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Sample data game strategy for active rendezvous with disturbance rejection

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ABSTRACT

In this paper, we investigate rendezvous problem between two spacecraft operating under J_2 perturbations and unknown control errors. As is well known, the main challenge is how to produce optimal actions against disturbance and limited measurements. Toward this, we propose a worst-case Nash strategy for the underlying problem, where each player is devoted to minimizing its own cost function while the disturbance works toward maximizing its influence on each player. In addition, a sampled-data game strategy is introduced to alleviate the seeking process of the Nash equilibrium (i.e., NE), and we proved rigorously that intermittently available measurements are sufficient to ensure the existence of the NE. Simulation results verified effectiveness of the proposed scheme.

1. Introduction

Spacecraft autonomous rendezvous is a critical technology that could benefit many space missions, such as repairing or refueling a spacecraft, in-space structure assembling, satellite networking, and object capturing [1–3]. In the past decades, a series of dynamical models for relative translation have been paid to the spacecraft translation issue in the previous literatures. Given that a reference spacecraft operating in a circular or elliptical orbit, then, other spacecraft with respect to it can be described by autonomous nonlinear differential equations such as Clohessy-Wiltshire (C-W) equations [4], Tschauner-Hempel (T-H) equations [5], and the line-of-sight (LOS) equations [6]. Without loss of any generality, the C-W equations are time invariant while T-H equations are varying periodically [7], and, they are extensively adopted in the relative motion control issue for the rendezvous and formation of the circular orbits and elliptic orbit respectively. In addition, the LOS-based model has been improved through the contributions of some researchers [8,9], which can be applied in the analysis of terminal stage of close-range rendezvous. Hence, these relative motion models are the efficient foundation in space tasks.

To address the relative motion problem, several effective solutions have been proposed based on impulse control during the past decades. For rendezvous control, the two-impulse control based on C-W or T-H frame has been focused on and developed into a mature theory [10–12], for which we have to consider the instantaneous change in velocity at the start and end time only. Considering the difficulty of implementing the ideal large instantaneous pulse, many researchers also paid significant attention to multi-impulse control [13–15]. However, in the aforementioned work, most achievements only include discrete impulse control which will lead to insufficient accuracy. Toward this, the continuous thrust control for micro-spacecraft has attracted much attention to the underlying problem. In particular, in [16] and [17], the solutions of shortest time transfer trajectory and minimum energy transfer trajectory are obtained, respectively, by means of multi-satellite optimal control. In addition, some direct strategies have been introduced in this field recently, such as artificial potential function, heuristic method and reinforcement learning based method. For example, an algorithm based on superquadric artificial potential fields is presented to support the autonomous on-orbit assembly of a large space structure in [18]; a particle swarm optimization (PSO) based scheme is implemented to solve optimal trajectories in [19], and it is significant due to the insensitive characteristic of singularity issue; a control policy is developed by implementing and evaluating Proximal Policy Optimization (PPO), and under the control policy the satellite is able to maneuver to within centimeters of the intended target without collision [20].

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Although various controllers as referred previously have been applied in the autonomous rendezvous missions effectively, there still some certain issues we should discuss further. In practice, several inevitable disturbances existing in space, that is, the oblateness of Earth, the atmospheric disturbance, the geomagnetism disturbance, and the control errors caused by the installation of the actuators, have a negative impact on the spacecraft trajectory [21]. Note that the presence of these external disturbances can affect the expected trajectory [22], such that it will cast a shadow over the mission of rendezvous. To overcome the negative effect, the minimax strategy has been widely discussed, where the cost function is formulated as a two-player zero-sum game with the controller being the minimizing player and the disturbance being the maximizing player [23]. In [24], the control input and the external disturbance signal are regarded as two players that compete each other for finding the minimax solution of the optimal game theoretic problem, and an analytic minimax solution of attitude control has been provided with the help of the inverse optimal approach. In [25], a differential evolution-based technique has been proposed to solve the minimax optimization problems, where the cost is measured by maximizing the objective function over the worst-case scenario. Moreover, the excellent performance of sliding mode control (SMC) [26][27] and model predictive control (MPC) [28] has also been discussed over the past decades. Apart from the above approaches, some advanced work on the basis of Nash game has been proposed to tackle the motion control issue with disturbance rejection. Specifically, the authors designed a robust control scheme under multiplicative noise which combines Nash game with H_2/H_∞ in [29]. Furthermore, the existence of a novel worst-case Nash equilibria in noncooperative multi-player differential games has been proved in [30], where the uncertain disturbance is seen as a malicious fictitious player trying to maximize the cost function of each player. Based on the contribution in [30], Nash/worst-case is extended to the spacecraft formation control in [21], which also introduces a novel distributed observer inspired by the contribution of [31]. Consequently, it is shown that differential game can deal with the disturbance rejection problem effectively, hence, we extend this theory to tackle the issue of rendezvous between two active spacecraft.

It should be pointed out that, albeit their effectiveness, most of aforementioned results requires continuous measurements on all entities, which could be cumbersome in practice and would otherwise lament the eventual accuracy. In addition, a critical issue that the passive spacecraft may deviate from the expected orbit by accidental disturbances should be paid significant attention to. In this paper, we propose a sample data based Nash strategy for rendezvous problem, where both parties maneuvers actively to achieve an optimal outcome. The advantage of the proposed strategy are threefold: 1) The proposed sample data strategy requires only intermittent measurements, which, as will be proved later, is theoretical enough to solve the underlying problem; 2) The proposed scheme is applicable to the active rendezvous mission as well as the conventional rendezvous mission performed by only one active spacecraft; 3) The proposed scheme ensures that both parties actively seek the equilibrium with minimal control effort against uncertain disturbances, and, by introducing disturbances as an auxiliary player, the solution is resilient against unpredictable uncertainties.

