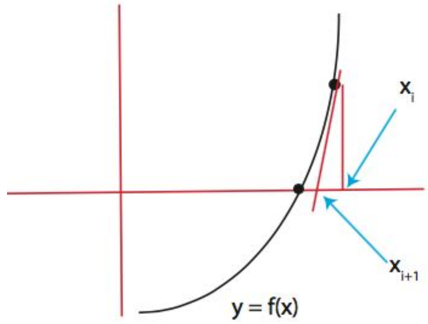
Lec11_Integer arithmetic, Karatsuba

Newton's Method

Find root of f(x) = 0 through successive approximation e.g. f(x) = x^2 -a



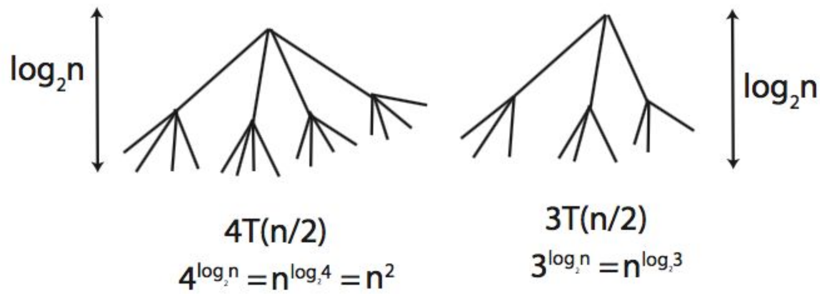
Tangent at (xi,(f(xi)) is line y = f(x) + f'(xi) * (x-xi) where f'(x) is the derivative. Xi+1 = intercept on x-axis

$$\boxed{x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}}$$

High Precision Multiplication

- Multiplying two n-digit numbers (radix r = 2,10)
- **Calculation**
  0 ≤ x, y < r^n
  x = x1 ·r^(n/2) +x0 x1 = high half
  y = y1·r^(n/2)+y0 x0=low half
  0 ≤ x0,x1<r^(n/2)
  0 ≤ y0,y1<r^(n/2)
  z=x·y=x1y1 ·r^n +(x0 ·y1 +x1 ·y0)*r^(n/2) +x0 ·y0
  → 4 multiplications of half-sized #'s ⇒ quadratic algorithm Θ(n^2) time
     T(n) = 4(T(n/2)) + Θ(n)

Karatsuba’s Method



- **Calculation**
  z0=x0·y0
  z2=x1·y1
  z1= (x0 +x1)·(y0 +y1)−z0 −z2 = x0y1+x1y0
  z = z2·rn+z1·rn/2+z0
- **Running Time**
  T(n) : time to multiply two n digit #’s
  = 3T(n/2) + Θ(n)
  = Θ(n^lg(3)) = Θ(n^1.5849625⋯) < Θ(n^2)