Signals and System

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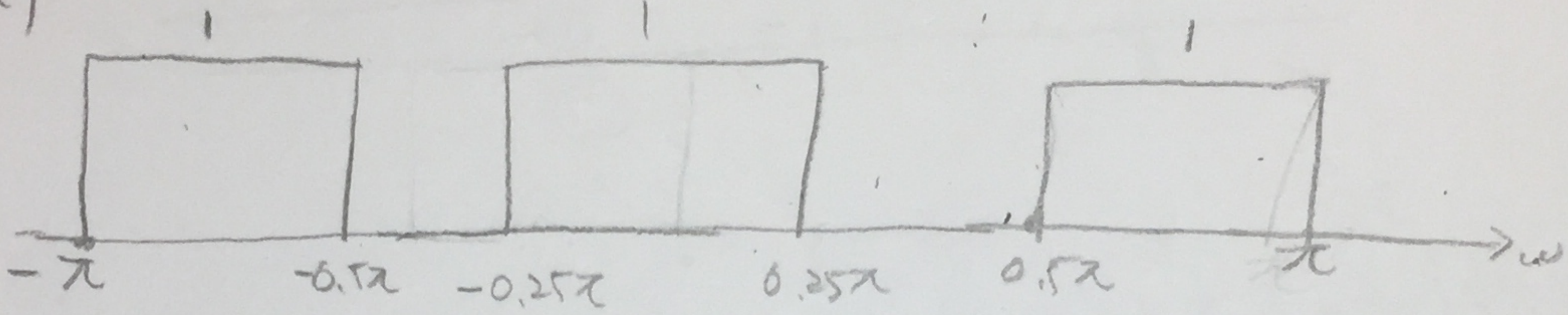
## 1.8.9

$$H'(\omega) = \begin{cases} 1, & |\omega| \le 0.25\pi \\ 0, & 0.25\pi < |\omega| \le 0.5\pi \\ 1, & 0.5\pi < |\omega| \le \pi \end{cases}$$

(a)




(b) $x(n) = 3 + 2\cos(0.3\pi n) + 2\cos(0.7\pi n) + (-1)^n$

$$y(n) = 3 + 2\underbrace{|H^f(0.3\pi n)|}_{=0}\cos(0.3\pi n)$$
$$+ 2\underbrace{|H^f(0.7\pi n)|}_{=1}\cos(0.7\pi n) + \underbrace{(-1)^n}_{=0}$$

$\Rightarrow y(n) = 3 + 2\cos(0.7\pi n)$

$\therefore \boxed{y(n) = 3 + 2\cos(0.7\pi n)}$

(c) The system is band stop filter

because $H'(\omega) = 0$ at $0.25\pi < |\omega| \le 0.5\pi$

## 1.8.22

| Frequency Response | Pole-Zero Diagram |
|---|---|
| 1 | 2 |
| 2 | 6 |
| 3 | 7 |
| 4 | 3 |
| 5 | 5 |
| 6 | 4 |
| 7 | 1 |
| 8 | 8 |

1.8.24

| Frequency Response | Pole-Zero Diagram |
|---|---|
| 1 | 2 |
| 2 | 4 |
| 3 | 8 |
| 4 | 5 |
| 5 | 6 |
| 6 | 3 |
| 7 | 1 |
| 8 | 7 |

