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HIGH FREQUENCY VIBRATION OF AIRCRAFT STRUCTURES

B. L. CLARKSON
Institute of Sound and Vibration Research,
University of Southampton, Southampton SO9 5NH, England

AND

D. J. MEAD
Department of Aeronautics and Astronautics,
University of Southampton, Southampton SO9 5NH, England

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Research on high frequency vibration of aircraft structures at Southampton, like that on jet noise, pre-dated the foundation of the Institute of Sound and Vibration Research by some ten years. Accordingly, the development of the subject over the past twenty years is reviewed in this paper, with emphasis on both fundamental and applied aspects with which I.S.V.R. research has been particularly concerned. Topics discussed include the following: vibration of stiffened skin panels, vibration of curved panels, vibration of skin-rib structures, damping of structures, wave propagation in periodic structures (one-, two- and three-dimensional), vibration design data, dynamic properties of carbon fibre reinforced plastics, acoustically induced fatigue crack propagation.

1. INTRODUCTION

In the early 1950's some failures were noticed in parts of aircraft structures close to the jet efflux. Further study showed that these failures were almost certainly caused by high frequency vibration of the local structure excited by the high noise level in the region of the jet. Following these incidents, a study of acoustically induced vibration of aircraft structures began. The framework for these studies was first formulated by Powell [1] using the normal mode theory. This formulation had the advantage of showing clearly the important parameters of the problem. The excitation forces were defined by their cross spectral density and the structural dynamic properties by the natural frequencies and predominant mode shapes. This led to an extensive programme of theoretical and experimental work which has continued throughout the decade.

Interest has been centred on the development of simplified models of practical structures. Measurements on full-scale aircraft have been used to suggest models which can predict the predominant features of the response to jet noise. The Powell theory also showed that the response to broadband noise pressures was inversely proportional to the modal damping. Thus an increase in damping was seen to be an efficient way of reducing the response to noise. A comprehensive experimental and theoretical study of damping mechanisms and practical methods of increasing damping has been completed. Studies of the initiation and propagation of fatigue cracks in structures excited by random forces have continued throughout the period. It is now possible to make the first estimates of fatigue life and crack propagation rates in typical structures which are subject to vibration in the audio frequency range at the low levels of stress associated with jet noise excitation.

Powell showed that the response spectrum due to a random excitation involved a double summation over all normal modes of the structure. Furthermore, each term in the double infinite summation involved a double integral over the surface of the structure. Thus whilst it is formally possible to estimate the response, the task is practically impossible for a large complex structure. An investigation into the effect of the cross terms which arise in the double summation was made by Mercer [2] who considered the case of a lightly damped continuous beam, resting on many supports, excited by an acoustic pressure field. In the case showing maximum coupling, the cross terms only contributed an additional 20% to the overall r.m.s. response level. In more typical cases the contribution was of the order of 5%.

Full-scale measurements on aircraft structures [3, 4] have shown that the predominant modes of vibration in the audio frequency range are those in which the major displacements are either in the skin or in the frame or rib support structure. Wavelengths of these vibrations are of the order of the stiffener spacing. Thus it is not possible to "smear" the stiffness and mass of the stiffeners to get an equivalent uniform plate model of the structure. The two main types of structure which have been the subject of the majority of the work in the I.S.V.R. are stiffened skins and box-type structures with relatively light internal connecting members.

2. VIBRATION OF STIFFENED SKIN PANELS

In the majority of aircraft structures manufactured to date the skin structure is stiffened internally by means of open or closed section stringers. These stringers are either riveted or bonded on to the skin and the complete stiffened skin is then supported on ribs or frames which are considerably stiffer than the stringers. The individual plates themselves usually have a high-aspect ratio and thus the effect of the fixing conditions at the frame or rib (i.e., short edges of the plates) is small. The predominant panel vibration has been shown [3] to be one in which adjacent plates are vibrating in association with the flexible stringers.

By considering an infinitely long row of plates simply supported between two stiff frames and by careful examination of the time varying boundary conditions at the edge of each panel due to the stringers, Lin [5] was able to show that the response of a skin-stringer panel array fell into distinct frequency bands with definite limiting modes. That is, for each half sine wave ($m = 1, 2, 3$, etc.) between the frames there is a definite frequency band in which all the natural frequencies of the row of panels would lie. This result paralleled much earlier work (e.g., that of Ayre and Jacobsen [6]) on multi-span beams. In the beam studies it had also been shown that within any one band there are as many natural frequencies as there are spans. The frequency and mode shape of the lower bounding mode was shown to be given by allowing the stringer to twist without bending and the upper bound was obtained by allowing the stringer to bend without twisting. In the theoretical model (infinite number of bays in the row) there are an infinite number of natural frequencies in-between these bounds.

The case of a finite number of plates in the row was considered by Mercer and Seavey [7] who applied the transfer matrix method. It is also possible to consider the practical case of unequal width plates. The basic principle of the transfer matrix theory is to relate physical quantities at one point in a structure to the same quantities at some other point in the structure. That is, the conditions at, say, the right-hand end of the panel are expressed in terms of those at the left-hand end. A similar transfer matrix may be developed relating the same quantities across a stringer by assuming the stringer acts as a line stiffness. Application of the assumed boundary condition (simple supports, fully fixed or free, say) will give all the natural frequencies and normal modes of the finite length panel row. It is apparent that each span (and each stringer) may be of different physical characteristics. Whilst the method is analytically elegant it does have, as described above, a fatal flaw in that it is numerically unstable. Mercer [8] investigated this problem and showed that by developing the transfer matrix chain in the form of one large tri-diagonal matrix whose elements were themselves 2×2 matrices this numerical problem may be easily surmounted. The essential difference is that in the classical transfer matrix formulation all intermediate deflections, slopes, etc., are eliminated whereas in the tri-diagonal matrix from this intermediate information is used to maintain stability. Typical results from the analysis of a six span (equi-spaced) skin stringer structure are shown in Figure 1.

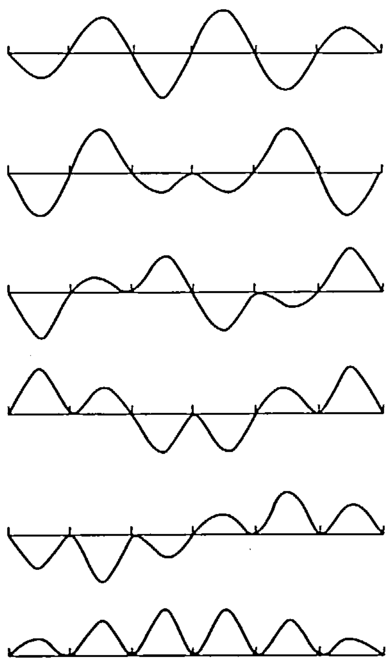

Figure 1. Normal modes for six spans with equal stringer spacing.

In the above, attention has been confined to considering flat skin stringer panel rows. In his early work Lin [5] also considered curved panels. He used an energy method and substituted the flat panel modes for the radial direction. Similarly the transfer matrix work has been developed to treat the curved panel case. The mode shapes are similar to those given by the flat panel modes as in general aircraft fuselages have a fairly shallow curvature.

