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Hybrid Ant Algorithm and Applications for Vehicle Routing Problem

Zhang Xiao,Wang Jiang-qing

School of Computer Science,
South-Central University for Nationalities,
Wuhan, China

Abstract

Ant colony optimization (ACO) is a metaheuristic method that inspired by the behavior of real ant colonies. ACO has been successfully applied to several combinatorial optimization problems, but it has some short-comings like its slow computing speed and local-convergence. For solving Vehicle Routing Problem, we proposed Hybrid Ant Algorithm （HAA） in order to improve both the performance of the algorithm and the quality of solutions. The proposed algorithm took the advantages of Nearest Neighbor (NN) heuristic and ACO for solving VRP, it also expanded the scope of solution space and improves the global ability of the algorithm through importing mutation operation, combining 2-opt heuristics and adjusting the configuration of parameters dynamically. Computational results indicate that the hybrid ant algorithm can get optimal resolution of VRP effectively.

*Keywords:*ant colony optimization, vehicle routing problem, nearest neighbor heuristic, mutation operation, 2-opt heuristics

1. Introduction

VRP is an NP-Hard problem, which was much concerned in recent years and it can be briefly described as follows: Given one or more depots, a fleet of vehicles, homogeneous or not, and a set of customers with known or forecast demands, finding a set of closed routes, originating and, generally, ending at one of the depots, to serve all customers at the lowest cost, while being within vehicle and depot capacity constraints. Other characteristics and requirements may also be taken into account, such as service and travel time restrictions, multiple commodities with different transportation requirements, time-dependent and uncertain demands or travel times, etc., yielding a rich set of VRP variants [1-2].

ACO based on foraging behavior of real ants was represented in the early 1990s by Dorigo and Caro (1999) [3]. On the basis of the traditional ACO and Nearest Neighbor (NN) heuristic [4] for solving VRP, we introduce the new hybrid algorithm of ACO, and further propose the mutation operation and 2-opt

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heuristics [5]. These strategies have proved to be helpful in selecting suitable search candidates and reducing algorithm computational time.

This thesis consists of five parts, including introduction and conclusion. The first part is the introduction, in which the aims and the structure of this thesis are stated. The second part describes the definition of VRP. A brief review for ant colony optimization is put forward in the third part. The fourth part is the main body of this thesis in which the proposed algorithm is developed while computational results are described and compared to other approaches. In the last part, conclusions are drawn.

2. Mathematical Model Of VRP

The objective of VRP is to minimize the traveling cost of vehicles while adhering to capacity restrictions. The VRP can be demonstrated as follows:

$$\text{Minimize } Z = \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{n=1}^{K} C_{ij} X_{ij}^{n} \tag{1}$$

Subject to:

$$\sum_{i=0}^{N} \sum_{j=0}^{N} X_{ij}^{n} d_i \le Q^n, 1 \le \text{n} \le \text{K}, \tag{2}$$

$$\sum_{i=0}^{N} \sum_{j=0}^{N} X_{ij}^{n} t_{ij} \le T^n, 1 \le \text{n} \le \text{K}, \tag{3}$$

$$\sum_{j=1}^{N} X_{ij}^{n} = \sum_{j=1}^{N} X_{ji}^{n} \le 1, \text{ for } i = 0 \text{ and } \text{n} \in \{1, \ldots, \text{K}\}, \tag{4}$$

$$\sum_{n=0}^{K} \sum_{j=1}^{N} X_{ij}^{n} \le K \quad \text{for } i = 0, \tag{5}$$

$$X_{ij}^{n} \in \{0,1\}, \ 1 \le \text{n} \le \text{K}, 0 \le i, j \le \text{N}, \tag{6}$$

where C_{ij} represents the cost incurred on customer i to customer j, N is the number of customers, K is the number of vehicles, d_i represents the demand of customer i, Q^n indicates the loading capacity of vehicle n, T^n is the most distance restriction of vehicle n, and t_{ij} stands for the distance from customer i to customer j. Equation (1) is the objective function of the problem. Equation (2) means the load of every vehicle cannot exceed the limit of capacity. Equation (3) represents the total distance cannot exceed the distance limit of the vehicle. Equation (4) ensures every route starts and ends at the delivery depot. Equation (5) specifies that there are maximum K routes going out of the delivery depot. Equation (6) describes that $X_{ij}^{n} = 1$ if vehicle n directly travels from customer i to customer j, otherwise $X_{ij}^{n} = 0$.

