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Tensor completion-based trajectory imputation approach in air traffic control

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The flight trajectory in air traffic control systems usually misses some updating positions because of unexpected errors. In this paper, a tensor completion-based approach is proposed to recover missing positions from a whole trajectory dataset. Considering the trajectory dependencies among different operations, the flight trajectories with the same flight number are organized as a three-dimensional tensor. A trace norm minimizing based tensor completion method is performed on the trajectory tensor to achieve the imputation task, in which the Block Coordinate Descent algorithm is applied to optimize the tensor model. Unlike other data-driven algorithms, the proposed approach captures the global information (route similarity and transition patterns) from the whole tensor, which is further applied to estimate the missing values in a training-free manner. Several experiments are designed to validate the proposed approach, including the padding methods, the dataset size, and the imputation performance on different missing patterns and rates. Experimental results on real-world flight trajectories show that the proposed approach can (1) estimate missing positions with high accuracy even on a small dataset, (2) recover missing positions even if the random missing rate up to 90%, (3) overcome the situation of the flight chain missing and block missing, which are the barriers of existing methods. The proposed approach serves as a post-processing procedure of air traffic data and can further provide high-quality data to other air traffic studies.

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1. Introduction

The flight trajectory, a time series with motion states of an aircraft, is essential data in air traffic control systems (ATCSs). The discrete positions of the flight trajectory are collected by surveillance infrastructures, such as radar or automatic dependent surveillance-broadcast (ADS-B). Based on the planning theory, different devices are installed at different locations, which are further formulated as a monitoring network to track flying targets in the air continuously. Meanwhile, the on-board ADS-B device periodically sends the motion information of the aircraft to ground systems in a cooperative manner. Thus, numerous flight positions are generated by different devices with high updating period (typically 1-4 seconds), which are further converted into flight trajectory by the processing of detection, filtering, tracking, and fusion, *etc.* [1,2].

The flight trajectory provides data to drive other downstream applications in ATCSs, such as flight trajectory prediction [3–6], air traffic flow prediction [7], conflict detection [8], air traffic management [9], etc. Especially after the emerging of machine learning methods in air traffic related studies, the flight trajectories, which are used to train machine learning models, become more and more important [10]. However, in practice, due to system errors or other malfunctions, the flight positions collected by the surveillance equipment are usually missing or invalid at some various or continuous moments. Even though the popularization of ADS-B devices greatly reduces the difficulty of target monitoring in air traffic, it still fails to be applied in many developing countries due to economic or security issues. Furthermore, the communication reliability of the ADS-B device is always a major obstacle to its applications [11]. The flight trajectory with missing positions not only reduces the accuracy and robustness of monitoring the real-time traffic situation [12], but also prevents further data mining and processing in air traffic research [13].

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Nomenclature

TrajDB	Trajectory database for all flights
F_i	Trajectory dataset for a certain flight number
f_j	Flight trajectory for a certain flight j, from its origin to destination airport
p_t	Trajectory position
$\boldsymbol{\chi}$	Tensor
$\lVert\boldsymbol{\chi}\rVert$	Norm of a tensor
$\lVert\boldsymbol{\chi}\rVert_F$	Frobenius norm of a tensor
$(\boldsymbol{\chi} \times_n \mathbf{A})$	Multiplication of a tensor $\boldsymbol{\chi}$ with a matrix $\mathbf{A}$ at n-mode
Ω	Index set indicating the known data in a tensor
M_n	Unfolded matrix from a tensor at n-mode
$\sum$	Singular value matrix
U, V	Left and right eigenvector of SVD
K	Maximum iterations of the optimization
$d(p_\mathrm{t}, p_\mathrm{e})$	Euclidean distance between two flight positions
μ_d	Mean of estimation errors
L_M	Number of missing positions in a tensor

With this focus, a common solution, i.e., data augmentation, has been used to enhance the data size, which aims at improving the final performance to a certain extent. However, it also fails to provide high diversity to achieve the real-time safety-critical real-world operations with considerably high confidence. Therefore, trajectory imputation, as a primary task of post data cleaning, has attracted more and more attention in air traffic-related fields all over the world. In addition, it is also very helpful to support the post-accident analysis by estimating the missing positions to locate the accident [14,15]. In general, the goal of trajectory imputation is to estimate missing positions of a flight trajectory based on the flight plan and its historical paths, i.e., trajectories with the same flight number on earlier days [16,17]. The core problem of imputation lies in how to build up the relationship between the known data and unknown ones. An intuitive and effective assumption is that the missing data mainly depends on its neighbor known positions, which is also known as the interpolation-based approach [18]. Basically, the longer the distance between two flight positions is, the smaller the correlation between them becomes. In practice, missing flight positions are usually continuous since the recovery of monitoring equipment in ATCSs is really difficult, which brings unsolvable problems to existing methods like the interpolation.

To this end, many advanced techniques were proposed and improved to estimate the missing positions of the trajectory data. In general, learning from other air traffic studies [19,20], the data-driven mechanism is indispensable to deal with the pattern diversity of the imputation task, in which the missing data are estimated by capturing the patterns from other known observations. In this paper, a novel approach based on tensor completion is proposed to estimate missing positions of the flight trajectory. We mainly focus on recovering the positional components of the flight trajectory, i.e., the longitude, latitude, and altitude. The trajectories of flights with a same flight number on different days share similar transition patterns, and the positions are correlated to their neighbors both temporally and spatially. Thus, the flight trajectories are constructed into a three-dimensional (3-D) tensor which represents different flight executions, position sequence of a single trajectory, and attributes of each position, respectively. A tensor model is able to fully capture and illustrate inherent structures of multi-mode and multi-pattern data [21,22], which is consistent with the global patterns of the flight trajectory. The length of the trajectory for different flight operations may be varied depending on their flight time. Thus, the linear interpolation method is applied to fix the updating interval (temporal resolution) along the flight trajectory dimension. To achieve the imputation task, the Block Coordinate Descent (BCD) algorithm is employed to optimize the target function and obtain desired estimation performance [23]. The "rank" is a powerful tool to capture some type of global information among different dimensions, and further be applied to estimate the missing values, i.e., imputation. Experimental results on real flight trajectories with different missing patterns and rates demonstrate that the trajectory imputation in the ATCSs is achieved with considerably high performance. Compared to other machine learning approaches, the proposed tensor completion-based approach achieves the imputation task in a training-free manner. The proposed approach takes all the flight trajectories as input to extract the global information, not only captures the low-level patterns (route similarity), but also the high-level transition patterns (with complicated transformation), such as velocity or altitude change. All in all, our original contributions of this paper are summarized as follows.

a) To the best of our knowledge, this is the first work that proposes a tensor completion-based approach to solve the trajectory imputation issue in air traffic research, which can provide high-quality data to support other downstream data-driven based models or methods.

b) Unlike other machine learning approaches, the proposed approach is able to achieve a comparable performance even facing a small dataset, i.e., less than 1000 meters for 12 trajectories and 60% random missing rate.

c) The proposed approach is capable of recovering the missing flight positions with high confidence even if the random missing rate up to 90%. The results show that a mean estimation error of a few hundred meters can be achieved.

d) Facing severe data missing, such as block missing, the proposed approach is able to overcome the data limitation and obtain the desired performance. As to the flight chain missing, the missing flight positions can also be reconstructed with lower error in the climb and descent phase of the flight.

