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Research Article

Incremental Optimization of Hub and Spoke Network for the Spokes' Numbers and Flow

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Hub and spoke network problem is solved as part of a strategic decision making process which may have a profound effect on the future of enterprises. In view of the existing network structure, as time goes on, the number of spokes and the flow change because of different sources of uncertainty. Hence, the incremental optimization of hub and spoke network problem is considered in this paper, and the policy makers should adopt a series of strategies to cope with the change, such as setting up new hubs, adjusting the capacity level of original hubs, or closing some original hubs. The objective is to minimize the total cost, which includes the setup costs for the new hubs, the closure costs, and the adjustment costs for the original hubs as well as the flow routing costs. Two mixed-integer linear programming formulations are proposed and analyzed for this problem. China Deppon Logistics as an example is performed to present computational analysis, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on network.

1. Introduction

Hub and spoke network is widely used in areas such as public transportation, telecommunications, and logistics distribution systems [1]. It consists of a set of nodes and some flow shipped between them. Some nodes chosen to be the hubs will consolidate, process, and redistribute the flow; others not chosen are called spokes. One important reason for considering hub and spoke network is the possibility of taking advantage of economies of scale.

Hub and spoke network was first put forward by Goldman in 1969, and then O'Kelly et al. proposed the mathematic model for hub and spoke network [2], which draw worldwide concerns and deepen research from numerous scholars. Hub and spoke network can be classified according to different standards. One important perspective concerns the allocation pattern of the spokes to the hubs. Based on this standard, single allocation and multiple allocation are often considered. Regarding the single allocation, each spoke is connected to a single hub. An extension of the capacitated single-allocation hub location problem was considered in which the capacity of the hubs was part of the decision making process and balancing requirements were imposed on the network [3, 4]. An efficient procedure that concurrently generated outer-approximation and benders cuts was devised to tackle the single-allocation hub location problem under congestion [5]. de-Camargo and Miranda [6] considered the single-allocation hub location problem under congestion from different perspectives of network owner and user. Moreover, Labbé et al. [7] developed a branch-and-cut algorithm for the hub location problems with single assignment. Puerto et al. [8] presented a new formulation for the single-allocation ordered median hub location problem and a branch-and-bound-and-cut (B&B&Cut) based algorithm to solve optimally this model. For the multiple allocation, a spoke node can send and receive flow from more than one hub. Some works considering this allocation pattern, for instance, Boland et al. [9], considered preprocessing and cutting for multiple allocation hub location problem. Contreras et al. [10] proposed a Lagrangian relaxation, based on the four-index formulation, that exploited the structure of the problem by decomposing it into independent subproblems which could be solved quite efficiently. García et al. [11] dealt with the uncapacitated multiple allocation p-hub median problem using a branch-and-cut algorithm. Hamacher et al. [12] also examined the feasibility polyhedron of the uncapacitated hub location problem with multiple allocation, which has applications in the fields of air passenger and cargo transportation, telecommunication, and postal delivery services. Kratica [13] dealt with the uncapacitated multiple allocation p-hub median problem and proposed an electromagnetism-like (EM) method for solving this NP-hard problem. Marín [14] formulated and solved splittable capacitated multiple allocation hub location problems. Sender and Clausen [15] presented a capacitated multiple allocation hub location problem, which arose from a network design problem in German wagonload traffic. Some works address both allocation patterns as it is the case in the papers [1, 2, 16]. When considering some type of constraint exists in the flow through the network, capacity constraints may indicate the nodes or the edges. Some works regarding nodes capacity are presented in [3, 4, 15]. Besides, capacity constraints both in the hubs and in the links have been studied together in research [17]. Most of the above literature is about the static hub and spoke network problems using a variety of models and algorithms. They ignore the number of spokes and flow change with the times and circumstances, and there is little research on incremental optimization of hub and spoke network driven by the number of spokes and the flow.

This paper studies the incremental optimization problem of the single-allocation hub and spoke network with a limited capacity, which means that each spoke is allocated to one and only one hub and also that there are capacity constraints. In real life, most enterprises have constructed their initial hub and spoke network in public transportation, urban distribution, and logistics areas. In view of the existing network structure, as time goes on, it is worth mentioning that the number of spoke nodes and the flow routing through the network change because of different sources of uncertainty. For instance, China Deppon Logistics, a leading integrated service-oriented logistics provider with the national 5A qualification, is dedicated to the domestic road and air freight services. Deppon has constructed the initial hub and spoke network, which includes about 50 hub centers and 4800 service points within China by the end of July 2014. With the growth in demand, its service points are extended from the original 4800 to the present 5400 in the last months, and the corresponding goods flows have also changed. In such cases, it makes sense to research the incremental optimization of the capacitated single-allocation hub and spoke network problem in this paper. As the spokes' number and flow change during each time period, the policy makers should adopt a series of strategies to cope with the change, and the decisions to be made comprise (i) whether new hubs need to be set up, and their selection, (ii) whether the capacity levels of original hubs need to be adjusted, and their new capacity levels, (iii) whether the original hubs need to be closed, (iv) the new allocation of the spoke nodes to the hubs, and (v) the flow distribution through the subnetwork defined by all the hubs. Setting up new hubs or closing original hubs need to spend sizable amounts of money on infrastructure and consume huge manpower physical resource. The adjustment of capacity level for original hub also gives rise to the expenditure. Therefore, the decision makers need to make choice by comprehensive consideration, to minimize the total cost and meet the needs.

The remainder of this paper is organized as follows. In Section 2, two mixed-integer linear programming formulations are proposed and analyzed for this problem. One is the initial optimization model as the base, and the other is the incremental optimization model of hub and spoke network. In Section 3, China Deppon Logistics as an example is performed to present computational analysis, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on network. The paper ends with some conclusions drawn from the work presented and some directions for further research.

2. Formulations and Properties

In this section, we introduce the basic setting for our analysis. Two mixed-integer linear programming formulations for the capacitated single-allocation hub and spoke network problem are proposed and discussed. We consider an initial optimization model in order to get a more focused demonstration of the incremental optimization model proposed.

