Contents


```
%%%% Problem Set I
%%%% Q1

% Jaeho Cho, Sep 23

clc; clear; close all;
```

(a) Create a vector of the complex points in the constellation.

```
d_min = 1;

% QPSK (4-PSK)

M_QPSK = 4;
theta_QPSK = 2*pi/M_QPSK; % Angle between points on unit circle
A_QPSK = d_min / abs(1 - exp(1j * theta_QPSK));
s_QPSK = A_QPSK * exp(1j * (0:M_QPSK-1) * 2*pi / M_QPSK);

disp('QPSK Constellation Points:');
disp(s_QPSK);

% 8-PSK

M_8PSK = 8;
theta_8PSK = 2*pi/M_8PSK; % Angle between points on unit circle
A_8PSK = d_min / abs(1 - exp(1j * theta_8PSK));
s_8PSK = A_8PSK * exp(1j * (0:M_8PSK-1) * 2*pi / M_8PSK);

disp('8-PSK Constellation Points:');
disp(s_8PSK);

% 16-QAM

M_16QAM = 16;
I_16QAM = -1.5:1:1.5;
Q_16QAM = -1.5:1:1.5;
[I_grid, Q_grid] = meshgrid(I_16QAM, Q_16QAM);
s_16QAM = I_grid(:) + 1j*Q_grid(:);

disp('16-QAM Constellation Points:');
disp(s_16QAM);

% 32-QAM
```

```
M_32QAM = 32;
I_32QAM = -2.5:1:2.5;
Q_32QAM = -2.5:1:2.5;
[I_grid_32, Q_grid_32] = meshgrid(I_32QAM, Q_32QAM);
s_32QAM = I_grid_32(:) + 1j*Q_grid_32(:);

% Exclude the four corners
corner_indices = find(abs(I_grid_32(:)) == 2.5 & abs(Q_grid_32(:)) == 2.5);
s_32QAM(corner_indices) = [];

disp('32-QAM Constellation Points:');
disp(s_32QAM);
```

```
QPSK Constellation Points:
   0.7071 + 0.0000i   0.0000 + 0.7071i  -0.7071 + 0.0000i  -0.0000 - 0.7071i

8-PSK Constellation Points:
  Columns 1 through 4

   1.3066 + 0.0000i   0.9239 + 0.9239i   0.0000 + 1.3066i  -0.9239 + 0.9239i

  Columns 5 through 8

  -1.3066 + 0.0000i  -0.9239 - 0.9239i  -0.0000 - 1.3066i   0.9239 - 0.9239i

16-QAM Constellation Points:
  -1.5000 - 1.5000i
  -1.5000 - 0.5000i
  -1.5000 + 0.5000i
  -1.5000 + 1.5000i
  -0.5000 - 1.5000i
  -0.5000 - 0.5000i
  -0.5000 + 0.5000i
  -0.5000 + 1.5000i
   0.5000 - 1.5000i
   0.5000 - 0.5000i
   0.5000 + 0.5000i
   0.5000 + 1.5000i
   1.5000 - 1.5000i
   1.5000 - 0.5000i
   1.5000 + 0.5000i
   1.5000 + 1.5000i

32-QAM Constellation Points:
  -2.5000 - 1.5000i
  -2.5000 - 0.5000i
  -2.5000 + 0.5000i
  -2.5000 + 1.5000i
  -1.5000 - 2.5000i
  -1.5000 - 1.5000i
  -1.5000 - 0.5000i
  -1.5000 + 0.5000i
  -1.5000 + 1.5000i
  -1.5000 + 2.5000i
  -0.5000 - 2.5000i
  -0.5000 - 1.5000i
  -0.5000 - 0.5000i
```

```
  -0.5000 + 0.5000i
  -0.5000 + 1.5000i
  -0.5000 + 2.5000i
   0.5000 - 2.5000i
   0.5000 - 1.5000i
   0.5000 - 0.5000i
   0.5000 + 0.5000i
   0.5000 + 1.5000i
   0.5000 + 2.5000i
   1.5000 - 2.5000i
   1.5000 - 1.5000i
   1.5000 - 0.5000i
   1.5000 + 0.5000i
   1.5000 + 1.5000i
   1.5000 + 2.5000i
   2.5000 - 1.5000i
   2.5000 - 0.5000i
   2.5000 + 0.5000i
   2.5000 + 1.5000i
```

