An SVD Based Algorithm to Estimate the Parameters of Vibration

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***Abstract*—Micro-Doppler features induced by micromotions of an object or structures of an object are a unique signature which is very useful in target identification and recognition. This paper introduces an algorithm based on singular value decomposition (SVD) to estimate the parameters of vibration. Firstly, the modulation induced by vibration is derived to be sinusoidal frequency modulation (SFM). Then, the modulation frequency is determinated through singular value ratio (SVR) spectrum. Afterwards, the frequency modulation index and the initial phase are estimated through two-dimension searching. Finally, this algorithm is demonstrated to be effective through several simulations under different SNRs (signal to noise ratio). Compared to the Hough transform (HT), the SVD based algorithm performs better in terms of computational complexity, computer storage and so on.**

Keywords-micro-doppler; vibration; sinusoidal frequency modulation (SFM); singular value decomposition (SVD); singular value ratio (SVR) spectrum

I. INTRODUCTION

If a target or any structure on the target has mechanical vibration or rotation in addition to its bulk translation, it might induce a frequency modulation (FM) on the returned signal that generates sidebands about the target's Doppler frequency shift. This is called the micro-Doppler (m-D) effect [1]. Micro-Doppler features can be regarded as a unique signature of an object with micromotions, providing additional information for the target classification, recognition and identification [2]. The m-D modulations induced by vibrations generated by a vehicle engine can be detected in the radar echoes returned from the surface of the vehicle, from which we can distinguish whether it is a gas turbine engine of a tank or the diesel engine of a bus [1].

Since Chen introduced the m-D effect into the radar imaging in 2000 [3], the m-D effects have attracted great research attention. Many techniques have been developed to extract the time-varying m-D signatures. One main kind of them is based on joint time-frequency distribution [4]-[6]. The image processing algorithms have also been employed, such as the Radon transform [7] and the Hough transform [8], [9]. In [10], a time-varying auto regressive (TVAR) model is utilized to estimate the micromotion parameters.

As reported in [9], the frequency modulation of the m-D effects is often of mono- or multi-component of sinusoidal frequency modulation (SFM), linear frequency modulation (LFM) or their combination. The m-D modulation model of vibrating targets in airborne SAR systems are verified to be SFM in [11] through experiment results. An algorithm based on sinusoidal frequency modulation Fourier transform (SFMFT) is presented in [12] to generate the frequency spectrum of vibration traces with the operations acting directly on the phase term of SFM signal. References [13] and [14] report the efficiency of the Fourier–Bessel transform (FBT) based method in extracting micro-Doppler signatures. The discrete sinusoidal frequency modulation transform (DSFMT) is proposed in [15] to separate multiple component SFM signals. In [16], a method of source separation and parameter estimation for multi-component SFM signals based on generalized periodicity and SVD is proved to be effective.

This paper introduces a method to estimate the vibration parameters based on SVD. In Section II, we derive that the signal model of the m-D modulation induced by vibration is SFM. An SVD based algorithm is introduced to estimate the FM frequency, the FM index and the initial phase in Section III. This algorithm performs better in many terms compared to the Hough transform. The simulation results are demonstrated in Section IV and Section V gives the conclusion.

II. SIGNAL MODEL

The geometry and dynamic model for radar and the vibrating scatterer has been constructed in [1]. Assume that the carrier frequency of the transmitted signal is f_c and a point scatterer P is vibrating at a frequency f_v with an amplitude D_v, and the unit vector of the radar line of sight (LOS) and the unit vector of the direction of vibrating are $\boldsymbol{n}_{LOS}$ and $\boldsymbol{n}_v$, respectively. After down-converting, the received signal becomes

$$s_R(t) = \rho \exp\left[j \frac{4\pi f_v D_v}{c} \sin(2\pi f_v t + \varphi_0) \boldsymbol{n}_v \cdot \boldsymbol{n}_{LOS} \right] \quad (1)$$

where ρ is the reflectivity of the point scatterer and φ_0 is the initial phase. Thus, the micro-Doppler frequency induced by the vibration is

$$f_{mD} = \frac{4\pi f_v D_v}{c} \cos(2\pi f_v t + \varphi_0) \boldsymbol{n}_v \cdot \boldsymbol{n}_{LOS} \quad (2)$$

Consequently, the received signal from a vibrating scatterer after preprocessing is derived to be in SFM model. The parameters of vibration can be obtained from the FM frequency, the FM index and the initial phase.

