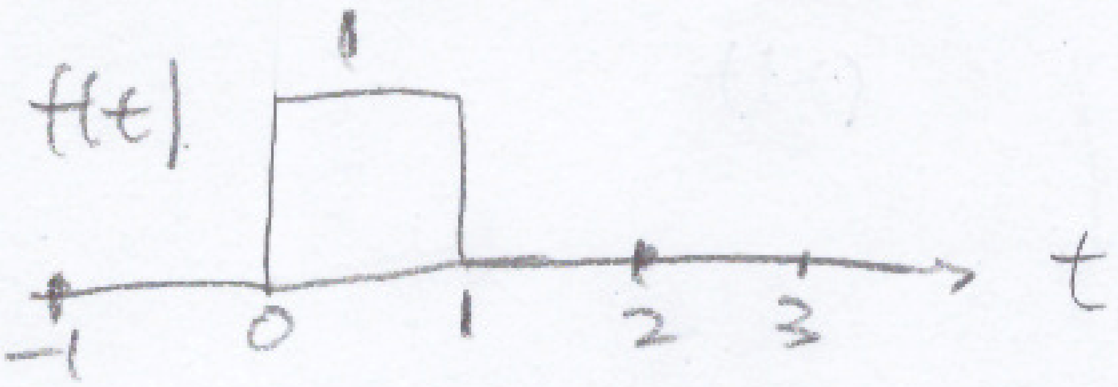
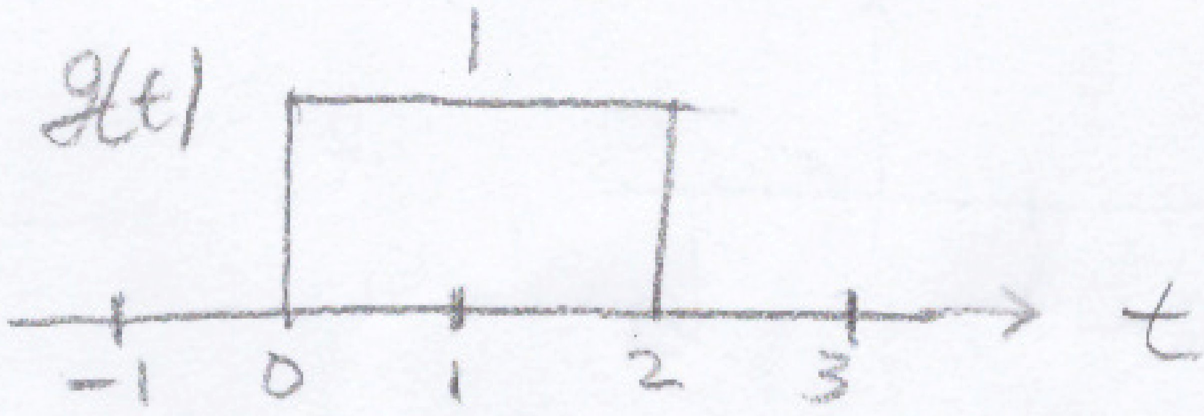
Jaeha Huh

2.3.2.





