HW9 Jaeha Huh.

2.4.5

$h(t) = \left(\frac{1}{2}\right)^t u(t)$ $\qquad L(e^{at}) = \frac{1}{s-a}$

$$L\{e^{\ln(\frac{1}{2})t}\} = \frac{1}{s - \ln(\frac{1}{2})}$$

$$\therefore \boxed{H(s) = \frac{1}{s - \ln(\frac{1}{2})}}$$

2.4.6

$r(t) = t\,u(t)$ $\qquad g(t) = r(t-2) = (t-2)\,u(t)$

$$r(t) \xrightarrow{L.T} R(s) = \frac{1}{s^2}$$

$$g(t) \xrightarrow{L.T} G(s) = e^{-2s}\,R(s)$$

$$\boxed{\therefore\ G(s) = \frac{e^{-2s}}{s^2}}$$

2.5.1

$h(t) = 3e^{-2t}u(t) - e^{-3t}u(t)$ $\qquad h(t) \xrightarrow{L.T} H(s)$

(a) $3e^{-2t}u(t) \xrightarrow{L.T} \frac{3}{s+2}$, $\quad e^{-3t}u(t) \xrightarrow{L.T} \frac{1}{s+3}$

$$H(s) = \frac{3}{s+2} - \frac{1}{s+3} = \frac{3(s+3) - (s+2)}{(s+2)(s+3)} = \frac{3s+9-s-2}{(s+2)(s+3)}$$

$$\therefore \boxed{H(s) = \frac{2s+7}{(s+2)(s+3)}}$$

(b) $H(s) = \frac{Y(s)}{X(s)} = \frac{2s+7}{(s+3)(s+2)} = \frac{2s+7}{s^2+5s+6}$

$Y(s)(s^2+5s+6) = X(s)(2s+7)$

$s^2Y(s) + 5sY(s) + 6Y(s) = 2sX(s) + 7X(s)$

⇒ Use Inverse Laplace Transforms

⇒ $\boxed{y''(t) + 5y'(t) + 6y(t) = 2x'(t) + 7x(t)}$

(c) poles : $s_1 = -2$, $s_2 = -3$ lie on left side of s-plane

∴ $\boxed{\text{The system is stable}}$

2.5.2

$y''(t) + y'(t) - 2y(t) = x(t)$

(a) $s^2Y(s) + sY(s) - 2Y(s) = X(s)$

$(s^2+s-2)Y(s) = X(s)$

∴ $\boxed{H(s) = \frac{Y(s)}{X(s)} = \frac{1}{s^2+s-2}}$

(b) $H(s) = \frac{1}{s^2+s-2} = \frac{1}{(s+2)(s-1)} = \frac{A}{s+2} + \frac{B}{s-1}$

$A = \frac{1}{s-1}\Big|_{s=-2} = -\frac{1}{3}$ $\quad B = \frac{1}{s+2}\Big|_{s=1} = \frac{1}{3}$

$H(s) = \frac{1}{3}\left(-\frac{1}{s+2} + \frac{1}{s-1}\right) \xrightarrow{\text{inverse LT}} h(t) = \frac{1}{3}(e^{t} - e^{-2t})u(t)$

∴ $\boxed{h(t) = \frac{1}{3}(e^{t} - e^{-2t})u(t)}$

(c) $s=1$ is right side of s-plane

∴ $\boxed{\text{The system is unstable}}$

2.5.3

$$h(t) = \delta(t) + 2e^{-t}u(t) - e^{-2t}u(t) \xrightarrow{L.T}$$

$$H(s) = 1 + \frac{2}{s+1} - \frac{1}{s+2} = 1 + \frac{2s+4-s-1}{(s+1)(s+2)} = 1 + \frac{s+3}{(s+1)(s+2)}$$

$$H(s) = \frac{Y(s)}{X(s)} = \frac{s^2+4s+5}{s^2+3s+2}$$

$$\Rightarrow s^2Y(s) + 3sY(s) + 2Y(s) = s^2X(s) + 4sX(s) + 5X(s)$$

$$\boxed{y''(t) + 3y'(t) + 2y(t) = x''(t) + 4x'(t) + 5x(t)}$$

2.5.4

(a) $2y'(t) + y(t) = 3x(t) \xrightarrow{L.T} 2sY(s) + Y(s) = 3X(s)$

$$Y(s) = \frac{3(x)}{2s+1}, \quad x(t) = 5e^{-3t}u(t) \xrightarrow{L.T} \frac{5}{s+3}$$