In some recent designs of aircraft structure the stiffened skin is made by machining a very thick slab of aluminium on numerically controlled milling machines. The result is that the stiffeners are now an integral part of the structure and no bonding or riveting is required. For simplicity the stiffeners are usually of deep rectangular cross section and closer together than the typical open or closed section stringers in the built-up design. Clarkson and Cicci [9] investigated the response of this type of skin structure. The transfer matrix method described above can again be used to compute the normal modes and natural frequencies but in this case the stringer torsion mode becomes the upper bound and the stringer bending mode becomes the lower bound. This is because the integral stiffener has a low bending stiffness compared with the built-up designs. The mode shapes are no longer dominated by the positions of the stiffeners. In the lower modes the lateral bending displacement of the stiffeners is nearly as great as the lateral displacement of the centres of the skin plates.

3. VIBRATION OF CURVED PANELS

For panels of higher curvature, such as may occur at the rear of fuselages, the transfer matrix method may not be the most suitable method of analysis. An alternative approach was taken by studying single panels. Initial work in this area concentrated on singly-curved rectangular panels. Various theoretical methods were used to predict the vibration characteristics of a clamped panel. At the same time, experiments were performed on such a panel. It was found [10] that both an extended Rayleigh–Ritz procedure and the finite element method gave highly accurate frequencies. In the case of single curved plates the extended Rayleigh–Ritz method is the easier one to use. However, for doubly curved plates of arbitrary planform the finite element method has a distinct advantage. The Kantorovich method [11] proved to be a relatively slow method computationally, and gave results which were not as accurate as the previous two methods. Recent work has shown that the finite strip method is more efficient and gives the same results if the same physical assumptions are made. Greater accuracy can be obtained by modifying the assumed deformation in the straight direction. The effects of static membrane stresses on natural frequencies have been determined theoretically. The results await the completion of model tests for verification.

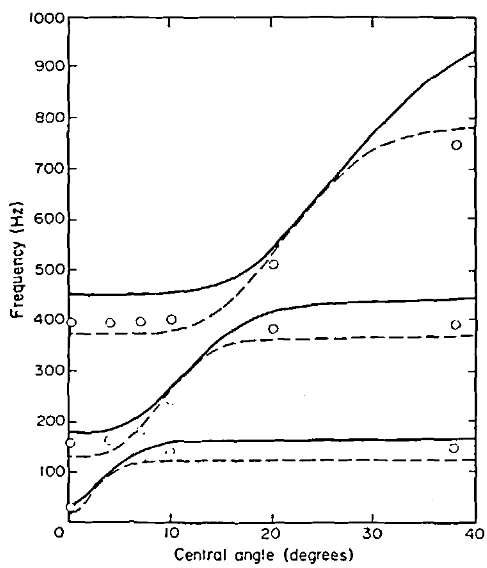

Figure 2. Symmetric frequencies of curved beams. ——, Clamped; ----, hinged; o, experimental.

The investigations on singly curved plates produced unexpected results for some of the modes. In an attempt to explain this behaviour the in-plane vibrations of curved beams were studied [12]. It was shown that, due to curvature effects, the symmetric modes interact to produce bending mode shapes different from those of a straight beam. These findings have also been confirmed theoretically. As a first step towards determining the vibrations of a singly curved stiffened panel, a multi-supported five bay curved beam has been studied [13]. It has been found that for all subtended angles the frequencies occur in groups of five (see Figure 2) as do the frequencies of a straight beam. The variation of frequency with subtended angle is similar to that of a single bay. Also, for all subtended angles, the frequencies of each group are bounded above and below by the frequencies of a single bay with clamped and hinged ends.

In order to be able to analyse doubly curved panels of arbitrary planform, doubly curved finite element models of both quadrilateral and triangular planform have been developed [14]. Tests on both singly and doubly curved panels have shown the elements to be highly accurate.

Recent work by Espindola has shown that the wave propagation method (described in section 6) can be used to compute the vibration characteristics of stiffened curved plates. In this work a transfer matrix approach is used to set up the description of one period of the periodic structure.

4. VIBRATION OF SKIN-RIB STRUCTURES

In many aircraft configurations the tail plane and elevator or fin and rudder are situated in regions of high jet noise levels. The tail plane and fin structures usually have relatively heavy stiffened skins to carry the overall bending loads. The skins have interconnecting ribs (chordwise) and spars (spanwise). The ribs generally carry relatively small shear forces and so are much lighter in construction than the skin. They usually have lightening holes and often have stiffeners between these holes.

When such a structure is excited by broadband jet noise pressures the skins vibrate as interconnected stiffened plates between the ribs. The rib structure is also forced to vibrate by rotation of its connections to the skin. There is now a danger of fatigue failure in the rib at the attachment point to the skin. Experimental studies of stresses in this internal structure have been made [3, 15]. A NASTRAN finite element analysis of the vibration characteristics of a box structure was made by Clarkson [16] but this did not prove to be a practical method of analysis because of the large number of normal modes which occur in the frequency range of interest.

Rudder and elevator structures generally have simpler skin and rib designs but the rib itself is triangular because of the tapering section of the structure. To study the vibration characteristics of this type of structure a two-dimensional, frame model was developed. For this model the two skins were assumed to be parallel, and effects caused by the finite distance between adjacent spars were neglected. A beam element with three degrees of freedom per node was used in the analysis. The natural frequencies and vibration modes of the frame were studied for different numbers of bays. The frequencies obtained from the analysis occurred in groups, with the number of frequencies in each group dependent on the number of bays. A distinct pattern was also observed in the occurrence of the modes of vibration.

Two plate bending elements were developed to investigate the vibration of three-dimensional, plate models of the skin-rib structure. The elements were a quadrilateral one, and a triangle. To study the performance of these elements, natural frequencies and nodal patterns were obtained for trapezoidal and triangular plates [17]. The plates chosen had a high aspect ratio, typical of the dimensions of an aileron rib. Both clamped and simply supported boundaries were considered.

A multicellular box construction formed by interconnecting plates was chosen as another model of the skin-rib structure. The outer skin plates were assumed to be parallel. In-plane motion of the ribs and skins was neglected. The coupling between the skins and ribs was assumed to be entirely due to the rotation of the skin rib junction. A five bay structure was considered. Because of the symmetry of the structure it was then only necessary to analyse one-eighth. Natural frequencies and mode shapes were obtained when this section was described by a mesh of 32 quadrilateral elements. This mesh was adequate to give the frequencies of the first few modes within about 2% of an exact solution.

The effect of non-parallel skins on the free vibration characteristics of these structures was studied for an aileron. The seven ribs of the aileron were triangular, with a height to base ratio of approximately 5. Frequencies and mode shapes were obtained for one, two and three bay sections.

All these models required a lengthy finite element analysis involving many degrees of freedom. The free vibration of periodic structures may also be described by a wave approach (as described in detail in section 6). A complex propagation constant that is a function of frequency is found for the infinite structure. The frequencies of the finite structure can be determined from the imaginary part of this propagation constant. An approximate method of obtaining the imaginary part of the propagation constant by the finite element technique has been developed. The method was simple to apply as it was only necessary to consider a single repetitive section of the structure. In addition, only slight modifications to the existing routines were required.

