1.(a) $m(t)$ has spectrum $M(f)$ which has a bandwidth of W

$$(m(t))^n = \underbrace{m(t)\cdot m(t)\cdots m(t)}_{n \text{ times}} \Rightarrow M_n(f) = \underbrace{M(f) * M(f) * \cdots * M(f)}_{n \text{ times}}$$

$\therefore (m(t))^n$ has bandwidth $\boxed{nW}$

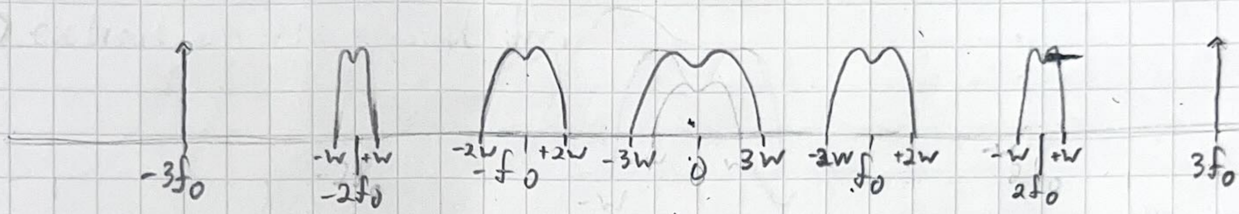
(b) $$V_{out}(t) = a[m(t) + A_0\cos(2\pi f_0 t)] + b[m(t) + A_0\cos(2\pi f_0 t)]^3$$
$$= a m(t) + aA_0\cos(2\pi f_0 t) + b\, m^3(t) + b\,3\, m^2(t) A_0 \cos(2\pi f_0 t) + \cdots$$
$$\cdots\, bA_0^3 \cos^3(2\pi f_0 t)$$

(baseband) $= a m(t) + b m^3(t) + \frac{3}{2} b m(t) A_0^2 + \ldots$

(carrier) $\ldots\, aA_0\cos(2\pi f_0 t) + 3bm^2(t)A_0\cos(2\pi f_0 t) + \frac{3}{4} bA_0^3\cos(2\pi f_0 t) + \cdots$

(harmonics) $\cdots\, \frac{3}{2} b m(t) A_0^2 \cos(4\pi f_0 t) + \frac{1}{4} b A_0^3 \cos(6\pi f_0 t)$

(c)



(c) An AM signal cannot be extracted from the signal due to nonlinearity (no term takes the form: $A_c[1 + k_a m(t)]\cos(2\pi f_c t)$)

(d) In the case of a DSB-SC signal: can be extracted from 2^{nd} harmonic

carrier frequency: $\boxed{2f_0}$

Bandpass: $\boxed{f_0 + 2W < |f| < 3f_0}$

W Constraints: $f_0 + 2W < 2f_0 - W \;\wedge\; 2f_0 + W < 3f_0$

$W < f_0/3 \;\wedge\; W < f_0 \equiv \boxed{W < f_0/3}$

4.(a) $P(R > \rho)\ \int_\rho^\infty p(r)\,dr = \int_\rho^\infty \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)dr$ $\qquad u = \frac{r^2}{2\sigma^2} \Rightarrow r = \sigma\sqrt{2u}$

$dr = \sigma\sqrt{\frac{2}{\sigma^2}}\,du = \frac{\sigma}{\sqrt{2u}}\,du$

$$\Rightarrow P(R > \rho) = \int_{\frac{\rho^2}{2\sigma^2}}^{\infty} \frac{\sigma\sqrt{2u}}{\sigma^2} e^{-u} \frac{\sigma}{\sqrt{2u}}\,du = \int_{\frac{\rho^2}{2\sigma^2}}^{\infty} e^{-u} = \boxed{e^{-\frac{\rho^2}{2\sigma^2}}}$$

(b) $10^{-4} = e^{-\frac{\rho^2}{2\sigma^2}} \Rightarrow -\frac{\rho^2}{2\sigma^2} = \ln(10^{-4}) \Rightarrow \frac{\rho}{\sigma} = \sqrt{-2\ln(10^{-4})} \approx 4.29$