The rest of this paper is organized as follows. The related works are reviewed in Section 2. The proposed approach is detailed in Section 3. The experimental design and configurations are listed in Section 4. Experimental results are reported and discussed in Section 5. The conclusion is provided in Section 6.

2. Related works

The flight trajectory plays an important role in air traffic control-related studies, such as real-time trajectory processing and 4D trajectory prediction. The kinematics dynamic models were studied and improved to predict the flight trajectory with different environmental factors such as wind and traffic operation [24–28]. Hidden Markov Models were applied to build the flight transition of motion states and further predict the flight trajectory [4]. For the probabilistic and machine learning models, related works studied hybrid models to estimate the flight trajectory in the local area based on historical operating data [29–32].

As to the trajectory imputation, a common analysis tool for vertical flight efficiency was proposed to reveal the importance of this task [33]. Park et al. focused on the performance of an adaptive trajectory prediction algorithm for climbing flights [34]. Mondoloni reviewed the application of the key performance indicators for the

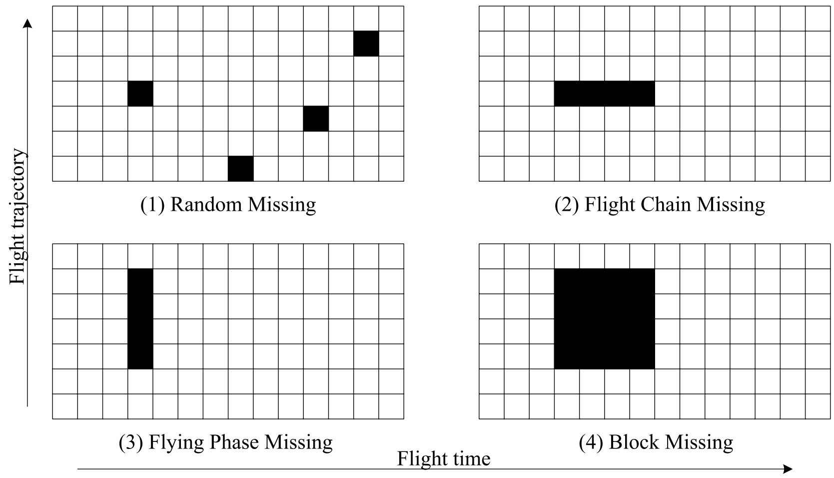

Fig. 1. Missing patterns of flight trajectory.

trajectory estimation in air traffic control [35]. The MITRE Corporation also proposed interpolation-based methods to recover missing positions of the flight trajectory and built a flight path library in the terminal area to further support air traffic-related studies [36]. However, existing interpolation-based methods can only deal with the random missing pattern, the performance and robustness of them for dealing with severe data missing are really poor [37]. In addition, the performance of interpolation-based methods is not stable and limited by the source data, which is usually better in the cruise phase compared to that of the climb or descent phase. Therefore, it is urgent to develop a new approach to solve the trajectory imputation issue for the research of air traffic control.

In this study, a tensor completion-based approach is proposed to estimate missing flight trajectory positions, which has been applied to achieve the imputation task in other fields. Silva et al. [38] conducted a simulation study on multiple imputation methods for handling missing values in longitudinal data in the presence of a time-varying covariate with a non-linear association. The tensor-based methods were originated with Hitchcock in 1927 [39], and over the past decade, the research interests on tensor have been expanded to other fields, such as signal processing [40], numerical linear algebra [41], computer vision [23], numerical analysis [42], data mining [43], graph analysis [44], and neuroscience [45]. Many researchers also applied this method or its variations to solve existing problems in the ground transportation study, which inspires us to reconsider the flight trajectory imputation task in air traffic control systems. Tan et al. applied a dynamic tensor completion approach to estimate the short-term travel time of the freeway [21]. A similar tensor-based approach was proposed [46] to reconstruct data collections of loop detectors in the ground transportation system. An imputation method was proposed in [47] to estimate the traffic speed based on the tensor completion and showed desired performance superiority compared to other methods. A tensor-based method was proposed in [48] to estimate traffic volume in ground transportation systems. A method [49] using the tensor completion method was proposed to achieve better coverage of traffic state estimation from sparse floating car data and further solve some special issues in the field of transportation research. Sun et al. proposed a probabilistic tensor factorization framework to understand urban mobility patterns [50]. To our limited knowledge, there is no research that applies tensor completion-based approaches to solve the imputation task for air traffic issues currently.

3. Methodology

3.1. Preliminary

In this study, the trajectory dataset is organized as a top-down architecture, as shown in Eq. (1). There are N sets of flights with the same flight number in our database *TrajDB*, i.e., N unique flight numbers. F_i is a sub-set of the trajectory database for the i-th flight number, which was carried out M_i times in the history. f_j saves all flight positions with respect to the flight time for the j-th flight execution, i.e., a flight trajectory from its origin to destination airport. The f_j flight trajectory consists of T_j collected flight positions. Each collected flight position is characterized by the following attributions: collecting time t, unique flight number id, the three-dimensional position x, y, z and their velocity, i.e., vx, vy, vz.

$$\begin{aligned}
&\mathrm{Traj}DB = \{F_1, F_2, \cdots, F_N\} \\
&F_i = \left\{f_1, f_2, \cdots, f_{M_i}\right\}, \ \forall i \in [1, N] \\
&f_j = \left\langle p_1, p_2, \cdots, p_{T_j}\right\rangle, \ \forall j \in [1, M_j] \\
&p_t = [t, id, x, y, z, vx, vy, vz], \ \forall t \in [1, T_\bullet]
\end{aligned} \tag{1}$$

Based on the trajectory imputation task in the ATCSs, the missing patterns of a flight trajectory can be defined as four types, as shown in Fig. 1. Each row in those figures denotes a flight trajectory, and the squares are flight positions. The white squares save observed positions in the trajectory database, while black ones represent unobserved ones that need to be estimated. In general, the pattern 2) and 4) are the most common and complicated ones, while the pattern 1) and 3) are similar and can be made up by existing approaches.

1) Random missing: random missing at different moments of different trajectories due to random system errors.

2) Flight chain missing: continuous missing moments of a single flight trajectory, which may happen at the local area due to equipment errors, blind spots, or communication errors someday.

3) Flying phase missing: a single missing moment at the same flight phase of different trajectories, which may be caused by blind spots at continuous flight executions.

4) Block missing: continuous phase missing of different trajectories, which is the combination of 2) and 3).

