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Convex Model Predictive Control for Rocket Vertical Landing

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Abstract: Rocket landing guidance under complex constraints is significant for reusable rocket. This paper presents a convex model predictive control strategy to generate the guidance command for fuel-optimal rocket landing problem. The performance of MPC is directly limited to its calculation speed and convergence, however the characteristics of convex optimization can make up for this problem. In this paper, we formulate the rocket landing problem in the longitudinal plane with nonlinear dynamics and nonconvex constraints, which could be convexified using successive linearization and relaxation respectively. Due to the free terminal time of this landing process, its time domain is mapped to a specific interval, and the terminal time is defined as additional control variable. A novel receding horizon strategy is proposed to obtain appropriate guidance command by changing the location of collocation points and objective function. Simulation results demonstrate that the proposed convex MPC method is very competitive that can make the rocket landing safely under complex constraints.

Key Words: model predictive control, sequential convex optimization, computational guidance

1 Introduction

The purpose of reducing the cost for launching and space exploration has always been focused worldwide. As of November 2017, SpaceX has successfully landed Falcon 9 first stage 19 times, three of which used reflight first stage rocket. Precision guidance is one of the key technologies to realize the rocket vertical landing process, which is a significant development direction in the aerospace field. However the fundamental task of launch vehicle is to send the payload into target orbit, and the primary requirement for this landing process is fuel optimal. In order to achieve a safe landing, the terminal position, velocity and rocket attitude constraints should be taken into account. According to the various rocket separation states and the aforementioned constraints, computing guidance command onboard is a proper strategy.

As technology improves and mission complexity increases, most spacecraft guidance problems are eager to achieve the optimal solution. Analytical guidance law is difficult to find by the traditional variation method and maximum principle, therefore the application of numerical optimization method to generate appropriate guidance command onboard has attracted wide attention. Computational guidance and control is proposed by Lu that extensively relies on onboard computation and often involving iterations[1]. Convex optimization, as one of the popular direction of computer guidance, is an attractive method that can theoretically guarantee the convergence and computational efficiency for aerospace autonomous operations. Liu sums up the aerospace application of convex optimization and its various lossless convexification techniques for original nonconvex problems in guidance, path planning and control problems[2]. Ref. [3]-[7] present the sequential convex programming method for various spacecraft in different scenarios. Trajectory optimization methods for asteroid landing and power descent with thrust constraints have been studied in Ref. [8]-[10], which transforms the original problem into convex optimization problem based on convexification techniques. In Ref. [11], the fuel-optimal rocket landing trajectory optimization in longitudinal plane is approximated in a second order cone problem, which is solved in polynomial time by primal-dual interior point method. This paper considers the same scenario with Ref. [11], and additionally discusses the terminal time free optimal problem and the guidance command generation with MPC (Model Predictive Control) method.

MPC has found wide applications in recent years due to its unique capability with respect to constraints satisfaction, optimized performance and adaptability for model uncertainty. It has advantages of handling constraints and optimization performance simultaneously via repeatedly solving an optimal control problem. Guo presents the moving horizon control method for ground vehicles[12]. Eren discusses the capability and theoretical foundation of MPC, and also considers its opportunities and challenges in control of aerospace systems[13]. Mesbahi solves the small body precision landing problem via convex MPC to achieve a successful landing[14], and develops unit dual quaternions in MPC for powered descent guidance within piecewise affine framework[15]. In Ref. [16], MPC is used for Asteroid landing by dividing the maneuver into a circumnavigation and a landing phase. Ref [17] presents MPC algorithm for guidance and reconfiguration of swarms of spacecraft using minimal fuel and computational resources. The formal verification of convex optimization algorithms and credible implementation for MPC are discussed in Ref. [18].

This paper presents a convex MPC method for rocket landing guidance by using novel receding horizon strategy. In the remaining of this paper, we will first formulate the nonconvex nonlinear optimization problem with landing

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dynamics, control constraints, terminal constraints and performance index. Then we present the convexification method and terminal time free linearized dynamics for the problem. In addition, we provided a novel MPC strategy for optimal guidance command that can satisfy the safety constraints. Finally, the numerical results are discussed to demonstrate the effectiveness of the proposed method.