5.(a) $f_{LO} = f_{RF} \pm f_{IF} = \boxed{19.4\text{ GHz},\ 20.6\text{ GHz}}$

(b) $f_{IMAGE} = |f_{RF} \pm 2f_{IF}| = 18.8\text{ GHz},\ 21.2\text{ GHz}$

$(f_{LO} = 19.4\text{ GHz}) \Rightarrow f_{IMAGE} = \boxed{18.8\text{ GHz}}$

$(f_{LO} = 20.6\text{ GHz}) \Rightarrow f_{IMAGE} = \boxed{21.2\text{ GHz}}$

```
clc; clear; close all;

kf = 3e6; % Frequency sensitivity (Hz/V)
cases = [
    2,  15e6;   % Case 1: Amplitude 2 V,  frequency 15 MHz
    10, 10e6;   % Case 2: Amplitude 10 V, frequency 10 MHz
    2,  20e6    % Case 3: Amplitude 2 V,  frequency 20 MHz
];

universal_curve_estimates = [
    0.3, 12;
    0.4, 10;
    3, 4;
];

results = cell(size(cases, 1), 5); % Use a cell array for mixed data types
for i = 1:size(cases, 1)
    A = cases(i, 1); % Amplitude (V)
    fm = cases(i, 2); % Modulating frequency (Hz)

    delta_f = kf * A; % Frequency deviation (Hz)
    beta = delta_f / fm; % Modulation index

    if beta <= 1 % Narrowband condition
        bw_type = "Narrowband";
    else
        bw_type = "Wideband";
    end

    % Carson’s Rule
    BW_carson = 2 * (delta_f + fm); % Approximate bandwidth (Hz)

    % Calculate bandwidth using Universal Curve (estimate)
    bt_over_delta_f = universal_curve_estimates(universal_curve_estimates(:, 1) == beta, 2);
    if isempty(bt_over_delta_f) % Handle cases where no match is found
        BW_universal = NaN;
    else
        BW_universal = bt_over_delta_f * delta_f;
    end

    results(i, :) = {delta_f, beta, bw_type, BW_carson, BW_universal}; % Store results in the cell array
end

% Results table
T = cell2table(results, ...
    'VariableNames', {'Freq_Deviation_Hz', 'Mod_Index', 'Is_Wideband', 'Bandwidth_Carson_Hz', 'Universal_Curve_Hz'});
T.Amplitude_V = cases(:, 1);
T.Frequency_Hz = cases(:, 2);
T = T(:, {'Amplitude_V', 'Frequency_Hz', 'Freq_Deviation_Hz', 'Mod_Index', 'Is_Wideband', 'Bandwidth_Carson_Hz', 'Universal_Curve_Hz'});
disp(T);
```

```
    Amplitude_V    Frequency_Hz    Freq_Deviation_Hz    Mod_Index    Is_Wideband     Bandwidth_Carson_Hz    Universal_Curve_Hz
    ___________    ____________    _________________    _________    ____________    ___________________    __________________

         2           1.5e+07             6e+06             0.4       "Narrowband"          4.2e+07                 6e+07      
        10             1e+07             3e+07               3       "Wideband"              8e+07               1.2e+08      
         2             2e+07             6e+06             0.3       "Narrowband"          5.2e+07               7.2e+07      
```

