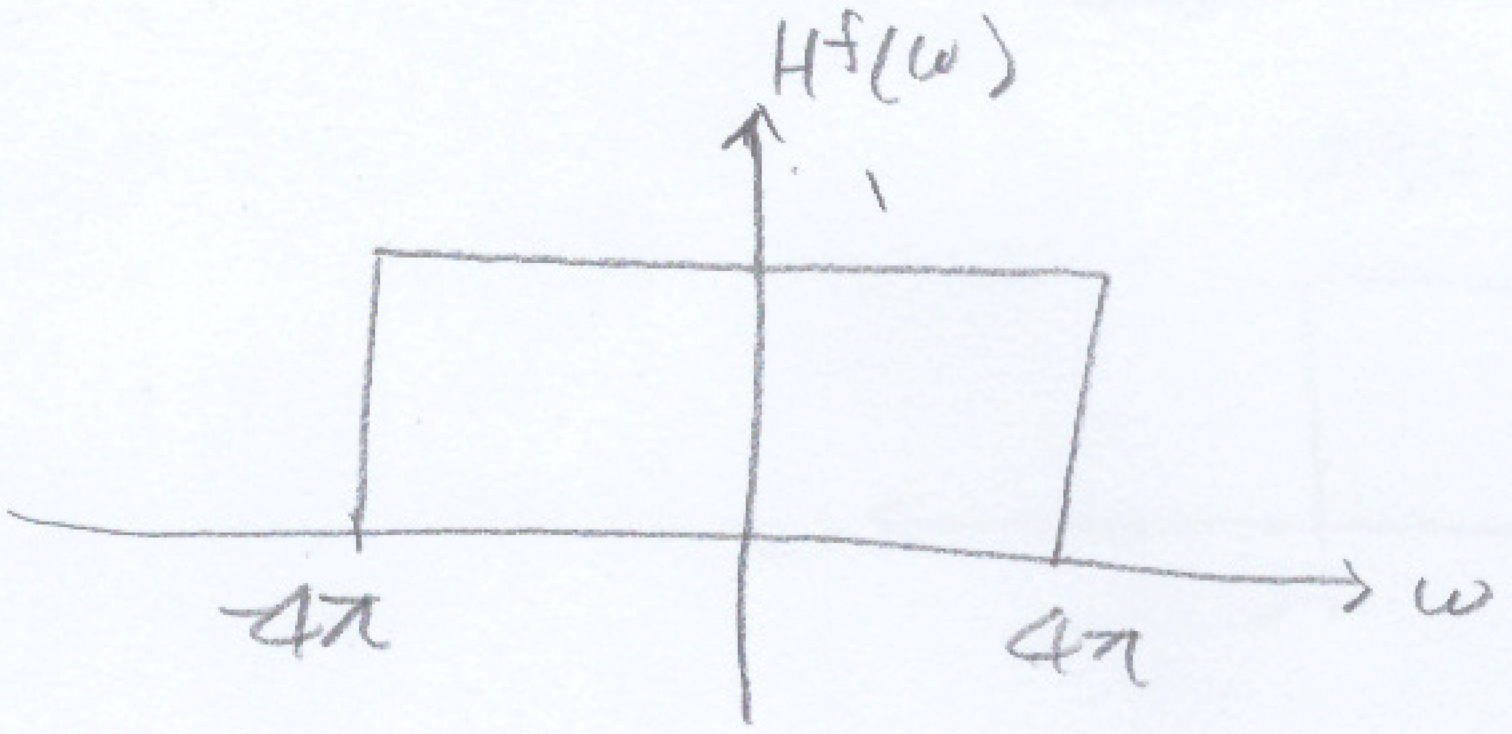
 Jaeha Huh.

2.7.1

$$H^f(\omega) = \begin{cases} 1 & \text{for } |\omega| < 4\pi \\ 0 & \text{for } |\omega| \geq 4\pi \end{cases}$$

(a)



(b) $x(t) = 3\cos(2\pi t) + 6\sin(5\pi t)$

$\xrightarrow{f.t} x(\omega) = 3\pi(\delta(\omega-2\pi)+\delta(\omega+2\pi)) + 6\pi(\delta(\omega-5\pi)+\delta(\omega+5\pi))$

the gain of the filter ($H^f(\omega)$) for $\omega = \pm 5\pi$ is $\emptyset$.

for $\omega = \pm 2\pi$ is 1.

$\Rightarrow \boxed{Y(t) = 3\cos(2\pi t)}$

2.7.2

$$H^f(\omega) = \begin{cases} \frac{6}{4\pi}(\omega+4\pi), & -4\pi < \omega < 0 \\ -\frac{6}{4\pi}(\omega-4\pi), & 0 < \omega < 4\pi \end{cases}$$

$x(t) = 3\cos(2\pi t) + 6\sin(5\pi t)$

$H^f(\omega) = 0$ for $|\omega| > 4\pi$.

$\rightarrow Y(t) = 3\cos(2\pi t)\,|H^f(\omega)|_{\omega=2\pi}$

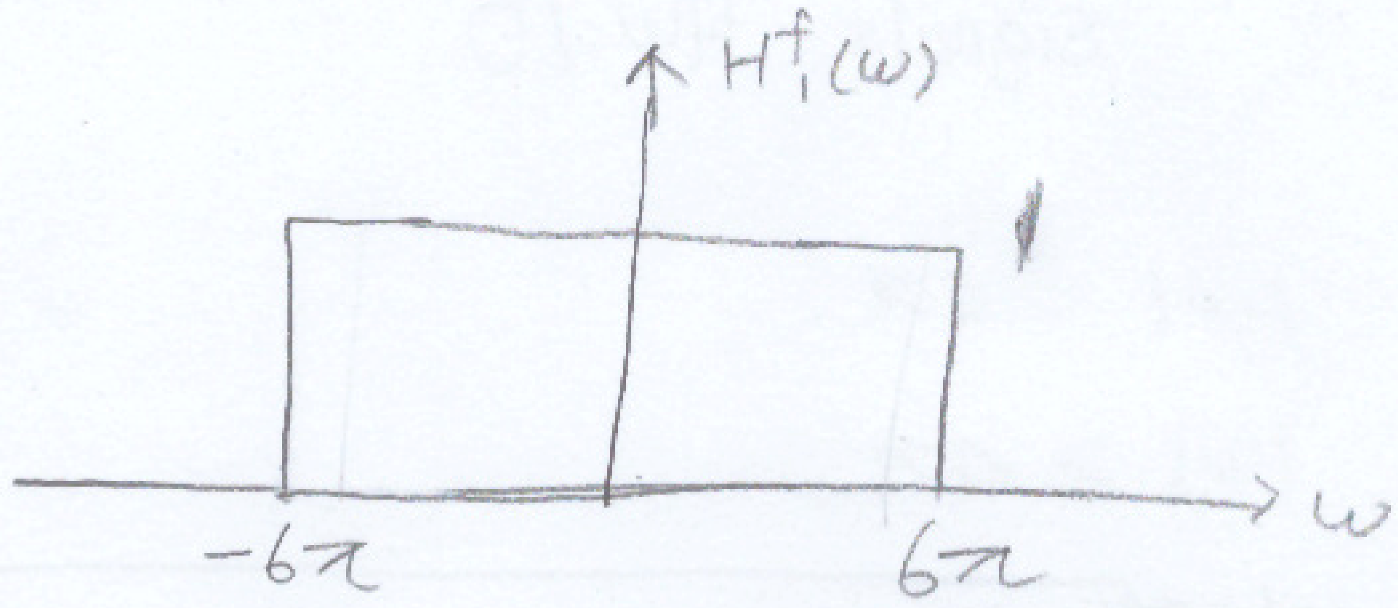
$|H^f(\omega)|_{\omega=2\pi} = -\frac{6}{4\pi}(2\pi-4\pi) = -\frac{6}{4\pi}(-2\pi) = 3.$