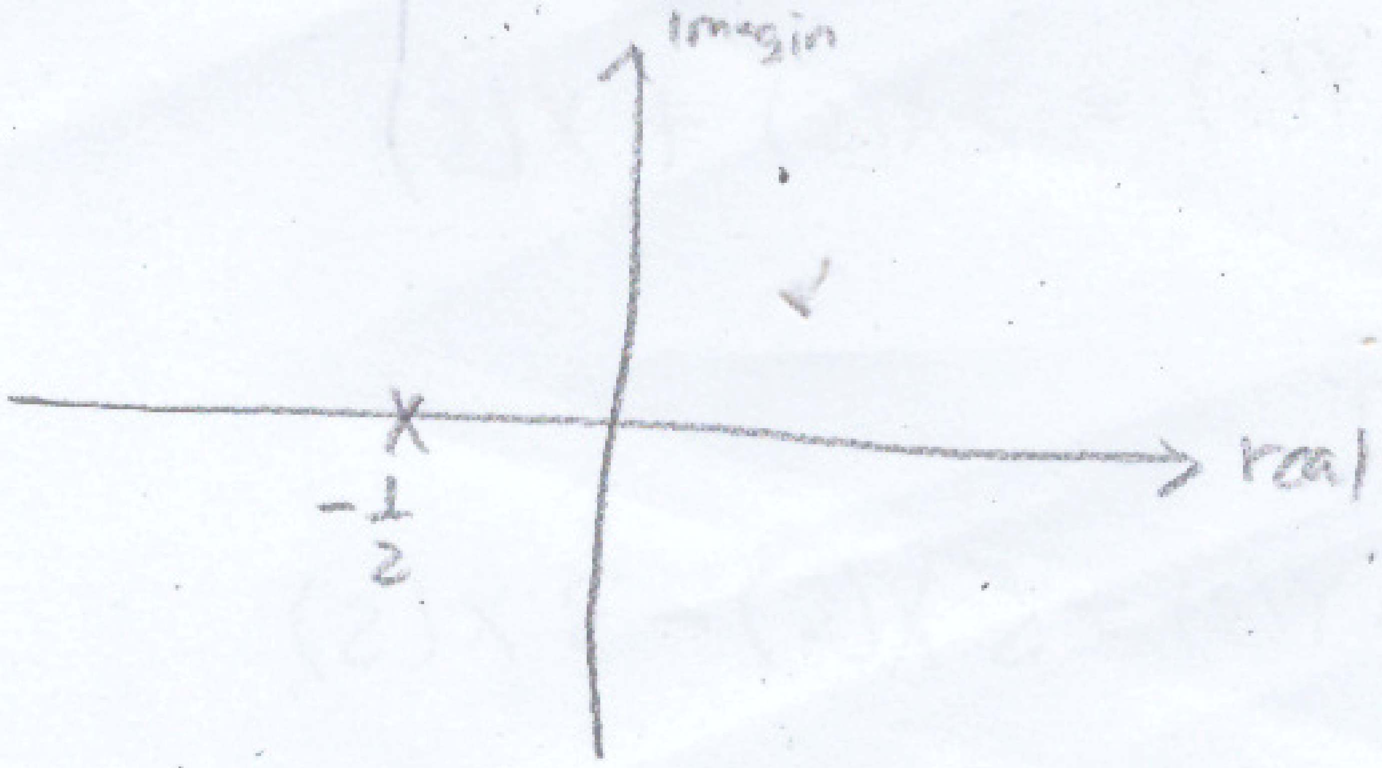
$$Y(s) = \frac{3}{2s+1}\left(\frac{5}{s+3}\right) = \frac{15}{(2s+1)(s+3)} = 3\left(\frac{1}{2s+1} - \frac{1}{s+3}\right)$$