(b) Compute Eb.

```
% Eb = Es / log2(M)
% Function to compute Eb given constellation points and bits per symbol
compute_Eb = @(s, m) mean(abs(s).^2) / log2(m);

% Compute Eb for each constellation
Eb_QPSK = compute_Eb(s_QPSK, M_QPSK);
Eb_8PSK = compute_Eb(s_8PSK, M_8PSK);
Eb_16QAM = compute_Eb(s_16QAM, M_16QAM);
Eb_32QAM = compute_Eb(s_32QAM, M_32QAM);

% Display Eb values
fprintf('E_b for QPSK: %f\n', Eb_QPSK);
fprintf('E_b for 8-PSK: %f\n', Eb_8PSK);
fprintf('E_b for 16-QAM: %f\n', Eb_16QAM);
fprintf('E_b for 32-QAM: %f\n', Eb_32QAM);
```

```
E_b for QPSK: 0.250000
E_b for 8-PSK: 0.569036
E_b for 16-QAM: 0.625000
E_b for 32-QAM: 1.000000
```

(c) Compute bits per dimension for each constellation.

```
% Bits per dimension = log2(M)/2

bpd_QPSK = log2(M_QPSK) / 2;
bpd_8PSK = log2(M_8PSK) / 2;
bpd_16QAM = log2(M_16QAM) / 2;
bpd_32QAM = log2(M_32QAM) / 2;

fprintf('Bits per dimension for QPSK: %f\n', bpd_QPSK);
fprintf('Bits per dimension for 8-PSK: %f\n', bpd_8PSK);
```

```
fprintf('Bits per dimension for 16-QAM: %f\n', bpd_16QAM);
fprintf('Bits per dimension for 32-QAM: %f\n', bpd_32QAM);
```

```
Bits per dimension for QPSK: 1.000000
Bits per dimension for 8-PSK: 1.500000
Bits per dimension for 16-QAM: 2.000000
Bits per dimension for 32-QAM: 2.500000
```

(d) Which of the four is the most power-efficient?

```
% The most power-efficient modulation is the one with the lowest Eb.

Eb_values = [Eb_QPSK, Eb_8PSK, Eb_16QAM, Eb_32QAM];
modulations = {'QPSK', '8-PSK', '16-QAM', '32-QAM'};

[~, idx_min_Eb] = min(Eb_values);
fprintf('\nThe most power-efficient modulation is %s.\n', modulations{idx_min_Eb});
```

```
The most power-efficient modulation is QPSK.
```

(e) Which of the four is the most spectrally efficient?

```
% The most spectrally efficient modulation is the one with the highest bits per dimension.

bpd_values = [bpd_QPSK, bpd_8PSK, bpd_16QAM, bpd_32QAM];

[~, idx_max_bpd] = max(bpd_values);
fprintf('\nThe most spectrally efficient modulation is %s.\n', modulations{idx_max_bpd});
```

```
The most spectrally efficient modulation is 32-QAM.
```

(f) Is it true that in every case a higher spectral efficiency corresponds to a higher power requirement? Identify any exceptions to this.

```
fprintf('\nComparison of Modulation Schemes:\n');
for i = 1:length(modulations)
    fprintf('%s: E_b = %f, Bits per symbol per dimension = %f\n', ...
        modulations{i}, Eb_values(i), bpd_values(i));
end

fprintf('\nRelationship between spectral efficiency and power requirement:\n');
for i = 1:length(modulations)-1
    delta_bps = bpd_values(i+1) - bpd_values(i);
    delta_Eb = Eb_values(i+1) - Eb_values(i);
    fprintf('From %s to %s: Δbps = %f, ΔE_b = %f\n', ...
        modulations{i}, modulations{i+1}, delta_bps, delta_Eb);
end

% While higher spectral efficiency often requires higher power, this is not always proportionally true.
```

