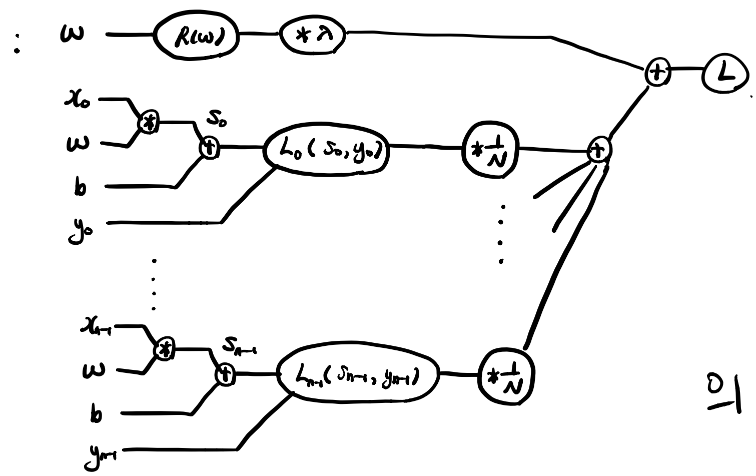
$$L = \frac{1}{N}\sum_{i=1}^{N} L_i(Wx_i + b,\ y_i) + \lambda R(W).$$

:



의 Gradient ?!?

1) $\frac{\partial L}{\partial W}$

① Regularization. $R(W) = \sum_k \sum_l W_{kl}^2$.

$$\frac{\partial R}{\partial W_{kl}} = 2W_{kl}$$ 이므로,

$$\frac{\partial}{\partial W}(\lambda R(W)) = 2\lambda \cdot W.$$

이때, $s = Wx_i + b$.

② Loss Function. (H-SVM). $L_i(s, y_i) = \sum_{j \neq y_i} \max(0, s_j - s_{y_i} + \Delta)$

a)
$$\frac{\partial L_i}{\partial s_k} = \begin{cases} -\sum_{j \neq y_i} 1 \cdot (s_j - s_{y_i} + \Delta > 0) & \text{if } k = y_i \\ 1 \cdot (\max(0, s_k - s_{y_i} + \Delta) > 0) & \text{otherwise} \end{cases}$$

then, $\frac{\partial L_i}{\partial s} = [\ ,\ ,\ ,\ \cdots\ ]^T$ (column vector)

b) next, apply Chain Rule

$$\therefore \frac{\partial L_i}{\partial W} = \frac{\partial L_i}{\partial s} \times \frac{\partial s}{\partial W} = \frac{\partial L_i}{\partial s} x_i^T$$

$$= \begin{bmatrix} \frac{\partial L_i}{\partial s_0} \\ \frac{\partial L_i}{\partial s_1} \\ \frac{\partial L_i}{\partial s_2} \end{bmatrix} [x_0 \ x_1 \ x_2 \ x_3] = \begin{bmatrix} x_0 \frac{\partial L_i}{\partial s_0} & x_1 \frac{\partial L_i}{\partial s_0} & x_2 \frac{\partial L_i}{\partial s_0} & x_3 \frac{\partial L_i}{\partial s_0} \\ & \vdots & & \end{bmatrix}$$

row 만 뜯어서 보면,

$$\frac{\partial L_i}{\partial w_k} = \begin{cases} -\left(\sum_{j \neq y_i} 1\cdot(s_j - s_{y_i} + \Delta > 0)\right) x_i^T & \text{if } k \neq y_i \\ (1\cdot(s_k - s_{y_i} + \Delta > 0)) x_i^T & \text{otherwise} \end{cases}$$

③ Loss Function (Cross-Entropy) $L_i = -\log\left(\frac{e^{s_{y_i}}}{\sum_j e^{s_j}}\right)$

$$= -s_{y_i} + \log\left(\sum_j e^{s_j}\right)$$

a) $$\frac{\partial L_i}{\partial s_k} = \begin{cases} -1 + \frac{e^{s_k}}{\sum_j e^{s_j}} & \text{if } k = y_i \\ \frac{e^{s_k}}{\sum_j e^{s_j}} & \text{otherwise.} \end{cases}$$

b) $$\frac{\partial L_i}{\partial W} = \frac{\partial L_i}{\partial s_k} \times x^T$$

row 만 보면, $$\frac{\partial L_i}{\partial w_k} = \begin{cases} -x_i + \frac{e^{s_k}}{\sum_j e^{s_j}} x_i & \text{if } k = y_i \\ \frac{e^{s_k}}{\sum_j e^{s_j}} x_i & \text{otherwise.} \end{cases}$$

then, $$\frac{\partial L}{\partial W} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial L_i}{\partial W} + 2\lambda \cdot W$$

M-SVM, CE 따라 다름..

2) $\frac{\partial L}{\partial b}$.

$$\frac{\partial L}{\partial b} = \frac{1}{N}\sum_{i=1}^{N}\frac{\partial L_i}{\partial b} = \frac{1}{N}\sum_{i=1}^{N}\left(1 \times \frac{\partial L_i}{\partial s}\right)$$