$v(t) = f(t) * g(t)$

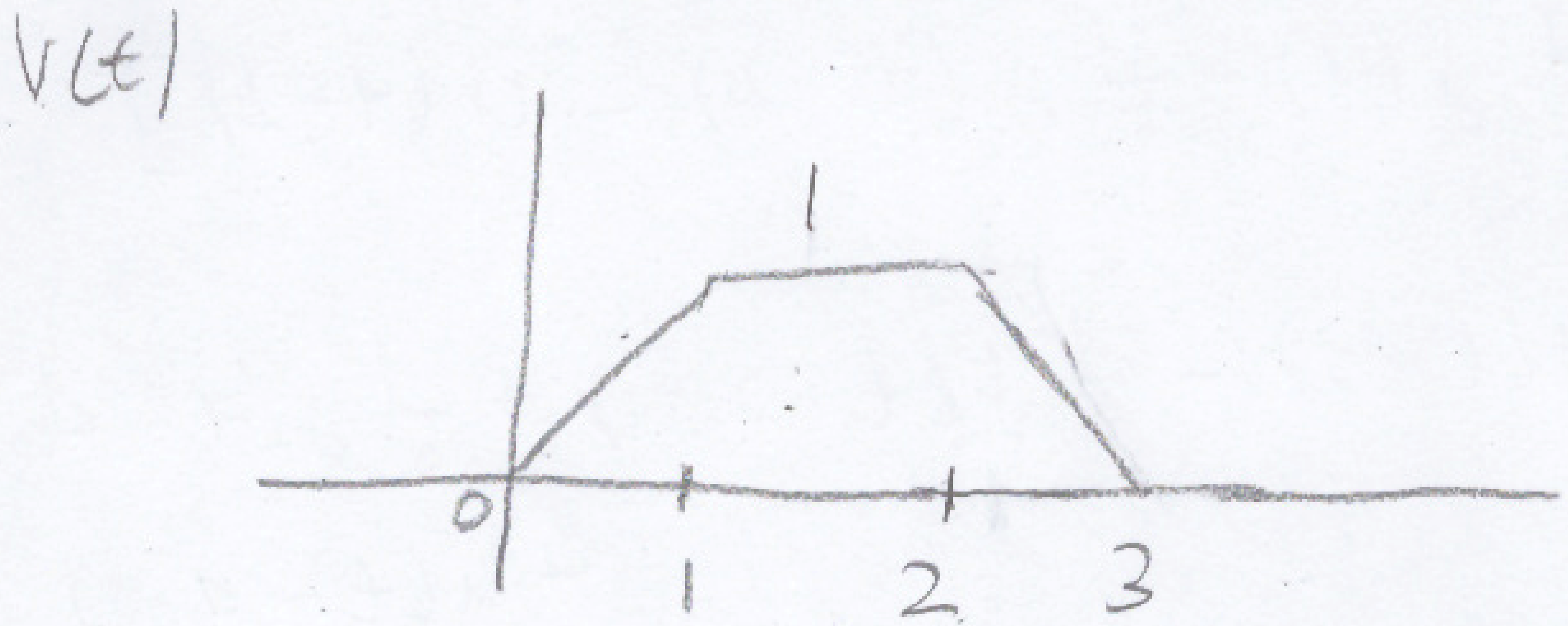
(i) for $t < 0 \Rightarrow$ $v(t) = 0$

(ii) for $0 < t < 1$, $v(t) = \int_0^t (1)\, dt = t$

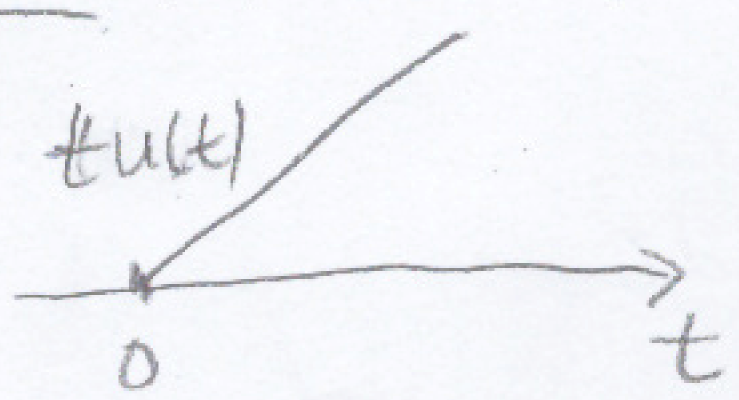
(iii) for $1 < t < 2$, $v(t) = 1$

(iv) for $2 < t < 3$, $v(t) = -t$

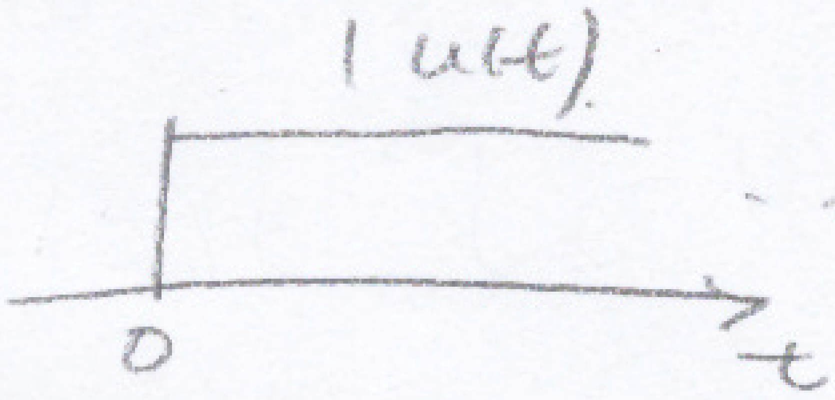
(v) for $t > 3$, $v(t) = 0$



2.3.4



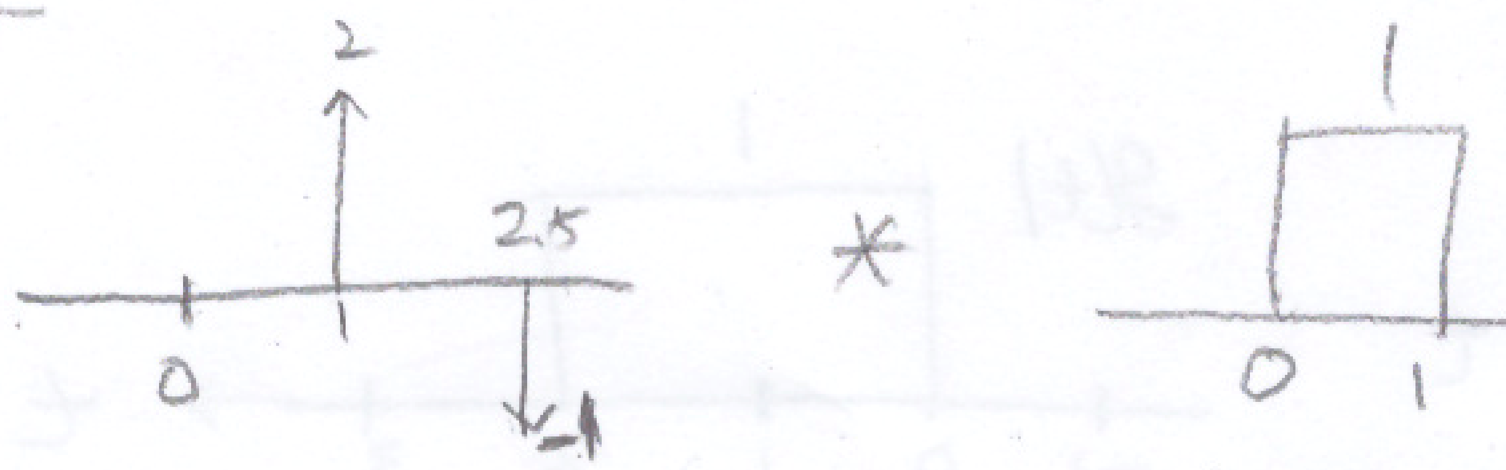
$*$



$$\boxed{\therefore\ t \cdot u(t) * u(t) = \frac{t^2}{2} u(t)}$$

2.3.5

(a)



