

Lecture 4. Neural Network Basics



Review

1. Intro to ML

2. Linear Classifn

1. Optimization

- SGD vs. ~~Random Search~~
- Analytic Gradient vs. ~~Numeric Gradient~~

2. Backpropagation

- Chain Rule
- Computational Graph
 - scalar
 - vector
 - matrix
- Implementation

Review

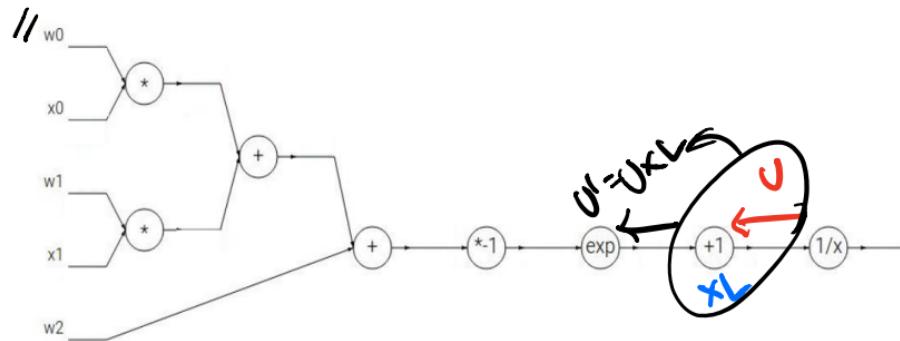
✓

$$w_{\text{new}} = w_{\text{old}} - lr \cdot \frac{\partial L}{\partial w} \quad |_{w=w_{\text{old}}}$$

✓

$$b_{\text{new}} = b_{\text{old}} - lr \cdot \frac{\partial L}{\partial b} \quad |_{b=b_{\text{old}}}$$

$$f(w, x) = \frac{1}{1 + e^{-(w_0x_0 + w_1x_1 + w_2x_2)}}$$



Backprop.

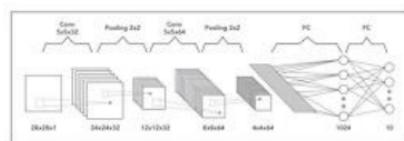
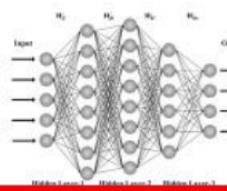
Review

Optimization : Calculating Gradient

IDEA 2. Analytic

지금은 simple linear classifier만 다루고 있지만,

앞으로 다른 model은



then, how should we derive the gradient?

Multilayer Perceptron

Linear Classifier 단독으로는

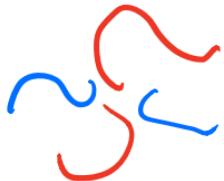
풀 수 없는 문제들 존재...

여러 개를 중첩시켜 보자!

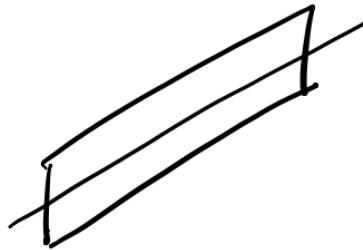
Today's Contents

- 1. Limitation of Linear Classifier
- 2. "Perceptron"
- 3. "MLP(Multilayer Perceptron)"
- 4. "Limitations of MLP"

Limitation of Linear Classifier



Kirby -



What's wrong with our "Linear" Classifier?

What about Nonlinear Classification Problems?

