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On a periodic vehicle routing problem☆

S Coene*, A Arnout and FCR Spieksma

Katholieke Universiteit Leuven, Leuven, Belgium

This paper deals with a study on a variant of the Periodic Vehicle Routing Problem (PVRP). As in the traditional Vehicle Routing Problem, customer locations each with a certain daily demand are given, as well as a set of capacitated vehicles. In addition, the PVRP has a horizon, say T days, and there is a frequency for each customer stating how often within this T-day period this customer must be visited. A solution to the PVRP consists of T sets of routes that jointly satisfy the demand constraints and the frequency constraints. The objective is to minimize the sum of the costs of all routes over the planning horizon. We develop different algorithms solving the instances of the case studied. Using these algorithms we are able to realize considerable cost reductions compared to the current situation.

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Introduction

In this paper we study a routing problem of a Belgian company collecting waste at slaughterhouses, butchers, and supermarkets. Planning of the routes occurs over a time period of several days (time horizon) in which customers are visited with different frequencies. For instance, supermarkets might request service every day, while for a small butcher one collection a week suffices. The resulting problem is a variant of the Periodic Vehicle Routing Problem (PVRP). As in the traditional Vehicle Routing Problem (VRP), customer locations each with a certain demand function are given, as well as a set of capacitated vehicles. In addition, the PVRP has a horizon, say T days, and there is a frequency for each customer stating how often within this T-day period this customer must be visited. A solution to the PVRP consists of T sets of routes that jointly satisfy the demand constraints and the frequency constraints. The objective is to minimize the sum of the costs of all routes over the planning horizon. Obviously, this problem is at least as hard as the VRP.

PVRP and variants

Several variants of the PVRP are described in literature. A classification of the different variants of the PVRP can be found in a survey by Mourgaya and Vanderbeck (2006). Different objective functions are distinguished. There are the classical objectives such as minimizing the distance travelled, the driving time, or total transportation cost. However, many other aspects such as regionalization of routes, an even spread of workload over the vehicles, the number of vehicles, and service quality can be part of an optimization function. The different constraints can be divided in three categories: constraints concerning (i) the planning of the visits (different frequencies, restrictions on certain days, etc), (ii) the type of demand (constant or variable; we return to this issue later), and (iii) the vehicles. Where the PVRP is mostly situated on tactical and operational level, Francis *et al* (2006) include strategic decisions. In their model frequencies of service are considered as variables within the model, instead of given parameters. A variant defined as the PVRP with intermediate facilities is described by Angelelli and Speranza (2002), Kim *et al* (2006), and Alonso *et al* (2008). Intermediate facilities are locations where vehicles can unload (or reload) and thus renew capacity during a route; this also applies to our case as we will see in the problem description.

Applications

The PVRP is a relevant problem; it occurs in companies that have to carry out periodic repair and maintenance activities or that collect/deliver goods periodically. Blakeley *et al* (2003) describe a case for periodic maintenance of elevators at different customer locations. More case studies describing problems such as waste collection and road sweeping can be found in Beltrami and Bodin (1974) and Eglese and Murdock (1991). Claassen and Hendriks (2007) describe a milk collection problem; in this case it is important that the goods are collected when fresh. Other products such as raw materials for a manufacturer of auto parts, on the contrary, have a very long time horizon, see Alegre *et al* (2007). Hemmelmayr *et al* (2008) investigate the periodic delivery of blood products to hospitals by the Austrian Red Cross. In this case, the regularity of deliveries is of uttermost importance. Ronen and Goodhart

☆This work grew out of the Master Thesis of Arnout (Arnout, 2007).

**Correspondence: S Coene, Katholieke Universiteit Leuven, Operations Research Group, Naamsestraat 69, B-3000 Leuven, Belgium.*

(2008) describe a problem for the tactical planning of store deliveries from several distribution centres. Many other case studies are described in Francis *et al* (2008) and the references contained therein.

Solution methods

The PVRP is situated on the border between tactical and operational planning, combining the classical VRP with planning over a time horizon. That is why solution methods often consist of two phases. Beltrami and Bodin (1974) consider two approaches. In a first approach, routes are developed and then assigned to days of the week; in a second approach customers are assigned to days in a first phase and in a second phase the routing problem for every single day is solved using classical techniques for solving VRPs. This second approach is used in many papers, such as Baptiste *et al* (2002), Tan and Beasley (1984), and Christofides and Beasley (1984). Tan and Beasley (1984) first solve an assignment problem to assign customers to days such that total demand in each day does not exceed demand capacity while taking pairwise distances between customers into account. In a second stage they solve a VRP for each day in the planning horizon. Thus, they approach this problem as an extension of the assignment problem with a routing component. Christofides and Beasley (1984) on the other hand formulate the PVRP as a routing problem with a selection decision. Customers are ordered in descending order of 'importance', depending on the demands. Then, customers are selected for a route on a certain day depending on the increase in total cost for the whole period. The approaches described above are the more classical solution strategies. Recent PVRP literature has focused on metaheuristic methods and mathematical-based approaches to solve the problem; we refer to Francis *et al* (2008) for an overview.

The rest of this paper is structured as follows. We first introduce the problem under consideration in further detail, then we describe our model, and we propose a solution method; in the final section we give some computational results and formulate a conclusion.