1.9.3

$y(n) = x(n) + 0.5x(n-1) + 0.2y(n-1)$

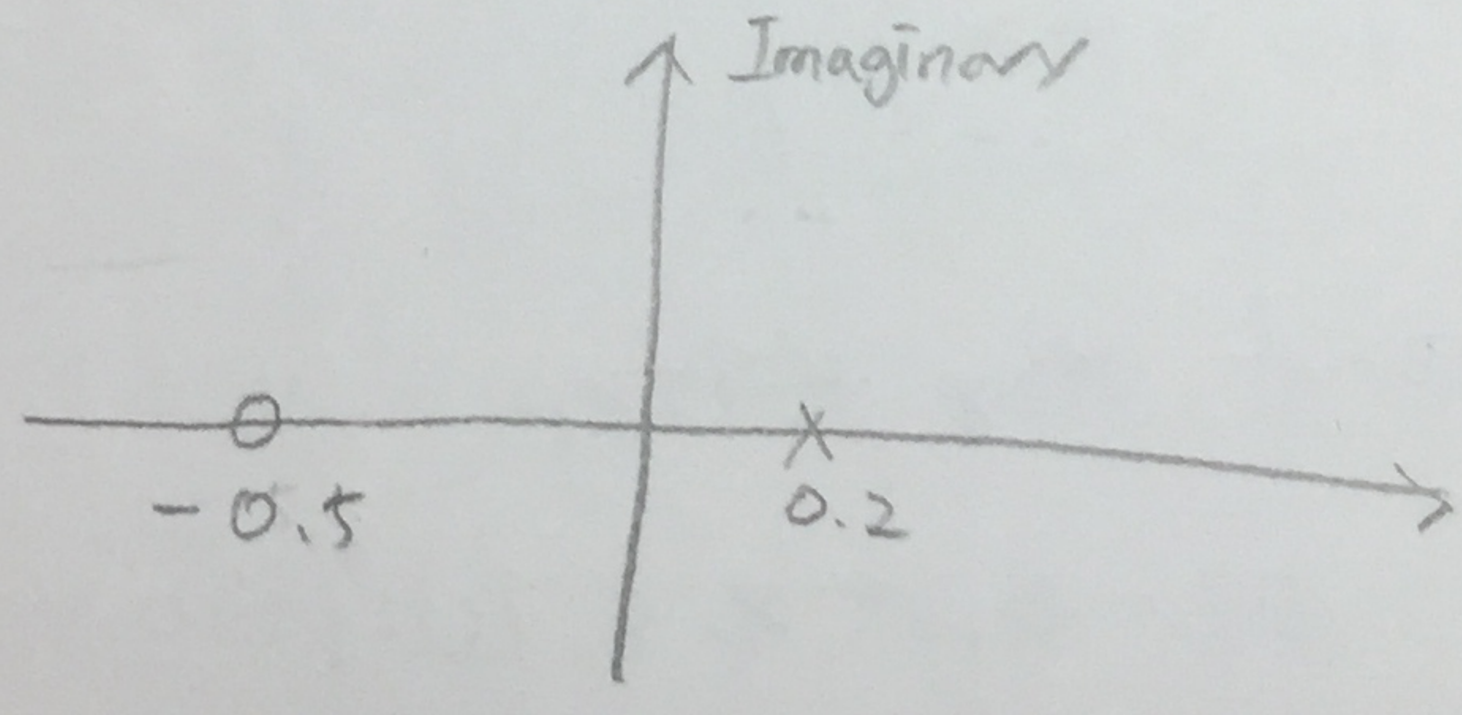
(a) $Y(z) = X(z) + 0.5z^{-1}X(z) + 0.2z^{-1}Y(z)$