2 Problem Formulation

In this section, we formulate the fuel-optimal rocket landing problem in longitudinal plane. It is assumed that the thrust direction of the engine always coincides with rocket axial direction, and the thrust vector is used to describe the rocket's attitude. Because the rocket is a cylinder, this paper only considers the influence of atmospheric drag and neglects the lift.

2.1 Dynamics and Constraints

The dimensionless dynamic equations of rocket landing are established in Eq. (1).

$$\begin{aligned} \dot{r} &= V\sin\gamma \\ \dot{s} &= V\cos\gamma \\ \dot{V} &= \frac{-T\cos\alpha - D}{m} - \frac{\sin\gamma}{r^2} \\ \dot{\gamma} &= \frac{-T\sin\alpha}{mV} - \frac{\cos\gamma}{r^2 V} \\ \dot{m} &= -T/I_{sp} \end{aligned} \tag{1}$$

Where $(r, s)^T$ refers to the position vector of the rocket, V is the velocity, γ is the flight path angle, m is the rocket mass. Control variables T and α represent thrust magnitude and angle of attack, respectively. The dimensionless scale for each variable can be found in Ref. [11]. D is the atmospheric drag expressed in Eq. (2). Where ρ is the air density, S_{ref} is the reference area, C_D is the drag coefficient, ρ_0 is the sea level density, β is the density scale, and h is the flight altitude, R_e is the earth radius.

$$D = 0.5\rho V^2 S_{ref} C_D, \quad \rho = \rho_0 \exp(-\beta h), \quad h = r - R_e \tag{2}$$

The boundary constraints include the initial conditions and terminal conditions shown in Eq. (3). In order to ensure the safe landing of the rocket, terminal constraints should cover landing velocity, attitude, and fuel consumption.

$$\begin{aligned} \mathbf{x_0} &= \left[r(t_0), s(t_0), V(t_0), \gamma(t_0), m(t_0)\right]^T \\ \mathbf{x_f} &= \left[r(t_f), s(t_f), V(t_f), \gamma(t_f), m(t_f)\right]^T \end{aligned} \tag{3}$$

$$V_f \le V_{safe}, \quad \gamma_f = 90^\circ, \quad \|\alpha_f\| \le \alpha_{safe}, \quad m_f \ge m_{dry} \tag{4}$$

In Eq. (4), V_{safe} is the maximum landing speed, α_{safe} is the maximum landing angle of attack, and m_{dry} is the rocket structural weight. We limit the γ_f and α_f to guarantee the rocket vertical landing.

The process constraints consist of thrust constraints and angle of attack constraints shown in Eq. (5), where subscripts '$_{\min}$' and '$_{\max}$' are the minimum and maximum of control variables, respectively.

$$\begin{aligned} T_{\min} &\le T \le T_{\max} \\ \alpha_{\min} &\le \alpha \le \alpha_{\max} \end{aligned} \tag{5}$$

2.2 Performance Index

The performance index of rocket landing problem can be selected as minimizing the fuel consumption during the landing process, which is equal to maximize the residual mass of the rocket at final time. Therefore the objective function is expressed in Eq. (6).

$$J = -m_f \tag{6}$$

From the above, fuel-optimal rocket landing problem in longitudinal plane with nonlinear dynamics, boundary constraints and process constraints is described as *Problem0*.

$$\begin{aligned} \mathit{Problem0}: \quad &\min \quad Eq.(6) \\ &s.t. \quad Eqs.(1)(3)(4)(5) \end{aligned} \tag{7}$$

2.3 Convexification

The fuel consumption for rocket landing is related to the flight time. It makes the *Problem0* be a terminal time free optimization problem, which does not apply to convex optimization. Thus the time domain $[t_0, t_f]$ is mapped to a specific domain [0, 1] by transformation Eq.(8), where τ is the new independent variable of the dynamic equations, and the terminal time is defined as additional control variable.

$$\frac{t - t_0}{t_f - t_0} = \frac{\tau - 0}{1 - 0} \tag{8}$$

The augmented dynamics with additional state variable flight time t and control variable t_f is shown in Eq. (9).

$$\begin{aligned} \dot{r} &= t_f V\sin\gamma \\ \dot{s} &= t_f V\cos\gamma \\ \dot{V} &= t_f\left(\frac{-T\cos\alpha - D}{m} - \frac{\sin\gamma}{r^2}\right) \\ \dot{\gamma} &= t_f\left(\frac{-T\sin\alpha}{mV} - \frac{\cos\gamma}{r^2 V}\right) \\ \dot{m} &= -t_f T/I_{sp} \\ \dot{t} &= t_f \end{aligned} \tag{9}$$