2. Problem formulation

2.1. Model of relative motion

In this paper, we assume that the rendezvous point in a circular orbit as the origin of the Local Vertical Local Horizontal (LVLH) frame, and the motion of spacecraft can be expressed relatively to the reference origin. Then, the LVLH frame (i.e., $o - xyz$) is defined on the basis of the reference origin, where the x-axis points along the direction from the center of the Earth to the origin, the y-axis is along the orbital track of it and the z-axis completes the right-hand orthogonal coordinate system. Consequently, to generate high-precision maneuver trajectories for the jth spacecraft, its dynamics with respect to the origin can be expressed as follows [32]

$$\begin{aligned}
\ddot{x}_j &= 2\dot{\theta}\dot{y}_j + \dot{\theta}^2 x_j - \frac{\mu_e(r_0 + x_j)}{r_j^3} + \frac{\mu_e}{r_0^2} + J_{xj} + a_{xj} \\
\ddot{y}_j &= -2\dot{\theta}\dot{x}_j + \dot{\theta}^2 y_j - \frac{\mu_e y_j}{r_j^3} + J_{yj} + a_{yj} \\
\ddot{z}_j &= -\frac{\mu_e z_j}{r_j^3} + J_{zj} + a_{zj}
\end{aligned} \tag{1}$$

where μ_e is the gravitational constant, r_0 is the orbit radius of the reference point and $\dot{\theta} = \sqrt{\frac{\mu_e}{r_0^3}}$ indicates the orbital angular velocity of reference origin, $r_j = \sqrt{(r_0 + x_j)^2 + y_j^2 + z_j^2}$ represents the spatial distance between the jth spacecraft and the center of Earth. Furthermore, $J_{(\cdot)}$ refers to the disturbance with the inclusion of J_2 perturbation in corresponding $(\cdot)$ direction, and $a_{(\cdot)}$ is defined as the acceleration inputs. Before proceeding further, the influence of J_2 perturbation of spacecraft j can be given as follows:

$$\begin{aligned}
J_{xj} &= \frac{\mu_e\left(r_0 + x_j\right)}{r_j^5}\left[\frac{3}{2}J_2 R_e^2\left(\frac{5z_j^2}{r_j^2} - 1\right)\right] + \frac{3}{2}\frac{\mu_e J_2 R_e^2}{r_0^4} \\
J_{yj} &= \frac{\mu_e y_j}{r_j^5}\left[\frac{3}{2}J_2 R_e^2\left(\frac{5z_j^2}{r_j^2} - 1\right)\right] \\
J_{zj} &= \frac{\mu_e z_j}{r_j^5}\left[\frac{3}{2}J_2 R_e^2\left(\frac{5z_j^2}{r_j^2} - 3\right)\right]
\end{aligned} \tag{2}$$

where R_e and J_2 are the mean radius and second zonal harmonic coefficient of the Earth, respectively.

Then, under the conjecture that the J_2 perturbation and the nonlinear terms in Eq. (1) are regarded as known disturbances, the relative motion of the jth spacecraft can be described as a linear system with specified disturbances,

$$\dot{X}_j = AX_j + B\begin{bmatrix} a_{xj} \\ a_{yj} \\ a_{zj} \end{bmatrix} + B\begin{bmatrix} d_{xj} \\ d_{yj} \\ d_{zj} \end{bmatrix} \tag{3}$$

where $X_j = [x_j, y_j, z_j, \dot{x}_j, \dot{y}_j, \dot{z}_j]^T \in \mathbb{R}^{6\times1}$ and $d_j = [d_{xj}, d_{yj}, d_{zj}]^T \in \mathbb{R}^{3\times1}$, and

$$\begin{aligned} d_{xj} &= J_{xj} - \frac{\mu_e\left(r_0 + x_j\right)}{r_j^3} + \frac{\mu_e}{r_0^2} \\ d_{yj} &= J_{yj} - \frac{\mu_e y_j}{r_j^3} \\ d_{zj} &= J_{zj} - \frac{\mu_e z_j}{r_j^3} \end{aligned} \tag{4}$$

and

$$A = \begin{bmatrix} 0_3 & I_3 \\ M_1 & M_2 \end{bmatrix}, \quad B = \begin{bmatrix} 0_3 \\ I_3 \end{bmatrix} \tag{5}$$

$$M_1 = \begin{bmatrix} \dot{\theta}^2 & 0 & 0 \\ 0 & \dot{\theta}^2 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad M_2 = \begin{bmatrix} 0 & 2\dot{\theta}^2 & 0 \\ -2\dot{\theta}^2 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{6}$$

2.2. Preliminaries on open-loop Nash equilibrium

Consider a linear system with N players, and the dynamics are governed by [33]

$$\dot{x}(t) = A(t)x(t) + \sum_{j=1}^{N} B_j(t)u_j(t), \quad x(t_0) = x_0 \tag{7}$$

where x is the state, and $u_j(t)$ is the input corresponding to payer j. In what follows, notation of matrices $A(t)$ and $B_j(t)$ will be simplified to A and B_j for the sake of brevity. In particular, for the jth player, its performance index can be chosen as

$$J_j = \frac{1}{2}x^T(t_f)S_jx(t_f) + \frac{1}{2}\int_{t0}^{tf} u_j^T R_j u_j dt \tag{8}$$

where S_j and R_j for all $j = 1, \cdots, N$ are positive definite matrices with appropriate dimensions.