The problem

A general description

As mentioned, we study a problem that is encountered by a Belgium transportation company. This company, which we call company A for confidentiality reasons, is responsible for the collection of waste at slaughterhouses, butchers, and supermarkets. This company has clients all over Belgium and in some areas of northern France.

Legislation that originated from the BSE-epidemic (Bovine Spongiform Encephalopathy, commonly known as mad cow disease) in the 1990s, stipulates that (i) there are three categories of animal waste, depending on the risk of containing BSE; and (ii) waste from different categories has to be collected separately. Company A only collects waste from two categories: category 1 (high-risk waste) and category 3 (low-risk waste). All high-risk waste is collected in order to be destroyed, while low-risk waste can be further processed into for example pet food. Vehicles assigned to collect high-risk waste cannot be used to collect low-risk waste and vice versa. In fact, this implies that company A needs to solve two different instances; one instance for the periodic collection of high-risk waste and a second instance for the periodic collection of low-risk waste.

In the current planning process of company A, routes are constructed manually on a regular basis, for example every month. During that period minor modifications to the routes can be made depending on changes in the set of customers. Company A wishes to decrease the dependence upon human expertise and wants to professionalize the planning procedure. Also, the management of company A wishes to plan the routes more efficiently in order to reduce travel time and travel distance. Several opportunities need to be explored: (i) can total driving time be decreased? (ii) can the routing be done using smaller vehicles? and (iii) is it possible to decrease the vehicle fleet? All of this could be possible through more efficient planning, but obviously, the same level of service towards the customers should be retained. It is clear that it would not be possible to change the vehicle fleet whenever the routing plan changes. Our routing plan, however, should allow the management to 'assemble' a good vehicle fleet in the long term. In the short term, the current vehicle fleet must be respected when constructing a routing plan.

The low-risk waste instance: details

Here we describe some properties of the instance corresponding to the low-risk waste. There are 48 customers, spread out over Belgium and northern France (see Figure 1). The planning period is 1 week (actually 6 days), and each customer requests a certain frequency of visits over the planning period. Table 1 gives an overview of how often each frequency occurs. How these frequencies can be obtained is explained in the following section. There are five different frequencies and a frequency of 4 days does not occur.

Company A has three trucks available for collecting low-risk waste; their capacities are 12, 22, and 26 tons, respectively. Some clients are located in the centre of a city and cannot be reached by a truck of 22 tons or bigger. There is a central depot where each route starts in the morning and ends in the evening. When a truck is fully loaded, the driver can unload at a disposal facility (represented by a square in Figure 1) and then continue its route. Notice that these disposal facilities can be seen as intermediate facilities, see Angelelli and Speranza (2002). The trucks do not need to return empty in the evening, they can also dispose of their load during the tour of the following day. Only when a truck does not drive on the following day it needs to be emptied before returning to the depot. An affiliated company processes the waste and has one disposal facility where the

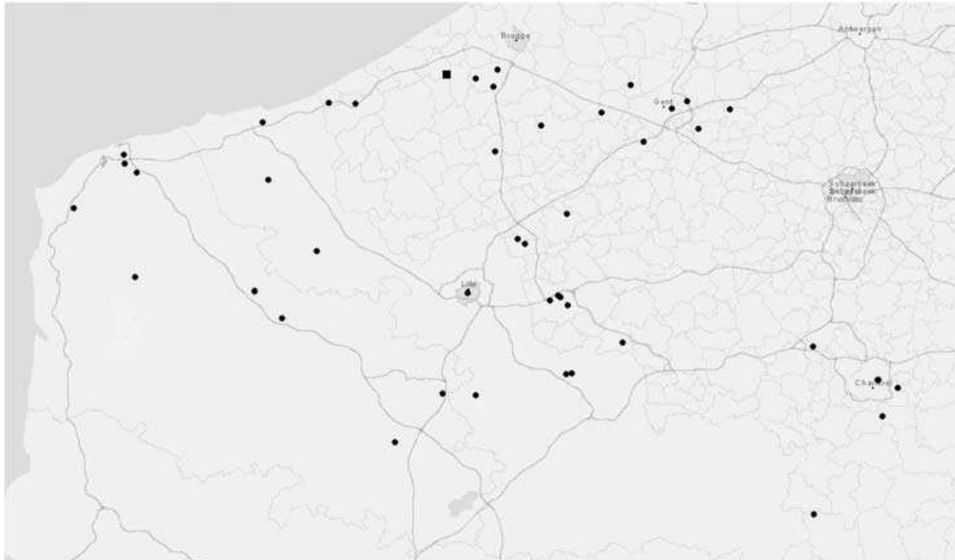

Figure 1 Locations of low-risk waste customers.

Table 1 Frequencies low-risk waste

Frequency (visits per period)	1	2	3	5	6	
nr of customers	21	15	5	5	2	48

trucks can unload 24 h a day. Loading and unloading times depend on the volume. Legally, the maximum driving time for a driver is restricted to 90 h within 2 weeks. On top, the company restricts the daily driving hours to 10 h. Notice that these are only driving hours, they do not include loading and unloading times.

The high-risk waste instance: details

The instance corresponding to the collection of high-risk waste contains 262 customers, distributed all within Belgium (see Figure 2). The planning period is 2 weeks (10 days), and again customers require certain frequencies of visits within that period. In Table 2 we give an overview of the different frequencies for the high-risk waste instance.

