

Training neural networks (2)

EECE454 Intro. to Machine Learning Systems

Fall 2024

Recap

- **Last class.** Setting up the training
 - Gradients and activation functions
 - Data preprocessing
 - Normalization layers
 - Parameter Initialization

Recap

- **Last class.** Setting up the training
 - Gradients and activation functions
 - Data preprocessing
 - Normalization layers
 - Parameter Initialization
- **Today.** Tuning the training process
 - Learning rate & Batch size
 - Optimizers
 - Regularizers
 - Hyperparameter tuning

Learning rate & Batch size

SGD

- **SGD.** Recall that the can be written as

$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta} \left(\sum_{i=1}^B \ell(y_i, f_{\theta}(\mathbf{x}_i)) \right)$$

- There are two key hyperparameters
 - Learning rate η
 - Batch size B

SGD

- **SGD.** Recall that the can be written as

$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta} \left(\sum_{i=1}^B \ell(y_i, f_{\theta}(\mathbf{x}_i)) \right)$$

- There are two key hyperparameters
 - Learning rate η
 - Batch size B
- **Question.** How do we **tune** the hyperparameters?
 - Usually by trial and error, with validation sets (discussed soon)
 - Guideline. Choose the largest possible B , and then tune the η

Reason: Fast training

RAM constraints + Generalization

Learning rate vs. Loss

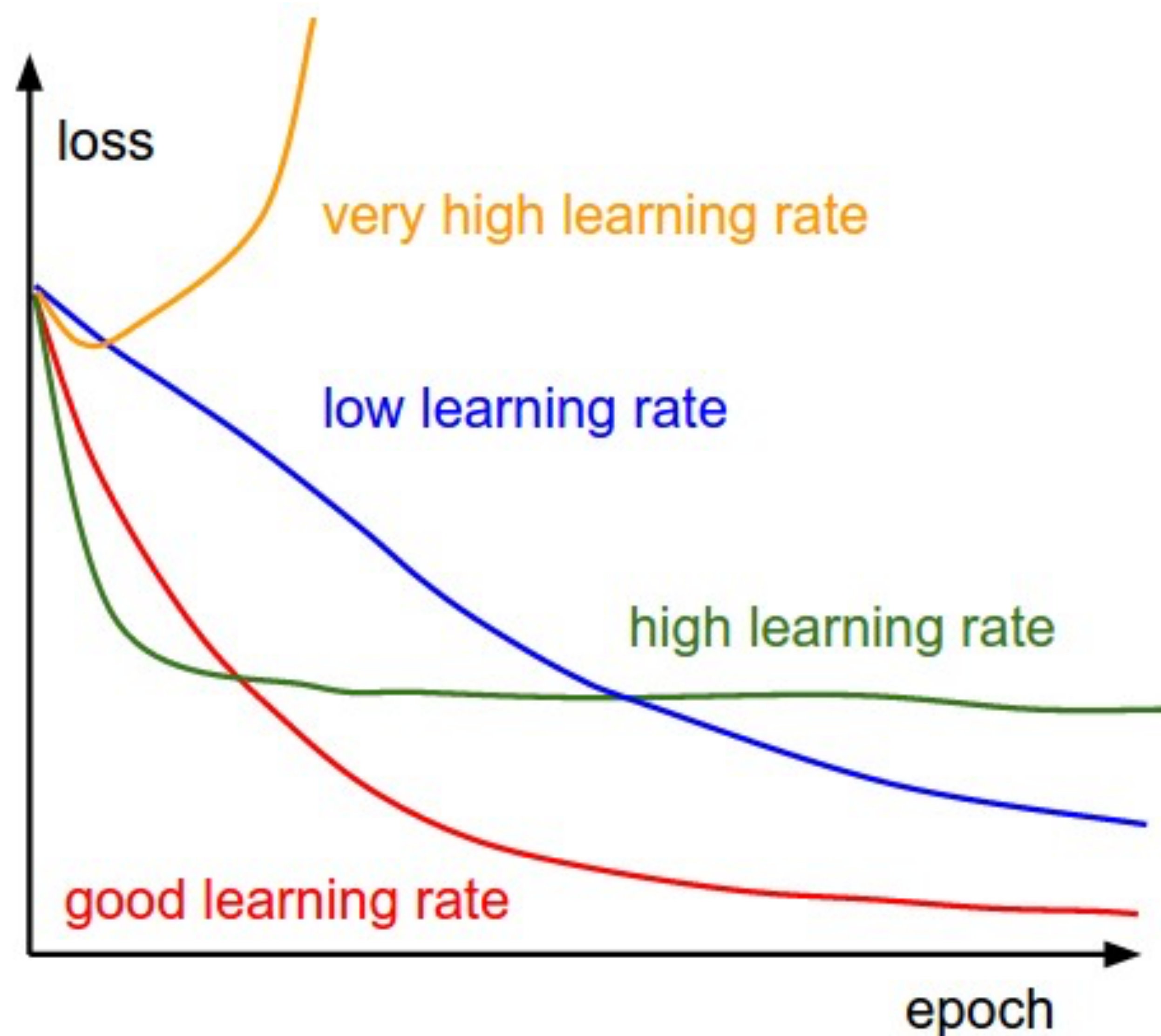
- **High LR.**

- Faster convergence 👍
- High final loss 👎

- **Low LR.**

- Slower convergence 👎
- Low final loss 👍

- **Question.** Can we get the best of both worlds?

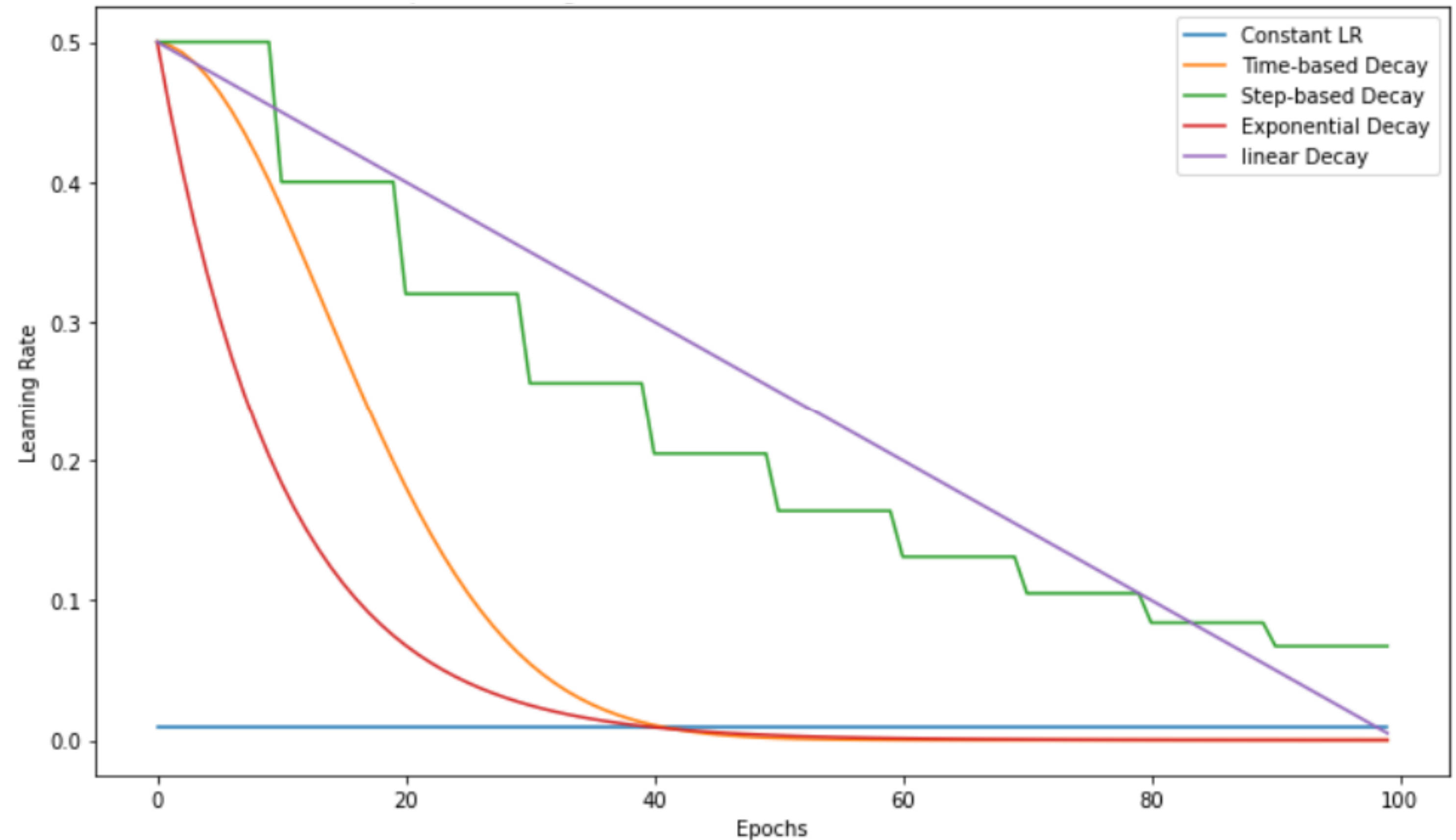


Learning rate scheduling

- **Idea.** Decay the learning rate.

