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# The real-time time-dependent vehicle routing problem

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**Abstract**

In this article, the real-time time-dependent vehicle routing problem with time windows is formulated as a series of mixed integer programming models that account for real-time and time-dependent travel times, as well as for real-time demands in a unified framework. In addition to vehicles routes, departure times are treated as decision variables, with delayed departure permitted at each node serviced. A heuristic comprising route construction and route improvement is proposed within which critical nodes are defined to delineate the scope of the remaining problem along the time rolling horizon and an efficient technique for choosing optimal departure times is developed. Fifty-six numerical problems and a real application are provided for demonstration.


*Keywords:* Time-dependent travel time; Real time; Time window constraints; Vehicle routing problem

## 1. Introduction

The Vehicle Routing Problem (VRP) has been and remains a rich topic for researchers and practitioners. Given a set of geographically dispersed customers, each showing a positive demand

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for a given commodity, the VRP consists of finding a set of tours of minimum length for a fleet of vehicles initially located at a central depot, such that the customers' demands are satisfied and the vehicles' capacities are not exceeded. Taking into consideration the time windows of the customers and the depot(s) extends the problem into the vehicle routing problem with time windows (VRPTW). There are many possible extensions of the VRPTW, such as pickup and delivery, VRPTW with backhauls, inventory routing, period routing, and so on. Most research focuses on static or deterministic vehicle routing in which all information is known at the time of planning.

In most real-life applications, real-time information is generated only when the vehicles are en route. Real-life examples of real-time vehicle routing problems include the distribution of oil to private households, the pick-up of courier mail/packages and the dispatching of buses for the transportation of elderly and handicapped people. Since customer service-oriented policy is a current focal point in business administration, more and more companies offer a service to customers within a few hours of the time a request is received. In such an operating environment, the demand profile, including customer location, actual demand, and the service time window, may not be known until the request is actually received. Furthermore, ordinary urban traffic congestion, either recurrent or non-recurrent, can seriously affect the vehicle routing plan. Time-dependent travel time function captures recurrent congestion to a certain degree. However, non-recurrent congestion still cannot easily be predicted in advance and the corresponding travel time needs to be updated in a real-time manner. The dynamism of such a system requires bringing the complexity of real time and time dependency into the VRP in the form of the real-time time-dependent vehicle routing problem with time windows (RT-TDVRPTW), which is the focus of this paper.

The paper is divided into six sections. After this introductory section and the literature review in Section 2, a version of the RT-TDVRPTW is formulated and discussed in Section 3. In Section 4 we propose and elaborate on a heuristic that includes methods for route construction and improvement. In Section 5, we demonstrate and validate our model based on, with minor modifications, fifty-six test problems created by Solomon. At the end of Section 5, we test our model on an actual case—a professional logistic company in Taiwan. The last section discusses our conclusions.

## 2. Literature review

The VRP and their variations are principally static; all information about customers and travel times are known a priori. The literature related to the dynamic vehicle routing problem (DVRP) is relatively scarce. Some of the earliest work on the DVRP was conducted by Bertsimas and Ryzin (1991, 1993). In their studies, the orders/demands arrive randomly in time and the dispatching of vehicles is a continuous process of collecting demands, forming routes and dispatching vehicles. Further work regarding stochastic and dynamic network and routing can be found in Powell et al. (1995). A survey of the DVRP, including the delivery of petroleum products or industrial gases, courier services, intermodal services, tramp ship operations, pickup and delivery services, management of container terminals, and so forth, was presented by Psaraftis (1995). Bertsimas and Simchi-Levi (1996) conducted a survey of the dynamic and stochastic vehicle routing problem and examined the worst and average-case behaviors of known algorithms. Gendreau and Potvin (1998) also recently surveyed the DVRP and pointed out the importance of considering other sources of uncertainty in the DVRP, for instance, request cancellations and service delays. Larsen (2001) dedicated his Ph.D. Thesis to the DVRP. Generally speaking, most of the DVRP focus on determining an a priori solution by considering the uncertainty of service requests.

Currently, it has become possible to obtain a large amount of online information due to the progress of communication and positioning technologies. Regarding real-time demand, Gendreau et al. (1999) studied real-time vehicle routing and dispatching, which deals with customers who generate requests in real time and must be served within a given soft time window. A heuristic using tabu search with a parallel processing technique was proposed for the solution. Shieh and May (1998) studied the VRPTW with real time demands and proposed an online algorithm for it. Liu (2000) tackled a traveling salesman problem with both time dependent travel times and real-time demands. Chen et al. (2002) developed an algorithm for the time-dependent vehicle routing problem with time-windows but did not validate it using real problems. Chang et al. (2003) tackled the real-time VRPTW with simultaneous delivery/pickup demand, proposing a heuristic comprising route construction, route improvement and tabu search.

Time-dependent travel time, in addition to real-time information, is another important issue in our study. Time-dependent travel times differ according to the sequence of service (Picard and Queyranne, 1978; Fox et al., 1980; Lucena, 1990) or time of day (Malandraki and Daskin, 1992). A restricted dynamic programming method was proposed by Malandraki and Dial (1996) for solving a time-dependent TSP. Nonetheless, due to the computational complexity involved, papers on the time-dependent VRP are relatively few.

To the authors' knowledge, the topic of this paper—the VRPTW with real-time and time-dependent travel times in addition to real-time demands (i.e., the RT-TDVRPTW)—has previously not been addressed by other researchers.

## 3. Model formulation

### 3.1. Problem description

In the RT-TDVRPTW, a planner may not be aware of all information for route planning at the time the routing process begins. Moreover, information, including demands and time-dependent travel times, may change after the initial routes are constructed, and such information cannot be known in advance. When a new customer appears, the main task of the dispatching center is to include the new customer into the current routing plan. Yet, if time dependent travel times have changed due to an unexpected incident(s), in order to fulfill time window constraints and achieve a lower travel cost objective, scheduled incoming customers must be re-scheduled based on the position and loading of en route vehicles. The dispatching center needs a very quick answer in order to respond in real-time to both the real-time demands and time-dependent travel times.

In order to capture the dynamic characteristics of the RT-TDVRPTW, we employ the concept of time rolling horizon and hence adopt a series of mixed integer programming submodels to

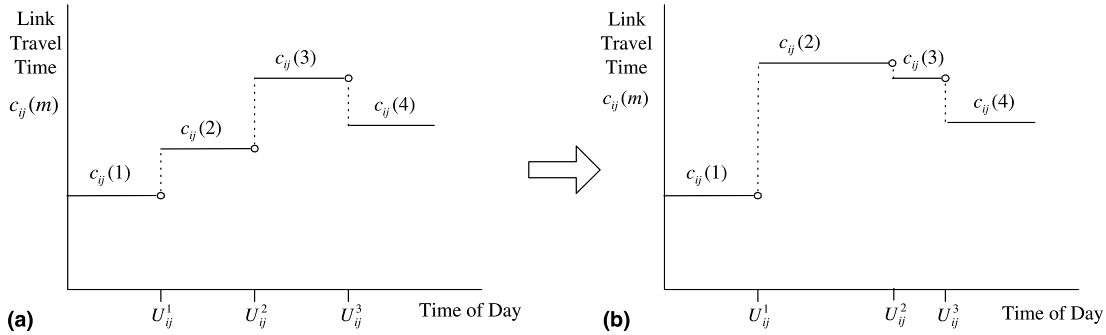


Fig. 1. Change of time-dependent travel time for link $i \rightarrow j$ in real-time.

formulate the problem. Each submodel represents a *special* VRPTW at a particular point in time when travel times and/or demands have changed. Each *special* VRPTW involves vehicles that may depart from current customers (called critical nodes, not the actual depot) and return to the actual depot, because those vehicles are already on their routes when the travel times and/or demands have changed. The goal of the RT-TDVRPTW at any particular time is to find a set of minimum cost vehicle routes, which originate from the critical nodes or the depot, $N_c(\tau) \cup \{0\}$, visit each unserviced node, $N_u(\tau)$, and terminate at the depot. The cost of the vehicle routes consists of travel times and waiting times, and each waiting time resulting from early arrival before a time window and/or from delaying departure after a service has finished. The reader may refer to Appendix A for a list of the notation used in this paper.

Time-dependent travel time and critical node are two important characteristics of the RT-TDVRPTW that need to be discussed in more detail, as follows:

(a) Time-dependent travel times are characterized by a step-wise function, as shown in Fig. 1(a), which represents predictive travel times at different time intervals. Once an unexpected incident happens, the predictive travel times need to be updated in real time, as shown in Fig. 1(b). As a result, the scheduled vehicle routes need to be updated, too. Note that the number of time intervals and the travel time in each time interval may change if non-recurrent congestion occurs.
(b) Once a customer has been serviced or the vehicle has left, that customer will be removed from the planned route thereafter. Only customers who have not been serviced will be considered in the planning of vehicle routes. Critical node plays an important role in distinguishing between a serviced customer and an unserviced customer. A critical node is defined as a customer who is currently using a vehicle or to whom a vehicle is heading. Critical nodes need to be identified instantly when a real-time demand or travel time has changed so that the route can be reconstructed.

Critical nodes can be identified either by using a graphical method or by a mathematical method. Using a graphical method, we can assume two vehicles are on their routes at time $\tau$, as shown in Fig. 2. One is on its way to node 4, and the other is staying at node 8. The critical nodes are nodes 4 and 8, respectively. We can identify critical nodes $N_c(\tau) = \{4, 8\}$ and $N_u(\tau) = \{5, 9, 10\}$.

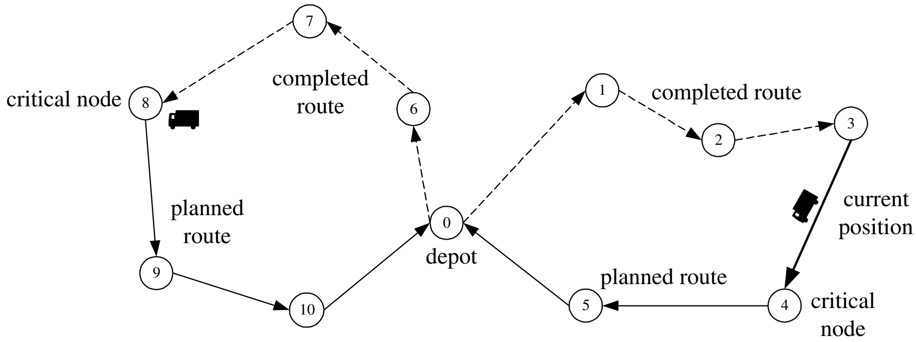


Fig. 2. Vehicle routes and critical nodes.