Suppose there are no direct links between spoke nodes, and the connection must be realized by hubs. We assume the hub level network is a complete graph. It should be noted that, for each $k \in N$, the following relation is assumed: $T_k^{q_1} < T_k^{q_2}$ for $q_1, q_2 \in Q$, such that $q_1 < q_2$. Regarding the fixed setup cost F_k^q, it can also include fixed operation costs for the hubs (dependent on the capacity level) when they exist.

Hereafter, the following notation is considered:

$N = \{1, \ldots, n\}$: set of nodes.

$Q = \{1, \ldots, q\}$: set of capacity levels.

F_k^q: fixed setup cost for installing a hub with capacity of level q at node k $(k \in N,\ q \in Q)$.

T_k^q: capacity of a hub installed at node k with a level of capacity q $(k \in N,\ q \in Q)$.

w_{ij}: flow originated at node i that is destined to node j $(i, j \in N)$.

$O_i = \sum_{j=1}^{N} w_{ij}$: total flow originated at node i $(i \in N)$.

$D_i = \sum_{j=1}^{N} w_{ji}$: total flow destined to node i $(i \in N)$.

CC: collection cost per unit of flow and per unit of distance between a spoke and a hub.

DC: distribution cost per unit of flow and per unit of distance between a hub and a spoke.

TC: transfer cost, that is, cost per unit of flow and per unit of distance between hubs. It is assumed that TC is smaller than CC and DC.

d_{ij}: distance between nodes i and j. It is assumed that $d_{ii} = 0$ and that the distances satisfy the triangle inequality $(i, j \in N)$.

λ: the weight of setup costs.

θ: the weight of flow shipment costs.

2.1. The Initial Optimization Model. The basic decisions to be made in the initial hub and spoke network problem comprise the selection of the nodes that should become hubs and the way flow should be routed through the network. The objective is to minimize the overall cost, which includes setup costs for the hubs and flow shipment costs. With regard to the latter, they consist of collection costs, transfer costs, and distribution costs, which are, respectively, for the flow sent from the spokes to the hubs, shipped between hubs and sent from the hubs to the spokes.

Denote by x_{ik} $(i, k \in N)$ a binary variable equal to 1 if node i is assigned to hub k and 0 otherwise. For $k \in N$, $x_{kk} = 1$ indicates that node k is a hub. Regarding the flows, we consider continuous variables y^i_{ks} $(i, k, s \in N)$ representing the amount of flow with origin at i that goes through hubs k and s. Finally, due to the capacity choice that is included in the decision making process, we consider z^q_k $(k \in N,\ q \in Q)$ as a binary variable equal to 1 if node k receives a hub with capacity level q and 0 otherwise.

The problem can be formulated as follows:

$$\min \quad f = \lambda \sum_{k=1}^{N} \sum_{q=1}^{Q} F^q_k z^q_k + \theta \left(\sum_{i=1}^{N} \sum_{k=1}^{N} \mathrm{CC} d_{ik} O_i x_{ik} + \sum_{i=1}^{N} \sum_{k=1}^{N} \sum_{s=1}^{N} \mathrm{TC} d_{ks} y^i_{ks} + \sum_{i=1}^{N} \sum_{k=1}^{N} \mathrm{DC} d_{ki} D_i x_{ik} \right) \quad (1)$$

$$\text{s.t.} \quad \sum_{k=1}^{N} x_{ik} = 1, \quad i \in N \quad (2)$$

$$x_{ik} \le x_{kk}, \quad i, k \in N \quad (3)$$

$$\sum_{s=1}^{N} y^i_{ks} - \sum_{s=1}^{N} y^i_{sk} = O_i x_{ik} - \sum_{j=1}^{N} w_{ij} x_{jk}, \quad i, k \in N \quad (4)$$

$$\sum_{\substack{s=1 \\ s \ne k}}^{N} y^i_{ks} \le O_i x_{ik}, \quad i, k \in N \quad (5)$$

$$\sum_{i=1}^{N} O_i x_{ik} \le \sum_{q=1}^{Q} T^q_k z^q_k, \quad k \in N \quad (6)$$

$$\sum_{q=1}^{Q} z^q_k \le 1, \quad k \in N \quad (7)$$

$$x_{ik} \in \{0, 1\}, \quad i, k \in N \quad (8)$$

$$y^i_{ks} \ge 0, \quad i, k, s \in N \quad (9)$$

$$z^q_k \in \{0, 1\}, \quad k \in N,\ q \in Q. \quad (10)$$

The objective function (1) represents minimizing the total cost. Constraints (2) assure that each node is a hub or is allocated to a single hub. Constraints (3) make sure that a spoke node can only be allocated to the opened hubs. Constraints (4) are flow balance constraints. Constraints (5) ensure that y^i_{ks} can only be different from 0 if x_{ik} is equal to one and, in this case, all the flow originated in node i is sent to hub k. Constraints (6) are the capacity constraints. Constraints (7) guarantee that for each hub at most one size is chosen. Finally, constraints (8), (9), and (10) are domain constraints.

2.2. The Incremental Optimization Model. Based on the existing network structure in Section 2.1, as time goes on, the number of spokes and the flow will change due to different sources of uncertainty. We propose the incremental optimization model for the capacitated single-allocation hub and spoke network problem. As the spokes' number and flow change during each time period, the policy makers should adopt a series of strategies to cope with the change, such as setting up new hubs, adjusting the capacity level of original hubs, or closing some original hubs. Concerning the cost structure of the incremental optimization model, we consider setup costs for new hubs, adjustment costs for original hubs, closure costs for original hubs, and all flow routing costs.

Beyond using the variables introduced above, a new set of variables should be added to describe the incremental optimization problem. Hereafter, a few new variables can be considered in the following notation:

$M = \{1, \ldots, m\}$: set of initial hubs.

$N = \{1, \ldots, n\}$: set of spoke nodes.

AC_m^q: adjustment cost of initial hub m for changing its original capacity level to the current level q ($m \in M$, $q \in Q$).

G_m^q: closure cost for closing the initial hub m with capacity level q ($m \in M$, $q \in Q$).

Denote by H_m^q ($m \in M$, $q \in Q$) a binary variable which is equal to 1 if the initial hub m changes its original capacity level to the current level q and 0 otherwise.