$$\Rightarrow \boxed{y(t) = 3\left(e^{-\frac{1}{2}} - e^{-3t}\right)u(t)}$$

(b) $H(s) = \dfrac{3}{2s+1} \Rightarrow s = -\frac{1}{2}$ (left side of s-plane)

$\therefore$ $\boxed{\text{The system is stable}}$

(c) $H(s) = \dfrac{3}{2s+1} \Rightarrow s = -\frac{1}{2}$ (pole)



2.5.5

$$y''(t) + 4y'(t) + 3y(t) = 2x'(t) + 3x(t)$$

$$s^2 Y(s) + 4s Y(s) + 3Y(s) = 2s X(s) + 3X(s)$$

(a) $$H(s) = \frac{Y(s)}{X(s)} = \frac{2s+3}{s^2+4s+3}$$

$$x(t) = 3e^{-2t}u(t) \xrightarrow{L.T} X(s) = \frac{3}{s+2}$$

$$Y(s) = \frac{2s+3}{s^2+4s+3} X(s) = \frac{2s+3}{(s+1)(s+3)} \cdot \frac{3}{s+2} = \frac{6s+9}{(s+1)(s+2)(s+3)}$$

$$Y(s) = \frac{3/2}{s+1} + \frac{3}{s+2} - \frac{9/2}{s+3}$$

$$\therefore \boxed{y(t) = \left(\frac{3}{2}e^{-t} + 3e^{-2t} - \frac{9}{2}e^{-3t}\right)u(t)}$$

(b) $$x(t) = 3u(t) \xrightarrow{L.T} X(s) = \frac{3}{s}$$

$$Y(s) = \frac{2s+3}{s^2+4s+3} X(s) = \frac{2s+3}{(s+1)(s+3)} \cdot \frac{3}{s} = \frac{6s+9}{s(s+1)(s+3)}$$

$$Y(s) = \frac{3}{s} - \frac{3/2}{s+1} - \frac{3/2}{s+3}$$

$$\therefore \boxed{y(t) = \left(3 - \frac{3}{2}e^{-t} - \frac{3}{2}e^{-3t}\right) \cdot u(t)}$$

2.5.12

$$T_1 = y''(t) + 3y'(t) + 7y(t) = 2x'(t) + x(t)$$

$$T_2 = y''(t) + y'(t) + 4y(t) = x'(t) - 3x(t)$$

$$T_1 = s^2 Y(s) + 3s Y(s) + 7Y(s) = 2X(s) + X(s)$$

$$H_1(s) = \frac{Y(s)}{X(s)} = \frac{2s+1}{s^2+3s+7}$$

$$T_2 = s^2 Y(s) + s Y(s) + 4Y(s) = s X(s) - 3X(s)$$

$$H_2(s) = \frac{Y(s)}{X(s)} = \frac{s-3}{s^2+s+4}$$

(a) $x(t) \rightarrow \boxed{H_1} \rightarrow \boxed{H_2} \rightarrow Y(t)$

$$H_1(s) = \frac{2s+1}{s^2+3s+7}, \quad H_2 = \frac{s-3}{s^2+s+4}$$

$$H_{Tot}(s) = H_1(s) \cdot H_2(s) = \frac{2s+1}{s^2+3s+7} \cdot \frac{s-3}{s^2+s+4}$$