Using a mathematical method, we can suppose the route for vehicle $k$ is from node $i$ to node $j$ via intermediate node $h$, i.e., $i\forall h\forall j$. The departure times of each node are $d_i$, $d_h$ and $d_j$, respectively. The critical node at time $\tau$ is identified as $h$ if $d_i < \tau \leqslant d_h$, or $j$ if $d_h < \tau \leqslant d_j$.

### 3.2. Mathematical model

Whenever real-time demands have appeared or time-dependent travel times have changed, the RT-TDVRPTW needs to be solved within estimated computation time $\rho$ for an initial solution. Suppose $\tau$ is the time when real-time information (i.e., travel times or demands) becomes available plus an estimated time to compute the model. At time $\tau$, the RT-TDVRPTW can be formulated as a mixed integer problem, as follows:

$$\min_{\{x_{ijk}(m), w1_i, w2_i\}\in\Omega} z(\tau) = \alpha \sum_{i\in N_{\mathrm{cu0}}(\tau)} \sum_{j\in N_{\mathrm{u0}}(\tau)} \sum_{k} \sum_{m\geqslant m_{ij}(\tau)} c_{ij}(m)x_{ijk}(m) + \beta \sum_{i\in N_{\mathrm{u}}(\tau)} w1_i + \gamma \sum_{i\in N_{\mathrm{cu}}(\tau)} w2_i \tag{1}$$

where $\Omega$ is the feasible region represented by the following constraints.

Flow conservation constraints:

$$\sum_{j}\sum_{k}\sum_{m\geqslant m_{ij}(\tau)} x_{ijk}(m) = 1 \quad \forall i \in N_{\mathrm{cu}}(\tau) \tag{2}$$

$$\sum_{i}\sum_{k}\sum_{m\geqslant m_{ij}(\tau)} x_{ijk}(m) = 1 \quad \forall j \in N_{\mathrm{u}}(\tau) \tag{3}$$

$$\sum_{i}\sum_{m\geqslant m_{ih}(\tau)} x_{ihk}(m) - \sum_{j}\sum_{m\geqslant m_{hj}(\tau)} x_{hjk}(m) = 0 \quad \forall h \in N_{\mathrm{u}}(\tau),\ \ k\in K \tag{4}$$

$$\sum_{j}\sum_{m\geqslant m_{ij}(\tau)} x_{ij\bar{k}_i}(m) = 1 \quad \forall i \in N_{\mathrm{c}}(\tau) \tag{5}$$

$$\sum_{j}\sum_{m\geqslant m_{0j}(\tau)} x_{0jk}(m) \leqslant 1 \quad \forall k \in K_0(\tau) \tag{6}$$

Time window and departure time constraints:

$$a_i \leqslant l_i \quad \forall i \in N_{\mathrm{u}}(\tau) \tag{7}$$

$$a_{0k} \leqslant l_0 \quad \forall k \in K \tag{8}$$

$$U_{ij}^{m-1} \leqslant d_i < U_{ij}^{m} \quad \text{if } x_{ijk}(m) = 1 \quad \forall i \in N_{\mathrm{cu}}(\tau),\ j \in N_{\mathrm{u0}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{9}$$

$$d_i - (a_i + w1_i + s_i) \geqslant 0 \quad \forall i \in N_{\mathrm{cu}}(\tau) \tag{10}$$

$$d_i - \tau \geqslant 0 \quad \forall i \in N_{\mathrm{c}}(\tau) \tag{11}$$

$$U_{0j}^{m-1} \leqslant d_{0k} < U_{0j}^{m} \quad \text{if } x_{0jk}(m) = 1 \quad \forall j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{12}$$

$$d_{0k} - \tau + \lfloor 1 - x_{0jk}(m) \rfloor M \geqslant 0 \quad \forall j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{13}$$

Vehicle capacity constraints:

$$\sum_{i \in N_{\mathrm{u}}(\tau)} q_i \sum_{j \in N_{\mathrm{u0}}(\tau)} \sum_{m \geqslant m_{ij}(\tau)} x_{ijk}(m) \leqslant Q_k - \overline{Q}_k(\tau) \quad \forall k \in K \tag{14}$$

Definitional constraints:

$$a_j = d_i + c_{ij}(m) \quad \text{if } x_{ijk}(m) = 1 \quad \forall i \in N_{\mathrm{cu0}}(\tau),\ j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{15}$$

$$a_j = d_{0k} + c_{0j}(m) \quad \text{if } x_{0jk}(m) = 1 \quad \forall j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{0j}(\tau) \tag{16}$$

$$a_{0k} = d_i + c_{i0}(m) \quad \text{if } x_{i0k}(m) = 1 \quad \forall i \in N_{\mathrm{cu}}(\tau),\ k \in K,\ m \geqslant m_{i0}(\tau) \tag{17}$$

$$w1_i = \max\{0, e_i - a_i\} \quad \forall i \in N_{\mathrm{cu}}(\tau) \tag{18}$$

$$w2_i = d_i - (a_i + w1_i + s_i) \quad \forall i \in N_{\mathrm{cu}}(\tau) \tag{19}$$

$$x_{ijk}(m) = \{0, 1\} \quad \forall i \in N_{\mathrm{cu0}}(\tau),\ j \in N_{\mathrm{u0}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{20}$$

The objective of the RT-TDVRPTW, as shown in Eq. (1), is to construct it as a weighted function of travel times for all links, waiting times before service, and waiting times before departure at all nodes. The respective weights are $\alpha$, $\beta$ and $\gamma$, with the relation of $\alpha > \beta > \gamma$. This relation results from the following facts: (a) for each vehicle, the moving cost is generally higher than the stopping cost, because movement involves gasoline use, depreciation, and additional social costs from such things as traffic congestion, air pollution and risk of traffic incidents; (b) a vehicle waiting for departure has more flexibility than one waiting for service, because the former can receive a new command under real-time dispatching.

Eq. (2) requires that only one vehicle can leave from a critical node or unserviced node $i$ once. Eq. (3) denotes that only one vehicle can arrive at unserviced node $j$ once. Eq. (4) states that for each unserviced node $h$, the entering vehicle must eventually leave this node. Eq. (5) requires that vehicle $\bar{k}_i$ which has arrived at or is approaching a critical node, must also leave this node once. Note that vehicle $\bar{k}_i$ is known at time $\tau$. Eq. (6) designates that each vehicle can leave the depot once at most.

Eq. (7) requires that for each node, the service should take place no later than the end of its time window. Eq. (8) indicates that all vehicles must return to the depot before the depot is closed. Eq. (9) indicates that if $x_{ijk}(m) = 1$, then the departure time $d_i$ of node $i$ must be within time interval $m$. This logical equation can be transferred to the following two expressions by adopting a very big number, $M$.

$$d_i - U_{ij}^m - [1 - x_{ijk}(m)]M < 0 \quad \forall i \in N_{\mathrm{cu}}(\tau),\ j \in N_{\mathrm{u0}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{21}$$

$$d_i - U_{ij}^{m-1} + [1 - x_{ijk}(m)]M \geqslant 0 \quad \forall i \in N_{\mathrm{cu}}(\tau),\ j \in N_{\mathrm{u0}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{22}$$

Eq. (10) requires that departure time $d_i$ must be later than or equal to the completion time of service, $a_i + w1_i + s_i$. Eq. (11) indicates that for critical node $i$, departure time $d_i$ must be later than or equal to time $\tau$. Eq. (12) indicates that if $x_{0jk}(m) = 1$, then the time when vehicle $k$ leaves the depot, $d_{0k}$, must be within time interval $m$. Eq. (13) indicates that vehicles cannot leave the depot earlier than or equal to time $\tau$. Eq. (14) states that the vehicle capacity cannot be exceeded. Eqs. (15) and (16) define the arrival time at node $j$. Eq. (17) defines the arrival time at the depot for each vehicle. Eqs. (15)–(17) can be transferred to equations without using logical expressions. For example, Eq. (15) is equivalent to the following two equations.

$$d_i + c_{ij}(m) - a_j + \lfloor 1 - x_{ijk}(m) \rfloor M \geqslant 0 \quad \forall i \in N_{\mathrm{cu0}}(\tau),\ j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{23}$$

$$d_i + c_{ij}(m) - a_j + \lfloor 1 - x_{ijk}(m) \rfloor M \leqslant 0 \quad \forall i \in N_{\mathrm{cu0}}(\tau),\ j \in N_{\mathrm{u}}(\tau),\ k \in K,\ m \geqslant m_{ij}(\tau) \tag{24}$$

Eq. (18) defines the waiting time for service at node $i$ due to early arrival. Eq. (19) defines the waiting time for departing at node $i$. Note that arrival time $a_i$ at critical node $i$ is known already and is treated as a constant in Eqs. (18) and (19), whereas for unserviced node $i$, arrival time $a_i$ is a variable as defined in Eqs. (15) and (16). Eq. (20) designates $x_{ijk}(m)$ as a 0–1 integer variable. $x_{ijk}(m)$ equals 1 if vehicle $k$ departs node $i$ toward node $j$ during time interval $m$. Otherwise, $x_{ijk}(m)$ equals 0.

Note that sometimes real-time demands may not be able to be serviced within their time windows if the end of a time window is close to the time when demands appear. In this case, the model could be infeasible. In order to get practicable routes, we must reject the orders from the customers in set $N_{\mathrm{u}}(\tau)$ whose time windows are impossible to fulfill, and solve the RT-TDVRPTW for the remaining customers. However, if the rejected customers must still be serviced, the customers have to postpone their time windows or pay an additional charge for special express service. Once the customers postpone their time windows, we can add them back to the set $N_{\mathrm{u}}(\tau)$ and solve the RT-TDVRPTW again. Another possible strategy is to modify our model into a soft time window model. The soft time window model and special express service are good directions for further studies.

### 3.3. Time-space network

For a better understanding of the relation between variables in the RT-TDVRPTW, we assume, in Fig. 3(a), a two-link three-node network, where node 0 denotes the depot and nodes $i, j$ are customers. When a temporal dimension is considered, the time-space network can be drawn, in Fig. 3(b), where the horizontal axis denotes spatial distance, and the vertical axis represents time of day.

In Fig. 3(b), travel time between the depot and node $i$ is the same within each time interval $m$, as can be observed by the identical slopes. If $x_{ijk}(m1) = 1$, the departure time $d_{0k}$ will be within time interval $m1$. Customers departing at different time intervals would experience different travel times. In Fig. 3(b), the solid trajectory depicts how a customer is being serviced. Suppose vehicle $k$ is departing from the depot at time 1, $d_{0k} = 1$, and is arriving at destination $i$ at time 2, $a_i = 2$, with 1 unit of travel time. The vehicle must wait 1 unit of time, $w1_i = 1$, before the starting of time window [3, 4] is reached. After 2 units of time spent on service, the vehicle should wait 1 unit of time, $w2_i = 1$, before departing at $d_i = 6$.