III. SVD BASED ALGORITHM

A. Period of Discrete SFM Sequences

An SFM signal can be expressed as

$$\tilde{s}(t) = A\exp\left[jm_0\sin\left(2\pi f_m t + \varphi_0\right)\right] \tag{3}$$

here A is the amplitude; m_0 is the FM index; f_m is the FM frequency and φ_0 is the initial phase of FM. In practice, the signal is typically sampled and digitally processed. Supposing the interval of sampling is T_s and the time span is T, then the sampling frequency is $f_s = 1/T_s$ and the length of the sequence is $N = T/T_s$. The representation for an SFM sequence $s[n]$ is

$$s[n] = \tilde{s}(nT_s) = A\exp\left[jm_0\sin\left(2\pi f_m n/f_s + \varphi_0\right)\right] \tag{4}$$

Let $N_m = f_s/f_m$ and k denotes a positive integer. Obviously, if N_m is an integer, we have $s[n+kN_m] = s[n]$. Thus $s[n]$ is periodic whose period is N_m.

B. Determinating Period Length Using the SVR Spectrum

The SVD of an $m\times n$ matrix $\boldsymbol{A}$ is defined as

$$\boldsymbol{A} = \boldsymbol{U\Sigma V}^T \tag{5}$$

where $\boldsymbol{U} = [\boldsymbol{u}_1, \boldsymbol{u}_2, \cdots, \boldsymbol{u}_m] \in \boldsymbol{R}^{m\times m}$, $\boldsymbol{V} = [\boldsymbol{v}_1, \boldsymbol{v}_2, \cdots, \boldsymbol{v}_n] \in \boldsymbol{R}^{n\times n}$, and $\boldsymbol{U}^T\boldsymbol{U} = \boldsymbol{I}$, $\boldsymbol{V}^T\boldsymbol{V} = \boldsymbol{I}$. $\boldsymbol{\Sigma}$ is $\left[\mathrm{diag}\left(\sigma_1, \sigma_2, \cdots, \sigma_p\right); \boldsymbol{O}\right]$ or its transpose, depending on whether $m < n$ or $m \ge n$, where $p = \min(m, n)$. The singular values of $\boldsymbol{A}$ are represented by $\sigma_1 \ge \sigma_2 \ge \cdots \ge \sigma_p \ge 0$ [17].

Supposing that $\boldsymbol{x}[k]$ is a perfectly periodic sequence whose period is l, then the measured signal equals

$$\boldsymbol{Y}[k] = \boldsymbol{x}[k] + \boldsymbol{e}[k] \tag{6}$$

where $\boldsymbol{e}[k]$ is additive noise. Assume that observations from m periods are available. Then, we can build the data matrix $\boldsymbol{Y} \in \boldsymbol{R}^{m\times l}$ by partitioning the signal into periods and placing each period (in phase) as a row of $\boldsymbol{Y}$, as shown below [17]:

$$\boldsymbol{Y} = \begin{bmatrix} y[1] & y[2] & \cdots & y[l] \\ y[l+1] & y[l+2] & \cdots & y[2l] \\ \vdots & \vdots & \ddots & \vdots \\ y[(m-1)l+1] & y[(m-1)l+2] & \cdots & y[ml] \end{bmatrix} \tag{7}$$

In the absence of noise, the rank of $\boldsymbol{Y}$ will be one (i.e., $\sigma_2 = 0$). In the presence of weak noise, σ_1/σ_2 would be very large. However, a slightly erroneous selection of row length would not produce a large σ_1/σ_2 due to misalignment. Hence, the profile of variation of this ratio with row length can be used to detect periodicity in a signal. This profile is called the SVR spectrum of a signal [17].

Assuming the position of the peak in the SVR spectrum is $\hat{N}_m$, then the estimation of the modulation frequency equals

$$\hat{f}_m = f_s/\hat{N}_m \tag{8}$$

Although the f_s/f_m may not be an integer, the estimation of f_m is sufficiently accurate since f_s is generally much larger than f_m.

The SVR spectrum can be used to not only detect periodicity but also determine the period length. However, this method of period determination involves the following issues.

- Large SVR occurs when the row length is small.
- Peaks occur not only at the position of the period but also at its higher multiples [17].

The procedure for period determination can be summed up as follows:

- Discard the initial range of the SVR spectrum where it decreases monotonically.
- Choose the first peak determined by its difference in height from its surrounding values [17].