Limitation of Linear Classifier

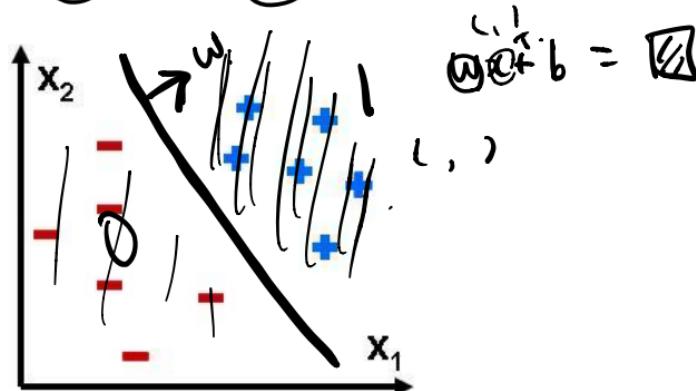
T/F

Binary Classification : AND / OR / XOR Gate with Linear Classifier

$$h(w^T x + b)$$

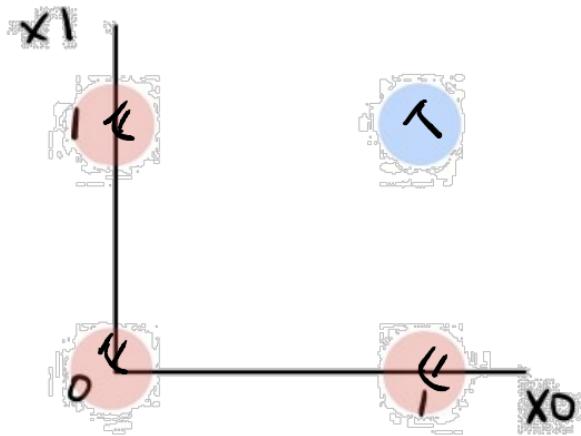
$x > 0$ otherwise

$$h(x) = \begin{cases} 1 & x > 0 \\ 0 & \text{otherwise} \end{cases}$$



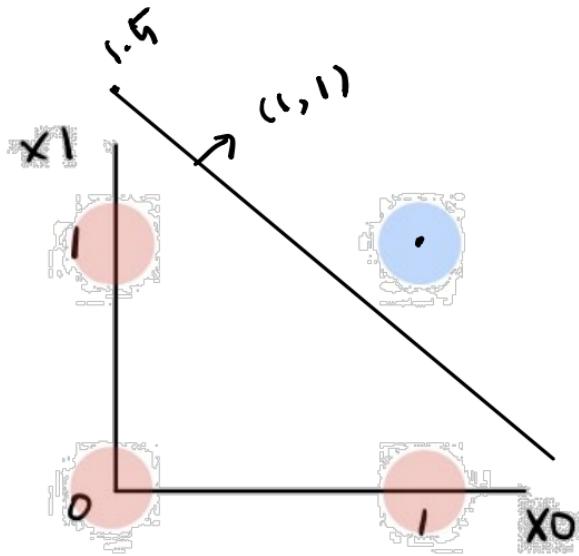
Define $h(x) = 1$ if $x > 0$, 0 otherwise

Linear Classification : AND Gate “ $T=1 \cdot F=0$ ”



x_0	x_1	ANS
1	1	T
1	0	F
0	1	F
0	0	F

Linear Classification : AND Gate



$$x_0 + x_1 - 1.5 = 0$$

$$(1,1) \rightarrow 1+1-1.5 = 0.5$$

$h(0.5) \rightarrow "1" . T$

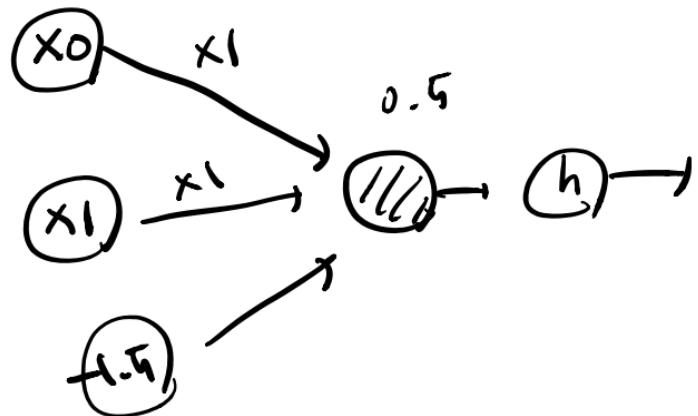
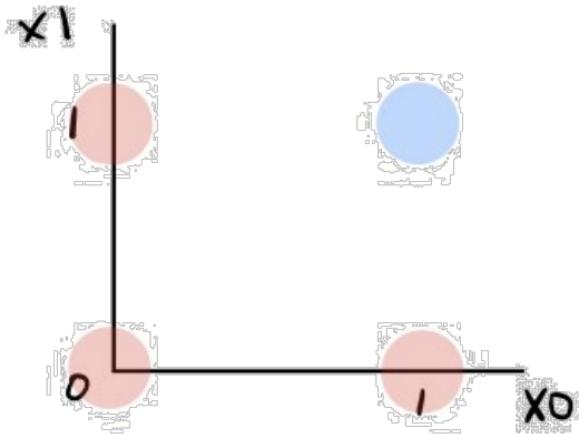
$$(1,0) \rightarrow 1+0-1.5 = -0.5$$

$h(-0.5) = "0" . F$

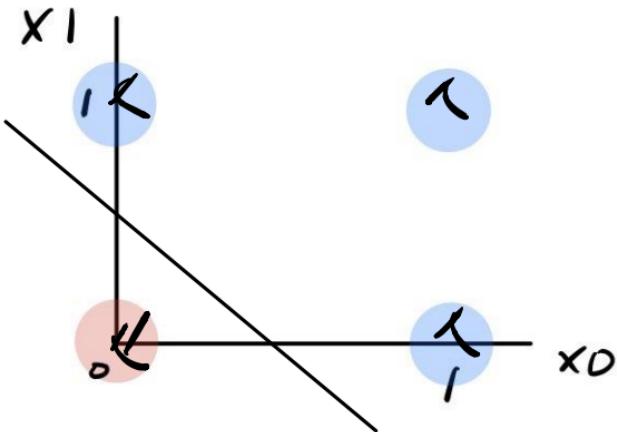
Linear Classification : AND Gate

(1.1)