$Y(z) - 0.2z^{-1}Y(z) = X(z) + 0.5z^{-1}X(z)$

$Y(z)(1-0.2z^{-1}) = X(z)(1+0.5z^{-1})$

$$\frac{Y(z)}{X(z)} = \frac{1+0.5z^{-1}}{1-0.2z^{-1}} = \frac{z+0.5}{z-0.2}$$

Zero : $z = -0.5$ , Pole : $z = 0.2$

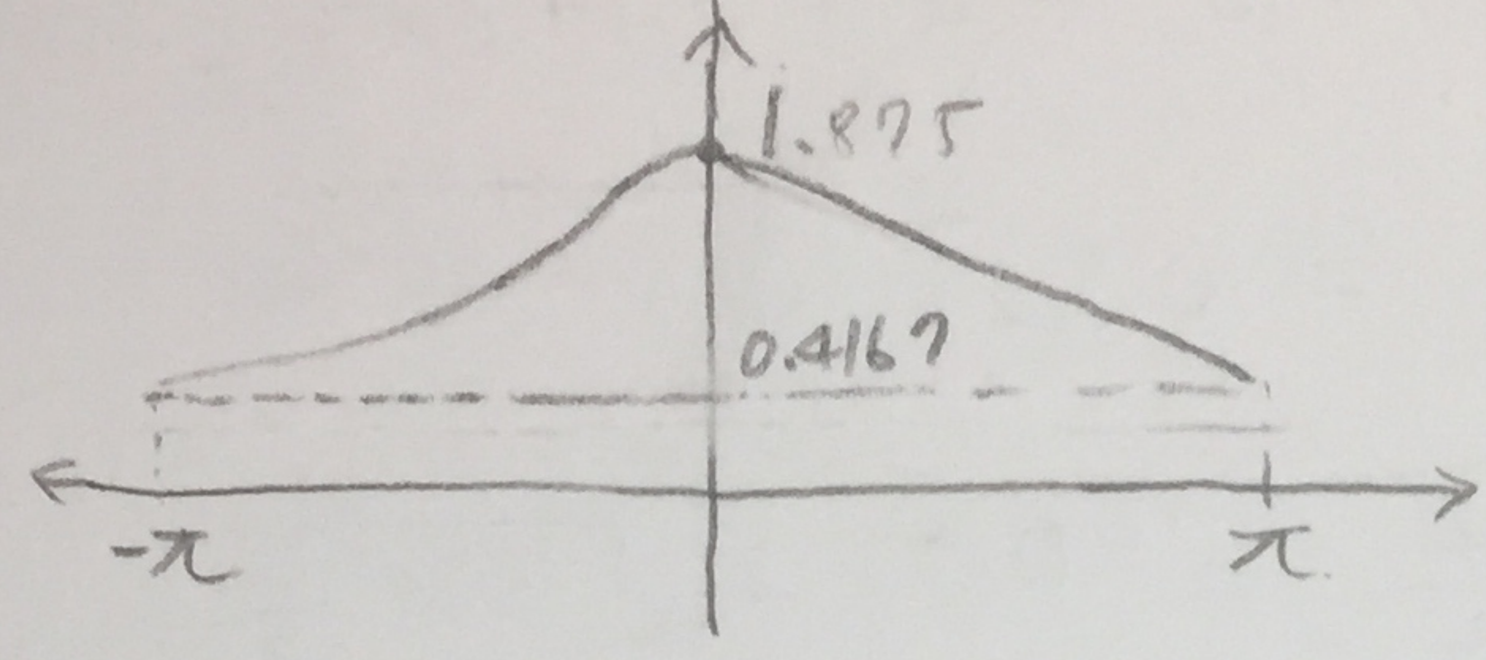



(b) $$H(e^{j\omega}) = \frac{e^{j\omega}+0.5}{e^{j\omega}-0.2}$$ DC gain $\div$ $\omega = 0$

$$H(e^{j\omega})\Big|_{\omega=0} = \frac{1+0.5}{1-0.2} = \frac{1.5}{0.8} = \boxed{1.875}$$

(c) $H(e^{j\omega})\big|_{\omega=\pi} = \dfrac{-1+0.5}{-1-0.2} = 0.4167$

(d)




(e) $x(n) = 2\left(\frac{1}{3}\right)^n u(n)$

$$X(z) = \frac{2}{1-\frac{1}{3}z^{-1}} \qquad H(z) = \frac{1+0.5z^{-1}}{1-0.2z^{-1}}$$

$$Y(z) = \frac{2}{1-\frac{1}{3}z^{-1}} \cdot \frac{1+0.5z^{-1}}{1-0.2z^{-1}} = \frac{A}{1-\frac{1}{3}z^{-1}} \cdot \frac{B}{1-0.2z^{-1}}$$

$$= A\left(\frac{1}{3}\right)^n u(n) + B(0.2)^n u(n)$$

1.9.8

$y(n) = 0.5\,x(n) - 0.5\,x(n-1)$

(a) $Y(z) = 0.5\,X(z) - 0.5z^{-1}X(z) = (0.5 - 0.5z^{-1})X(z)$

$\dfrac{Y(z)}{X(z)} = 0.5 - 0.5z^{-1}$. $\qquad H(z) = 0.5 - 0.5z^{-1}$

(b) $h(n) = 0.5\,\delta(n) - 0.5\,\delta(n-1)$

(c) $H^f(\omega) = H(e^{j\omega}) = 0.5 - 0.5e^{-j\omega}$

(d) $e^{-j\omega} = \cos w - j\sin w$

$H(e^{jw}) = 0.5(1-\cos w) + j\,0.5\sin w$

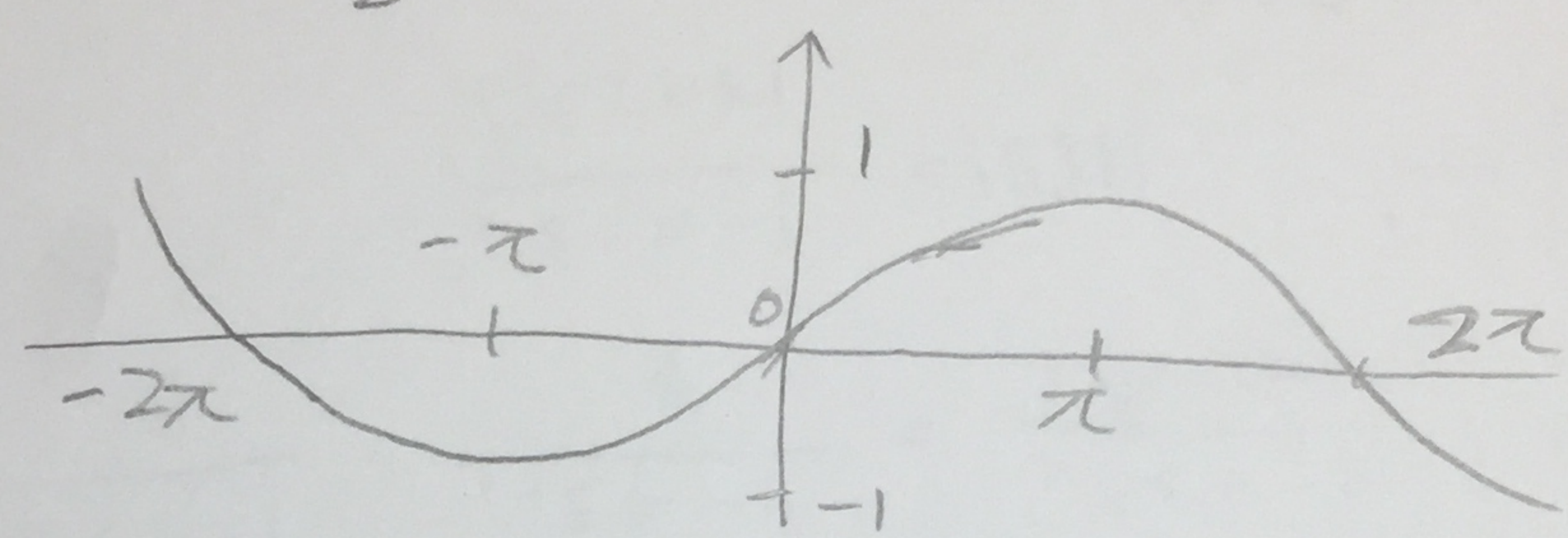
$$|H(e^{j\omega})| = |0.5(1-\cos\omega) + j0.5\sin\omega)|$$