C. Estimating the FM Index and the Initial Phase

Now we have obtained the estimation of the modulation frequency $\hat{f}_m$ of the SFM signal. Structure a reference signal $\hat{s}_{m,\varphi}[n] = \exp\left[-jm\sin\left(2\pi\hat{f}_m n/f_s + \varphi\right)\right]$ and multiply $s[n]$ by $\hat{s}_{m,\varphi}[n]$. Here we have $m \in \left(0, f_s/\left(4\hat{f}_m\right)\right)$ and $\varphi \in [0, 2\pi)$. Then

$$\begin{aligned} s[n]\cdot\hat{s}_{m,\varphi}[n] = A\exp\left[jm_0\sin\left(2\pi f_m n/f_s + \varphi_0\right)\right]\cdot \\ \exp\left[-jm\sin\left(2\pi\hat{f}_m n/f_s + \varphi\right)\right] \end{aligned} \tag{9}$$

Since $\hat{f}_m \approx f_m$, if $m = m_0$ and $\varphi = \varphi_0$, we have

$$s[n]\cdot\hat{s}_{m,\varphi}[n] \approx A \tag{10}$$

Here A is a constant, whose spectrum is the impulse function. Define the following function for every m and φ:

$$P(m,\varphi) = \max\left[\left|\mathrm{FFT}\left(s[n]\cdot\hat{s}_{m,\varphi}[n]\right)\right|\right] \tag{11}$$

Then we can estimate the modulation index and the initial phase by searching for the maximum value of $P(m,\varphi)$, i.e.,

$$(\hat{m}_0, \hat{\varphi}_0) = \arg\max_{m,\varphi}\left[P(m,\varphi)\right] \tag{12}$$

D. Performance of the SVD Based Algorithm

1) Error analysis of FM frequency estimation

Supposing that the FM frequencies of two signals are f_1 and f_2 respectively and $f_2 = f_1 + \Delta f$, and the difference between the two periods is 1, then we have

$$\frac{f_s}{f_1} = \frac{f_s}{f_1 + \Delta f} + 1 \tag{13}$$

Thus

$$\frac{\Delta f}{f_1} = \frac{f_1}{f_s - f_1} \approx \frac{f_1}{f_s} \tag{14}$$

In practice, f_s is generally hundreds of times larger than f_1. That is to say, the percent error is less than 1%.

TABLE I. PERFORMANCE COMPARISON OF THE SVD BASED ALGORITHM AND THE HOUGH TRANSFORM

Performance parameters	**Algorithms**	
	SVD	***HT***
high accuracy	yes	yes
computational complexity	low	high
computer storage	small	large
fuzziness of results	no	yes
dependence on high resolution time-frequency distribution	no	yes

2) *The analysis of computational complexity*

Suppose that the ranges of the FM period, the FM index and the initial phase are $\left[N_{m_{\min}}, N_{m_{\max}}\right]$, $\left[m_{\min}, m_{\max}\right]$ and $\left[\varphi_{\min}, \varphi_{\max}\right]$, and the numbers of discrete points of each range are K_{N_m}, K_m and K_φ. The SVD based algorithm introduced above consists of K_{N_m} times of SVD and $K_m \cdot K_\varphi$ times of FFT.

In comparison, when using the Hough transform, supposing that the number of $\left(f_{mD} - n\right)$ pairs [8] is K_{HT}, then the number of HT operations is $K_{HT} \cdot K_m \cdot K_\varphi$. Generally K_{HT} is several times larger than K_{N_m}, meaning that the computational complexity of HT is much larger.

3) *Performance comparison against HT*

The performance comparison of the SVD based algorithm and the Hough transform is shown in TABLE I. Consequently, the SVD based algorithm has a better performance than the Hough transform in many terms in extracting the m-D features.

IV. SIMULATION RESULTS AND ANALYSIS

A. *Micro-Doppler Effect Induced by Vibration*

Assume that the radar operates at 10 GHz and transmits a pulse waveform whose PRF equals $2000Hz$. Given $D_v = 0.01m$, $f_v = 5Hz$, the azimuth and elevation angle of the vibration direction in the reference coordinates $\alpha_p = 30^\circ$ and $\beta_p = 30^\circ$, and the center of the vibration at $\left(U = 1000m, V = 5000m, W = 5000m\right)$, as shown in Fig. 1, the simulation result of m-D effect induced by vibration is identical to the theoretical result.