Propagation constants for a beam on simple supports, and a skin rib structure with parallel skins have been obtained by this method. They were compared with the exact solutions. The comparison showed that the approximate, finite element technique yielded satisfactory values for the imaginary part of the propagation constant. This technique was then applied to the frame and plate models of the skin-rib structure previously analysed by conventional finite element methods. Frequencies for the finite structures obtained from the graph of the imaginary propagation constants, agreed with those obtained previously. Propagation constants for the aileron have also been evaluated by this method.

5. DAMPING OF STRUCTURES

As the excitation has broadband frequency characteristics, it is not possible to reduce the response by "detuning" the structure. In some cases an increase in stiffness may even increase the response because the generalized force may be greater at the increased frequency. In this situation damping plays an important role and it may be that the most efficient structure is one in which the damping is increased artificially by the addition of energy absorbers.

In typical aircraft structures the total damping comes from several energy absorbing mechanisms. Much work on this topic has been carried out by Mead (see, for example, reference [18] in which the topic is reviewed). The internal hysteresis in the metal itself usually provides an energy loss which is at least an order of magnitude lower than that from the other mechanisms. The damping resulting from radiation of sound and the energy absorption at joints are the two major contributors. The acoustic radiation damping for flat and curved rectangular plates has been computed by Mead [19] and some experimental confirmation obtained. Work on joint damping, by Mead and Eaton [20], shows that the damping is dependent on several parameters. The most important appear to be joint shear load, joint compression amplitude and time. It is also shown that the joint damping and fatigue life can both be increased substantially by the addition of visco-elastic inserts.

The damping of stiffened skin designs can be increased considerably by the use of visco-elastic sandwich material in place of the single metal sheet skin. In these designs the visco-elastic material is sandwiched between two thin sheets of metal. This composite can then be formed and assembled with rivets being used in the normal way. Even greater damping can be achieved if the joints do not provide a rigid connection between the two face sheets of the sandwich. This can be done by bonding the inner face sheet to the support or stiffener structure or by riveting only the inner face to the support. These procedures may be unacceptable for primary structure which is designed to carry static loads (the visco-elastic material creeps under static load) but in limited regions where acoustic loading predominates it may produce the most efficient design. A systematic study of the use of such sandwich material for stiffened skins has been described by Mead [19]. During the past few years this work has been considerably refined and extended.

Mead and Markus [21–24] have conducted extensive investigations into the three-layer sandwich plate, while Clarkson and Cicci [9] and Coote [25] have developed constrained layer additive dampers for use on integrally-machined skin-stringer panels.

Mead and France [26] started on the problem of predicting the damping and resonant frequencies of a three-layer sandwich beam, fully fixed at both ends. The infinitely long sandwich beam had been adequately analysed in the preceding decade by Kerwin, Ungar and others at Bolt, Beranek and Newman, Inc., and the finite simply supported beam and plate by Mead. The normal modes of the sandwich beam with zero damping in the central core become coupled when the core is allowed to be visco-elastic: i.e., when damping is introduced. It is then insufficient to specify the damping of the beam by the loss factor of a single mode, as energy can be lost from one mode of vibration by being transmitted to and dissipated in another mode.

This problem was overcome by Mead and Markus [21], who discovered a set of orthogonal damped forced modes of vibration, in terms of which the general response of the sandwich beam (or plate) could be expressed (see also the paper by Mead [27]). The modes can only exist when each point on the surface is excited by an external pressure which is in phase with the local velocity but proportional to the local inertia loading. Each mode resonates at its characteristic resonant frequency, and the ratio of the local external pressure amplitude to the local inertia loading can be identified with the modal loss factor. This loss factor is a unique specification of the plate damping for a particular mode. The response of the plate to an arbitrary harmonic loading can be expressed as a simple series of the damped forced modes, the coefficients of which are determined from a set of uncoupled equations.

The loss factors, as identified above, can be found as the imaginary part of a complex natural frequency. The real part of this corresponds to the resonant frequency of the particular mode. Loss factors and resonant frequencies for three-layered sandwich beams and plates have thus been found by computer calculation for a wide range of boundary conditions. The fully-fixed beam has been studied in detail [22] and the relationship between loss factor and core properties has been compared with that of the well-known simply supported beam.

Also investigated has been the effect on the plate damping of adding torsional and flexural constraints to the edge of the plate [23, 24]. If these constraints are attached to the sandwich by rivets or bolts which pass right through the central visco-elastic core, shearing of the core will be locally restricted and the damping mechanism would be expected to be impaired. This has been found to be true under some conditions of torsional constraint and core shear stiffness, but with other combinations the damping is actually increased (see Figure 3). As the torsional stiffness of the edge constraints is increased, the general damping capacity of the whole plate has been found to increase. The computer programmes developed for this investigation can be used to obtain the optimum design of sandwich plates to withstand given acoustic or vibratory environments.

The concept of the damped normal modes has been combined with the methods of analysing periodic structures described in section 6. In this way, a method has been found of predicting loss factors and resonant frequencies of sandwich plates stiffened by a regular array of stiffeners. Iterative methods of solution have been developed for the computer and have been successfully run. Experimental confirmation of the loss factors and resonant frequencies has been sought from tests on sandwich plates with multiple stiffeners [28]. These tests have confirmed the impossibility of measuring the high damping of modes of vibration whose natural frequencies lie close together! The tests have therefore been used solely to measure response levels excited by noise, and to compare these with theoretical predictions. This work is continuing, with the theoretical work following the methods of periodic system analysis.

In the case of the integrally stiffened skin structures described at the end of section 2, there are fewer joints than in the built-up designs and consequently the inherent damping in the structure is low. The damping of such structures can be increased considerably by the addition of damping links across the tops of the stiffeners [9] as shown in Figure 4. In the predominant modes of vibration the stringers bend and twist. The resultant displacement of the tops of the stringers strains the damping links and vibration energy is absorbed. The dramatic reduction in response levels is shown in the figure. A study of the damping properties of materials suitable for this application has been made by Coote [25].

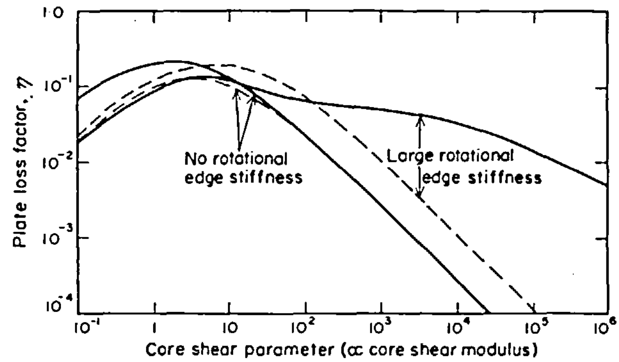

Figure 3. Effect of rotational stiffness and edge rivets on the loss factor of a damped three-layered sandwich plate. ----, With rivets; ——, no rivets.