$$[2\delta(t-1) - \delta(t-2.5)] * [u(t) - u(t-1)]$$

$$= 2u(t-1) - 2(t-2) - u(t-2.5) + u(t-3.5)$$

(b)

$$[2\delta(t-1) - \delta(t-2.5)] * [\tfrac{1}{2}\delta(t) - \tfrac{1}{2}\delta(t-2) - u(t-2)]$$

$$= \delta(t-1) - \delta(t-3) - 2u(t-3) - \tfrac{1}{2}\delta(t-2.5) + \tfrac{1}{2}\delta(t-4.5) + u(t-4.5)$$

2.3.6

$x(t) = f(t) * g(t)$

$f(t) = u(t) \qquad g(t) = 3e^{-2t}u(t)$

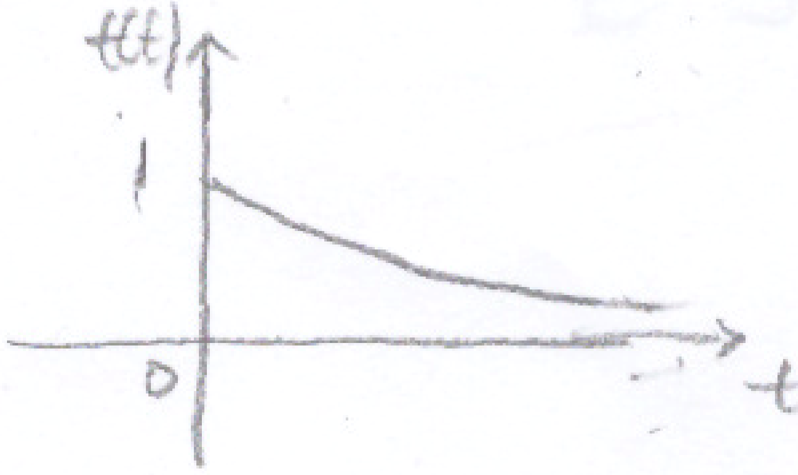
$$x(t) = f(t) * g(t) = \int_{-\infty}^{\infty} g(\tau) f(t-\tau)\, d\tau$$

$$= \int_0^t 3e^{-2t}\, dt = -\frac{3}{2}[e^{-2t} - 1]$$

$$x(t) = \frac{3}{2}(1 - e^{-2t})$$

2.3.8

$f(t) = e^{-t} u(t) \qquad g(t) = e^{-t} u(t)$



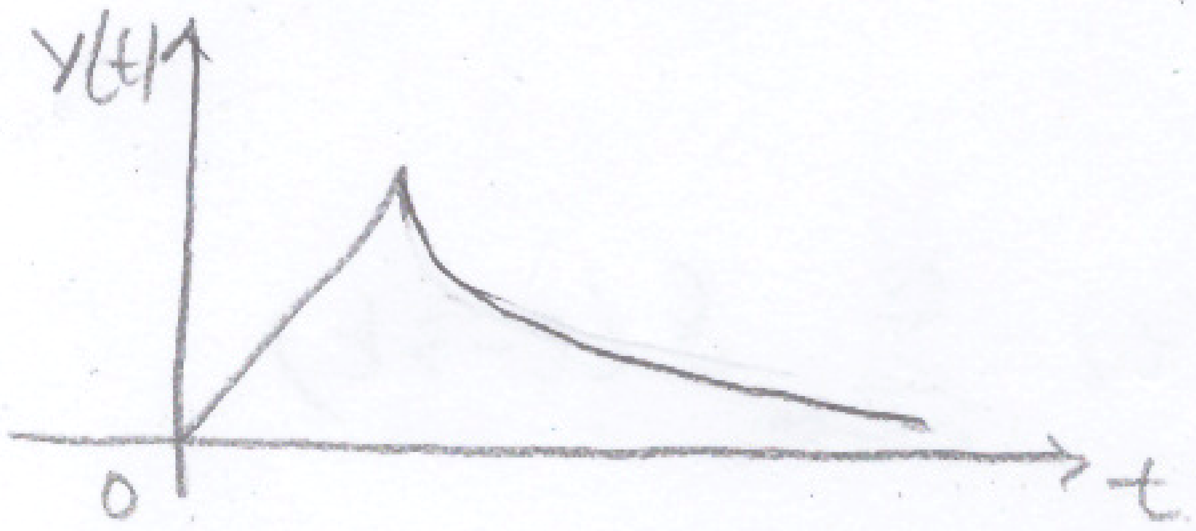
$y(t) = f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$