$Y(t) = 3\cos(2\pi t)(3)$

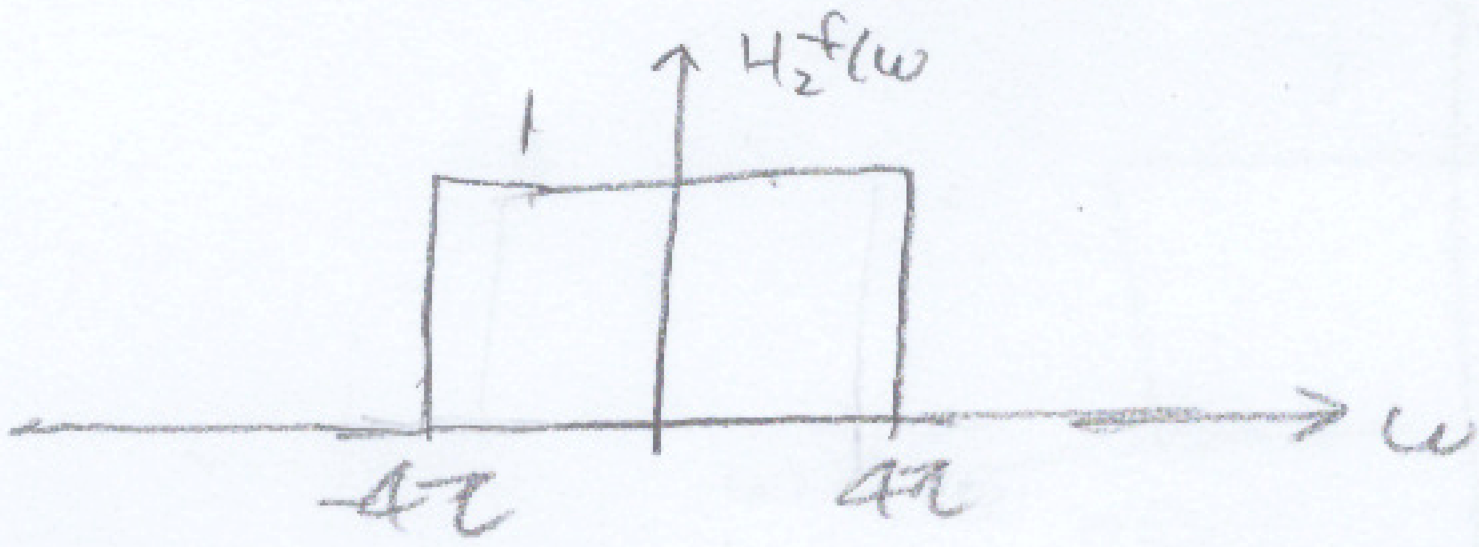
$\therefore \boxed{Y(t) = 9\cos(2\pi t)}$

2.7.3

(a) sys 1



sys 2



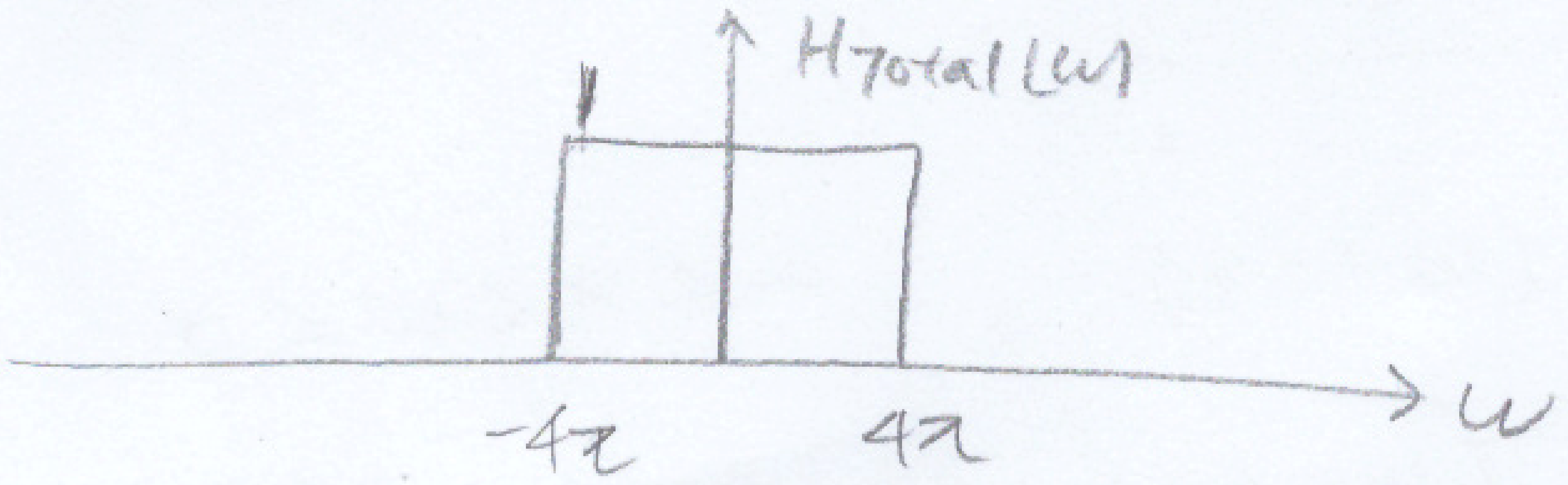
(b) $x(t) = 2\cos 3\pi t - 3\sin 5\pi t + 4\cos(7\pi t)$

$H_{Total}(\omega) = 0$ for $|\omega| > 4\pi$

so, $\boxed{y(t) = 2\cos 3\pi t}$

(c) $H_{Total}(\omega) = H^f_1(\omega) \cdot H^f_2(\omega) = H^f_2(\omega)$

$\therefore H_{Total}(\omega) = \begin{cases} 0 & \text{for } |\omega| < 4\pi \\ 1 & \text{for } |\omega| > 4\pi \end{cases}$