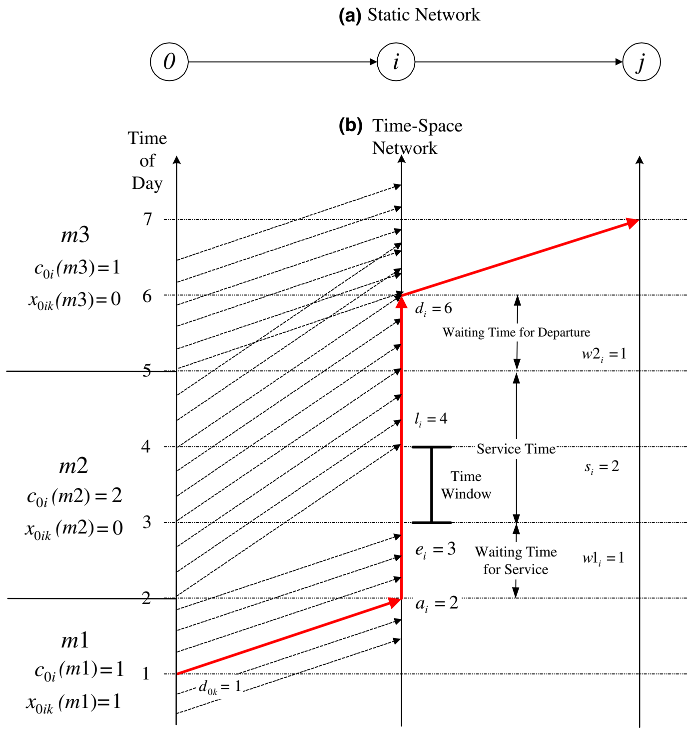


Fig. 3. Time-space network.

## 4. Solution algorithm

Because of the NP-hardness and the high complexity of the RT-TDVRPTW, it is difficult to solve the problem within a reasonable time by an exact algorithm, especially for large problems. Considering both the computational efficiency and the real-time response requirement, a heuristic comprising routes construction and routes improvement is proposed for the RT-TDVRPTW. Insertion cost calculation and departure time choice are important in route construction and are described in Section 4.1. In Section 4.2 a systematic solution procedure, embedding both route construction and route improvement, is illustrated. An efficient strategy is proposed for route construction and is elaborated in Section 4.3.

### 4.1. Calculation of insertion cost and choice of departure time

#### 4.1.1. Calculation of insertion cost

According to the objective function shown in Eq. (1), the generalized cost $C_{ij}(t)$ between nodes $i$ and $j$ at time $t$ can be defined in Eq. (25) as a weighted function of travel time $c_{ij}(m)$, waiting time $w1_j$ for service at node $j$ and waiting time $w2_i$ for departure at node $i$.

$$C_{ij}(t) = \alpha c_{ij}(m) + \beta w1_j + \gamma w2_i \quad \text{where } m = m_{ij}(t) \tag{25}$$

Consider a vehicle route, $(\ldots i, j, j+1, j+2, \ldots, j+n, \ldots, 0)$, where 0 represents the depot. If we insert node $h$ between nodes $i$ and $j$, the total increased cost can be expressed in terms of nodes $i$, $h$, and $j$ as

$$\Delta C = C_{ih}(t'_i) + C_{hj}(t_h) - C_{ij}(t_i) + \delta_j(t'_j) \tag{26}$$

where $t_i$ and $t'_i$ denote the optimal departure times from node $i$ before and after the insertion of node $h$, and $t_h$ represents the optimal departure time from node $h$. The last term $\delta_j(t'_j)$ in the above equation is the increased cost at node $j$ due to the insertion of node $h$ between nodes $i$ and $j$, which is essentially a recursive function of the increased cost occurred at node $j+1$.

$$\delta_j(t'_j) = C_{j,j+1}(t'_j) - C_{j,j+1}(t_j) + \delta_{j+1}(t'_{j+1}) \tag{27}$$

By recursively replacing $\delta_j(t'_j)$, we derive total increased cost, $\Delta C$, as follows:

$$\Delta C = C_{ih}(t'_i) + C_{hj}(t_h) - C_{ij}(t_i) + \sum_{p=j}^{j+n}(C_{p,p+1}(t'_p) - C_{p,p+1}(t_p)) + \delta_{j+n+1}(t'_{j+n+1}) \tag{28}$$

Once the increased cost at node $j+n+1, \delta_{j+n+1}(t'_{j+n+1})$ is known, the total increased cost $\Delta C$ can be calculated. If node $j+n+1$ denotes the depot, or the customer at which the vehicle's departure time is not affected by the insertion of a node, let $\delta_{j+n+1}(t'_{j+n+1}) = 0$. However, if the time window constraint at node $j+n+1$ is violated, let $\delta_{j+n+1}(t'_{j+n+1}) = \infty$ and, as a result, $\Delta C = \infty$. The infinitive total increased cost implies that the insertion of node $h$ is infeasible.

*4.1.2. Choice of departure time*

When node $h$ is inserted between nodes $i$ and $j$, the optimal departure times $t'_i,\ t_h,\ t'_p$ for nodes $i$, $h$, $p$, where $p = j, j+1, \ldots,$ can be obtained by minimizing the total increased cost $\Delta C$, as expressed in Eq. (28).

$$(t'_i, t_h) = \underset{(x,y)}{\arg}\min \Delta C = \{C_{ih}(x) + C_{hj}(y) - C_{ij}(t_i) + \delta_j(t'_j)\} \tag{29}$$

$$t'_p = \underset{x>\bar{d}'_p}{\arg}\min\{\delta_p(x)\} \quad \forall p \geqslant j = \underset{x>\bar{d}'_p}{\arg}\min\{C_{p,p+1}(x) - C_{p,p+1}(t_p) + \delta_{p+1}(t'_{p+1})\} \tag{30}$$

where $t_i$ and $t_p$ are the optimal departure times for nodes $i$ and $p$ before the insertion of node $h$, and $\bar{d}'_p$ and $t'_p$ denote the earliest and optimal departure times, respectively, for node $p$ after the insertion of node $h$. To derive the optimal departure times $t'_i,\ t_h,\ t'_p$ at nodes $i$, $h$, $p$, respectively, is not easy because the time is continuous. Therefore, finding ways to reduce the computation workload so that the RT-TDVRPTW can be solved within a reasonable time is necessary.

To reduce the computation time for choosing optimal departure times $t'_i,\ t_h,\ t'_p$ at nodes $i$, $h$, $p$, we propose in Appendix B a technique for deriving these optimal departure times.

*4.2. Unified framework of solution procedure*

For solving real-time operations, an *anytime* algorithm is desired. In other word, the solution procedure must have the ability to stop at any time and provide an acceptable solution. We describe a unified solution procedure for the RT-TDVRPTW in the following. During the solution process, we constantly check whether (1) departure time for critical nodes is up, (2) new demand occurs, and (3) time-dependent travel time has changed. The strategies of dispatching en route or on-call vehicles at the right time to the assigned customers, reconstructing routes, improving the quality of the existing routes, and so forth, are repeatedly applied. Note that the computation time allowed for route construction and improvement is usually short due to the requirement of real-time response. It would be natural to adopt a "quicker" solution algorithm for use. In the following, the insertion method is used for route construction, while the Or-opt node exchange algorithm is adopted for route improvement. However, more studies are necessary on selecting more efficient algorithms for practical large-scale applications. The steps of the unified framework of solution procedure are drawn in Fig. 4.

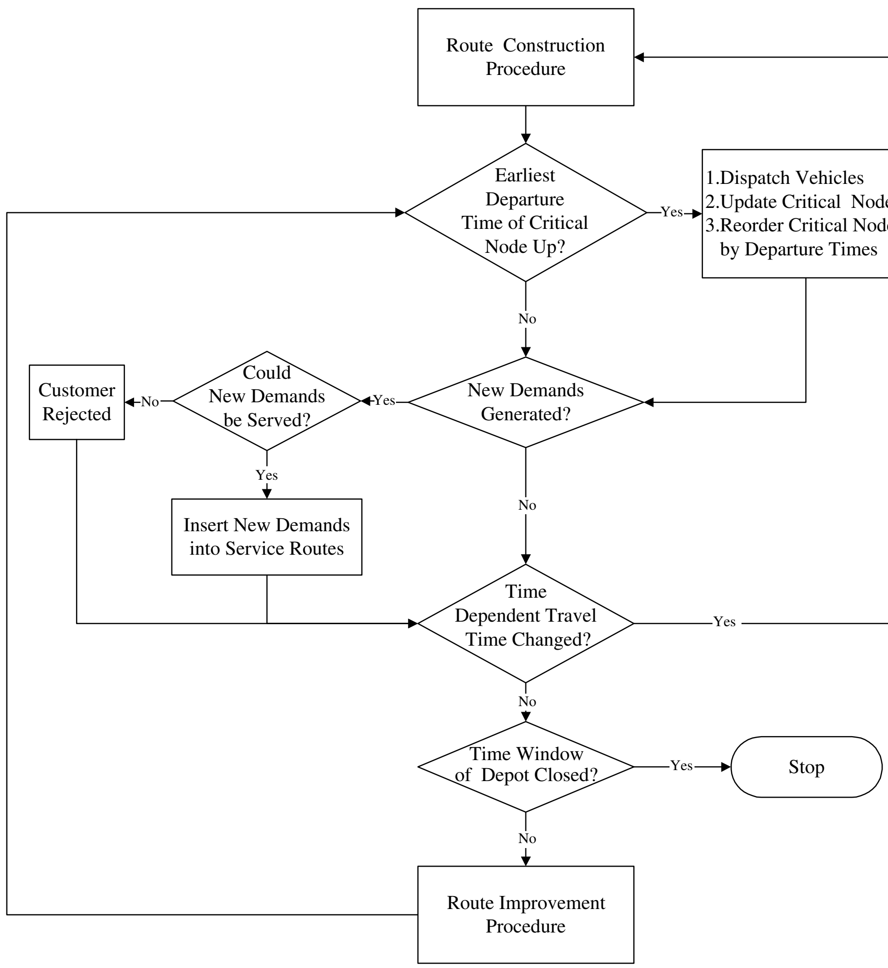


Fig. 4. Unified framework of solution procedure.

### 4.3. Method for route construction

For the VRPTW, the insertion method has been proven effective in constructing static routes (Solomon, 1987). So we adopted the insertion method with modifications for route construction, as follows:

*Step 1. Input data*
Input real-time demands $\{q_i\}$ as well as their time windows and service times. Input time-dependent travel times $\{c_{ij}(m)\}$ and estimated time $\tau$.