The capacities of the three trucks available are 9, and twice 12 tons, respectively. The collected waste must be delivered at two disposal facilities of an external company (represented by squares in Figure 2). Note that time windows apply to these facilities. Loading and unloading times are constant, 10 min for loading and 30 min for unloading. Restrictions on driving hours and depot locations are the same as for low-risk waste.

Visit-frequencies

An important aspect in this problem are the 'visit-frequencies'. They can be determined before solving the problem, which is the case for the problem studied here; or they can be part of the routing problem as in for example Francis *et al* (2006). In our case these frequencies are determined as follows. Based on historical data, it can be accurately predicted how much waste a customer produces per day, and how much waste can be stored at a customer's site. On the basis of these data and in consultation with the customer, a *visit-frequency* is computed for each customer: the number of times a collection is done during the planning period. We also compute a demand (q_i) depending upon this frequency. As an example, consider some customer i that generates 10 tons of waste per day, and has a storage capacity of 30 tons, while the planning period is $T = 5$. We compute the corresponding visit-frequency f_i for this client i as follows: $f_i = \lceil \frac{10T}{30} \rceil = 2$. More in particular, the visits must be spread well-balanced over the planning period. Hence, this customer can be visited on days 1 and 3, but also on days 1 and 4, 2 and 4, 2 and 5, and 3 and 5; but not on any two consecutive days. The demand of this customer equals $q_i = \frac{10T}{f_i} = 25$. Notice that the actual amount of waste collected at customer i may differ from the predicted amount q_i.

For every frequency, we define all possible combinations of days in the planning period, and we call them scenarios. Then, for every customer we need to select exactly one scenario from the set of scenarios that correspond to the frequency associated with the customer. How exactly the scenarios are assigned to customers will be an important part of our solution method.

Model: notation and description

We develop a model capturing all aspects of the problem; the mathematical formulation can be found in the appendix. We define a network $G = (V, A)$. The customers to be visited are represented by nodes 1 to N, the depot is represented by node

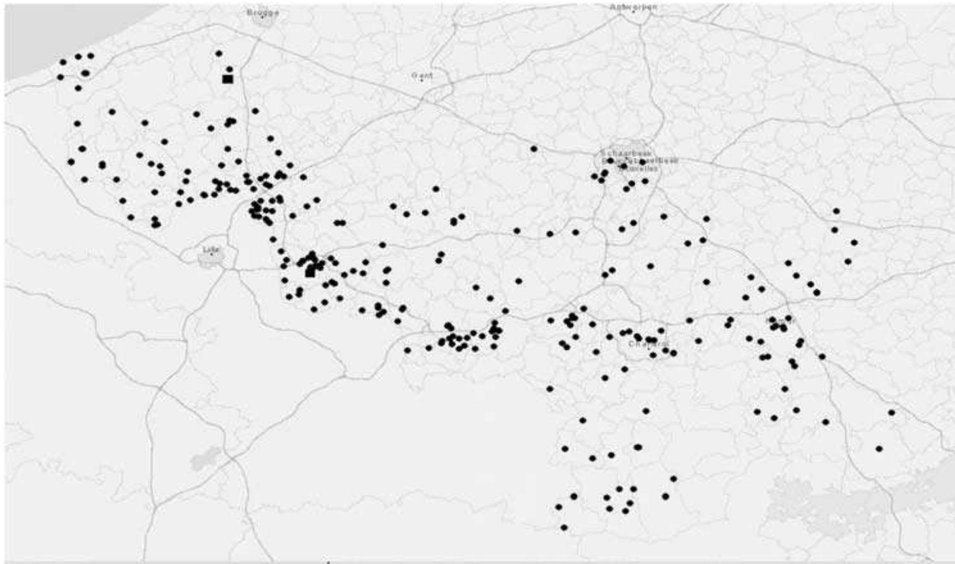

Figure 2 Locations of high-risk waste customers.

Table 2 Frequencies high-risk waste

Frequency (visits per period)	1	2	4	
nr of customers	62	186	14	262

0 and the disposal facilities by nodes $N+1$ to $N+M$. Thus $V=\{0, 1, 2, \ldots, N, N+1, \ldots, N+M\}$ is the vertex set and A is the arc set with for each arc (i, j) a travel distance d_{ij} and a travel time $c_{ij}(i, j \in V)$. The planning period has a length of T days and every customer is visited within this period according to a certain scenario $c \in C$. Essentially, a scenario is a set of days within $\mathscr{T}=\{1, \ldots, T\}$; choosing a scenario for a customer means that the customer is visited during these days. We let f^c equal the number of visiting days in scenario c. There are K vehicles, each with a certain capacity Q_k. For each customer $i \in \mathscr{N}=\{1, \ldots, N\}$, a frequency f_i, a quantity q_i, and a loading time l_i are given. Quantity q_i is the quantity to be collected at each visit; this number is based on the average amount of waste that is collected at each visit (see our discussion earlier). We can use this average amount to approximate reality because visits are spread evenly over the planning period. In a mathematical model different constraints are developed to capture the specific demands of the problem studied. Here we will just shortly describe these constraints. The complete model can be found in the appendix.