2.7.5

$h(t) = \delta(t) - e^{-t}u(t)$

(a) $h_1(t) = \delta(t)$, $h_2(t) = -e^{-t}u(t)$

$h(t) = \delta(t) \xrightarrow{f.t.} H_1(\omega) = e^{-j\omega(0)} = 1$

$h_2(t) = -e^{-t}u(t) \xrightarrow{f.t} H_2(\omega) = -\frac{1}{1+j\omega}$

$H(\omega) = H_1(\omega) + H_2(\omega) = 1 - \frac{1}{1+j\omega}$

$\boxed{H(\omega) = \frac{j\omega}{1+j\omega}}$

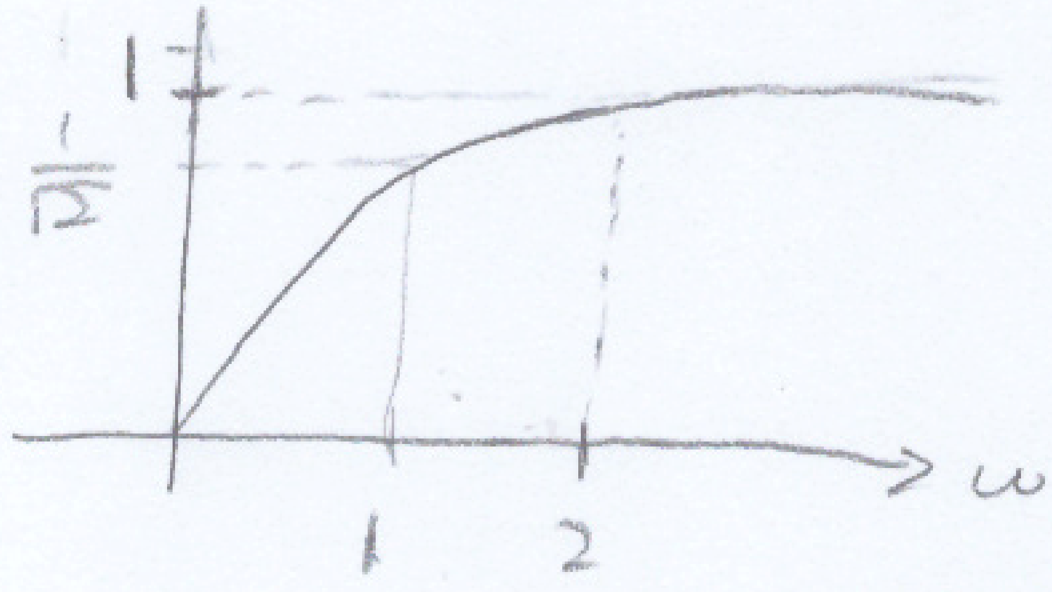
(b) $|H(\omega)| = \frac{\omega}{\sqrt{1+\omega^2}}$ $\quad \angle H(\omega) = 90^\circ - \tan^{-1}\omega$

at $\omega = 0$, $|H(0)| = 0$, $\angle H(\omega) = 90^\circ$

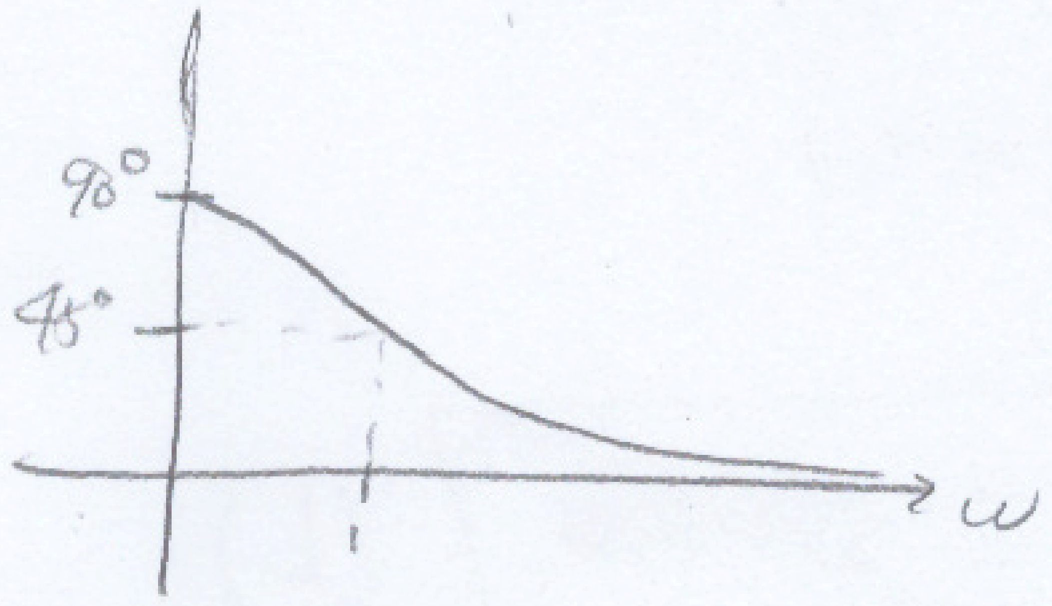
at $\omega \to \infty$, $|H(\infty)| = 1$, $\angle H(\omega) = 0^\circ$

at $\omega = 1$, $|H(1)| = \frac{1}{\sqrt{2}}$, $\angle H(1) = 90^\circ - 45^\circ = 45^\circ$

$|H(\omega)|$



$\angle H(\omega)$



(c) It is High pass filter.

