Problem Set III: Information Theory

```
clc; clear; close all;
```

Problem 1

```
disp('Problem 1')
```

```
Problem 1
```

```

% Define symbols and respective probabilities
symbols =   {'a',  'b',  'c',  'd',  'e',  'f',  'g',  'h' };
p       =   [0.24, 0.21, 0.18, 0.15, 0.07, 0.06, 0.05, 0.04];

%%% Part (a)
```

Part (a)

```
disp('Part (a)')
```

```
Part (a)
```

```
% Codeword lengths
l = ceil(-log2(p));

% Kraft sum
kraft_sum = sum(2.^(-l));

disp('Codeword lengths (l):');
```

```
Codeword lengths (l):
```

```
disp(l);
```

```
     3     3     3     3     4     5     5     5
```

```

% Show that Kraft inequality is satisfied
disp('Kraft Inequality:');
```

```
Kraft Inequality:
```

```
disp([num2str(kraft_sum) ' <= 1']);
```

```
0.65625 <= 1
```

```
if kraft_sum <= 1
    disp('The code lengths satisfy the Kraft inequality.');
else
    disp('The code lengths do not satisfy the Kraft inequality.');
end
```

```
The code lengths satisfy the Kraft inequality.
```

Part (b)

```
disp('Part (b)')
```

```
Part (b)
```

```
% Entropy H(X)
H_X = sum(-p .* log2(p));

% Average code length L_bar
L_bar = sum(p .* l);

disp('Entropy H(X):');
```

```
Entropy H(X):
```

```
disp(H_X);
```

```
    2.7367
```

```
disp('Average code length L_bar:');
```

```
Average code length L_bar:
```

```
disp(L_bar);
```

```
    3.3700
```

```

% Verify that H(X) <= L_bar < H(X) + 1
disp([num2str(H_X) ' <= ' num2str(L_bar) ' < ' num2str(H_X + 1)]);
```

```
2.7367 <= 3.37 < 3.7367
```

```
if H_X <= L_bar && L_bar < H_X + 1
    disp('The inequality H(X) <= L_bar < H(X) + 1 is satisfied.');
else
    disp('The inequality H(X) <= L_bar < H(X) + 1 is not satisfied.');
end
```

```
The inequality H(X) <= L_bar < H(X) + 1 is satisfied.
```

```