The goal of our problem is to minimize total travel distance taking into account that:

— each customer should be assigned a scenario that corresponds to its requested frequency,
— each customer then needs to be visited on the days of the scenario assigned to it,
— the constructed tours cannot contain subtours,
— each vehicle can be assigned only a single route per day,
— the number of driving hours per day and per week are restricted,
— the vehicles have limited capacity,
— the vehicles can dispose of their load during the route at the disposal facilities,
— vehicles do not always need to be empty at the end of the day,
— there are time windows at certain disposal facilities.

Solving this model (using ILOG's Cplex) in order to obtain an optimal solution for a small data set of less than 10 customers already takes a very long time. That is why in the following section we develop a heuristic solution approach.

Solution approach

We study methods that consist of two phases: in one phase customers are assigned to days, and in another phase a VRP-instance is solved. In a first subsection we investigate methods that first assign customers to days, and next solve a VRP-instance. We consider two ways of assigning customers to days, namely striving for an 'even spread' of the number of visits on a day, or using a geographically based clustering approach. Finally, we describe how we use ILOG Dispatcher to solve a VRP-instance. In a second subsection we describe a method that first solves a VRP-instance, and then assigns customers to days.

First assign customers to days, then route

Assigning customers to days: algorithm ES Here, we focus on the spread of visits over the planning horizon. The following problem (problem ES) is solved. We are given a set V of customers, with for each customer i an associated frequency f_i. Further there is a set C of scenarios, where a scenario consists of a set of f^c days, meaning that in that scenario C, visits are performed on f^c different days. These scenarios need to be assigned to customers such that the frequency of each customer equals the frequency of the scenario assigned to the customer. The goal is to minimize the maximum number of customers visited on each day of the planning horizon. We refer to this problem as problem ES and we claim the following:

Fact 1 *Problem ES is NP-hard.*

Proof We prove NP-hardness of problem ES by a reduction from Exact Cover by 3-sets (X3C). In X3C a finite set X containing $3n$ elements and a collection C of 3-element subsets of X are given. The question is whether there exists a subset C' of C such that every element of X occurs in exactly one triple of C'. This problem is proven to be NP-complete by Garey and Johnson (1979). For every instance of X3C we can create an instance of problem ES as follows. There are n customers, each with frequency f_i equal to 3. The planning period lasts $3n$ days and each triple from C corresponds to a scenario consisting of 3 days corresponding to the elements of the triple. Now, if an assignment of scenarios to customers can be found such that exactly one customer is visited on each day (even spread), then also a solution for X3C exists, and *vice versa*. □

Thus, this proves that in general problem ES is a hard problem to solve. However, for our purposes, due to the relatively small size of our instances (we have $T = 5$ (10), $|C| = 15$ (20), $N = 48$ (262)) problem ES can be solved in reasonable time using the following integer programme (IP). Define parameter a_{ct}, which is equal to 1 if day t is visited in scenario c, and 0 otherwise. Variable y_{ic} is equal to 1 if customer i is assigned scenario c and 0 otherwise. z_t is an integer variable representing the number of customers visited on day t. Notice that this variable is not strictly needed; we add it to make the model more clear.

$$\text{Minimize} \quad w \tag{1}$$

subject to

$$w \geqslant z_t \qquad \forall t \in \mathscr{T}, \tag{2}$$

$$\sum_{c \in C: f^c = f_i} y_{ic} = 1 \qquad \forall i \in \mathscr{N}, \tag{3}$$

$$\sum_{i \in V} \sum_{c \in C: f^c = f_i} a_{ct} y_{ic} = z_t \qquad \forall t \in \mathscr{T}, \tag{4}$$

$$y_{ic} \in \{0, 1\} \qquad \forall i \in \mathscr{N};\ \forall c \in C, \tag{5}$$

$$z_t, w \in \mathbb{Z} \qquad \forall t \in \mathscr{T}. \tag{6}$$

The goal of this IP is to assign a scenario to every customer such that visits are spread evenly over the planning period. This is enforced by minimizing the maximal number of customers visited on a day during the planning period (2). In that way more or less the same amount of customers will be visited on each day. Constraint (3) makes sure that every customer is visited according to exactly one scenario that matches its frequency. We solve this IP optimally using ILOG Cplex 10.2.

Notice that the location of the customers is completely ignored in this approach. Clearly, this implies that there is a risk that customers positioned close to each other are scheduled on different days. In the next section, we assign scenario's to customers such that this potential disadvantage is dealt with.

Assigning customers to days: algorithm CL In a second algorithm, algorithm CL (CLuster), the customers are partitioned using the algorithm '*k*-medoids clustering' before scenarios are assigned to customers. This clustering algorithm partitions the locations into k clusters and attempts to minimize the squared error, that is the squared distances between points labelled to be in a cluster and a point designated as the centre of that cluster. k-medoids chooses data points as centres. We use this particular clustering method because it is suited for cases where the distance matrix is used (instead of the locations), as is the case here. We use the k-medoid method as it is defined in the C Clustering Library by de Hoon *et al* (2008). The algorithm starts with an arbitrary selection of k data points that will act as centres (medoids) and then tries to improve by swapping the medoids with other points.