$$x_0 - x_1 - 1.5$$



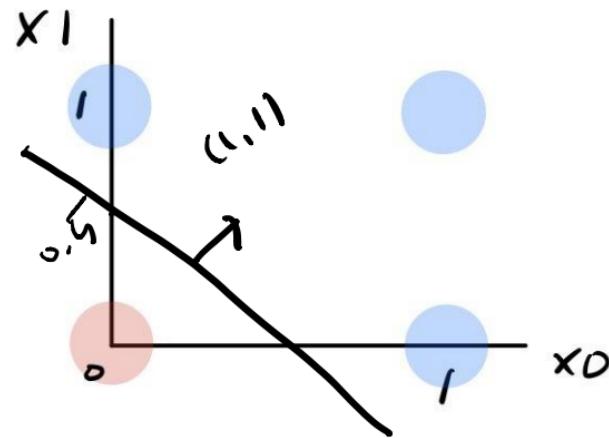
Linear Classification : OR Gate



x0	x1	ANS
1	1	T
1	0	T
0	1	T
0	0	F

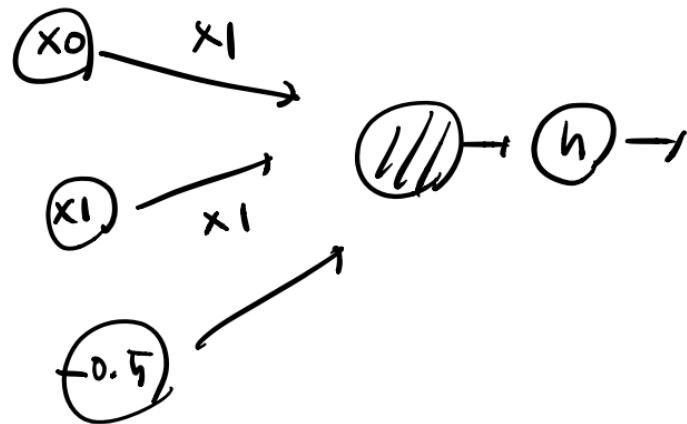
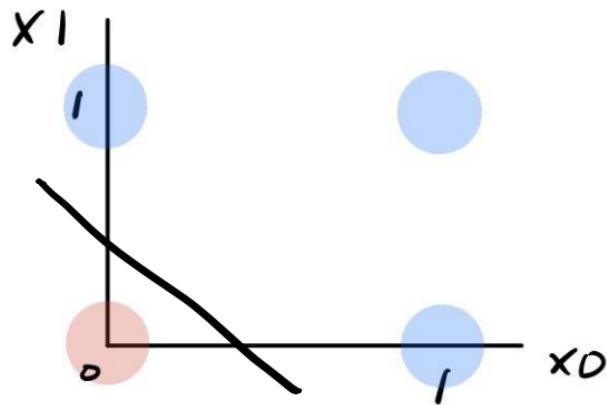
Linear Classification : OR Gate

$$x_0 + x_1 - 0.5 = 0$$



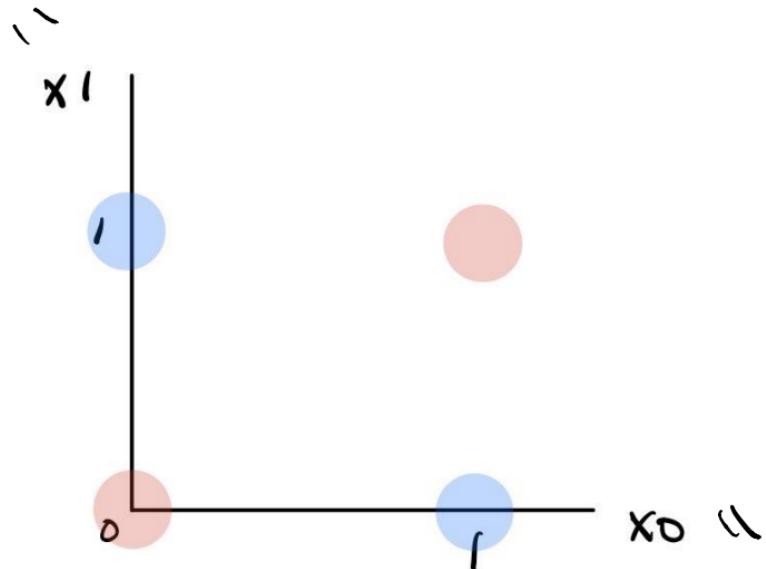
Linear Classification : OR Gate

$$x_0 + x_1 - 0.5 = 0$$



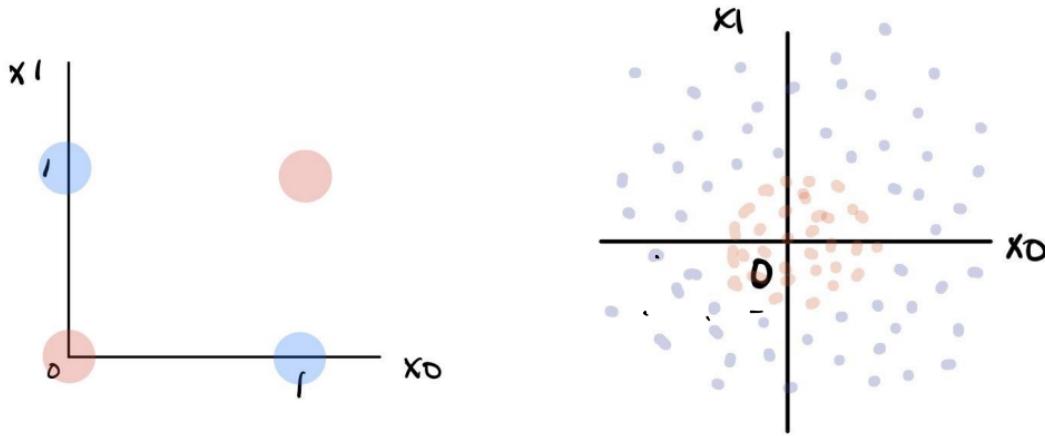
Linear Classification : XOR Gate

"Exclusive"