This paper defines a new state variable z, and three new control variables $[u_1, u_2, u_3]$ in Eq. (10).

$$\begin{aligned} z &= \ln(m) \\ u_1 &= T\cos\alpha/m \\ u_2 &= T\sin\alpha/m \\ u_3 &= T/m \end{aligned} \tag{10}$$

Following nonlinear constraint should be satisfied that the new dynamics is equivalent to Eq. (9).

$$u_1^2 + u_2^2 = u_3^2 \tag{11}$$

The new objective function with new state variable z is described in Eq. (12).

$$J = -z_f \tag{12}$$

The terminal constraints of mass and angle of attack are transformed in Eq. (13), where z_{dry} is equal to $ln(m_{dry})$.

$$\left\| u_2/u_1 \right\|_{t_f} \le \tan\left(\alpha_{safe}\right), \quad e^z \ge e^{z_{dry}} \tag{13}$$

The new process constraints are described in Eq. (14).

$$\begin{aligned} &T_{\min} e^{-z} \le u_3 \le T_{\max} e^{-z} \\ &\tan\alpha_{\min} \le u_2/u_1 \le \tan\alpha_{\max} \end{aligned} \tag{14}$$

Thus Eq. (9) can be simplified as Eq. (15).

$$\begin{aligned} &\dot{r} = t_f V \sin\gamma \\ &\dot{s} = t_f V \cos\gamma \\ &\dot{V} = t_f\left(-u_1 - \frac{D}{m} - \frac{\sin\gamma}{r^2}\right) \\ &\dot{\gamma} = t_f\left(-\frac{u_2}{V} - \frac{\cos\gamma}{r^2 V}\right) \\ &\dot{z} = -t_f u_3 / I_{sp} \\ &\dot{t} = t_f \end{aligned} \tag{15}$$

Based on the above substitution, *Problem1* presents the new terminal time free optimal control problem.

$$\begin{aligned} \textit{Problem1}: &\ \min \quad Eq.(12) \\ &\ s.t. \quad Eqs.(3)(11)(13)(14)(15) \end{aligned} \tag{16}$$

Successive linearization and relaxation are two major convexification methods to approximate the nonlinear dynamics and nonconvex constraints. Eq. (15) is converted into a linear system as

$$\dot{x} = A\left(x^k, u^k\right)x + B\left(x^k, u^k\right)u + C\left(x^k, u^k\right) \tag{17}$$

Where $x := [r, s, v, \gamma, z, t]^T$, $u := [u_1, u_2, u_3, t_f]^T$, and the coefficient matrix of Eq. (17) are defined as

$$A\left(x^k, u^k\right) = \begin{bmatrix} 0 & 0 & \partial\dot{r}/\partial V & \partial\dot{r}/\partial\gamma & 0 & 0 \\ 0 & 0 & \partial\dot{s}/\partial V & \partial\dot{s}/\partial\gamma & 0 & 0 \\ \partial\dot{V}/\partial r & 0 & \partial\dot{V}/\partial V & \partial\dot{V}/\partial\gamma & 0 & 0 \\ \partial\dot{\gamma}/\partial r & 0 & \partial\dot{\gamma}/\partial V & \partial\dot{\gamma}/\partial\gamma & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{18}$$

$$B\left(x^k, u^k\right) = \begin{bmatrix} 0 & 0 & 0 & \partial\dot{r}/\partial t_f \\ 0 & 0 & 0 & \partial\dot{s}/\partial t_f \\ \partial\dot{V}/\partial u_1 & 0 & 0 & \partial\dot{V}/\partial t_f \\ 0 & \partial\dot{\gamma}/\partial u_2 & 0 & \partial\dot{\gamma}/\partial t_f \\ 0 & 0 & \partial\dot{z}/\partial u_3 & \partial\dot{z}/\partial t_f \\ 0 & 0 & 0 & \partial\dot{t}/\partial t_f \end{bmatrix} \tag{19}$$

$$C\left(x^k, u^k\right) = \dot{x}\left(x^k, u^k\right) - A\left(x^k, u^k\right)x^k - B\left(x^k, u^k\right)u^k \tag{20}$$