Definition 1. For system (7) with performance index defined in (8), strategies $(u_1^*,\cdots,u_N^*)$ form an open-loop Nash equilibrium (i.e., NE) if the following inequality holds for $u_j \in U_j$ for all $j = 1, \cdots, N$,

$$J_j(u_1^*, \cdots, u_j^*, \cdots, u_N^*) \le J_j(u_1^*, \cdots, u_j, \cdots, u_N^*) \tag{9}$$

where $U_j = \left\{u_j(t, x_0)|t \in [t_0, t_f]\right\}$ is the admissible set of input for controller j. □

Obviously, the open-loop NE could be given by using Pontryagin's minimum principle, and

$$u_j^* = -R_j^{-1}B_j^T P_j x^*, \quad j = 1, \cdots, N \tag{10}$$

where $x^*(t) = \Phi(t, t_0)x_0$, and $\Phi(t, t_0)$ is the state-transition matrix, $P_j (j = 1, \cdots, N)$ are the solutions to the coupled Riccati equations, that is

$$\dot{P}_j = -A^T P_j - P_j A + P_j B_j R_j^{-1} B_j^T P_j + P_j \sum_{k=1, k\neq j}^{N} B_k R_k^{-1} B_k^T P_k, \quad P_j(t_f) = S_j \tag{11}$$

The following lemma summarizes performance of the proposed NE, its proof can be found in [34] and thus omitted here for the sake of brevity.

Lemma 1. *Consider system (7), and let (10) be its control strategy. Then, performance index (8) becomes*

$$J_j^* = \frac{1}{2}x_0^T K_j(t_0)x_0, \quad j = 1, \cdots, N \tag{12}$$

where $K_j(t)$ for all $j = 1, \cdots, N$ are specified by

$$\begin{aligned}\dot{K}_j = &-\left(A - B_j R_j^{-1} B_j^T P_j - \sum_{k=1,k\neq j}^{N} B_k R_k^{-1} B_k^T P_k\right)^T K_j \\ &- K_j\left(A - B_j R_j^{-1} B_j^T P_j - \sum_{k=1,k\neq j}^{N} B_k R_k^{-1} B_k^T P_k\right) \\ &- P_j^T B_j R_j^{-1} B_j^T P_j, \qquad K_j(t_f) = S_j \qquad \square\end{aligned} \tag{13}$$

2.3. Formulation of active rendezvous

In this paper, rendezvous problem of two active spacecraft under J_2 perturbation and uncertain disturbances will be discussed. For active rendezvous, we assumed both spacecraft maneuver actively to ensure the discrepancy between their relative position and velocity vanish over time, while taking action based on their expectations concerning the worst disturbances that will occur.

In LVLH frame, the relative dynamics between two spacecraft can be expressed as follows:

$$\begin{bmatrix}\dot{p}\\ \dot{v}\end{bmatrix} = \begin{bmatrix}0_3 & I_3\\ M_1 & M_2\end{bmatrix}\begin{bmatrix}p\\ v\end{bmatrix} + B_1(u_1 + \hat{d}_1) + B_2(u_2 + \hat{d}_2) \tag{14}$$

where $p = [x_1 - x_2, y_1 - y_2, z_1 - z_2]^T \in \mathbb{R}^{3\times 1}$ and $v = [\dot{x}_1 - \dot{x}_2, \dot{y}_1 - \dot{y}_2, \dot{z}_1 - \dot{z}_2]^T \in \mathbb{R}^{3\times 1}$ denote the relative distance and relative velocity, respectively, and $B_1 = -B_2 = B$, and $u_j = [a_{xj}, a_{yj}, a_{zj}]^T \in \mathbb{R}^{3\times 1}$. Moreover, $\hat{d}_j \in \mathbb{R}^{3\times 1}$ denotes the uncertain disturbances including the nonlinear terms (i.e., Eq. (4)) and unknown control disturbances in practice, which is defined as

$$\hat{d}_j = [d_{xj}, d_{yj}, d_{zj}]^T + [e_{xj}, e_{yj}, e_{zj}]^T, \quad j = 1, 2, \tag{15}$$

where $e_{(\cdot)}$ denotes the uncertain disturbance errors in $(\cdot)$ direction.

Remark 1. Note that, under the relative dynamic (14), the spacecraft are not intended to rendezvous at the origin, but to find the optimal trajectories that minimize the relative state error.

Then, performance index J_j for spacecraft j can be chosen as follows:

$$J_j = \frac{1}{2}[p^T(t_f)S_{pj}p(t_f) + v^T(t_f)S_{vj}v(t_f)] + \frac{1}{2}\int_{t0}^{tf}(u_j^T R_{uj} u_j - \hat{d}_j^T R_{dj}\hat{d}_j)dt, \quad j = 1, 2, \tag{16}$$

where $S_{pj} \in \mathbb{R}^{3\times 3}$, $S_{vj} \in \mathbb{R}^{3\times 3}$, $R_{uj} \in \mathbb{R}^{3\times 3}$, $R_{dj} \in \mathbb{R}^{3\times 3}$ are positive definite matrices to be specified. It should be pointed out that, due to inevitable nature of the disturbance, the corresponding disturbance is also treated as a malicious/auxiliary player trying to maximize the performance index. Consequently, the physical significance behind performance index (16) is that, each spacecraft intends to minimize the final error and energy consumption against a worst-case assumption of the disturbance.