x_0	x_1	ANS
1	1	F
1	0	T
0	1	T
0	0	F

Limitation of Linear Classifier : Overcoming



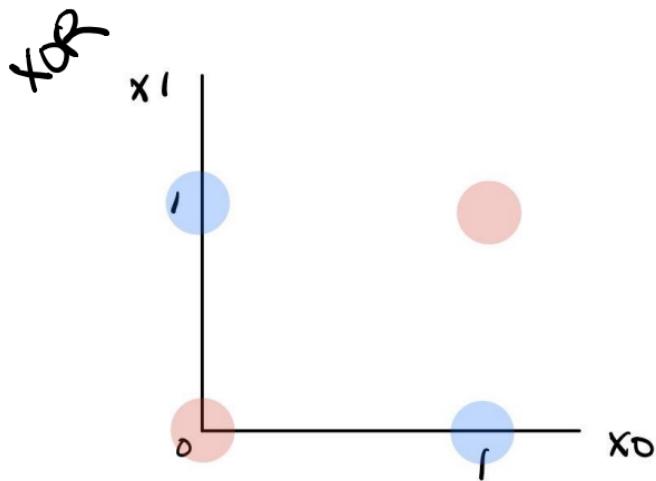
Linear Classifier 단독으로는 풀지 못하는 classification 문제 존재...!

then, how should we classify them?

Limitation of Linear Classifier : Overcoming

“IDEA. Mapping the Data into Other Dimension”

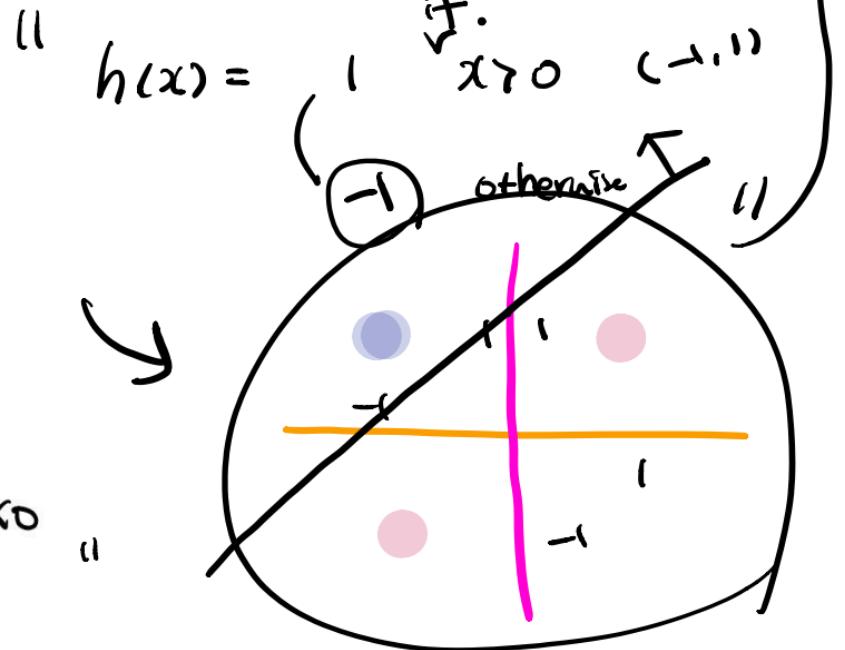
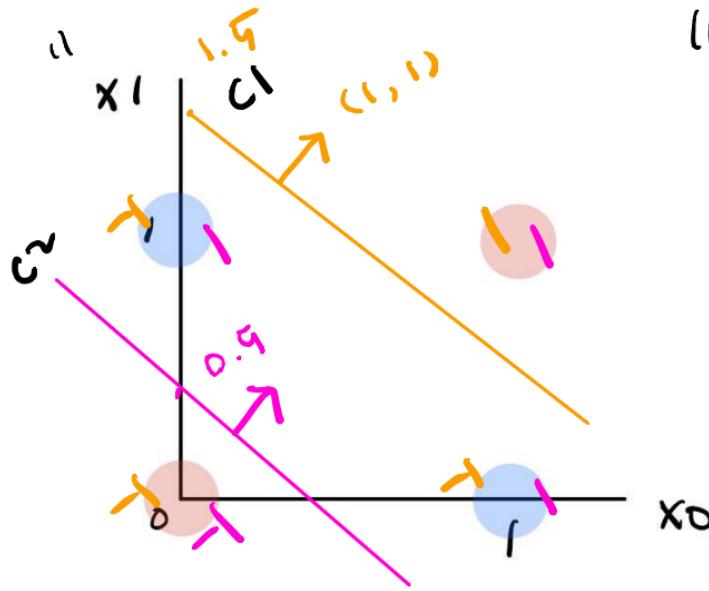
Map the Feature into other dimension, so that it becomes Linearly Separable