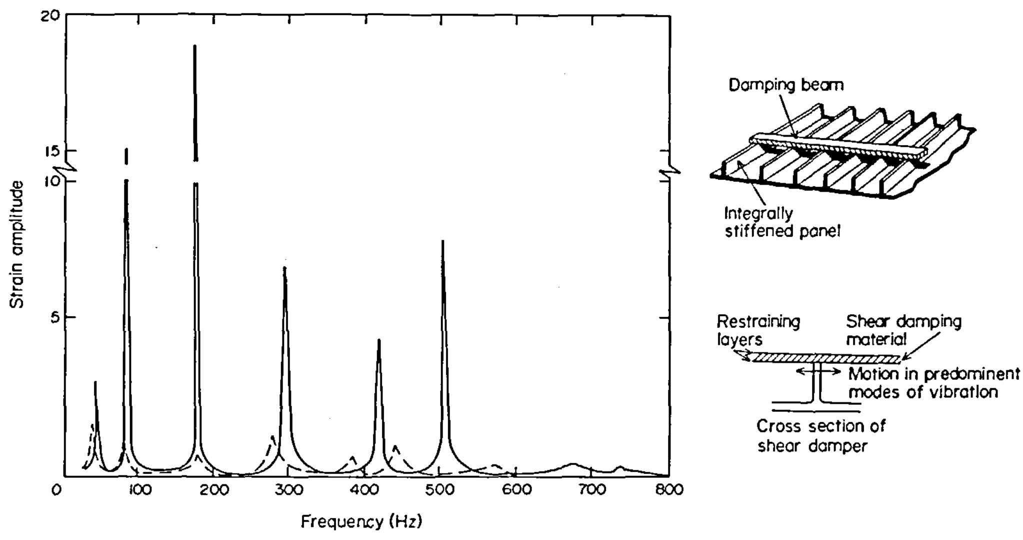

Figure 4. Effect of a damping layer on the response of an integrally stiffened panel.

6. WAVE PROPAGATION IN PERIODIC STRUCTURES

As has been described above, the normal mode method has formed the basis of much of the analytical work. Increasingly complex structures have been analysed in attempts to get closer to practical structures. At the same time, methods of damping the structural vibration by the addition of visco-elastic layers to the surfaces were being actively investigated. Efficient additive-damping mechanisms had become well known and were included in the study of stiffened plates.

Two problems have emerged. On the experimental side, it has been found to be impossible to measure the damping of the modes of the heavily-damped structures. On the theoretical and computational side the high damping meant that the structural response at one resonant frequency involved substantial and important components from off-resonant modes. The computation of the response therefore became a lengthy and tedious problem involving "modal overlap" and the determination of numerous "cross-acceptances". The fact that significant damping coupling might now exist between the "normal" modes had to be ignored.

Now it is well known that when a highly-damped structure is excited at a single point its response is confined near to the excitation point. The damping causes the outgoing wave motion from the point to decay as it propagates, and any wave motion which does reach a boundary is reflected and further attenuated by damping before returning to the source. If the damping is large enough, the returning wave motion will be negligible and the boundaries have virtually no influence on the response at the source. The structure may then be regarded as infinite in extent, having no discrete normal modes and is better analysed in terms of travelling wave motion than of normal modes.

This last feature led to the development of a wave-type approach to the estimation of response levels in acoustically-excited, highly-damped and multi-stiffened plates. The method which has been developed applies, in the first instance, to plates which are stiffened at regular intervals such that a "periodic structure" is formed: i.e., the stiffening breaks up the structure into a number of identical basic structural units (or "elements") which are attached side-by-side and end-to-end. The method has been found to be equally applicable to periodic systems with low damping. Furthermore, although developed for thin aero-space structures, the method is readily applicable to the study of vibrating periodic structures of any sort, including ships' hulls, multi-storey buildings, etc.

6.1. WAVE MOTION IN INFINITE ONE-DIMENSIONAL PERIODIC STRUCTURES

Much can be understood about the vibration of an orthogonally-stiffened plate by considering a type of wave motion which propagates in a direction parallel to one of the sets of stiffeners. In the simplest case, the plate between a pair of these stiffeners can then be reduced to a unit strip in the direction of propagation resting at regular intervals on springs which represent the stiffeners in the other direction. The strip undergoes flexural vibration as a beam. The springs exert both rotational and transverse constraint on the beam, in order to represent the stiffeners adequately.

The free waves generated in this simple structure by, say, a single point harmonic force have been extensively studied [29]. In certain frequency bands, the wave motion cannot spread out from the source without decaying in amplitude from one bay to the next. This is true whether damping is present or not. In the alternate frequency bands, flexural wave motion can propagate from bay to bay without decay (in the absence of damping), and each bay vibrates identically with its neighbour, apart from a characteristic phase difference, ε, known as the "propagation constant". This varies with frequency, from zero at the frequency at one end of the frequency propagation band, to π at the other end (see Figure 5). The propagating flexural wave motion is not a simple sinusoidal wave, for such motion is necessarily interrupted and restricted at the supports. However, it can be analysed into an infinite series of sinusoidal waves [30] each of which travels at a different phase velocity. The wave should therefore be regarded as a wave group, the group velocity of which is the frequency times the length of the periodic element times $2\pi/\varepsilon$.

The nature of the response of an infinite periodic beam to such a harmonic point force has been studied as a basic "brick" in the understanding of the response of periodic structures to more general harmonic and random excitation fields. The undamped beam undergoes infinite response at the bounding frequencies of each frequency propagation band, but between these frequencies the response is finite. When point forces act on each bay of the beam, at the same position in each bay but with a fixed phase difference between one force and the next, the infinite harmonic response occurs at that intermediate frequency at which the propagation constant is equal to the imposed phase difference between adjacent forces.

The response of a periodic beam to a convected harmonic pressure distribution (e.g., a propagating plane sound wave) is very easily calculated in a closed form [31]. In the absence of damping, an infinite response can occur in each propagation band, at the frequency at which the wave group velocity is equal to the convection velocity. The infinite response becomes finite as damping (acoustic or structural) is introduced into the system.

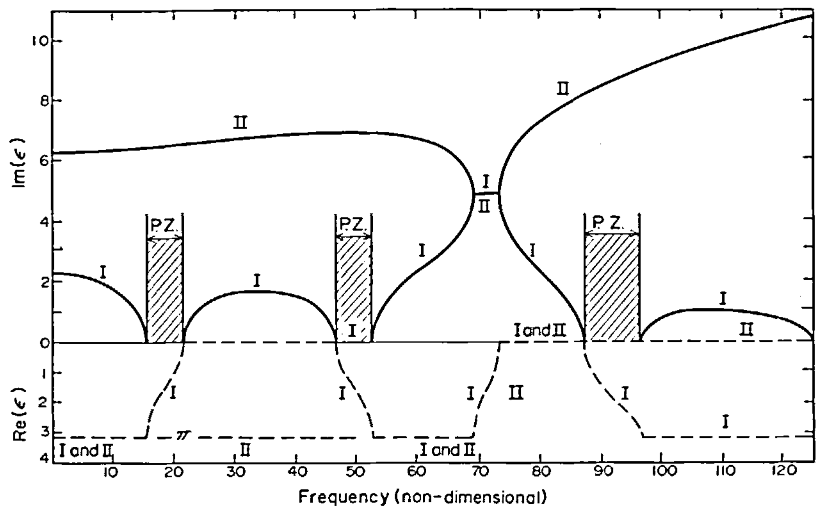

Figure 5. The two branches (I and II) of the complex wave propagation constants for a periodically supported beam on flexible supports, plotted against non-dimensional frequency. ----, Real part: i.e., the phase constant; ——, imaginary part: i.e., the attenuation constant. P.Z. is a propagation zone.