*Step 2. Classify customers and calculate earliest departure times*
Step 2.1. Classify customers into critical nodes $N_c(\tau)$ and unassigned nodes $N_u(\tau)$.
Step 2.2. For dispatched vehicle $k$, calculate the earliest departure time at critical node $i$.

$$\bar{d}_i = \max(\tau, a_i + w1_i + s_i) \quad \forall i \in N_c(\tau) \tag{31}$$

*Step 3. Find the optimal place and departure time for each unassigned node and calculate its accrued cost*
For each unassigned node $u \in N_u(\tau)$, find the optimal place and departure time for insertion and calculate its accrued cost.
Step 3.1. Set $k = 1$.
Step 3.2. If vehicle $k$ is en route, check its capacity constraint. If vehicle $k$ has surplus capacity to accommodate unassigned node $u$, then
Step 3.2.1. Calculate the insertion cost and check the time window constraint for unassigned node $u$ in all possible places along the moving route of vehicle $k$.
Step 3.2.2. Record the minimal insertion cost and the associated place for unassigned node $u$ along the moving route of vehicle $k$.
Step 3.3. If $k = K$, continue. Otherwise, set $k = k + 1$ and go to Step 3.2.
Step 3.4. If some vehicles are available for dispatching in the depot, calculate the accrued cost and check the time window constraint for unassigned node $u$ for the newly dispatched vehicle.
Step 3.5. Select the minimal insertion cost and its place for node $u$ in all possible routes. If either the capacity constraint or the time constraint cannot be fulfilled, exclude that customer from the service list.
Step 3.6. If every unassigned node $u \in N_u(\tau)$ has been examined, continue. Otherwise, go back to Step 3.1 for the next unassigned node $u$.

*Step 4. Insert the customer and update the relevant data*
Update the system by inserting node $u$ into the place with minimal accrued cost in an appropriate route (of vehicle $k^*$) and redefine relevant data, such as departure time and arrival time for each affected node. Once inserted, node $u$ is then removed from $N_u(\tau)$.

*Step 5. Stopping check for assignment*
If set $N_u(\tau)$ is empty, enter the unified framework of the solution procedure. Otherwise, do the following:

Step 5.1. For each unassigned node $u$, compute the insertion cost and its corresponding place along the route of vehicle $k^*$. Note that for unassigned nodes, their corresponding cost and optimal place for insertion in all routes, other than that taken by vehicle $k^*$, are calculated already and unchanged, and hence need not be recomputed.

Step 5.2. Compare and select the optimal place for inserting unassigned node $u$ with the minimal increased costs among all possible routes. If unassigned node $u$ cannot be included for service subject to the current vehicle capacity and time window constraints, exclude node $u$ from the service list. When set $N_{\mathrm{u}}(\tau)$ is empty, enter the unified framework of the solution procedure. Otherwise, go to Step 4.

The above steps are depicted in Fig. 5.

## 5. Computational results

### 5.1. Test problem set

Fifty-six problems created by Solomon (1987) are adopted with an additional column added to accommodate real-time demands. We assume the first 50 customers appeared at time 0, and other customers are generated by the following formula:

$$\max(0, e_i - \theta t_{0i} - r) \tag{32}$$

where $t_{0i}$ denotes the distance between the depot 0 and node $i$, parameter $r$ is a random number smaller than $e_i - \theta t_{0i}$, and adjustment parameter $\theta$ is designed to avoid generating new demands in the neighborhood of the time window at node $i$. Here we set $\theta = 1.5$.

Time-dependent travel times are characterized by a step-wise function. We arbitrarily divide the entire analysis time into four time intervals for each link. Suppose $Dist_{ij}$ denotes the distance between node $i$ and node $j$, and random parameters $\varepsilon 1$, $\varepsilon 2$, $\varepsilon 3$ and $\rho 1$, $\rho 2$ are generated within the range of $-(l_0 - e_0)/16 \leqslant \varepsilon 1, \varepsilon 2, \varepsilon 3 \leqslant (l_0 - e_0)/16$ and $1.1 \leqslant \rho 1, \rho 2 \leqslant 1.5$, respectively. The time-dependent travel time for each link and time interval is hypothesized in Table 1.

The assumed weights for travel time, waiting time for service, and waiting time for departure are $\alpha = 0.7$, $\beta = 0.2$, $\gamma = 0.1$, and the estimated time for computing the initial solution of the RT-TDVRPTW is set as $\Delta = 5$ units.

### 5.2. Test results

With the above input data, two scenarios were further hypothesized for testing: Scenario 1, in which time dependent travel times are updated at time $(l_0 - e_0)/5$; and Scenario 2, in which time dependent travel times are updated at times $(l_0 - e_0)/5$, $2(l_0 - e_0)/5$, $3(l_0 - e_0)/5$, $4(l_0 - e_0)/5$. The updating rule for time dependent travel times is set in Table 1.

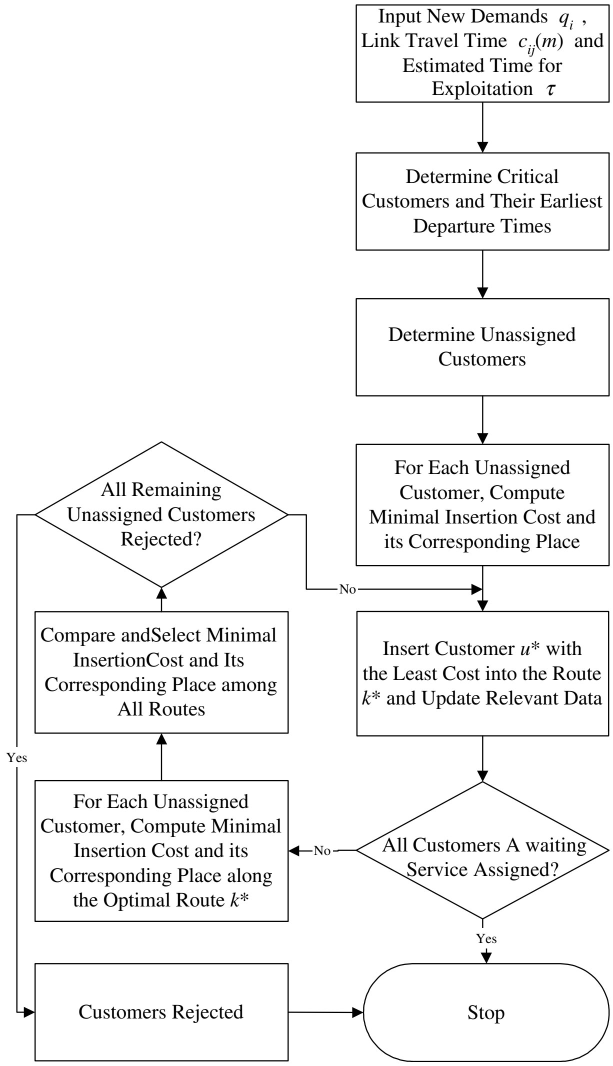


Fig. 5. Flow chart for route construction.

The two scenarios of the RT-TDVRPTW were solved and compared with a benchmark solution. The benchmark solution is obtained from a problem that accounts for real-time demands, but does not update the time-dependent travel times. As a result, the original scheduled routes persist. Under these circumstances, customers who cannot comply with the time window constraint will be rejected. In addition, for a node whose scheduled departure time is no longer valid under the new time-dependent travel times, the vehicle must depart to the next assigned node immediately after the service. Appendix C compares the solution of Scenario 1 with the benchmark solution in terms of objective value, travel time, waiting time for service, waiting time before departure, number of rejected customers and number of dispatched vehicles. Appendix D

Table 1
Time dependent travel time for link $i \to j$ and time interval $m$

| Time interval $m$ | Lower limit of time interval | Travel time $c_{ij}(m)$ |
|---|---|---|
| 1 | 0 | $Dist_{ij}$ |
| 2 | $(l_0 - e_0)/4 + \varepsilon 1$ | $Dist_{ij} * \rho 1$ |
| 3 | $2(l_0 - e_0)/4 + \varepsilon 2$ | $Dist_{ij} * \rho 2$ |
| 4 | $3(l_0 - e_0)/4 + \varepsilon 3$ | $Dist_{ij}$ |

compares the solution of Scenario 2 with the benchmark solution. The superiority of the solutions of Scenarios 1 and 2 over the benchmark solution in terms of some statistical measures is summarized in Table 2.

The objective value has been decreased 64 and 78 units on average, and the average number of rejected customers reduced 4.3 and 3.8 customers for Scenarios 1 and 2, respectively. The number of problems improved in either objective value or rejected customers or both are also notable, as shown in Table 2. The total number of problems not improved at all is only 5 and 2, respectively, out of 56 testing problems for Scenarios 1 and 2. In other words, only 7 out of 112 testing problems were not improved at all in objective value, rejected customers, or both. All customers in the C2 problem type can be serviced. The number of dispatched vehicles is not listed and compared,

Table 2
Scenarios 1 and 2 versus benchmark solution

| Problem type<br>Number of problems | | R1<br>12 | R2<br>11 | RC1<br>8 | RC2<br>8 | C1<br>9 | C2<br>8 | Total<br>56 | Average |
|---|---|---|---|---|---|---|---|---|---|
| Change of objective value | Scenario 1 | −57.84 (−4%) | −47.97 (−4%) | 57.27 (4%) | −81.49 (−6%) | −112.68 (−8%) | −145.71 (−13%) | – | −64 (−5%) |
| | Scenario 2 | −69.37 (−5%) | −16.86 (−1%) | 75.77 (5%) | −50.96 (−4%) | −177.03 (−13%) | −247.60 (−22%) | – | −78 (−6%) |
| Change of rejected customers | Scenario 1 | −8.2 | −4.2 | −4.9 | −6.1 | −0.9 | −0.5 | – | −4.3 |
| | Scenario 2 | −6.9 | −2.8 | −4.1 | −6.4 | −0.7 | −0.8 | – | −3.8 |
| No. of problems improved in both objective value and rejected customers | Scenario 1 | 9 | 8 | 1 | 5 | 3 | 3 | 29 | – |
| | Scenario 2 | 9 | 6 | 1 | 6 | 5 | 4 | 31 | – |
| No. of problems improved in objective value[a] | Scenario 1 | 0 | 0 | 0 | 0 | 2 | 3 | 5 | – |
| | Scenario 2 | 1 | 0 | 1 | 0 | 4 | 4 | 10 | – |
| No. of problems improved in rejected customers | Scenario 1 | 3 | 3 | 6 | 3 | 2 | 0 | 17 | – |
| | Scenario 2 | 2 | 3 | 6 | 2 | 0 | 0 | 13 | – |
| No. of problems not improved | Scenario 1 | 0 | 0 | 1 | 0 | 2 | 2 | 5 | – |
| | Scenario 2 | 0 | 2 | 0 | 0 | 0 | 0 | 2 | – |

[a] The number of problems improved in objective value can only be compared with the same number of rejected customers.

since the total number of vehicles ($K = 25$) is sufficient and the dispatching cost for additional vehicles is not taken into account in the objective function.