- This requires a careful scheduling of rates

- Step decay
- Linear decay
- Exponential decay
-

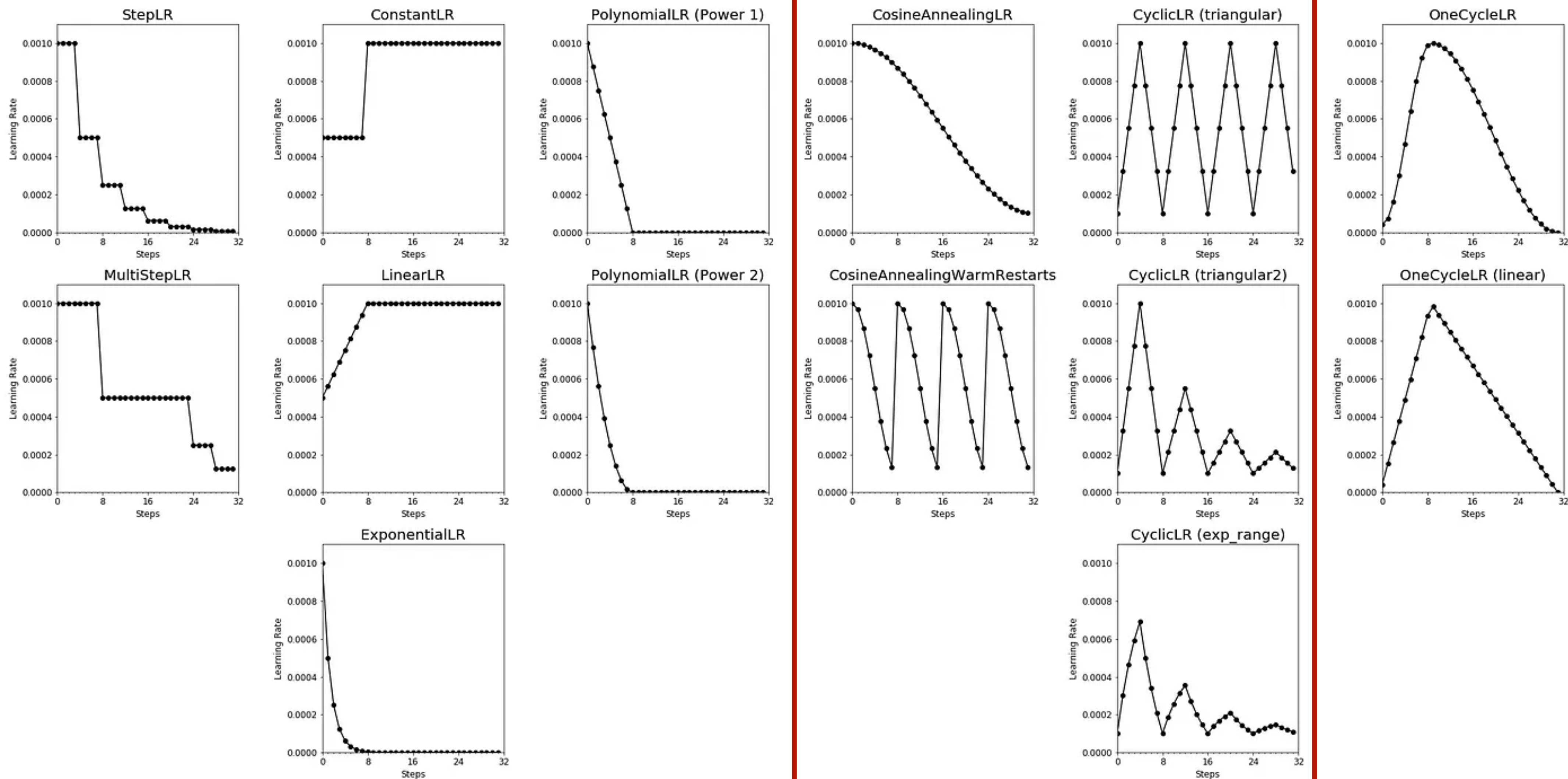


- Note. Optimizers have different sensitivities to LR decay

- e.g., less critical issues in Adam than SGD + Momentum

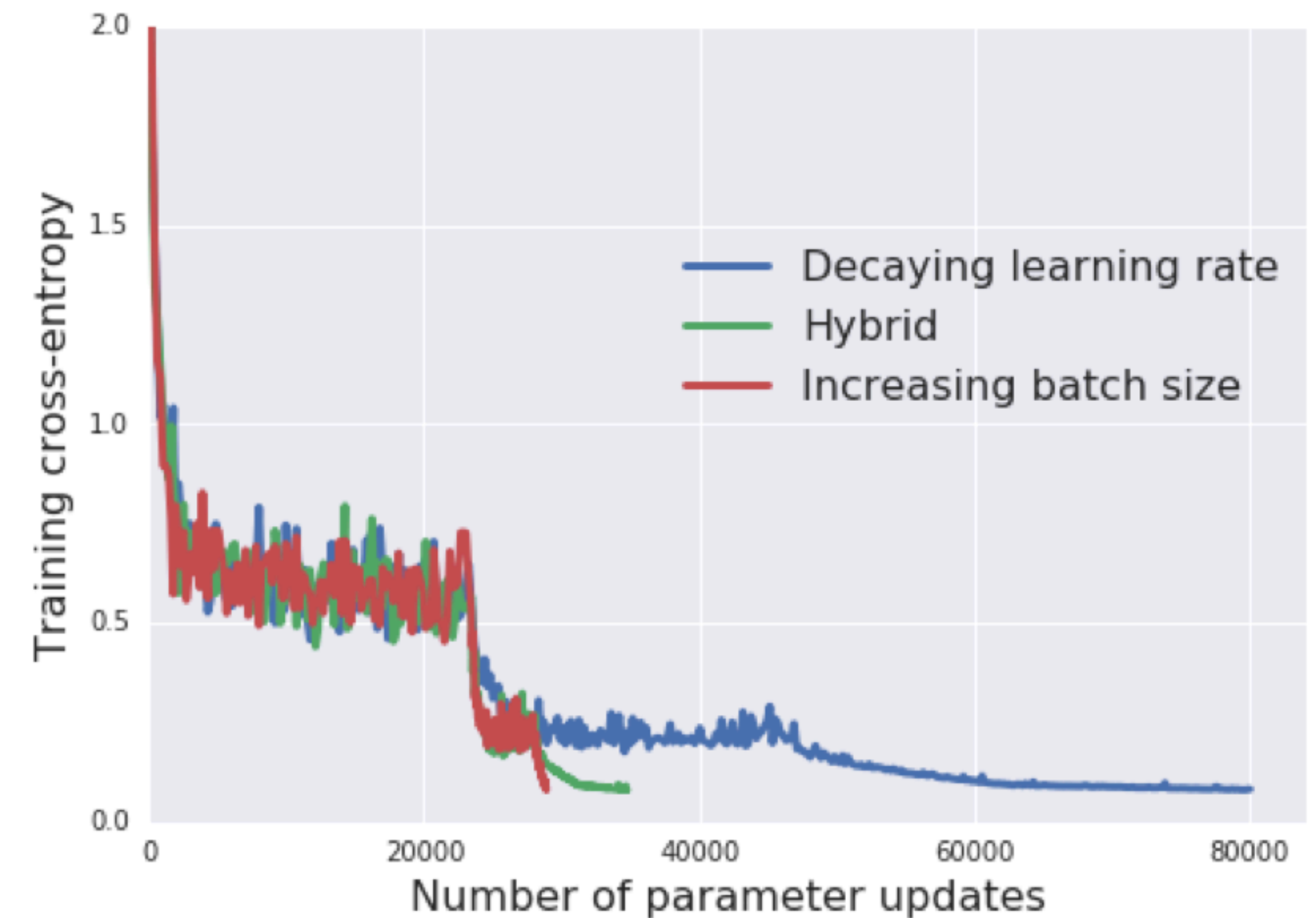
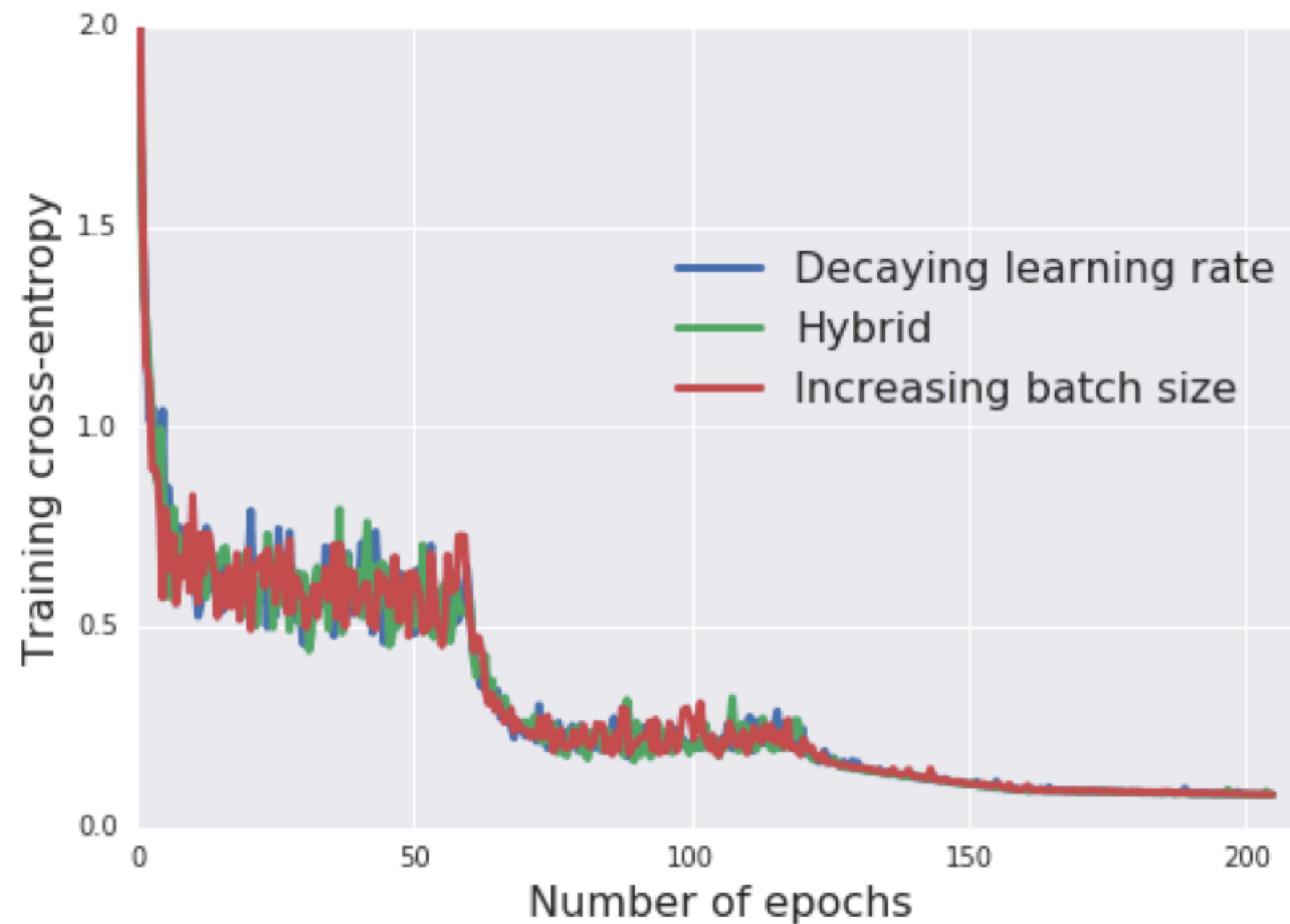
Learning rate scheduling

- **Popular.** Quite common to use cosine annealing / cyclic LR (Optional: Warm restart, Warmup)



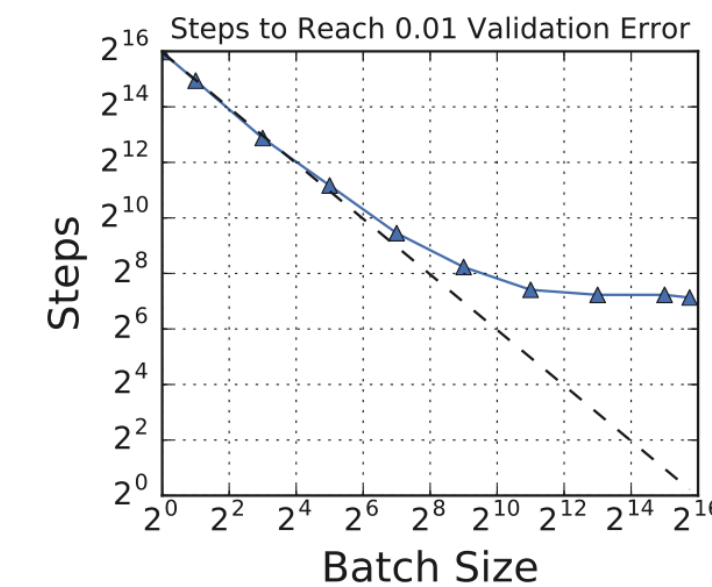
Learning rate vs. Batch size

- Empirically, **increasing the batch size** has a similar effect to **decreasing the learning rate**

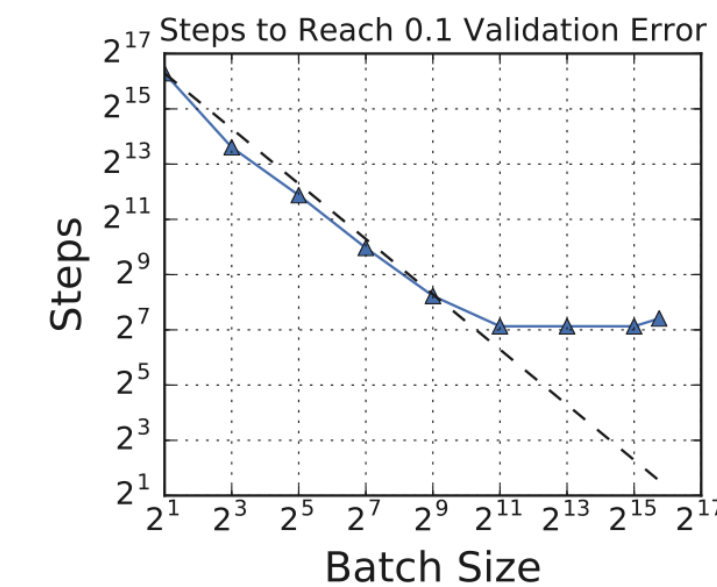


Learning rate vs. Batch size

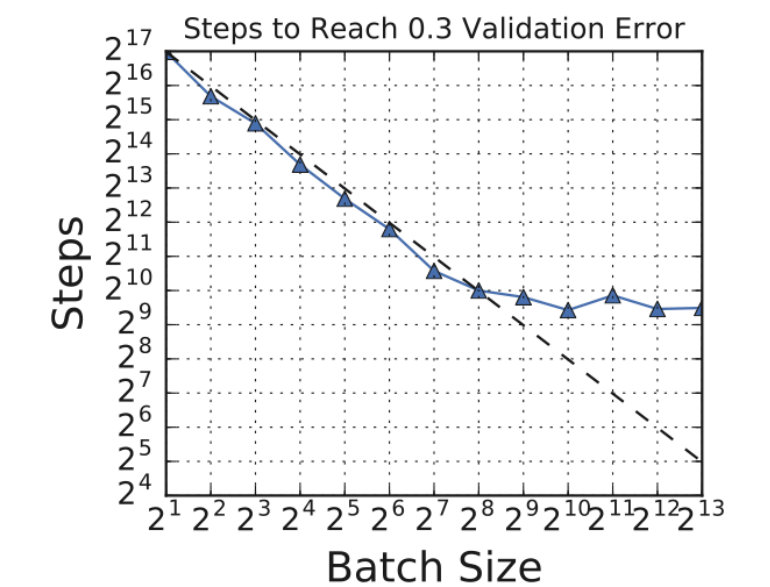
- With a **larger batch size**, we expect
 - Reduced #SGD steps needed to achieve the similar test performance
- Optimal LR scales linearly with batch size
- Eventually the benefit saturates