Now a random and diffuse sound field, or a convected boundary-layer pressure field may be decomposed into a spectrum of simple convected harmonic pressure distributions. The spectrum of the periodic beam response is related to this excitation spectrum through the beam response function corresponding to the simple harmonic pressure distribution. The beam response spectrum caused by a random pressure field can therefore be determined (on a computer) from a closed-form solution [32].

Computer studies based on this method have been undertaken to investigate the response of a simple "one-dimensional plate" to such pressure fields. The effects of beam damping, the rotational stiffness at the supports and the mean velocity of convection of a boundary-layer-type excitation have all been considered. Design optimization studies have been made possible, whereby the structural configuration for minimum stress response may be found for a given excitation field. Unfortunately, these invariably indicate that minimum dynamic stresses are obtained with the minimum of constraint and stiffness at the supports: i.e., a plate subjected to high frequency random pressures will have the lowest stresses when its stiffeners have zero torsional stiffness. This optimum design configuration was also deduced by Clarkson [4] from consideration of the types of modes excited.

Aeronautical "rib-skin" structures under acoustic excitation are well-suited to this method of analysis [33, 34]. The flexural wave motion can travel in both top and bottom skins of the structure, the waves in both skins being coupled through the ribs. At the periodic support (i.e., rib location) of the skins, there are three freedoms—rotation of each skin and in-plane deflection of the rib. Three waves can therefore exist at each frequency. In computer studies that have been undertaken, however, the in-plane rib motion has been ignored. The responses of the skins and ribs to homogenous noise fields have been calculated from closed-form solutions, and the influence on the response of adding damping to either the ribs or skins has been investigated. In some frequency bands, the addition of damping to the ribs has been found to be detrimental.

6.2. WAVE MOTION IN FINITE PERIODIC STRUCTURES

When an infinite periodic structure is subjected to a convected homogeneous noise field, the response is readily computed (as above) by making use of the fact that any bay of the structure must undergo the same response as all the other bays, albeit with appropriate phase differences. This is not true for a finite periodic structure, in which the response may be greater at one end than at the other. Nevertheless, the total response of the finite structure may be regarded as the forced wave motion of the infinite structure, together with a set of free waves generated by the impinging of the forced wave motion on the extreme ends of the finite structure [31, 32]. Combined together, this total wave motion yields the exact response, the amplitude of which varies from bay to bay. Closed-form solutions to the response of finite periodic structures to random noise have thereby been obtained, and computer studies undertaken. The average responses of quite short finite structures (of only five bays) have been found to be surprisingly close to the responses of their infinite counterparts (even with light damping), especially when the noise field convects very rapidly over the structure (see Figure 6). This has been confirmed experimentally by O'Keefe [35].

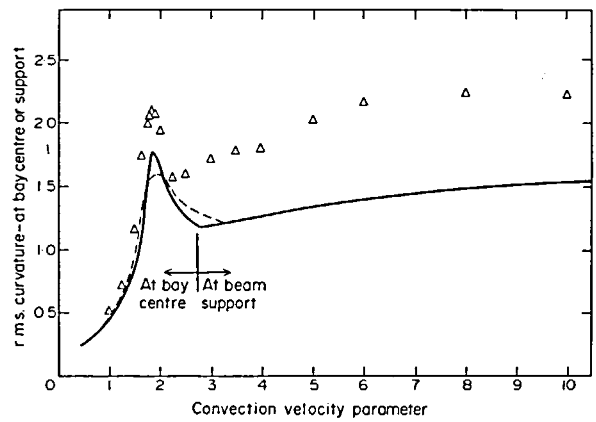

Figure 6. Comparison of r.m.s. responses of finite and infinite periodic beams to convected random acoustic plane waves. Beam loss factor 0·25. ——, Infinite beam; ----, finite, 5-bay beam (mean value); △, finite beam (maximum values).

If the natural modes and frequencies of finite periodic structures are required, use may be made of the propagation constants of the system. For a system with only one degree of freedom at each periodic support, the natural modes consist of an equal and opposite pair of propagating waves which combine at a certain frequency to satisfy the extreme boundary conditions and to form a standing wave. At this frequency, the total phase shift associated with the wave travelling backwards and forwards once along the system, together with the phase shift due to its reflection at the extreme boundaries, is equal to an integral multiple of 2π. This concept has been developed to yield a very accurate method of finding natural frequencies of, say, continuous beams on rigid supports [36], and has been used as the basis of ESDU Data Sheets [37] for the estimation of natural frequencies of stiffened plates.

6.3. WAVE MOTION IN TWO- AND THREE-DIMENSIONAL PERIODIC STRUCTURES

Thin plates which are reinforced by regular arrays of stiffeners in two directions fall in the class of "two-dimensional" periodic structures. As with one-dimensional systems, free flexural waves in the plate can propagate without decay only in certain frequency bands. Whereas closed-form solutions can be used for the one-dimensional structures to determine these bands and the propagation constants within them, such solutions cannot be used for the two-dimensional structures. Sen Gupta [38] developed a series form approach, and verified its accuracy in a limiting condition by using it to make good predictions of the natural frequencies of fully-fixed flat rectangular plates. Pujara [39] used an alternative series form to predict the response of an orthogonally stiffened flat plate to convected sound fields. Pujara's method had the additional advantage that it readily permitted the estimation of the sound transmission through the plate. It further showed how sound can be radiated from an infinite stiffened plate excited by a subsonic boundary-layer pressure field.

An "energy method" of determining the curve of propagation constant *versus* frequency for general periodic systems (one-, two- and multi-dimensional) has now been formulated. Abrahamson [40] used the Hamiltonian approach and, using an appropriate approximate mode for the propagating wave, found the relationship between propagation constant and frequency for a rib-skin structure. Mead [41] used a generalized coordinate, Lagrangian approach for a periodic system with multiple-coupling between the elements and arrived at a similar relationship. In both cases, a Rayleigh–Ritz type of modification has been developed from the basic method. Appropriate approximate wave forms are assumed for the motions in each element, and the phase difference between the motion at the ends of the element is assigned. The approximate frequency at which wave motion with this phase difference will occur is then obtained from an energy relationship, and this frequency is minimized as in the Rayleigh–Ritz method.

These developments apply to multi-dimensional structures, and seem particularly well suited to the study of vibration and noise transmission in modular-type multi-storey buildings, and in other structures having the periodic property. The sound transmission through double-walls connected by studs at regular intervals has also been analysed by using the methods for periodic systems. Fahy [42] has carried out such an investigation to predict sound transmission coefficients.