Definition 2. Consider system (14) with performance index defined in (16), let $u = (u_1, u_2)$ be input to system (14). Then, u and $\hat{d}_j^*(u)$ form a worst-case strategy if the following inequality holds [30]

$$J_j(u, \hat{d}_j^*(u)) \geq J_j(u, \hat{d}), \quad j = 1, 2 \tag{17}$$

where $\hat{d} = [\hat{d}_1, \hat{d}_2]$.

Consequently, by invoking Definition 1, it can be concluded that control strategies u_1^* and u_2^* form a Nash/worst-case equilibrium, that is

$$J_j(u^*, (d_j, \hat{d}_{-j}^*)) \leq J_j(u^*, \hat{d}^*(u^*)) \leq J_j((u_j, u_{-j}^*), \hat{d}_j^*(u_j, u_{-j}^*)), \quad j = 1, 2 \tag{18}$$

where the subscript (-j) represents the agent other than j in the corresponding set.

In order to cope with the disadvantage of open-loop strategy as well as to facilitate implementation, we propose a sample data game strategy for the underlying problem, aiming for a piecewise-continuous feedback solution [35], which relies on the intermittent measurements over the entire horizon.

Assumption 1. Spacecraft can obtain measurement of the state information at n discrete instants, namely, $t_i \in [t_0, t_f], t_i < t_{i+1} (i = 0, 1, \cdots, n-1)$. $\square$

Definition 3. Suppose that control strategies of form $u_i^* = (u_{1i}^*(i, t, x(t_i)), u_{2i}^*(i, t, x(t_i)))$ with worst-case disturbance $\hat{d}_{ji}^*(u_i^*)$ for all $t \in [t_i, t_{i+1})$, there exists a sampled-data Nash/worst-case equilibrium if

$$J_j(u_i^*, \hat{d}_i^*(u_i^*)) \leq J_j((u_{ji}, u_{(-j)i}^*), \hat{d}_{ji}^*(u_{ji}, u_{(-j)i}^*)) \tag{19}$$

hold for each admissible control strategy u_i and its corresponding worst-case disturbance $\hat{d}_{ji}\left(u_{ji}, u_{(-j)i}^*\right)$. $\square$

In what follows, notation of $u_{ji}(i,t,x(t_i))$ and $\hat{d}_{ji}(i,t,x(t_i))$ will be simplified to u_{ji} and $\hat{d}_{ji}$, respectively, for the sake of notation brevity.

3. Main results

In this section, we proceed to solve the rendezvous problem with sample data game strategy. In particular, with the inclusion of the penalty term on disturbance, we reason its existence as a player in the game and attempt to solidify its influence on individual decisions. Namely, each player is devoted to minimizing its own performance function while the disturbance works toward maximizing its influence on each player. The following theorem summarizes the existence and uniqueness of sampled-data Nash/worst-case equilibrium in the underlying problem, and its proof follows immediately.

Theorem 1. *Consider the rendezvous problem of two spacecraft in the specified LVLH frame, suppose dynamics of active rendezvous system can be expressed by (14) with performance index defined in (16). Then, there exists a sampled-data Nash/worst-case equilibrium if u_{1i} and u_{2i} for $t \in [t_i, t_{i+1})$, for all $i = 0, 1, \cdots, n-1$, are chosen according to*

$$u^*_{ji} = -R^{-1}_{uj} B^T_j P_{ji} \Phi(t,t_i) \begin{bmatrix} p(t_i) \\ v(t_i) \end{bmatrix} \tag{20}$$

$$\hat{d}^*_{ji} = R^{-1}_{dj} B^T_j P_{ji} \Phi(t,t_i) \begin{bmatrix} p(t_i) \\ v(t_i) \end{bmatrix} \tag{21}$$

where $\Phi(t,t_i) \in \mathbb{R}^{6\times 6}$ is the state-transition matrix function, and $P_{ji}(t) \in \mathbb{R}^{6\times 6}$ for $t \in [t_i, t_{i+1})$ are time-varying gains to be specified, and

$$\Phi(t,t_i) = \begin{bmatrix} \phi_1(t,t_i) & \phi_2(t,t_i) \\ \phi_3(t,t_i) & \phi_4(t,t_i) \end{bmatrix} = \exp\left\{ \left[A - \sum_{j=1,2} B_j R_{udj} B^T_j P_{ji} \right] (t - t_i) \right\} \tag{22}$$

$$\dot{P}_{ji} = -A^T P_{ji} - P_{ji} A + P_{ji} \sum_{k=1,2} \left(B_k R_{udk} B^T_k P_{ki} \right), P_{ji}(t_{i+1}) = K_{j,i+1}(t_{i+1}) \tag{23}$$

where $R_{udj} = R^{-1}_{uj} - R^{-1}_{dj}$, and the boundary condition $K_{j,i+1}$ satisfies

$$\begin{aligned} \dot{K}_{ji} = &- \left(A - B_j R_{udj} B^T_j P_{ji} - B_{(-j)} R_{ud(-j)} B^T_{(-j)} P_{(-j)i} \right)^T K_{ji} \\ &- K_{ji} \left(A - B_j R_{udj} B^T_j P_{ji} - B_{(-j)} R_{ud(-j)} B^T_{(-j)} P_{(-j)i} \right) \\ &- P^T_{ji} B_j R_{udj} B^T_j P_{ji} \end{aligned} \tag{24a}$$

$$K_{ji}(t_{i+1}) = K_{j,i+1}(t_{i+1}), \quad K_{jn}(t_f) = S_j \tag{24b}$$

where subscript $(-j)$ and j are mutually exclusive, namely, $j \cup (-j) = \{1, 2\}$.