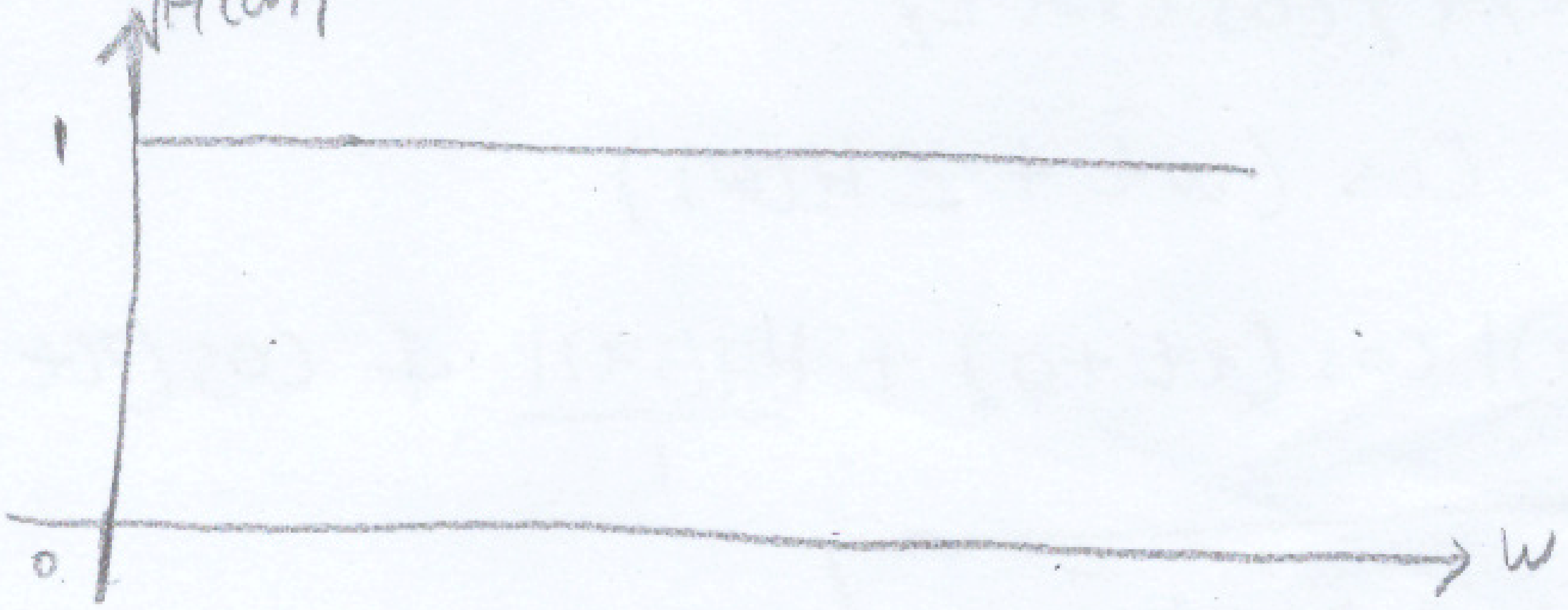
2.7.6

$h(t) = \delta(t-2)$, $x(t-t_0) = e^{-j\omega t_0}$. $\frac{X(\omega)}{=1}$

(a) $\boxed{H(\omega) = e^{-j2\omega}}$

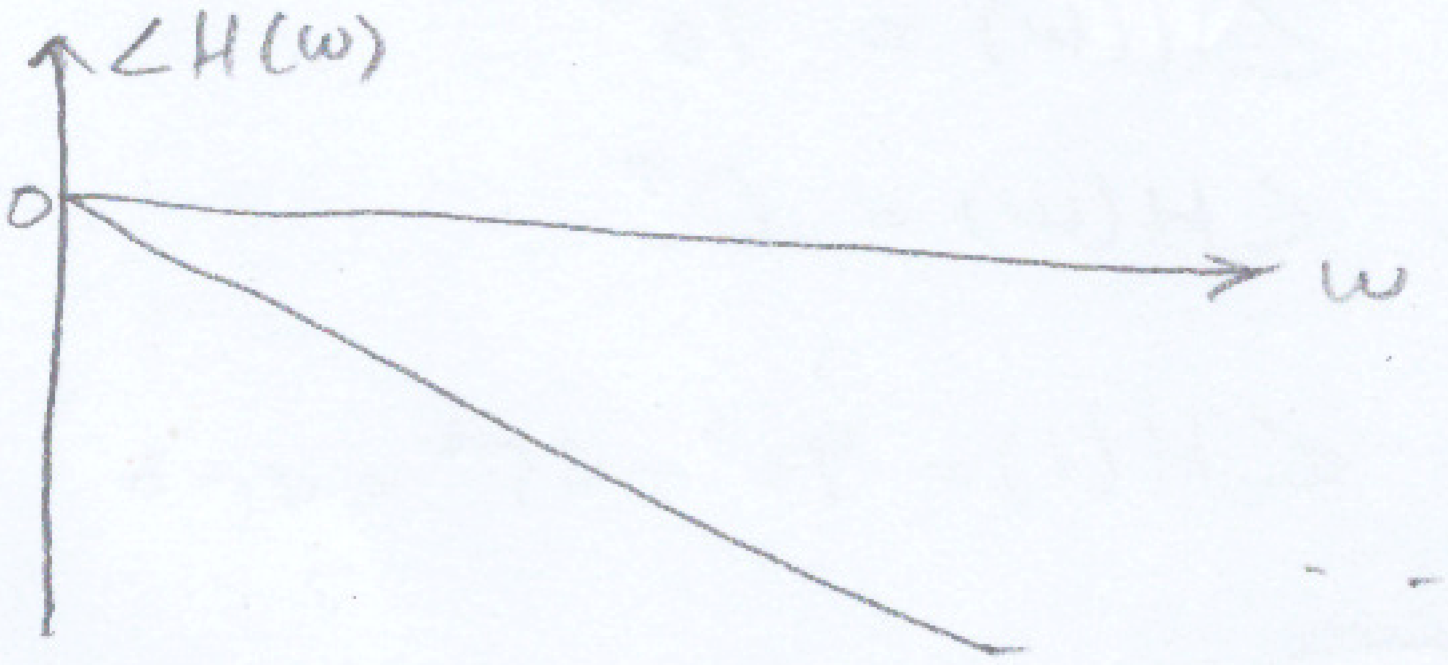
(b) $|H(\omega)| = |e^{-j2\omega}| = 1$

at $\omega = 0$, $|H(0)| = 1$; at $\omega \to \infty$, $|H(\infty)| = 1$



(c). $\angle H(\omega) = -2\omega$

at $\omega = 0$, $\angle H(0) = 0$; at $\omega \to \infty$, $\angle H(\infty) = -\infty$



(d) It is all pass filter.