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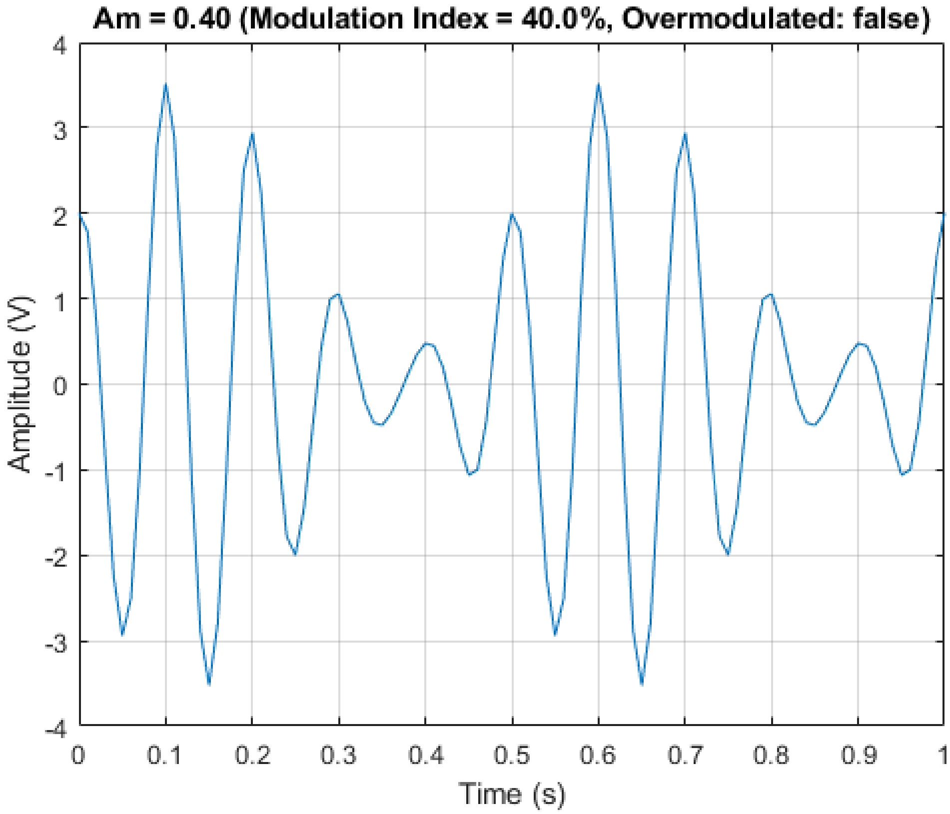
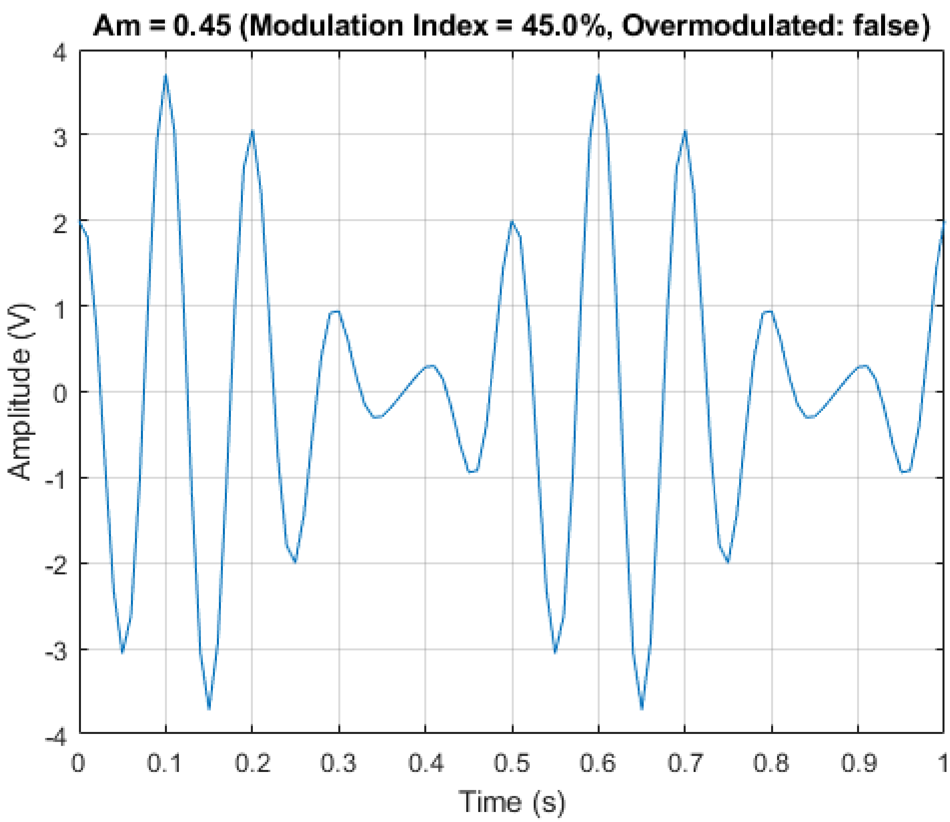
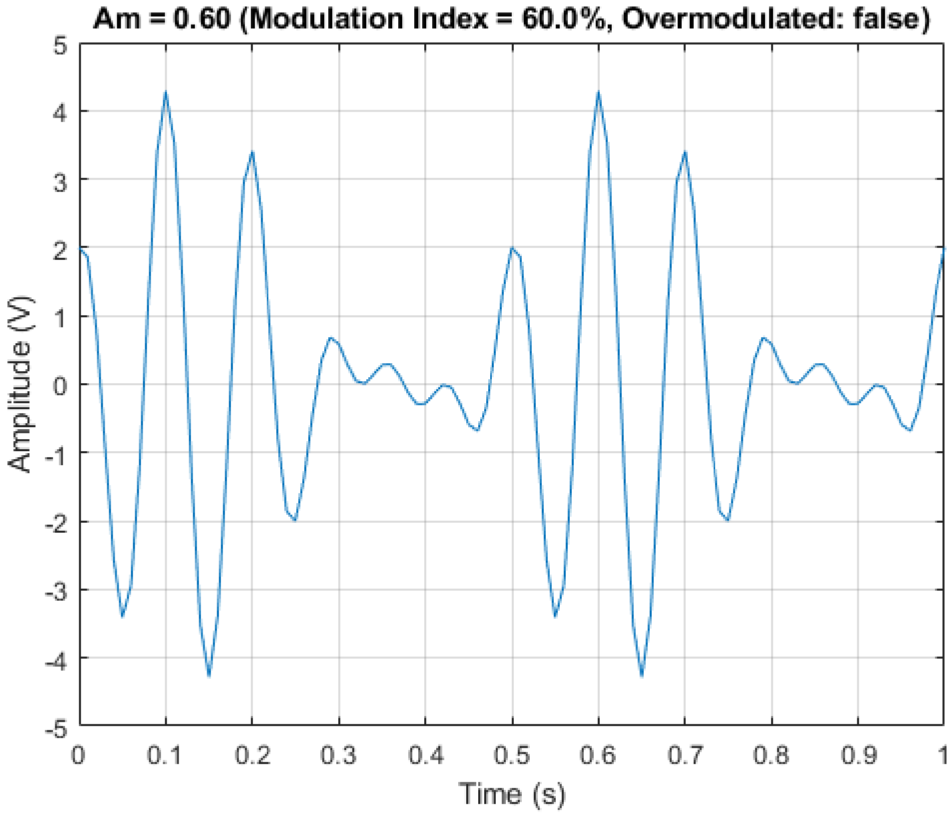
```matlab
clc; clear; close all;

fc = 10;        % Carrier frequency (Hz)
Ac = 2;         % Carrier amplitude (V)

fm = 2;         % Modulating frequency (Hz)
mu_sens = 2;    % Amplitude sensitivity (V/V)

fs = 100;       % Sampling frequency (Hz)
ts = 1/fs;      % Sampling time
n = 2000;       % Number of samples
t = (0:n-1)*ts; % Time vector

Am_values = [0.4, 0.45, 0.6];

for i = 1:length(Am_values)
    Am = Am_values(i);

    % Modulation index calculation
    beta = mu_sens * Am / Ac * 100;
    is_overmodulated = beta > 100;

    m_t = Am * sin(2*pi*fm*t);

    % AM signal
    s_t = Ac * (1 + mu_sens * m_t) .* cos(2*pi*fc*t);

    figure;
    plot(t, s_t);
    title(sprintf('Am = %.2f (Modulation Index = %.1f%%, Overmodulated: %s)', ...
        Am, beta, string(is_overmodulated)));
    xlabel('Time (s)');
    ylabel('Amplitude (V)');
    xlim([0 1]);
    grid on;
end
```