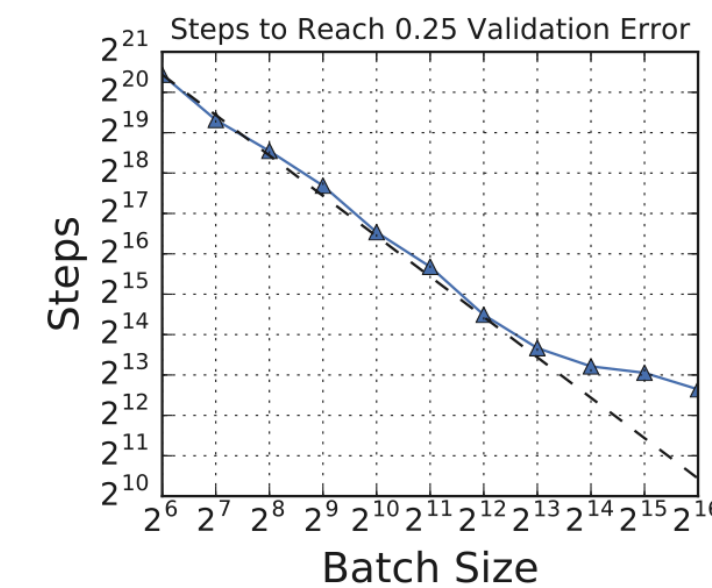
(a) Simple CNN on MNIST



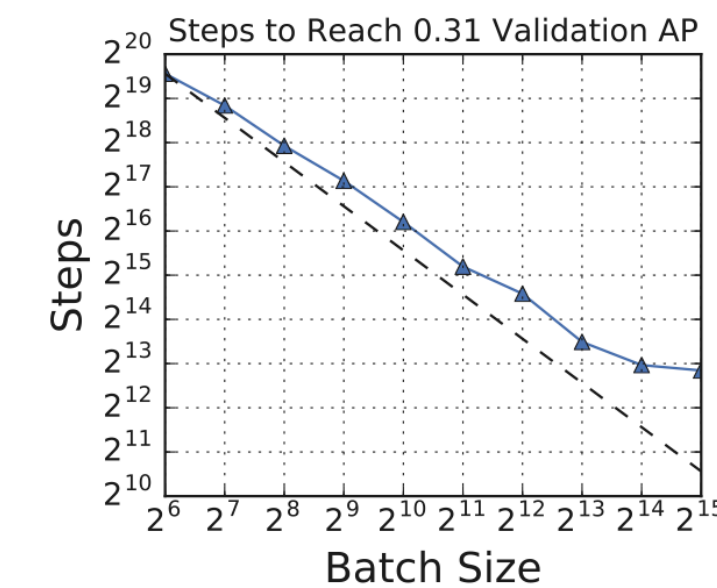
(b) Simple CNN on Fashion MNIST



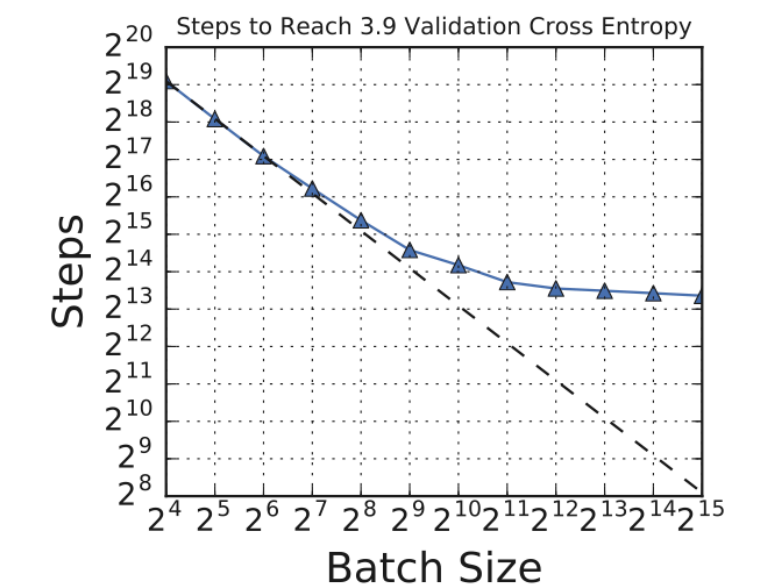
(c) ResNet-8 on CIFAR-10



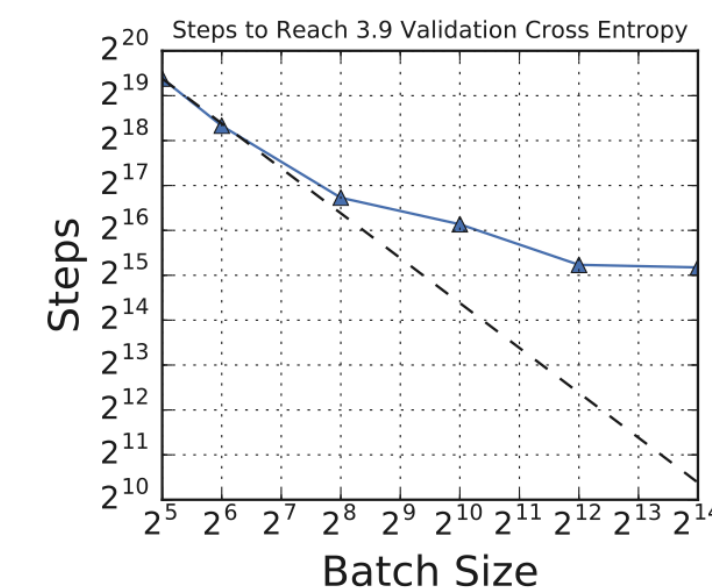
(d) ResNet-50 on ImageNet



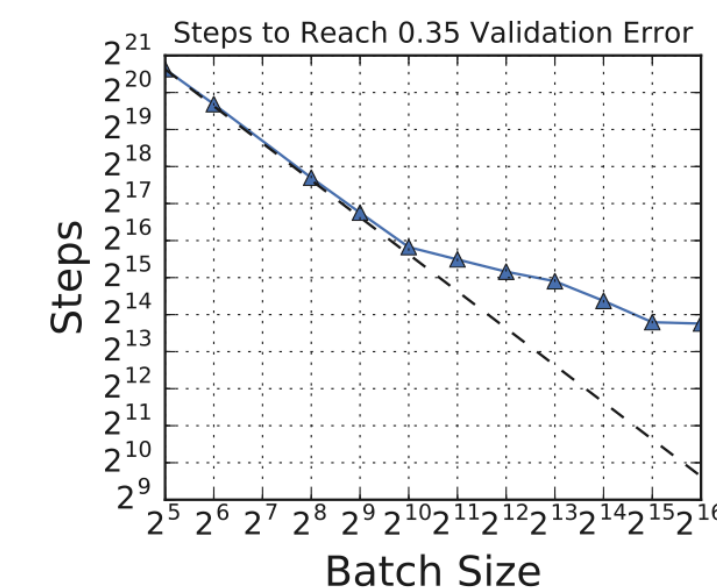
(e) ResNet-50 on Open Images



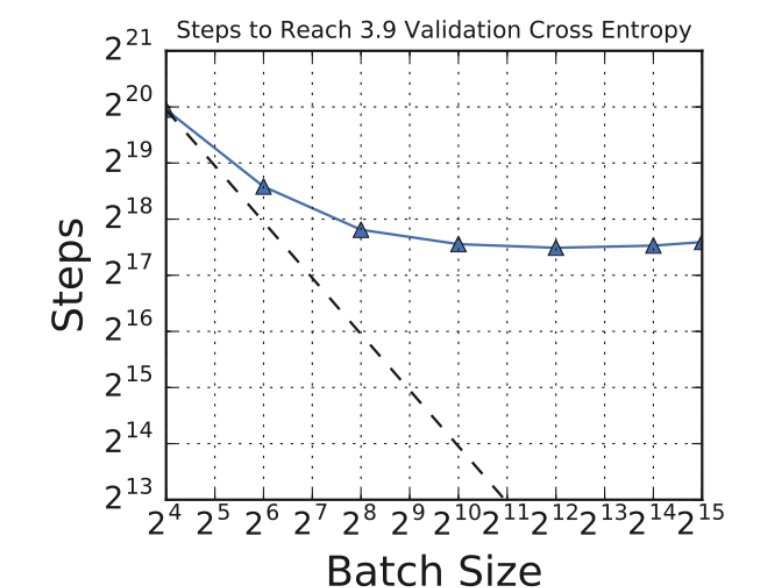
(f) Transformer on LM1B



(g) Transformer on Common Crawl



(h) VGG-11 on ImageNet



(i) LSTM on LM1B

Optimizers

Optimizers

- We rarely use the “vanilla” version of the SGD

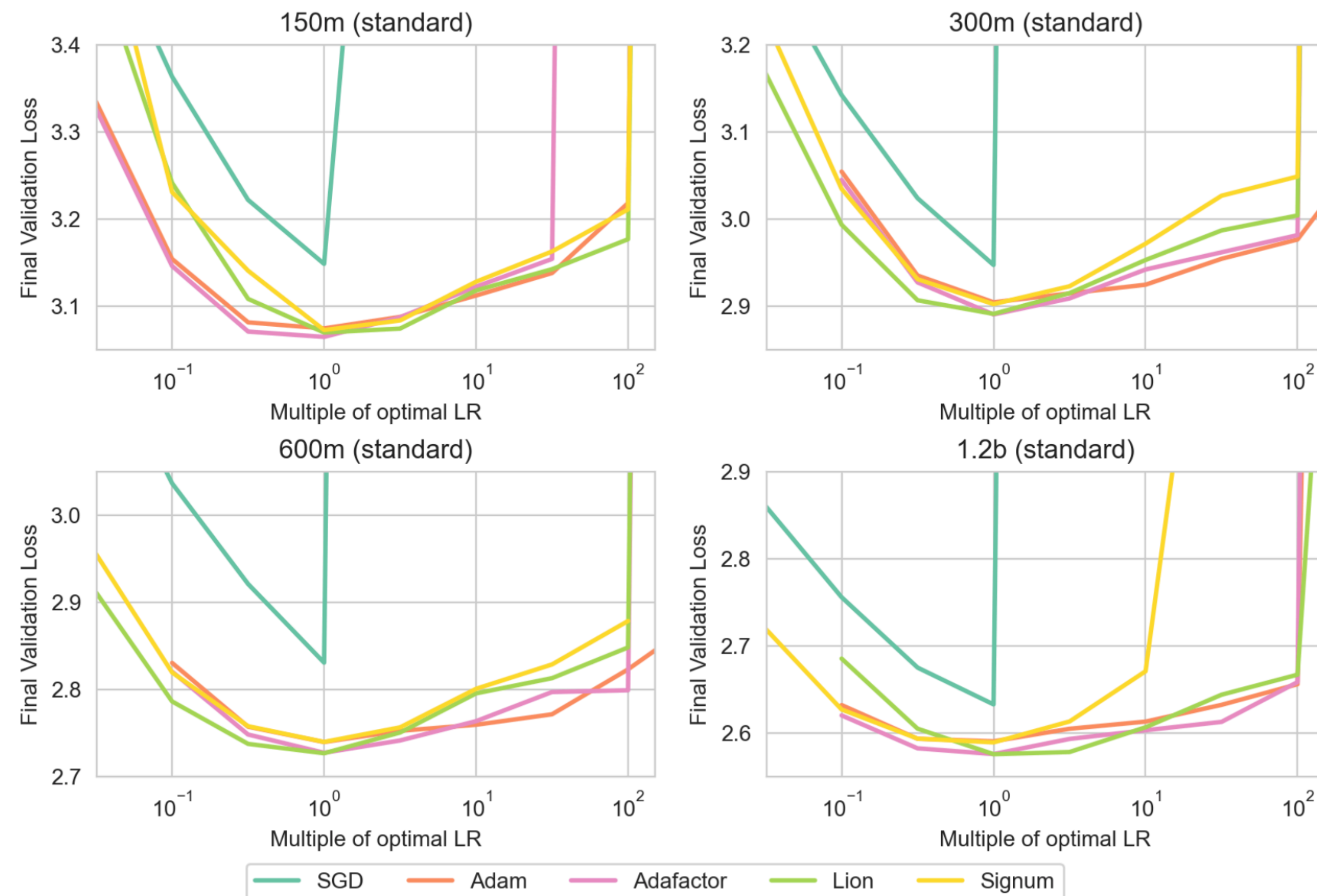
$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta} L(\theta^{(t)})$$

- There are many alternatives:

- PyTorch native.** AdaDelta, AdaFactor, AdaGrad, Adam AdamW, SparseAdam, AdaMax, ASGD, LBFGS, NAdam, RAdam, RMSProp, RProp, ...

- More recent.** Shampoo, Lion, Signum, ...

- Now.** Understand key concepts

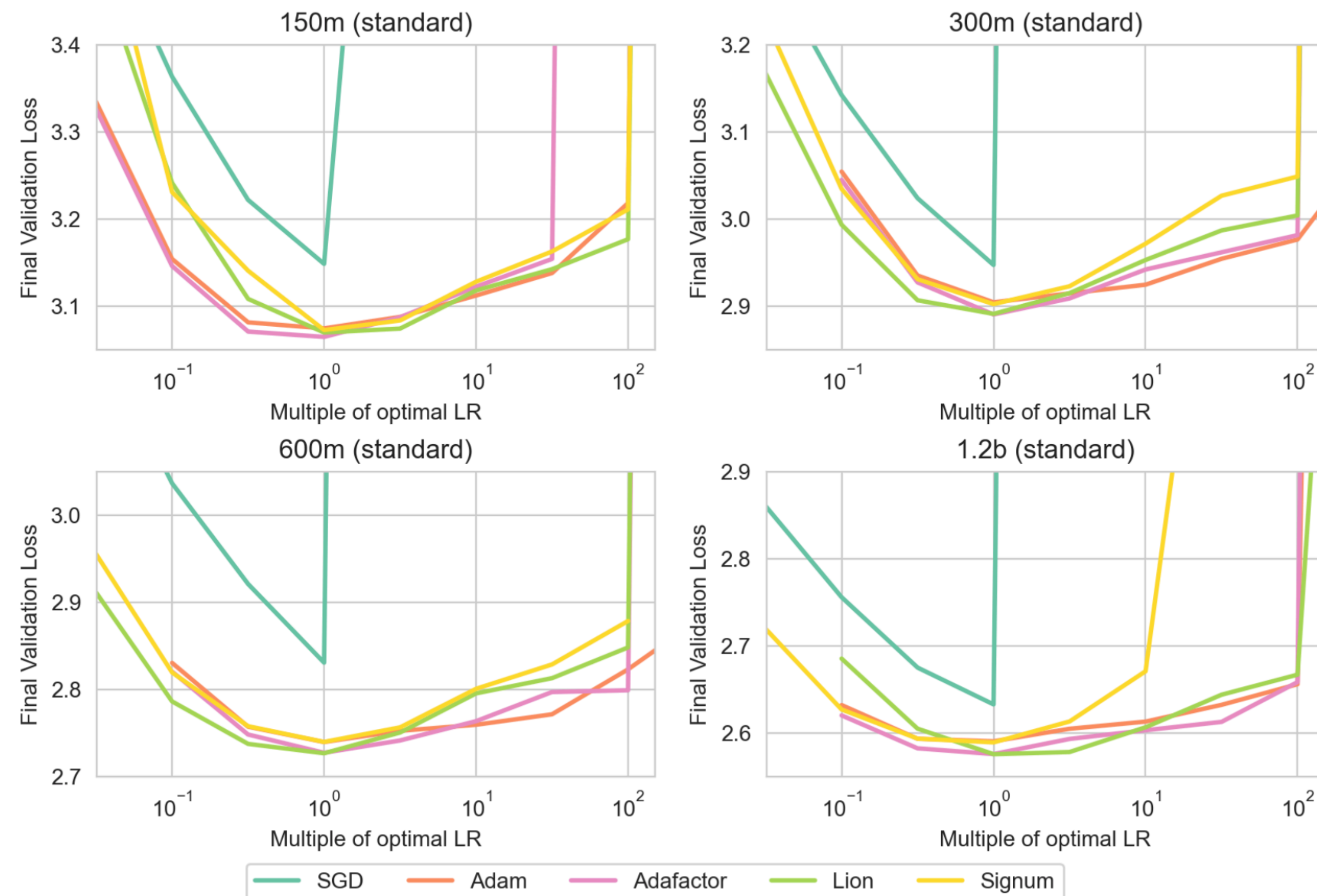


Optimizers

- We rarely use the “vanilla” version of the SGD

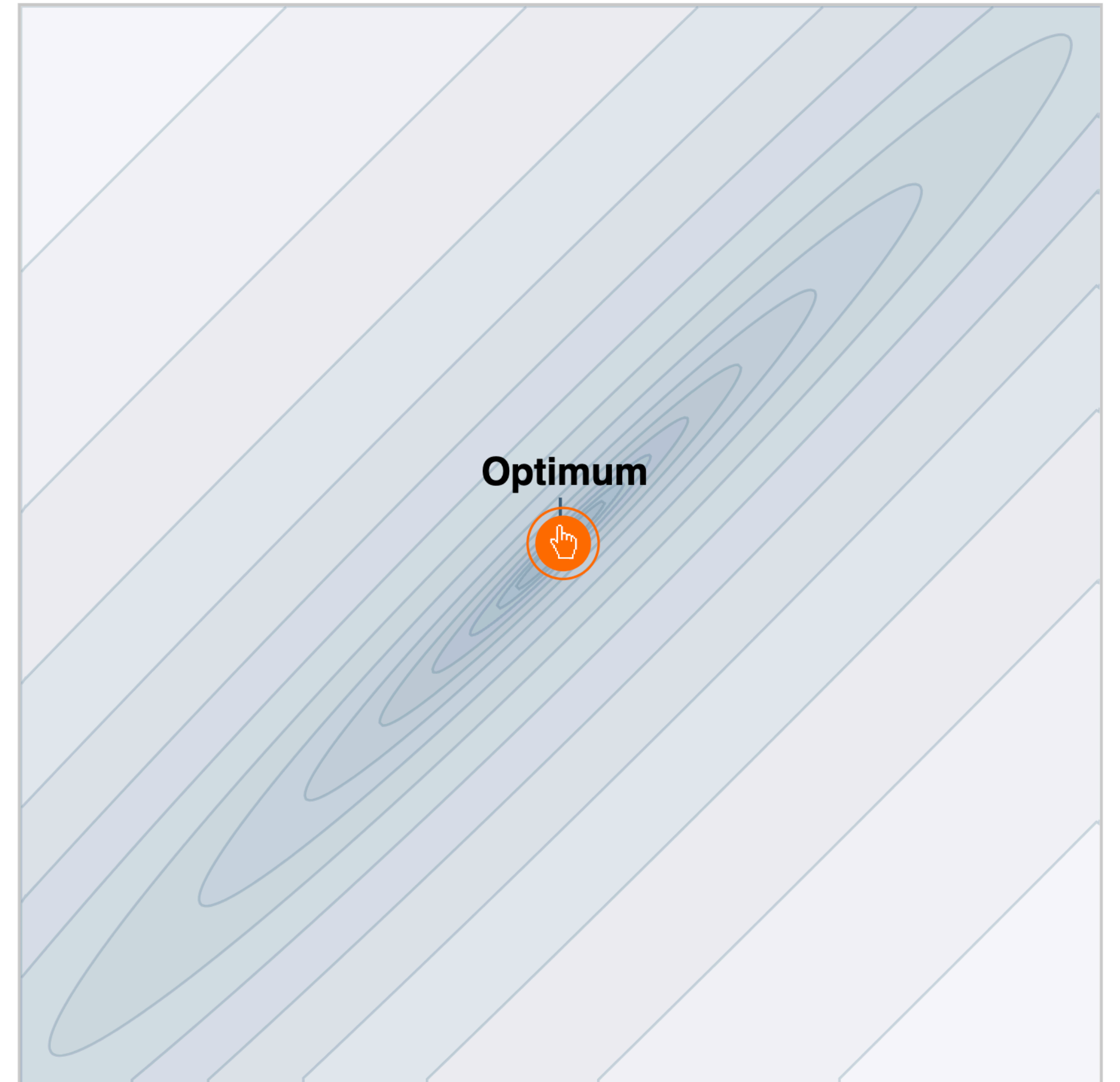
$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta} L(\theta^{(t)})$$