Except for some irregular conditions (flight returning and alternating), the aircraft of the same flight number on different days usually travels along the same waypoint sequence and the transition patterns of these flight trajectories are similar [4]. More

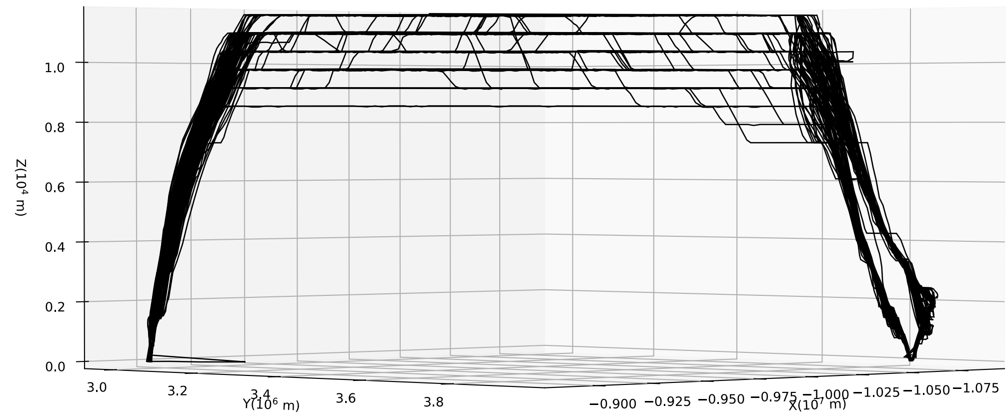

Fig. 2. Historical trajectories of a certain flight number.

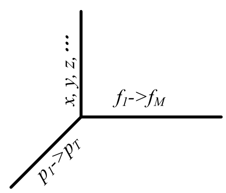

Fig. 3. Tensor architecture in this work.

specifically, the flight trajectory of each execution for the same flight number has similar transition patterns, such as velocity or altitude change. The historical trajectories of a given flight number are shown in Fig. 2. As can be seen from the figure, the flight trajectories are similar to each other among different executions, which demonstrates that missing positions of a flight trajectory can be estimated not only by the trajectory of the current day, but also by the transition patterns of the flight trajectory of other historical operations.

3.2. Tensor algebra

A tensor is a multi-dimensional array, which is an element of the product of multiple vector spaces. In this paper, a third-order tensor with size $L_i \times L_j \times L_k$ is applied to characterize a three-mode flight trajectory with a same flight identity on different days, just as described in Fig. 3. The tensor is denoted by a bold and italic notation, such as $\boldsymbol{\chi}$, wherein the elements are represented by x_{ijk}, $\forall i \in [1, L_i]$, $j \in [1, L_j]$, $k \in [1, L_k]$. The matrices in this paper are denoted by bold capital notations, such as A, B and **C**. The norm of a tensor $\boldsymbol{\chi} \in R^{L_i \times L_j \times L_k}$ is defined in Eq. (2), while the Frobenius norm of a tensor [39] is defined as Eq. (3).

$$\|\boldsymbol{\chi}\| = \sqrt{\sum_{k=1}^{L_k}\sum_{j=1}^{L_j}\sum_{i=1}^{L_i} x_{ijk}^2} \tag{2}$$

$$\|\boldsymbol{\chi}\|_F = \sqrt{\sum_{k=1}^{L_k}\sum_{j=1}^{L_j}\sum_{i=1}^{L_i} |x_{ijk}|^2} \tag{3}$$

There are some common operations that are applied to implement a tensor completion method. Firstly, the unfolding operation reorders the elements of an N-way array into a matrix at a certain dimension. For instance, a 3-D $L_i \times L_j \times L_k$ tensor can be unfolded to a 2-D tensor (matrix) whose size may be $(L_i \times L_j) \times L_k$, $(L_i \times L_k) \times L_j$, or $L_i \times (L_j \times L_k)$ based on the unfolded dimension. Generally, in the mode-n matricization of a tensor $\boldsymbol{\chi} \in R^{I_1 \times I_2 \times \cdots \times I_N}$, the element with indices $(i_1, i_2, \cdots, i_N)$ in the original tensor will be arranged to the element with indices $(i_n, i_x), n \in [1, N]$ in the unfolding matrix, where i_x is represented as Eq. (4). The folding operation is defined as the inverse of the unfolding operation.

$$i_x = \left[\sum_{l=1, l\neq n}^{N} (i_l - 1) J_l\right] + 1, \quad J_l = \prod_{m=1, m\neq n}^{l-1} I_m \tag{4}$$

Another common operation for tensor completion is the n-mode multiplication, i.e., multiplying a tensor by a matrix (or a vector) at the n$^{\text{th}}$ mode [51]. For any tensor $\boldsymbol{\chi} \in R^{I_1 \times I_2 \times \cdots \times I_N}$, the n-mode multiplication with a matrix $\mathbf{A} \in R^{J \times I_n}$ is denoted by $\boldsymbol{\chi} \times_n \mathbf{A}$ whose size is $I_1 \times \cdots \times I_{n-1} \times J \times I_{n+1} \times \cdots \times I_N$. The elementwise rule for the n-mode multiplication can be formulated as Eq. (5), which is also applicable when **A** is a vector ($J = 1$). The n-mode multiplication also has the property of Eq. (6), i.e., the commutative law of the common multiplication.

$$(\boldsymbol{\chi} \times_n \mathbf{A})_{i_1 i_2 \cdots i_{n-1} j i_{n+1} \cdots i_N} = \sum_{i_n=1}^{I_N} x_{i_1 i_2 \cdots i_N} a_{j i_n}, a_{j i_n} \in \mathbf{A} \tag{5}$$

$$\boldsymbol{\chi} \times_n \mathrm{A} \times_m B = \begin{cases} \boldsymbol{\chi} \times_m B \times_n \mathrm{A}\ (m \neq n) \\ \boldsymbol{\chi} \times_n (\mathrm{BA})\ (m = n) \end{cases} \tag{6}$$

3.3. Proposed tensor completion model

The flight trajectory is a kind of high-dimensional data inherently, whose multi-mode and multi-pattern characteristics are the key to achieve the imputation task. In this paper, a tensor model is proposed to fully capture the multi-mode and multi-pattern features of the high-dimensional trajectory data. By constructing the trajectories with the same flight number on different days into a tensor, the flight positions are represented as tensor elements. Thereby, the issue of recovering missing positions for the flight trajectory can be addressed by estimating the missing tensor element, i.e., tensor completion. To this end, a trace norm minimizing based tensor completion method is proposed in this study, whose goal is to minimize the rank of the studied tensor, as shown in Eq. (7):

$$\min_{\boldsymbol{\chi}} : \ rank(\boldsymbol{\chi}), \ \text{s.t.} : \boldsymbol{\chi}_\Omega = \boldsymbol{\tau}_\Omega \tag{7}$$

where, $\boldsymbol{\chi}$ and $\boldsymbol{\tau}$ have the same size. The elements of $\boldsymbol{\tau}$ in the index set Ω are observed while the remaining elements are missing and need to be completed. The missing values are estimated such that the rank of the tensor is as low as possible. A low-rank tensor