3. Ant colony optimization

ACO simulates the behavior of ant colonies in nature as they forage for food and find the most efficient routes from their nests to food sources. As some ants travel, they deposit a constant amount of pheromone trail that other ants are attracted to follow them. The rise in pheromone increases the likelihood of the next ants selecting the path. Over time, as more ants are able to complete the shorter route, pheromone accumulates faster on shorter paths and longer paths are less reinforced. The ants are capable of not only finding the shortest path from a food source to the nest, but also adapting to changes in the environment once the old one is no longer feasible due to a new obstacle. This natural behavior of ants can be used to explain reason that they can find the shortest path. Recently, ACO is widely employed to solve optimization problems and have satisfied performances in various applications. Many researchers also use ACO to obtain near optimal solutions or even global optimal solutions for VRP. Bullnheimer et al. used a nearest neighbor heuristic for VRP in ant systems [6]. Bell and McMullen [7]applies ant colony optimization to an established set of vehicle routing problems. Thereafter, ACO was also proposed for VRP [8–12].

4. The Hybrid ant algorithm ON VRP

In this paper, a hybrid ant algorithm (HAA) is proposed to solve VRP. It has the advantage of NN heuristic and ACO, the ability to find feasible solutions, and that of mutation operation and 2-opt heuristics, the ability to avoid premature convergence and then search over the subspace. The key processes of HAA for VRP involve construct_solutions, mutation operation, 2-opt heuristics and update_pheromone. During the solution construction phase, each ant of the colony attempts to construct a complete solution, which is represented as a long, single route, consisting of a number of vehicle routes. In this context, a vehicle route is considered completed when the depot node is visited, and a new vehicle route is started. The solution obtained by the ant is then subjected to mutation operation and 2-opt heuristics, to improve the solution further. Finally, pheromone trail of all sides is update.

4.1 Construct_solutions

In the process of construct_solutions, ants will utilize pheromone trail and heuristic information to build feasible solutions. The k*th* ant at time t positioned on node r moves to the next node s with the rule governed by

$$s = \begin{cases} \arg\{\max_{v=allowed_k(t)}[\tau_{rv}(t)\eta_{rv}^{\beta}]\} & \text{when } (q \le q_0) \\ S & ,otherwise. \end{cases} \quad (7)$$

where $\tau_{rv}(t)$ is the pheromone trail at time *t*, η_{rv} is heuristic information, β is a parameter representing the importance of heuristic information, q is a random number uniformly distributed in [0,1], q_0 is a pre-specified parameter $0 \le q_0 \le 1$, $allowed_k(t)$ is the set of feasible nodes currently not assigned by ant k at time t, and S is an index of node selected from $allowed_k(t)$ according to the probability distribution given by

$$p_{rs}^{k}(t) = \begin{cases} \dfrac{\tau_{rv}(t)\eta_{rv}^{\beta}}{\sum_{v\in allowed_k(t)}[\tau_{rv}(t)\eta_{rv}^{\beta}]} & \text{if } \ s \in allowed_k(t) \\ 0 & ,otherwise. \end{cases} \quad (8)$$

4.2 Mutation operation

Mutation operation refers to genetic algorithm [13] alters each child at a predefined probability. The operations can help the HAA to reach further solutions in the search space. The idea of the mutation operation is to randomly mutate the tour and hence produce a new solution that is not very far from the original one. Taken an example solution of VRP with two routes is represented as (0, 2, 4, 3, 6, 0, 1, 3, 5, 10, 0). The first route contains customers (0, 2, 4, 3, 6, 0), the second route contains customers (0, 1, 3, 5, 10, 0).

Mutation operation includes crossover mutation and insertion mutation, Fig. 1 shows the representation of the mutation procedure.