- There are many alternatives:
 - PyTorch native.** AdaDelta, AdaFactor, AdaGrad, Adam AdamW, SparseAdam, AdaMax, ASGD, LBFGS, NAdam, RAdam, RMSProp, RProp, ...
 - More recent.** Shampoo, Lion, Signum, ...
- Now.** Understand a key concept; momentum



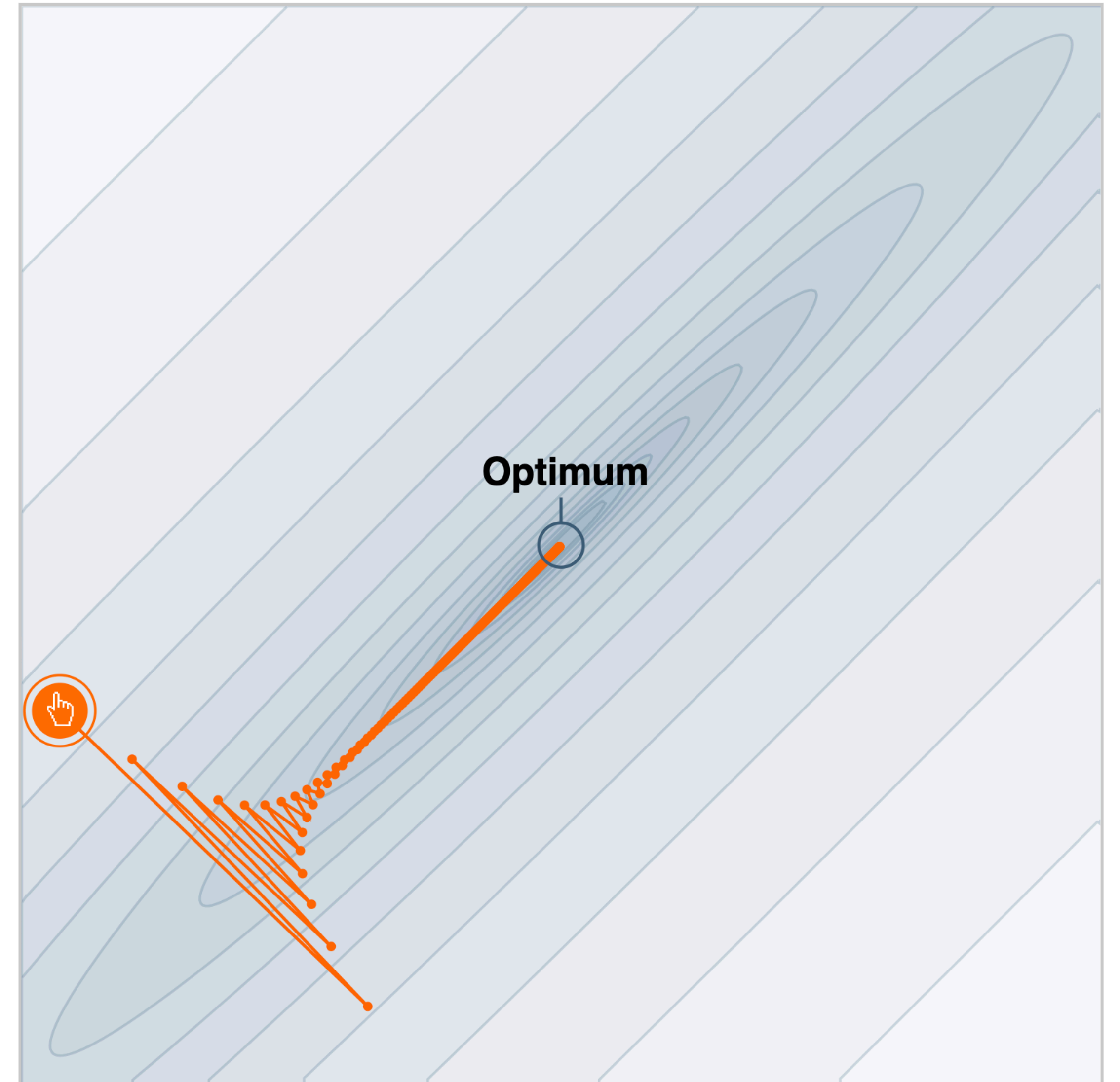
Momentum

- **Motivation.** Suppose that the risk
 - Changes fast in one direction
 - Changes slowly in another direction
- **Question.** What would happen?



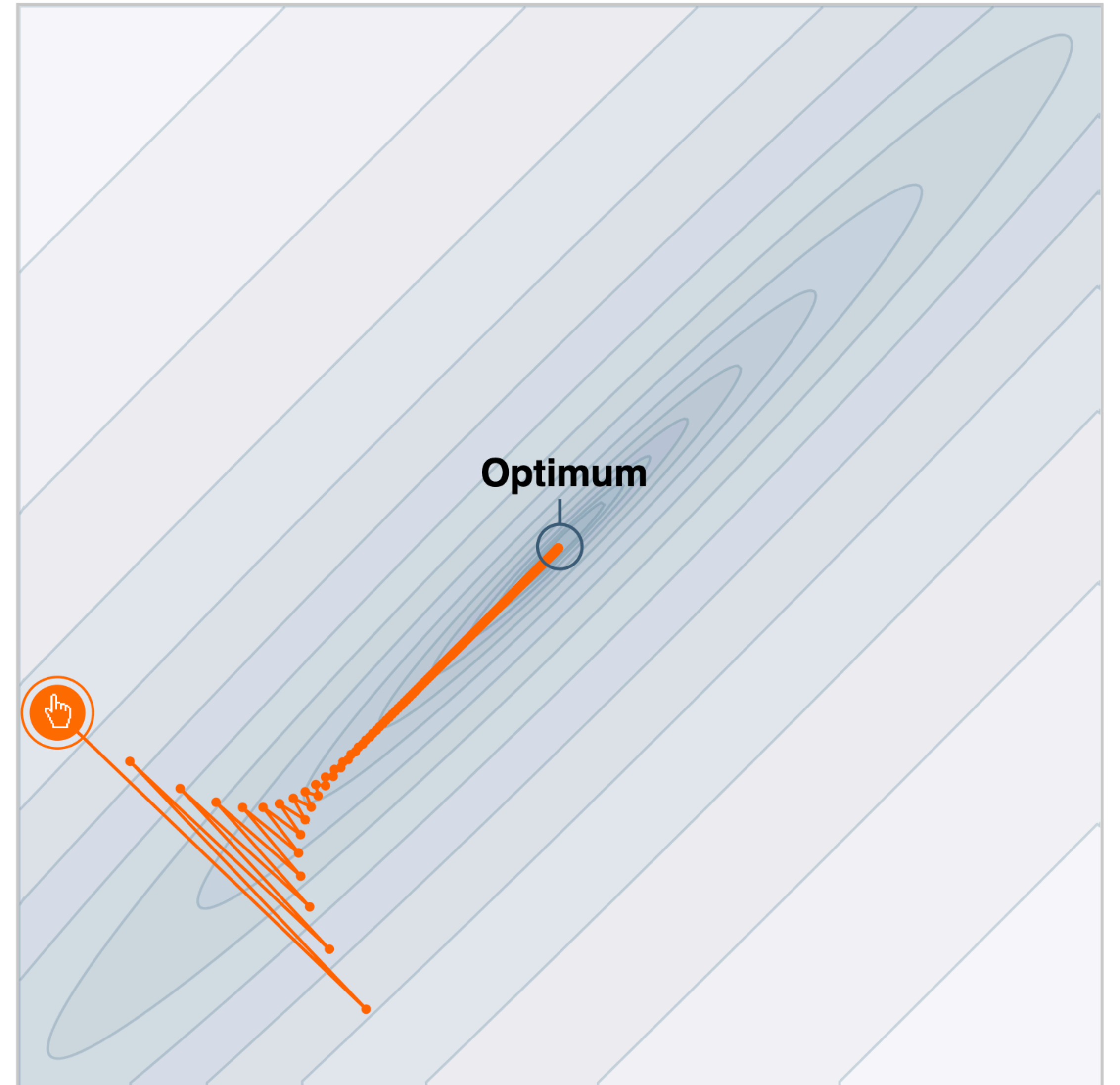
Momentum

- **Motivation.** Suppose that the risk
 - Changes fast in one direction
 - Changes slowly in another direction
- **Question.** What would happen?
- **Observation.** GD evolves as...
 - **Slow progress** in shallow direction
 - **High jitter** in steep direction
- Note. The loss has a large “condition number”



Momentum

- **Idea.** Let our GD have an **inertia**
 - If we were moving to one direction consistently, move more faster in that direction



Momentum

- **Idea.** Let our GD have an inertia
 - If we were moving to one direction consistently, move more faster in that direction

- Original GD

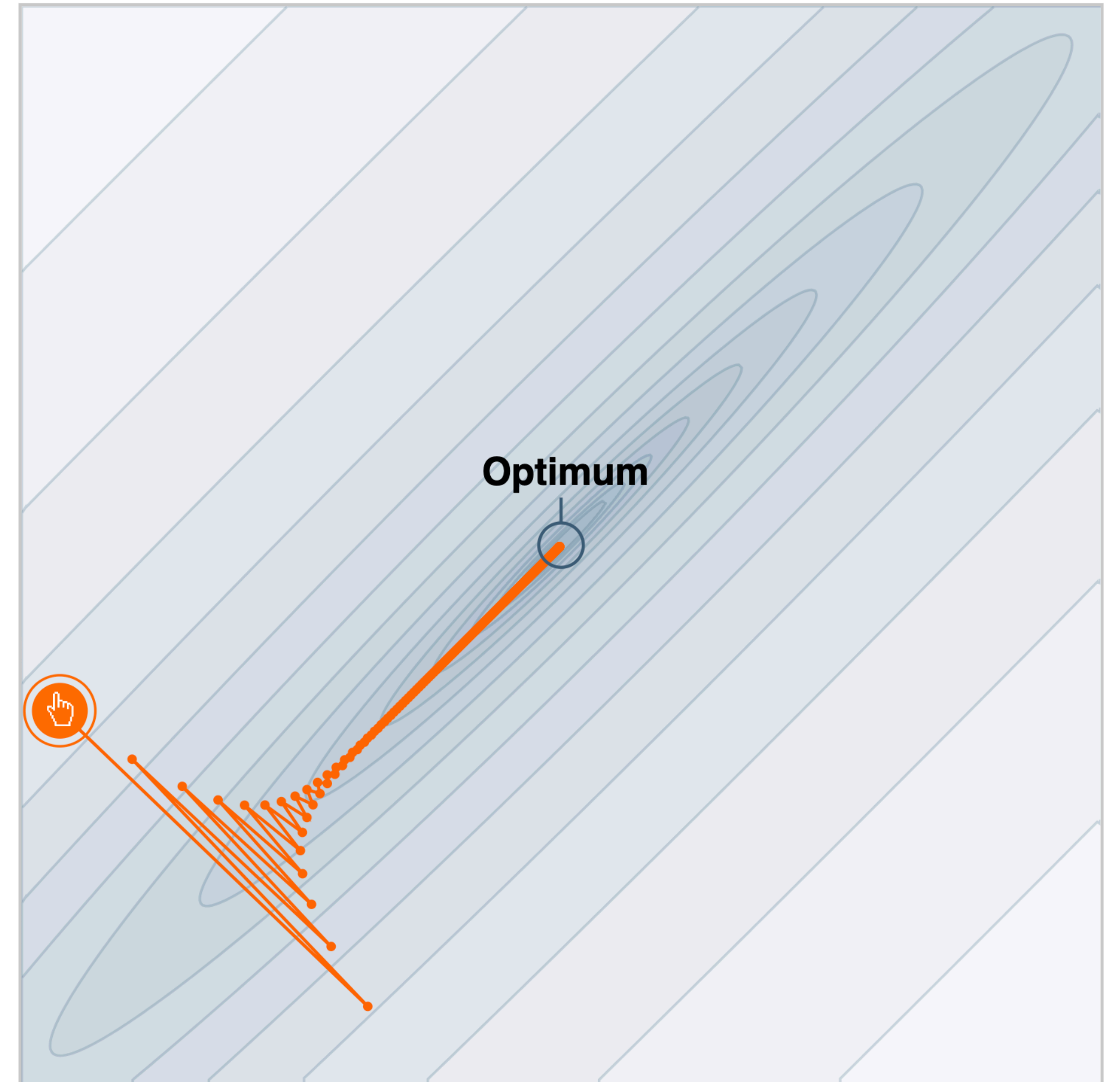
$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta} L(\theta^{(t)})$$

- GD + Momentum

$$v^{(t+1)} = \beta \cdot v^{(t)} + \nabla_{\theta} L(\theta^{(t)})$$

Accumulated gradients

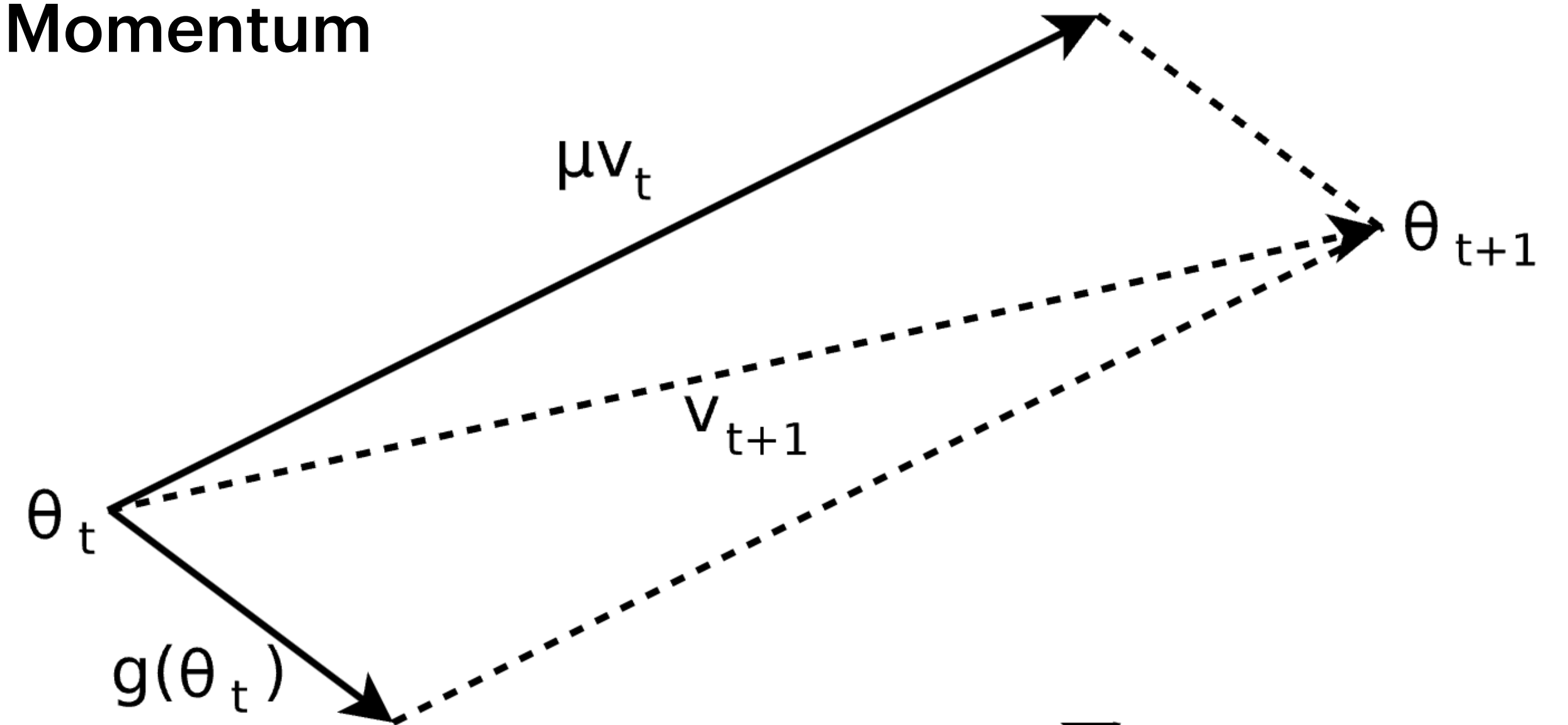
$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot v^{(t+1)}$$



