1.

(A) $AC + A'B'C = C(A + A'B')$

$= C(A + B')$ (∵ Redundancy law)

(B) $A'B' + A'BC' + (A+C')' = A'B' + A'BC' + A'C$

$= A'(B' + BC' + C)$

$= A'(B' + C' + C)$ (∵ Redundancy law)

$= A'(B' + 1)$

$= A'$

(C) $ABCD' + A(BCD)' + (A+B+C+D)' = ABCD' + A(B'+C'+D') + A'B'C'D'$

$= ABCD' + AB' + AC' + AD' + A'B'C'D'$

$= AD'(BC + 1) + AB' + AC' + A'B'C'D'$

$= AB' + AC' + AD' + A'B'C'D'$

$= AB' + AC' + D'(A + A'B'C')$

$= AB' + AC' + D'(A + B'C')$ (∵ Redundancy law)

2.

(A)

A_1	A_0	B_1	B_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0
0	0	1	1	0	0	0	0
0	1	0	0	0	0	0	0
0	1	0	1	0	0	0	1
0	1	1	0	0	0	1	0
0	1	1	1	0	0	1	1
1	0	0	0	0	0	0	0
1	0	0	1	0	0	1	0
1	0	1	0	0	1	0	0
1	0	1	1	0	1	1	0
1	1	0	0	0	0	0	0
1	1	0	1	0	0	1	1
1	1	1	0	0	1	1	0
1	1	1	1	1	0	0	1

(B)

B_1B_0 \ A_1A_0	00	01	10	11
00	0	0	0	0
01	0	0	0	0
11	0	0	0	1
10	0	0	0	0

$Y_3 = A_1A_0B_1B_0$

B_1B_0 \ A_1A_0	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	0	0	0	1
10	0	0	1	1