Am = 0.40 (Modulation Index = 40.0%, Overmodulated: false)
Amplitude (V)
Time (s)
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1


Am = 0.45 (Modulation Index = 45.0%, Overmodulated: false)
Amplitude (V)
Time (s)
4
3
2
1
0
-1
-2
-3
-4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1



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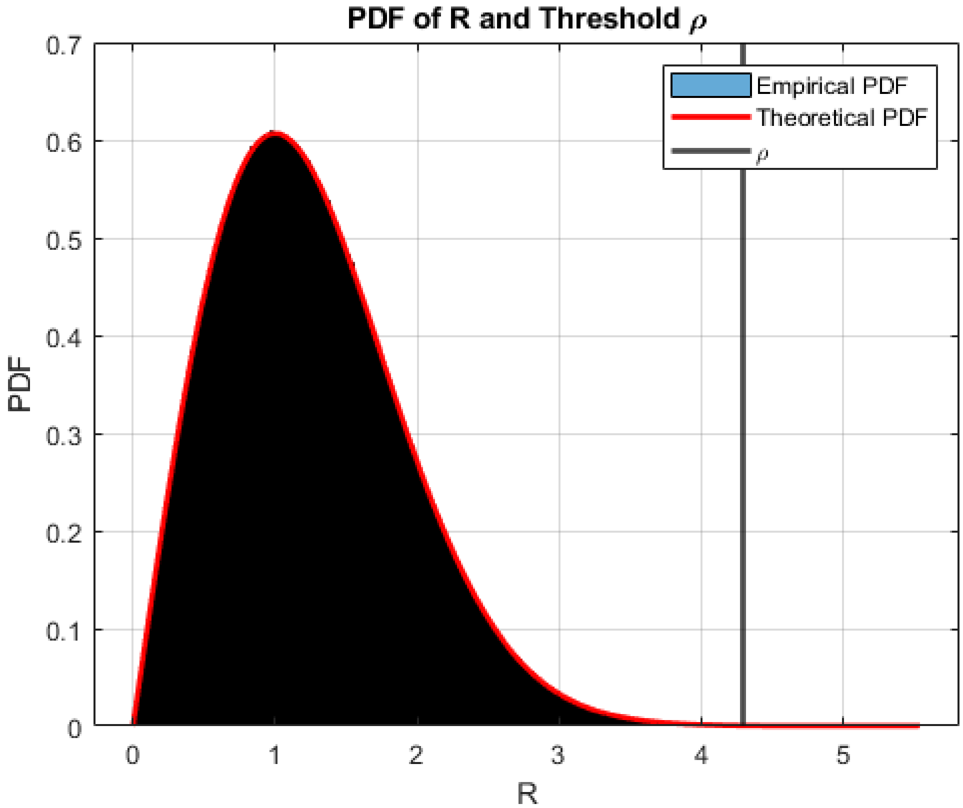
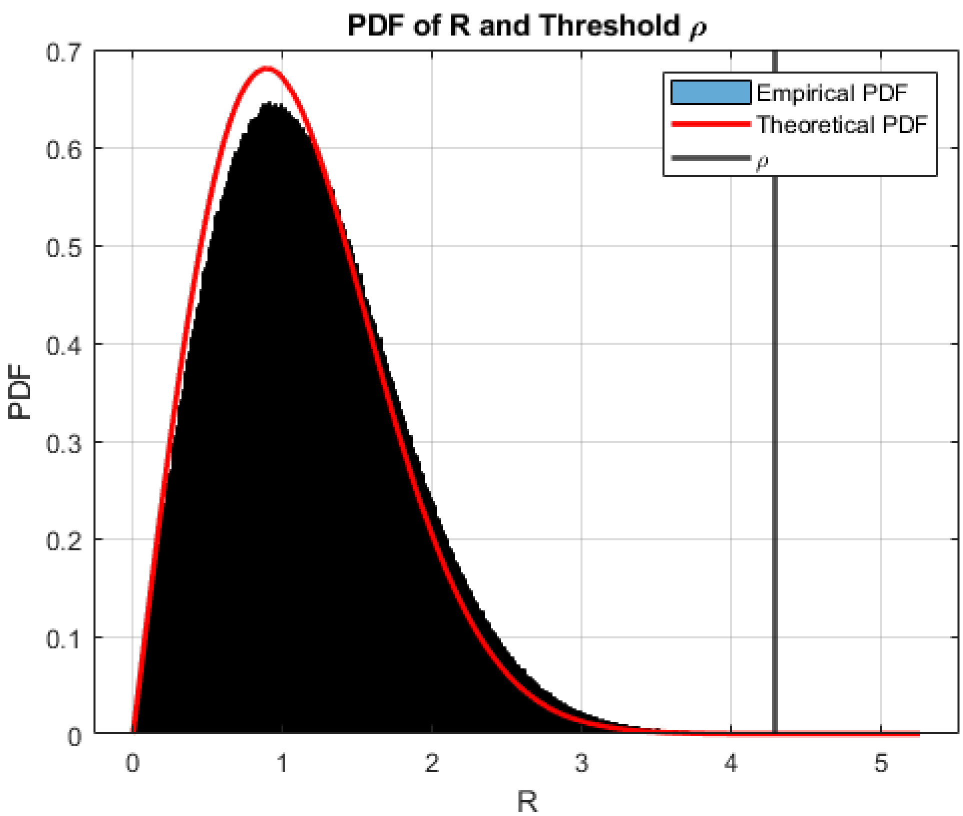
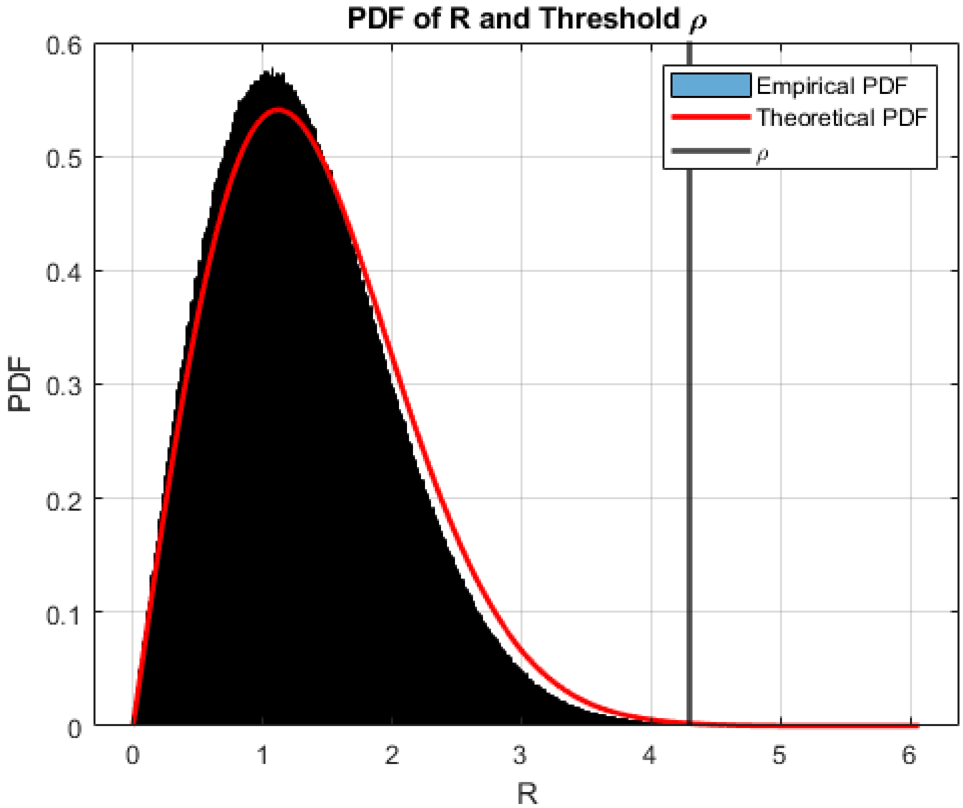
```
clc; clear; close all;

% (a) Compute P(R > rho)
function P_R_greater_rho = P_R_greater_rho_(rho, sigma2)
    % Compute the probability P(R > rho)
    P_R_greater_rho = exp(-rho^2 / (2 * sigma2));
end

% (b) Find rho/sigma so that P(R > rho) = 10^-4
function rho_sigma_ratio = rho_sigma_ratio_(rho_threshold)
    % Find rho/sigma so that P(R > rho) = 10^-4
    rho_sigma_ratio = sqrt(-2 * log(rho_threshold));
end

rho_threshold = 10^-4;
rho_sigma_ratio = rho_sigma_ratio_(10^-4);
fprintf('rho/sigma ratio: %.5f\n', rho_sigma_ratio);

% (c) Sketch R = |n_I + jn_Q|
function R = generateNoise(num_samples, sigma2)
    % Generate noise components