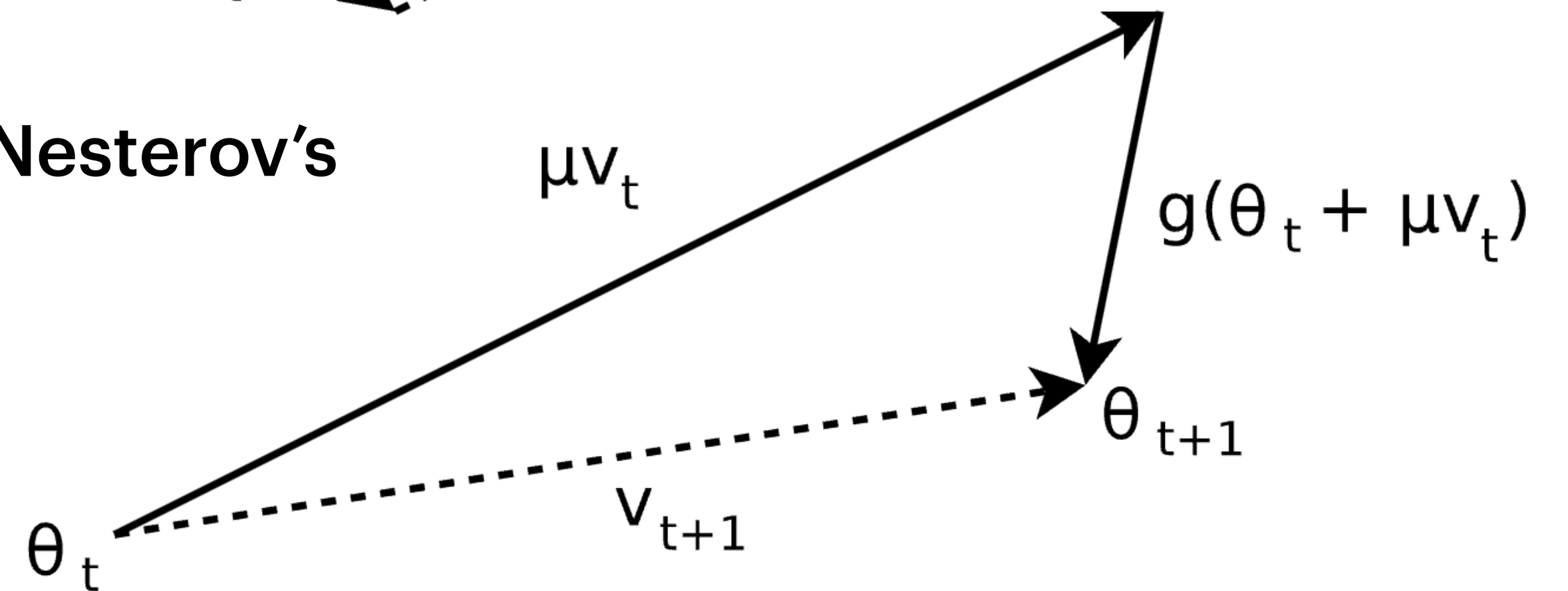
Nesterov's momentum

- A small fix to the momentum
- **Idea.** Evaluate the gradients at the **parameter + momentum**, not the current parameter
- Interpretation. Looking ahead any hardship that will come next

Momentum



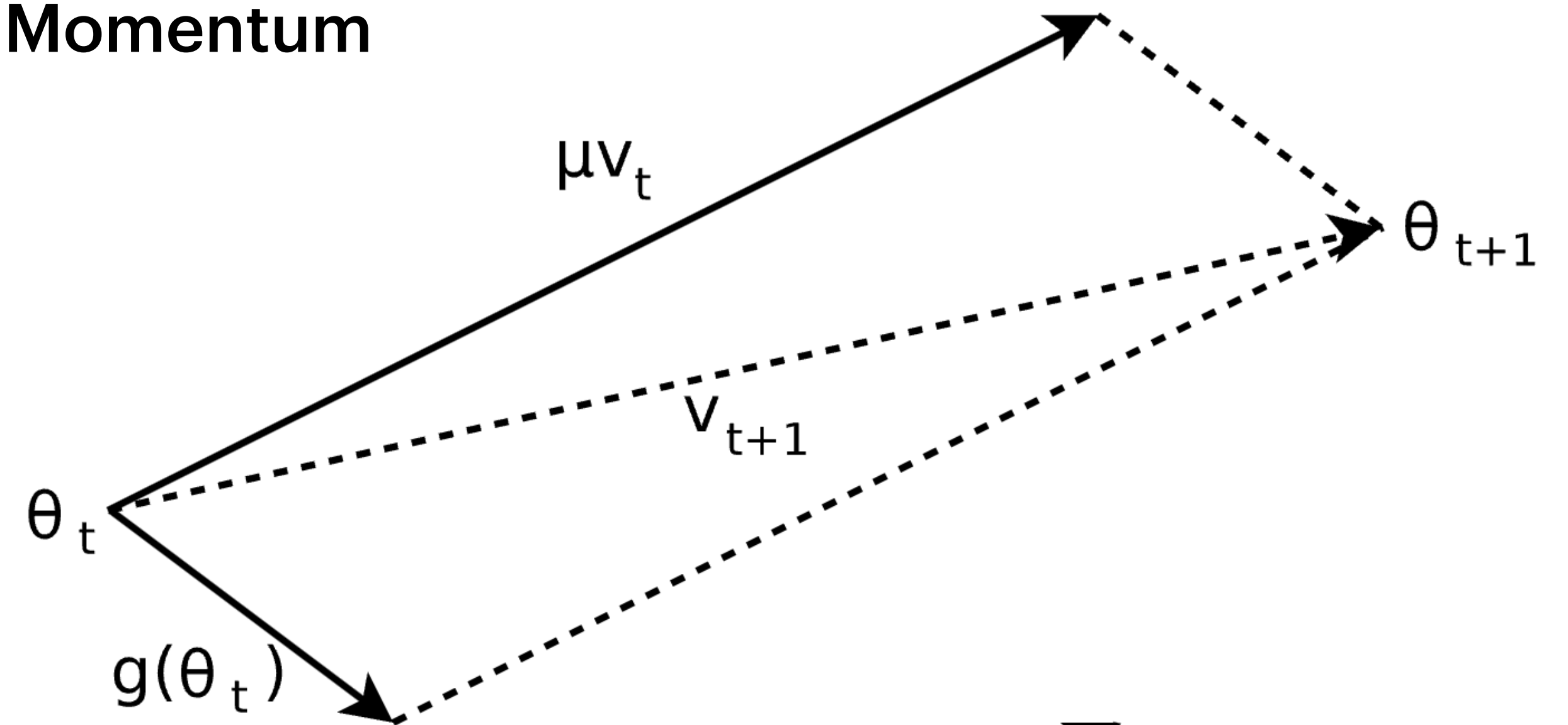
Nesterov's



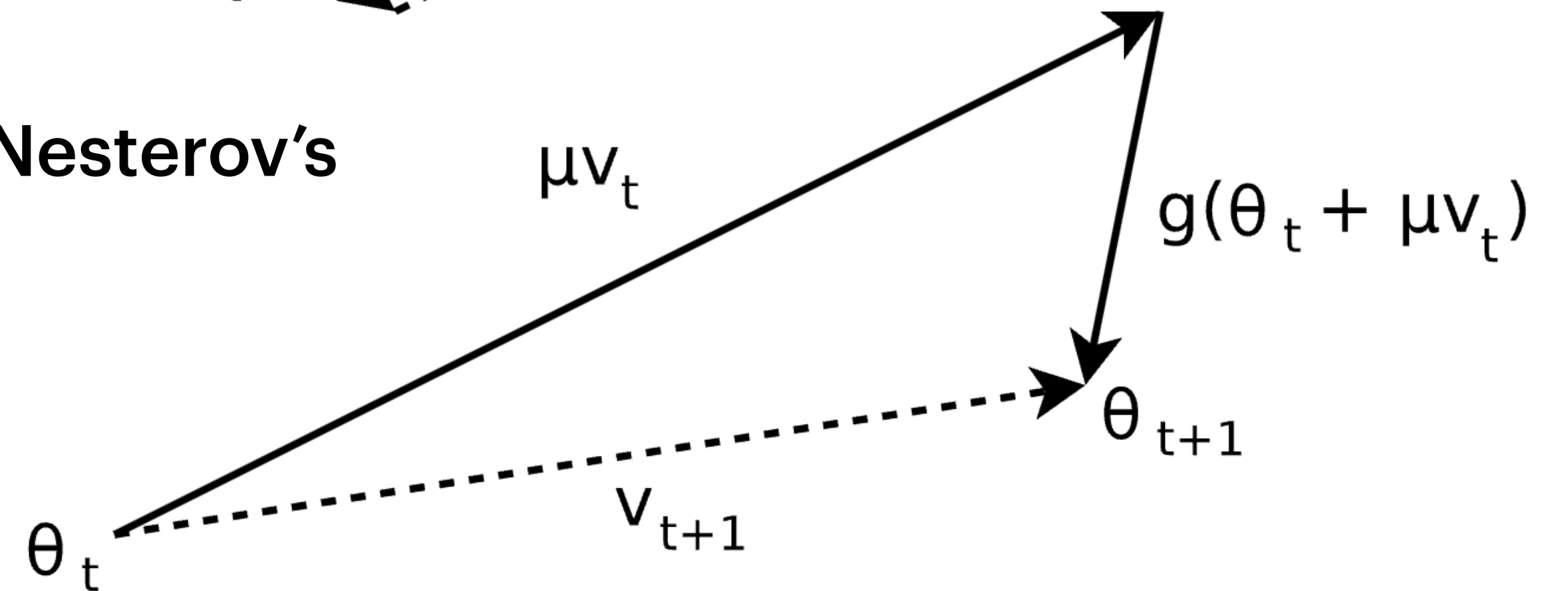
Nesterov's momentum

- A small fix to the momentum
- **Idea.** Evaluate the gradients at the parameter + momentum, not the current parameter
 - Interpretation. Looking ahead any hardship that will come next
- **Empirically.** Neat idea, but does not always guarantee a better convergence rate 😓
- Thus, try both

Momentum



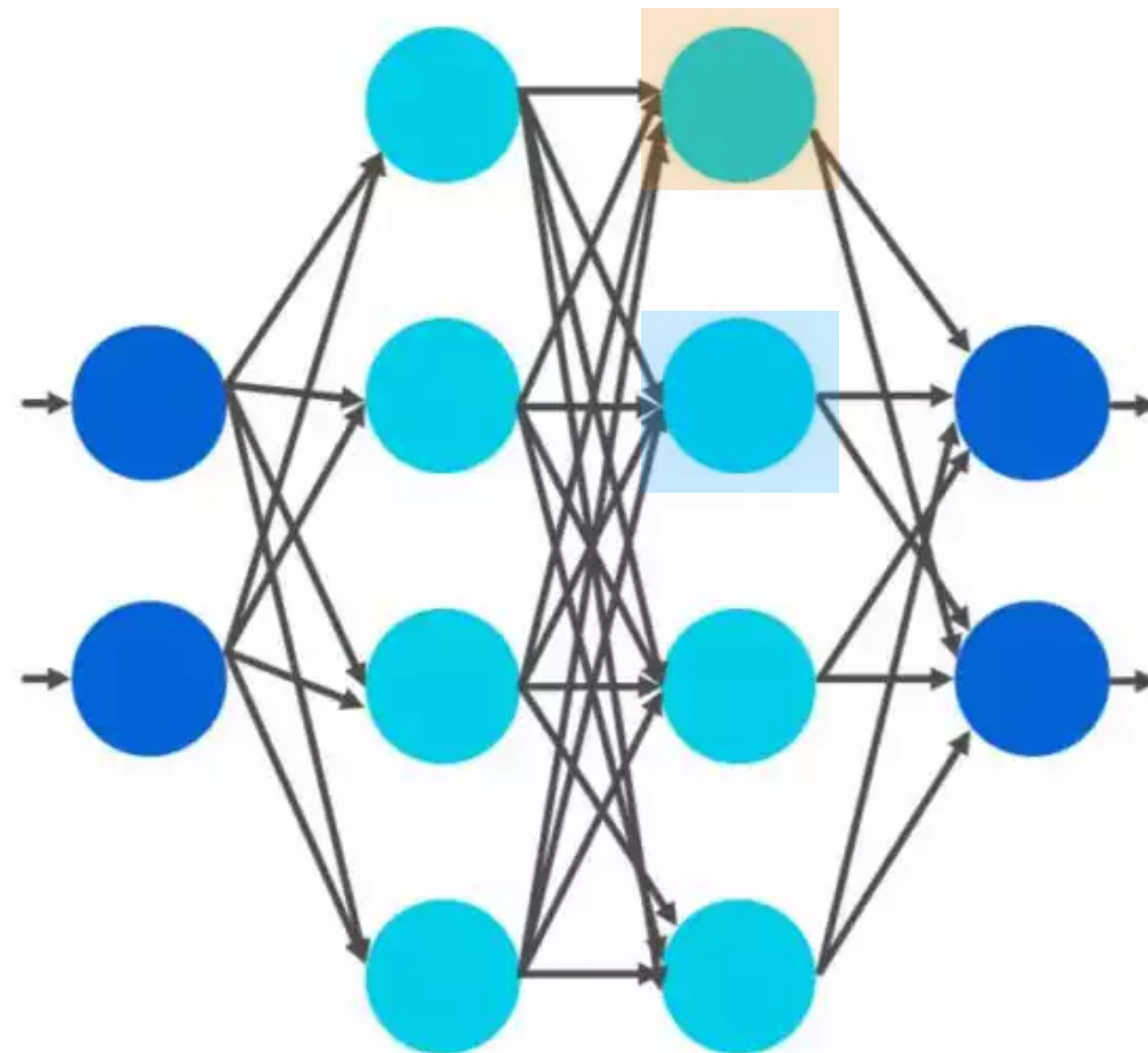
Nesterov's



Adaptive learning rate

- **Motivation.** Single learning rate may not work well for all parameters.
 - Example. Suppose that we have a “cat neuron” and a “dog neuron.”