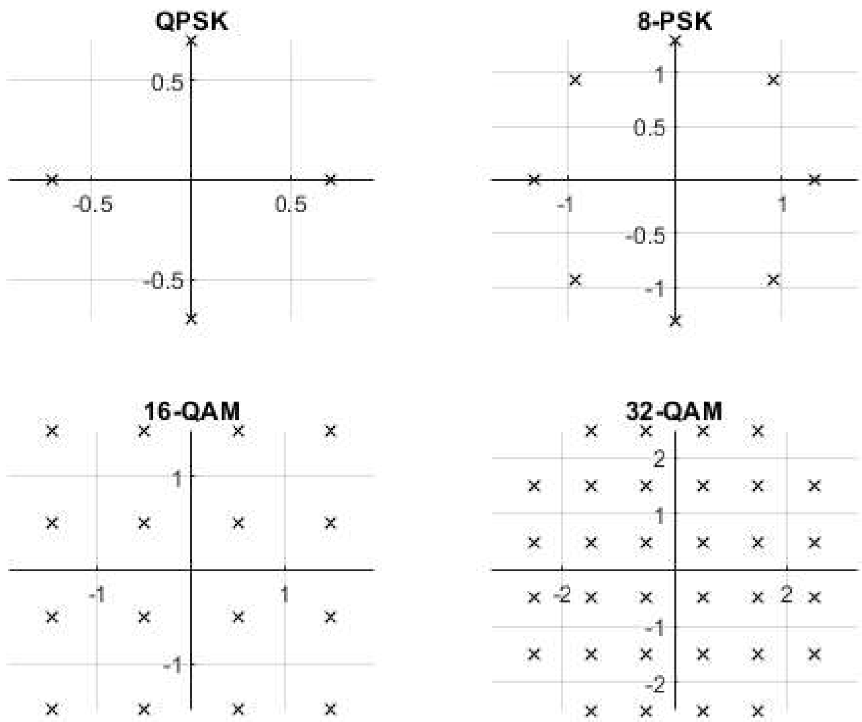
```
% The increase in Eb from 8-PSK to 16-QAM is minimal compared to the increase in spectral efficiency
% Therefore it is not always true that a higher spectral efficiency corresponds to a higher power requirement.
```

```
Comparison of Modulation Schemes:
QPSK: E_b = 0.250000, Bits per symbol per dimension = 1.000000
8-PSK: E_b = 0.569036, Bits per symbol per dimension = 1.500000
16-QAM: E_b = 0.625000, Bits per symbol per dimension = 2.000000
32-QAM: E_b = 1.000000, Bits per symbol per dimension = 2.500000

Relationship between spectral efficiency and power requirement:
From QPSK to 8-PSK: Δbps = 0.500000, ΔE_b = 0.319036
From 8-PSK to 16-QAM: Δbps = 0.500000, ΔE_b = 0.055964
From 16-QAM to 32-QAM: Δbps = 0.500000, ΔE_b = 0.375000
```

Constellation Plots

```
figure;

% QPSK
subplot(2,2,1);
plot(real(s_QPSK), imag(s_QPSK), 'kx');
title('QPSK');
grid on;
axis equal;
axh = gca;
axh.XAxisLocation = 'origin';
axh.YAxisLocation = 'origin';
axh.Box = 'off';

% 8-PSK
subplot(2,2,2);
plot(real(s_8PSK), imag(s_8PSK), 'kx');
title('8-PSK');
grid on;
axis equal;
axh = gca;
axh.XAxisLocation = 'origin';
axh.YAxisLocation = 'origin';
axh.Box = 'off';

% 16-QAM
subplot(2,2,3);
plot(real(s_16QAM), imag(s_16QAM), 'kx');
title('16-QAM');
grid on;
axis equal;
axh = gca;
axh.XAxisLocation = 'origin';
axh.YAxisLocation = 'origin';
axh.Box = 'off';

% 32-QAM
subplot(2,2,4);
plot(real(s_32QAM), imag(s_32QAM), 'kx');
title('32-QAM');
grid on;
axis equal;
```

```
axh = gca;
axh.XAxisLocation = 'origin';
axh.YAxisLocation = 'origin';
axh.Box = 'off';
```