$$= 0.5\sqrt{(1-\cos\omega)^2 + \sin^2\omega} = 0.5\sqrt{1+\cos^2\omega - 2\cos\omega + \sin}$$

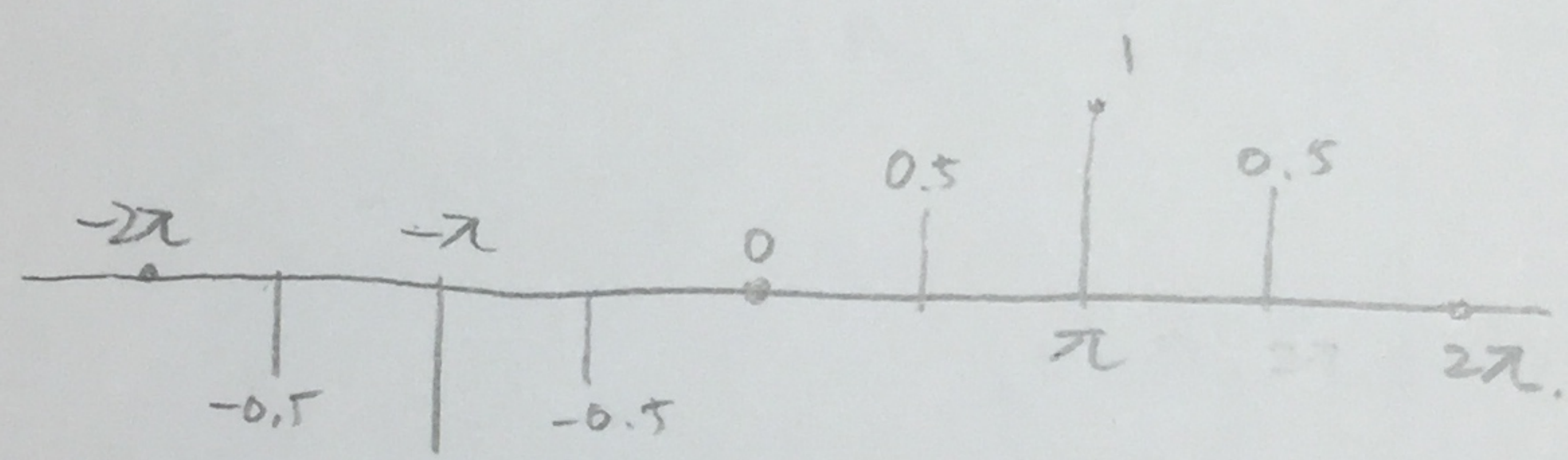
$$= 0.5\sqrt{2-2\cos\omega} = 0.5\sqrt{2}\sqrt{1-\cos\omega}$$

$$= 0.5\sqrt{2}\sqrt{2\sin^2\frac{\omega}{2}} = 0.5\cdot 2\sqrt{\sin^2\frac{\omega}{2}}$$