Wave propagation in periodic orthogonally-stiffened cylindrical shells has also been studied. The initial methods in which closed-form solutions were used for the wave motion led to equations for the propagation constants which were very poorly conditioned, and unreliable results were obtained from the computer. Subsequent developments have incorporated a transfer matrix approach, and this has yielded consistent results for the propagation constants and is now being successfully used to predict response levels in the cylindrical shell due to external convected pressure fields.

The finite element method has also been used (as described towards the end of section 4), to determine the propagation constants in a wedge-type structure.

7. DESIGN DATA

While the above methods are being developed, there is a need for a method which can be used in design. Even when the more sophisticated methods are available, there will continue to be a need for a "back of the envelope" estimation procedure for initial design studies. Such a method will also be needed for empirical modification to the more accurate overall methods in order to estimate the effects of structural details such as cleats, cut-outs, local stiffening, etc.

A simplified theory has been derived by Clarkson [43, 44] on the assumption that the major part of the response results from the contribution of one predominant mode. Tests on full-scale structures have shown that in certain types of structure (usually large skin plates) the response spectrum may only have one major peak. In other structures, such as control surfaces, there may be many peaks in the response spectrum. Even in this latter case, however, the simple theory gives a reasonable estimate of the overall stress level.

On the assumption that there is only one significant mode contributing to the response and that the acoustic pressures are exactly in phase over the section of structure, a simplified expression for the r.m.s. stress in a directly excited skin structure can be written as

$$\sqrt{\overline{\sigma^2(t)}} = \left|\frac{\pi}{4\zeta} f_r G_p(f_r)\right|^{1/2} \sigma_0,$$

where ζ is the viscous damping ratio (usually in the range 0·01 to 0·02), f_r is the frequency of the predominant mode, $G_p(f_r)$ is the spectral density of pressure at frequency f_r, and σ_0 is the stress at the point of interest due to a uniform static pressure of unit magnitude.

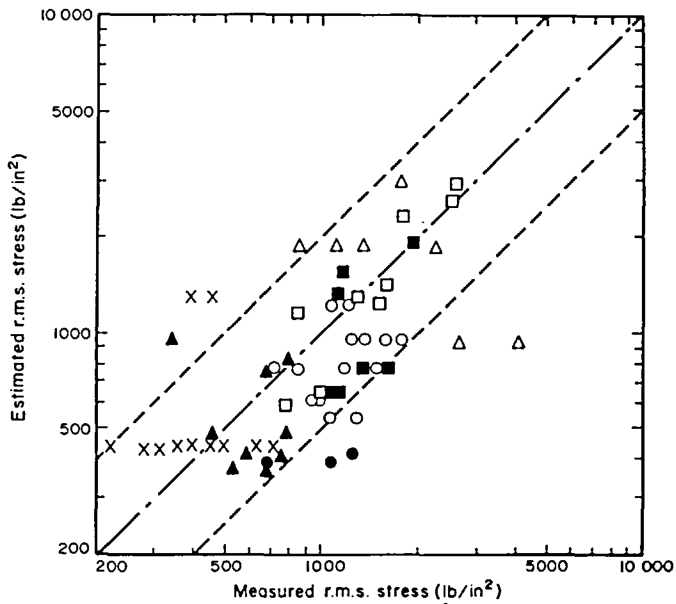

Figure 7. Comparison of estimated and measured r.m.s. stress. Control surfaces. (Based on estimated frequency.) Test series: ○, 5; △, 6; □, 7; ×, 8; ●, 9; ▲, 10; ■, 11.

The natural frequency f_r and the unit stress parameter σ_0 are derived from simplified models of the structure. For example, in the case of a stiffened skin, the individual plate members between stiffeners are assumed to be single isolated plates with fully-fixed boundaries. This is a very severe assumption, but the over-estimate of stress at the boundary (due to the assumption of full fixity) is compensated by the under-estimate due to considering only one mode instead of the several which will contribute. These counter-balancing assumptions result in an estimate which is generally within a factor of 2 of the measured result. Figure 7 shows a comparison of estimated and measured results from a wide range of structural designs [45].

This method can be extended by using a Rayleigh–Ritz method to estimate f_r and σ_0 for a more representative model of the structure.

8. CARBON FIBRE REINFORCED PLASTICS

Carbon fibres may be used to reinforce synthetic resins to produce structures offering considerable weight savings compared with conventional steel or aluminium alloys. It has also been shown during feasibility studies that the cost of a structure may be reduced by using carbon fibre reinforced plastics. Static strength and stiffness are the major advantages of the materials but in the high-performance aerospace structures which are proving to be the first major application, dynamic behaviour is of importance. The properties of greatest relevance in this area are the vibration damping and the fatigue resistance. Facilities have therefore been established at the I.S.V.R. to manufacture and test the materials. The test results have then been compared with theoretical predictions.

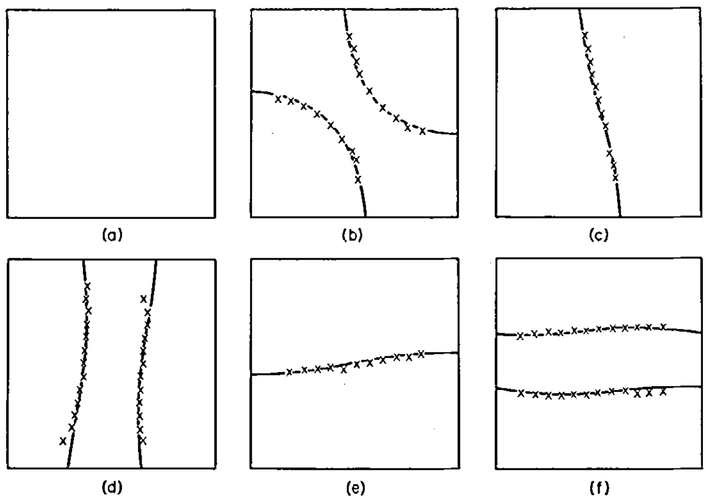

Figure 8. Comparison of experimental and theoretical results for a glass-fibre reinforced plate. (a) Predicted 50·4 Hz, experimental 49 Hz; (b) predicted 143 Hz, experimental 142 Hz; (c) predicted 91 Hz, experimental 89 Hz; (d) predicted 161 Hz, experimental 148 Hz; (e) predicted 108 Hz, experimental 104 Hz; (f) predicted 195 Hz, experimental 177 Hz.

The first task was to set up the manufacturing facilities for both unidirectionally reinforced beams and multidirectional plates. The beams have been made in simple open-ended moulds since the closed mould led to severe fibre misalignment. The plate moulding proved more difficult; Chamis [46] has shown that the control of thickness is of major importance for in-plane behaviour. Also for bending the thickness becomes a dominant parameter. The best method for moulding plates accurately is the use of a press and a suitable machine has been built at the I.S.V.R. This press is capable of moulding 600 × 600 mm sheets with an accuracy on thickness of ±0·025 mm. The sheets are also flat, which is important for vibration testing. Plates made by an industrial source by using the alternative autoclave process were not flat and the thickness varied by ±0·125 mm. This variation is not acceptable for dynamic testing.