$$(2s+1)(s-3) = 2s^2-5s-3$$

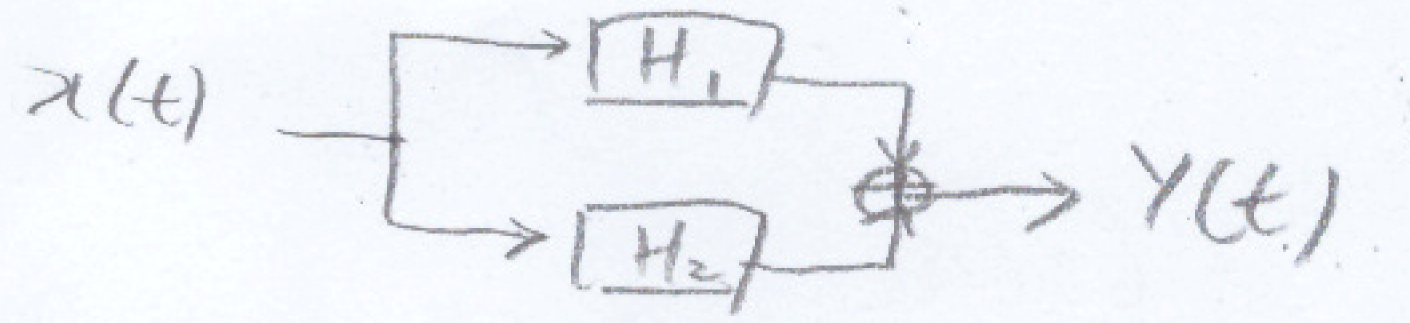
$$(s^2+3s+7)(s^2+s+4) = s^4+4s^3+14s^2+19s+28$$

$$H_{Tot}(s) = \frac{2s^2-5s-3}{s^4+4s^3+14s^2+19s+28} = \frac{Y(s)}{X(s)}$$

$$s^4Y(s) + 4s^3Y(s) + 14s^2Y(s) + 19sY(s) + 28Y(s)$$
$$= 2s^2X(s) - 5sX(s) - 3X(s)$$

$$\therefore y^{(4)}(t) + 4y'''(t) + 14y''(t) + 19y'(t) + 28y(t)$$
$$= 2x''(t) - 5x'(t) - 3x(t)$$

(b).



$$H_{Tot}(s) = H_1(s) + H_2(s) = \frac{2s+1}{s^2+3s+7} + \frac{s-3}{s^2+s+4}$$

$$= \frac{(2s+1)(s^2+s+4) + (s-3)(s^2+3s+7)}{(s^2+3s+7)(s^2+s+4)} = \frac{3s^2+3s^2+7s-17}{s^4+4s^3+14s^2+19s+28}$$

$$s^4Y(s) + 4s^3Y(s) + 14s^2Y(s) + 19sY(s) + 28Y(s)$$
$$= 3s^3X(s) + 3s^2X(s) + 7sX(s) - 17X(s)$$

$$\therefore y^{(4)}(t) + 4y'''(t) + 14y''(t) + 19y'(t) + 28y(t)$$
$$= 3x'''(t) + 3x''(t) + 7x'(t) - 17x(t)$$

2.5.14

$h(t) = 3 \cdot e^{-t}u(t) + 2e^{-2t}u(t) + e^{-t}u(t)$

$\Rightarrow h(t) = 4e^{-t}u(t) + 2e^{-2t}u(t)$

(a) $H(s) = \frac{4}{s+1} + \frac{2}{s+2} = \frac{4(s+2)+2(s+1)}{(s+1)(s+2)} = \boxed{\frac{6s+10}{s^2+3s+2}}$

(b) $H(s) = \frac{Y(s)}{X(s)} = \frac{6s+10}{s^2+3s+2}$

$s^2Y(s) + 3sY(s) + 2Y(s) = 6sX(s) + 10X(s)$

$\therefore \boxed{y''(t) + 3y'(t) + 2y(t) = 6x'(t) + 10x(t)}$

(c). $H(s) = \frac{6s+10}{s^2+3s+2} = \frac{6s+10}{(s+1)(s+2)}$

$\boxed{\text{Poles} : s = -1, s = -2}$

(d). DC gain : $\lim_{s\to 0} \left(\frac{6s+10}{s^2+3s+2} \right) = \frac{10}{2} = \boxed{5}$