We apply this method for several values of k, and then we solve (7)–(10) (which is a modification of (1)–(6)), which takes clusters into account when assigning customers to days. Define parameter o_{il}, which is equal to 1 if customer $i \in \mathscr{N}$ is in cluster $l \in 1, \ldots, k$, with k the number of clusters, and 0 otherwise. Variable z'_{tl} is equal to 1 if and only if a customer of cluster l is visited on day t. Now, our goal is to minimize the total sum of the number of clusters visited each day. A cluster is visited if a customer of that cluster is visited. Customers that belong to the same cluster will thus be as much as possible assigned to the same day.

$$\text{Minimize} \quad z'_{tl} \tag{7}$$

subject to

$$\sum_{c \in C: f^c = f_i} y_{ic} = 1 \qquad \forall i \in \mathscr{N}, \tag{8}$$

$$\sum_{c \in C: f^c = f_i} \sum_{i \in V} a_{ct} y_{ic} o_{il} \leqslant z'_{tl} \qquad \forall t \in \mathscr{T}; \quad \forall l \in \{1, \ldots, k\}, \tag{9}$$

$$y_{ic}, z'_{tl} \in \{0, 1\} \qquad \forall i \in \mathscr{N}; \quad \forall c \in C; \quad \forall t \in \mathscr{T}; \quad \forall l \in \{1, \ldots, k\}. \tag{10}$$

Notice that a potential risk associated to the outcome of model (7)–(10) is that the driving time needed to visit all the customers assigned to a certain day can exceed the maximal driving time allowed per day. To prevent this, a constraint on the number of customers visited on each day is added; we come back to this issue when discussing the computational results.

Routing using ILOG Dispatcher Having assigned all customers to scenarios, either using ES or CL, we know which customers need to be visited every day. Thus, for each day in the planning horizon we now need to solve a, more or less standard, VRP. Some specific constraints do need to be taken into account, such as vehicles that have different capacities, vehicles that can dispose of their load during the day and then continue the route (they can collect more than their capacity on one day), some customers can only be reached by small vehicles, vehicles do not need to unload at the end of the day, and the number of driving hours per vehicle per day is limited. A solution algorithm for this VRP is implemented using ILOG Dispatcher 4.4, which is a C++ library based on ILOG Solver and that offers features especially adapted to solving problems in vehicle routing. We implement all the standard VRP constraints, as well as the problem-specific constraints mentioned above.

In the resulting routing algorithm, an initial solution to the VRP is constructed using a standard savings heuristic, which makes a trade off between more vehicles with shorter routes and fewer vehicles with longer routes. Then, ILOG's dispatcher performs neighbourhood search to improve this solution. Both intra-route and inter-route neighbourhoods are considered. As intra-route neighbourhoods, 2-Opt and Or-Opt are used. ILOG's dispatcher also interchanges between routes: the 'relocate' neighbourhood (inserting a customer in another route); the 'exchange' neighbourhood (swapping two customers from different routes); and the 'cross' neighbourhood (exchanging the ends of two routes). We refer to ILOG Dispatcher user's manual (ILOG, 2005) for a more elaborate description. We have restricted ourselves here to these methods to improve the routes.

First route, then assign customers to days: algorithm MR

In this approach, called algorithm MR (Mega Route), we first construct large routes visiting all the customers and the disposal facilities. To accomplish this, we use the same VRP heuristic as described in the previous section. Then, on the basis of these routes, customers are assigned to days using model (7)–(10). Customers belonging to the same route are then visited as much as possible on the same day. Then, we resolve a VRP for each day in the planning horizon in order to obtain better routes. Algorithm MR is similar to algorithm CL, where we first cluster the customers, as geographically close customers tend to end up being visited on the same day. Routing can yield a very different grouping of the customers though.

Computational results

The different algorithms have been implemented in Microsoft Visual C++2005, in combination with ILOG CPLEX 10.2 and ILOG Dispatcher 4.4. All algorithms are run on a personal computer with a 2.80 GHz Intel Pentium IV processor and 504 MB of RAM. Notice that we have not been concerned with running times of our approach. This is mostly because the application did not enforce strong limitations on the amount of computing time used. In the following subsections computational results for the low-risk waste instance and for the high-risk waste instance, respectively, are given. In a final subsection, we discuss the performance of the different methods for the two instances.

The low-risk waste instance: results

In Table 3 some computational results are shown. Applying algorithm ES, we find a set of routes requiring three vehicles. On several days only two of them are being used. Notice that visits are well spread over the week, as each day a comparable distance is travelled. Between vehicles though, there is a rather uneven workload. Total travelling time is 5859 min in 6 days.

As can be seen in Table 3, a solution by algorithm CL is obtained using all three vehicles. Comparing this solution with the previous one we can see that the variation in route length over the days is higher and that total travelling time decreased to only 5551 min.

The last columns in Table 3 show the results of algorithm MR. Here also three vehicles are used and total travelling time is rather high (5934 min). Thus, routing the customers before assigning them to the different days in the planning horizon does not yield a good solution for this instance.