$Y_2 = A_1B_1\overline{B_0} + A_1\overline{A_0}B_1$

$= A_1B_1(\overline{A_0} + \overline{B_0})$

B_1B_0 \ A_1A_0	00	01	11	10
00	0	0	0	0
01	0	0	1	1
11	0	1	0	1
10	0	1	1	0

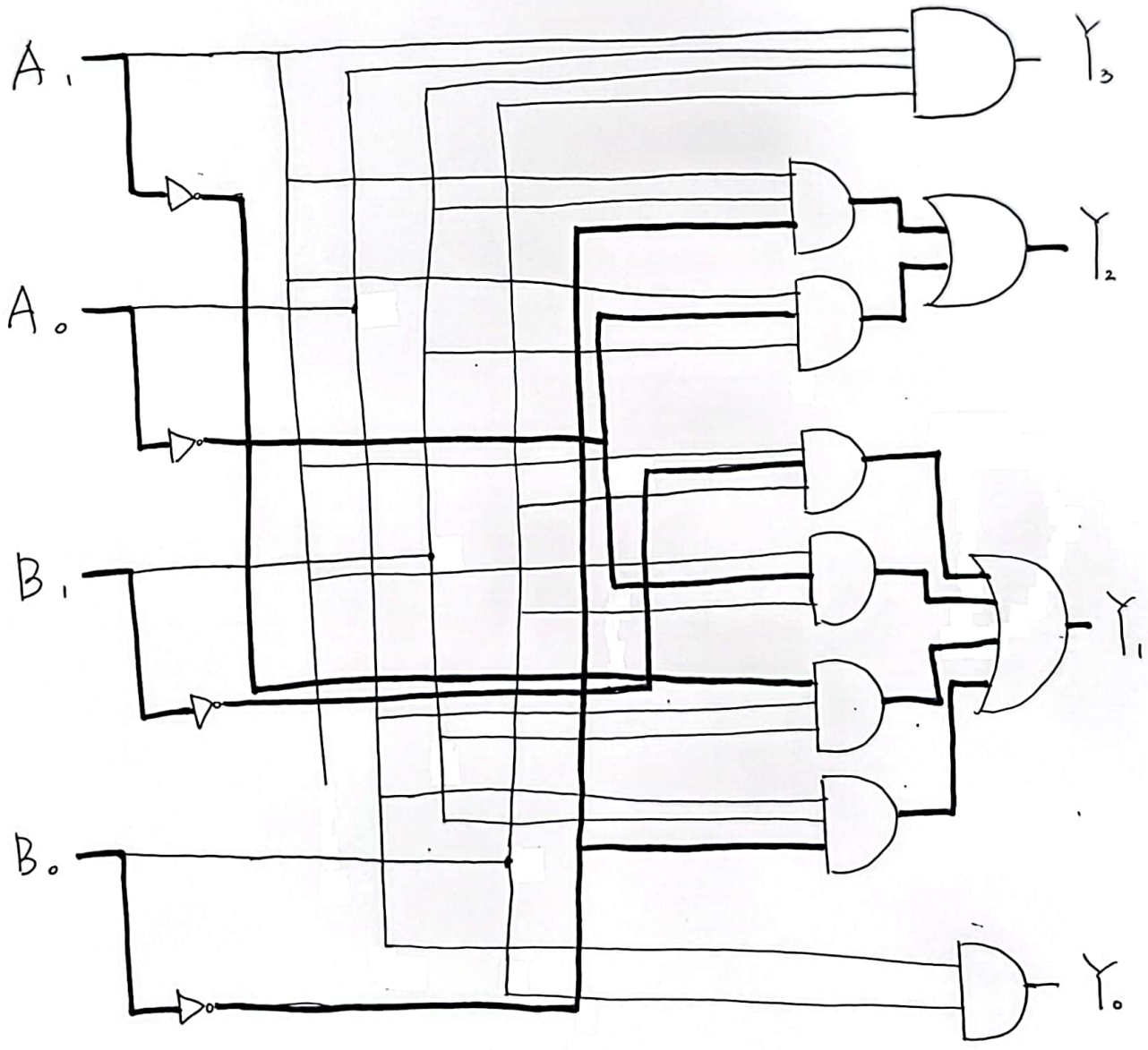
B_1B_0 \ A_1A_0	00	01	11	10
00	0	0	0	0
01	0	1	1	0
11	0	1	1	0
10	0	0	0	0

$$Y_1 = A_1\overline{B_1}B_0 + A_1\overline{A_0}B_0 + \overline{A_1}A_0B_1 + A_0B_1\overline{B_0}$$

$$= A_1B_0(\overline{A_0} + \overline{B_1}) + A_0B_1(\overline{A_1} + \overline{B_0})$$

$$Y_0 = A_0B_0$$

(c)



(D) (C)에서 굵게 그은 선들이 critical path 다. 거쳐가는 게이트가 NOT, AND, OR 세 개이므로

$t_{pd} = t_{pd\_NOT} + t_{pd\_AND} + t_{pd\_OR}$ 다.

다른 path 들은 1개 혹은 2개의 게이트만을 지나므로 굵게 그은 선들이 critical path다.

(E) 전파 지연 시간의 최댓값이 최소가 되도록 레지스터를 삽입하면 클럭 주기가 최소가 됩니다.

AND Gate 이전 부분에 레지스터를 삽입합니다.

3.

K-map

CD \ AB	00	01	11	10
00	0	X	0	X
01	X	0	1	1 → $A\bar{C}D$
11	0	X	X	0
10	1	X	1	X → $C\bar{D}$