2.7.7

$x(t) = \sin(10\pi t) + \cos(5\pi t)$

$Y(t) = 3\sin(10\pi t) + 2\cos(7\pi t)$

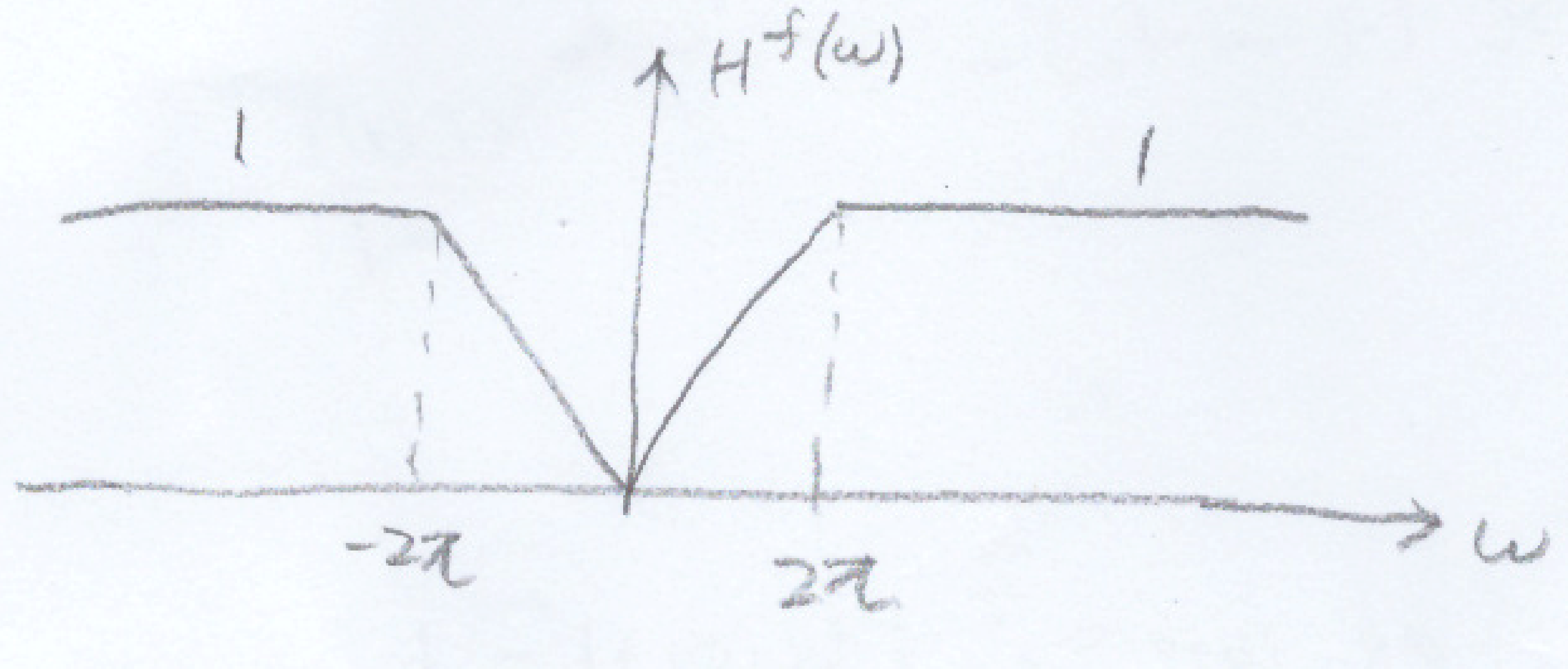
ω_0 is changed $5 \to 7$

$\Rightarrow$ $\boxed{\text{The system is not L.T.I system}}$ (b)

2.7.9

$$H^f(\omega) = \begin{cases} \frac{|\omega|}{2\pi}, & |\omega| \le 2\pi \\ 1, & |\omega| > 2\pi \end{cases}$$

(a)



(b) $x(t) = 1 + 2\cos(\pi t) + 4\cos(3\pi t)$

$Y(t) = |H(\omega)| A \cos(\omega t + \angle H(\omega))$

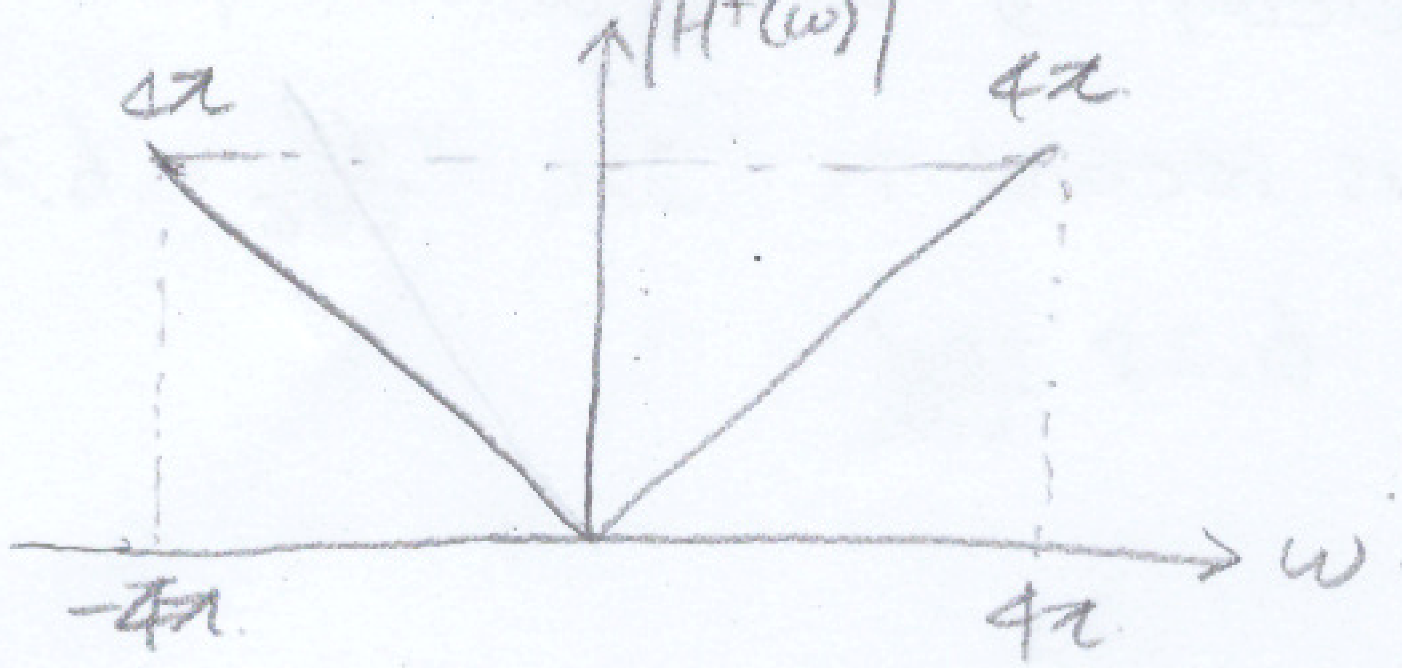
$Y(t) = 0 + 2\underbrace{|H(\pi)|}_{\frac{1}{2}} \cos(\pi t + 0) + \underbrace{|H(3\pi)|}_{1} 4 \cos(\pi t + 0)$