Table 3 Low-risk waste routes (travelling time in minutes)

Algorithm	*ES*				*CL*				*MR*			
Vehicle Day	*12T*	*22T*	*26T*	*TOT*	*12T*	*22T*	*26T*	*TOT*	*12T*	*22T*	*26T*	*TOT*
1	452	391	240	1083	495	413	476	1384	339	235	567	1141
2	509		551	1060	409		279	688	339	252	220	811
3	511	378	240	1129	416	439	237	1092	439	288	588	1315
4	427	280	482	1189	512	374	214	1100	354	364	276	994
5	561		482	1140	460	163	406	1029	584	510	321	1415
6			258	258			258	258			258	258
				5859				5551				5934

Table 4 High-risk waste routes (travelling time in minutes)

Algorithm	*ES*			*CL*			
Vehicle Day	*9T*	*12T*	*TOT*	*9T*	*9T*	*12T*	*TOT*
1	558	350	908	311		385	696
2	662	495	1157	387		495	882
3	596	342	938				
4	587	570	1157	579	583	398	1560
5	450	549	999	551			551
6	308	599	907	445		423	868
7	641	434	1075	494	619	115	1228
8	599	334	933				
9	592	607	1199	585	577	356	1518
10	450	549	999	311		116	427
			10 272				7730

Table 5 High-risk waste routes (travelling time in minutes)

Algorithm	*CL-limited clients per day*				*MR-limited clients per day*		
Vehicle Day	*9T*	*9T*	*12T*	*TOT*	*9T*	*12T*	*TOT*
1	540		279	819	527		527
2	510		655	1165	452	673	1125
3	506		225	731	504		504
4					538	675	1213
5	544		675	1219			
6	541		289	830	577		577
7	551	203	519	1273	455	673	1128
8	542		228	770	546		546
9					514	686	1200
10	591		615	1206	257	578	835
				8013			7655

Concluding, this suggests that using the geographic structure yields a better routing plan. Running times vary from about 1 h for ES, to a few hours for CL and up to 48 h for MR. We experience a large variation in time needed by the VRP solver. Finally, notice that the current routes used in practice have a total travelling time of 6565 min; using algorithm CL we could improve them by 15.5%.

The high-risk waste instance: results

Let us now apply the same algorithms to the instance for the collection of high-risk waste. The results are summarized in Tables 4 and 5. Using algorithm ES, every day the same amount of customers is visited, unloading only occurs once in 2 days and on Fridays, and only two vehicles are required. Total travel time is then 10 272 min for 2 weeks.

Two large clusters are obtained when applying algorithm CL. Based on these two clusters, all the customers are assigned to the days of the planning horizon. This results in a very uneven spread of the customers; on some days up to 100 customers are visited. As a result three vehicles are required on these days, while on other days no customers are visited. Total travelling time is then 7730 min, which is clearly much better than ES; however, this leads to the usage of three vehicles. In order to see whether the instance could be solved using two vehicles only, we add a constraint to (7)–(10) bounding the number of customers assigned to 1 day. Solving this model, we obtain routes with a total travel time of 8013 min and using only two vehicles on most days.

Finally, we also apply algorithm MR to this instance. Two giant routes are created, visiting all the customers and the disposal facilities. Again, when dividing the customers over the days, we obtain a highly uneven spread of customers. Visits are only performed on 4 out of 10 days, with up to 171 customers on 1 day. In fact, as mentioned as a potential risk in a previous section, this is not feasible given the available driving time; relaxing these constraints yields a total time travelled of 6591 min. This is not a feasible solution though, thus we need to limit the number of customers visited per day. We set this limit equal to 75, which is more or less the amount of customers that can be served on 1 day by two vehicles. This yields a total travelling time of 7655 min and utilizes two vehicles. More details are shown in Table 5.

Concluding, for this instance it is important to take into account geography. Running times are limited to seconds for all three algorithms. Compared to the original routes of company A which take 8434 min, algorithm ES performs badly; MR improves the currently used routes by 9% and uses one vehicle less.

Discussion

Depending on the instance, the methods used above perform differently. Clustering the customers yields a better result for both instances compared to algorithm ES, but the effect is larger for the high-risk waste instance. Algorithm MR performs comparably as CL for this instance but a lot worse than ES and CL for the low-risk waste instance. Explanation of this phenomenon is that there are customers in the low-risk instance with a very high frequency (see Table 1) and these clients are rather spread over the country (see Figure 1). Ignoring the geographic locations has then only a small impact on the solutions, as on each day the vehicles have to travel in different directions anyway. There is a large difference in the visit frequencies of the high-risk waste instance compared to the low-risk waste instance (see Tables 1 and 2). In the first instance, many customers require the same visit frequency, and no customer has a very high visit frequency (maximally four visits in 10 days). Together with the fact that the instance is larger, this explains why clustering has a vast effect on the solutions. As mentioned before, we obtain routes that might yield a rather uneven workload for the different vehicles. Company A does not consider this as a problem. We propose routes using three vehicles for the small instance. Not every day, though, all vehicles are needed, thus it might be interesting to consider renting a vehicle and driver at an external company for the days necessary. Changing the volume of the vehicles in order to diminish the number of vehicles does not seem really useful. For the low-risk waste instance a vehicle with capacity of 12 tons must be available for customers not reachable with a larger vehicle. For the high-risk waste instance, the volumes to be picked up are rather small and unloading is only necessary after 2 days. It is not the volume but the time restrictions that have the highest impact on travel times. That is also the reason why running times differ a lot between the low-risk waste instance and high-risk waste instance. In the low-risk waste instance the volumes to be picked up are higher such that vehicles unload several times during the day. This complicates the VRP model and increases running times.

Conclusion

We consider a problem occurring in practice, and we model it as a PVRP. Using different approaches (including ILOG's dispatcher), we were able to improve the current routes of Company A, using one vehicle less for one of the instances. This not only means a reduction in cost due to the gain in travel time, but also a reduction in wage costs and material costs. We use rather simple algorithms to assign the customers to the days and to solve the VRPs. The use of clustering customers depends highly on the dispersion of the customers and on their frequencies. We deal with this instance of the PVRP by considering the problems of assigning customers to days and routing the customers independently. For those interested to further study the problem, the instances can be found on the following website: http://www.econ.kuleuven.ac.be/public/N05012/.