$= \int_0^t (e^{-\tau}) \cdot e^{-(t-\tau)}\, d\tau$

$= e^{-t} \int_0^t e^{-\tau} \cdot e^{\tau}\, d\tau$

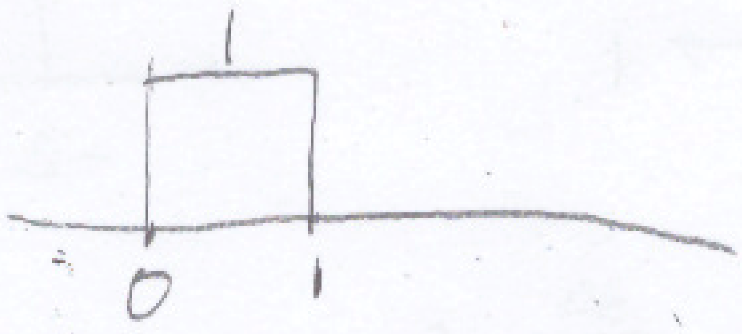
$= e^{-t} \int_0^t d\tau$

$y(t) = t e^{-t} u(t)$



2.3.10

$g(t) = e^{-2t} u(t) \qquad f(t)$



$y(t) = \int_{-\infty}^{\infty} f(\tau)\, g(t-\tau)\, d\tau$

(i) $t < 0$, $\quad y(t) = 0$

(ii) $0 \leq t < 1$,

$y(t) = \int_0^t e^{-2(t-\tau)}\, d\tau = e^{-2t} \int_0^t e^{2\tau}\, d\tau$

$= \frac{1}{2} e^{-2t} [e^{2t} - 1]$

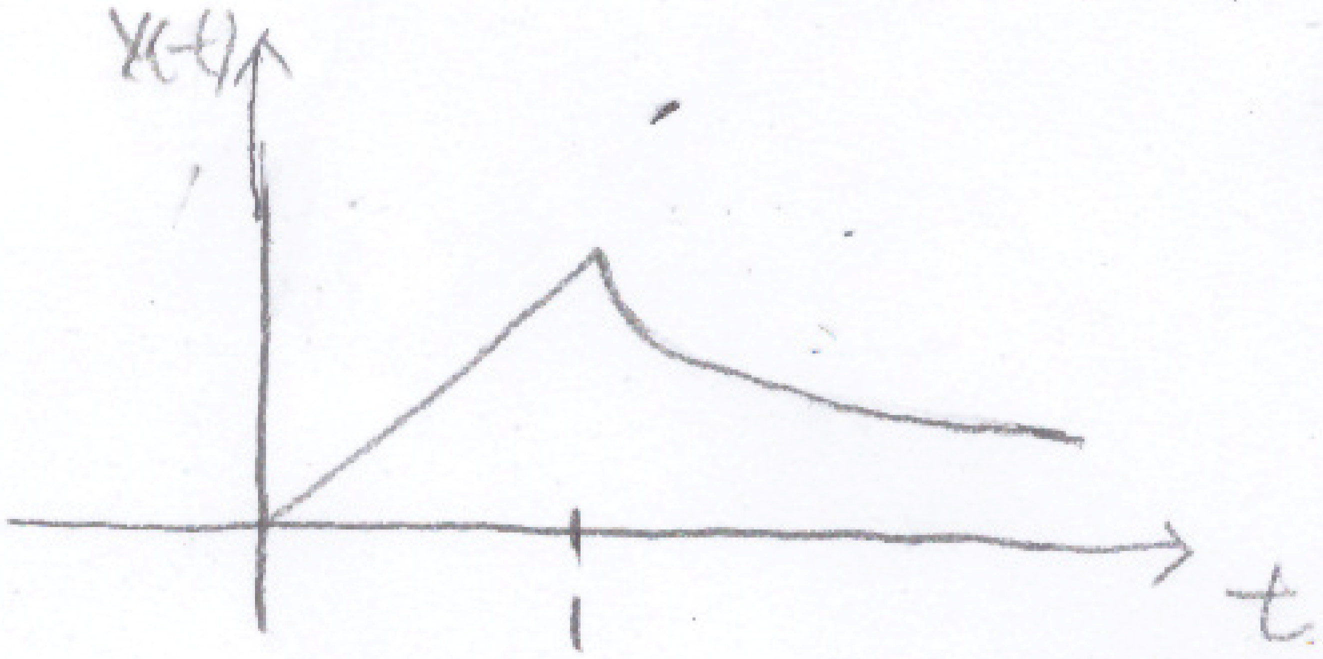