The successive linearization model is equal to the nonlinear dynamic system when the optimal solution is achieved. For the nonlinear equations including e^{-z}, we linearize them as follows

$$e^{-z} = e^{-z^k} - e^{-z^k}\left(z - z^k\right) \tag{21}$$

The superscripts k represents the value of each variable in the k^{th} iteration. The control equality constraint Eq. (11) is relaxed in a second order cone constraint as shown in Eq. (22). Ref. [11] demonstrates that the optimal solution is always active on the boundary of the second order cone. Therefore this relaxation is a lossless convexification.

$$u_1^2 + u_2^2 \le u_3^2 \tag{22}$$

Thus the original rocket landing problem is convexified as a convex optimization problem described in Eq. (23).

$$\begin{aligned} \textit{Problem2}: &\ \min \quad Eq.(12) \\ &\ s.t. \quad Eqs.(3)(13)(14)(17)(21)(22) \end{aligned} \tag{23}$$

Problem2 could be iteratively solved in polynomial time by the primal-dual interior point method. This feature provides a strong support for generating guidance command onboard by using MPC. Iteratively solving this problem until convergence we can obtain the optimal solution of the original *problem0*. The stopping criteria of sequential convex optimization is shown in Eq. (24), which means the optimal states and control sequence is convergent.

$$\max\left|x_i^{k+1} - x_i^k\right| \le \varepsilon_x, \quad \max\left|u_i^{k+1} - u_i^k\right| \le \varepsilon_u \tag{24}$$

3 Model Predictive Control

The control input of MPC is obtained by optimizing a finite horizon open-loop optimal control problem, and only implementing the first step of the optimal control sequence. The purpose of MPC is to minimize the tracking error with regard to reference trajectory that can satisfy the constraints and correct uncertainties. The reference state variables at each discrete points act as the prediction model to construct optimization problem, and its prediction horizon moves forward a sampling period after each iteration.

The objective function of a typical MPC is to minimize the tracking error of the state variables and control variables as described in Eq. (25). The dynamics and constraints have the same expression as presented in *Problem2*.

$$\begin{aligned} &J_j = \sum_{k=1}^{N_p}\left(\Delta x^T Q \Delta x + \Delta u^T R \Delta u\right) \\ &\begin{cases} \Delta x = x\left(j+k|j\right) - x_{ref}\left(j+k|j\right) \\ \Delta u = u\left(j+k|j\right) - u_{ref}\left(j+k|j\right) \end{cases} \end{aligned} \tag{25}$$

Where j is the current time, N_p is the range of prediction horizon and k is a specific time in the prediction horizon. We define that $x(j+k\,|\,j)$ indicates the states and controls of $j+k$ moment, which is estimated at the current time. x_{ref} and Δx indicate the reference states and the tracking error of states, respectively. The definition of u is the same as x. In the objective function, Q and R are the weighting coefficients that express the relative importance between states and controls.

For typical MPC the discrete interval of reference states and controls equals to sampling time of the discretization optimization problem. However for launch vehicle trajectory optimization, in order to rapid calculation, the number of discrete points is less and the arrangement is sparse. The most direct approach is to apply linear interpolation to calculate the control variables for each guidance period. Then the reference trajectory is achieved by numerical integration such as Runge-Kutta method. An obvious deficiency of this method is that the process constraints at the interpolation point are not considered. In particular, it is unreasonable to use linear interpolation in the discrete interval where the control variables change dramatically. Although MPC method tracking reference trajectory can satisfy the process constraints, it is difficult to determine the exact turning point of control variable.

This paper demonstrates a novel receding horizon strategy to obtain appropriate guidance command by changing the location of discrete points and objective function. Due to the control for every guidance period is the first step of the optimal control sequence, a simple but effective idea of discrete points configuration is illustrated in Fig. 1.