(e) $x(t) = 1 \Rightarrow X(s) = \frac{1}{s}$

$Y(s) = \frac{6s+10}{s^2+3s+2} \cdot X(s) = \frac{6s+10}{s^2+3s+2} \cdot \frac{1}{s}$

$\therefore \boxed{Y(s) = \frac{6s+10}{s(s^2+3s+2)}}$

(f)

$y(t) = \lim_{s\to 0} sY(s) = s \cdot \frac{6s+10}{s(s^2+3s+2)} = \frac{10}{2} = \boxed{5}$

2.6.1

$y''(t) + y(t) = x(t)$

(a) $s^2 Y(s) + Y(s) = X(s)$ $\quad \frac{Y(s)}{X(s)} = \frac{1}{s^2+1}$

Poles : $s^2+1=0$ $\quad s = \pm j$

$\therefore H(s) = \frac{1}{s^2+1}$, ROC: $Re(s) > 0$, Poles: $s=j$, $s=-j$

(b) $X(s) = 1$

$h(t) = y(t) = \sin t$

(c) both poles lie on imaginary axis

$\therefore$ The system is stable

(d). $x(t) = u(t)$ $\quad X(s) = \frac{1}{s}$

$Y(s) = \frac{1}{s^2+1} X(s) = \frac{1}{s^2+1} \cdot \frac{1}{s} = \frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}$

$y(t) = u(t) - \cos t\, u(t)$

$\therefore y(t) = (1 - \cos t)\, u(t)$

2.6.2

(a) $h(t) = \sin 3t\, u(t)$

$\sin \omega t\, u(t) \xrightarrow{L.T} \frac{\omega}{s^2+\omega^2}$

$\sin 3t\, u(t) \xrightarrow{L.T} \frac{3}{s^2+9}$, $Re(s) > 0$

(b) $h(t) = e^{-\frac{1}{2}t} \sin 3t\, u(t)$

$e^{-\frac{1}{2}t} \sin 3t\, u(t) \xrightarrow{L.T} \frac{3}{(s+\frac{1}{2})^2+9}$, $Re(s) > -\frac{1}{2}$

(c) $h(t) = e^{-t} u(t) + e^{-\frac{1}{2}} \cos(3t)\, u(t)$

$\xrightarrow{L.T} \frac{1}{s+1} + \frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+9}$, $Re(s) > -\frac{1}{2}$

2.6.3

$y''(t) + 2y'(t) + 5y(t) = x(t)$

$s^2 Y(s) + 2s Y(s) + 5 Y(s) = X(s)$

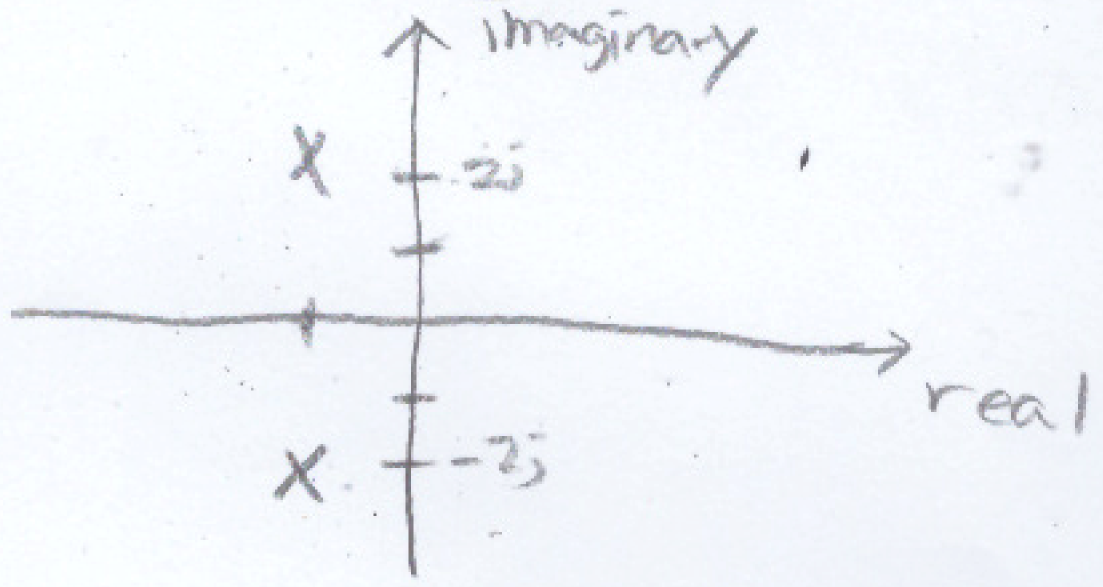
$$\frac{Y(s)}{X(s)} = \frac{1}{s^2+2s+5}$$