Remark 2. R_{dj} could be interpreted as a risk aversion parameter matrix, and, it could be designed to be a large R_{dj} if it is negligible for the expectation of disturbance of spacecraft j.

Proof. Suppose there exist sampled-data based control strategies (u^*_1, u^*_2) and $(\hat{d}^*_1, \hat{d}^*_2)$ over time horizon $[t_{i+1}, t_f)$. Then, the value of performance index can be written as, at $t \in [t_{i+1}, t_f)$

$$V_j = \frac{1}{2}[p^T(t_f) S_{pj} p(t_f) + v^T(t_f) S_{vj} v(t_f)] + \frac{1}{2} \int_t^{tf} (u^{*T}_j R_{uj} u_j - \hat{d}^{*T}_j R_{dj} \hat{d}_j) dt \tag{25}$$

for $j = 1, 2$. Consequently, performance index over interval $[t_i, t_{i+1})$ can be rewritten as, at t_i,

$$J_{ji} = V_{j,i+1} + \frac{1}{2} \int_{t_i}^{t_{i+1}} (u^T_{ji} R_{uj} u_{ji} - \hat{d}^T_{ji} R_{dj} \hat{d}_{ji}) dt \tag{26}$$

where $V_{j,i+1} = V_j(p(t_{i+1}), v(t_{i+1}), t_{i+1})$ refers to the value of (25) at t_{i+1}, and, u_{ji} and $\hat{d}_{ji}$ for $j = 1, 2$ are defined over $[t_i, t_{i+1})$ only.

As such, the Hamiltonian over $[t_i, t_{i+1})$ can be defined as

$$H_{ji} = \frac{1}{2} u^T_{ji} R_{uj} u_{ji} - \frac{1}{2} \hat{d}^T_{ji} R_{dj} \hat{d}_{ji} + \eta^T_{ji} v + \lambda^T_{ji} (M_1 p + M_2 v + u_{1i} + \hat{d}_{1i} - u_{2i} - \hat{d}_{2i}) \tag{27}$$

where η_{ji} and λ_{ji} are the Lagrangian multipliers (i.e., the costate vectors). Without loss of any generality, the control strategies and its worst-case disturbance should satisfy the following conditions,

$$\begin{cases} \dfrac{\partial H_{ji}}{\partial u_{ji}} = 0 \\ \dfrac{\partial H_{ji}}{\partial \hat{d}_{ji}} = 0 \end{cases} \tag{28}$$

Substituting (27) into (28) for $j = 1, 2$, we obtain

$$\begin{cases} u_{1i} = -R_{u1}^{-1}\lambda_{1i} \\ \hat{d}_{1i} = R_{d1}^{-1}\lambda_{1i} \end{cases} \tag{29a}$$

$$\begin{cases} u_{2i} = R_{u2}^{-1}\lambda_{2i} \\ \hat{d}_{2i} = -R_{d2}^{-1}\lambda_{2i} \end{cases} \tag{29b}$$

Furthermore, substituting (27) into the costate dynamics, we have

$$\begin{cases} \dot{\eta}_{ji} = -\dfrac{\partial H_{ji}}{\partial p} = -M_1^T\lambda_{ji} \\ \dot{\lambda}_{ji} = -\dfrac{\partial H_{ji}}{\partial v} = -\eta_{ji} + M_2^T\lambda_{ji} \end{cases}, \quad j = 1, 2 \tag{30}$$

Subsequently, the above equations can be rewritten as

$$\begin{bmatrix} \dot{\eta_{ji}} \\ \dot{\lambda_{ji}} \end{bmatrix} = -A^T \begin{bmatrix} \eta_{ji} \\ \lambda_{ji} \end{bmatrix} \tag{31}$$

Then, by invoking *Lemma 1*, $V_{j,i+1}$ could be formulated as

$$V_{j,i+1} = \frac{1}{2}\left[p^T(t_{i+1}), v^T(t_{i+1})\right] K_{j,i+1} \begin{bmatrix} p(t_{i+1}) \\ v(t_{i+1}) \end{bmatrix} \tag{32}$$

where $K_{j,i+1} \in \mathbb{R}^{6\times 6}$ is the positive definite matrix. Then, the boundary condition of costate vectors can be given by

$$\begin{bmatrix} \eta_{ji}(t_{i+1}) \\ \lambda_{ji}(t_{i+1}) \end{bmatrix} = \begin{bmatrix} \dfrac{\partial V_{j,i+1}}{\partial p(t_{i+1})} \\ \dfrac{\partial V_{j,i+1}}{\partial v(t_{i+1})} \end{bmatrix} = K_{j,i+1}(t_{i+1}) \begin{bmatrix} p(t_{i+1}) \\ v(t_{i+1}) \end{bmatrix} \tag{33}$$