Limitation of Linear Classifier : Overcoming

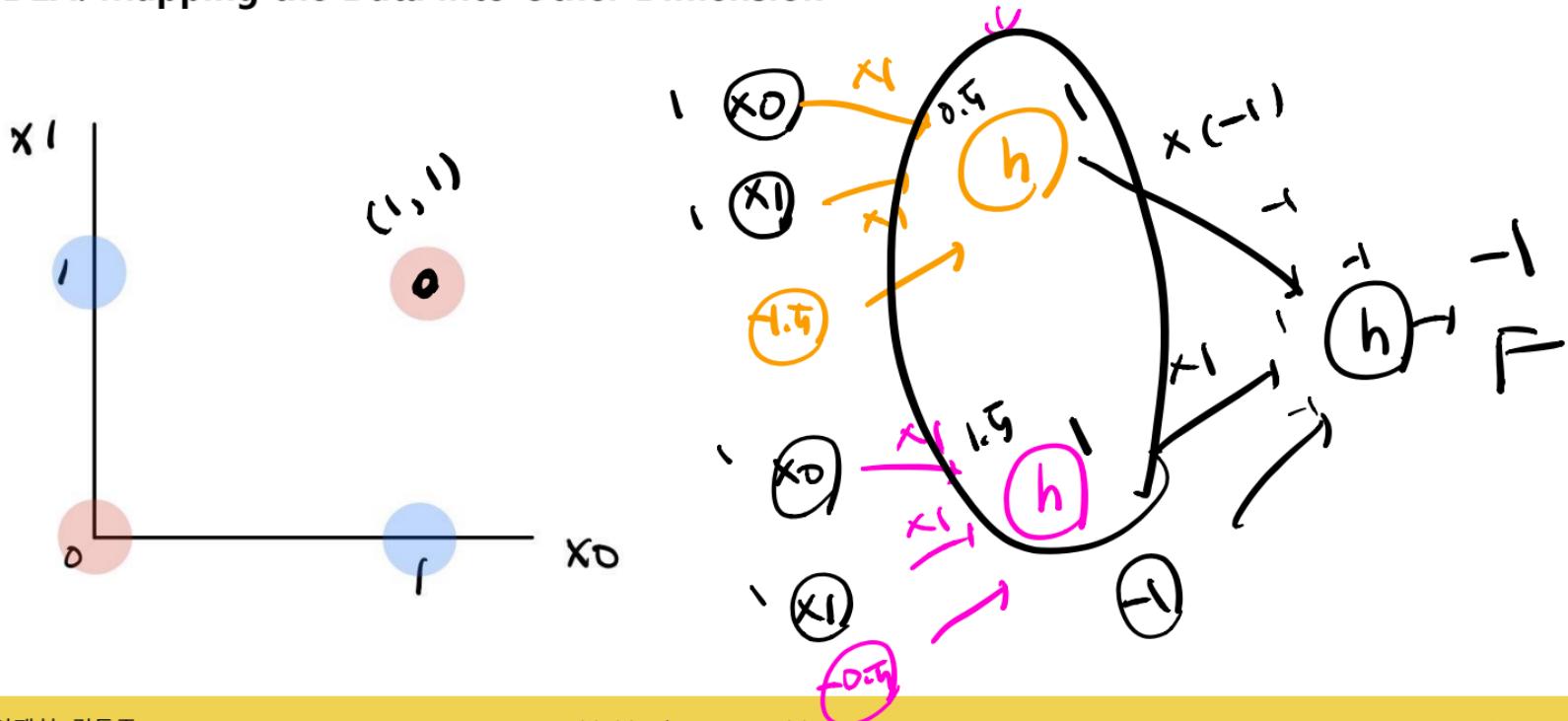
$$\underline{-x_0 + x_1 - 1 = 0}$$

IDEA. Mapping the Data into Other Dimension



Limitation of Linear Classifier : Overcoming

IDEA. Mapping the Data into Other Dimension



Limitation of Linear Classifier : Overcoming

IDEA. Mapping the Data into Other Dimension

$$f = h(w_2 \cdot h(w_1 \cdot x + b_1) + b_2)$$

Linear Classifier로 다른 차원에 Map된 Input을,

또 다른 Linear Classifier로 Classify!

This model is called the **Multilayer Perceptron**, or **Artificial Neural Network**.

MLP

ANN

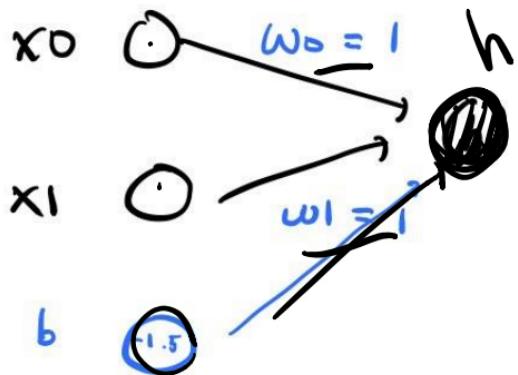
Limitation of Linear Classifier : Overcoming

What is a "Perceptron" ...?