$Y = A\bar{C}D + C\bar{D}$

Quine-McCluskey

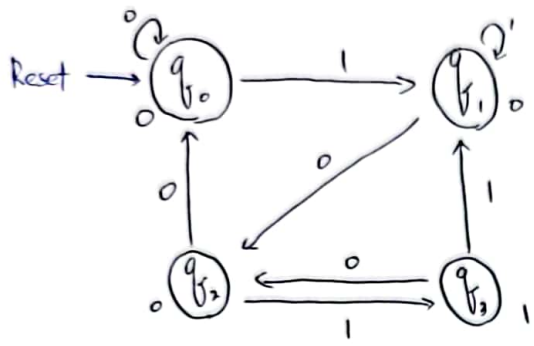
	Column I	Column II	Column III
1	0 0 0 1 ✓	_ 0 0 1 *	_ _ 1 0 *
2	0 0 1 0 ✓	0 _ 1 0 ✓	~~_ _ 1 0~~
4	0 1 0 0 ✓	_ 0 1 0 ✓	_ 1 1 _ *
8	1 0 0 0 ✓	0 1 _ 0 *	~~_ 1 1 _~~
6	0 1 1 0 ✓	1 0 0 _ *	
9	1 0 0 1 ✓	1 0 _ 0 *	
10	1 0 1 0 ✓	0 1 1 _ ✓	
7	0 1 1 1 ✓	_ 1 1 0 ✓	
13	1 1 0 1 ✓	1 _ 0 1 *	
14	1 1 1 0 ✓	1 _ 1 0 ✓	
15	1 1 1 1 ✓	_ 1 1 1 ✓	
		1 1 _ 1 *	
		1 1 1 _ ✓	

true minterm	0010	1001	1101	1110
_001		↓		
01_0		↓		
100_		↓		
10_0		↓		
PIs 1_01		↓	↓	
$A\bar{C}D$ 11_1			↓	
$C\bar{D}$ __10	↓			↓
_11_				

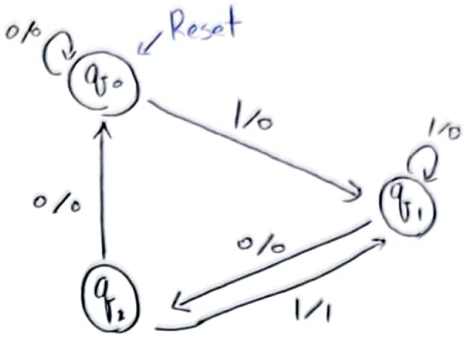
$Y = A\bar{C}D + C\bar{D}$

4.

(A) i) Moore machine



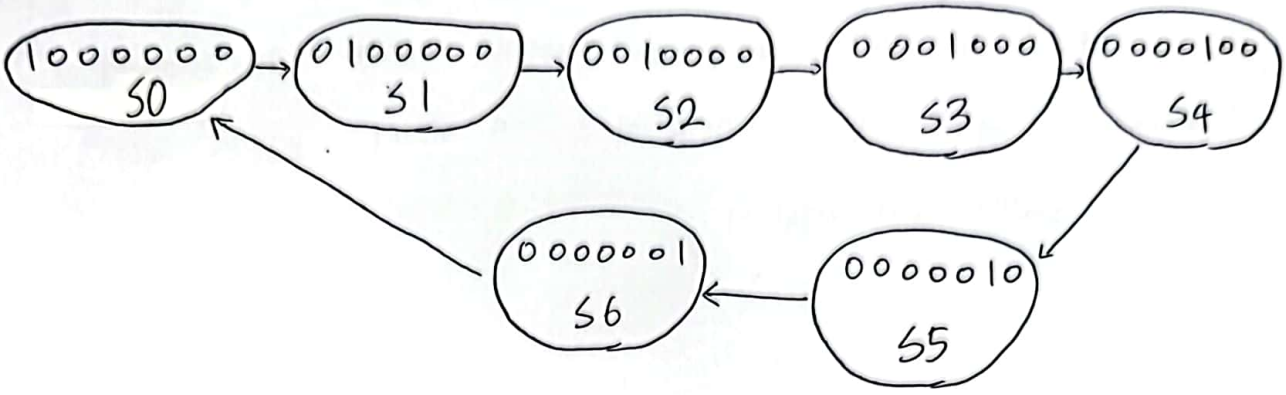
ii) Mealy Machine



(B)

5.

