 0.5z^{-1} - 0.1z^{-2})} \qquad z = e^{j\omega}$$

$$\boxed{\therefore H^f(\omega) = \frac{0.5 + 0.2e^{-j\omega}}{1 + 0.5e^{-j\omega} - 0.1e^{-2j\omega}}}$$

1.8.5

$$Y^f(\omega) = H^f(\omega) \cdot G^f(\omega)$$

(a) $Y^f(\omega)$



(b) only $\frac{\pi}{2}$ component will pass

$$|Y^f(\omega)|\Big|_{\omega=\frac{\pi}{2}} = 0.5 \quad , \quad \angle Y^f(\omega)\Big|_{\omega=\frac{\pi}{2}} = 0^\circ$$

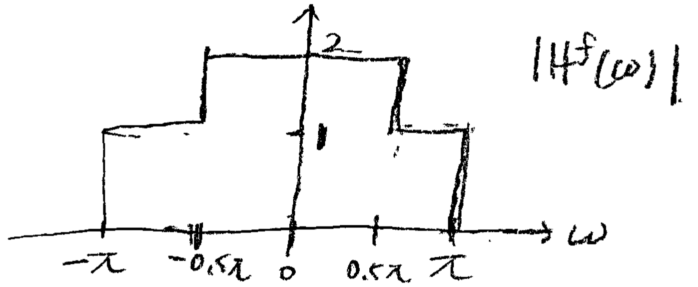
$$Y(n) = 3\,(0.5)\cos\left(\frac{\pi n}{2} + 0^\circ\right)$$

$$\boxed{\therefore Y(n) = 1.5\cos\left(\frac{\pi n}{2}\right)}$$

1.8.8

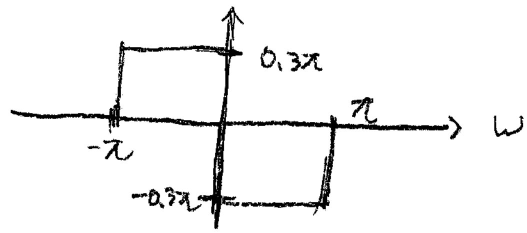
$$|H^f(\omega)| = \begin{cases} 2 & \text{for } |\omega| < 0.5\pi \\ 1 & \text{for } 0.5\pi < |\omega| < \pi. \end{cases}$$

(a)



magnitude Response

(b) $\angle H^f(\omega) \begin{cases} 0.3\pi & \text{for } -\pi < \omega < 0 \\ -0.3\pi & \text{for } 0 < \omega < \pi \end{cases}$



(c)

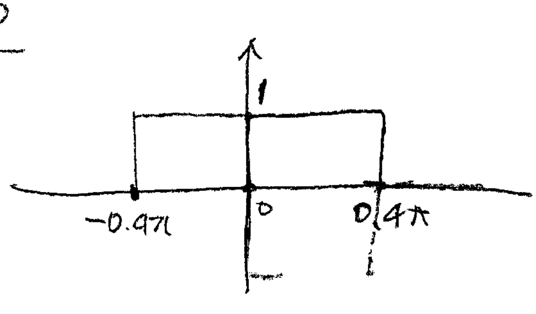
$Y(n) = |H^f(0.2\pi)|\ 2\sin(0.2\pi n + \angle H^f(0.2\pi)) +$
$|H^f(0.6\pi)|\ 3\cos(0.6\pi n + \angle H^f(0.6\pi) + 0.2\pi)$

$Y(n) = 2\ (2\sin(0.2\pi n + (-0.3\pi)) + 1\ (3\cos(0.6\pi n + (-0.3\pi) + 0.2\pi))$

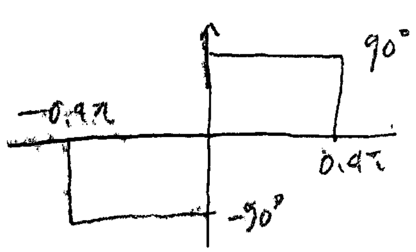
$\therefore \boxed{Y(n) = 4\sin(0.2\pi n - 0.3\pi) + 3\cos(0.6\pi n - 0.1\pi)}$

1.8.10

(a)



(b



$\frac{\pi}{4} = 0.25\pi.$

(c) $Y(n) = |H^f(0.3\pi)| \left(2\cos(0.3\pi n + \angle H^f(0.3\pi))\right)$

$+ |H^f(0.7\pi)| \, 2\cos(0.7\pi n + \angle H^f(0.3\pi)) + \cos \pi n$

$Y(n) = 1\left(2\cos(0.3\pi n + 0.25\pi)\right) + \emptyset + \emptyset$

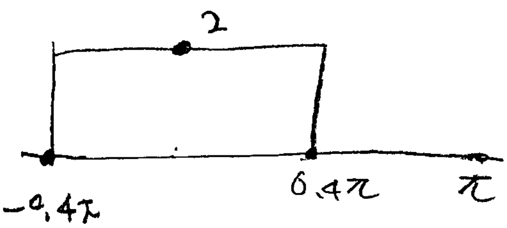
$\therefore \boxed{Y(n) = 2\cos(0.3\pi n +}$

!

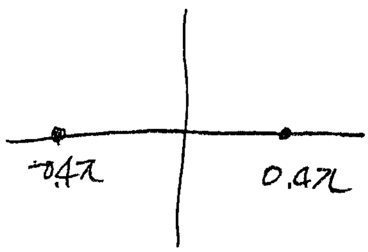
1.8.12

$H^f(\omega) = \begin{cases} 2e^{-j1.5\omega}, & \text{for } |\omega| \le 0.4\pi \\ 0, & \text{for } 0.4\pi < |\omega| \le \pi. \end{cases}$

(a)



(b).



1.8.15

System	output.
A	4
B	1
C	2
D	3

1.8.16

Input signal	System	output signal
1	1	2
2	1	4
3	1	3
1	2	6
2	2	5
3	2	1

1.8.18

Input signal	System	output signal
1	1	3
1	2	4
2	1	2
2	2	1

1.8.1

$y(n) = |H^f(0.3\pi)|\ 1.2\cos(0.3\pi n + \angle H^f(0.3\pi n))$

$\therefore y(n) = 1.2\cos(0.3\pi n + 0.25\pi n)$