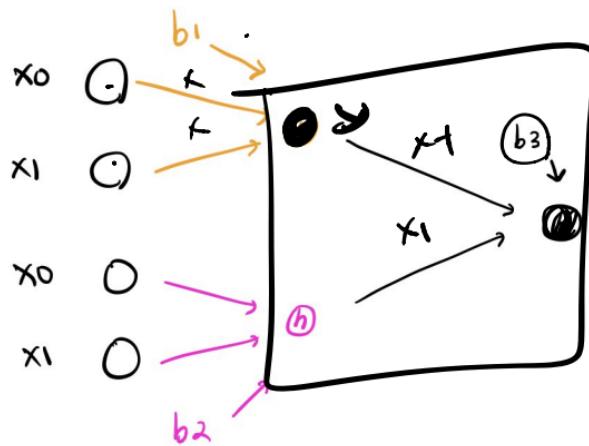
What does "Neural" mean...?

2 Perceptron : A Linear Classifier

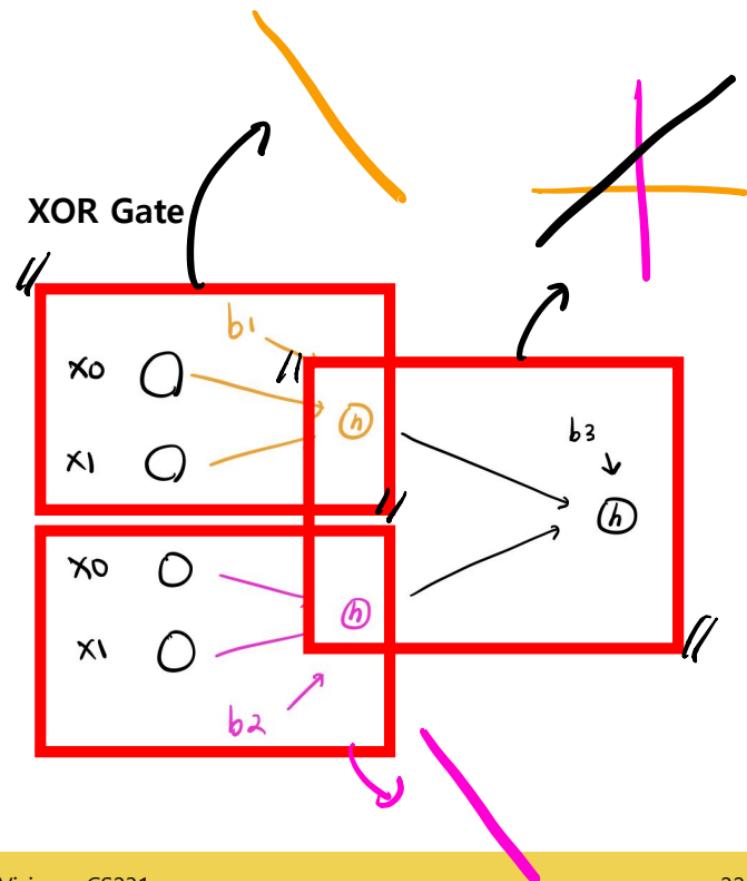
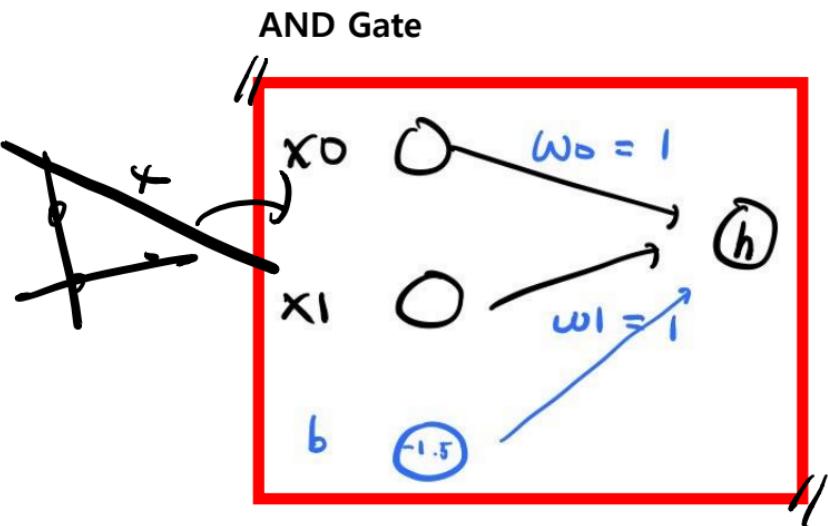
AND Gate