$\therefore$ $\boxed{Y(t) = \cos \pi t + 4\cos(3\pi t)}$

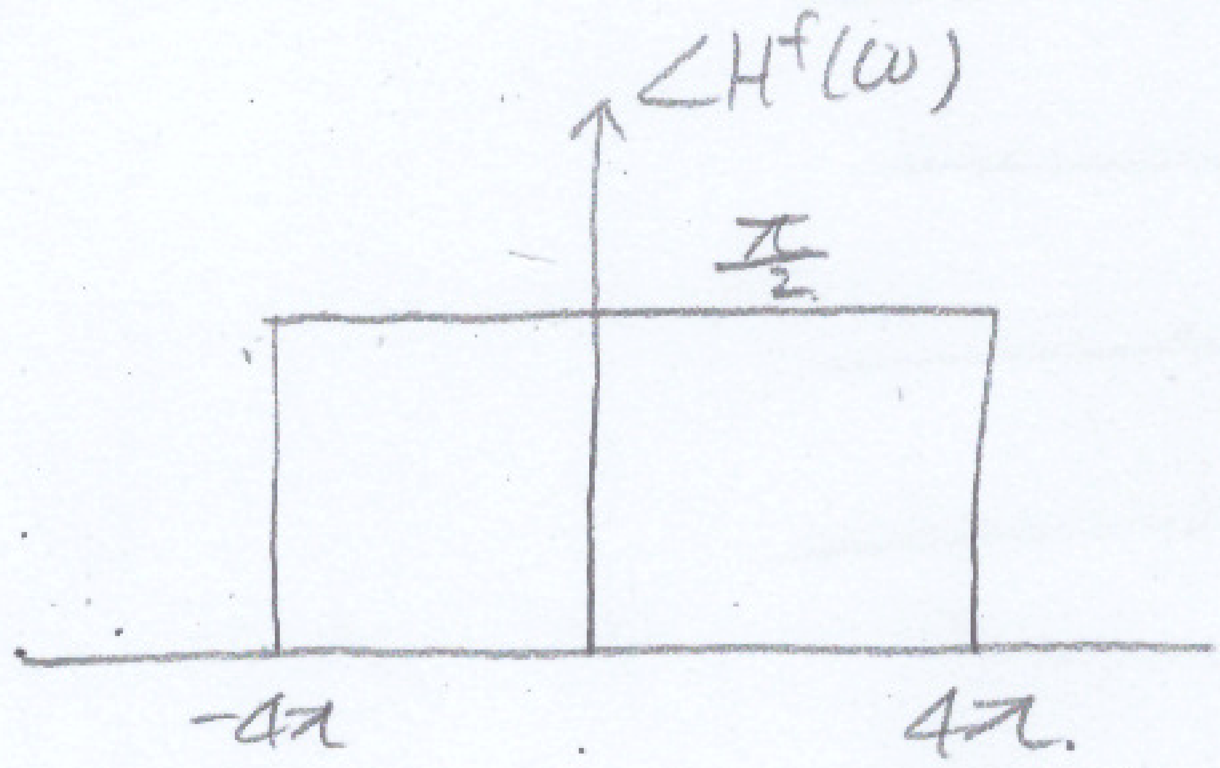
2.7.17

$$H^f(\omega) = \begin{cases} j\omega, & |\omega| \le 4\pi \\ 0, & |\omega| > \pi \end{cases}$$

(a)



(b)



(c) $x(t) = 2 + \cos(\pi t) + 0.5\sin(2\pi t) + 3\cos(6\pi t)$

$Y(t) = |H(0)| \times 2 + |H(\pi)| \times 1 \cos(\pi t + \frac{\pi}{2}) + 0.5 \times |H(2\pi)| \times \sin(2\pi t + \frac{\pi}{2}) + 3\,|H(6\pi)| \times \cos(6\pi t + 0)$

$= 0 + \pi\cos(\pi t + \frac{\pi}{2}) + \pi\sin(2\pi t + \frac{\pi}{2}) + 0$

$\therefore \boxed{Y(t) = \pi\cos(\pi t + \frac{\pi}{2}) + \pi\sin(2\pi t + \frac{\pi}{2})}$

(d). It is low pass differentiator because

$x_1(t) = 2 \rightarrow \frac{d}{dx} x_1(t) = 0 \Rightarrow Y_1(t) = 0.$

$x_2(t) = \cos(\pi t) \rightarrow \frac{d}{dx} x_2(t) = -\pi\sin(\pi t) = \pi\cos(\pi t + \frac{\pi}{2})$

$\Rightarrow Y_2(t) = \pi\cos(\pi t + \frac{\pi}{2})$

$x_3(t) = 0.5\sin(2\pi t) \rightarrow \frac{d}{dx} x_3(t) = \pi\cos(2\pi t) = \pi\sin(2\pi t + \frac{\pi}{2})$

$\Rightarrow Y_3(t) = \pi\sin(2\pi t + \frac{\pi}{2})$

$Y_4(t) = 0$; $x_4(t) = 3\cos 6\pi$

2.7.21

The answer is $\boxed{\text{Frequency Response D}}$. From the impulse response, the response is exponentially decaying with a time period $= 1s$. So, frequency in radius second is $2\pi \frac{rad}{sec} = 6.28$

Thus, figure has a peak at $6.28 \frac{rad}{sec}$.

2.7.23

Pole-zero diagram	Frequency Response
1	3
2	1
3	2
4	4

2.7.24

Pole-zero diagram	Frequency Response
1	3
2	8
3	2
4	7
5	4
6	1
7	5
8	6

2.7.29 $x(t) = 1 + 2\cos(\pi t) + 0.5\cos(10\pi t)$

system	output signal
1	3
2	4
3	6
4	1
5	5
6	2

2.9.2

$$y''(t) + 5y'(t) + 6y(t) = Ax''(t) + Bx'(t) + Cx(t)$$

$$H(s) = \frac{As^2 + Bs + C}{s^2 + 5s + 6} \quad ; \quad \text{let } s = j\omega.$$

$$H(\omega) = \frac{-A\omega^2 + j(B\omega) + C}{(6-\omega^2) + j5\omega}$$

(a) 1 kill $f = 5\text{ Hz} \Rightarrow \omega = 10\pi \ \frac{\text{rad}}{\text{sec}}$