$$= \sin\frac{\omega}{2}$$

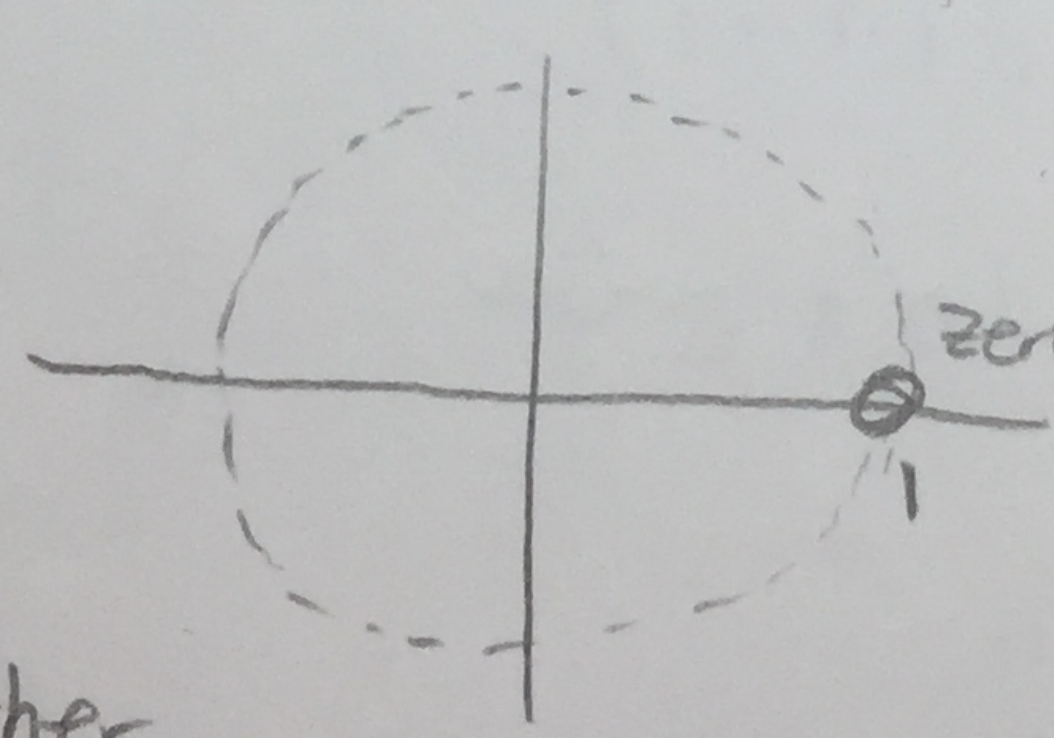



(e)




(f) $H(z) = 0.5 - 0.5z^{-1}$

Zero : $z - 1 = 0$ $\quad$ $z = 1.$




(g) neither

1.10.1

a) $H(z) = k \dfrac{(z-e^{j0.75\pi})(z-e^{-j0.75\pi})}{z^2}$

$= k \dfrac{z^2 - z(e^{j0.75\pi} + e^{-j0.75\pi}) + e^{j0.75\pi} \cdot e^{-j0.75\pi}}{z^2}$

$= k \dfrac{z^2 - 2z\cos(0.75\pi) + 1}{z^2}$

$\therefore H(z) = k(1 - 2\cos(0.75\pi) z^{-1} + z^{-2})$

b) $k(1 - 2\cos(0.75\pi) z^{-1} + z^{-2}) = 1$

$\Rightarrow k = \dfrac{1}{3.4142} = 0.29$

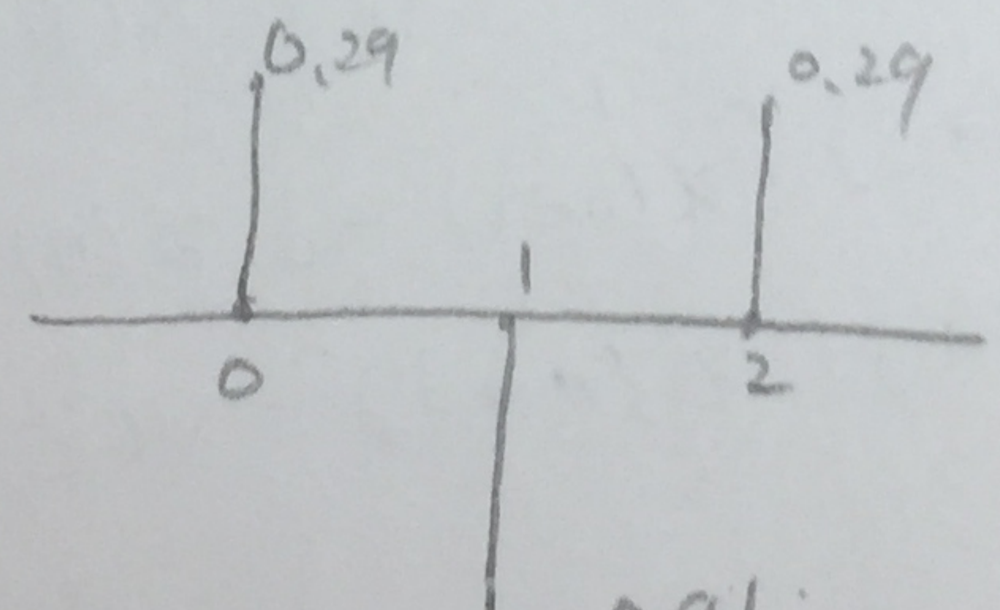
$H(z) = 0.29 - 0.41 z^{-1} + 0.29 z^{-2}$