XOR Gate



Perceptron : A Linear Classifier



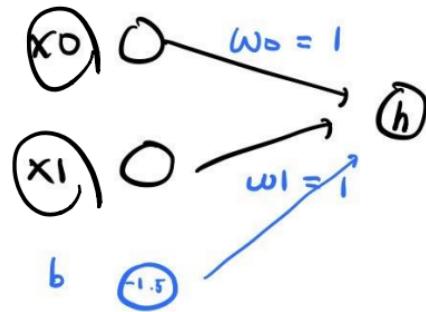
Perceptron : A Linear Classifier

on

Each represents a (Decision Hyperplane) where

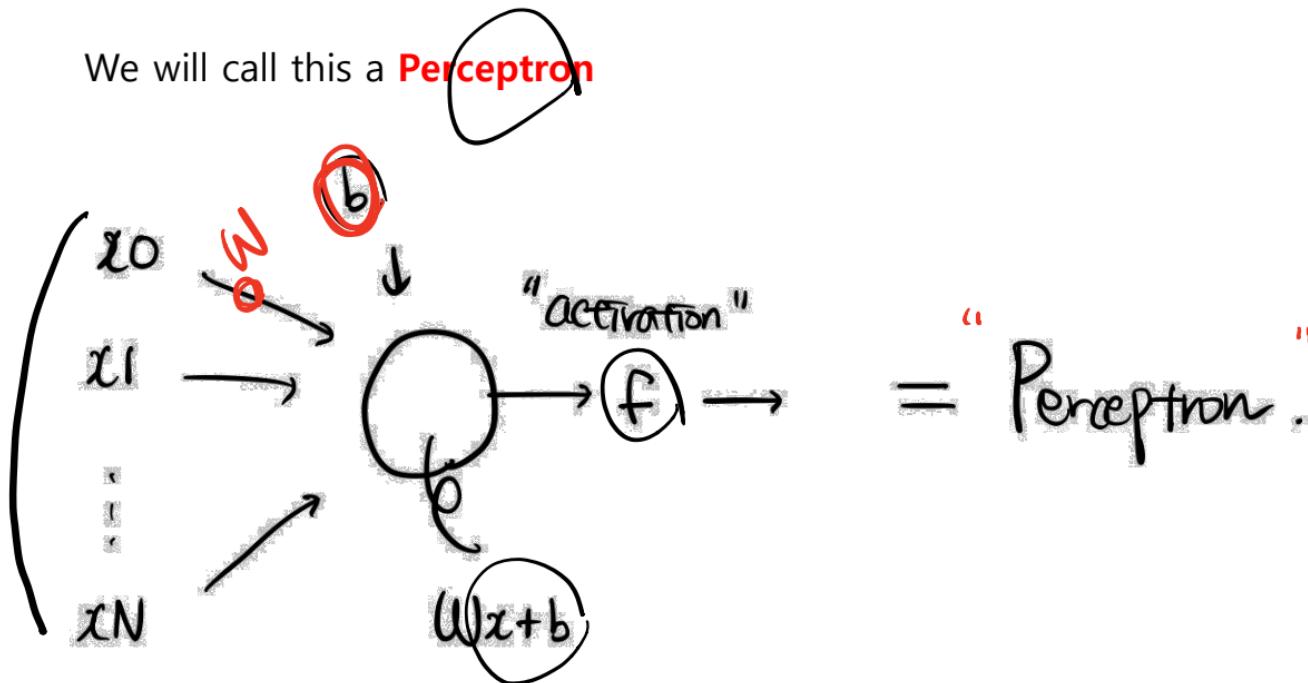
1. Input들에 가중치를 곱한 값을 받아서,
2. 그들의 Linear Combination을 취한 뒤,
3. 특정 함수를 통과시켜 다음 Node의 Input으로 보냄

