HW4

Optimization

Jaeho, Chang

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Implement the followings. Here, denote the elementwise product as $*$.

1. $\sum_i \mathbf{a}_i^\top w_i \mathbf{b}_i$
2. $\sum_i y_i \mathbf{x}_i^\top \beta \mathbf{v}_i$

Solutions

1. Let A, B be $m \times n$ matrices. Then, we can write $A = [\mathbf{a}_1, \cdots, \mathbf{a}_n]$ and $B = [\mathbf{b}_1, \cdots, \mathbf{b}_n]$. So,

$$\sum_i \mathbf{a}_i^\top w_i \mathbf{b}_i = \text{rowSums}(A * w * B)$$

.

```
rm(list = ls())
m <- 8 ; n <- 7
A <- matrix(rbinom(m*n, 1, c(.5, .5)), nrow = m)
B <- matrix(rbinom(m*n, 1, c(.5, .5)), nrow = m)
w <- matrix(rep(1:n, m), nrow = m, byrow = T)
print(w)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]    1    2    3    4    5    6    7
## [2,]    1    2    3    4    5    6    7
## [3,]    1    2    3    4    5    6    7
## [4,]    1    2    3    4    5    6    7
## [5,]    1    2    3    4    5    6    7
## [6,]    1    2    3    4    5    6    7
## [7,]    1    2    3    4    5    6    7
## [8,]    1    2    3    4    5    6    7
```

```
print(A*B)
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]    0    0    0    1    0    0    0
## [2,]    0    1    1    0    0    1    1
## [3,]    0    0    1    0    0    0    0
## [4,]    1    1    0    0    0    0    0
## [5,]    0    0    1    1    1    0    0
## [6,]    0    0    0    0    0    0    0
## [7,]    1    0    1    0    0    0    0
## [8,]    0    0    0    0    0    0    1
```

```
print((A*w*B))
```

```
##      [,1] [,2] [,3] [,4] [,5] [,6] [,7]
## [1,]    0    0    0    4    0    0    0
## [2,]    0    2    3    0    0    6    7
## [3,]    0    0    3    0    0    0    0
```

```
## [4,]    1    2    0    0    0    0    0
## [5,]    0    0    3    4    5    0    0
## [6,]    0    0    0    0    0    0    0
## [7,]    1    0    3    0    0    0    0
## [8,]    0    0    0    0    0    0    7
```

```
print(rowSums((A*w*B)))
```

```
## [1]  4 18  3  3 12  0  4  7
```

2. Let X, V be $n \times p, p \times n$ matrix each and $\beta \in \mathbb{R}^p$. Then, we can write $X = \begin{bmatrix} \mathbf{x}_1^\top \\ \vdots \\ \mathbf{x}_n^\top \end{bmatrix}$ and $V = [\mathbf{v}_1, \cdots, \mathbf{v}_n]$.

   So, by this construction,

$$\sum_{i=1}^{n} \mathbf{v}_i (y_i \mathbf{x}_i^\top \beta) = V(\mathbf{y} * X\beta)$$

   .

```
rm(list = ls())
p <- 4 ; n <- 10
X <- matrix(rnorm(n*p), nrow = n)
V <- matrix(rnorm(p*n), nrow = p)
y <- 1:n
b <- 1:p

X
```

```
##             [,1]       [,2]        [,3]       [,4]
##  [1,] -0.6247397 -0.3404307 -0.28200856 -1.7673618
##  [2,]  0.1895947  1.0884902 -0.98366780 -0.1071718
##  [3,] -0.1720387  0.5268641  1.30649353  0.6198702
##  [4,] -0.7960356 -0.3641588  1.27955388  1.2531027
##  [5,] -0.1472626  0.6630641 -1.10989268  0.8486934
##  [6,]  0.6877962 -0.7971126 -0.22982087 -0.1327617
##  [7,]  0.7907681 -0.1062897  0.48078109  2.5848416
##  [8,]  0.5877268  0.8516989 -0.09014114  0.2326556
##  [9,] -0.9371826  1.5300621  1.13489084 -0.6085247
## [10,]  0.7669446 -0.4993664 -1.07443563 -0.9083465
```

```
V
```

```
##            [,1]       [,2]       [,3]        [,4]        [,5]       [,6]
## [1,] -0.1593051 -1.3474066  1.0992659 -2.31441930 -2.12430196  0.1284391
## [2,]  0.7563666 -0.1841082 -0.2940178  0.07563853 -0.04643338 -0.4107802
## [3,]  1.6581456 -0.7876686  0.4964714  0.85427316  2.00363818 -0.5960531
## [4,]  0.2268107  0.1850109  0.2136720  1.35892087  0.67490634 -1.5000500
##            [,7]       [,8]       [,9]      [,10]
## [1,] -1.9567085 -0.4662249 -1.4362125 -0.3551911
## [2,]  0.5139216  0.8751158  0.7869646  0.3450167
## [3,] -1.8218583  2.5367773  0.3485598 -0.2772278
## [4,] -0.6591133 -1.1635587  0.3957781 -0.7462532
```

```
y
```

```
##  [1]  1  2  3  4  5  6  7  8  9 10
```

```
b
```

```
## [1] 1 2 3 4
```

```
Xb <- X %*% b
Xb
```

```
##            [,1]
##  [1,] -9.221074
##  [2,] -1.013116
##  [3,]  7.280651
##  [4,]  7.326719
##  [5,]  1.243961
##  [6,] -2.126938
##  [7,] 12.359898
##  [8,]  2.951324
##  [9,]  3.093515
## [10,] -7.088481
```

```
y*Xb
```

```
##             [,1]
##  [1,]  -9.221074
##  [2,]  -2.026231
##  [3,]  21.841953
##  [4,]  29.306878
##  [5,]   6.219805
##  [6,] -12.761630
##  [7,]  86.519289
##  [8,]  23.610590
##  [9,]  27.841637
## [10,] -70.884812
```

```
V %*% (y*Xb)
```

```
##            [,1]
## [1,] -249.58072
## [2,]   56.72682
## [3,]  -26.12041
## [4,]   44.78610
```