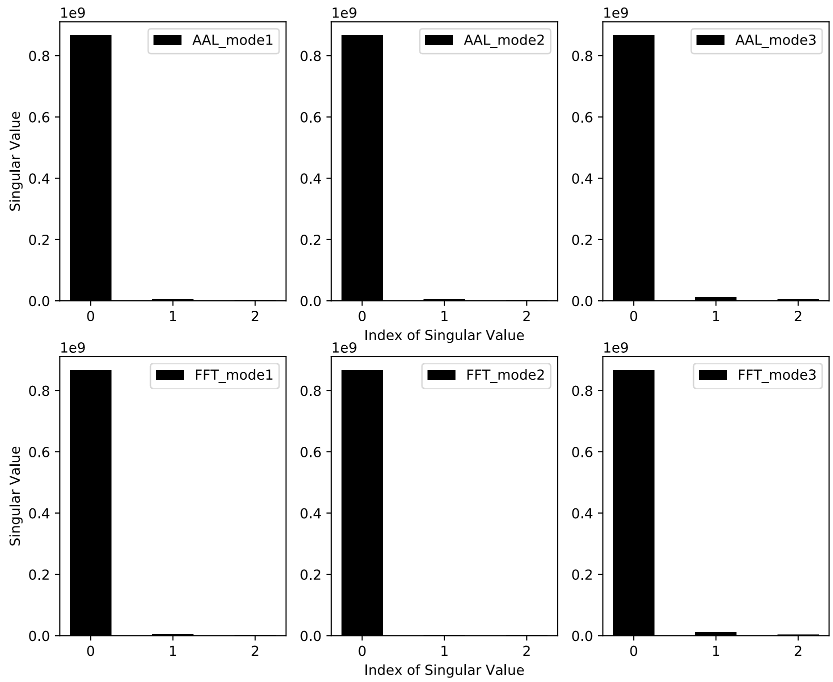

Fig. 4. Results of SVD analysis.

indicates that elements among different modes have high similarity on transition patterns, and the whole data can be represented by the linear combination of related rows. Specifically, historical trajectories and neighboring positions of current flight can be used to estimate missing flight positions in this paper. In this study, the singular value decomposition (SVD) algorithm is applied to validate the low-rank property of the trajectory tensor, in which the tensor is unfolded into different modes. As demonstrated for the flight number AAL2720 and FFT1310 in Fig. 4, the first singular value is much larger than other ones for both flights and their tensor modes, which means that the rank of trajectory tensor is very low and further supports the motivation of applying the tensor completion-based approach to achieve the trajectory imputation task.

Since the rank of tensor in Eq. (7) is nonconvex [23], the key of the tensor completion is to create an alternative optimization function to approximate the tensor rank in the original optimization. A common solution is the trace norm ($\|\boldsymbol{\chi}\|_*$), the tightest convex envelop of the tensor rank, as shown below:

$$\|\boldsymbol{\chi}\|_* = \sum_{i=1}^{n} \alpha_i \left\|\boldsymbol{\chi}_{(i)}\right\|_* \tag{8}$$

$$\left\|\boldsymbol{\chi}_{(i)}\right\|_* = \sum_{r=1}^{R} \delta_r u_r \circ v_r \tag{9}$$

wherein the factor parameters must meet the following requirements: $\alpha_i \geq 0$ and $\sum_{i=1}^{n} \alpha_i = 1$. $\boldsymbol{\chi}_{(i)}$ is the unfolded matrix along i^{th}-mode and the trace norm of an unfolded matrix is decomposed as $\boldsymbol{\chi}_{(i)} = U\Sigma V^T$ by SVD. R is the rank of the unfolded matrix. δ_r is the singular value in the Σ and meet the requirement of $\delta_1 \geq \delta_2 \geq \cdots \geq \delta_R$. u_r and v_r are the vectors in U and V, respectively. Operator $\circ$ is the outer product of two matrices. To eliminate the dependencies among matrices on unfolded modes, M_i is applied to replace $\boldsymbol{\chi}_{(i)}$ and the optimization in Eq. (7) will be refined as Eq. (10). n is the dimension of the tensor, i.e., 3 in this work. Moreover, an alternative solution was also proposed to improve the performance of this issue, as shown in Eq. (11) [52], where thr_i is a threshold value that can be assigned manually. However, since manually assigned parameters are usually not the optimal ones, the optimization function is converted into an equivalent problem as Eq. (12), which is the final version of our optimization target. β_i is a certain positive value (penalty factor) to measure the dependencies between the optimization target and each unfolded matrix.

$$\min_{\boldsymbol{\chi}, \mathrm{M}_i} : \sum_{i=1}^{n} \alpha_i \|\mathrm{M}_i\|_*, \text{ s.t.}: \boldsymbol{\chi}_{(i)} = M_i, \forall i \in [1, n], \boldsymbol{\chi}_\Omega = \boldsymbol{\tau}_\Omega \tag{10}$$

$$\min_{\boldsymbol{\chi}, \mathrm{M}_i} : \sum_{i=1}^{n} \alpha_i \|\mathrm{M}_i\|_*, \text{ s.t.}: \left\|\boldsymbol{\chi}_{(i)} - M_i\right\|_F^2 \leq thr_i, \quad \forall i \in [1, n], \boldsymbol{\chi}_\Omega = \boldsymbol{\tau}_\Omega \tag{11}$$

$$\min_{\boldsymbol{\chi}, \mathrm{M}_i} : \sum_{i=1}^{n} \alpha_i \|\mathrm{M}_i\|_* + \frac{\beta_i}{2} \left\|\boldsymbol{\chi}_{(i)} - M_i\right\|_F^2, \text{ s.t.}: \boldsymbol{\chi}_\Omega = \boldsymbol{\tau}_\Omega \tag{12}$$

3.4. Optimization solution

Upon the optimization target is determined, the tensor completion can be implemented by employing the Block Coordinate Descent (BCD) algorithm [53]. The core idea of the BCD algorithm is to optimize one block of unknown variables while fixing other ones. From the optimization function, the whole tensor can be divided into $n+1$ blocks, including $\boldsymbol{\chi}$ and $\mathrm{M}_1, M_2, \cdots, M_n$. During the optimization, we first compute $\boldsymbol{\chi}$ with all fixed matrices M_i by solving the sub-problem of Eq. (13), whose solution is given by Eq. (14). $\Gamma_i(\bullet)$ is the folding operation of a matrix. After the block $\boldsymbol{\chi}$ is optimized, the M_i can be obtained by the sub-problem of Eq. (15), and solution details can be found in [54,55], as shown in Eqs. (16) and (17):

$$\min_{\boldsymbol{\chi}} : \sum_{i=1}^{n} \frac{\beta_i}{2} \left\|\boldsymbol{\chi}_{(i)} - M_i\right\|_F^2, \text{ s.t.}: \boldsymbol{\chi}_\Omega = \boldsymbol{\tau}_\Omega \tag{13}$$