(a) $x(s) = 1$

$$Y(s) = \frac{1}{s^2+2s+5} = \frac{1}{(s+1)^2+2^2}$$

$$y(t) = \frac{1}{2} e^{-t} \sin 2t$$

(b)



Poles: $s = -1 + 2j$

$s = -1 - 2j$

(c) DC gain: $\lim_{s \to 0} \left(\frac{1}{s^2+2s+5} \right) = \frac{1}{5}$

(d) The system is stable.

(e) $x(s) = \frac{1}{s}$

$$Y(s) = \frac{1}{s^2+2s+5} x(s) = \frac{1}{s^2+2s+5} \cdot \frac{1}{s}$$

$$Y(s) = \frac{1}{s^3+2s^2+5s}$$

2.6.4

(a) $y''' + 3y'' + 2y' = x'' + 6x' + 6x$

$s^3 Y(s) + 3s^2 Y(s) + 2sY(s) = s^2 X(s) + 6s X(s) + 6X(s)$

$(s^3 + 3s^2 + 2s) Y(s) = (s^2+6s+6) X(s)$

$$\frac{Y(s)}{X(s)} = \frac{s^2+6s+6}{s^3+3s^2+2s} = \frac{s^2+6s+6}{s(s^2+3s+2)} = \frac{s^2+6s+6}{s(s+1)(s+2)}$$

$$H(s) = \frac{3}{s} + \frac{-1}{s+1} + \frac{-1}{s+2}$$

$$\boxed{h(t) = 3 - e^{-t} - e^{-2t}}$$

ROC: $s > -2$

(b) $y''' + 8y'' + 46y' + 68y = 10x'' + 53x' + 144x$

$$(s^3+8s^2+46s+68)\,Y(s) = (10s^2+53s+144)\,X(s)$$

$$\frac{Y(s)}{X(s)} = \frac{10s^2+53s+144}{s^3+8s^2+46s+68} = \frac{3}{s+2} + \frac{7s+21}{s^2+6s+34}$$