If we see much less “dogs” than “cats,”
then maybe using **higher LR** for “dogs” will help run faster.



Adaptive learning rate

- **Motivation.** Single learning rate may not work well for all parameters.

- Example. Suppose that we have a “cat neuron” and a “dog neuron.”

If we see much less “dogs” than “cats,”
then maybe using higher LR for “dogs” will help run faster.

- **RMSProp.** Keep the moving average of gradient², and divide the LR by it (do this elementwise)

$$g^{(t+1)} = \gamma \cdot g^{(t)} + (1 - \gamma) \cdot (\nabla_{\theta} L(\theta^{(t)}))^2$$

$$\theta^{(t+1)} = \theta^{(t)} - \frac{\eta}{\sqrt{g^{(t+1)} + \epsilon}} \cdot \nabla_{\theta} L(\theta^{(t)})$$

tiny value, for avoiding division by zero

Adaptive learning rate

- **Motivation.** Single learning rate may not work well for all parameters.

- Example. Suppose that we have a “cat neuron” and a “dog neuron.”

If we see much less “dogs” than “cats,”
then maybe using higher LR for “dogs” will help run faster.

- **RMSProp.** Keep the moving average of gradient², and divide the LR by it.

$$g^{(t+1)} = \gamma \cdot g^{(t)} + (1 - \gamma) \cdot (\nabla_{\theta} L(\theta^{(t)}))^2$$

$$\theta^{(t+1)} = \theta^{(t)} - \frac{\eta}{\sqrt{g^{(t+1)} + \epsilon}} \cdot \nabla_{\theta} L(\theta^{(t)})$$

- **Adam.** RMSProp + Momentum (most cited paper in last 10 years)

Remarks

- **Memory.** *Optimizer states* should also be stored on memory!
 - For each parameter (32bits), we keep ...
 - Gradient (32bits)
 - Momentum (32bits)
 - Adaptive LR (32bits)
- **Tuning.** Advanced optimizers introduce *additional hyperparameters* to tune
 - Much training computation needed for the best performance

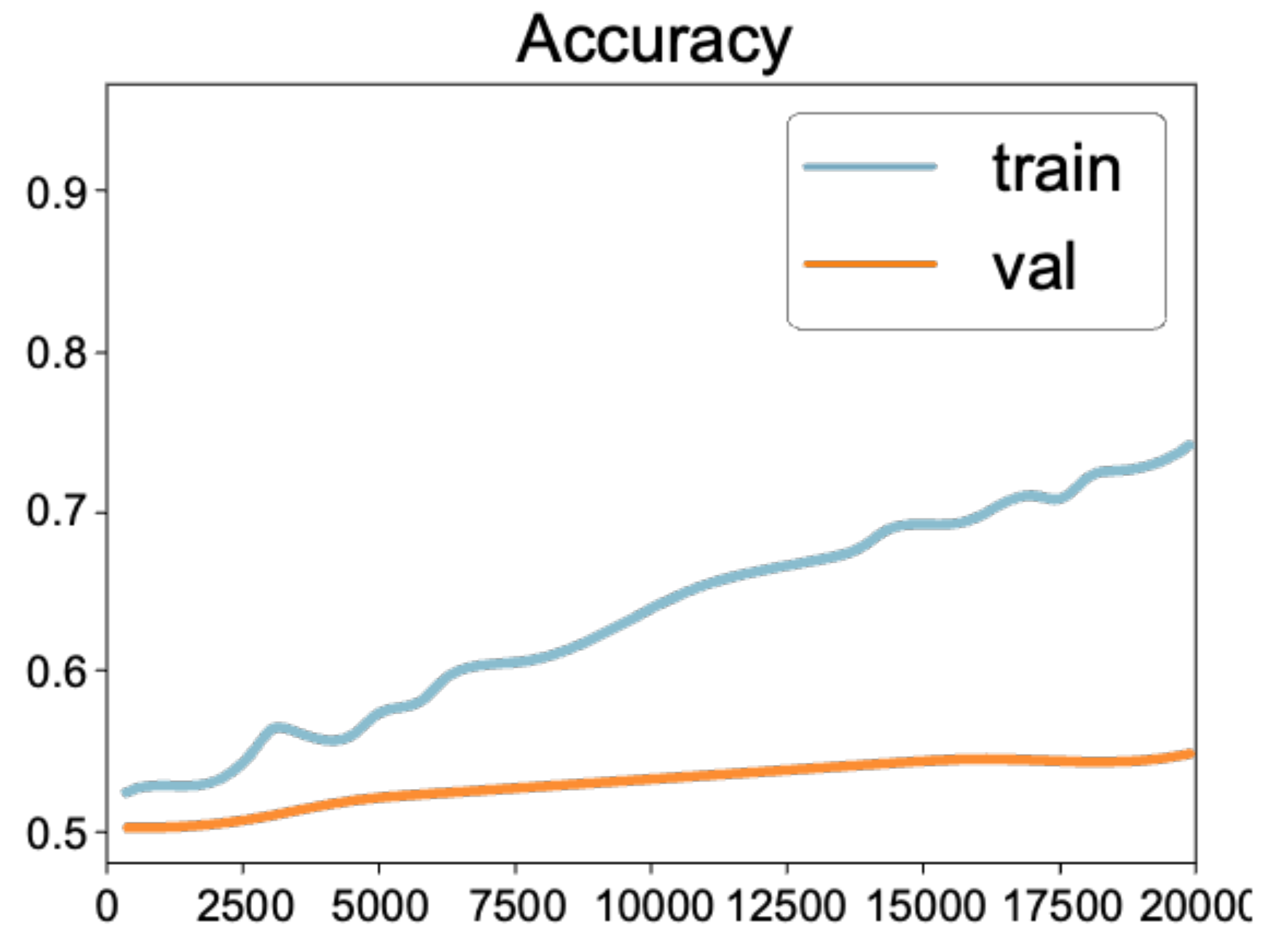
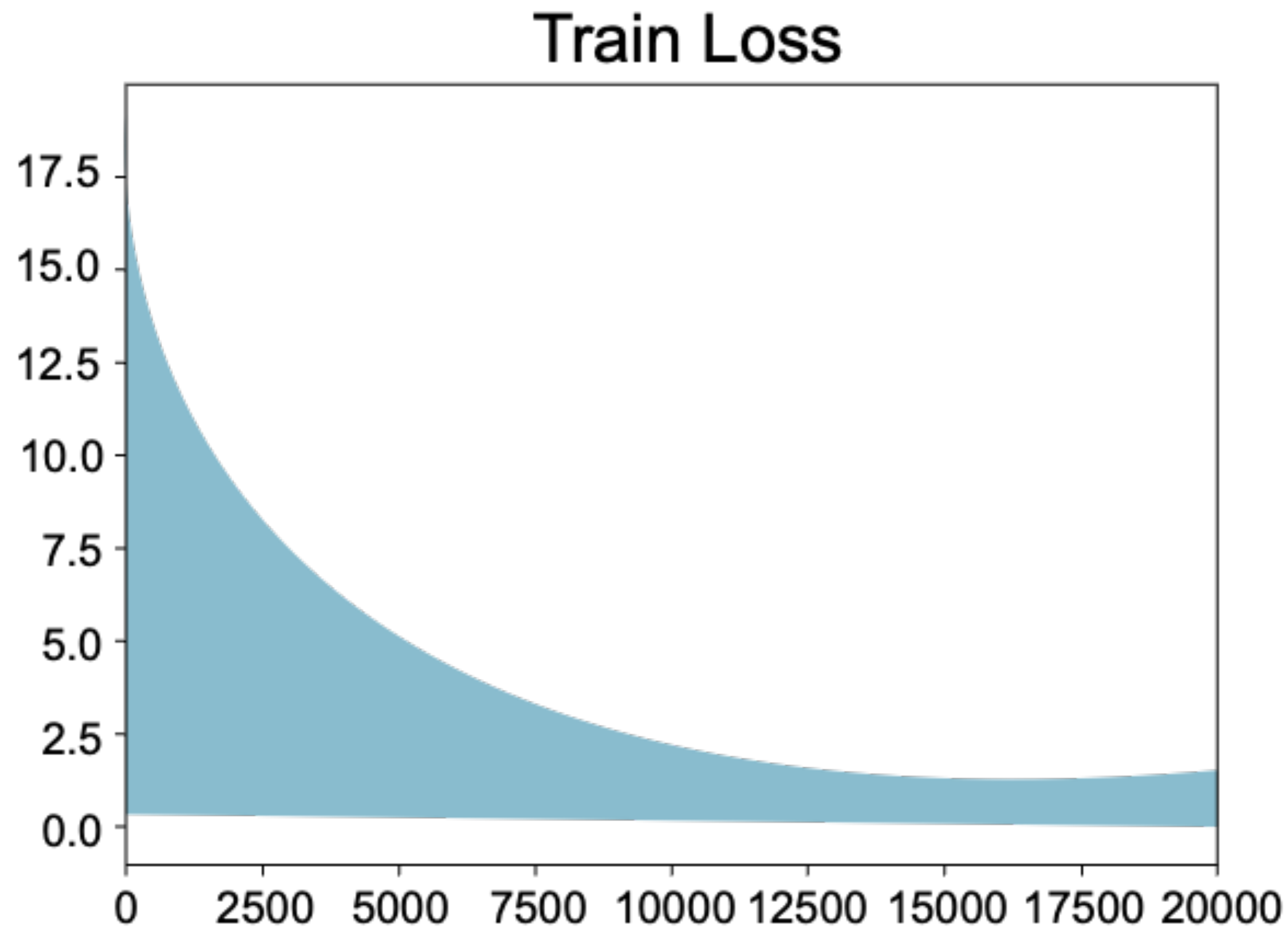
References

- **Momentum.** <https://distill.pub/2017/momentum/>
- **Adam.** <https://optimization.cbe.cornell.edu/index.php?title=Adam>
- **Others.** <https://cs231n.github.io/neural-networks-3/>

Regularization

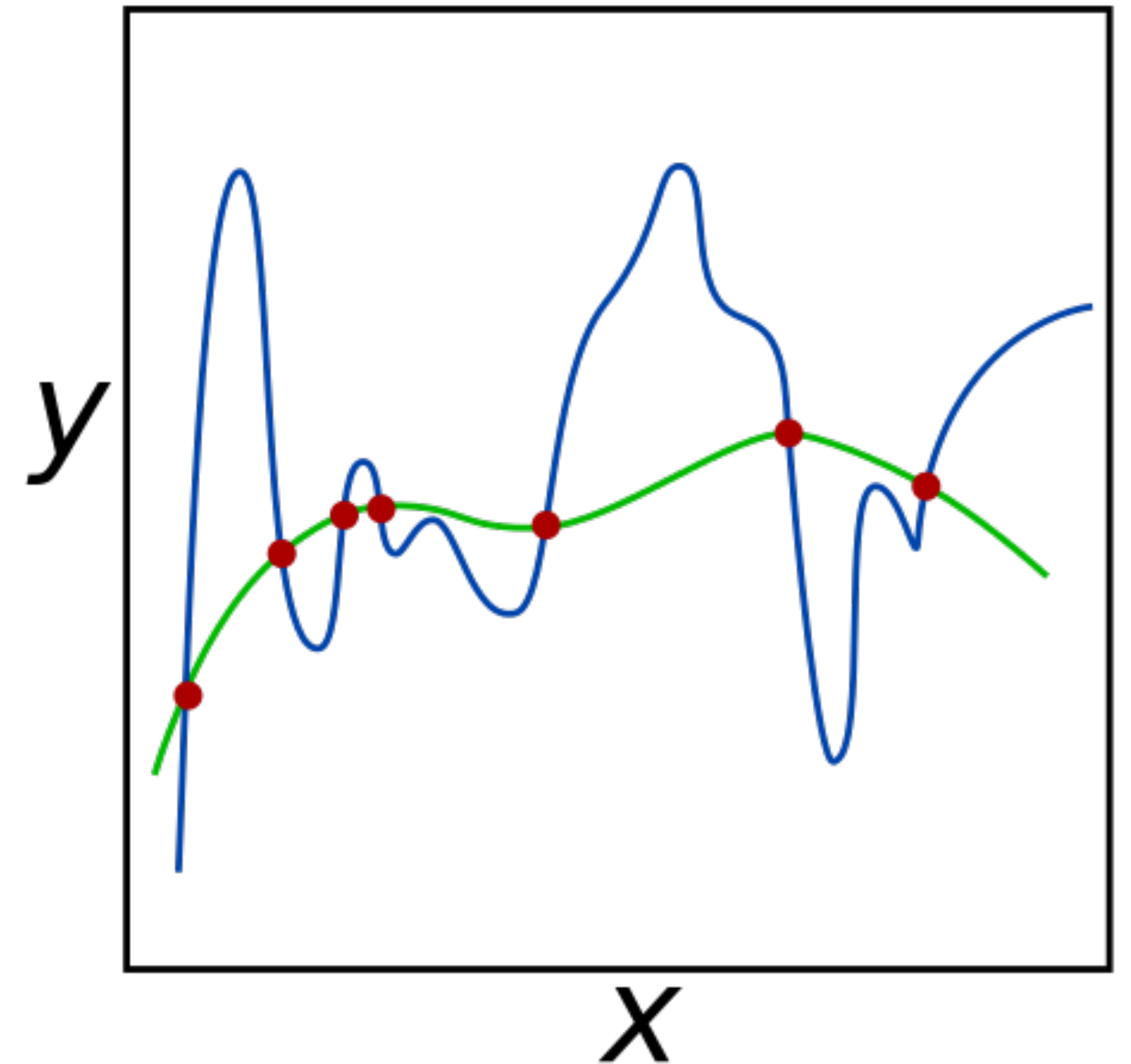
Beyond training error

- Better optimization algorithms help reduce the **training loss**
 - But we actually care about the **test performance** — how can we reduce the gap?