    n_I = randn(num_samples, 1) * sqrt(sigma2);
    n_Q = randn(num_samples, 1) * sqrt(sigma2);
    % Compute the magnitude R
    R = abs(n_I + 1j * n_Q);
end

function plot_pdf(R, rho, sigma2, num_samples)
    % Plot the empirical PDF
    figure;
    histogram(R, 'Normalization', 'pdf');
    hold on;

    % Compute theoretical PDF
    r_vals = linspace(0, max(R), num_samples);
    pdf_vals = (r_vals / sigma2) .* exp(-r_vals.^2 / (2 * sigma2));
    plot(r_vals, pdf_vals, 'r-', 'LineWidth', 2);

    % Add a vertical line at rho
    xline(rho, 'k', 'LineWidth', 2);

    title('PDF of R and Threshold \rho');
    xlabel('R');
    ylabel('PDF');
    legend('Empirical PDF', 'Theoretical PDF', '\rho');
    grid on;
end

% (d) Fraction of time R > rho
function analysis(sigma2, rho, num_samples)
    R = generateNoise(num_samples, sqrt(sigma2));
    plot_pdf(R, rho, sigma2, num_samples);

    frac_R_greater_rho = mean(R > rho);

    % Display results
```

```
    fprintf('Empirical fraction R > rho: %.5f\n', frac_R_greater_rho);
    fprintf('Theoretical P(R > rho): %.5f\n', P_R_greater_rho_(rho, sigma2));
end

sigma2 = 1;