The incremental optimization problem can be formulated as follows:

$$\begin{aligned}
\min \quad f &= \sum_{i=1}^{N}\sum_{m=1}^{M}\mathrm{CCD}_{im}O_i x_{im} + \sum_{i=1}^{N}\sum_{k=1}^{N}\mathrm{CCD}_{ik}O_i x_{ik} \\
&+ \sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{l=1}^{M}\mathrm{TC}d_{ml}y_{ml}^{i} \\
&+ \sum_{i=1}^{N}\sum_{m=1}^{M}\sum_{k=1}^{N}\mathrm{TC}d_{mk}y_{mk}^{i} \\
&+ \sum_{i=1}^{N}\sum_{k=1}^{N}\sum_{s=1}^{N}\mathrm{TC}d_{ks}y_{ks}^{i} \\
&+ \sum_{i=1}^{N}\sum_{m=1}^{M}\mathrm{DCD}_{mi}O_i x_{im} \\
&+ \sum_{i=1}^{N}\sum_{k=1}^{N}\mathrm{DCD}_{ki}O_i x_{ik} + \sum_{k=1}^{N}\sum_{q=1}^{Q}F_k^q z_k^q \\
&+ \sum_{m=1}^{M}H_m^q\mathrm{AC}_m^q + \sum_{m=1}^{M}\sum_{q=1}^{Q}G_m^q \\
&* \left(1 - \sum_{i=1}^{N}x_{im}\right)
\end{aligned} \tag{11}$$

$$\text{s.t.} \quad \sum_{k=1}^{M}x_{im} + \sum_{k=1}^{N}x_{ik} = 1, \quad i \in N \tag{12}$$

$$x_{ik} \le x_{kk}, \quad i,k \in N \tag{13}$$

$$\sum_{l=1}^{M}y_{ml}^{i} - \sum_{l=1}^{M}y_{lm}^{i} = O_i x_{im} - \sum_{j=1}^{N}w_{ij}x_{jm}, \quad i \in N,\ m \in M \tag{14}$$

$$\sum_{m=1}^{M}y_{km}^{i} - \sum_{m=1}^{M}y_{mk}^{i} = O_i x_{ik} - \sum_{j=1}^{N}w_{ij}x_{jk}, \quad i,k \in N \tag{15}$$

$$\sum_{s=1}^{N}y_{ks}^{i} - \sum_{s=1}^{N}y_{sk}^{i} = O_i x_{ik} - \sum_{j=1}^{N}w_{ij}x_{jk}, \quad i,k \in N \tag{16}$$

$$\sum_{\substack{l=1\\ l\neq m}}^{M}y_{ml}^{i} \le O_i x_{im}, \quad i \in N,\ m \in M \tag{17}$$

$$\sum_{m=1}^{M}y_{km}^{i} \le O_i x_{ik}, \quad i,k \in N \tag{18}$$

$$\sum_{\substack{s=1\\ s\neq k}}^{N}y_{ks}^{i} \le O_i x_{ik}, \quad i,k \in N \tag{19}$$

$$\sum_{i=1}^{N}O_i x_{im} \le \sum_{q=1}^{Q}T_m^q H_m^q, \quad m \in M \tag{20}$$

$$\sum_{i=1}^{N}O_i x_{ik} \le T_k^q z_k^q, \quad k \in N \tag{21}$$

$$\sum_{q=1}^{Q}H_m^q \le 1, \quad m \in M \tag{22}$$

$$\sum_{q=1}^{Q}z_k^q \le 1, \quad k \in N \tag{23}$$

$$x_{ik}, x_{im} \in \{0,1\}, \quad i,k \in N,\ m \in M \tag{24}$$

$$y_{ml}^{i}, y_{mk}^{i}, y_{ks}^{i} \ge 0, \quad i,k \in N,\ m,l,s \in M \tag{25}$$

$$z_k^q, H_m^q \in \{0,1\}, \quad k \in N,\ m \in M,\ q \in Q. \tag{26}$$

The objective function (11) represents the total cost of incremental optimization to be minimized. Constraints (12) assure that each node is a hub or is allocated to a hub. Constraints (13) make sure that a spoke node is only allocated to operating hubs. Constraints (14), (15), and (16) are flow balance constraints. Constraints (17), (18), and (19) ensure that y_{ml}^{i}, y_{km}^{i}, and y_{ks}^{i} can only be different from 0 if x_{im}, x_{ik} are, respectively, equal to one. Constraints (20) and (21) are the capacity constraints. Constraints (22) and (23) guarantee that for each hub at most one size is chosen. Finally, constraints (24), (25), and (26) are domain constraints.

3. Computational Experience

3.1. The Initial Optimization Analysis and Conclusion. In this section, we take China Deppon Logistics as an example to present computational analysis with the initial and incremental optimization models, to assess the effects of change of spokes' number and flow on the resulting solutions. The service network of Deppon Logistics has covered 31

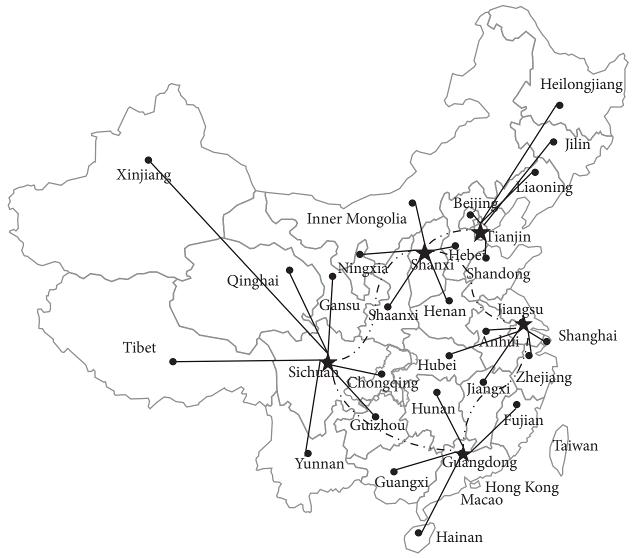


Figure 1: The initial hub and spoke network structure.