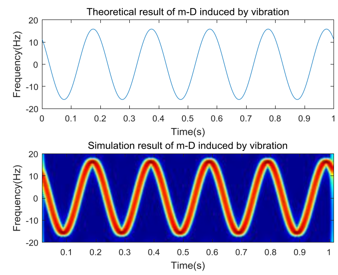
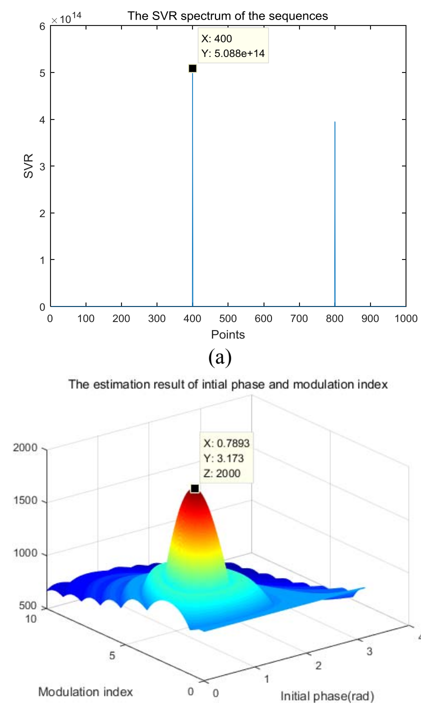


Figure 1. The theoretical and simulation results of m-D effect induced by vibration.

B. *Parameters Estimation under Different SNRs*

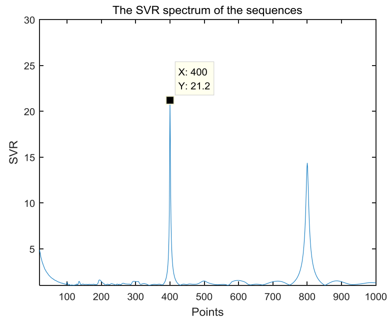
To verify the effectiveness of the algorithm introduced in Section III, we implement several simulations under different SNRs. Assuming that the initial phase $\varphi_0 = \pi/4 \approx 0.7854$ and the other parameters of vibration is the same as that in the above section. Then the FM index equals

$$m_0 = \frac{4\pi f_v D_v}{c} \boldsymbol{n}_v \cdot \boldsymbol{n}_{LOS} \approx 3.1762 \tag{15}$$

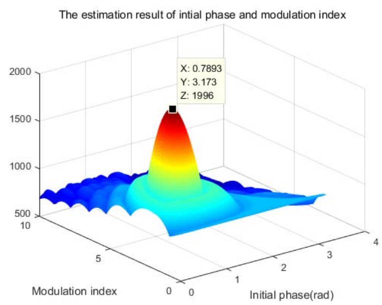


(b)

Figure 2. Estimation results of (a) FM frequency; (b) FM index and initial phase (no noise).

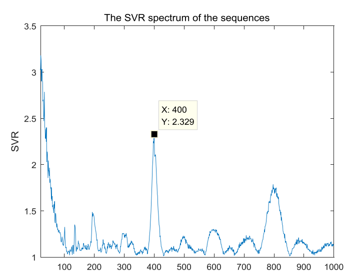


(a)

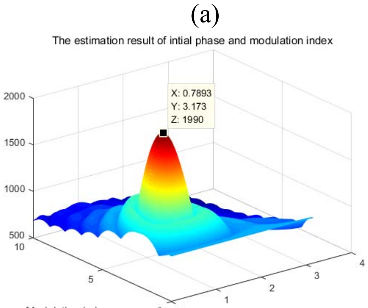


(b)

Figure 3. Estimation results of (a) FM frequency; FM index and initial phase ($SNR = 20dB$).



(a)



(b)

Figure 4. Estimation results of (a) FM frequency; (b) FM index and initial phase ($SNR = 0dB$).

The simulation results shown in Fig. 2, Fig. 3 and Fig. 4 correspond to the estimation results and the echoes whose SNRs are 20 dB and 0dB, respectively. The results indicate that the SVD based algorithm can determine the FM frequency and estimate the FM index and the initial phase accurately even that the SNR equals 0 dB.

V. CONCLUSION

The m-D effect has been verified to be very useful for radar target classification, identification and recognition. This paper introduces a SVD based method to extract the m-D features of vibrating targets, which performs better in many terms compared to the Hough transform.

The dynamic model of vibration in this paper is simplified. In reality, most of the micromotions are complex, which should be modeled as a composer of multiple frequency components. Our future work will focus on the separation and parameters estimation of multiple component micro-Doppler modulation.

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