### 5.3. Real application

To evaluate the authenticity of our model and algorithm in the real world, we took a professional logistic company (an anonymous Company A) in Taiwan for testing. Company A was

Table 3
Demand information of company A

| Show up time (min) | Customer no. | Demand (chests) | Time window | | Service time (min) |
|---|---|---|---|---|---|
| | | | Lower end (min) | Upper end (min) | |
| 0 | 4 | 55 | 0 | 420 | 58 |
| 0 | 5 | 66 | 0 | 420 | 91 |
| 0 | 7 | 38 | 0 | 420 | 36 |
| 0 | 14 | 72 | 0 | 420 | 114 |
| 0 | 28 | 37 | 0 | 420 | 40 |
| 0 | 29 | 97 | 0 | 420 | 152 |
| 0 | 32 | 71 | 0 | 420 | 73 |
| 0 | 35 | 37 | 0 | 420 | 30 |
| 24 | 16 | 1 | 0 | 420 | 5 |
| 28 | 20 | 11 | 0 | 420 | 12 |
| 34 | 25 | 5 | 0 | 420 | 38 |
| 35 | 6 | 1 | 0 | 420 | 2 |
| 44 | 15 | 3 | 0 | 420 | 4 |
| 46 | 26 | 10 | 30 | 210 | 13 |
| 48 | 13 | 14 | 30 | 210 | 23 |
| 55 | 18 | 2 | 0 | 420 | 4 |
| 56 | 31 | 14 | 0 | 420 | 7 |
| 62 | 33 | 10 | 30 | 210 | 7 |
| 71 | 2 | 10 | 30 | 210 | 12 |
| 89 | 24 | 1 | 0 | 420 | 2 |
| 98 | 11 | 3 | 30 | 210 | 7 |
| 148 | 12 | 1 | 0 | 420 | 1 |
| 152 | 17 | 2 | 0 | 420 | 4 |
| 162 | 21 | 1 | 0 | 420 | 2 |
| 166 | 3 | 1 | 0 | 420 | 2 |
| 168 | 30 | 6 | 0 | 420 | 10 |
| 175 | 8 | 1 | 0 | 420 | 5 |
| 181 | 9 | 1 | 0 | 420 | 4 |
| 207 | 1 | 7 | 0 | 420 | 9 |
| 209 | 22 | 11 | 0 | 420 | 30 |
| 211 | 27 | 5 | 0 | 420 | 6 |
| 214 | 36 | 1 | 0 | 420 | 2 |
| 282 | 23 | 6 | 0 | 420 | 6 |
| 286 | 19 | 2 | 0 | 420 | 2 |
| 296 | 10 | 1 | 0 | 420 | 2 |
| 305 | 34 | 9 | 0 | 420 | 25 |

Table 4
Final results of the RT-TDVRPTW for company A

| Vehicle no. | Actual routes | | | | Loads |
|---|---|---|---|---|---|
| 1 | 0(–, 5)[a] | $\gamma$7(35, 70.7) | $\gamma$4(85.7, 143.5) | $\gamma$0(163.5, 173) | 93 |
| 1 | 0(163.5, 173) | $\gamma$28(209, 249) | $\gamma$36(279, 280.8) | $\gamma$12(310.8, 311.8) | 65 |
| | $\gamma$10(316.8, 318.8) | $\gamma$34(338.8, 363.5) | $\gamma$23(378.5, 384.5) | $\gamma$19(389.5, 391.5) | |
| | $\gamma$9(401.5, 405.5) | $\gamma$1(415.5, 424.4) | $\gamma$0(454.4, –) | | |
| 2 | 0(–, 5) | $\gamma$5(28, 119.1) | $\gamma$11(129.1, 136) | $\gamma$2(151, 163) | 79 |
| | $\gamma$0(198, –) | | | | |
| 3 | 0(–, 5) | $\gamma$14(41, 155.5) | $\gamma$17(165.5, 169.5) | $\gamma$18(174.5, 178.4) | 96 |
| | $\gamma$16(188.4, 192.9) | $\gamma$15(197.9, 202.1) | $\gamma$13(207.1, 230.1) | $\gamma$3(250.1, 252.2) | |
| | $\gamma$8(277.2, 287) | $\gamma$0(319, –) | | | |
| 4 | 0(–, 5) | $\gamma$ 35(63, 92.6) | $\gamma$33(107.6, 114.4) | $\gamma$26(124.4, 137.6) | 86 |
| | $\gamma$25(142.6, 180.4) | $\gamma$30(195.4, 205.7) | $\gamma$21(215.7, 218) | $\gamma$22(223, 252.9) | |
| | $\gamma$27(262.9, 269.2) | $\gamma$24(274.2, 276.2) | $\gamma$0(311.2, –) | | |
| 5 | 0(–, 5) | $\gamma$29(45, 197.9) | $\gamma$6(212.9, 214.9) | $\gamma$0(239.9, –) | 98 |
| 6 | 0(–, 5) | $\gamma$32(55, 128.1) | $\gamma$31(133.1, 140) | $\gamma$ 20(150, 174.1) | 96 |
| | $\gamma$0(206.1, –) | | | | |

[a] Node (arrival time, departure time).

established in 1990 and has 4 distribution centers, providing an island-wide, punctual and real-time service. The service includes foods, pharmacy, household products, cosmetics, computers, clothes, and so on. We adopted relevant data with slight modification to account for real-time and time-dependent information. Data on only one day's worth of orders of Company A were collected for use. The day's orders include 36 customers with their demands, time windows, and service times. The vehicle capacity is 100 chests. The business hours are ten. For real-time consideration, we randomly choose a show-up time for each customer whose demand is less than one-third of vehicle capacity. Other customers are supposed to give their orders before the business hour. The customers' information is listed in Table 3.

Company A also provided the average travel time between each customer. In fact, the travel time varies according to urban traffic congestion. However, the provided travel times by Company A fall with the normal traffic conditions. We generated the step-wise travel time function according to the traffic flow profile experienced in the real situation.

The final routes are listed in Table 4. The test results show that all customers can be serviced by 6 vehicles with 7 routes. This is because the first vehicle accomplishes two routes without violating any time window constraint. The total travel time is 832 min, which is smaller than the result of manual planning by Company A (i.e. 875 min), even under real-time information and consideration of traffic conditions. In addition, the total waiting time for service is zero; the total waiting time for departure is 16.82 min.

To better understand the real-time dispatching, we illustrate the planned routes solved by the RT-TDVRPTW at specific instants in Fig. 6. When the routing schedule is changed, our program will automatically output the new schedule to notify the dispatching center. Some of these instants

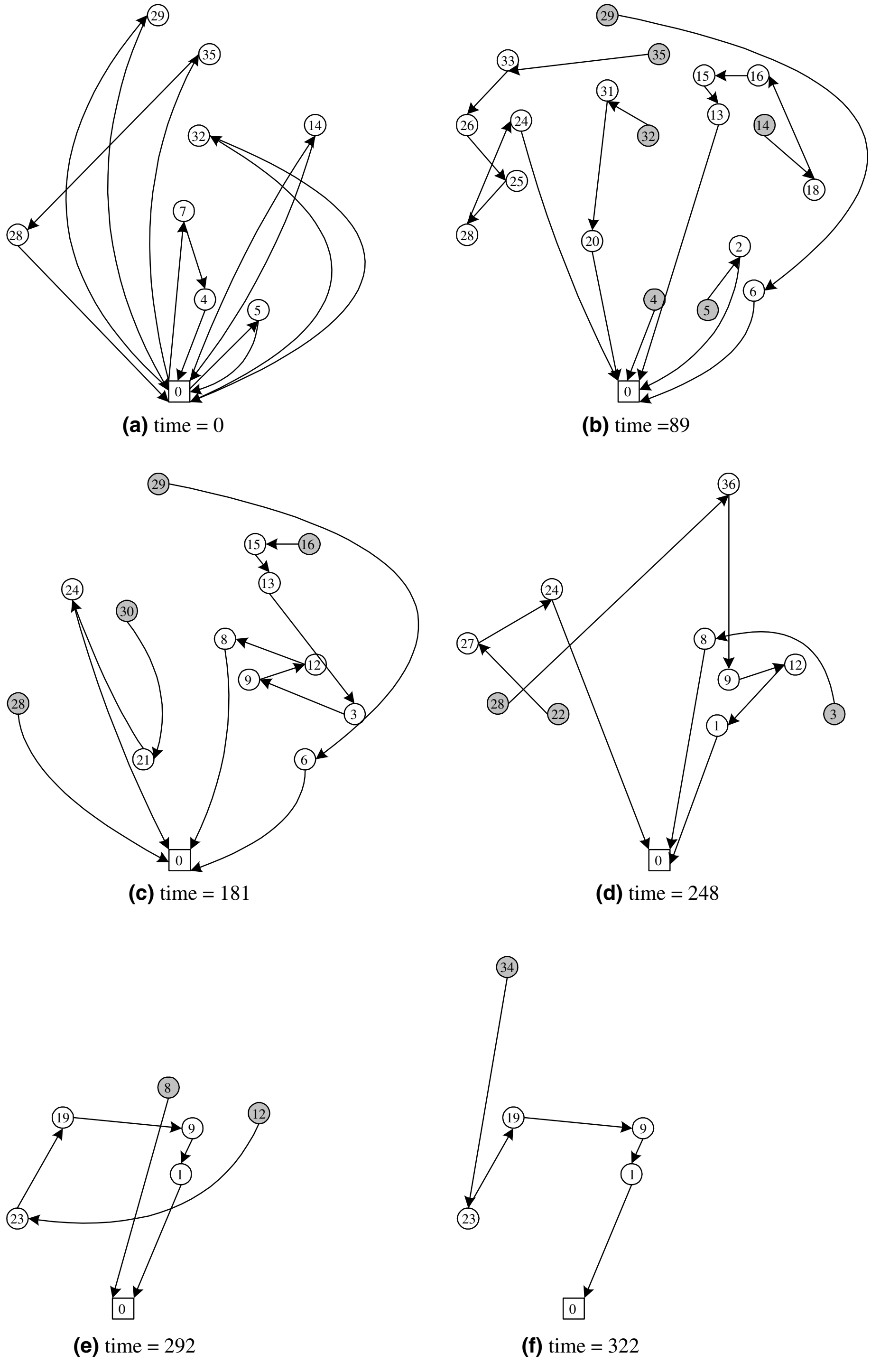


Fig. 6. The planned routes solved by the RT-TDVRPTW at selected instants.

include times 0, 89, 181, 248, 292, and 322 in the real application. Note that one shadowed node represents a critical node, at which the vehicle is staying or on its way at that moment. Having the aid of our solution results, the dispatching center can effortlessly give orders to the driver through an advanced communication device. Which customer should be serviced next? When should the vehicle depart? We can foresee that the planned routes may change later in time due to new customer requests, the real-time variation of travel time or the improvement of routes. Note that once the customer has been serviced and the vehicle has left, the customer will be removed from the unplanned route thereafter.