provinces all over China and the total cargo quantity is nearly 60000 tons a day. Figure 1 shows the node distribution of Deppon in 31 provinces across China. In order to estimate the requirements of nodes, which means the highway freight volume of 31 provinces, as determined by the following steps, first select the related indicators of highway freight volume, such as GDP, population, total social retail sales, fixed assets investment, industrial added value, and output value of agricultural and sideline products, and these related indicators of each province can be obtained from the website http://www.stats.gov.cn/. Then, analyze and find out the better indexes which have bigger degree of relation with that of highway freight volume by grey relational grade. Next, the fuzzy linear regression model that was proposed by [18] can be evolved into the forecasting model of highway freight volume and according to certain standards to evaluate the membership function of the forecasting model. It is generally believed the fitting accuracy is higher when the membership function is greater than 0.5. At the same time, examine the significance level of difference between the simulation result and the actual value by t-test. Again, to compare the prediction error of the fuzzy linear regression model and its prediction method, the greatest relative error, the mean absolute error, and the average relative error can be adopted. Finally inquire the market share of highway freight volume of Deppon in 31 provinces according to China Deppon Logistics official website http://www.deppon.com/, and the highway freight demand of each province and demands between provinces for Deppon can be calculated as seen in the appendix.

The distance between provinces is estimated and calculated using a geography information system (GIS). The collection cost and distribution cost per unit are taken, respectively, equal to 3 and 2; that is, $CC = 3, DC = 2$. For the value of transfer cost per unit TC, assume it is smaller than the collection cost and distribution cost per unit, so we let TC to be taken equal to 0.75. The weights of setup costs and flow shipment costs are taken equal to 0.5. Four capacity levels are available for each hub which are equal to 6000, 8000, 10000, and 12000. The fixed setup cost and closure cost for the hub depend on the capacity level chosen, which can be seen in Table 1. Adjustment costs for the hub depend on the original capacity level and adjusted capacity level. Adjustment costs between different capacity levels are as seen in Table 2.

Table 1: Capacity, fixed setup cost, and closure cost for hub.

	Capacity level			
	1	2	3	4
Capacity	6000	8000	10000	12000
Fixed setup cost	100000	120000	140000	160000
Closure cost	3000	3600	4200	4800

Table 2: Adjustment costs for different capacity levels.

Adjusted capacity level	Original capacity level			
	1	2	3	4
1	2500	5000	7000	8000
2	5000	2500	6000	7000
3	7000	6000	2500	6500
4	8000	7000	6500	2500

Deppon needs to choose some provinces as hubs to construct distribution centers and deliver goods to meet demands. This is the initial hub and spoke network. Then, as time goes on, the number of spokes or flow changes because of different sources of uncertainty. The leader should make choice to meet new demands and decrease the total cost.

The instance of the initial optimization is solved to optimality with MATLAB version 7.0. We do not report the CPU times separately for each instance since all the instances are solved within a few seconds.

We present the results in Figure 1 for the initial optimization of hub and spoke network problem. In Figure 1, the provinces of Shanxi, Sichuan, Guangdong, Jiangsu, and Tianjin are chosen to be the hubs (represented by stars) that consolidate, process, and redistribute the flow; others not chosen are spokes (represented by spots). The capacity levels of these hubs are 1, 2, 1, 1, and 1. The allocation patterns between spokes and hubs are described by solid line. The connections between hubs are described by dotted line. This indicates that Deppon should set up four hubs with the capacity of 6000 and one hub with the capacity of 8000. The total cost of the initial hub and spoke network is 1290000, and setup costs for the hubs and flow shipment costs are 520000 and 770000.

3.2. The Incremental Optimization Analysis and Conclusion. Based on the initial network structure in Section 3.1, considering the actual operation of the process, as time goes on, the number of spokes and the flow change because of different sources of uncertainty, such as the uncertainty of the market and the adjustment of enterprise strategic layout; the number of spokes may be increased or decreased; the flow will also change, so Deppon needs to do corresponding measures in different periods to cope with changing circumstances. This paper designs a series of related experiments for the incremental optimization, respectively, for spoke nodes, flow change scenarios, and studying the influence of the variables in the model on the network.

Experiment One: Incremental Optimization Analysis under the Change of the Number of Spoke Nodes. As time goes on, the number of spokes may change because of different sources of uncertainty, such as the uncertainty of the market and the adjustment of enterprise strategic layout, the number of spokes may be increased or decreased, so decision-maker need to do corresponding measures to cope with changing circumstances. Based on the initial hub and spoke network in Figure 1, two spokes are increased or decreased at each time, and the corresponding demands are generated in the same way as shown in Section 3.1. Assume these new increased nodes in turn include Suzhou in Jiangsu, Guangan in Sichuan, Pudong in Shanghai, Xiamen in Fujian, Shuozhou in Shanxi, Linyi in Shandong, Dalian in Liaoning, and Yueyang in Hunan. And the original nodes such as Jilin, Guangxi, Ningxia, and Jiangxi will be canceled as followed. Then, we solved the incremental optimization model to optimality using MATLAB version 7.0 on the same computer with the properties given in Section 2.2. The results are presented in Figure 2.

Experiment Two: Incremental Optimization Analysis under the Change of Flow. As far as the demands are concerned, they may be estimated in advance; however, the time elapsed between the moment the decision is made and the moment the network starts operating may make such information completely obsolete. Based on the initial hub and spoke network in Figure 1, the flow changes in certain proportion at each time. Then, we solved the incremental optimization models to optimality using MATLAB version 7.0 on the same computer with the properties given in Section 2.2. The results are presented in Figure 3.

Experiment Three: Pareto Analysis for Setup Costs and Flow Shipment Costs of the Network. Based on the initial hub and spoke network in Figure 1, for the value of λ, let $\lambda \in \{0.00, 0.01, \ldots, 1.00\}$ and $\theta = 1 - \lambda$, and then we solved the incremental optimization models to optimality using MATLAB software. The results are presented in Figure 4.

Experiment Four: Analysis of the Influence of Fixed Setup Cost on the Network. In practice, the actual fixed setup cost varies due to many factors such as the price of the property or the price of the raw materials. Considering the variation in the fixed setup cost, we take the fixed setup cost randomly from the interval $F_k^q \in [0.2F_k^q, 2F_k^q]$ for each potential hub location. We then solved the optimization models to optimality using MATLAB software on the same computer. The results are presented in Figure 5.