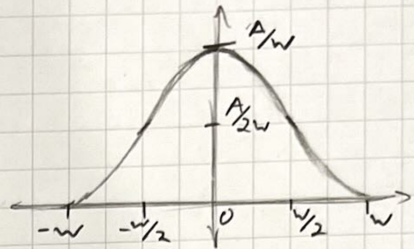


2.

$$X(f) = \frac{A}{2W}\left[1+\cos\left(\frac{\pi f}{W}\right)\right]\Pi\left(\frac{f}{2W}\right)$$

a) $X(0) = \frac{A}{2W}[1+1](1) = \frac{A}{W}$, $X(\pm W/2) = \frac{A}{2W}[1+0](1) = \frac{A}{2W}$

$\lim_{f\to\pm W} X(W) = 0$



b) $X(f) = \frac{A}{2W}\left[1+\cos\left(\frac{\pi f}{W}\right)\right]\Pi\left(\frac{f}{2W}\right) = A\left[\frac{1}{2W}\Pi\left(\frac{f}{2W}\right) + \frac{1}{2W}\cos\left(\frac{\pi f}{W}\right)\Pi\left(\frac{f}{2W}\right)\right]$

$\mathcal{F}^{-1}\left(\frac{1}{2W}\Pi\left(\frac{f}{2W}\right)\right) = \mathrm{sinc}(2Wt)$

$\Downarrow$

$\mathcal{F}^{-1}\left(\frac{1}{2W}\cos\left(\frac{\pi f}{W}\right)\Pi\left(\frac{f}{2W}\right)\right) = \frac{1}{2}\left[\mathrm{sinc}\left(2W\left(t-\frac{1}{2W}\right)\right) + \mathrm{sinc}\left(2W\left(t+\frac{1}{2W}\right)\right)\right]$

$x(t) = A\left[\mathrm{sinc}(2Wt) + \frac{1}{2}\left(\mathrm{sinc}\left(2W\left(t-\frac{1}{2W}\right)\right) + \mathrm{sinc}\left(2W\left(t+\frac{1}{2W}\right)\right)\right)\right]$

c) $\mathrm{sinc}(0) = \lim_{\xi\to 0}\frac{\sin(\pi\xi)}{\xi} = 1$ $\quad\sin(\pi n) = 0$

$\mathrm{sinc}(n) = 0$ for integers $n \neq 0$

d) $x(t) = A\left[\mathrm{sinc}(t/T) + \frac{1}{2}\left(\mathrm{sinc}\left(1/T\,(t-T)\right) + \mathrm{sinc}\left(1/T\,(t+T)\right)\right)\right]$

$x(0) = A\left[1 + \frac{1}{2}(0+0)\right] = A$

$x(-T) = A\left[0 + \frac{1}{2}(0+1)\right] = A/2$

$x(T) = A\left[0 + \frac{1}{2}(1+0)\right] = A/2$

e) $x(kT) = A\left[\mathrm{sinc}(k) + \frac{1}{2}\left(\mathrm{sinc}(k-1) + \sin(k+1)\right)\right]$

for all integers $k \neq 0, \pm 1$, $\mathrm{sinc}(k) = 0 \wedge \mathrm{sinc}(k\pm 1) = 0$

$x(kT) = A\left[0 + \frac{1}{2}(0+0)\right] = 0$

3.

$u(t) = \frac{1}{1+t^2}$, $v(t) = \frac{t}{1+t^2}$

a)

$s_{USSB}(t) = u(t) + j v(t) = \frac{1}{1+t^2} + \frac{jt}{1+t^2} = \frac{1+jt}{1+t^2} = \frac{1+t^2}{(1+t^2)(1-jt)} = \frac{1}{1-jt}$

zero: No zeroes , poles: $t = -j$ => closed upper half plane

$s_{LSSB}(t) = u(t) - j v(t) = \frac{1}{1+t^2} - \frac{jt}{1+t^2} = \frac{1-jt}{1+t^2} = \frac{1+t^2}{(1+t^2)(1+jt)} = \frac{1}{1+jt}$

zero: No zeroes, poles: $t = j$ => closed lower half plane

b)

$|s_{USSB}(t)| = \frac{|1+jt|}{|1+t^2|} = \frac{\sqrt{1+t^2}}{1+t^2} = \frac{1}{\sqrt{1+t^2}}$ => decays at $\frac{1}{|t|}$