## 6. Conclusion

In this paper, the RT-TDVRPTW was studied and formulated as a mixed integer programming model, which accounts for real-time and time-dependent travel times, and real-time demands. Note that both real-time and time-dependent travel times and real-time demands are uncertain in advance. An *anytime* algorithm comprising route construction and route improvement was proposed and validated for the RT-TDVRPTW. Fifty-six problems created by Solomon were taken with minor modifications, upon which two scenarios were further differentiated for testing. The results obtained were compared with a specially designed benchmark solution, and the superiority of our model was clearly justified. A real application to a professional logistic company in Taiwan was also provided.

The main contributions of the paper can be summarized as follows:

1. The departure time at each node is treated as a decision variable with which there is no need to impose numerous subtour (a circle disconnected from the depot) elimination constraints in the constraint set. See Eqs. (10) and (15) for examples.
2. A clear definition of critical node is proposed, which defines the scope of the remaining problem at a particular instant along the time horizon. The concept of critical nodes distinguishes the *real-time* VRPTW from the traditional VRPTW to a high degree.
3. A new insertion cost expression which appears to be a recursive function is proposed under time-dependent travel times. As a result, the computation time for choosing departure times can be greatly reduced.
4. In contrast to the traditional VRP, the model does not require that a vehicle leaves the customer as soon as the service ends. By allowing delayed departure at each customer, more efficient schedules can be created.

## Acknowledgement

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## Appendix A. Notations

Parameters and constants:

$\alpha$ weight associated with link travel time in the objective function
$\beta$ weight associated with waiting time before service in the objective function
$\gamma$ weight associated with waiting time before departure in the objective function
$\rho$ estimated computation time for initial solution
$\tau$ the time when real-time information (travel times or demands) occurs plus an estimated computation time, $\rho$
$c_{ij}(m)$ travel time between nodes $i$ and $j$ during time interval $m$
$e_i$ the starting time of time window at node $i$
$l_i$ the ending time of time window at node $i$
$M$ a very big number
$m_{ij}(\tau)$ time interval subject to nodes $i$ and $j$ in which time $\tau$ occurs.
$q_i$ demand at node $i$
$Q_k$ capacity of vehicle $k$
$\overline{Q}_k(\tau)$ cumulated loads in vehicle $k$ at time $\tau$
$s_i$ service time at node $i$
$U_{ij}^m$ ending time of time interval $m$ between nodes $i$ and $j$

Set of nodes:

$\{0\}$ depot
$N_c(\tau)$ set of critical nodes at time $\tau$; critical node is defined as the node that the vehicle is staying at or on route to
$N_u(\tau)$ set of unassigned (or unserviced) nodes at time $\tau$
$N_{cu}(\tau)$ set of critical and unassigned nodes at time $\tau$
$N_{u0}(\tau)$ set of the depot and unassigned nodes at time $\tau$
$N_{cu0}(\tau)$ set of the depot, critical and unassigned nodes at time $\tau$

Set of vehicles:

$K$ set of all vehicles, which is the union of two sets, $K_0(\tau)$ and $\overline{K}_0(\tau)$
$K_0(\tau)$ set of vehicles in the depot at time $\tau$
$\overline{K}_0(\tau)$ set of vehicles en route from the depot at time $\tau$

Variable:

$a_i$ time arriving at node $i$
$a_{0k}$ time for vehicle $k$ to return to the depot
$d_i$ time to depart from node $i$
$\bar{d}_i$ earliest time allowed to leave node $i$
$d_{0k}$ time for vehicle $k$ to depart from the depot
$x_{ijk}(m)$ 1, if vehicle $k$ departs node $i$ toward node $j$ during time interval $m$; 0, otherwise

$w1_i$ waiting time for service at node $i$
$w2_i$ waiting time before departure at node $i$

## Appendix B. Choice of departure times

Two important assumptions are restated here for reference.

(1) Time-dependent travel times are characterized by a step-wise function.
(2) The weights associated with travel time, waiting time for service and waiting time before departure are in decreasing order, i.e., $\alpha > \beta > \gamma$.

The first assumption allows us to discuss the optimal departure time for each node, time interval by time interval, because within each time interval the link travel time would not change. The second assumption simply states the order of importance of travel time, waiting time for service, and waiting time before departure.

We first define the earliest departure time $\bar{d}_i$ from node $i$ as follows:

$$\bar{d}_i = \begin{cases} \max\{\tau, a_i + w1_i + s_i\} & \text{if } i \in N_\mathrm{c}(\tau) \\ a_i + w1_i + s_i & \text{if } i \in N_\mathrm{u}(\tau) \end{cases}$$

For link $i \to j$ and time interval $m$, given the lower limit $U_{ij}^{m-1}$, the upper limit $U_{ij}^{m}$ (not included) and the travel time $c_{ij}(m)$, the optimal departure time with the assumption of $U_{ij}^{m} > \bar{d}_i$ is discussed by three situations, as follows:

(1) $e_j \geqslant U_{ij}^{m} + c_{ij}(m)$

As shown in Fig. B1(a), the range of departure time $d_i$ is $\lfloor \bar{d}_i, U_{ij}^{m} - \varepsilon \rfloor$, where $\varepsilon$ is a small positive number. Since $w1_j > 0$, $\beta > \gamma$ and $c_{ij}(m)$ is constant, the departure time can be delayed to $U_{ij}^{m} - \varepsilon$ so as to decrease the objective value with the increment of $w2_i$ equal to decrement of $w1_j$. Here, $U_{ij}^{m}$ is not considered because its corresponding travel time $c_{ij}(m + 1)$ is determined within time interval $m + 1$.

(2) $U_{ij}^{m} + c_{ij}(m) > e_j \geqslant U_{ij}^{m-1} + c_{ij}(m)$

Let $d_i'$ be the departure time from node $i$ such that the range of arrival time at node $j$ is the lower limit of the time window, i.e., $d_i' = e_j - c_{ij}(m)$. As such, two cases can be considered:

Case 1: $d_i' > \bar{d}_i$: As shown in Fig. B1(b), the range of departure time $d_i$ is $[\bar{d}_i, U_{ij}^{m} - \varepsilon]$. With $w1_j > 0$, the objective value can be decreased by delaying the departure time from $\bar{d}_i$ to $d_i'$. It is not necessary to delay the departure time further than $d_i'$ because it will increase the waiting time for departure at node $i$.

Case 2: $\bar{d}_i \geqslant d_i'$: As shown in Fig. B1(c), the range of departure time $d_i$ is $[\bar{d}_i, U_{ij}^{m} - \varepsilon]$. Again, there is no need to postpone the departure time since it will increase the waiting time for departure from node $i$.

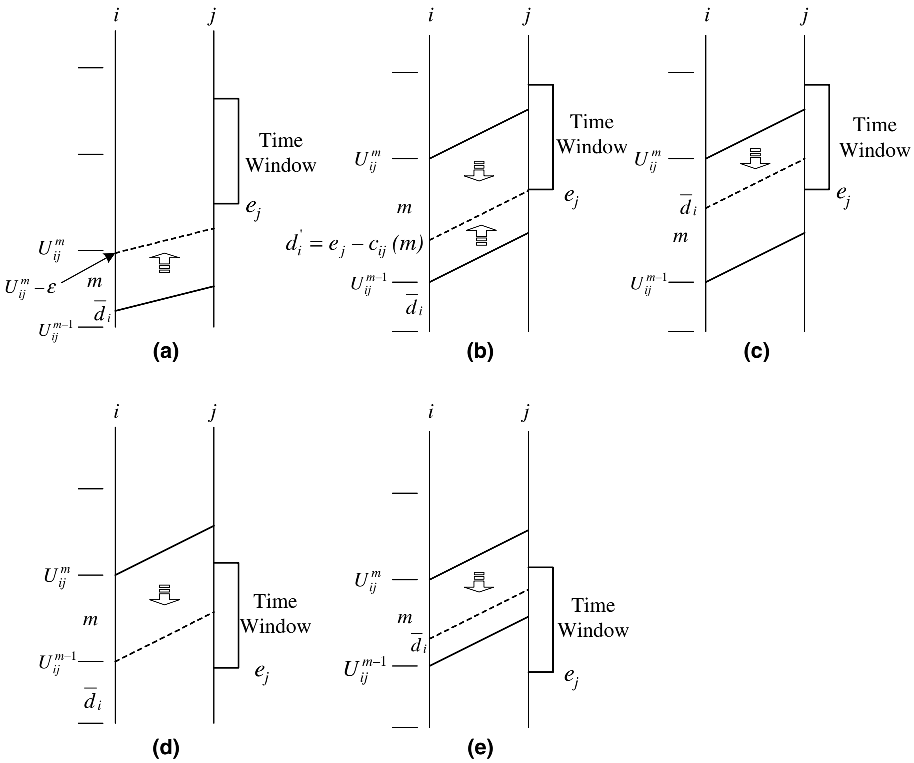


Fig. B1. Choice of departure times.

By considering the above two cases, the optimal departure time under the condition $U_{ij}^m + c_{ij}(m) > e_j \geqslant U_{ij}^{m-1} + c_{ij}(m)$ is $\max(\bar{d}_i, d_i')$.

(3) $U_{ij}^{m-1} + c_{ij}(m) > e_j$

Two cases can be considered:

Case 1: $U_{ij}^{m-1} \geqslant \bar{d}_i$: As shown in Fig. B1(d), the range of departure time $d_i$ is $[U_{ij}^{m-1}, U_{ij}^m - \varepsilon]$. Since $w2_i > 0$ and $w1_j = 0$, the optimal departure time from node $i$ would be $U_{ij}^{m-1}$ in terms of objective value.

Case 2: $\bar{d}_i \geqslant U_{ij}^{m-1}$: As shown in Fig. B1(e), the range of departure time $d_i$ is $[\bar{d}_i, U_{ij}^m - \varepsilon]$. Since $w2_i > 0$ and $w1_j = 0$, the optimal departure time from node $i$ would be $\bar{d}_i$ in terms of objective value.