Based on the four experiments, by analysis and calculation, this paper draws the relevant conclusions as follows.

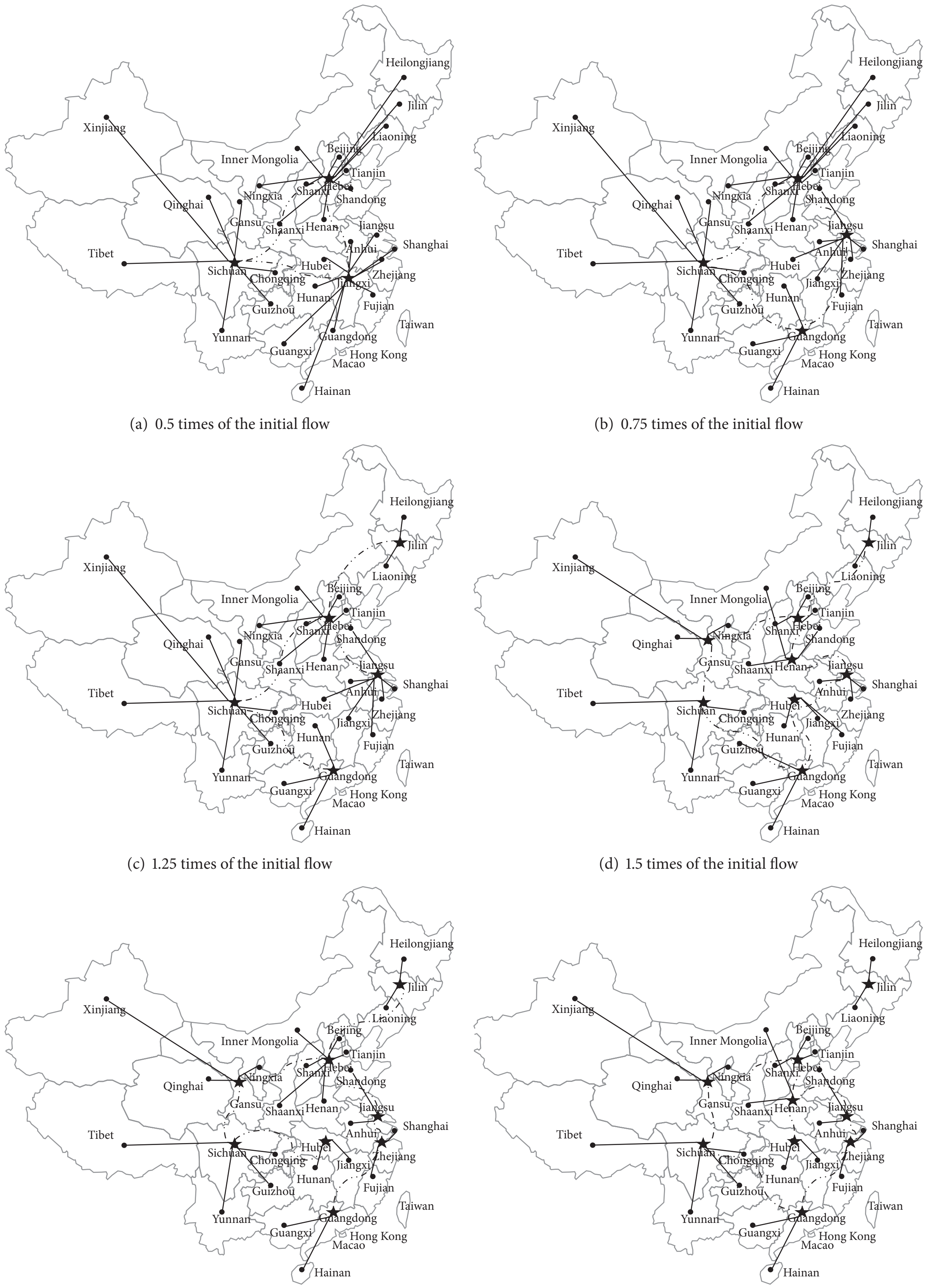
From Experiment One results can be seen, in Figures 2(a) to 2(f), showing the selection of hubs and the allocation between spokes and hubs under the changes of the number of spokes at each period of time. The new network structure at each time is determined by the changes of spoke nodes for achieving the goal of the minimum total cost including the setup costs, flow shipment costs, adjustment costs, and closure costs for the decision makers of Deppon. Figure 2 shows that when the number of spokes increases by 2, 4, 6, and 8, in turn the numbers of nodes chosen as the hubs are 6, 7, 8, and 8. Compared with the initial network, the number, location, and capacity levels of hubs are all changed, and the allocation relationship between hubs and spokes also changed. Similarly, when the number of spoke nodes decreased by 2 and 4, in turn the numbers of nodes chosen as the hubs are 5 and 4, and the location, capacity level, and allocation relationship between hubs and spokes are also changed.

From Experiment Two results can be seen, in Figures 3(a) to 3(f), showing the selection of hubs and the allocation between spokes and hubs under the changes of the flow at each period of time. The new network structure is determined at each time by the changes of demands for achieving the goal of the minimum total cost. Figure 3 shows that when the flow changes by 0.5, 0.75, 1.25, 1.5, 1.75 and 2 times, in turn the numbers of nodes chosen as the hubs are 3, 4, 5, 8, 8 and 9. The hub location, capacity level, and allocation relationship between hubs and spokes are also changed.

From Experiment Three results can be seen, in Figure 4; there is a reciprocal relationship between setup costs and flow shipment costs. When the setup costs increase by 933%, the corresponding flow shipment costs fall by 211%. With the changes of weight coefficients, the variation of setup costs becomes more obvious. Taking the cases under the different weights to analyze, when the weight coefficient of setup costs is 0.28, the setup costs and flow shipment costs are 800000 and 600390, the eight hubs selected are the provinces of Xinjiang, Jilin, Hubei, Sichuan, Guangdong, Jiangsu, Hebei,

(a) Adding two spokes
(b) Adding four spokes
(c) Adding six spokes
(d) Adding eight spokes
(e) Removing two spokes
(f) Removing four spokes

FIGURE 2: Incremental optimization results under the change of the number of spokes.