(a) $\dfrac{Y(z)}{X(z)} = 0.29 - 0.41 z^{-1} + 0.29 z^{-2}$

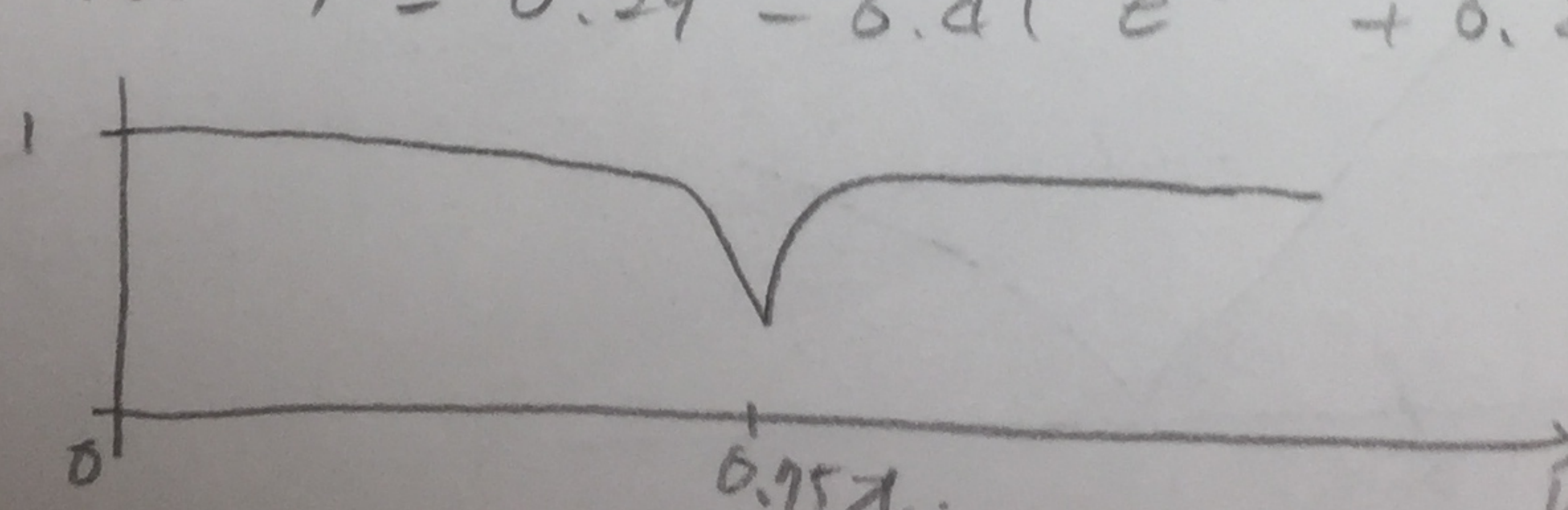
$Y(z) = 0.29 X(z) - 0.41 z^{-1} X(z) + 0.29 z^{-2} X(z)$

$Y(n) = 0.29 x(n) - 0.41 x(n-1) + 0.29 x(n-2)$

(b).




(c) $H(e^{j\omega}) = 0.29 - 0.41 e^{-j\omega} + 0.29 e^{-j2\omega}$



1.10.3

a) $z = j/2$ , $z = z = -j/2$

$$H(e^{j\omega}) = \frac{\phi(e^{j\omega})}{(e^{j\omega} - j/2)(e^{j\omega} + j/2)} = \frac{\phi(e^{j\omega})}{e^{2j\omega} + \frac{1}{4}}$$

b) $\cos(0.5\pi n) \rightarrow \omega_0 = \frac{\pi}{2}$

$$\therefore H(e^{j\omega}) = \frac{(e^{j\omega} - j)k}{(e^{2j\omega} + \frac{1}{4})}$$

c) $H(\omega)\big|_{\omega=0} = 1$ (DC gain)

$$\frac{(1-j)k}{1+\frac{1}{4}} = 1 \Rightarrow k = \frac{4(1+j)}{10} = 0.4(1+j)$$

$$\therefore H(e^{j\omega}) = \frac{0.4(1+j)(e^{j\omega} - j)}{(e^{2j\omega} + \frac{1}{4})}$$

(a) $$\frac{Y(e^{j\omega})}{X(e^{j\omega})} = \frac{0.4(1+j)(e^{j\omega} - j)}{(e^{2j\omega} + \frac{1}{4})}$$

$Y(e^{j\omega})(0.25 + e^{2j\omega}) = [0.4(1+j)(e^{j\omega} - j)]\, X(e^{j\omega})$

$0.25\, y(n) + y(n+2) = 0.4(1+j)[x(n+1) - j\, x(n)]$

$\therefore y(n) + 4y(n+2) = 0.16(1+j)\, x(n+1) - 0.16(j-1)\, x(n)$

(b)

(b) $|H(\omega)|\Big|_{\omega=\pi} = \left| \dfrac{0.4\,(1+j)(e^{j\pi}-j)}{(e^{2j\pi}+\frac{1}{4})} \right|$

$= |-0.64j| = 0.64$

(c) $H(\omega)\Big|_{\omega=\pi/2} = \dfrac{0.4\,(1+j)\,(e^{j\pi/2}-j)}{(e^{2j\frac{\pi}{2}}+\frac{1}{4})}$