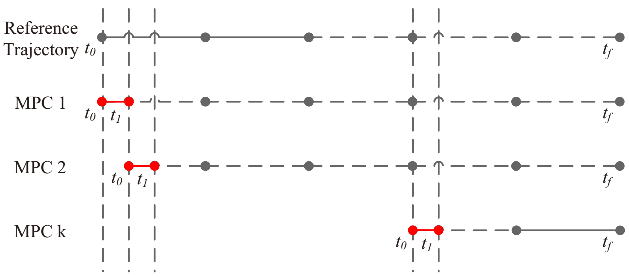

Fig. 1: Diagram of novel receding horizon strategy

As can be seen in Fig. 1, based on the discrete point sequence of trajectory optimization, a specific discrete point is added at the next sampling moment, and the location of other points remains unchanged. We use t_0 and t_1 respectively to represent current moment and next moment, which are transferred forward iteratively, and t_f indicates the terminal time of launch vehicle landing. Instead of the tracking error, the objective function is defined as minimizing the fuel consumption of the landing process. Therefore the new MPC problem is in line with the *Problem2* except the additional discrete point.

The solution procedure of MPC is discrete time, and in this paper we discretize *Problem*2 by the trapezoidal rule. Assuming that we have N discrete intervals with N+1 discrete points, and the step size for each interval is Δt_i, then the linear system in Eq. (17) can be transcribed as

$$x_i = x_{i-1} + \frac{\Delta t_i}{2}\left(\dot{x}_{i-1} + \dot{x}_i\right), \quad i = 1, 2, \ldots, N \tag{26}$$

The optimization variable vector contains the states and controls at each discrete points as in Eq. (27). The boundary conditions, process constraints and objective function can be described by this optimization variable vector.

$$Variable_{opt} = \left[x_0^T, \ldots x_N^T, u_0^T, \ldots, u_N^T\right]^T \tag{27}$$

The advantage of this proposed MPC method is that the objective function of each receding horizon is the global optimization objective. When large deviations occur during the flight, the new MPC controller does not rigidly follow the original reference trajectory, but generates the guidance command with the optimal performance index as the target.

We bring the optimal solution of the previous period as the initial guess to the next optimization problem, so this novel MPC strategy has good computational efficiency for convex optimization in every iteration.

4 Numerical Simulation

In this section, numerical examples are provided to validate the proposed method for rocket landing trajectory optimization and computational guidance problems. The following simulations results are obtained using the software MOSEK, and run on a desktop with Intel Core i7-4790 3.60GHz and 4GB RAM. The parameters of rocket are given in Table 1.

Table 1: Rocket Parameters

Symbol	Value
m_0	55000kg
m_{dry}	49000kg
S_{ref}	8m^2
C_D	0.25
$[T_{min}, T_{max}]$	[412.7, 1375.6]kN
I_{sp}	443s

4.1 Trajectory optimization

The initial and terminal conditions of rocket landing problem, and the process constraints are listed in Table 2. We define 21 discrete points to solve the trajectory optimization problem, which consists of 126 state variables and 63 control variables.

Table 2: Conditions and Constraints of Rocket Landing

h_0/h_f	s_0/s_f	V_0	γ_0	$[\alpha_{min}, \alpha_{max}]$	α_{safe}	V_{safe}
3/0km	0/1km	280m/s	-65°	[-10°,10°]	2°	1m/s

The simulation results of rocket landing problem are illustrated in Fig.2 and Fig. 3. First we assume the landing process costs 20s, and optimize *Problem0*. The optimal solution with fixed terminal time is represented by a blue line. Then we optimize *Problem1* with a free terminal time that is represented by the green line. In addition, we compute the optimal landing trajectory with Runge-Kutta method represented by a red dash line, which integration step is chosen as 0.1s.

Fig. 2 shows the states of the rocket vertical landing problem, and Fig. 3 shows the thrust magnitude and angle of attack, where the nodes on the thrust curve represent 21 discrete points of the optimization problem. From the simulation results, all initial and terminal conditions and process constraints are satisfied for both terminal time fixed and free problem. In t_f free condition, the optimal flight time is 19.28s with 5410kg fuel consumption, which is less than the 5489kg fuel consumption in t_f fixed condition. Due to the integration step is 0.1s, we use linear interpolation to achieve the control of each step, and the flight time is 19.3s. Most states agree with the planning results, except for the terminal flight path angle that is caused by a large discrete interval of the optimization problem and the approximate flight time. In next subsection, MPC method can eliminate this error.