$y(t) = \frac{1}{2} [1 - e^{-2t}]$

(iii) $t \geq 1$

$$Y(t) = \int_0^1 e^{-2(t-\tau)} d\tau = \frac{1}{2} e^{-2t} [e^2 - e^0]$$

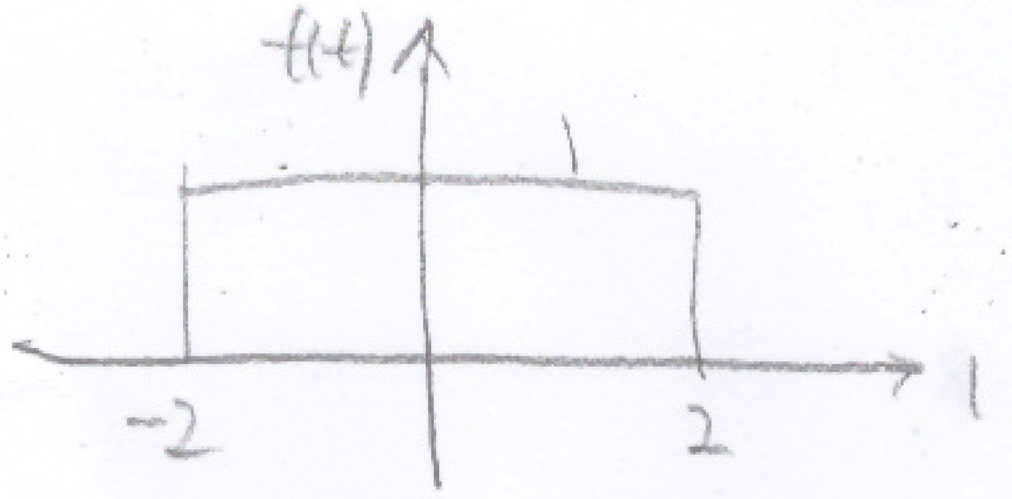
$$= \frac{1}{2} e^{-2t} [e^2 - 1]$$

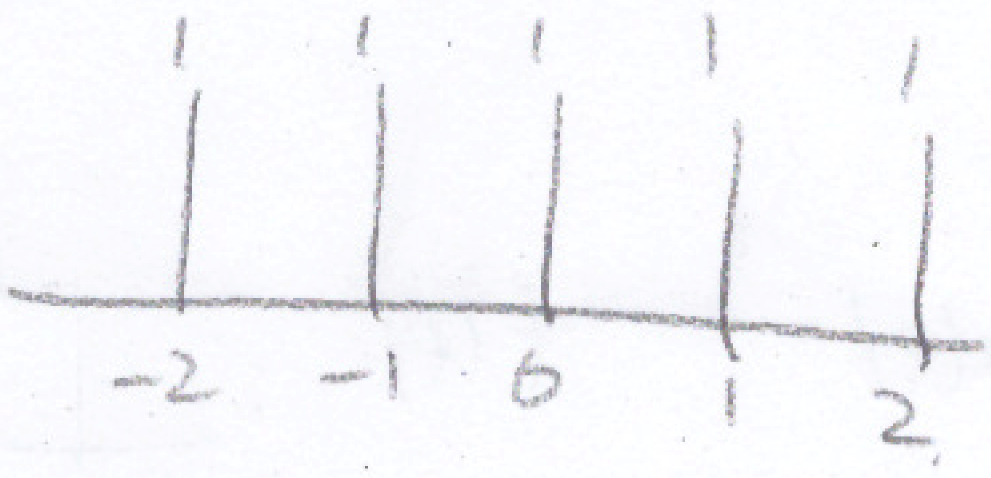
$$\Rightarrow \quad Y(t) \begin{cases} 0, & t < 0 \\ \frac{1}{2}(1 - e^{-2t}) & 0 \leq t < 1 \\ \frac{1}{2} e^{-2t} [e^2 - 1] & t \geq 1 \end{cases}$$



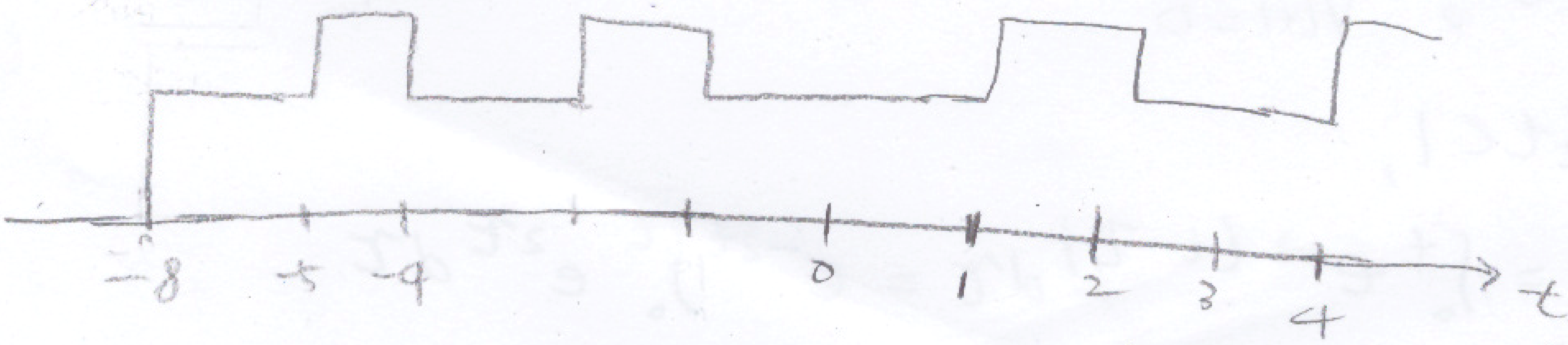
2.3.11

$$f(t) = u(t+2) - u(t-2), \quad g(t) = \sum_{k=-\infty}^{\infty} \delta(t - 3k)$$