According to (31) and (33), the function between costate vectors and expected state trajectory can be formulated as:

$$\begin{bmatrix} \eta_{ji}(t) \\ \lambda_{ji}(t) \end{bmatrix} = P_{ji}(t) \begin{bmatrix} p(t) \\ v(t) \end{bmatrix} \tag{34}$$

where $P_{j,i+1} \in \mathbb{R}^{6\times 6}$ is the positive semi-definite matrix. Then, substituting (34) into (29a) and (29b), and again admitting the results to (14), we obtain the expected state trajectory as

$$\begin{bmatrix} \dot{p} \\ \dot{v} \end{bmatrix} = (A - E_1P_{1i} - E_2P_{2i}) \begin{bmatrix} p \\ v \end{bmatrix} \tag{35}$$

where $E_j = B_jR_{udj}B_j^T$ for $j = 1, 2$. Then, we have

$$p(t) = \phi_1(t, t_i)p(t_i) + \phi_3(t, t_i)v(t_i) \tag{36a}$$

$$v(t) = \phi_2(t, t_i)v(t_i) + \phi_4(t, t_i)v(t_i) \tag{36b}$$

where $\phi_{(\cdot)} \in \mathbb{R}^{3\times 3}$ for $t \in [t_i, t_{i+1})$ are given by

$$\begin{bmatrix} \phi_1(t, t_i) & \phi_2(t, t_i) \\ \phi_3(t, t_i) & \phi_4(t, t_i) \end{bmatrix} = \exp\left[(A - E_1P_{1i} - E_2P_{2i})(t - t_i)\right] \tag{37}$$

Consequently, substituting (36) into (34) and (29), we obtain the control strategies (20) and its worst-case disturbance (21). Moreover, combining (34) and (31), we have

$$\dot{P_{1i}} = -A^TP_{1i} - P_{1i}A + P_{1i}E_1P_{1i} + P_{1i}E_2P_{2i} \tag{38a}$$

$$\dot{P_{2i}} = -A^TP_{2i} - P_{2i}A + P_{2i}E_2P_{2i} + P_{2i}E_1P_{1i} \tag{38b}$$

Finally, substituting (32) into (25) and then taking derivatives on both sides, we have

$$\begin{aligned} &\frac{1}{2}\left[p^T, v^T\right](A^TP_{ji} - P_{1i}^TE_1K_{ji} - P_{2i}^TE_2K_{ji} + \dot{K_{ji}} + K_{ji}A - K_{ji}E_1P_{1i} - K_{ji}E_2P_{2i})\left[p^T, v^T\right]^T = \\ &-\frac{1}{2}\left[p^T, v^T\right]P_{ji}^TE_jP_{ji}\left[p^T, v^T\right]^T \end{aligned} \tag{39}$$

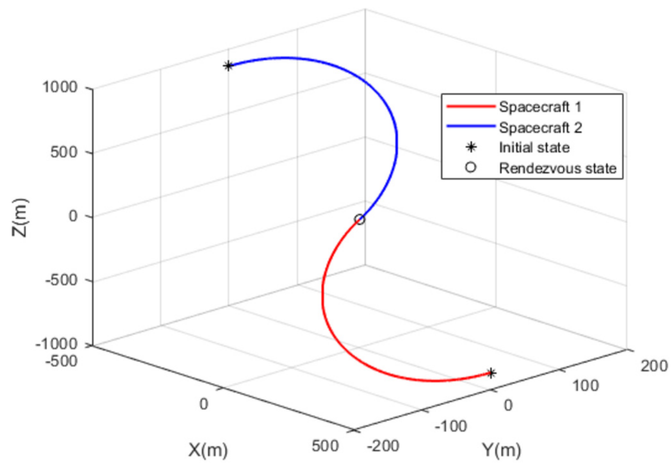

Fig. 1. Rendezvous trajectories under sampled-data Nash strategy without uncertain disturbances. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

and then

$$\dot{K}_{1i} = -(A - E_1 P_{1i} - E_2 P_{2i})^T K_{1i} - K_{1i}(A - E_1 P_{1i} - E_2 P_{2i}) - P_{1i}^T E_1 P_{1i} \tag{40a}$$

$$\dot{K}_{2i} = -(A - E_2 P_{2i} - E_1 P_{1i})^T K_{2i} - K_{2i}(A - E_2 P_{2i} - E_1 P_{1i}) - P_{2i}^T E_2 P_{2i} \tag{40b}$$

and the boundary conditions can be concluded that

$$P_{ji}(t_{i+1}) = K_{j,i+1}(t_{i+1}), \quad K_{ji}(t_{i+1}) = K_{j,i+1}(t_{i+1}), \quad K_{jn}(t_f) = S_j \tag{41}$$

As a result, (23) and (24) can be concluded, which completes the proof. □

Remark 3. Obviously that the game is solvable should we find the solution to (38) for every sampling interval. In other words, the proposed sample data game strategy admits a shrinking horizon scheme that searches the optimal solution recursively from the last interval, which ensures the optimality of the overall strategy. In addition, the most salient feature of the proposed scheme is that it achieves the NE through a sequence of temporal measurements, thus offers great freedom to onboard sensors, and by taking disturbance into consideration of the game strategy, the solution is resilient against unpredictable uncertainties.