Table 1
Working steps of the proposed approach.

```
Algorithm 1: the proposed tensor completion approach
Input: χ with missing values, Ω, β_i, K
Output: χ with imputation values
for k in 1 to K; do
  for i in 1 to n; do
    update M_i = D_{α_i/β_i}(χ_(i)) by (15)
  end for n
  update χ by (14)
end for K
```

$$\boldsymbol{\chi}_{i_1 i_2 \cdots i_n} = \begin{cases} \left(\frac{\sum_i \beta_i \Gamma_i(M_i)}{\sum_i \beta_i}\right)_{i_1 i_2 \cdots i_n} & (i_1 i_2 \cdots i_n \notin \Omega) \\ \boldsymbol{\tau}_{i_1 i_2 \cdots i_n} \quad (i_1 i_2 \cdots i_n \in \Omega) \end{cases} \tag{14}$$

$$\min_{\mathrm{M}_i} : \sum_{i=1}^{n} \frac{1}{2} \left\| \boldsymbol{\chi}_{(i)} - M_i \right\|_F^2 + \frac{\alpha_i}{\beta_i} \left\| \mathrm{M}_i \right\|_* \tag{15}$$

$$M_i = \boldsymbol{D}_{\frac{\alpha_i}{\beta_i}}(\boldsymbol{\chi}_{(i)}) = \mathrm{U}\boldsymbol{D}_{\frac{\alpha_i}{\beta_i}}(\Sigma) V^T \tag{16}$$

$$\boldsymbol{D}_{\frac{\alpha_i}{\beta_i}}(\Sigma) = \mathrm{diag}\left\{ \max\left(0, \delta_r - \frac{\alpha_i}{\beta_i}\right)\right\} \tag{17}$$

where Eq. (16) is the SVD formulation of a matrix and δ_r in Eq. (17) denotes the singular values in Σ.

In summary, the proposed approach can be listed as the pseudo-code in Table 1. In this algorithm, the maximum optimization iterations K is set manually, and an early-stopping strategy is also applied, i.e., the estimation procedure will be terminated when the loss difference between two consecutive iterations is less than a given threshold (10^{-5} in this work).

4. Experiments

4.1. Data description

All flight trajectories used in this work are real-world data and downloaded from an open platform.[1] A total of 10 flight numbers are applied to evaluate the performance of the proposed approach. The departure and destination airport are ORD and MIA, respectively. The updating interval (temporal resolution) of the flight trajectory is about 30 seconds. The flight trajectories are collected from 11/01/2017 to 03/31/2018, and the specific period of each flight may be varied. Apart from canceled flights and low-quality data, 120 historical trajectories are extracted for each flight, which serve as the sample data for evaluating the performance of the proposed approach. All flight positions for data samples are available in the raw dataset. To evaluate the imputation performance of the proposed approach, some flight positions are randomly masked according to different missing patterns and rates. All flight positions quantified by latitude and longitude are converted into a same projected coordinate system to keep a unified unit with altitude (i.e., meter) and further be applied to measure the estimation error.

Since the length of each flight trajectory is different from each other, two approaches are applied to fix the data updating interval, including the linear interpolation or end padding. Linear interpolation is applied to insert values between observed values to meet the dimension requirements. Let $L_{\max}^{\mathrm{F}}$ be the length of the longest time series (flight trajectory) for flight F, the purpose is to interpolate trajectory positions to other trajectories of this flight number, extend their length to $L_{\max}^{\mathrm{F}}$ and keep an identical updating interval. End padding means that the last known observation is simply appended to the end of the flight position sequence until the tensor dimension meets the requirements.

4.2. Experimental configurations

In this work, all codes for the data pre-processing are programmed with Python and the proposed tensor completion algorithm is implemented with MATLAB. The main hardware of our experiments is configured as: Intel Core™ i7-7700HQ CPU @2.80GHZ, 8.00 GB memory with Windows10 operating system. Due to the trajectory attribution in this study, the Euclidean distance in the 3D earth space between the ground truth and reconstructed value for a given missing position is applied to evaluate the estimation error, and the mean estimation errors for all positions are regarded as the final measurements for a certain flight. In Eq. (18), $d(p_{\mathrm{t}}, p_{\mathrm{e}})$ denotes the Euclidean distance between two flight positions, where the subscript t and e represent the ground truth and recovered data, respectively. $x_{\bullet}$, $y_{\bullet}$ and $z_{\bullet}$ are the rectangular coordinates of the corresponding flight position. To evaluate the general performance, the mean and standard deviation (STD) of the estimation error are considered for all the trajectories, as shown in Eqs. (19) and (20). L_M saves the total number of missing positions in research tensors.

$$d(p_{\mathrm{t}}, p_{\mathrm{e}}) = \sqrt{(x_{\mathrm{t}} - x_{\mathrm{e}})^2 + (y_{\mathrm{t}} - y_{\mathrm{e}})^2 + (z_{\mathrm{t}} - z_{\mathrm{e}})^2} \tag{18}$$

$$\mu_d = \frac{1}{L_M} \sum_{i=1}^{L_M} d(p_{\mathrm{t}}^i, p_{\mathrm{e}}^i) \tag{19}$$

$$\delta_d = \sqrt{\frac{1}{L_M} \sum_{i=1}^{L_M} \left(d(p_{\mathrm{t}}^i, p_{\mathrm{e}}^i) - \mu_d\right)^2} \tag{20}$$

In addition to the specified parameters, there are some more hyper-parameters that need to be initialized in our experiments. The maximum optimization iteration K is set to 2000. According to data patterns, the imputation performance is evaluated with different hyperparameters α and β, and the best experimental result is regarded as the final measurement. The number of the running iteration and time cost for each flight number depends on the size of the data sample and the number of missing positions. The computing cost reaches the maximum when the full dataset (120 in this paper) with 90% missing rate is applied to evaluate the performance. By recording the experimental log, it can be seen that experiments with the full dataset and 90% missing rate are terminated after running about 600 iterations and the time cost is about 20 seconds under given hardware configurations.

4.3. Experiment design

According to the above-mentioned descriptions and the task of trajectory imputation, the following experimental procedures are designed to validate the proposed approach.

(a) Approach evaluation

a) Sequence padding: the linear interpolation and end padding are used to fix the data updating interval, respectively. In these experiments, the missing pattern is selected to random missing, while the missing rate are 60% and 90%, respectively.

b) Size of the dataset: After the padding method is determined, several experiments are conducted to check the imputation performance with different size of the dataset, i.e., 10%, 40%, 70% and 100% of the 120 flight trajectories. In these experiments, we also use the random missing pattern to test the proposed approach, whose missing rate is set to 60%.

(b) Performance evaluation

[1] https://zh.flightaware.com/.