$= -4.24\times10^{-4} - (4.25\times10^{-4})j$

$= 6\times10^{-4} \angle -134.9°$

$\Rightarrow 6\times10^{-4}\sin(0.5\pi - 0.75\pi)$

1.10.4

a) $A\cos(\omega_0 n) \Rightarrow \boxed{H(\omega)} \Rightarrow A|H(\omega)|\cos(n\omega_0 + \angle H(\omega)\big|_{\omega=\omega_0})$

$(-1)^n = \cos(n\pi) \Rightarrow \omega_0 = \pi$

$H(\omega)\big|_{\omega_0=\pi} = 0$

$\cos(0.5\pi n) \Rightarrow \omega_0 = \frac{\pi}{2}$ $\quad H(\omega)\big|_{\omega_0=\frac{\pi}{2}} = 0$

$\therefore H(e^{j\omega}) = k\,(e^{j\omega}+1)(e^{2j\omega}+1)$

b) $H(\omega)\big|_{\omega=0} = 1 \Rightarrow k(2)(2) = 1$ $\quad k = \frac{1}{4}$

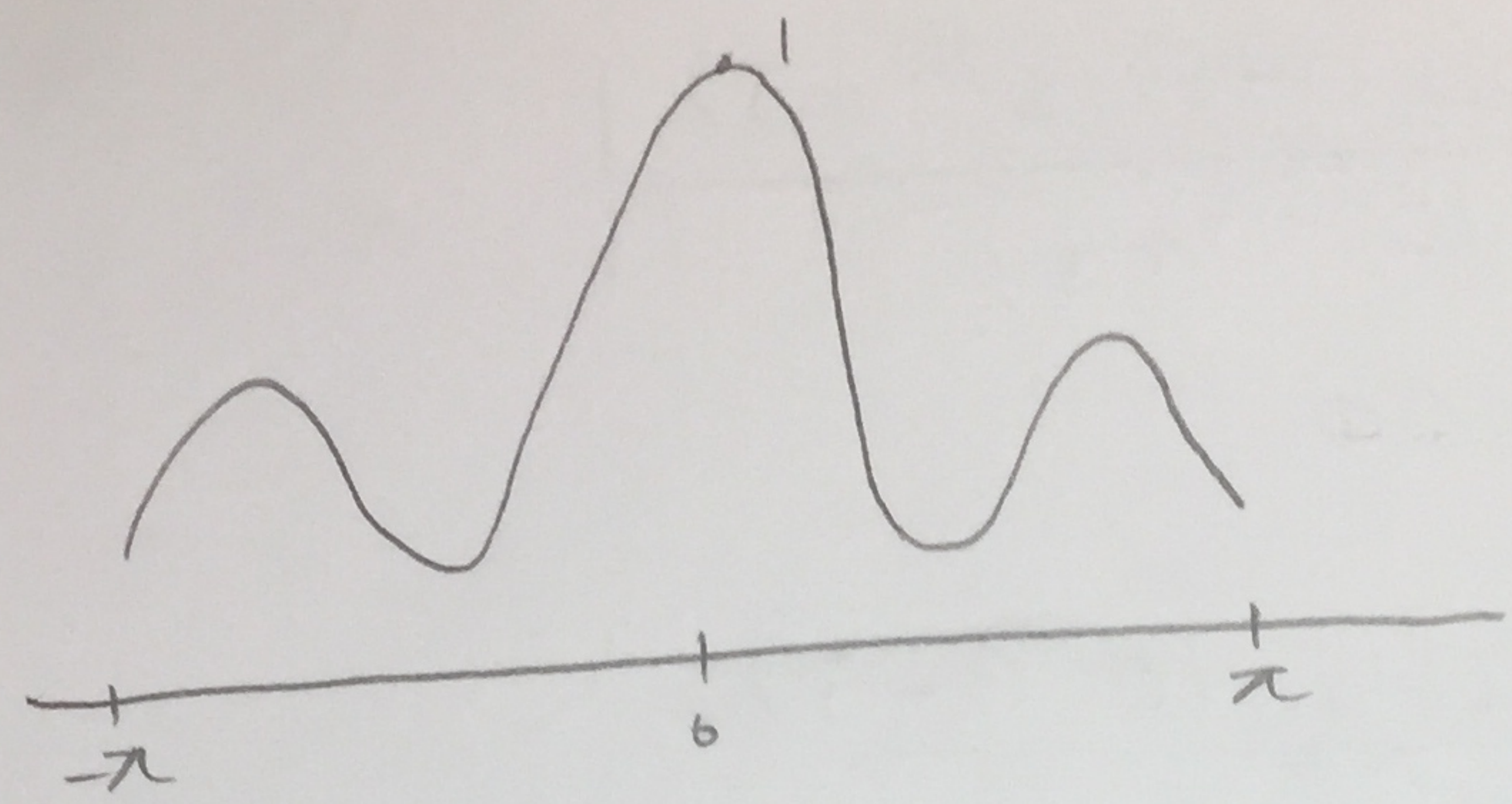
$\therefore H(e^{j\omega}) = \dfrac{(e^{j\omega}+1)(e^{2j\omega}+1)}{4}$