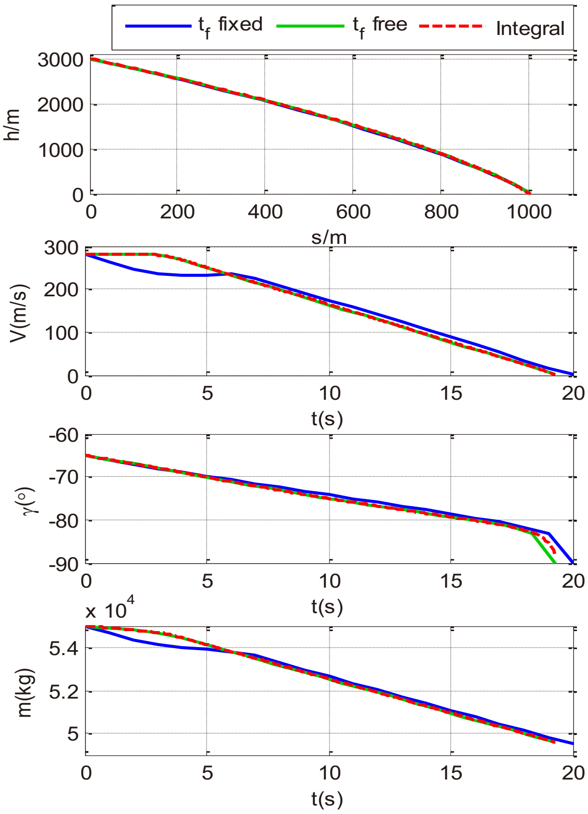

Fig. 2: The states of rocket landing

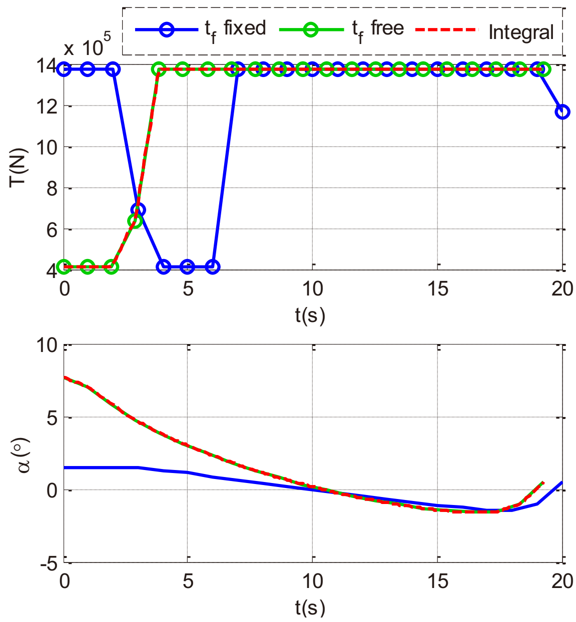

Fig. 3: The controls of rocket landing

4.2 MPC results

According to the optimal trajectory presented in the previous subsection, the novel MPC method is used to generate the guidance command. The guidance cycle of rocket landing problem is assumed as $0.1s$ less than the discrete interval of trajectory optimization, and two scenarios are given to show effectiveness of the proposed method in different situations. First we assume that the flight environment has no disturbances and uncertainties. At every guidance cycle we solve the MPC problem to achieve the optimal command based on the current actual states of the rocket. Second we suppose that the initial states of the rocket entering the vertical landing phase are uncertain because of the effect of interference. The deviation of the initial position, velocity and flight path angle and their magnitude are presented in Table 3. The impact of each deviation on fuel consumption is analyzed as well. In Table 3, the plus sign indicates that the fuel consumption rise, minus sign down. When these four deviations are positive, they all need to consume more fuel to complete the landing, that is, these deviations are positively related to fuel consumption.

Table 3: Initial Deviation Influence on Fuel Consumption

Deviation variables	Magnitude	Positive	Negative
Δh	$100m$	+	-
Δs	$100m$	+	-
ΔV	$20m/s$	+	-
$\Delta \gamma$	$3°$	+	-

Thus all deviations are positive or negative are two extreme conditions in this problem, and we use the novel MPC method generate guidance command online in the two cases. The simulation results for the two scenarios are illustrated in Fig.4 and Fig. 5.