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References

Alegre J, Laguna M and Pacheco J (2007). Optimizing the periodic pick-up of raw materials for a manufacturer of auto parts. *Eur J Opl Res* **179**: 736–746.

Alonso F, Alvarez MJ and Beasley JE (2008). A tabu search algorithm for the periodic vehicle routing problem with multiple vehicle trips and accessibility restrictions. *J Opl Res Soc* **59**: 963–976.

Angelelli E and Speranza MG (2002). The periodic vehicle routing problem with intermediate facilities. *Eur J Opl Res* **137**: 233–247.

Arnout A (2007). *Het periodieke routeplanningsprobleem: een case studie*. Master Thesis, Katholieke Universiteit Leuven, Belgium.

Baptiste S, Oliviera RC and Zúquete E (2002). A period vehicle routing case study. *Eur J Opl Res* **139**: 220–229.

Beltrami EJ and Bodin LD (1974). Networks and vehicle routing for municipal waste collection. *Networks* **4**: 65–94.

Blakeley F, Argüello B, Cao B, Hall W and Knolmajer J (2003). Optimizing periodic maintenance operations for Schindler Elevator Corporation. *Interfaces* **33**(1): 67–79.

Christofides N and Beasley JE (1984). The period routing problem. *Networks* **14**: 237–256.

Claassen GDH and Hendriks THB (2007). An application of special ordered sets to a periodic milk collection problem. *Eur J Opl Res* **180**: 754–769.

de Hoon M, Imoto S and Miyano S (2008). *The C Clustering Library*. The University of Tokyo, Institute of Medical Science, Human Genome Center: Tokyo.

Eglese RW and Murdock H (1991). Routing road sweepers in a rural area. *J Opl Res Soc* **42**: 169–194.

Francis P, Smilowitz K and Tzur M (2006). The period vehicle routing problem with service choice. *Transport Sci* **40**: 439–454.

Francis P, Smilowitz K and Tzur M (2008). The period vehicle routing problem and its extensions. In: Golden B, Raghavan S and Wasil E (eds). *The Vehicle Routing Problem*. Springer: US, pp 239–261.

Garey MR and Johnson DS (1979). *Computers and Intractability: A Guide to the Theory of NP-Completeness*. Freeman: San Francisco.

Hemmelmayr V, Doerner KF, Hartl RF and Savelsbergh MWP (2008). Delivery strategies for blood products supplies. *OR Spectrum* **31**: 707–725.

ILOG (2005). ILOG Dispatcher 4.1, User's manual.

Kim B, Kim S and Sahoo S (2006). Waste collection vehicle routing problem with time windows. *Comput Opns Res* **33**: 3624–3642.

Mourgaya M and Vanderbeck F (2006). Problème de tournées de véhicules multipériodiques: Classification et heuristique pour la planification tactique. *RAIRO Opns Res* **40**: 169–194.

Ronen D and Goodhart CA (2008). Tactical store delivery planning. *J Opl Res Soc* **59**: 1047–1054.

Tan CCR and Beasley JE (1984). A heuristic algorithm for the period vehicle routing problem. *Omega* **12**: 497–504.

Appendix

The mathematical model

We define the network $G=(V, A)$. The customers to be visited are represented by nodes 1 to N, the depot is represented by node 0 and the disposal facilities by nodes $N+1$ to $N+M$. Thus vertex set $V=\{0, 1, 2, \ldots, N, N+1, \ldots, N+M\}$ and A is the arc set with for each arc (i, j) a travel distance d_{ij} and a travel time $c_{ij}(i, j \in V)$. The planning period has a length of T days and a customer is visited within this period according to a certain scenario $c \in C$. Essentially, a scenario is a set of days within $\mathscr{T}=\{1, \ldots, T\}$; choosing a scenario for a customer means that the customer is visited during these days. We let f^c equal the number of visiting days in scenario c. We are given a number of vehicles K, each with a certain capacity Q_k. For each customer $i \in \mathscr{N}=\{1, \ldots, N\}$, a frequency f_i, a quantity q_i, and a loading time l_i are given. Quantity q_i is the quantity to be collected at each visit. Further, a driver may not drive longer than D_d h a day, and no longer than D_T h in the total planning period of T days. Disposal facilities can only be visited within their time windows $[r_i, s_i], i \in M=\{N+1, \ldots, N+M\}$. Finally, a_{ct} is equal to 1 if day $t \in \mathscr{T}$ is visited within scenario $c \in C$ and 0 otherwise. We then define the following 5 sets of variables: x_{ijkt} is a binary variable, which is equal to 1 if customer $i \in V$ is visited after customer $j \in V$ by vehicle $k \in \mathscr{K}$ at day $t \in \mathscr{T}$, 0 otherwise; y_{ic} is a binary variable, which is 1 if client $i \in V$ is visited according to scenario $c \in C$ and 0 otherwise; let L_{ikt} and T_{ikt} be the total load and the time, respectively, of vehicle $k \in \mathscr{K}$ at day $t \in \mathscr{T}$ after having visited customer $i \in V$; and finally, the load of a vehicle $k \in \mathscr{K}=\{1, 2, \ldots, K\}$ at the beginning of day $t \in \mathscr{T}$ is denoted by S_{1kt} and the load at the end of the day by S_{2kt}.