Remark 4. It should be pointed out that the proposed strategy could also be extended to the conventional rendezvous mission, where the LVLH frame is established on an ideally uncontrolled passive spacecraft. Furthermore, the proposed strategy provides a practical solution for the situation that the passive satellites accidentally deviate from its intended orbit.

4. Simulation results and discussions

In this section, we consider an active rendezvous scenario involving two spacecraft operating under J_2 perturbation and uncertain disturbances, and, verify the significance of the introduced sample data based Nash/worst-case control strategy. The orbit radius of reference origin is selected as 7378 km. The initial relative states with respect the reference origin of each spacecraft are given as

$$[x_1(0), y_1(0), z_1(0)]^T = [500, 0, -866.0254]^T \text{ m},$$
$$[\dot{x}_1(0), \dot{y}_1(0), \dot{z}_1(0)]^T = [0, -0.9962, 0]^T \text{ m/s},$$
$$[x_2(0), y_2(0), z_2(0)]^T = [-500, 0, 866.0254]^T \text{ m},$$
$$[\dot{x}_2(0), \dot{y}_2(0), \dot{z}_2(0)]^T = [0, 0.9962, 0]^T \text{ m/s}.$$

In what follows, parameters of (16) are given by

$$S_{p1} = S_{p2} = \text{diag}\{10, 10, 10\},$$
$$S_{v1} = S_{v2} = \text{diag}\{10, 10, 10\},$$
$$R_{u1} = R_{u2} = \text{diag}\{10, 10, 10\},$$
$$R_{d1} = R_{d2} = \text{diag}\{20, 20, 20\}.$$

In addition, we expect the rendezvous between two spacecraft will be completed at $t_f = 1000s$ and each spacecraft can obtain the state measurements at every second (i.e., $n = 1000$).

For comparison purpose, we first implement the control strategies defined in (20) and open-loop Nash strategy [36] without uncertain disturbances, namely $e_1 = e_2 = [0, 0, 0]^T$, over the entire horizon. Fig. 1 shows the rendezvous trajectories of each spacecraft under our

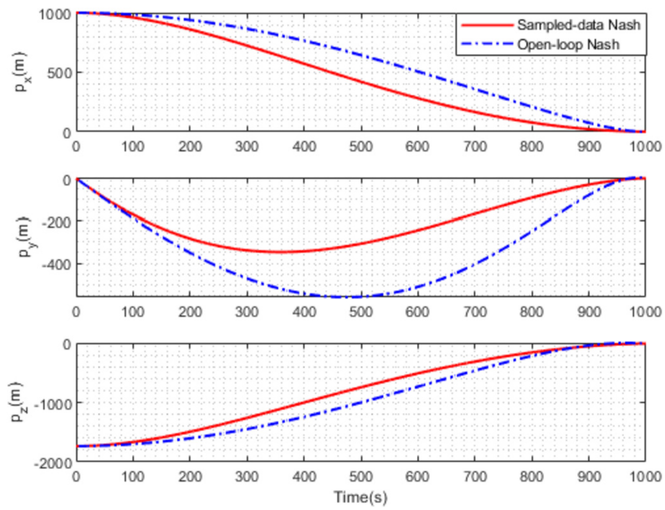

Fig. 2. Relative position under sampled-data Nash and open-loop Nash strategy without uncertain disturbances.

Table 1
Terminal relative states under sampled-data and open-loop strategy.

External control disturbance	Final state	Sampled-data strategy	Open-loop strategy
$e_1 = [0; 0; 0] m/s^2$ $e_2 = [0; 0; 0] m/s^2$	$p(t_f)(m)$	[0.0114; −0.0197; −0.0278]	[−0.4212; 2.9643; 3.9104]
	$v(t_f)(m/s)$	[−0.0014; 0.0024; 0.0034]	[−0.0597; −0.0072; −0.1057]
$e_1 = 0.1[\cos(2\pi t - \frac{\pi}{2}); \sin(2\pi t); \cos(2\pi t - \frac{\pi}{2})] m/s^2$ $e_2 = [0; 0; 0] m/s^2$	$p(t_f)(m)$	[0.0661; 0.0329; 0.0260]	[0.1914; 3.5040; 4.4685]
	$v(t_f)(m/s)$	[−0.0078; −0.0037; −0.0028]	[−0.0767; −0.0182; −0.1194]

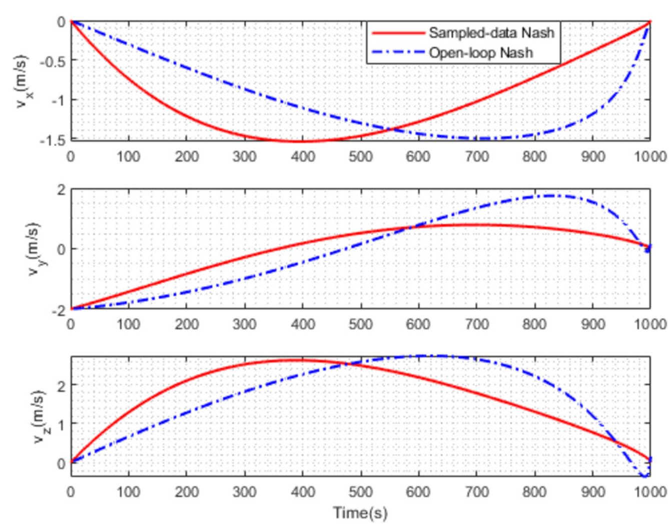

Fig. 3. Relative velocity under sampled-data Nash and open-loop Nash strategy without uncertain disturbances.

proposed control strategy in the specified LVLH coordinate. Figs. 2 and 3 show the relative position and velocity between the two spacecraft under both aforementioned strategies, respectively. Note that, although both strategies could guide spacecraft to rendezvous state, the sample data based Nash/worst-case strategy has the advantage in terminal relative state. Numerically, the terminal relative states have been shown in Table 1.