$|s_{LSSB}(t)| = \frac{|1-jt|}{|1+t^2|} = \frac{\sqrt{1+t^2}}{1+t^2} = \frac{1}{\sqrt{1+t^2}}$ => decays at $\frac{1}{|t|}$

$|u(t)| = \frac{1}{1+t^2}$ => decays at $\frac{1}{t^2}$

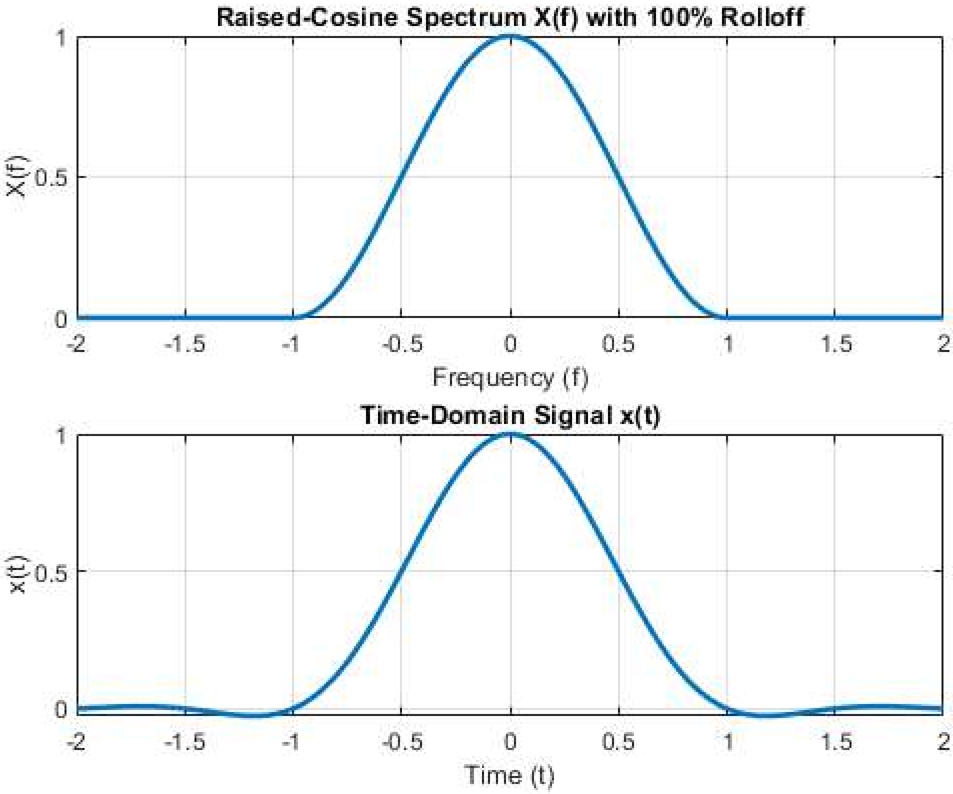
For $t > 1$, the envelopes of USSB and LSSB decay at a slower rate

Contents


```
%%%% Problem Set I
%%%% Q2

% Jaeho Cho, Sep 23

clc; clear; close all;
```

(f) Plot X(f) and x(t)

```
A = 1;
W = 1;

f = linspace(-2*W, 2*W, 1000);
X_f = (A / (2 * W)) * (1 + cos(pi * f / W)) .* (abs(f) <= (2*W)/2);

figure;
subplot(2, 1, 1);
plot(f, X_f, 'LineWidth', 2);
xlabel('Frequency (f)');
ylabel('X(f)');
title('Raised-Cosine Spectrum X(f) with 100% Rolloff');
grid on;

T = 1/(2*W);
t = linspace(-4*T, 4*T, 1000);

x_t = A * (sinc(t/T) + 0.5 * (sinc((1/T)*(t - T)) + sinc((1/T)*(t + T))));

subplot(2, 1, 2);
plot(t, x_t, 'LineWidth', 2);
xlabel('Time (t)');
ylabel('x(t)');
title('Time-Domain Signal x(t)');
grid on;
```



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