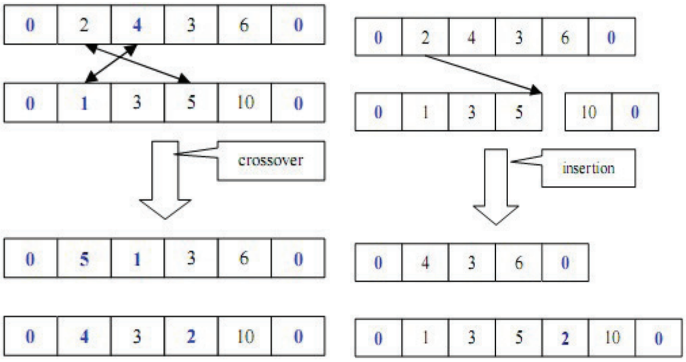

Figure 1. The mutation procedure.

4.3 2-opt heuristics

In the 2-opt heuristics, all possible pairwise exchanges of customers within an individual vehicle route are examined to see if an overall improvement in the objective function, that is, a shorter route distance, can be achieved by simply changing the order in which the customers are visited. The 2-opt heuristics is used to improve on the solution found by each ant after the solution construction procedure. They can be considered as ACO+2-opt. The representation of 2-opt heuristics is as Fig. 2:

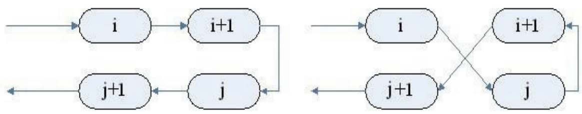

Figure 2. The 2-opt heuristics

4.4 Update_pheromone

In seeking for feasible solutions, ants perform the process of update_pheromone. This process consists of both pheromone evaporation and new pheromone deposition which can guide ants to explore possible paths and avoid trapping in locally optimal solutions. It can be stated as:

$$\tau_{rs}(t+1) = (1-\varphi)\tau_{ij}(t) + \varphi\tau_0 \tag{9}$$

where $0 < \varphi \leq 1$ is a constant, $\tau_0 = (m \cdot L_{NN})^{-1}$ is the initial value of pheromone trails, m is the number of ants, and L_{NN} is the fitness obtained by NN heuristic. When the best solution has been found, the pheromone trail update rule is performed as:

$$\tau_{rs}(t+1) = (1-\rho)\tau_{rs}(t) + \rho\Delta\tau_{rs}(t) \tag{10}$$

where $0 < \rho \leq 1$ is a parameter governing the pheromone decay process, $\Delta\tau_{rs}(t) = 1/L_{best}$ and L_{best} is the obtained best fitness from the beginning of the search process [14].

Moreover, in order to prevent from local optimization and increase the probability of obtaining a higher-quality solution, upper and lower limits $[\tau_{\min}, \tau_{\max}]$ are fixed to the updating equation:

$$\tau_{\max} = \frac{1}{2(1-\rho)} \frac{1}{L(S^{gb})}, \quad \tau_{\min} = \frac{\tau_{\max}}{5} \quad (11)$$

ρ is the constant that controls the speed of evaporation, $L(S^{gb})$ is the length of global optimal solution.

4.5 Detailed steps for solving VRP

Step1 Initialize parameters;
Step2 Calculate initial pheromone. Generate initial solution using NN heuristic; Initialize pheromone level for all cities $= \tau_0$;
Step3 Construct solutions. Each ant starts from distribution center and choose the service point according to (7) (8);
Step4 Try to improve the solution by mutation operation and 2-opt heuristics;
Step5 Repeat **step3** until all ants have constructed a solution;
Step6 Compare and contrast the paths of all ants, update the path of the minimum objective function value and paths. According to (9) (10) (11), update pheromone trail of all sides and limit the amount of pheromone trail with Max-Min thoughts;
Step7 Cumulative number of iterations;
Step8 If the number of iterations is larger than the scheduled number of iterations or all ant converge to the same path, then output the best solution, else goto **Step3**.

5. Simulation Comparative Study

Select CE-VRP (Christofides and Eilon) and att-n48-k4 problem(Converted TSPLIB Problems) from the VRPLIB(http://neo.lcc.uma.es/radiaeb/WebVRP/index.html), ACO , ACO+2-opt and this hybrid algorithm for solving the problems are achieved by C++, record intermediate data and the object function value. Compare the optimal results.

In the ACO and ACO+2-opt, we use ant cycle system, values for each parameter are: α=1, β=1, ρ=0.3, Q=10, max NC=100. The value of all parameters for proposed algorithm of this paper is: α=1, β=1, ρ=0.9, Q=1. Select the number of ants equal to the number of customers. Each algorithm runs 10 times and gets the best value. The experimental results obtained are shown in TABLE I.