(a) 0.5 times of the initial flow

(b) 0.75 times of the initial flow

(c) 1.25 times of the initial flow

(d) 1.5 times of the initial flow

(e) 1.75 times of the initial flow

(f) 2 times of the initial flow

Figure 3: Incremental optimization results under the change of flow.

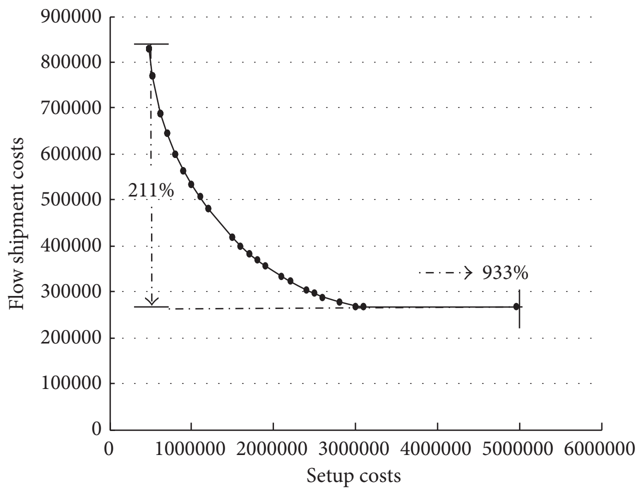


Figure 4: Pareto result.

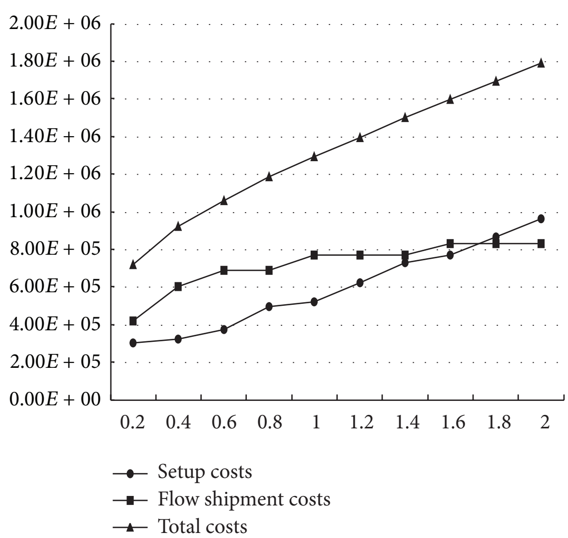


Figure 5: The impact of fixed setup cost on network.

and Gansu, and the capacity levels, respectively, are 1, 1, 1, 1, 1, 1, 1, and 1; when the weight coefficient is 0.6, the setup costs and flow shipment costs are 520000 and 770000, the five hubs selected are the provinces of Shanxi, Sichuan, Guangdong, Jiangsu, and Tianjin, and the capacity levels, respectively, are 1, 2, 1, 1, and 1. The setup costs in the above two scenarios are reduced by 53.9%, while the flow shipment costs correspondingly increase by 28.3%. In addition, during the incremental optimization we can see that, with the changes of the number of spokes and the flow at each time period, when the weight coefficient of setup costs is 0.28, the numbers of hubs change very quickly, and the capacity levels of initial hubs are almost unchanged; while the weight is 0.6, the capacity levels of original hubs are continuously changing from low level to high level, and the numbers of new or closed hubs rarely change.

This means that, for the incremental optimization of hub and spoke network under the changes of the number of spokes or the flow, the decision makers tend to be preferentially choosing to set up new hubs or close initial hubs to meet the needs in the scenarios of smaller setup cost weight. But for the scenarios of larger weight of setup costs, the priority should be given to change the capacity levels of original hubs, and the number of hubs does not change obviously. For the latter scenario, the main reason is that the setup costs have a large influence on the total cost. Therefore, the larger weight is given to the setup cost, and its influence on the total cost is greater. The adjustment cost of changing the capacity level of original hub is smaller than the fixed setup cost of new hub; hence, in order to seek the lowest total cost, the decision makers give priorities on changing the capacity level of original hub until the maximum capacity is exceeded and then choose to set up new hubs. On the contrary, for the former with smaller weight, the effect of setup costs on the total cost is small while the flow shipment costs become an important factor influencing the total cost; in this case only to change the original hub levels may cause a sharp increase in the flow shipment costs. It is effective on reducing the total cost of the optimization network that changes the number of hubs. Therefore, for the smaller weight scenarios, policy makers appear to be preferentially choosing to set up new hubs or close the initial hubs to meet the needs.

By the study on the results of Experiment Four, in Figure 5, the fixed setup cost will have a certain impact on the hub and spoke network. With the fixed setup cost changes from initial 0.2 times increase to 2 times, the setup costs first get slow and then fast while the flow shipment costs first get fast and then slow to be moderate, and the total cost is approximately linear upward trend. Before reaching 1 time, the fixed setup cost has an obvious influence on the three costs, especially for the flow shipment costs. When up to 1 time, with the increase of fixed setup cost, the setup costs and the total cost increase greatly, but the change rate of flow shipment costs remains basically unchanged. Therefore, in order to reduce the costs of the hub and spoke network, it is necessary to improve the management level and the operational efficiency, to make full use of hub capacity efficiently, and to increase the amount of transfer goods to meet the needs.

4. Conclusion

In this paper, we studied the incremental optimization problem of the capacitated single-allocation hub and spoke network. In view of the existing network structure, as time goes on, the number of spokes and the flow change because of different sources of uncertainty. The objective is to minimize the total cost, which includes the setup costs for the new hubs, the closure costs, and the adjustment costs for the original hubs as well as the flow routing costs. Two mixed-integer linear programming formulations are proposed and analyzed for this problem. China Deppon Logistics as an example is performed to present computational analysis, and we analyze the changes in the solutions driven by the number of spokes and the flow. The tests also allow an analysis to consider the effect of variation in parameters on network.

Table 3: Demands between provinces for Deppon.