$$H(s) = \frac{3}{s+2} + \frac{7(s+3)}{(s+3)^2+5^2}$$

$$\boxed{h(t) = 3e^{-2t} + e^{-3t}\cdot 7\cos(5t)}$$

2.6.6

$$y''(t) + 4y'(t) + 5y(t) = x'(t) + 2x(t)$$

(a) $s^2 Y(s) + 4sY(s) + 5Y(s) = sX(s) + 2X(s)$

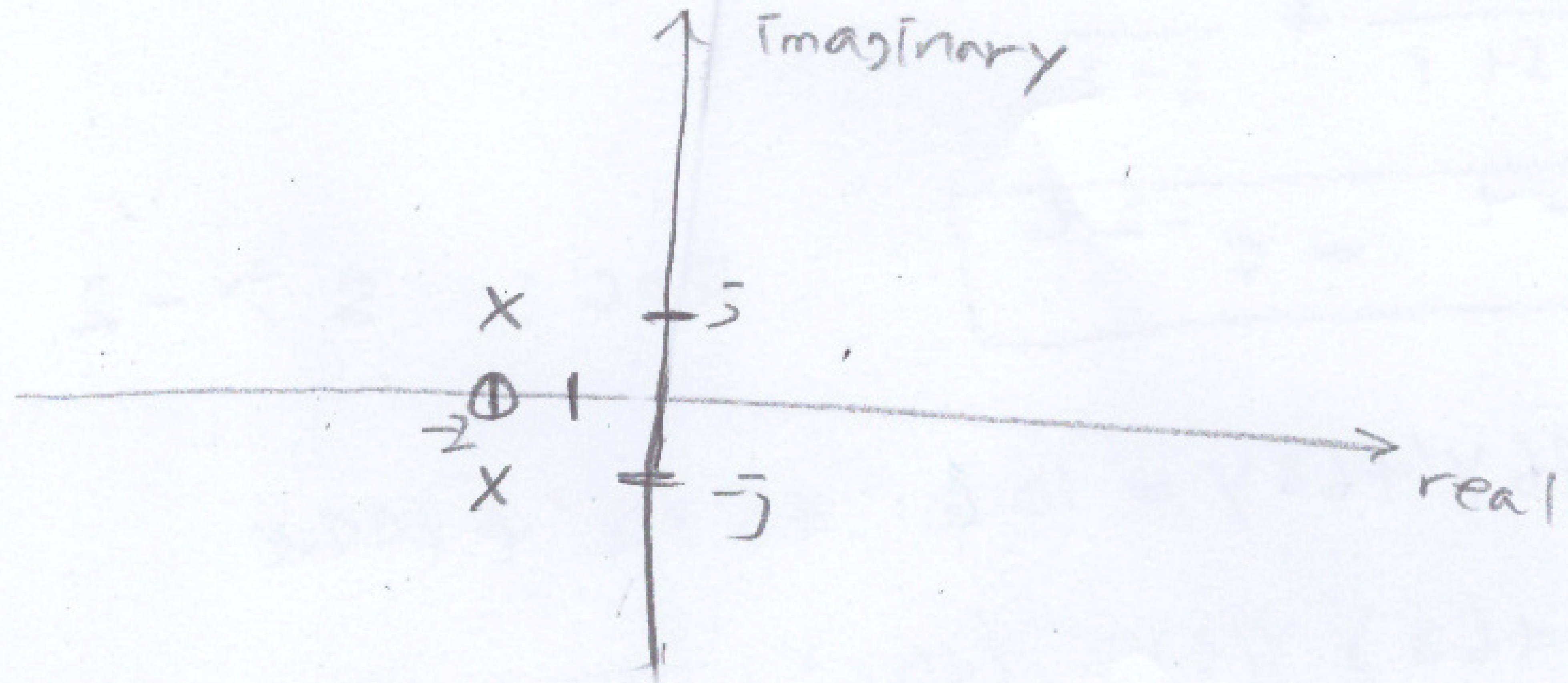
$$\frac{Y(s)}{X(s)} = \frac{s+2}{s^2+4s+5}$$ $X(s) = 1$ (impulse input)

$$Y(s) = \frac{s+2}{s^2+4s+5}X(s) = \frac{s+2}{s^2+4s+5} = \frac{s+2}{(s+2)^2+1}$$

$$\therefore \boxed{y(t) = e^{-2t}\cos t}$$

(b) Poles : $s^2+4s+5=0$; $s=-2+j$, $s=-2-j$

Zeros : $s+2=0$; $s=-2$



(c) dc gain : $\lim_{s\to 0} \dfrac{s+2}{s^2+4s+5} = \boxed{\dfrac{2}{5}}$

(d) System is stable because poles and zero lie on left side of s-plane.

2.6.10

impulse response	pole-zero Diagram
1	6
2	1
3	8
4	3
5	2
6	7
7	4
8	5