The testing of low-mass beams required the development of novel test techniques so that correction factors were eliminated. In the test technique a specially developed optical vibration transducer is used which, unlike most displacement devices, does not need a conducting material on the beam. The lightest beam tested so far had a mass of 1·8 g. These tests on beams showed that the damping of fibre reinforced plastics is not as high as had previously been thought. Loss factors as low as half those of aircraft type alloys have been measured. This feature of the material is somewhat alarming since there is a tendency to mould the structure with very few joints. Most of the damping in metal structures is due to the joints. Work is in progress to attempt to increase the damping of the material without losing too much of the performance.

Plate testing indicated considerable non-linear behaviour and also time-dependent resonance frequencies. The transient test technique [47] was therefore used to decrease the test time and reduce the response levels. A new analysis method for determining the modal damping ratios from the transient response has been developed. In this method the requirement for dynamic range necessary when using the Kennedy–Pancu vector diagram approach is relaxed. The method is an extended logarithmic decrement technique in which two successive Fourier transforms of the autocorrelated response are used. Dynamic ranges as low as 10 dB can be used. A method of economically using the repeatability of the rapid swept sine excitation to measure the modal deflection patterns has also been developed.

With the above techniques, tests on fibre reinforced plates have been made and some results are shown in Figure 8. The theoretical predictions also shown in Figure 8 are from a Rayleigh–Ritz procedure. This method of analysis has proved adequate for frequency and mode shape prediction. It is not suitable for stress analysis, however, because the solution does not converge rapidly when the number of terms is increased.

The research programme has recently been expanded and now includes three full-time researchers, an M.Sc. student and a part-time research fellow working in the associated field of the fatigue of carbon fibre reinforced sandwich panels.

9. ACOUSTICALLY INDUCED FATIGUE CRACK PROPAGATION

9.1. INTRODUCTION

The vibration of lightweight aircraft structural components can be induced by the acoustic pressure fluctuations generated by aircraft engines and turbulent boundary layers. Although the stresses associated with this acoustically induced vibration are usually only a few percent of the ultimate strengths of commonly used structural materials, the acoustic excitation can produce a rapid stress reversal rate, typically of the order of 10^6 reversals per hour. This creates a situation in which a fatigue crack may initiate and propagate. The geometry of an aircraft skin allows a long crack to be sustained and so the crack propagation phase of the fatigue process may occupy a considerable portion of the structural life. For these reasons the work done at the I.S.V.R. on acoustically induced fatigue crack propagation has been primarily concerned with the propagation of cracks in simulated aircraft skin panels.

9.2. CENTRALLY CRACKED PANELS WITH UNIAXIAL LOADING

The initial work was done by Clarkson [48] using centrally cracked panels tensioned by a static load applied in a direction normal to the crack. The static load simulated the effect of the hoop tensile stress generated in the fuselage skin by the pressurization. Superimposed on these static stresses were dynamic bending and in-plane stresses induced by the acoustic excitation. The acoustic excitation was obtained from a high-intensity noise facility based on an electrodynamic air modulator siren. The basic emphasis of this initial work was on obtaining an understanding of the structural dynamics aspects of the problem. Thus the effect of the static load on the natural frequencies and mode shapes of the cracked panel was investigated. Further, information on crack propagation rates in typical materials for various hoop stresses and excitation levels was obtained.

Petyt [49] introduced the use of the finite element method to the problem of a tensioned plate containing a central crack and by its use was able to find the static stress distribution in the plate and also to predict its natural frequencies and mode shapes.

A predominantly experimental investigation of this same system was made by Jost [50]. He considered the propagation of the crack in terms of the stress intensity factor. The out-of-plane vibrations of the plate induce dynamic bending and in-plane stresses in the plate and these affect the growth rate of the crack. A method of combining the dynamic in-plane and bending stress intensity factors was used to provide a rational means of expressing crack growth rate under these combined conditions.

All of the above investigators [48–50] have paid considerable attention to the problem of buckling of the panel which occurs because the static loading introduces compressive stresses along the crack edge.

9.3. CENTRALLY CRACKED PANELS WITH BI-AXIAL LOADING

A logical extension of the work done on uniaxially loaded plates was to consider the effects of biaxial loading and this was done by Mills [51] and MacDonald [52]. The biaxial loading provides a more realistic representation of the static loading on the aircraft fuselage in that longitudinal and hoop stresses are present in the cracked panel.

The aim of the work done by Mills [51] was to provide a means of predicting the crack growth rate in an aircraft fuselage. The basic method he evolved was to estimate the dynamic stress intensity factor associated with the fundamental mode response of the cracked panel and to use this to predict the crack growth rate.

MacDonald [52] used the finite element method to examine this problem. In particular he was concerned with determining the crack tip stresses associated with the various modes of vibration of the cracked bi-axially loaded plate. The problem was investigated by using triangular finite elements assembled such that there was a high element density in the region of the crack tip where the stresses vary rapidly.

9.4. EDGE CRACKED PANELS

Acoustically induced fatigue cracks in an aircraft skin usually grow in the skin close to the edges of stiffeners on the skin and so it was decided to try to extend the work done on centrally cracked panels to the situation where the crack is propagating along an edge of a panel. Byrne [53] modelled this edge-cracked panel configuration by a rectangular panel clamped on three edges and part of the fourth. The unclamped part of the fourth edge represented the crack. Nominal stresses near the crack tip were evaluated by using the Rayleigh–Ritz method to find the mode shapes and natural frequencies. These stresses have been found to agree reasonably well with measured values. Work on edge cracked panels is currently proceeding.

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REFERENCES

1. A. POWELL 1958 *Journal of the Acoustical Society of America* 30, 1130–1135. On the fatigue failure of structures due to vibration excited by random pressure fields.
2. C. A. MERCER 1965 *Journal of Sound and Vibration* 2, 293–306. Response of a multi-supported beam to a random pressure field.
3. B. L. CLARKSON and R. D. FORD 1962 *Journal of the Royal Aeronautical Society* 66, 31. The response of a typical aircraft structure to jet noise.