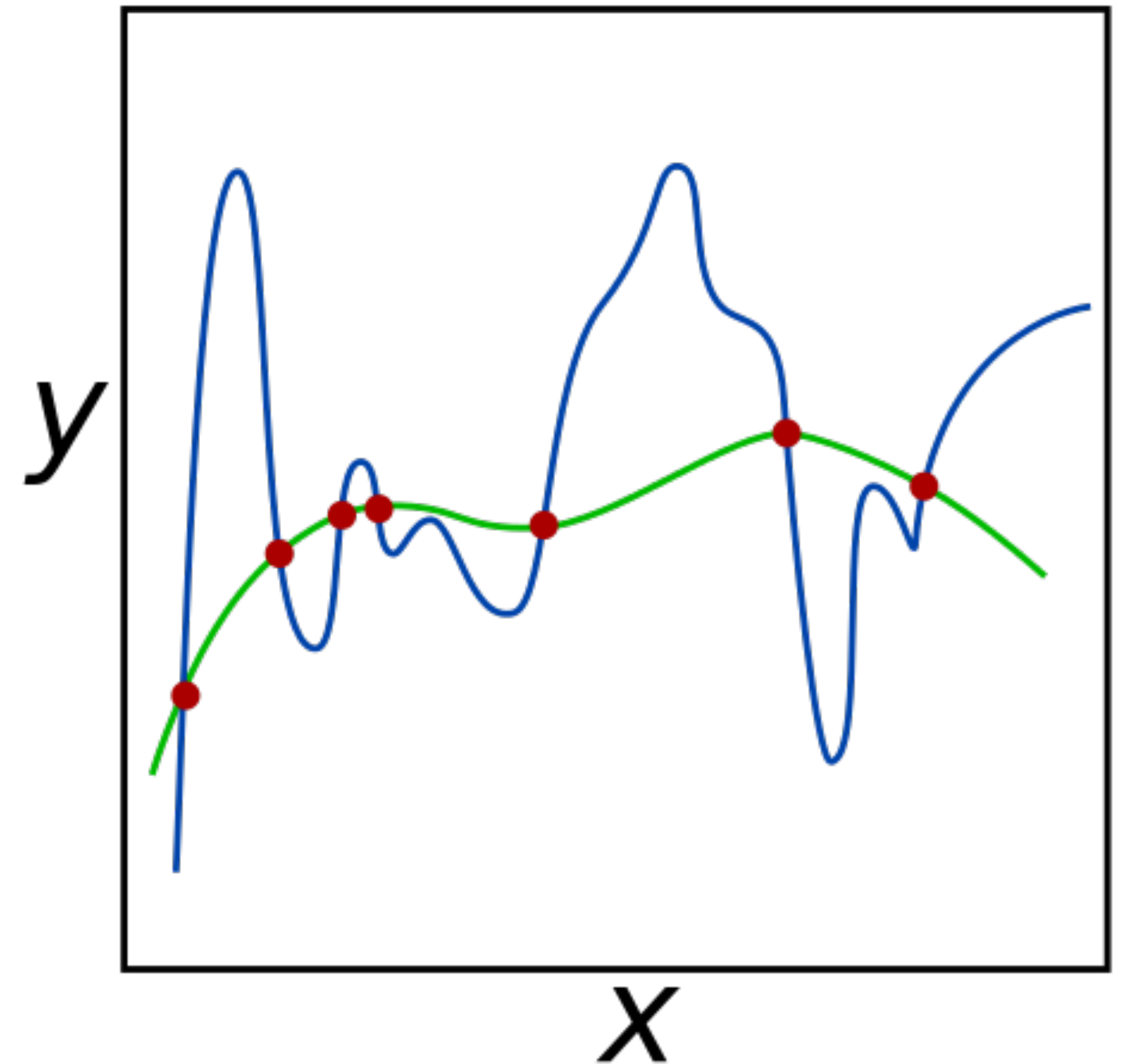
Beyond training error

- Better optimization algorithms help reduce the training loss
 - But we actually care about the test performance — how can we reduce the gap?
- **Core philosophy.** Most regularization methods follow the principle of **Occam's razor**
 - “Whenever possible, use simpler models”



Beyond training error

- Better optimization algorithms help reduce the training loss
 - But we actually care about the test performance — how can we reduce the gap?
- **Core philosophy.** Most regularization methods follow the principle of Occam's razor
 - “Whenever possible, use simpler models”
- Simplicity. Many definitions, including
 - Number of weight parameters
 - Norm of the weight parameters
 - Prediction confidence ...



Regularization (through loss)

- **Idea.** Add complexity to the loss function; doable for differentiable complexity measures

$$\frac{1}{n} \sum_{i=1}^n \ell(y_i, f_{\theta}(\mathbf{x}_i)) + \text{complexity}(\theta)$$

Regularization (through loss)

- **Idea.** Add complexity to the loss function; doable for differentiable complexity measures

$$\frac{1}{n} \sum_{i=1}^n \ell(y_i, f_{\theta}(\mathbf{x}_i)) + \text{complexity}(\theta)$$

- **Example.** L2 regularization; use smaller ℓ_2 norm solution, whenever possible.

$$\theta^{(t+1)} = \theta^{(t)} - \eta \cdot \nabla_{\theta}(L(\theta) + \lambda \cdot \|\theta\|_2)$$

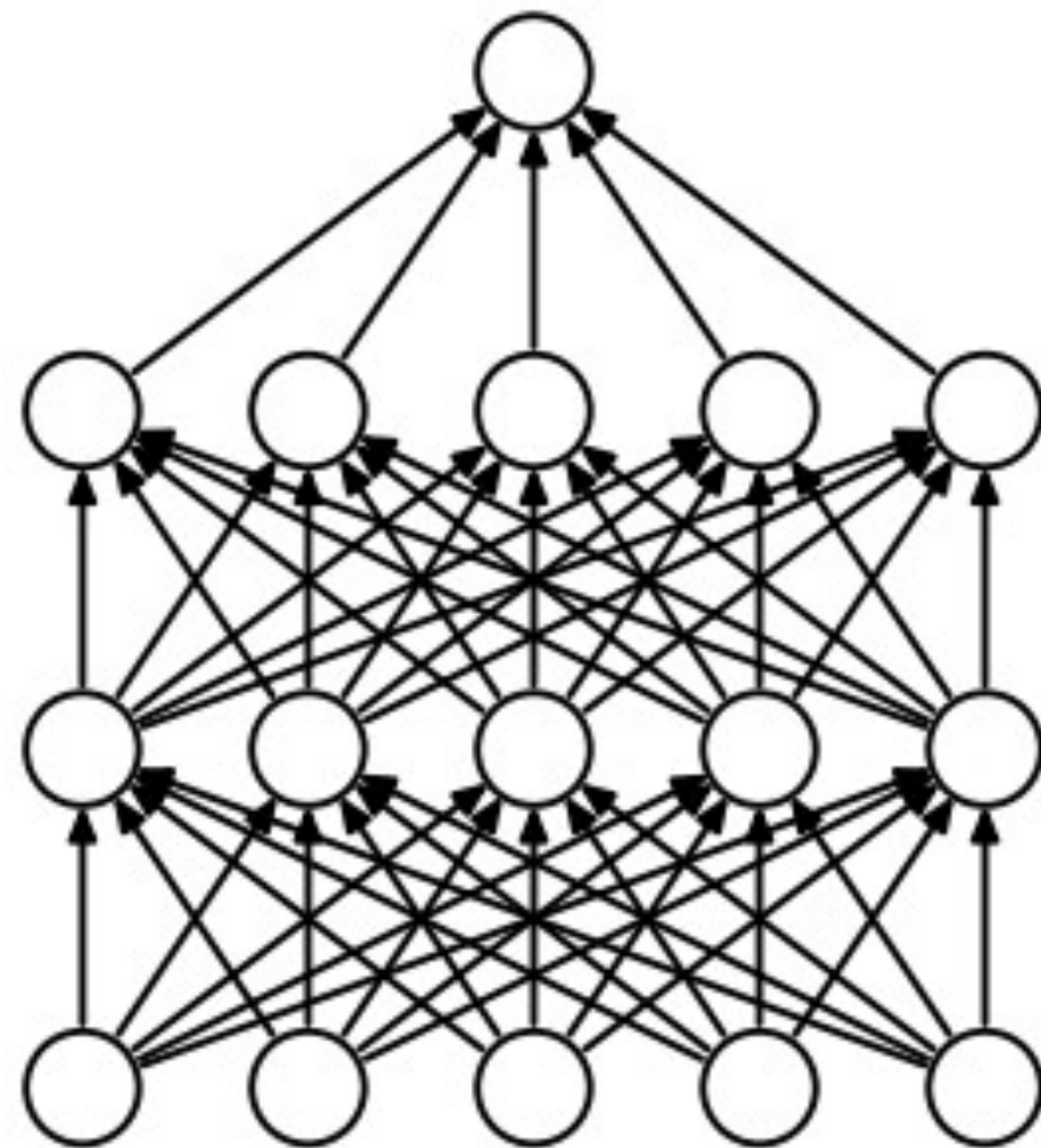
- This is equivalent to a simpler-to-implement form:

$$\theta^{(t+1)} = (1 - \eta\lambda)\theta^{(t)} - \eta \cdot \nabla_{\theta}L(\theta)$$

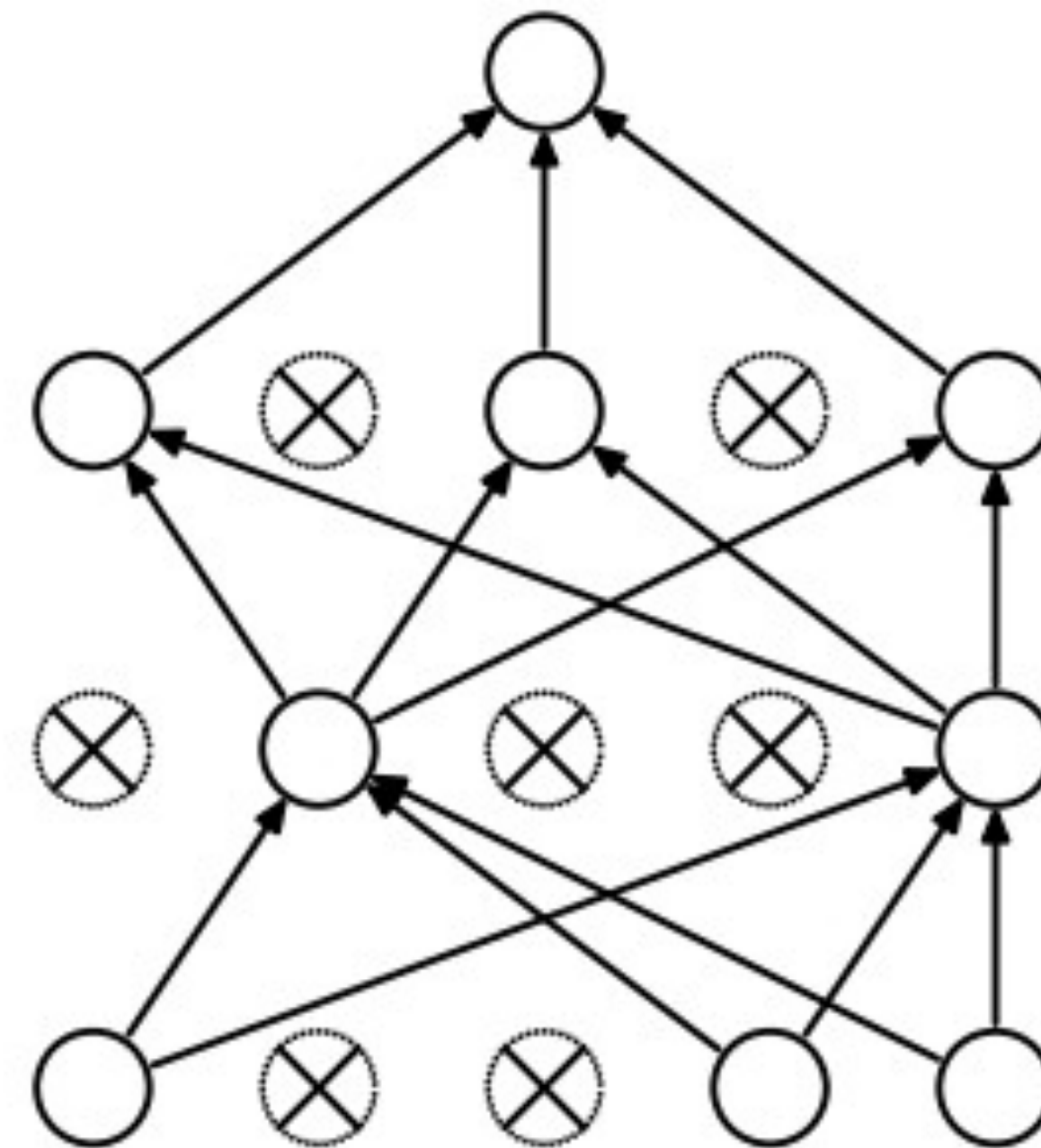
(thus often called “weight decay”)

Nondifferentiable complexities

- For non-differentiable complexities, design a customized algorithm.
- **Example.** Whenever possible, use a smaller number of parameters
 - Dropout. During the training, randomly remove each neuron, w.p. p
 - For the inference, rescale the weights back to $1/p$.



(a) Standard Neural Net



(b) After applying dropout.

Nondifferentiable complexities

- For non-differentiable complexities, design a customized algorithm.
- **Example.** Whenever possible, use a smaller number of parameters
 - Dropout. During the training, randomly remove each neuron, w.p. p
 - For the inference, rescale the weights back to $1/p$.
- **Example.** Whenever possible, use a parameter that can be discovered within a shorter time
 - Early stopping. Pause training when validation error does not drop anymore.

Remarks

- **Optimization.** Sometimes, regularization make the optimization easier.