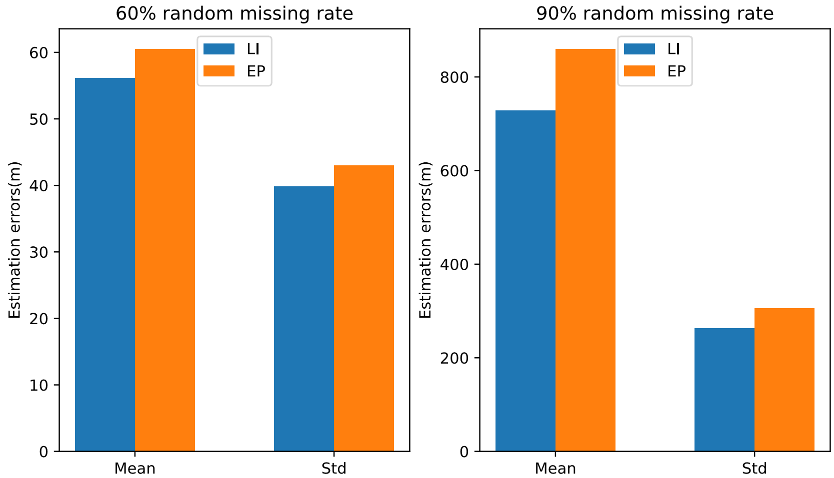

Fig. 5. Results of different padding methods.

As the core task in this study, several experiments are designed to evaluate the influence of the missing rate and missing pattern on the final estimation accuracy. Since the flying phase missing is very similar to the random missing, no any experiment is conducted for this missing pattern. Thus, the missing patterns in this section consist of the random missing, flight chain missing, and block missing. In these experiments, all the 120 flight trajectories are applied to optimize the proposed approach. To eliminate the randomness of masking data manually, all the experiments are conducted 10 times and the average estimation errors are regarded as the final measurements. The experimental configurations are summarized below.

a) Random missing: Missing positions are a sub-set of the whole flight trajectories. The missing rate indicates how many flight positions are missed in the whole set of flight trajectories. In this experiment, the imputation performance is reported when the missing rate ranges from 10% to 90% with a 10% step.

b) Flight chain missing: Missing positions are a sub-sequence of a certain flight trajectory. The missing rate indicates how many flight positions are missed for the given flight trajectory. In this experiment, the missing rate ranges from 10% to 50% with a 10% step. As to existing methods, the estimation errors of missing positions in the climb and descent phase are larger than that of in the cruise phase. To prove the performance of the proposed approach when missing positions distribute in different flight phases, the start index of missing sub-sequence is randomly selected from both the climb (first 100 positions) and cruise phases (index from 100 to 200).

c) Block missing: Missing positions are trajectory sub-sequences of some continuous flight operations. The missing rate indicates how many flight trajectories lose a trajectory sub-sequence. In this experiment, the length of missing sub-sequences for each flight number is set to 40% of its total length. The start index of missing sequences is set to 70, which means that some missing positions are in the climb phase to show the stability of the proposed approach. In this experiment, the missing rate ranges from 20% to 80% with a 10% step.

In addition, two comparative experiments are further designed to show the performance superiority of the proposed approach over other common training-free methods. The third-order spline interpolation[2] and flight trajectory planning approach [24] are regarded as comparative methods due to their training-free essence. All 120 trajectories for each flight number are used to optimize the spline interpolation based on their flight time. The flight plans are also extracted from the trajectory database, and the parameters of the kinematics model are drawn from the base of aircraft data (BADA [56]) based on the information of the flight plan.

5. Results and discussions

5.1. Sequence padding

In this section, the influence of two padding methods on final imputation results is evaluated, including the linear interpolation and end padding. The evaluation measurements (mean and STD) with different padding methods and random missing rates are reported in Fig. 5. The legend "LI" and "EP" in the figure denote the linear interpolation and end padding method, respectively. It can be seen that the imputation results for the two padding methods are almost the same when the random missing rate is 60%. The performance obtained by the LI is slightly better than that of the EP. When the random missing rate reaches 90%, there is a considerable performance gap between the two padding methods. The imputation performance obtained by the LI is obviously better than that of the EP, i.e., over 15% difference for the mean estimation error. In addition, the standard deviation of the estimation errors in these two experiments is similar to that of the mean of the estimation error. Based on the experimental results, the linear interpolation padding method is applied to meet the dimension requirements in the following experiments.

5.2. Size of dataset

In this section, several experiments are conducted to validate the robustness of the proposed approach when facing the problem of a small dataset. The data samples in this section are randomly selected from the full dataset. To eliminate the randomness, the designed experiments with the same data size are conducted 10 times and the average measurements are regarded as the final results. The experimental results (mean and STD) are reported in Fig. 6. As can be seen from the results, the proposed approach is able to obtain a comparable performance (<1000 meters) with a small dataset and severe data missing, i.e., only 12 flights and 60% random missing rate. As a comparison, the spline interpolation algorithm is optimized with all 120 flight trajectories and 60% random missing rate, whose final mean estimation error is about 630 meters (as shown in Fig. 7).

The proposed approach is able to recover the missing positions with a low estimation error (the mean is about 700 meters) even if

[2] https://www.scipy.org.

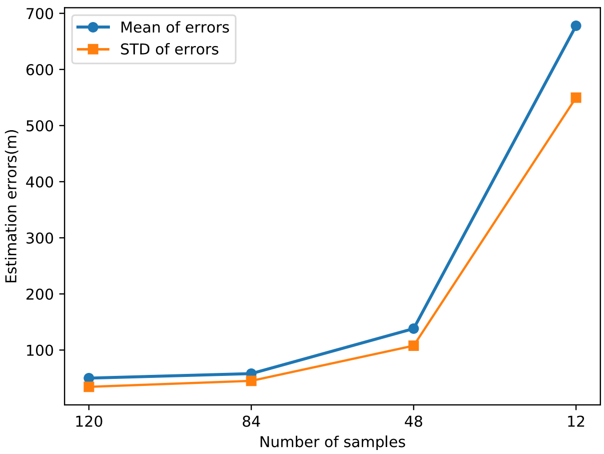

Fig. 6. Results of different dataset size.

the number of samples is only 12. When the data sample number is 48, the mean estimation error is only about 140 meters. The proposed approach is able to yield a similar performance with only 12 flight trajectories and 60% random missing rate. It is believed that the comparison of experimental results supports the robustness of the proposed approach when facing a comparative small dataset. Moreover, with the increasing of the samples, the standard deviation of the estimation error presents a similar trend with that of the mean estimation error, i.e., decreasing.

5.3. Missing pattern and missing rate

5.3.1. Random missing

In this section, we focus on checking the imputation performance for the random missing pattern with different missing rates. The estimation errors for different experimental configurations are reported in Fig. 7. As can be seen from the result that the proposed approach achieves a higher estimation accuracy among the three approaches, even if the random missing rate is as high as 90%. When missing rates are less than 80%, the mean estimation error is only about 200 meters, which is a very excellent accuracy for the flight trajectory imputation task. When the missing rate reaches 90%, the mean of the estimation errors surges to about 800 meters, which can still be considered sufficiently accurate for the trajectory imputation task in air traffic studies. It can be also seen that the standard deviations drop to the lowest point when the missing rate is 50% and show an upward trend with the increase of the missing rate. This fact can be attributed to the over-fitting problem, i.e., suffers from a performance fluctuation with a smaller missing rate from a global input.