```

Problem 2

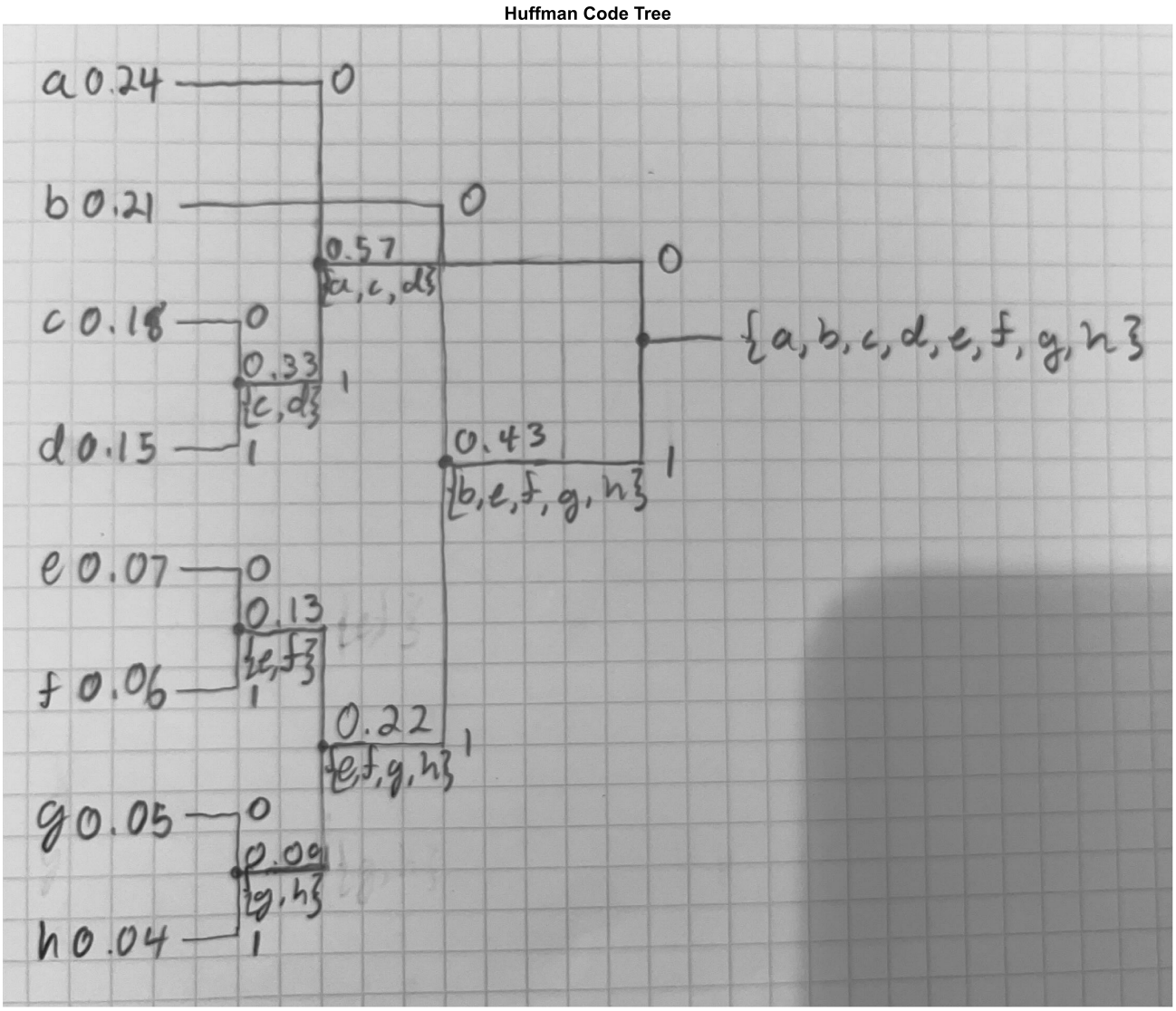
```
disp('Problem 2')
```

```
Problem 2
```

```

% Code Tree
img = imread('Code Tree.jpeg');
imshow(img);
```

```
title('Huffman Code Tree');
```




```
% Symbols         a     b     c      d      e       f       g       h
huffman_codes = {'00', '10', '010', '011', '1100', '1101', '1110', '1111'};

% Calculate Huffman code lengths
huffman_lengths = cellfun(@length, huffman_codes);

% Calculate -log2(p) for each probability
neg_log2_p = -log2(p);

% Code Table
code_table = table(symbols', p', huffman_codes', huffman_lengths',
round(neg_log2_p, 2)', 1', ...
                   'VariableNames', {'Symbol', 'Probability', 'Code', 'Length', '-
log2(p)', 'ceil(-log2(p))'});
```

```
disp('Huffman Code Table:');
```

```
Huffman Code Table:
```

```
disp(code_table);
```

Symbol	Probability	Code	Length	-log2(p)	ceil(-log2(p))
{'a'}	0.24	{'00' }	2	2.06	3
{'b'}	0.21	{'10' }	2	2.25	3
{'c'}	0.18	{'010' }	3	2.47	3
{'d'}	0.15	{'011' }	3	2.74	3
{'e'}	0.07	{'1100'}	4	3.84	4
{'f'}	0.06	{'1101'}	4	4.06	5
{'g'}	0.05	{'1110'}	4	4.32	5
{'h'}	0.04	{'1111'}	4	4.64	5

```
% Kraft inequality for Huffman codes
kraft_sum_huffman = sum(2.^(-huffman_lengths));
disp('Kraft inequality (Huffman):');
```

```
Kraft inequality (Huffman):
```

```
disp([num2str(kraft_sum_huffman) ' <= 1']);
```

```
1 <= 1
```

```
if kraft_sum_huffman <= 1
    disp('The Huffman code lengths satisfy the Kraft inequality.');
else
    disp('The Huffman code lengths do not satisfy the Kraft inequality.');
end
```

```
The Huffman code lengths satisfy the Kraft inequality.
```

```
% Average code length L_bar for the Huffman code
L_bar_huffman = sum(p .* huffman_lengths);

% Display entropy, average code length for Huffman, and comparison
disp('Average code length L_bar for Huffman code:');
```

```
Average code length L_bar for Huffman code:
```

```
disp(L_bar_huffman);
```

```
    2.7700
```

```
% Verify the inequality H(X) <= L_bar < H(X) + 1 for Huffman code
disp([num2str(H_X) ' <= ' num2str(L_bar_huffman) ' < ' num2str(H_X + 1)]);
```

```
2.7367 <= 2.77 < 3.7367
```

```
if H_X <= L_bar_huffman && L_bar_huffman < H_X + 1
```

```
    disp('The inequality H(X) <= L_bar < H(X) + 1 is satisfied for the Huffman
code.');
else
    disp('The inequality H(X) <= L_bar < H(X) + 1 is not satisfied for the Huffman
code.');
end
```

```
The inequality H(X) <= L_bar < H(X) + 1 is satisfied for the Huffman code.
```

Problem 3