- Example. Consider solving a least-square problem $\min_{\mathbf{w}} \|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2$

- Ordinary solution:

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X})^{-1} \mathbf{X}^\top \mathbf{y}, \quad (\text{no guarantee that } \mathbf{X}^\top \mathbf{X} \text{ is invertible})$$

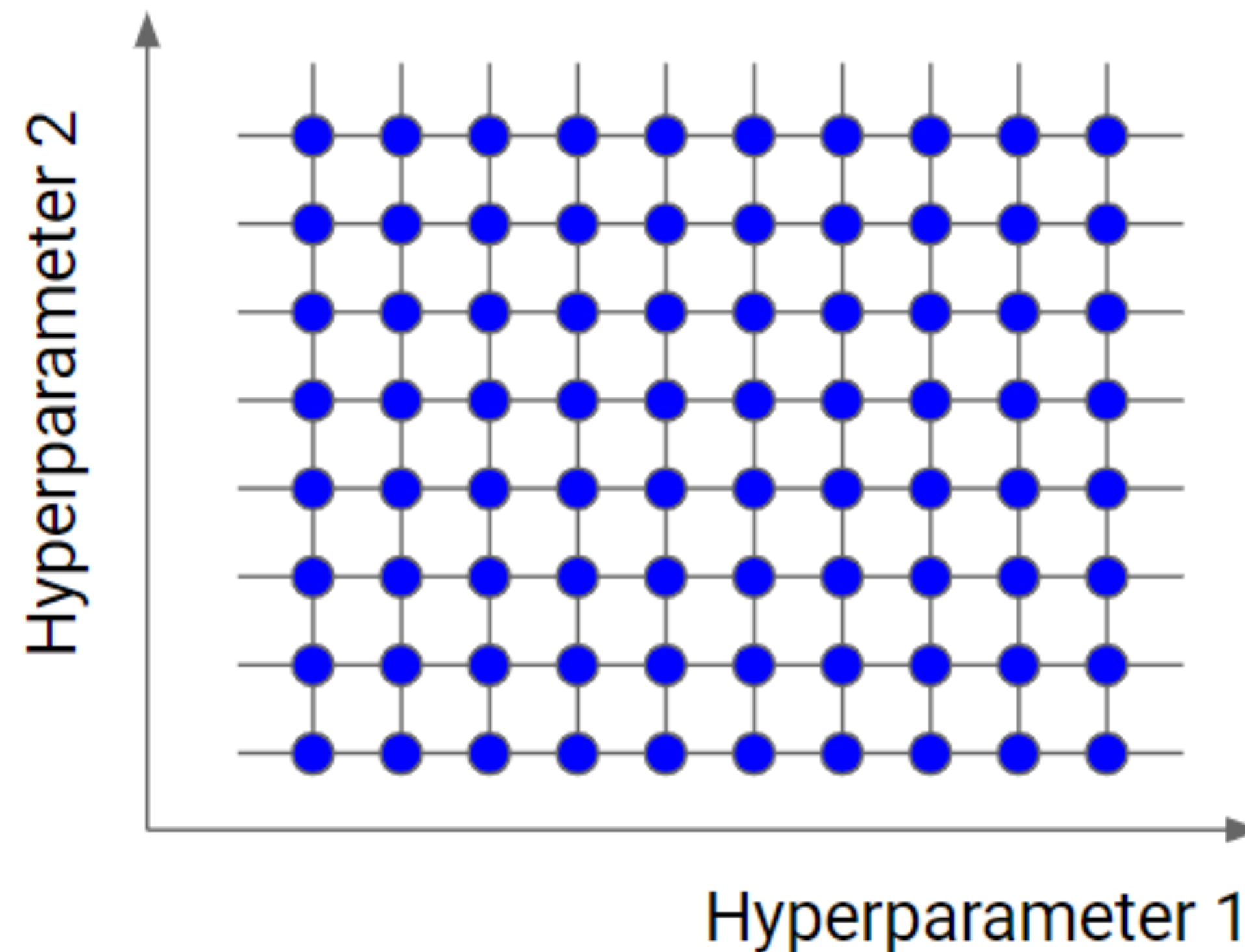
- Added ℓ_2 penalty: $\min_{\mathbf{w}} (\|\mathbf{y} - \mathbf{X}\mathbf{w}\|^2 + \lambda \|\mathbf{w}\|^2)$

$$\mathbf{w} = (\mathbf{X}^\top \mathbf{X} + \lambda \mathbf{I}_n)^{-1} \mathbf{X}^\top \mathbf{y} \quad (\text{invertible, if } \lambda \text{ is nonzero})$$

Hyperparameter tuning

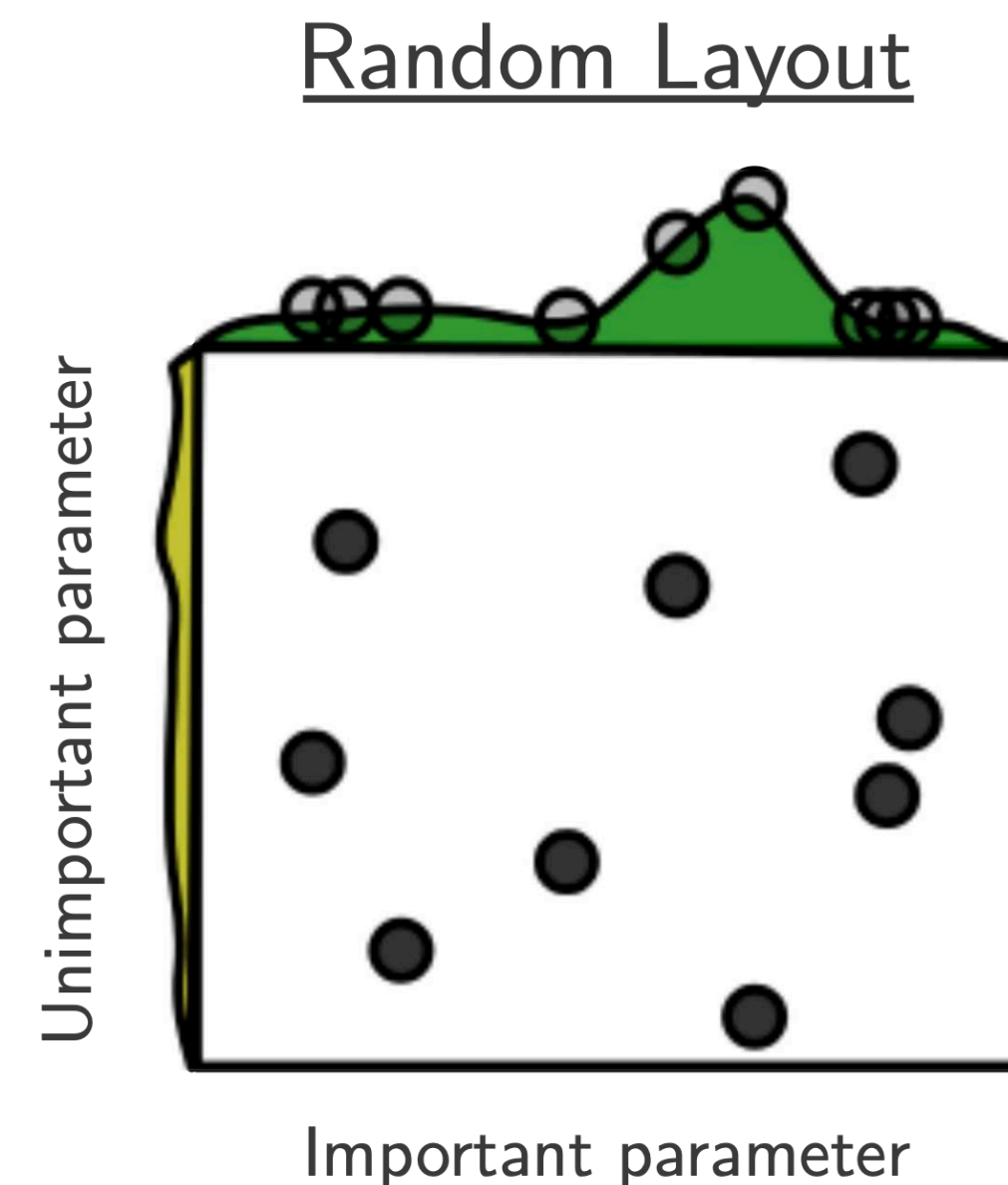
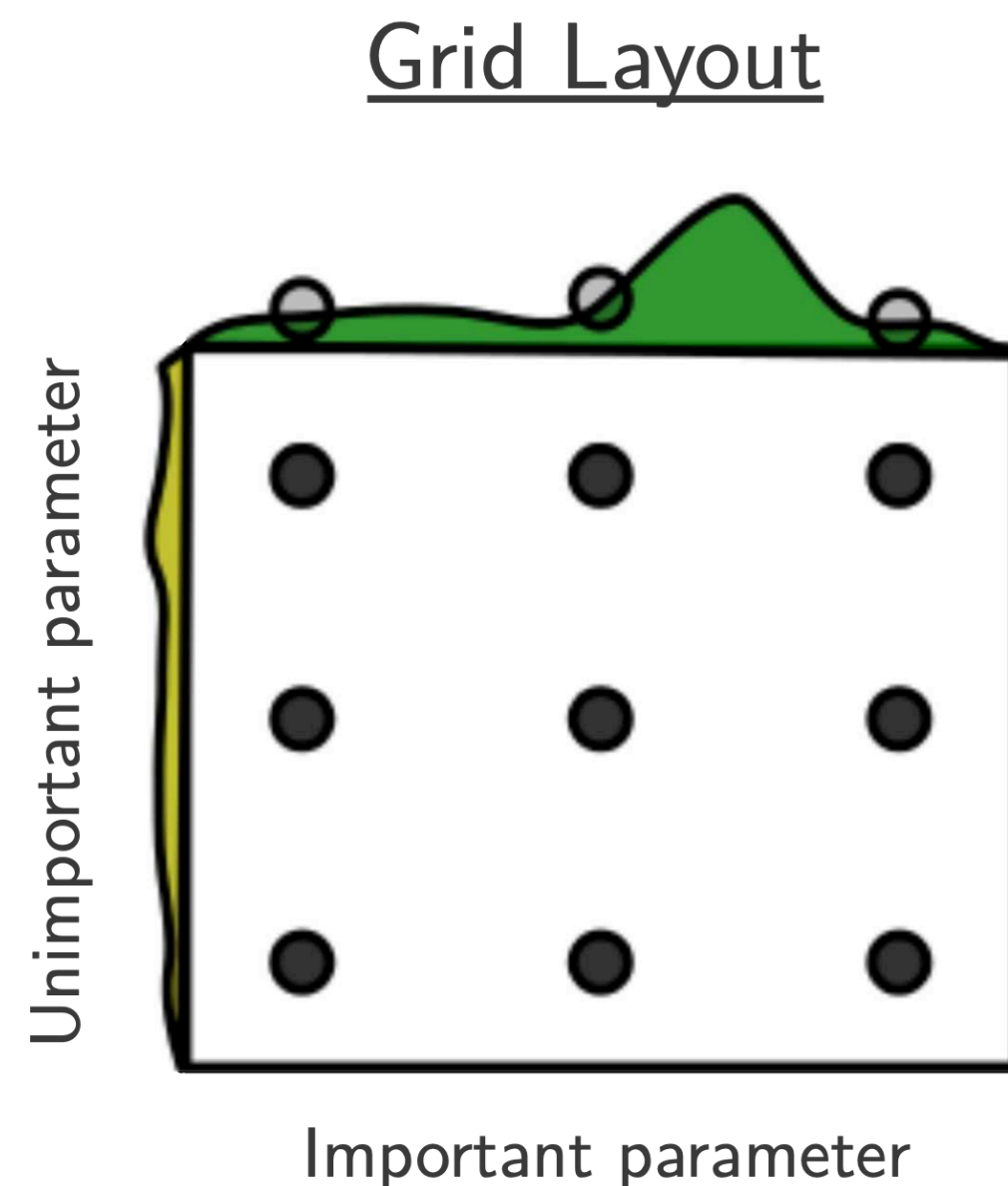
Strategy

- **Question.** How do we select the hyperparameter?
 - Grid search. Use coarse-to-fine grids, to reduce #trials.
 - Sometimes, use log-scales
(e.g., search LR from $\{10^{-2}, 10^{-3}, 10^{-4}, \dots\}$)



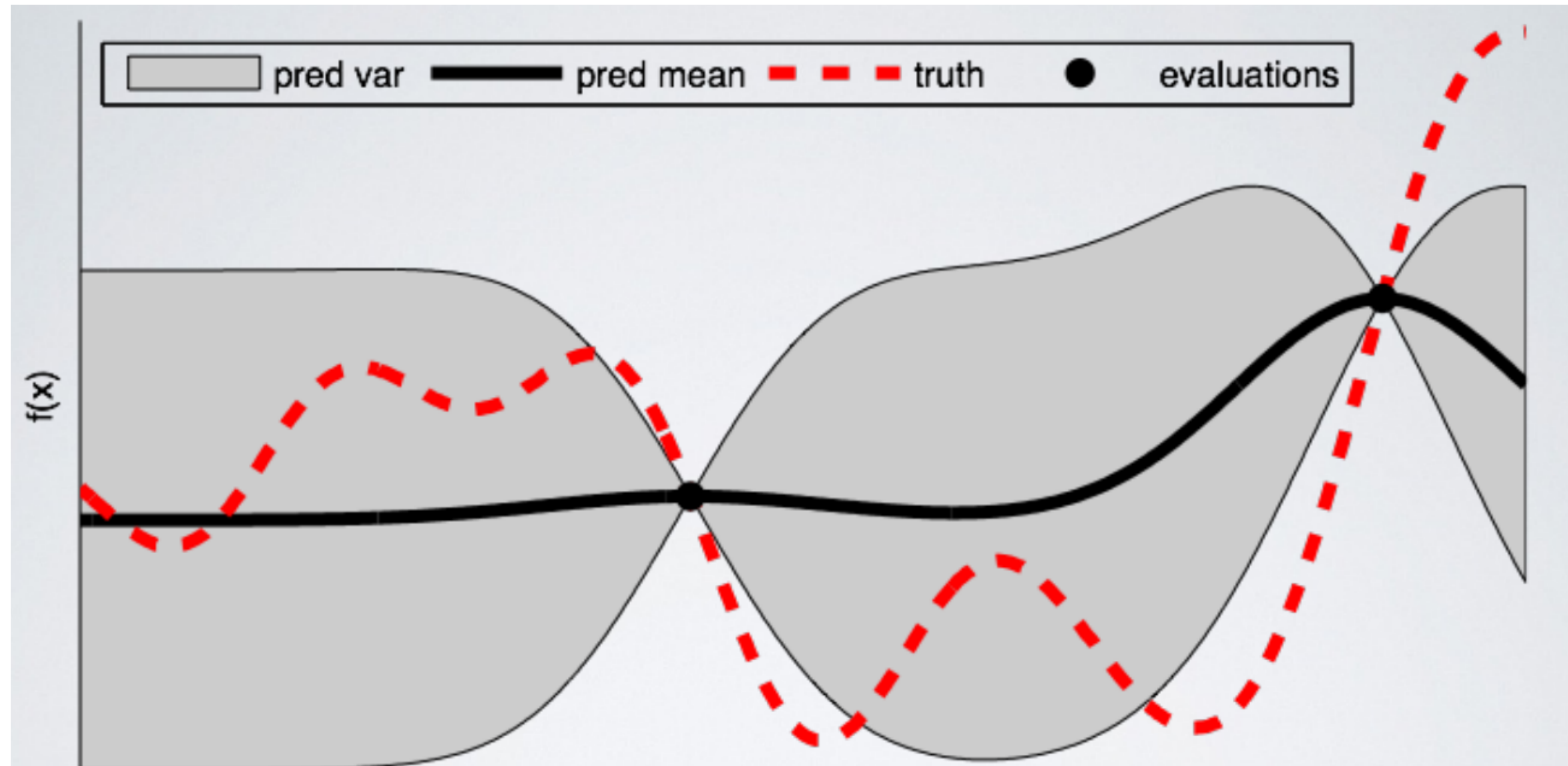
Strategy

- **Question.** How do we select the hyperparameter?
 - Grid search. Use coarse-to-fine grids, to reduce #trials.
 - Sometimes, use log-scales
(e.g., search LR from $\{10^{-2}, 10^{-3}, 10^{-4}, \dots\}$)
 - Random search. Use randomly sampled HPs
 - Has a larger “effective sample size”



Strategy

- **Sophisticated.** In some cases, we use Bayesian HP optimization techniques...
 - Idea. The performance-HP relationship may be a smooth function as well.
 - Predict the performance with Gaussian processes



Strategy

- Even more sophisticated. For LLMs, we **transfer** the hyperparameters.
 - Tune HPs on a small model, and use them on larger models
 - Requires a special parameterization (called μ -parameterization)



Figure 2: Illustration of μ Transfer

Cheers