1 LDA Likelihood

$$
\begin{aligned}
\pi := P(C=2),\ C-1 \sim Bern(\pi),\ C \in \{1,2\} \\
P(C=c) = \pi^c (1-\pi)^{1-c}
\end{aligned}
$$

$$
\begin{aligned}
f_1(\mathbf{x}_i) := P(\mathbf{x}_i|C_i=1) = |\det(2\pi\Sigma)|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{x}_i-\boldsymbol{\mu}_1)'\Sigma^{-1}(\mathbf{x}_i-\boldsymbol{\mu}_1)\right\} \\
f_2(\mathbf{x}_i) := P(\mathbf{x}_i|C_i=2) = |\det(2\pi\Sigma)|^{-\frac{1}{2}} \exp\left\{-\frac{1}{2}(\mathbf{x}_i-\boldsymbol{\mu}_2)'\Sigma^{-1}(\mathbf{x}_i-\boldsymbol{\mu}_2)\right\}
\end{aligned}
$$

Let $\boldsymbol{\beta} := \Sigma^{-1}(\boldsymbol{\mu}_2 - \boldsymbol{\mu}_1)$.

$$
\begin{aligned}
P(\mathbf{x}_i, c_i) = P(\mathbf{x}_i|c_i)P(c_i) &= \begin{cases} f_2(\mathbf{x}_i)\pi & ,\ c_i = 2 \\ f_1(\mathbf{x}_i)(1-\pi) & ,\ c_i = 1 \end{cases} \\
&= \{\pi f_2(\mathbf{x}_i)\}^{c_i-1}\{(1-\pi)f_1(\mathbf{x}_i)\}^{2-c_i}
\end{aligned}
$$