rho = rho_sigma_ratio * sqrt(sigma2);
num_samples = 1e7;

fprintf('\nFor sigma^2 = 1:');
analysis(sigma2, rho, num_samples);


% (e) Decrease sigma^2 by 1 dB
sigma2_decreased = sigma2 * 10^(-1/10);

fprintf('\nFor decreased sigma^2 by 1 dB:');
analysis(sigma2_decreased, rho, num_samples);

% (f) Increase sigma^2 by 1 dB
sigma2_increased = sigma2 * 10^(1/10); % Increase sigma^2 by 1 dB

fprintf('\nFor increased sigma^2 by 1 dB:');
analysis(sigma2_increased, rho, num_samples);
```

```
rho/sigma ratio: 4.29193

For sigma^2 = 1:Empirical fraction R > rho: 0.00010
Theoretical P(R > rho): 0.00010

For decreased sigma^2 by 1 dB:Empirical fraction R > rho: 0.00003
Theoretical P(R > rho): 0.00001

For increased sigma^2 by 1 dB:Empirical fraction R > rho: 0.00026
Theoretical P(R > rho): 0.00066
```


PDF of R and Threshold ρ
PDF
R
Empirical PDF
Theoretical PDF
ρ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
4
5


PDF of R and Threshold ρ
PDF
R
Empirical PDF
Theoretical PDF
ρ
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
1
2
3
4
5


PDF of R and Threshold ρ
Empirical PDF
Theoretical PDF
ρ
0.6
0.5
0.4
0.3
0.2
0.1
0
PDF
0
1
2
3
4
5
6
R