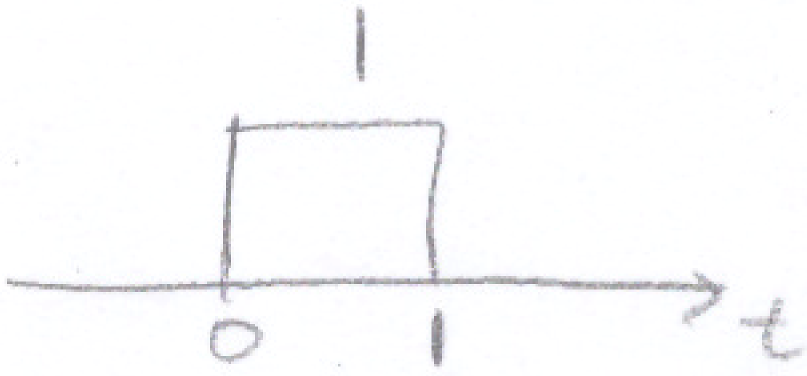


$$Y(t) = f(t) * g(t) = \sum_{-\infty}^{\infty} f(t - 3k)$$

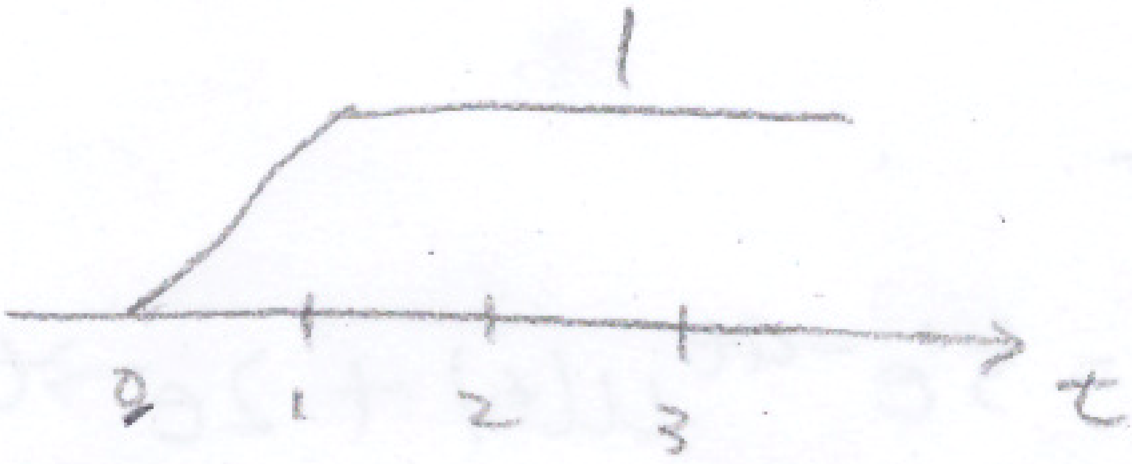


2.3.12

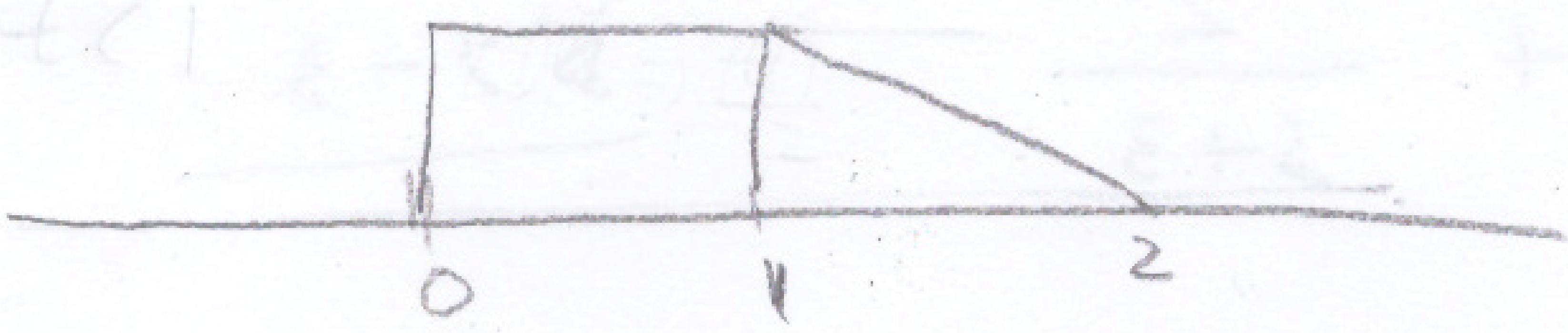
$f(t)$



$g(t)$

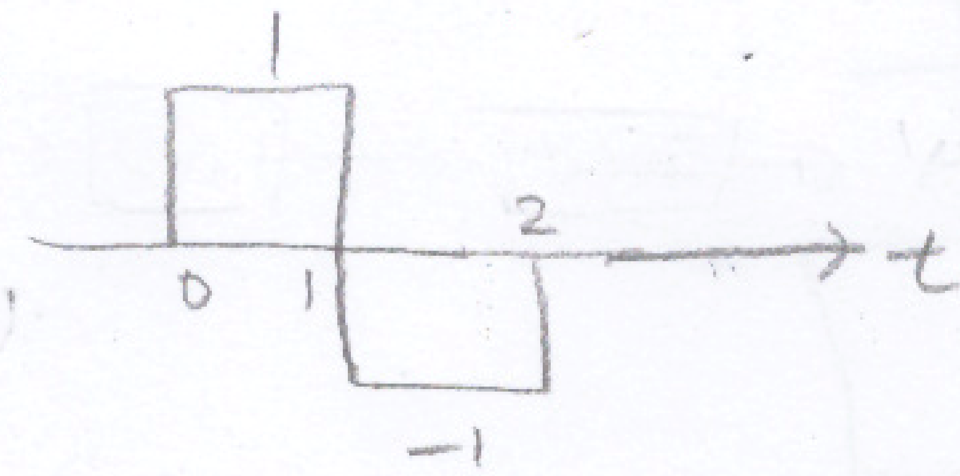


$f(t) * g(t)$