$$H(10\pi) = 0 \Rightarrow -A(100\pi^2) + j(10\pi B) + C = 0 + j0$$

2 D.C gain = 2 $\Rightarrow H(0) = \frac{C}{6} = 2. \quad C = 12$

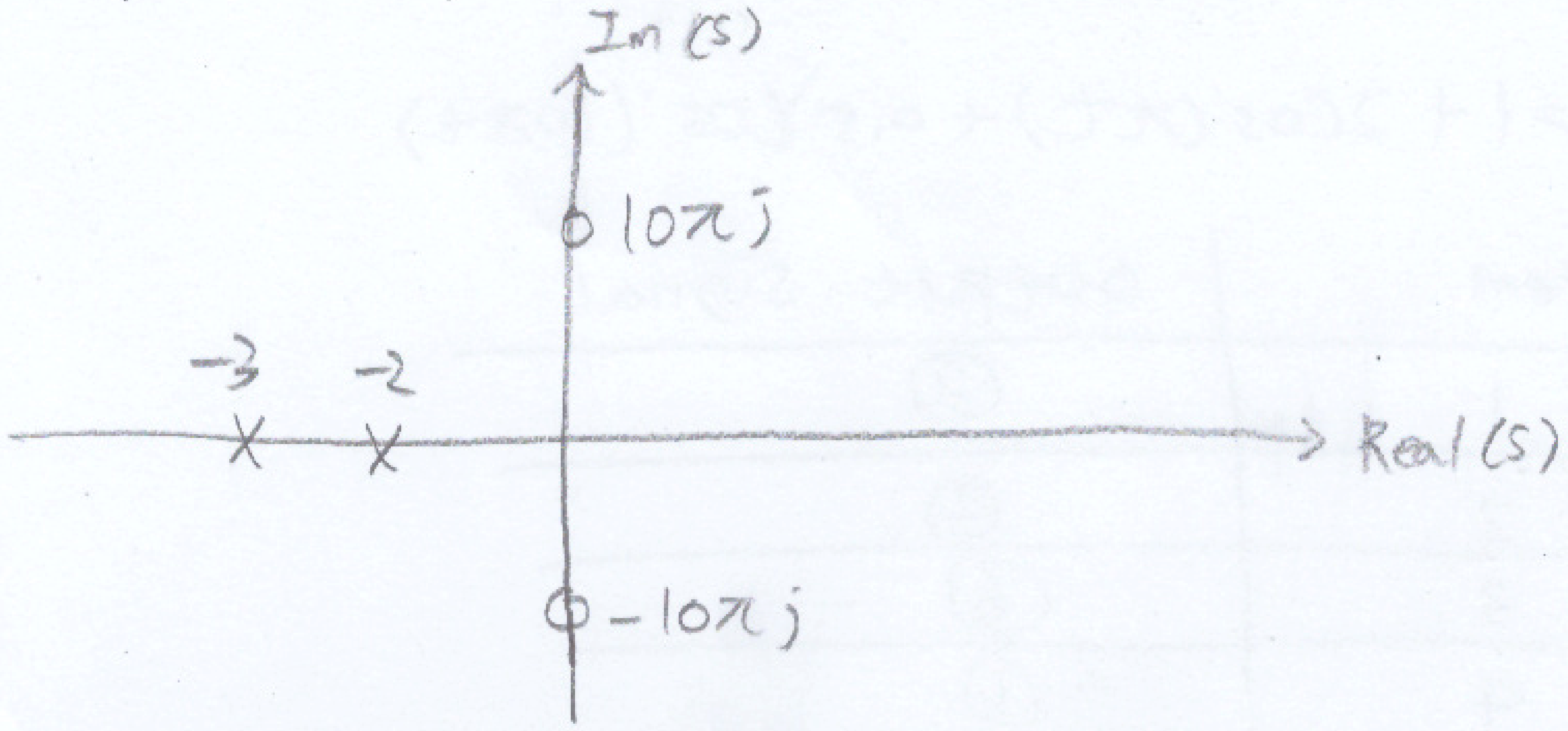
$$-A(100\pi^2) + j(10\pi B) + 12 = 0$$

$$\Rightarrow \boxed{A = \frac{12}{100\pi^2}, \quad B = 0, \quad C = 12}$$

(b) $$H(s) = \frac{\frac{12}{100\pi^2}s^2 + 12}{(s+2)(s+3)}$$

Zeros : $s^2 = -100\pi^2 \Rightarrow s = \pm 10\pi j$

Poles : $s = -2, \ s = -3$



(c) $H(s) = a + \frac{b}{s+2} + \frac{c}{s+3}$

$\Rightarrow h(t) = a\delta(t) + b e^{-2t} u(t) + c e^{-3t} u(t)$

2.9.3

$H(0) = 0$; system has a pole $s = -5$

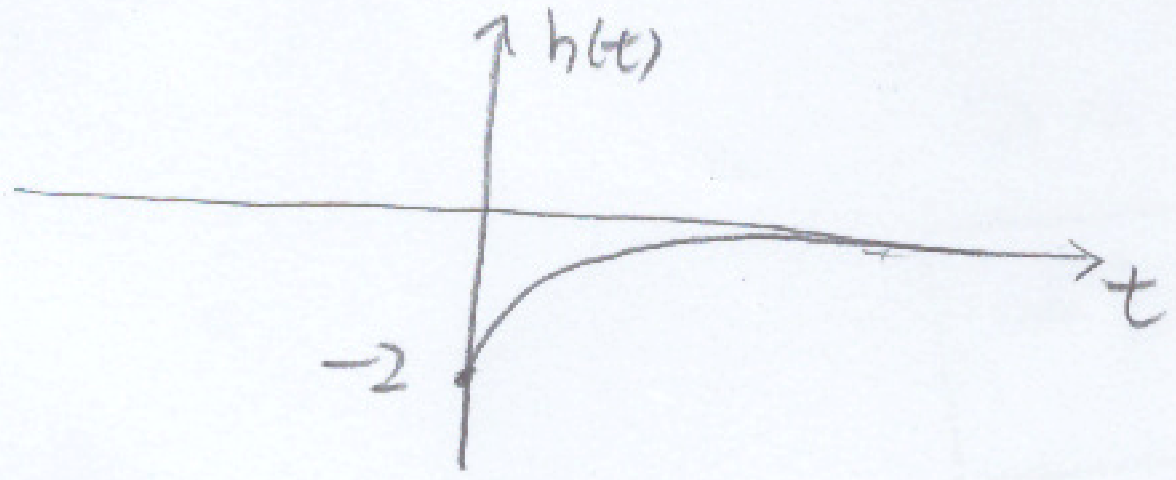
$H(\omega)|_{\omega=0} = 0.5$

(a) $H(s) = \frac{0.5s}{s+5} = \frac{Y(s)}{X(s)} \Rightarrow (s+5)Y(s) = 0.5s\,X(s)$

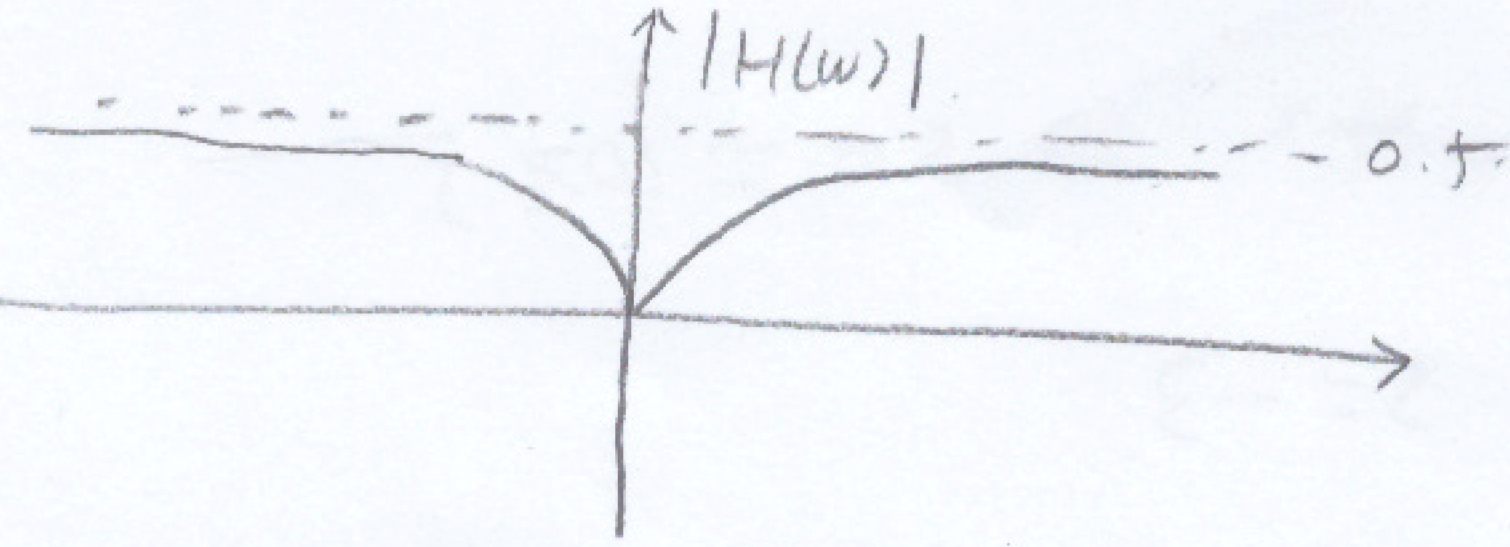
$\Rightarrow \boxed{y'(t) + 5y(t) = 0.5x'(t)}$