Then, we proceed to implement the solutions of both sampled-data Nash/worst-case and open-loop Nash strategy under specified control disturbances for spacecraft 1. Mathematically speaking, the control disturbances are defined as the following format in our simulation, where $e_1 = 0.1[\cos(2\pi t - \frac{\pi}{2}), \sin(2\pi t), \cos(2\pi t - \frac{\pi}{2})]^T$, for $t \in [t_0, t_f]$. In this scenario, the transfer trajectories under sampled-data Nash strategy of both spacecraft are shown in Fig. 4. And the relative position and velocity under both strategies are shown in Fig. 5 and Fig. 6, respectively. It should be pointed out that, according to the terminal relative state, the proposed strategy shows significant advantages over the open-loop Nash law, and the simulation results are included in Table 1.

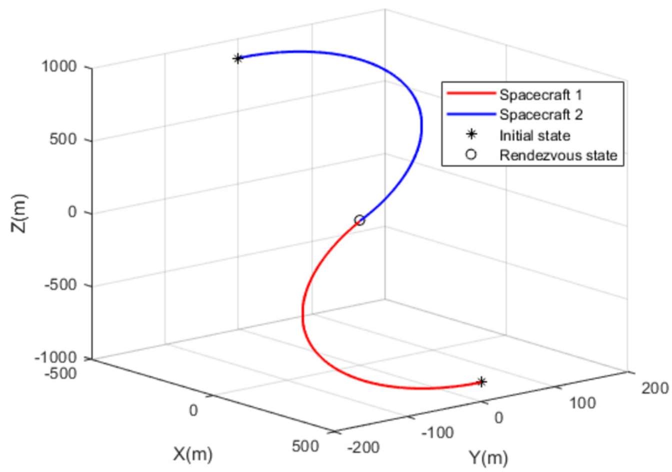

Fig. 4. Rendezvous trajectories under sampled-data Nash strategy with accidental disturbances.

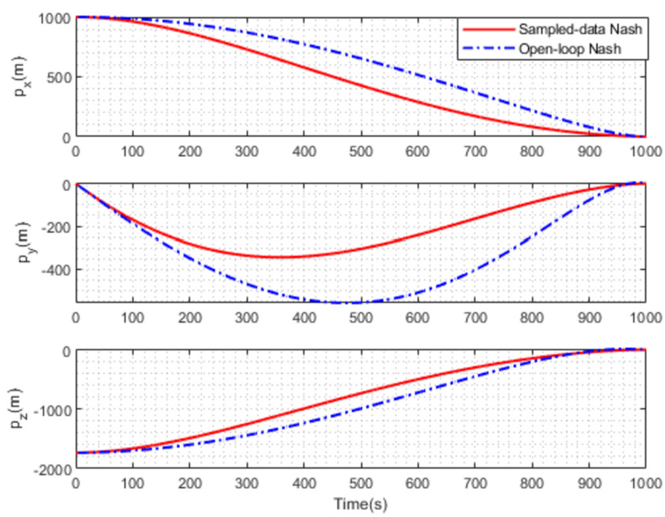

Fig. 5. Relative position under sampled-data Nash and open-loop Nash strategy with accidental disturbances.

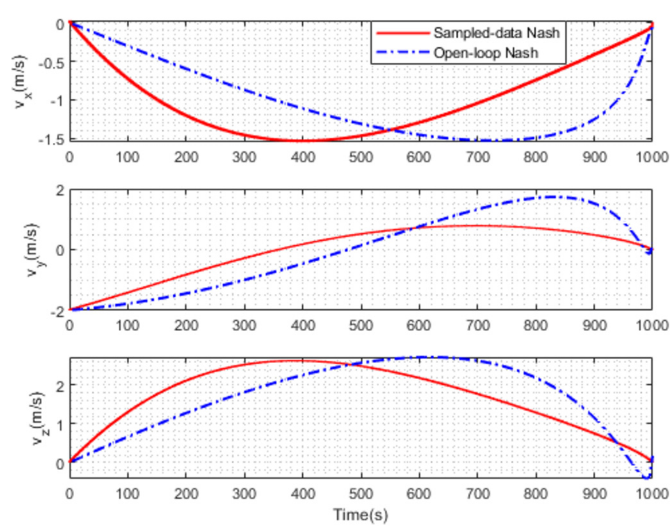

Fig. 6. Relative velocity under sampled-data Nash and open-loop Nash strategy with accidental disturbances.

5. Conclusions

In this paper, we extend a sampled-data Nash/worst case equilibrium strategy to deal with the rendezvous problem of two spacecraft under J_2 perturbation and unknown control errors. The underlying problem is considered as a non-zero-sum differential game, where each spacecraft is intended to minimize its own performance index without any information of disturbance. Consequently, the Nash/worst-case equilibrium has been proved rigorously for each sampling interval. Simulation results demonstrate that both spacecraft can approach each other with satisfactory accuracy with disturbance rejection.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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