Node	Province	1	2	3	4	5	6	7	8	9	10
1	Tibet	0.52	2.78	7.35	9.23	10.83	13.78	15.68	18.22	21.47	22.86
2	Ningxia	0.62	3.17	6.84	8.99	11.47	13.72	16.04	17.83	20.95	22.96
3	Xinjiang	0.98	3.41	7.25	8.97	11.34	13.67	16.43	17.56	21.31	22.74
4	Yunnan	0.76	2.92	6.88	9.43	10.61	13.88	15.55	17.57	21.13	23.08
5	Inner Mongolia	0.89	2.7	7.4	8.77	10.95	14.35	15.84	17.69	20.52	22.7
6	Chongqing	1.36	2.98	7.29	9.48	10.6	14.2	16.01	18.13	20.55	23.41
7	Jilin	0.99	3.16	6.81	9.2	10.73	13.86	15.92	18.19	21.05	23.42
8	Shanxi	0.74	3.35	7.35	8.85	10.51	13.84	15.79	18.02	20.72	23.02
9	Hunan	0.76	2.76	6.76	8.69	10.87	13.66	16.1	17.56	20.84	22.81
10	Anhui	0.8	2.64	7	8.56	11.44	14.35	15.62	18.26	20.75	22.72
11	Hubei	1	3.2	6.78	9.15	10.96	14.1	15.72	18.41	20.95	23.48
12	Sichuan	1.09	2.79	6.81	9.26	10.7	14.09	15.97	17.94	21	23.18
13	Henan	1.44	2.88	6.64	9.25	11.21	13.81	16.43	17.63	20.89	23.33
14	Shanghai	1	3.37	6.66	8.82	11.25	13.82	16.15	17.7	21.43	23
15	Zhejiang	0.55	3.18	6.65	8.51	10.64	14.22	16.06	17.63	20.73	23.43
16	Guangdong	0.53	2.9	7.33	8.75	10.8	14.37	15.69	18.2	20.99	23.07
17	Jiangsu	1.2	2.9	6.56	9.23	11.09	14.4	15.62	17.65	20.57	23.09
18	Shandong	1.28	3	6.99	9.32	11.06	13.68	15.98	17.71	20.67	23.38
19	Hebei	0.59	3.31	6.92	8.53	10.73	13.73	16.03	18.05	20.68	22.56
20	Beijing	0.93	3.37	7.07	8.74	11.21	14.43	16.37	17.62	20.88	23.28
21	Fujian	0.9	3.16	6.53	9.18	11.16	13.88	15.75	18.21	20.59	22.54
22	Liaoning	1.16	2.51	6.91	9.36	10.99	14.18	16.32	17.73	20.87	23.24
23	Guangxi	0.9	2.93	6.58	8.84	11.36	13.74	16.12	18.17	21.2	22.93
24	Jiangxi	0.52	2.91	7.43	9.31	10.56	13.88	15.8	17.77	21.03	23.48
25	Tianjin	0.51	2.87	6.8	8.62	10.81	13.94	16.18	18.14	20.64	23.49
26	Shaanxi	1.3	2.84	6.79	8.6	10.52	14.04	16.44	18.36	21.21	23.24
27	Heilongjiang	1.23	3.13	7.4	9.2	10.55	14.18	16.04	17.5	20.71	22.89
28	Guizhou	1.28	2.95	6.78	9.4	10.88	14.28	15.78	17.71	20.93	23.16
29	Gansu	1	3.04	6.75	8.54	11.29	13.53	16.22	17.67	20.65	23.5
30	Hainan	0.5	2.56	6.64	9.28	11.35	14.1	15.67	17.97	21.28	23.16
31	Qinghai	0.56	2.92	6.64	9.02	11.36	13.88	16.39	18.23	20.78	23.19
		11	12	13	14	15	16	17	18	19	20
1	Tibet	28.98	35.92	42.64	49.28	87.38	178.02	106.11	53.32	46.13	38.22
2	Ningxia	28.82	36.46	43.21	49.06	87.29	177.85	106.32	53.03	45.54	38.29
3	Xinjiang	29.23	35.74	43.45	48.78	86.9	178.15	105.6	52.75	45.62	37.92
4	Yunnan	29.49	36.34	43.31	49.01	86.56	178.41	105.76	52.73	45.7	38.4
5	Inner Mongolia	29	36.44	43.19	48.93	87.12	178.08	105.62	52.74	46.26	38.5
6	Chongqing	28.96	36.24	43.09	48.51	87.19	178.39	106.41	53.12	46.32	38.01
7	Jilin	29.07	35.65	43.44	48.78	87.08	177.96	106.27	53.47	45.94	38.41
8	Shanxi	28.9	36.23	42.68	49.36	86.51	177.8	106.38	53.37	46.24	37.98
9	Hunan	28.93	35.82	43.41	48.56	87.41	178	106.33	53.02	45.61	37.75
10	Anhui	28.74	36.17	43.43	49.4	87.25	177.65	106.17	52.96	45.96	38.22
11	Hubei	28.81	35.91	43.5	49.29	87.07	178.3	105.64	53.12	46.39	38.29
12	Sichuan	29.01	35.59	43.11	48.7	87.22	178	105.71	53.28	46.39	37.7
13	Henan	28.86	35.56	42.69	48.99	86.68	178.17	106.18	52.72	45.69	37.73
14	Shanghai	29.39	35.95	42.81	48.73	86.91	177.79	105.7	52.63	46.02	37.85
15	Zhejiang	28.89	36.3	43.01	48.55	86.85	178.08	105.95	52.72	45.57	37.56
16	Guangdong	28.68	35.83	42.68	48.86	87.35	178.46	105.66	53.04	46.17	37.74
17	Jiangsu	29.2	35.54	42.54	49.43	86.69	177.93	106	52.67	45.5	38.27

Table 3: Continued.