(b) $H(s) = \frac{0.5s}{s+5} = \frac{0.5(s+5-5)}{s+5} = 0.5 - \frac{2.5}{s+5}$

$h(t) = 0.5\delta(t) - 2.5e^{-5t}u(t)$

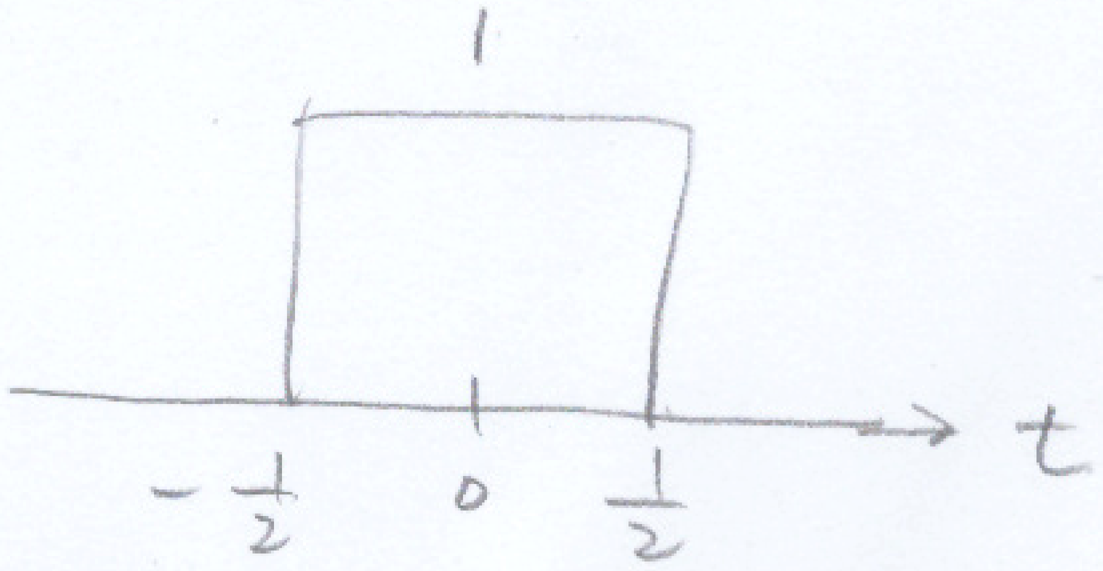


(c) $H(\omega) = \frac{0.5j\omega}{j\omega+5}$; $|H(\omega)| = \frac{0.5\omega}{\sqrt{5+\omega^2}}$



3.1.7

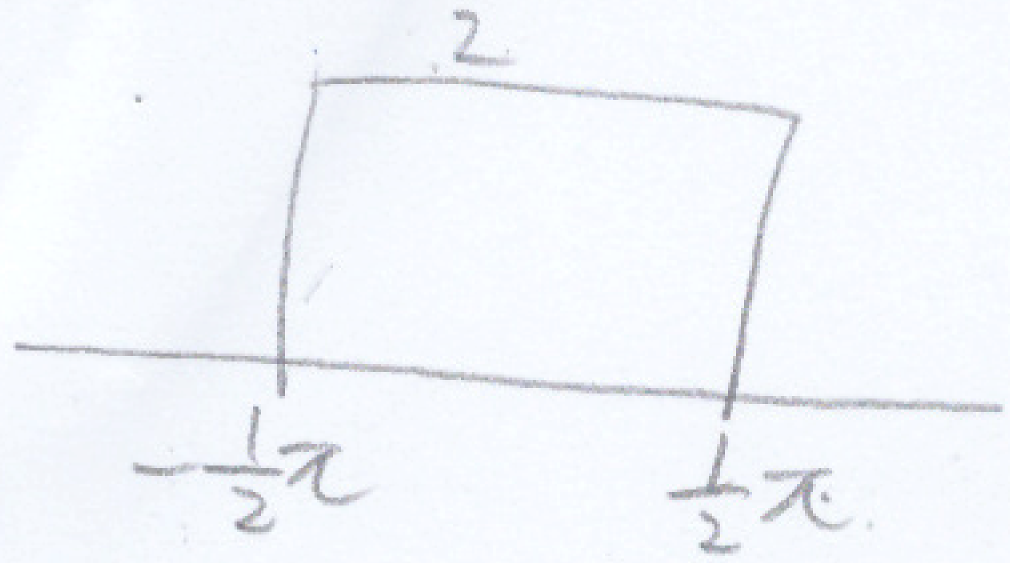
(a)



$$x(t) = 2\,\frac{\sin\frac{1}{2}\pi t}{\frac{1}{2}\pi t}$$

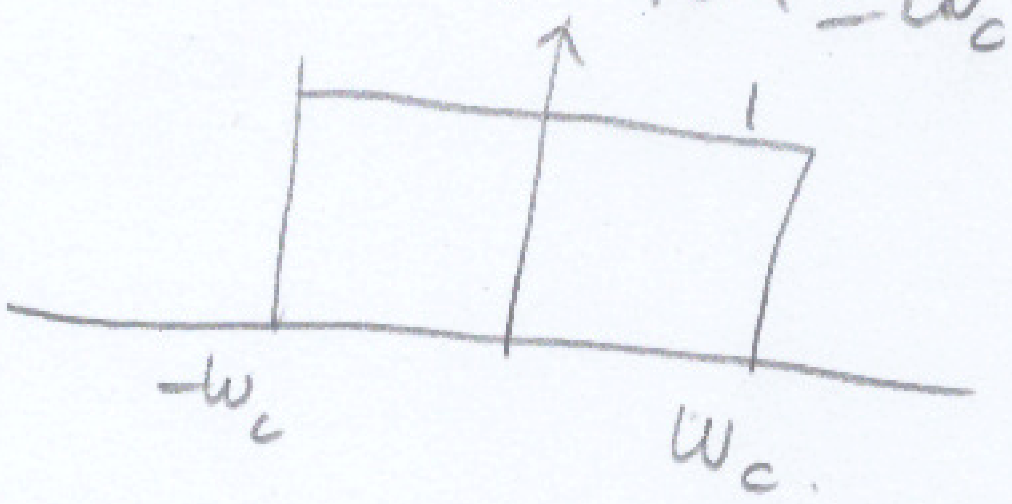
$$\Rightarrow \boxed{2\,\mathrm{sinc}\left(\tfrac{1}{2}\pi t\right)}$$

(b)



3.1.22

$$H_f(\omega) = \begin{cases} 0, & |\omega| \le \omega_c \\ 1, & |\omega| \ge \omega_c \end{cases}$$



$$H(\omega) = H_2(\omega) - H_1(\omega) \Rightarrow h(t) = h_2(t) - h_1(t)$$

$$h_1(t) = \frac{\sin\omega_c t}{\pi t} \qquad h_2 = \delta(t)$$

$$\therefore\ h(t) = \delta(t) - \frac{\sin\omega_c t}{\pi t}$$