The model

$$\text{(PVRP) Minimize} \quad \sum_{i\in V}\sum_{j\in V}\sum_{k\in\mathscr{K}}\sum_{t\in\mathscr{T}} d_{ij}x_{ijkt} \tag{11}$$

subject to

$$\sum_{c\in C: f^c=f_i} y_{ic}=1 \qquad \forall i \in \mathscr{N}, \tag{12}$$

$$\sum_{j\in V}\sum_{k\in\mathscr{K}} x_{ijkt} - \sum_{c\in C} a_{ct}y_{ic}=0 \qquad \forall i \in \mathscr{N};\ \forall t \in \mathscr{T}, \tag{13}$$

$$\sum_{i\in V} x_{ihkt} - \sum_{j\in V} x_{hjkt}=0 \qquad \forall h \in V;\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{14}$$

$$\sum_{j\in\mathscr{N}} x_{0jkt} \leqslant 1 \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{15}$$

$$\sum_{i\in V}\sum_{j\in V}\sum_{t\in\mathscr{T}} d_{ij}x_{ijkt} \leqslant D_T \qquad \forall k \in \mathscr{K}, \tag{16}$$

$$\sum_{i\in V}\sum_{j\in V} d_{ij}x_{ijkt} \leqslant D_d \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{17}$$

$$\sum_{i\in S}\sum_{j\in S} x_{ijkt} \leqslant |S|-1 \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}; S \subseteq \mathscr{N}; |S| \geqslant 2, \tag{18}$$

$$(L_{ikt}+q_j-L_{jkt}) \leqslant (1-x_{ijkt})Q_k \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T};\ \forall i, j \in \mathscr{N}, \tag{19}$$

$$L_{ikt} \leqslant Q_k \qquad \forall i \in \mathscr{N};\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{20}$$

$$L_{ikt}=0 \qquad \forall i \in \mathscr{M};\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{21}$$

$$S_{2kt}=S_{1k(t+1)} \qquad \forall k \in \mathscr{K};\ \forall t \in \{1, \ldots, T-1\}, \tag{22}$$

$$L_{ikt}-S_{2kt} \leqslant (1-x_{i0kt})Q_k \qquad \forall i \in V;\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{23}$$

$$S_{1kt}-L_{jkt} \leqslant (1-x_{0jkt})Q_k \qquad \forall j \in V;\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{24}$$

$$S_{2kt}=0 \qquad \forall t \in \{fridays\};\ \forall k \in \mathscr{K}, \tag{25}$$

$$S_{2kt} \leqslant Q_k z_{kt} \qquad \forall k \in \mathscr{K};\ \forall t \in \{1, \ldots, T-1\}, \tag{26}$$

$$1-\sum_{j\in V} x_{0jk(t+1)} \leqslant Q_k(1-z_{kt}) \qquad \forall k \in \mathscr{K};\ \forall t \in \{1, \ldots, T-1\}, \tag{27}$$

$$T_{ikt}+l_i+c_{ij}-T_{jkt} \leqslant (1-x_{ijkt})M \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T};\ \forall i, j \in V, \tag{28}$$

$$r_i x_{ijt} \leqslant T_{ikt} \leqslant s_i x_{ijkt} \qquad \forall k \in \mathscr{K};\ \forall t \in \mathscr{T};\ \forall i, j \in \mathscr{N}, \tag{29}$$

$$x_{ijkt}, z_{kt} \in \{0, 1\} \qquad \forall i, j \in V;\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}, \tag{30}$$

$$y_{ic} \in \{0, 1\} \qquad \forall i \in \mathscr{N};\ \forall c \in C, \tag{31}$$

$$L_{ikt}, T_{ikt}, S_{1kt}, S_{2kt} \geqslant 0 \qquad \forall i \in V;\ \forall k \in \mathscr{K};\ \forall t \in \mathscr{T}. \tag{32}$$

This model finds a scenario for every customer and a set of routes for each day of the planning period such that total travel distance is minimized. The first constraints (12) make sure that exactly one scenario is selected for every customer and in such a way that within this scenario the customer is visited according to its frequency. A customer then will be visited on the days of the selected scenario; this is ensured by constraints (13). Constraints (14) make sure that when a vehicle arrives at a customer, it also leaves from that customer. Constraints (15) impose that each vehicle can be used at most once every day. Constraints (16) and (17) keep the number of driving hours for every vehicle within the restrictions for the whole planning period, and within the daily restrictions, respectively. Constraints (18) are subtour elimination constraints. Correct counting of vehicle loads is ensured by constraints (19), and constraints (20) keep the amount of vehicle load within the capacity. Constraints (21) impose that vehicles are empty when they have visited a disposal facility. A vehicle does not need to dispose of all of its load at the end of the day. It follows that the load of a vehicle at the end of a day needs to be equal to the load of that vehicle at the start of the following day, this is insured by constraints (22) to (24). At certain moments in the week though vehicles do need to unload at the end of the day, for example at the end of the week (25) and when a vehicle is not used on the next day, see constraints (26) and (27). Equations (28) and (29) make sure that time windows for the disposal facilities are not violated. Finally, constraints (30) through (32) impose binary conditions and non-negativity conditions on the variable set.

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