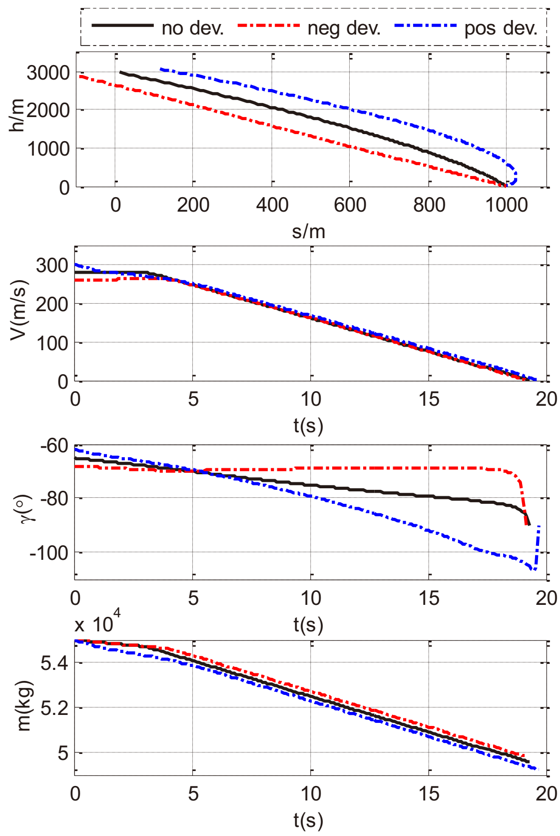

Fig. 4: The states of MPC solution

In the first condition, we assume that all deviations are zero, and the simulation results are represented by a black line. Its fuel consumption is $5419kg$, the thrust changed slowly, and the terminal flight path angle can achieve -90°. Then, we present the negative and positive deviation conditions in red and blue dot dash line, respectively. For the negative deviation, the rocket needs to consume 5198kg fuel with 19.2s. The angle of attack should be negative with a larger magnitude to correct the deviation during the landing process. For the positive deviation, the fuel consumption is 5777kg, and the flight time is 19.6s. The rocket need more fuel to eliminate a larger initial energy in less horizontal distance, therefore in the last few seconds of the landing process the rocket should fly over the landing point and land from the opposite direction. Correspondingly, the flight path angle will exceed -90° during the last few seconds and achieve -90° at the terminal time.

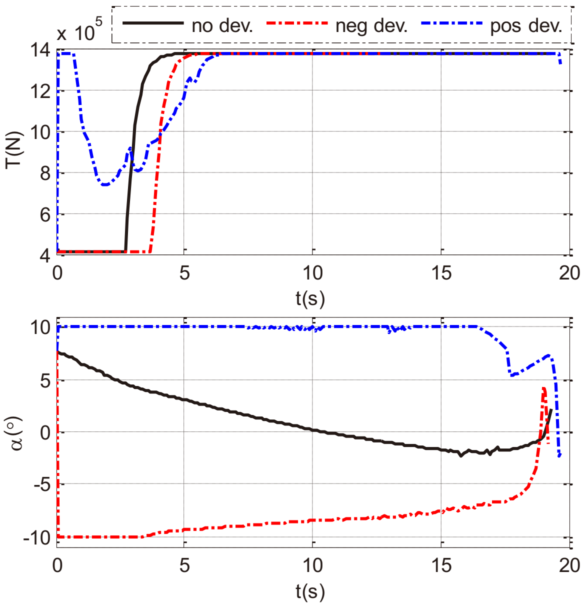

Fig. 5: The controls of MPC solution

5 Conclusion

This paper presents a terminal time free convex optimization method for fuel-optimal rocket landing problem, which is fuel-efficient than terminal time fixed method. This method introduces the terminal time as the independent variable to the optimization problem, and overcome the disadvantage that the convex optimization can only solve the fixed end point problem. According to the launch vehicle trajectory optimization characteristics that the discrete interval is larger than the control period, a novel convex MPC method with particular discrete point collocation and objective function is proposed to generate the guidance command onboard. This novel method uses less discrete points to optimize the guidance command, which can eliminate the solution error caused by discrete point sparsity while ensuring the rapid calculation. Besides under the conditions of deviation, the new guidance command can be generated with target of fuel optimal, rather than simply tracking the reference trajectory. Consequently applying this method to rocket vertical landing problem enables the rocket to achieve the optimal guidance command onboard with respect to its current states, and landing safely in the case of interference.

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