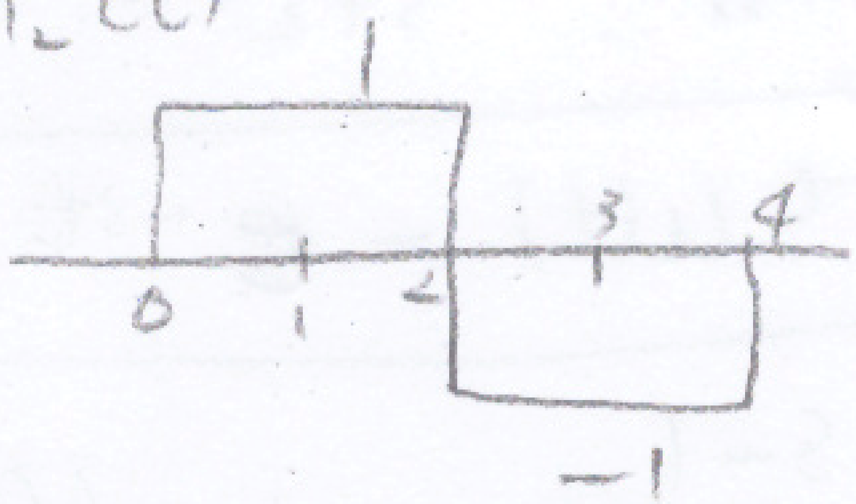


2.3.13

$h_1(t)$



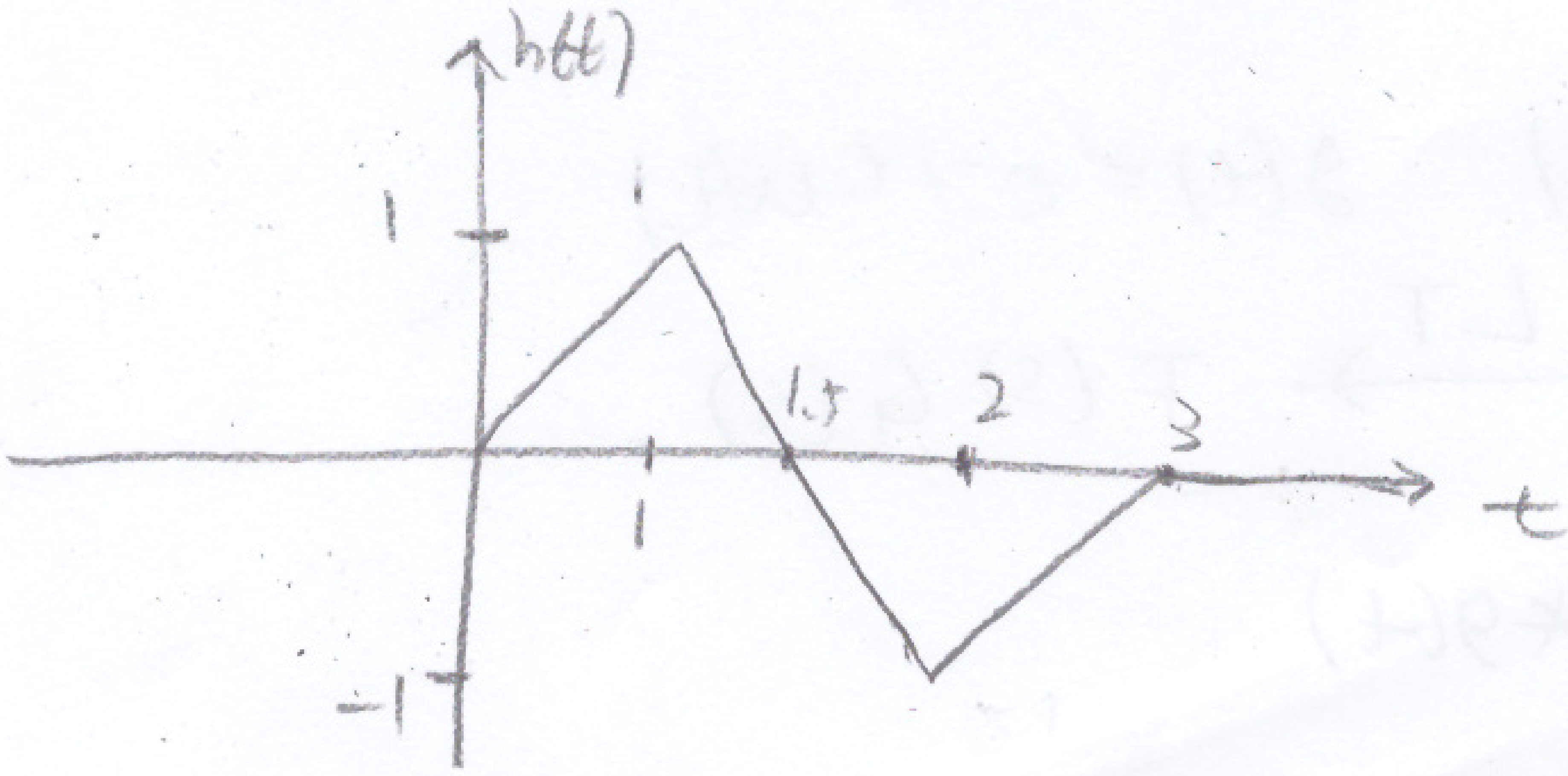
$h_2(t)$



$h(t) = h_1(t) * h_2(t)$

$$h(t) \begin{cases} t, & 0 \le t < 1 \\ -2t+3, & 1 \le t < 2 \\ t-3, & 2 \le t < 3 \\ 0, & \text{otherwise} \end{cases}$$