By considering the above two cases, the optimal departure time under the condition $U_{ij}^{m-1} + c_{ij}(m) > e_j$ is $\max(U_{ij}^{m-1}, \bar{d}_i)$.

In summary, for each time interval greater than or equal to $m_{ij}(\bar{d}_i)$, only one optimal departure time exists. Among all time intervals, a comparison is made in terms of total increased cost so that the time-space route with the minimal increased cost is chosen for use.

## Appendix C. Comparison of Scenario 1 and benchmark solution

| Problem set | Benchmark solution | | | | | | Scenario 1 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Objective value | Travel time | $\sum_i w1_i$ | $\sum_i w2_i$ | RN[a] | NV[b] | Objective value | Travel time | $\sum_i w1_i$ | $\sum_i w2_i$ | RN[a] | NV[b] |
| R101 | 1716.19 | 2351.3 | 15.8 | 671.2 | 10 | 25 | 1644.41 | 2230.5 | 87.6 | 655.37 | 3 | 25 |
| R102 | 1670.08 | 2273.6 | 43.8 | 698 | 9 | 23 | 1491.44 | 2031 | 36.7 | 624 | 2 | 21 |
| R103 | 1379.33 | 1868 | 54.4 | 608.5 | 8 | 19 | 1323.81 | 1799.5 | 79.6 | 482.4 | 4 | 19 |
| R104 | 1225.26 | 1698.3 | 3.9 | 356.7 | 7 | 15 | 1129.34 | 1656.3 | 9.2 | 317.92 | 2 | 15 |
| R105 | 1541.09 | 2124.5 | 32.4 | 474.6 | 4 | 20 | 1527.36 | 2126.8 | 16.8 | 352.4 | 2 | 20 |
| R106 | 1420 | 1975.9 | 9.5 | 349.7 | 7 | 19 | 1421.15 | 1946.3 | 17.6 | 552.23 | 0 | 20 |
| R107 | 1394.69 | 1925.3 | 30.5 | 408.8 | 4 | 17 | 1209.45 | 1669.9 | 25 | 355.2 | 1 | 16 |
| R108 | 1117.57 | 1567.1 | 18.8 | 168.4 | 10 | 14 | 1146.73 | 1595.8 | 0 | 296.73 | 0 | 14 |
| R109 | 1382.18 | 1919.9 | 2.1 | 378.3 | 15 | 17 | 1364.23 | 1914 | 4 | 236.32 | 0 | 19 |
| R110 | 1405.48 | 1970.2 | 15.4 | 232.6 | 12 | 17 | 1346.23 | 1884.7 | 21.3 | 226.79 | 0 | 18 |
| R111 | 1328.8 | 1853.2 | 17.2 | 281.2 | 10 | 17 | 1261.08 | 1755.4 | 10.9 | 301.25 | 0 | 17 |
| R112 | 1182.1 | 1667.3 | 5.4 | 139.1 | 16 | 13 | 1203.46 | 1671.5 | 0 | 334.11 | 0 | 17 |
| R201 | 1510.28 | 2054.5 | 8.2 | 704.9 | 2 | 6 | 1413.79 | 1882.8 | 3.4 | 951.5 | 0 | 7 |
| R202 | 1257.85 | 1627.4 | 2.5 | 1181.7 | 1 | 5 | 1279.78 | 1697.9 | 13 | 886.5 | 0 | 6 |
| R203 | 1360.42 | 1858.8 | 0 | 592.6 | 3 | 6 | 1289.09 | 1704.6 | 13.6 | 931.48 | 0 | 7 |
| R204 | 1034.44 | 1390 | 37.7 | 539 | 3 | 5 | 918.18 | 1247.6 | 4.2 | 440.16 | 0 | 5 |
| R205 | 1346.15 | 1876.1 | 0 | 328.8 | 4 | 5 | 1317.82 | 1777.4 | 0 | 736.42 | 0 | 5 |
| R206 | 1073.29 | 1501.6 | 0 | 221.7 | 8 | 4 | 1236.8 | 1692 | 32.3 | 459.42 | 0 | 8 |
| R207 | 1140.58 | 1574.1 | 0 | 387.1 | 3 | 3 | 1045.49 | 1350.1 | 0 | 1004.2 | 0 | 4 |
| R208 | 1191.5 | 1607.3 | 2.3 | 659.3 | 5 | 5 | 889.25 | 1238 | 0 | 226.5 | 0 | 4 |
| R209 | 1218.62 | 1650.2 | 2.5 | 629.8 | 8 | 5 | 1182.49 | 1653.9 | 0 | 247.59 | 0 | 6 |
| R210 | 1202.88 | 1662.1 | 0 | 394.1 | 3 | 4 | 1173.16 | 1599.3 | 0 | 536.52 | 0 | 5 |
| R211 | 1037.7 | 1404.9 | 5.5 | 531.7 | 5 | 4 | 1100.17 | 1493 | 0 | 550.72 | 0 | 5 |
| RC101 | 1671.64 | 2318 | 24.3 | 441.8 | 3 | 20 | 1760.83 | 2432.2 | 6.9 | 569.14 | 3 | 22 |
| RC102 | 1689.42 | 2370 | 2.5 | 299.2 | 13 | 19 | 1542.15 | 2155.6 | 0.3 | 331.66 | 5 | 19 |
| RC103 | 1390.1 | 1929.2 | 24.1 | 348.4 | 8 | 16 | 1501.66 | 2103.7 | 30 | 230.7 | 1 | 17 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RC104 | 1215.32 | 1696.3 | 0.6 | 277.9 | 5 | 14 | 1258.62 | 1761.9 | 3.1 | 246.69 | 0 | 14 |
| RC105 | 1674.82 | 2340.1 | 13.9 | 339.7 | 9 | 19 | 1695.54 | 2362.1 | 2.3 | 416.14 | 2 | 21 |
| RC106 | 1603.74 | 2247.8 | 10 | 282.8 | 4 | 18 | 1744.75 | 2454.6 | 7.6 | 250.13 | 1 | 19 |
| RC107 | 1508.86 | 2103.2 | 14.5 | 337.2 | 4 | 17 | 1618.24 | 2259.2 | 7.4 | 353.22 | 0 | 18 |
| RC108 | 1430.44 | 1999.2 | 4.7 | 300.6 | 5 | 16 | 1536.68 | 2149.1 | 0 | 323.15 | 0 | 18 |
| RC201 | 1703.79 | 2323.7 | 18.2 | 735.6 | 6 | 7 | 1697.14 | 2317.3 | 0 | 750.27 | 0 | 6 |
| RC202 | 1526.29 | 2082.4 | 3.8 | 678.5 | 7 | 5 | 1375.43 | 1814.2 | 22.8 | 1009.3 | 0 | 7 |
| RC203 | 1395.89 | 1860 | 1.8 | 935.3 | 6 | 5 | 1403.33 | 1809.9 | 0 | 1364 | 0 | 7 |
| RC204 | 1278.12 | 1722.8 | 4.7 | 712.2 | 6 | 4 | 918.18 | 1247.6 | 4.2 | 440.16 | 0 | 5 |
| RC205 | 1609.49 | 2141.3 | 7.4 | 1091 | 9 | 6 | 1552.79 | 1980.3 | 10.9 | 1644 | 0 | 8 |
| RC206 | 1461.4 | 2039.7 | 2.3 | 331.5 | 9 | 5 | 1508.59 | 2092.7 | 0 | 437.01 | 0 | 6 |
| RC207 | 1288.2 | 1779.9 | 1.5 | 419.7 | 4 | 6 | 1338.4 | 1847.8 | 0 | 449.4 | 0 | 5 |
| RC208 | 1296.34 | 1760.2 | 109.8 | 422.4 | 2 | 5 | 1113.78 | 1450.1 | 106.5 | 774.11 | 0 | 6 |
| C101 | 1126.63 | 1595.3 | 0 | 99.2 | 2 | 12 | 1013.1 | 1412.5 | 0 | 243.5 | 0 | 12 |
| C102 | 1476.12 | 1977.4 | 9.4 | 900.6 | 2 | 13 | 1476.29 | 1942.6 | 0 | 1164.8 | 0 | 14 |
| C103 | 1736.36 | 2250.1 | 20.2 | 1572.5 | 2 | 14 | 1322.1 | 1734.5 | 14.8 | 1049.9 | 0 | 13 |
| C104 | 1352.06 | 1807.9 | 0 | 865.3 | 0 | 11 | 1385.01 | 1899.1 | 0 | 556.37 | 0 | 12 |
| C105 | 1276.05 | 1776 | 3.8 | 320.9 | 1 | 12 | 1351.78 | 1907.2 | 0 | 167.4 | 0 | 14 |
| C106 | 1205.07 | 1679.7 | 8.2 | 276.4 | 0 | 12 | 936.28 | 1317.7 | 0 | 138.9 | 0 | 13 |
| C107 | 1375.57 | 1882.8 | 24.2 | 527.7 | 0 | 12 | 1207.23 | 1710 | 0 | 102.3 | 0 | 11 |
| C108 | 1383.79 | 1897 | 9.1 | 540.7 | 1 | 12 | 1177.44 | 1649.8 | 18.1 | 189.61 | 0 | 12 |
| C109 | 1245.08 | 1713.8 | 0.8 | 452.6 | 0 | 12 | 1293.42 | 1802.2 | 0 | 318.75 | 0 | 12 |
| C201 | 719.84 | 1002.6 | 0 | 180.2 | 0 | 6 | 581.04 | 830 | 0 | 0.4 | 0 | 5 |
| C202 | 1000.72 | 1177.1 | 7.8 | 1751.9 | 0 | 5 | 1009.8 | 1306.5 | 0 | 952.5 | 0 | 6 |
| C203 | 1396.85 | 1574.8 | 3.3 | 2938.3 | 0 | 6 | 1179.07 | 1500.7 | 0 | 1285.8 | 0 | 6 |
| C204 | 1607.19 | 1852.9 | 6 | 3089.6 | 0 | 5 | 1342.45 | 1706.2 | 0 | 1481.1 | 0 | 5 |
| C205 | 842.5 | 1180.7 | 0.6 | 158.9 | 0 | 6 | 917.54 | 1294.6 | 0 | 113.2 | 0 | 6 |
| C206 | 969.28 | 1186.6 | 0 | 1386.6 | 1 | 5 | 851.51 | 1163 | 0 | 374.1 | 0 | 5 |
| C207 | 1162.82 | 1562.2 | 6.5 | 679.8 | 2 | 6 | 970.59 | 1209.6 | 0 | 1238.7 | 0 | 6 |
| C208 | 1276.72 | 1539.4 | 0.2 | 1991 | 1 | 5 | 958.23 | 1279.4 | 0 | 626.49 | 0 | 5 |

[a] RN: number of rejected customers.