As to the baselines, in general, the proposed approach obtains higher performance on both the accuracy and the estimation stability (STD). Specifically, the performance of the flight trajectory planning approach is better than that of the spline interpolation approach. Different from the proposed approach, both the measurements obtained by the two approaches increase as the missing rates increase, while the proposed approach stably keeps them at a low level. The experimental results also support the robustness and applicability of the proposed approach.

5.3.2. Flight chain missing

The experimental results of flight chain missing with different missing rates obtained by different approaches are displayed in Fig. 8. The same dataset is used to conduct the designed experiments to ensure comparison fairness. In general, for all the three approaches, the means of the estimation errors are larger in the climb phase compared to those in the cruise phase, as well as the standard deviation of the errors. It is clear that the proposed approach obtains a better performance than that of other baselines, with all missing rates in both the climb and cruise phase. When the missing positions are distributed in the climb phase, the estimation errors obtained by the proposed approach range from 900 to 1400 meters with respect to the missing rates of 10% to 50%. The performance of spline interpolation and flight trajectory planning approach are similar and worse than that of the proposed approach, i.e., the mean estimation errors are from 1000 to 2800 meters corresponding to the 10% to 50% missing rate. With the same missing rate, the proposed approach can obtain the minimal estimation error, while the spline interpolation approach suffers from the maximum. The standard deviation of the estimation errors in the climb phase shows a similar trend to its mean, in which the proposed approach can obtain more stable estimation results (lower STD) compared to the other two approaches.

When the missing positions are in the cruise phase, the three approaches obtain comparable estimation accuracy with less than 30% missing rates. Moreover, the flight trajectory planning approach even obtains better performance than that of the proposed approach with 10% missing rate. However, with the increase of the missing rate, the proposed approach shows its superiority over the other two baselines and obtains the highest accuracy when the missing rates are 40% and 50%, i.e., leading about hundreds of meters. The standard deviation of the estimation errors in the cruise phase shows a different situation, where the flight trajectory planning approach can estimate the missing positions more stable compared to the other two comparative approaches.

The larger estimation errors for both the three approaches in the climb phase can be explained as the aircraft maneuverability. Since the aircraft needs to accelerate to its cruise speed at a desired location, the hidden linearity of flight transition patterns in this phase is relatively low, which further reduces the estimation accuracy. Fortunately, the proposed approach has the ability of capturing the multi-mode and multi-pattern features of the trajectory tensor (transition patterns), which promotes performance superiority over other methods. In the cruise phase, the aircraft flies at a specified speed, whose motion rules can be characterized with high linearity. The hidden patterns of flight trajectories in the cruise phase are easy to be recognized, which supports the similar performance of the three approaches. In addition, as can be seen from the results, the standard deviation of estimation errors in the cruise phase are less than that in the climb phase, which can also be attributed to the linear scarcity of the flight trajectory.

5.3.3. Block missing

The experiment results of the block missing with different missing rates are shown in Fig. 9. In most cases, the proposed approach is able to accomplish the trajectory imputation task with higher confidence compared to the two baselines. The mean estimation error is about 1000 meters when missing rates are less than 70%. We can safely draw a conclusion that the proposed approach applies to the flight trajectory imputation task with block missing in the air traffic control system. However, when the missing rates come to 70% and 80%, the mean estimation error reaches about 4000 meters and 7700 meters, respectively. Similarly, the proposed approach is also able to obtain a stable estimation accuracy compared to the two baselines, as demonstrated as the standard deviation of the errors.

Comparatively, the spline interpolation and flight trajectory planning approach estimate missing positions with inferior performance, in which the performance obtained by the flight trajectory planning approach is better than that of the spline interpolation approach. Unlike the proposed approach keeps the estimation errors at a low level when the missing rate is less than 70%, the estimation errors of the two comparative methods increase as the

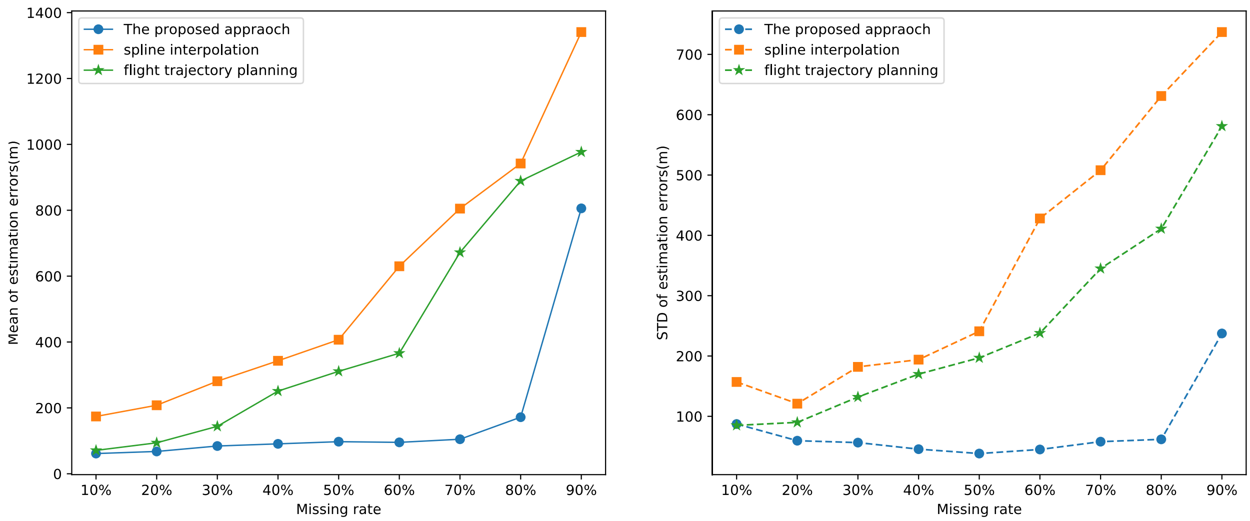

Fig. 7. Results of random missing with different missing rates.

