Lec1_Introduction and Peak Finding

Peak Finder

One-dimensional Version

- Question

    Position 2 is a peak if and only if $b \geq a$ and $b \geq c$. Position 9 is a peak if $i \geq h$.

    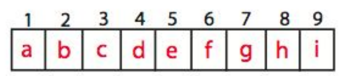

    Figure 1: a-i are numbers

    Problem: Find a peak if it exists (Does it always exist?)

- Answer

    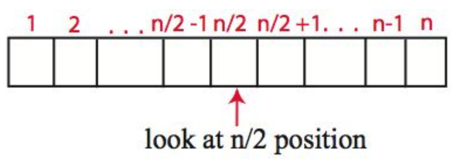

    Figure 3: Divide & Conquer

    - Strategies (Divide & Conquer)
        - If a[n/2] < a[n/2-1] : 1~ n/2 -1 : PEAK
        - Else If a[n/2] < a[n/2+1] : n/2+1 ~ n : PEAK
        - Else : n/2 : PEAK
    - Complexity
        - $T(n) = T(n/2) + \Theta(1) = \Theta(1) + \ldots + \Theta(1)$ (lg2(n) times) $= \Theta(\lg2(n))$

Two-dimensional Version

- Question

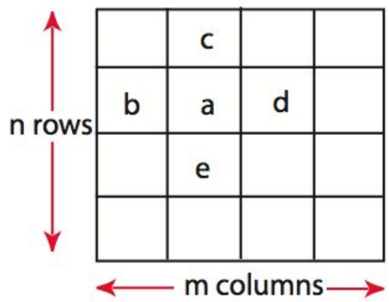

Figure 4: Greedy Ascent Algorithm: $\Theta(nm)$ complexity, $\Theta(n^2)$ algorithm if $m = n$

a is a 2D-peak iff $a \geq b, a \geq d, a \geq c, a \geq e$

- Answer
  - Strategy
    - Pick middle column j = m/2
    - Find global maximum on column j at (i,j)
    - Compare (i,j-1), (i,j), (i,j+1)
    - Pick bigger part and continue
      - If (i,j+1) > (i,j) >= ($\forall$ component j) --- cond(1)
  - Complexity
    - T(n,m) = T(n,m/2) + Θ(n) = Θ(n*lg2(m))
  - Question : What if we replaced global maximum with 1D-peak
    - Doesn't work
      If it is not global maximum, It is impossible to guarantee the condition(1).