[b] NV: number of dispatched vehicles.

## Appendix D. Comparison of Scenario 2 and benchmark solution

| Problem set | Benchmark solution | | | | | | Scenario 2 | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Objective value | Travel time | $\sum_i w1_i$ | $\sum_i w2_i$ | RN[a] | NV[b] | Objective Value | Travel Time | $\sum_i w1_i$ | $\sum_i w2_i$ | RN[a] | NV[b] |
| R101 | 1746.04 | 2395.8 | 15.5 | 658.8 | 11 | 25 | 1584.3 | 2163.6 | 58.3 | 581.17 | 5 | 25 |
| R102 | 1651.74 | 2244.9 | 49.2 | 704.7 | 7 | 23 | 1498 | 2052.4 | 39 | 535.22 | 2 | 22 |
| R103 | 1382.13 | 1873.2 | 57 | 594.9 | 9 | 19 | 1283.82 | 1743.8 | 104.8 | 422.03 | 5 | 19 |
| R104 | 1233.33 | 1709.7 | 3.9 | 357.6 | 6 | 15 | 1172.86 | 1646.7 | 9.4 | 182.93 | 1 | 15 |
| R105 | 1532.84 | 2113.7 | 30.2 | 472.1 | 2 | 20 | 1473.7 | 2050.5 | 18.5 | 346.5 | 2 | 20 |
| R106 | 1431.01 | 1992.6 | 12.8 | 336.3 | 8 | 19 | 1408.78 | 1951.3 | 12.1 | 404.52 | 1 | 20 |
| R107 | 1390.07 | 1919 | 34.6 | 398.5 | 9 | 17 | 1244.89 | 1727.4 | 32.7 | 291.67 | 1 | 16 |
| R108 | 1128.89 | 1586.6 | 14.2 | 154.3 | 10 | 14 | 1201.57 | 1680.2 | 0.2 | 253.95 | 0 | 15 |
| R109 | 1380.78 | 1918.1 | 2.1 | 376.9 | 12 | 17 | 1315.28 | 1849.5 | 1 | 204.33 | 1 | 19 |
| R110 | 1406.17 | 1970.2 | 14.8 | 240.7 | 10 | 17 | 1325.39 | 1858.6 | 15.9 | 211.94 | 1 | 19 |
| R111 | 1337.81 | 1867 | 17.9 | 273.3 | 6 | 17 | 1231.94 | 1725.5 | 10.9 | 219.13 | 1 | 17 |
| R112 | 1179.16 | 1662.2 | 5.4 | 145.4 | 14 | 13 | 1226.97 | 1711.2 | 0 | 291.34 | 1 | 17 |
| R201 | 1499.59 | 2038.5 | 11.7 | 703 | 3 | 6 | 1373.85 | 1847.5 | 3.7 | 798.6 | 0 | 7 |
| R202 | 1247.19 | 1610 | 3.8 | 1194.3 | 0 | 5 | 1297.82 | 1683.8 | 13.1 | 1165.37 | 0 | 7 |
| R203 | 1356.27 | 1850.5 | 0.1 | 609 | 3 | 6 | 1181.9 | 1550.7 | 15.1 | 933.86 | 0 | 7 |
| R204 | 1032.36 | 1387.6 | 37.7 | 535 | 0 | 5 | 1099.86 | 1462.3 | 35.3 | 691.92 | 0 | 5 |
| R205 | 1352.01 | 1885.6 | 1 | 318.9 | 5 | 5 | 1405.41 | 1910.8 | 0 | 678.48 | 0 | 6 |
| R206 | 1057.66 | 1477.9 | 0.9 | 229.5 | 4 | 4 | 1215.82 | 1669.8 | 38.1 | 393.43 | 0 | 8 |
| R207 | 1127.18 | 1552.3 | 0 | 405.7 | 1 | 3 | 1166.34 | 1546.9 | 38.6 | 757.94 | 0 | 6 |
| R208 | 1190.46 | 1607.3 | 2.3 | 648.9 | 3 | 5 | 1038.48 | 1415.4 | 0 | 477 | 0 | 4 |
| R209 | 1211.24 | 1640 | 0.4 | 631.6 | 4 | 5 | 1141.03 | 1539.7 | 0 | 632.42 | 0 | 5 |
| R210 | 1212.95 | 1676.2 | 1.5 | 393.1 | 5 | 4 | 1193.15 | 1604.4 | 0 | 700.7 | 0 | 5 |
| R211 | 1047.02 | 1420.1 | 3 | 523.5 | 3 | 4 | 1034.82 | 1395.6 | 0 | 579 | 0 | 5 |
| RC101 | 1658.95 | 2297.6 | 25.9 | 454.5 | 7 | 20 | 1726.73 | 2390.7 | 6.9 | 518.64 | 3 | 22 |
| RC102 | 1688.52 | 2369.6 | 0.7 | 296.6 | 12 | 19 | 1656.97 | 2313 | 0 | 378.66 | 6 | 21 |
| RC103 | 1392.51 | 1934.4 | 18.1 | 348.1 | 8 | 16 | 1298.69 | 1800.9 | 30 | 320.62 | 8 | 16 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RC104 | 1220.72 | 1703.7 | 0.2 | 280.9 | 5 | 14 | 1436.55 | 2009.1 | 0 | 301.8 | 2 | 17 |
| RC105 | 1676.73 | 2342.3 | 18.4 | 334.4 | 10 | 19 | 1734.96 | 2412.7 | 2.6 | 455.47 | 3 | 22 |
| RC106 | 1609.38 | 2255.6 | 9.9 | 284.8 | 5 | 18 | 1726.4 | 2432.1 | 7.6 | 224.13 | 4 | 20 |
| RC107 | 1522.54 | 2123.8 | 14.5 | 329.8 | 8 | 17 | 1620.64 | 2251.8 | 7.4 | 429 | 2 | 20 |
| RC108 | 1421.71 | 1986.7 | 4.4 | 301.4 | 8 | 16 | 1596.25 | 2249.2 | 0 | 218.08 | 2 | 18 |
| RC201 | 1719.44 | 2350.3 | 16.9 | 708.5 | 6 | 7 | 1873.26 | 2512.5 | 0 | 1145.05 | 1 | 7 |
| RC202 | 1520.09 | 2071.1 | 7.9 | 687.4 | 9 | 5 | 1297.82 | 1683.8 | 13.1 | 1165.37 | 0 | 7 |
| RC203 | 1397.62 | 1861.5 | 1.4 | 942.9 | 5 | 5 | 1285.18 | 1722.7 | 0 | 792.91 | 0 | 7 |
| RC204 | 1268.43 | 1706.2 | 9.2 | 722.5 | 6 | 4 | 1099.86 | 1462.3 | 35.3 | 691.92 | 0 | 5 |
| RC205 | 1647.96 | 2200.1 | 5.9 | 1067.1 | 12 | 6 | 1623.43 | 2132.2 | 10.9 | 1287.05 | 0 | 7 |
| RC206 | 1451.23 | 2024.7 | 0 | 339.4 | 8 | 5 | 1444.31 | 2014.1 | 0 | 344.43 | 0 | 5 |
| RC207 | 1288.53 | 1781 | 3.5 | 411.3 | 4 | 6 | 1426.08 | 1945.7 | 0 | 640.9 | 0 | 6 |
| RC208 | 1297.55 | 1761.5 | 109.8 | 425.4 | 2 | 5 | 1133.24 | 1489.4 | 106.5 | 693.57 | 0 | 7 |
| C101 | 1166.2 | 1652 | 0.3 | 97.4 | 2 | 12 | 1032.12 | 1413.3 | 0 | 428.1 | 1 | 12 |
| C102 | 1478.67 | 1978.1 | 11.4 | 917.2 | 1 | 13 | 1408.76 | 1928 | 0 | 591.63 | 1 | 14 |
| C103 | 1719.32 | 2224.1 | 21.3 | 1581.9 | 2 | 14 | 1298.39 | 1774.6 | 14.8 | 532.08 | 0 | 13 |
| C104 | 1348 | 1800.1 | 1.2 | 876.9 | 0 | 11 | 1241.74 | 1673.1 | 15.6 | 674.49 | 0 | 11 |
| C105 | 1285.65 | 1791.4 | 6 | 304.7 | 0 | 12 | 1139.41 | 1585 | 0 | 299.09 | 0 | 14 |
| C106 | 1217.29 | 1697.6 | 6.3 | 277.1 | 1 | 12 | 1048.35 | 1476.6 | 0 | 147.32 | 0 | 13 |
| C107 | 1391.69 | 1911 | 12.4 | 515.1 | 1 | 12 | 1312.19 | 1868.5 | 0 | 42.4 | 0 | 12 |
| C108 | 1367.48 | 1873.3 | 9.1 | 543.5 | 1 | 12 | 1095.11 | 1532 | 18.1 | 190.94 | 0 | 12 |
| C109 | 1265.14 | 1743.1 | 1.1 | 447.5 | 0 | 12 | 1070.06 | 1510.7 | 0 | 125.7 | 0 | 12 |
| C201 | 721.29 | 1005.4 | 0 | 175.1 | 0 | 6 | 584.64 | 833.9 | 0 | 9.1 | 0 | 5 |
| C202 | 1008.53 | 1190.5 | 7.5 | 1736.8 | 0 | 5 | 760.7 | 960 | 0 | 887 | 0 | 5 |
| C203 | 1402.47 | 1584 | 2.7 | 2931.3 | 0 | 6 | 912.39 | 1163.8 | 0 | 977.3 | 0 | 5 |
| C204 | 1618.7 | 1868.2 | 5.5 | 3098.6 | 2 | 5 | 1230.64 | 1492.9 | 0 | 1856.14 | 0 | 5 |
| C205 | 850.12 | 1191.4 | 0.6 | 160.2 | 0 | 6 | 822.65 | 1170.3 | 0 | 34.4 | 0 | 5 |
| C206 | 966.73 | 1183.8 | 0 | 1380.7 | 1 | 5 | 834.38 | 1130.3 | 0 | 431.7 | 0 | 6 |
| C207 | 1148.56 | 1539.9 | 9 | 688.3 | 2 | 6 | 947.43 | 1214.7 | 0 | 971.42 | 0 | 5 |
| C208 | 1262.89 | 1518.6 | 0 | 1998.7 | 1 | 5 | 905.68 | 1217.6 | 0 | 533.59 | 0 | 5 |

[a] RN: number of rejected customers.
[b] NV: number of dispatched vehicles.

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