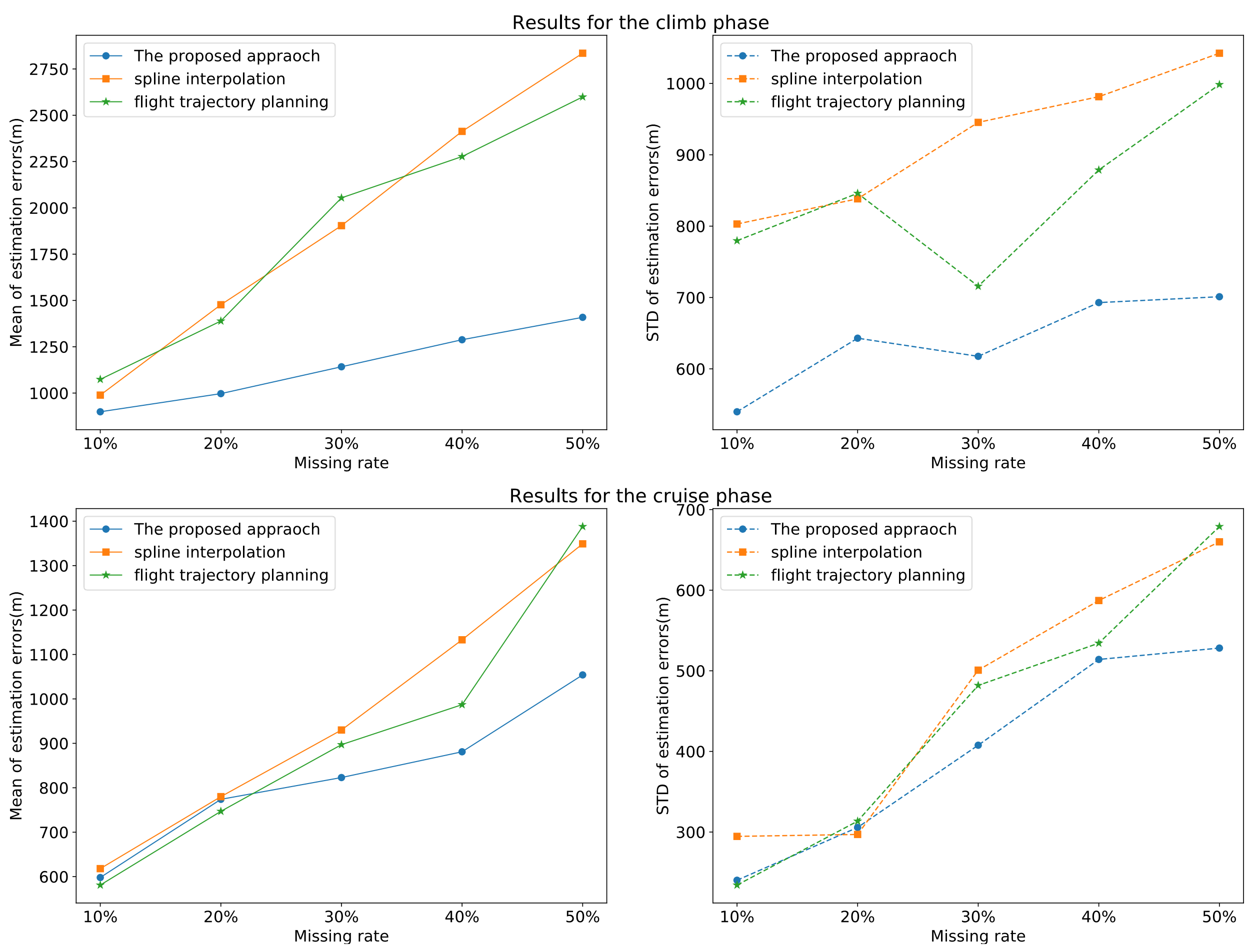

Fig. 8. Results with flight chain missing with different missing rates.

missing rate gets larger. In general, the baseline approaches take a local sequence of the trajectory (compared to the whole trajectories) as their input and achieves the estimation with static information (flight plan or the spline interpolation). That is to say, the baselines may fail to deal with the diversity of the flight trajectory and their estimation errors are also accumulated with the increased missing rate.

The mean estimation errors of the proposed approach are about 1000 meters, while the mean estimation errors of the two comparative methods range from hundreds to about 3000 meters. The performance superiority benefits from the intrinsic abstract representation ability of the proposed approach when processing the multi-mode data patterns, i.e., capturing trajectory patterns from historical paths on both temporal and spatial dependencies.

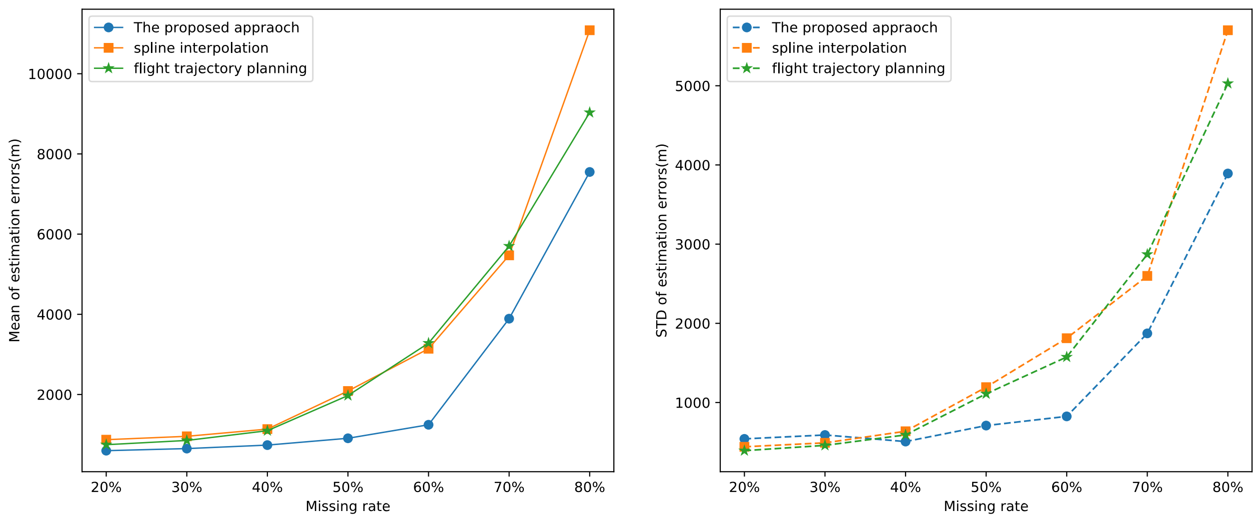

Fig. 9. Results with block missing with different missing rates.

6. Conclusions

In this work, a tensor completion-based approach is proposed to estimate missing positions of the flight trajectory in air traffic control systems. The global information of flight trajectories with the same flight number on different days is considered to achieve the imputation task. By constructing the flight trajectories into a 3-D tensor, the imputation is achieved by the tensor completion. The trace norm minimizing-based tensor completion is proposed and further resolved by the BCD algorithm. Real flight trajectories are applied to validate the performance under different configurations of missing pattern and rate. Experimental results demonstrate that the proposed approach achieves the trajectory imputation task with considerably high performance even with a small dataset and a high missing rate. A mean estimation error of a few hundred meters can be obtained in the case of 90% random missing rate. The proposed approach has the ability of overcoming the flight chain missing and block missing pattern for the flight trajectory. Moreover, the missing positions in the climb phase can also be recovered with high accuracy and stability. In summary, the proposed approach can serve as strong support of data-driven models in the field of air traffic research.

In the future, to improve the performance of the proposed approach, we plan to optimize the target function by considering the trajectory similarity when the flight travels along with different waypoint sequences.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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