4. B. L. Clarkson 1962 *Journal of the Royal Aeronautical Society* **66**, 603. The design of structures to resist jet noise fatigue.
5. Y. K. Lin 1960 *Journal of Applied Mechanics* **27**, 669. Free vibration of continuous skin-stringer panels.
6. R. S. Ayre and L. S. Jacobsen 1950 *Journal of Applied Mechanics* **17**, 391. Natural frequencies of continuous beams on uniform span lengths.
7. C. A. Mercer and C. Seavey 1967 *Journal of Sound and Vibration* **6**, 149–162. Prediction of natural frequencies and normal modes of skin-stringer panel rows.
8. C. A. Mercer 1965 *Revue Française de Mechanique* **13**, 15. On the application of transfer matrices.
9. B. L. Clarkson and F. Cicci 1969 *Transactions of the American Society of Mechanical Engineers, Journal of Engineering for Industry* **91B**, 1203–1209. Methods of reducing the response of integrally stiffened structures to random pressures.
10. M. Petyt 1971 *Journal of Sound and Vibration* **15**, 381–395. Vibration of curved plates.
11. M. Petyt and J. M. Deb Nath 1971 *Journal of Sound and Vibration* **13**, 485–497. Vibration analysis of singly curved rectangular plates.
12. M. Petyt and C. C. Fleischer 1971 *Journal of Sound and Vibration* **18**, 17–30. Free vibration of a curved beam.
13. M. Petyt and C. C. Fleischer 1973 Submitted to *Journal of Sound and Vibration*. Vibration of multi-supported curved beams.
14. M. Petyt and C. C. Fleischer 1972 In *The Mathematics of Finite Elements and Applications* (ed. J. R. Whiteman). Vibration of curved structures using quadrilateral finite elements. New York: Academic Press.
15. B. L. Clarkson 1972 *AGARD Symposium on Acoustic Fatigue, Toulouse, 26–27 September.* Estimates of the response of box type structures to acoustic loading.
16. B. L. Clarkson 1972 *National Symposium on Computerized Structural Analysis and Design, Washington, D.c., 27–29 March.* Computer based analyses of the response of box type structures to random processes.
17. Ruth M. Orris and M. Petyt 1973 *Journal of Sound and Vibration* **27**, 325–344. A finite element study of the vibration of trapezoidal plates.
18. D. J. Mead 1968 In *Noise and Acoustic Fatigue in Aeronautics* (eds. E. J. Richards and D. J. Mead). Chapter 18: The damping of jet excited structures. London: John Wiley and Sons, Ltd.
19. D. J. Mead 1965 In *Acoustical Fatigue in Aerospace Structures* (eds. W. J. Trapp and D. M. Forney). Chapter 26: The damping of stiffened plate structures. Syracuse University Press.
20. D. J. Mead and D. C. G. Eaton *ASD-TR*-61-667. Interface damping at riveted joints. Part II. Damping and fatigue measurements.
21. D. J. Mead and S. Markus 1969 *Journal of Sound and Vibration* **10**, 163–175. The forced vibration of a three-layer, damped sandwich beam with arbitrary boundary conditions.
22. D. J. Mead and S. Markus 1970 *Journal of Sound and Vibration* **12**, 99–112. Loss factors and resonant frequencies of encastré-damped sandwich beams.
23. D. J. Mead 1971 *Strojnicky Casopis XXII*, **1**, 53–67. The vibration characteristics of damped sandwich plates with stiffeners and various boundary conditions.
24. D. J. Mead 1972 *Journal of Sound and Vibration* **24**, 275–295. The damping properties of elastically supported sandwich plates.
25. C. Coote 1971 *Journal of Sound and Vibration* **21**, 133–147. Measurement of the damping properties of silicone-based elastomers over wide temperature ranges.
26. D. France 1968 *M.Sc. Thesis, University of Southampton.* A study of the vibration of a damped sandwich layer in non-sinusoidal modes.
27. D. J. Mead 1970 *Proceedings of the Symposium on Structural Dynamics, Loughborough University of Technology, Paper No.* C5. The existence of normal modes of linear systems with arbitrary damping.
28. D. J. Mead 1967 *Proceedings of the International Symposium on the Damping of Vibrations of Plates by Means of a Layer, Louvain, Belgium, September.* The use of stiffened sandwich plates on aircraft.
29. D. J. Mead and E. G. Wilby 1966 *Shock and Vibration Bulletin* **35**, 45–55. The random vibrations of a multi-supported heavily damped beam.
30. D. J. Mead 1970 *Journal of Sound and Vibration* **11**, 181–197. Free wave propagation in periodically supported infinite beams.
31. D. J. Mead 1971 *Journal of Engineering for Industry, Transactions of the American Society of Mechanical Engineers* **93**, 783–792. Vibration response and wave propagation in periodic structures.

32. D. J. MEAD and G. SEN GUPTA 1970 *Proceedings of Conference on Current Developments in Sonic Fatigue, University of Southampton.* Wave group theory applied to the response of finite structures.
33. D. J. MEAD and G. SEN GUPTA 1970 *United States Air Force Report AFML-TR*-70-13. Propagation of flexural waves in infinite, damped rib-skin structures.
34. D. J. MEAD and G. SEN GUPTA 1970 *Proceedings of the Symposium on Structural Dynamics, Loughborough University of Technology, March* 1970. Wave group theory applied to the analysis of forced vibrations of rib-skin structures.
35. J. O'KEEFE 1971 *M.Sc. Dissertation, University of Southampton.* A study of the forced response of a highly damped periodic structure.
36. G. SEN GUPTA 1970 *Journal of Sound and Vibration* **13**, 89–101. Natural flexural waves and the normal modes of periodically-supported beams and plates.
37. Engineering Sciences Data Unit (London). *Data Sheet No.* 72003. Natural frequencies of built-up, flat, periodic skin-stringer structures. Part 1: Stringers rigid in bending.
38. G. SEN GUPTA 1970 *Ph.D. Thesis, University of Southampton.* Dynamics of periodically stiffened structures using a wave approach.
39. D. J. MEAD and K. PUJARA 1971 *Journal of Sound and Vibration* **14**, 525–541. Space-harmonic analysis of periodically supported beams; response to convected random loading.
40. L. ABRAHAMSON 1971 *Ph.D. Thesis, University of Southampton.* The response of periodic structures to aero-acoustic pressures with particular reference to aircraft skin-rib-spar structures.
41. D. J. MEAD 1973 *Journal of Sound and Vibration* **27**, 235–260. A general theory of harmonic wave propagation in linear periodic systems with multiple coupling.
42. F. J. FAHY 1971 *Chalmers Technical University, Gothenberg, Sweden, Report No.* 70-37. Wave propagation in frame wall structures.
43. B. L. CLARKSON 1967 *AFML-TR*-67-199. Stresses in skin panels subjected to random acoustic loading.
44. Engineering Sciences Data Unit (London). *Data Sheet No.* 72005. The estimation of r.m.s. stress in stiffened skin panels subjected to random acoustic loading.
45. B. L. CLARKSON 1968 *Aeronautical Journal of the Royal Aeronautical Society* **72**, 1000–1010. Stresses in skin panels subjected to random acoustic loading.
46. C. C. CHAMIS 1969 *24th Annual Technical Conference on Reinforced Plastics/Composites Division, Society of the Plastics Industry, Paper* 18-E. Important factors in fibre composite design.
47. R. G. WHITE 1971 *Journal of Sound and Vibration* **15**, 147–161. Evaluation of the dynamic characteristics of structures by transient testing.
48. B. L. CLARKSON 1965 In *Acoustical Fatigue in Aerospace Structures* (eds. W. J. Trapp and D. M. Forney). The propagation of fatigue cracks in a tensioned plate subjected to acoustic loads. Syracuse University Press.
49. M. PETYT 1967 *AFML-TR*-67-396. Finite element vibration analysis of cracked plates in tension.
50. G. J. JOST 1969 *Ph.D. Thesis, University of Southampton.* Fatigue crack growth under combined random stress conditions.
51. D. M. MILLS 1970 *Ph.D. Thesis, University of Southampton.* Acoustically propagated cracks in biaxially tensioned plates.
52. I. J. MACDONALD 1972 *Ph.D. Thesis, University of Southampton.* Triangular finite elements for fatigue crack analysis.
53. K. P. BYRNE 1972 *Institute of Sound and Vibration Research Technical Report No.* 59. Strains affecting the growth rate of edge cracks in acoustically excited panels.