Node	Province											
18	Shandong	29.19	35.97	42.78	49.31	87.06	178.12	106.41	53.44	45.96	37.98	
19	Hebei	29.36	36.15	42.62	49.27	86.59	177.98	105.86	53.31	46.1	37.83	
20	Beijing	28.53	36.16	42.99	49.01	87.29	177.98	106.11	53.09	45.94	38.11	
21	Fujian	29.19	36.16	43.12	49.11	87.22	178.17	105.98	53.32	45.6	37.95	
22	Liaoning	28.68	35.51	42.84	48.59	87.2	177.85	105.61	53.13	45.61	38.35	
23	Guangxi	29.42	35.68	43.49	49.03	86.91	178.22	106.21	52.98	45.58	37.93	
24	Jiangxi	29.06	36.08	43.08	48.88	87.43	177.78	105.83	52.77	46.32	37.83	
25	Tianjin	28.61	35.9	43.2	48.9	86.67	177.94	105.65	53.38	45.66	37.92	
26	Shaanxi	29.03	35.63	43.08	48.52	86.66	177.88	105.86	52.99	46.08	37.59	
27	Heilongjiang	29.41	35.63	42.51	49.13	86.66	178.39	105.59	53.11	45.84	38.22	
28	Guizhou	29.14	36.18	43.32	48.99	87.02	177.72	105.71	52.73	45.83	38.39	
29	Gansu	28.53	36.09	43.09	48.57	87.4	178.27	105.7	53.13	46.41	38.41	
30	Hainan	28.57	35.61	43.16	49.15	86.74	178.05	106.39	53.09	45.52	38.28	
31	Qinghai	29.38	35.99	43.42	48.52	87.32	177.74	105.69	53.29	45.79	37.73	
		21	22	23	24	25	26	27	28	29	30	31
1	Tibet	36.32	26.21	23.26	17.75	16.89	16.08	11.63	9.46	7.59	5.86	1.92
2	Ningxia	36.03	25.94	23.26	18.25	16.78	16.28	11.6	9.14	7.86	6.19	2.25
3	Xinjiang	36.35	25.88	22.85	17.97	17.24	15.59	12.25	8.92	7.79	6.23	1.94
4	Yunnan	36.44	25.8	22.95	17.75	17.22	16.21	12.23	8.79	8.37	6.4	2.36
5	Inner Mongolia	36.36	26.03	22.65	18.25	17.36	15.5	12.1	9.08	8.34	6.33	2.46
6	Chongqing	35.58	26.14	23.35	17.7	16.58	16.12	11.69	9.44	8.38	5.9	2.07
7	Jilin	35.86	26.48	23.03	17.61	16.76	16.33	12.06	9.44	7.71	6.49	2.02
8	Shanxi	36.14	25.86	22.63	18.07	17	15.52	11.98	9.33	8.01	6.22	1.77
9	Hunan	35.68	25.84	23.34	17.63	16.55	16.01	12.32	9.18	7.72	5.95	1.58
10	Anhui	36.13	26.05	22.79	18.16	16.76	16.33	11.86	9.12	7.86	6.03	1.95
11	Hubei	36.33	25.79	22.98	18.38	17.14	15.57	11.51	9.14	7.8	6.31	1.86
12	Sichuan	36.45	25.53	22.5	17.61	16.71	15.88	12.3	8.96	8.07	5.59	2.19
13	Henan	36.47	25.69	22.91	17.87	17.25	16.42	12.45	9.03	7.92	5.89	2.19
14	Shanghai	36.13	25.76	22.55	17.67	17.03	16.19	11.95	9.3	7.98	6.37	2.18
15	Zhejiang	35.94	26.34	23.24	17.58	17.36	15.83	11.68	9.33	7.83	5.88	2
16	Guangdong	35.72	26.4	23.25	18.39	17.37	16.26	11.94	9.44	8.2	5.85	2.34
17	Jiangsu	36.48	25.98	22.89	17.56	16.98	15.93	11.85	8.97	7.64	5.96	1.82
18	Shandong	36.21	26.12	22.74	18.07	16.93	15.55	12.48	8.72	8.06	6.49	1.55
19	Hebei	35.52	26	22.91	17.56	17.17	15.79	11.6	9.05	8.04	6.04	2.33
20	Beijing	35.85	26	22.6	18.46	17.26	16.32	12.34	9.08	7.68	6.3	2.24
21	Fujian	36.37	25.93	23.33	17.78	16.74	15.76	12.04	9.24	7.64	6.14	1.84
22	Liaoning	35.97	26.39	22.97	17.79	17.01	16.43	11.98	8.75	7.63	6.19	2.41
23	Guangxi	36.29	25.7	23.34	18.21	16.53	15.67	12.09	9.48	7.94	5.6	2.06
24	Jiangxi	35.73	26.01	22.56	17.98	17.14	16.05	12.36	9.23	8.05	5.53	2.05
25	Tianjin	36.4	25.74	22.74	18.01	16.91	15.58	12.12	8.51	8.36	6.3	1.84
26	Shaanxi	36.25	25.92	22.72	18.05	17.03	15.56	11.68	9.47	7.63	5.57	2.02
27	Heilongjiang	36.07	25.93	22.61	17.65	17.18	15.77	12.26	8.5	8.42	5.62	1.7
28	Guizhou	36.13	25.78	22.93	17.83	17.21	15.71	11.76	9.25	8.15	6.49	1.66
29	Gansu	35.94	26.42	23.11	18.44	17.12	15.67	11.85	9.34	8.49	6.49	2.21
30	Hainan	35.51	25.85	23.36	18.28	16.71	16.33	11.52	8.97	7.87	6.24	1.53
31	Qinghai	35.5	26.34	23	17.99	16.55	15.6	12.1	9.14	7.94	5.68	2.45

The results show that when the number of spokes or flow changes, the hub location, capacity level, and allocation relationship between hubs and spokes are also changed. There is a reciprocal relationship between the setup costs and the flow shipment costs. When the setup costs increase by 933%, the corresponding flow shipment costs fall by 211%. For the instances of smaller weight of setup costs, the decision makers tend to be preferentially choosing to set up new hubs or close original hubs to meet the needs, but for the larger weight scenarios, the priority should be given to adjust the capacity levels of original hubs. The fixed setup cost has a great influence on the total cost and setup costs and has a certain influence on the flow shipment costs for the hub and spoke network.

Two lines of research can be drawn from here. One regards more complex incremental optimization problem of hub and spoke network such as problems with dynamic network design decisions. Another line of research regards the development of heuristic procedures for giving good quality feasible solutions for large instances of the problem.

Appendix

See Table 3.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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