(a) $\dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{(e^{j\omega}+1)(e^{2j\omega}+1)}{4}$

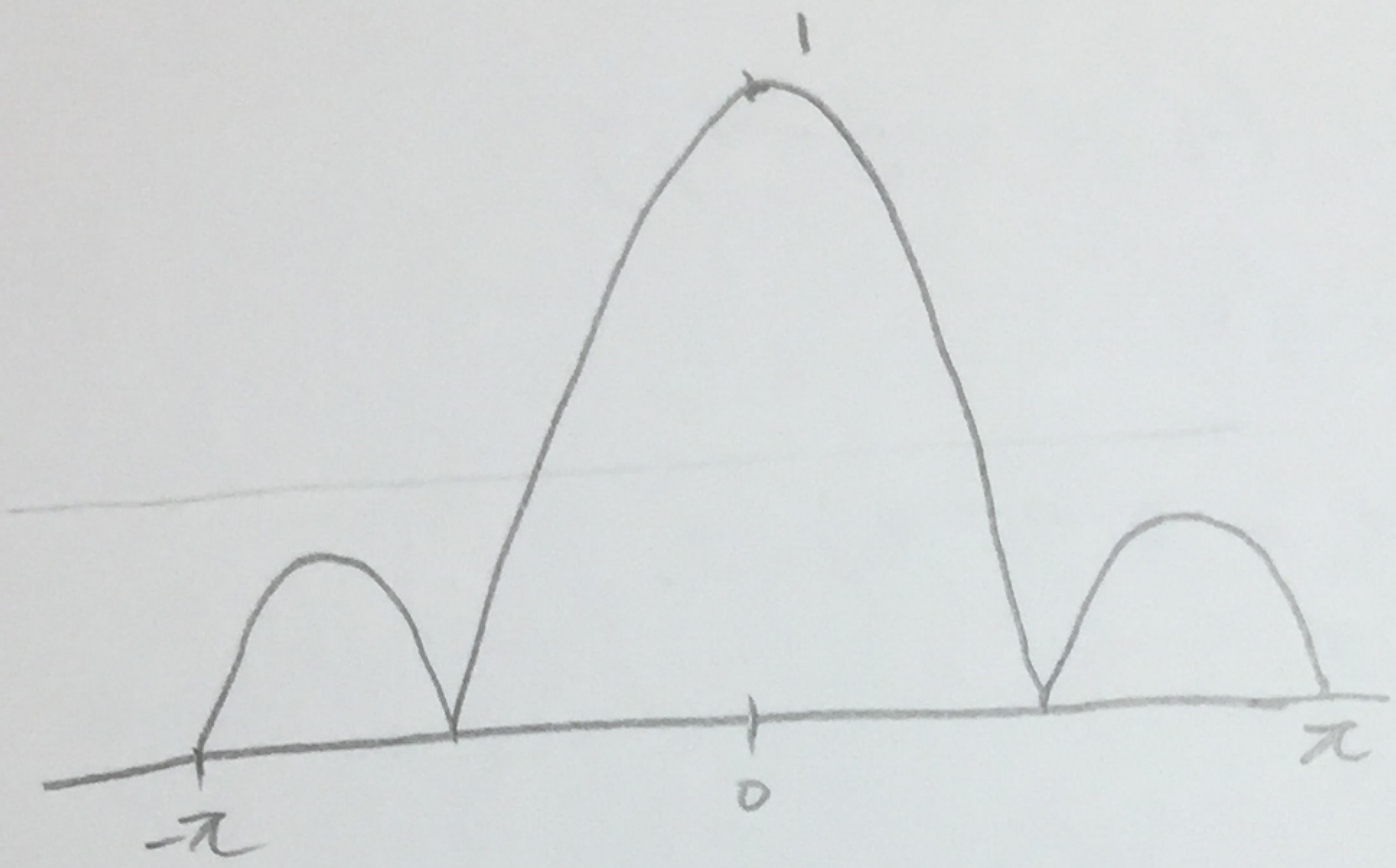
$4Y(e^{j\omega}) = [e^{3j\omega}+e^{j\omega}+e^{2j\omega}+1]\,X(e^{j\omega})$

$\therefore 4y(n) = x(n+3) + x(n+2) + x(n+1) + x(n)$

(b)




(c)




1.11.1

| Impulse Response | Frequency Response | Pole-Zero Diagram |
|---|---|---|
| 1 | 4 | 1 |
| 2 | 2 | 3 |
| 3 | 1 | 2 |
| 4 | 3 | 4 |