(A)



(B)

state transition table

Current state							Next state						
S_6	S_5	S_4	S_3	S_2	S_1	S_0	S_6'	S_5'	S_4'	S_3'	S_2'	S_1'	S_0'
1	0	0	0	0	0	0	0	1	0	0	0	0	0
0	1	0	0	0	0	0	0	0	1	0	0	0	0
0	0	1	0	0	0	0	0	0	0	1	0	0	0
0	0	0	1	0	0	0	0	0	0	0	1	0	0
0	0	0	0	1	0	0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0	0	0	0	0	0	1
0	0	0	0	0	0	1	1	0	0	0	0	0	0

state encoding table

State	Encoding
S0	1000000
S1	0100000
S2	0010000
S3	0001000
S4	0000100
S5	0000010
S6	0000001

output table

Current state							Output						
S_6	S_5	S_4	S_3	S_2	S_1	S_0	Y_6	Y_5	Y_4	Y_3	Y_2	Y_1	Y_0
1	0	0	0	0	0	0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0
0	0	1	0	0	0	0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0	0	0	1	0	0	0
0	0	0	0	1	0	0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0	0	0	0	0	1	0
0	0	0	0	0	0	1	0	0	0	0	0	0	1

$Y_n = S_n (n=0.1.2.3.4.5.6)$, $S_6' = S_0$, $S_5' = S_6$, $S_4' = S_5$

$S_3' = S_4$, $S_2' = S_3$, $S_1' = S_2$, $S_0' = S_1$

6. 플립플랍을 뒷쪽에서부터 f1, f2, f3 라고하자.

CLK1 과 CLK2 가 비동기일 가능성이 존재하므로 f2 에서 f3로 신호가 전달될 와중에 CLK2가 rising edge를 인식한다면 Metastable한 스테이트가 f3로부터 출력된다.

Metastable 한 state의 빈도수가 최소가 되도록 하기 위해서 플립플랍을 추가하여 MTBF를 증가시키면 문제가 해결됩니다.

7.

(A)

$T_c = \frac{1}{110}\mu s$

$T_c \geq t_{pcq} + N t_{pd\_cuB} + t_{setup}$

$\frac{10^3}{110} ns \geq (0.71 + 0.49N + 0.51)\, ns$

$N \leq 16.06$ $\therefore$ **maximum : 16**

(B) $t_{skew} < t_{ccq} + t_{cd\_cuB} - t_{hold}$

$t_{skew} < 0.31ns + 0.24ns - 0ns$

$t_{skew} < 0.55ns = 550ps$ **$t_{skew} < 550ps$**

(C) $T_c \geq t_{pcq} + 3t_{pd\_cuB} + t_{setup}$

$T_c \geq (0.71 + 3\times 0.49 + 0.51)\, ns$

$T_c \geq 2.69ns$

$f_c = \frac{1}{T_c} \leq \frac{1}{2.69} GHz = 0.37GHz$ **$\therefore$ maximum: 0.37 GHz**

8.

i) two-flipflop synchronizer

$P_E = \frac{400p}{2n} = 0.2$

$P_U = \frac{1}{3} e^{-\frac{1.6}{0.1}} = \frac{1}{3} e^{-16} = 0.4 \times 10^{-7}$

$(t_w = t_{ck} - t_{setup} - t_{dcq} = 2ns - 0.4ns = 1.6ns)$

$P_F = P_E P_U = 0.8 \times 10^{-8}$

$R_F = f_{input} \cdot P_F = 0.8 \times 10^{-2}$

MTBF ∿ 125 seconds

↳ 1 failure every 125 seconds ∿ every 2 minutes.

ii) three flipflop synchronizer

$P_E = 0.2$

$P_U = \frac{1}{3} e^{-\frac{3.2}{0.1}} = \frac{1}{3} e^{-32} = 4.2 \times 10^{-15}$

$(t_w' = t_w \times 2FF = 3.2ns)$

$P_F = P_E P_U = 8.4 \times 10^{-16}$

$R_F = f_{input} \cdot P_F = 8.4 \times 10^{-10}$

MTBF ∿ 1.2×10^9 seconds ∿ more than 38 years.