TABLE I. The Results Of Three Algorithms For VRP

Problem	Official best	Algorithm	Best solution	Deviation
E-n22-k4	4/375	ACO	4/418.3	11.5%
		ACO+2-OPT	4/376.5	0.4%
		HAA	4/376.5	0.4%
E-n23-k3	3/569	ACO	3/618.7	8.7%
		ACO+2-OPT	3/579.3	1.8%
		HAA	3/577.4	1.4%
E-n33-k3	4/835	ACO	4/880.4	5.0%
		ACO+2-OPT	4/865.2	3.6%
		HAA	4/842.5	0.8%

E-n51-k5	5/521	ACO	5/639.9	22.8%
		ACO+2-OPT	5/606.6	16.4%
		HAA	5/535.9	2.6%
E-n76-k8	8/735	ACO	8/944.9	28.5%
		ACO+2-OPT	8/911.5	24.0%
		HAA	8/783.1	6.5%
E-n76-k10	10/832	ACO	10/1113	33.7%
		ACO+2-OPT	10/1065	28.0%
		HAA	10/895.7	7.6%
E-n101-k8	8/817	ACO	8/1109	35.7%
		ACO+2-OPT	8/1103	35.0%
		HAA	8/844.5	3.3%
E-n101-k14	14/1077	ACO	14/1433	33.0%
		ACO+2-OPT	14/1431	32.8%
		HAA	14/1103	2.4%
att-n48-k4	4/40002	ACO	4/44970	12.4%
		ACO+2-OPT	4/43010	7.5%
		HAA	4/40891	2.2%

We can reach some conclusions by comparing the data in TABLE I: With regards to quality of solutions, the optimal solution obtained via this method is better than that of others. The deviations of solutions are also superior to that of others. All of these show that this method can be a better way for VRP. Fig. 3 demonstrates the optimal convergence curves of this method and the other two for att-n48-k4 problem, and Fig. 4 shows the optimal solution for att-n48-k4 in a route map. We can conclude that this method has a certain extent improvement than others in best result and the speed of convergence.

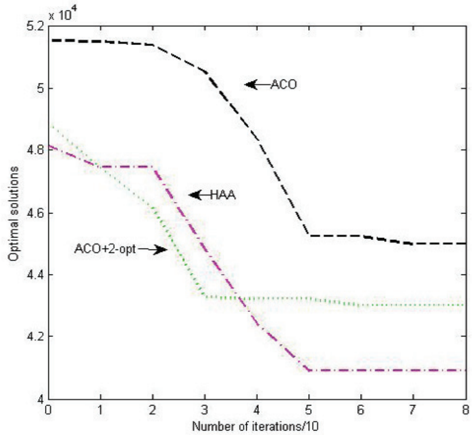

Figure 3. Trends of three Algorithms.

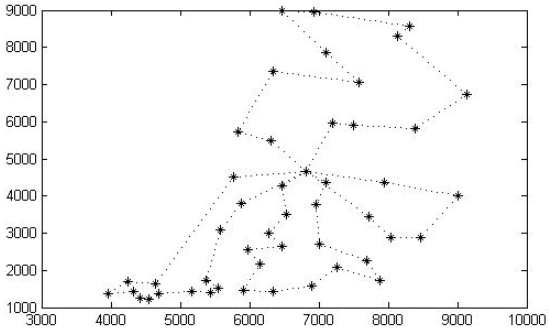

Figure 4. The route map of the optimal solution for att-n48-k4 instance.

6. Conclusions

VRP conceived as an important problem in the field of distribution and logistics. Since the delivery routs consist of any combination of customers, this problem belongs to the class of NP-hard problems. This paper presents a HAA with 2-opt heuristics and a mutation operation. It uses NN heuristic to get the initial solution and then initialize the pheromone trail for all edges. The computational results of 9 benchmark problems reveal that the proposed HAA is effective and efficient. However, there are some shortcomings, for instance, we did not study the effect of α, β, ρ on the performance of algorithm and the convergence of this method. Further research on additional modifications of the HAA to extensions of the vehicle routing problem with time windows or with more depots, are of interest. These issues will be discussed in the near future.

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