$h(t)$



2.4.1

$h(t) = 5e^{-4t}u(t) + 2e^{-3t}u(-t)$

$H(s) = \frac{5}{s+4} + \frac{2}{s+3}$ ROC $s > -3$

2.4.2

(a) $H(s) = \frac{s+4}{(s+2)(s+3)} = \frac{2}{s+2} + \frac{-1}{s+3}$

$\boxed{h(t) = 2e^{-t}u(t) - e^{-3t}u(t)}$

(b) $H(s) = \frac{s-1}{2s^2+3s+1} = \frac{1}{2}\frac{(s-1)}{(s+\frac{1}{2})(s+1)} = \frac{A}{s+\frac{1}{2}} + \frac{B}{s+1}$

$= \frac{-\frac{3}{2}}{s+\frac{1}{2}} + \frac{-2}{s+1}$

$\boxed{h(t) = -\frac{3}{2}e^{-\frac{t}{2}}u(t) - 2e^{-t}u(t)}$

2.4.3

$f(t) = e^{-t}u(t)$ $g(t) = e^{-2t}u(t)$

$f(t) * g(t) \xrightarrow{L.T} F(s)\,G(s)$

$y(t) = f(t) * g(t)$

2.4.3

$f(t) \longrightarrow F(s) = \frac{1}{s+1}$

$g(t) \longrightarrow G(s) = \frac{1}{s+2}$

$Y(s) = F(s) \cdot G(s) = \frac{1}{s+1} \cdot \frac{1}{s+2} = \frac{1}{(s+1)(s+2)}$

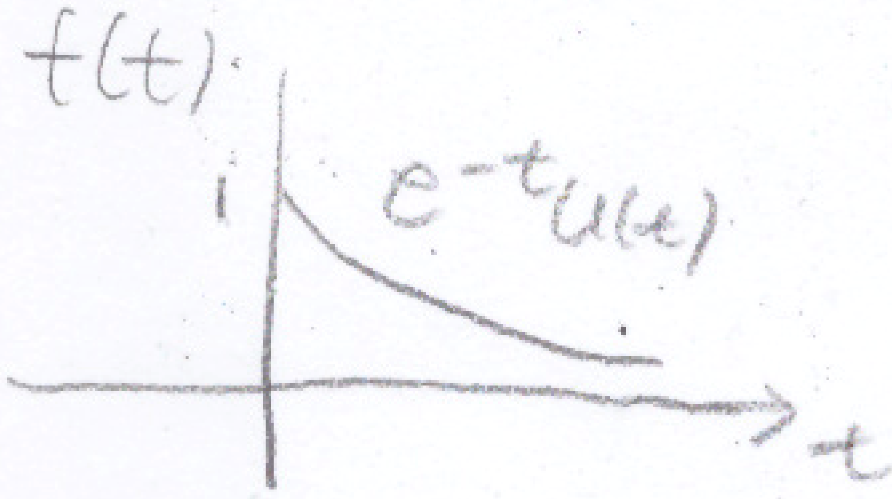
$Y(s) = \frac{A}{s+1} + \frac{B}{s+2} \qquad A = \left.\frac{1}{s+2}\right|_{s=-1}, \quad A = 1$

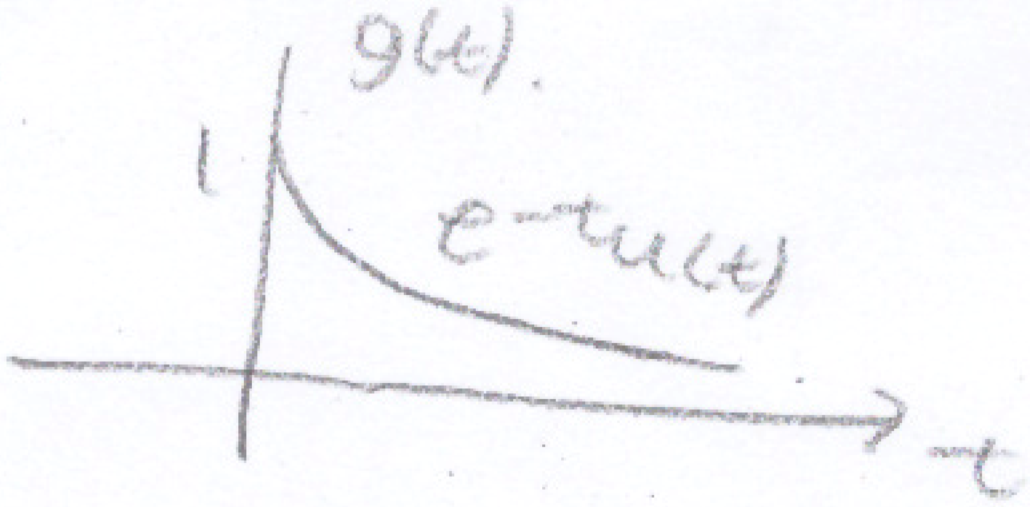
$B = \left.\frac{1}{s+1}\right|_{s=-2}, \quad B = -1$

$\Rightarrow Y(s) = \frac{1}{s+1} - \frac{1}{s+2}$

$$\boxed{y(t) = e^{-t}u(t) - e^{-2t}u(t)}$$

2.4.4





$y(t) = f(t) * g(t)$

$F(s) = \frac{1}{s+1}$; $G(s) = \frac{1}{s+1}$ $\qquad Y(s) = F(s) \cdot G(s) = \frac{1}{(s+1)^2}$

$\therefore y(t) = t e^{-t} u(t)$