h



Perceptron : A Linear Classifier

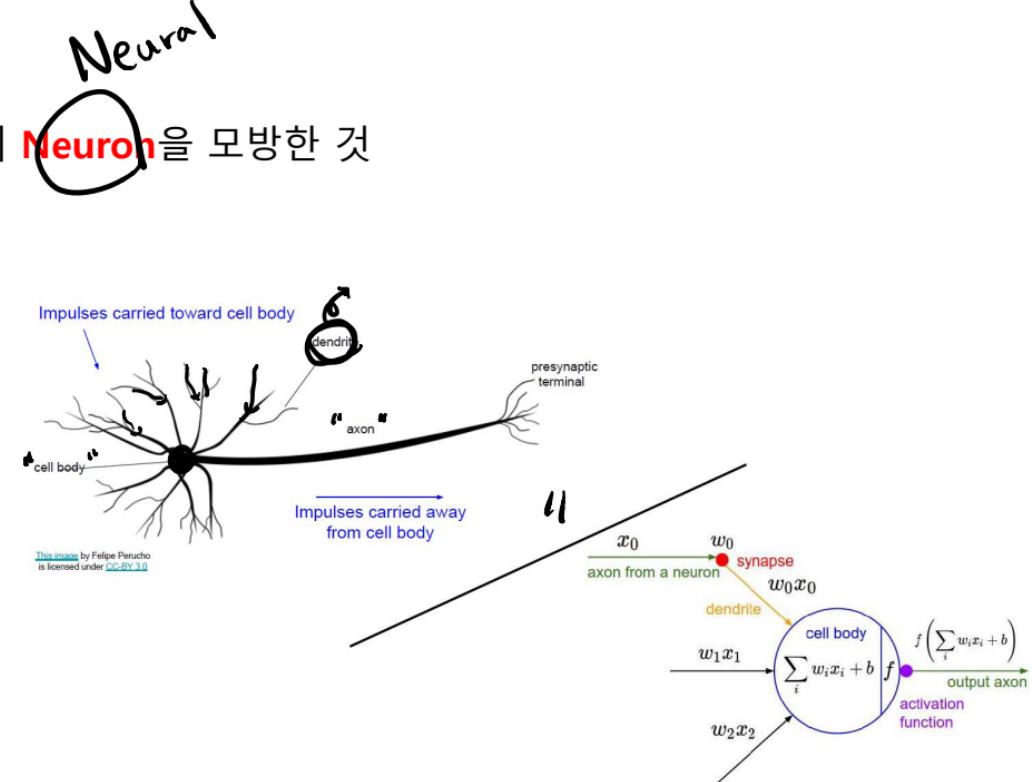
We will call this a **Perceptron**



Perceptron : Analogy to Neurons

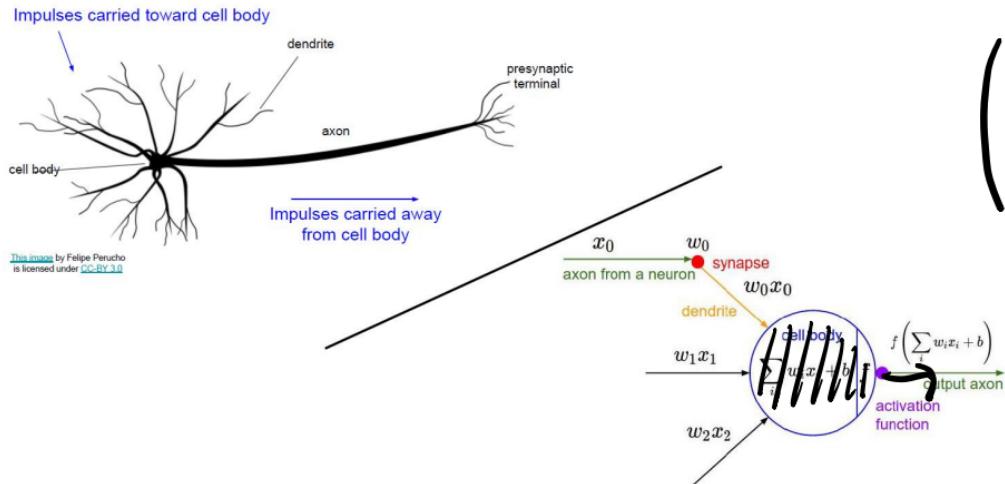
Perceptron Model은 신경계의 **Neuron**을 모방한 것

1. Dendrite로 신호 수신
2. Cell Body에서 신호 합침
3. 신호의 강도 조절하여, Axon을 통해 신호 출력



Perceptron : Analogy to Neurons

Perceptron Model은 신경계의 Neuron을 모방한 것



1. Input들에 가중치 곱한 값들 받아
2. 그들의 Linear Combination 취한 뒤,
3. 특정 함수를 통과시켜 출력

Perceptron : Activation Function

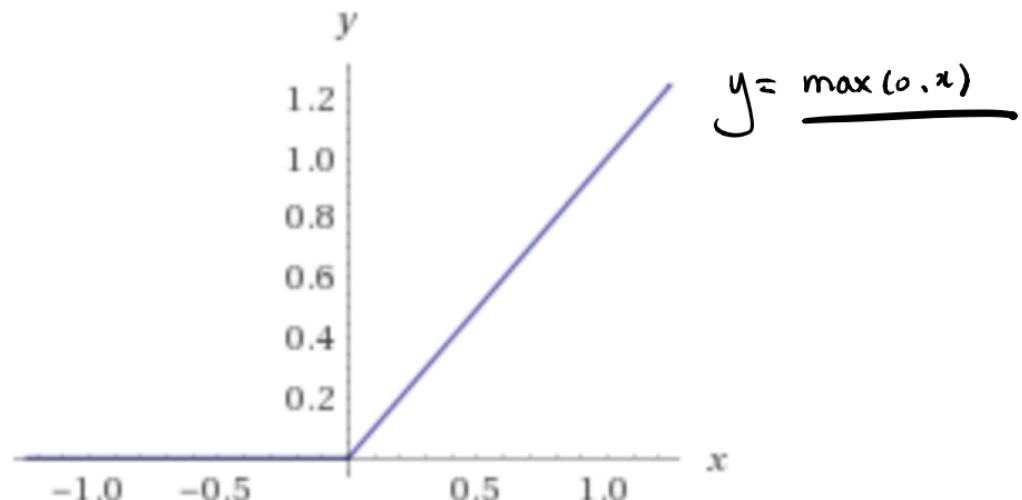
The diagram illustrates a perceptron model. It consists of two main components: a linear classifier and an activation function. The linear classifier is represented by a horizontal oval containing the text "Linear Classifier". An arrow points from this oval to the right, leading into a second oval. This second oval is labeled "Neuron H." and contains the text "Activation Function". Below the diagram, the text "Perceptron = Linear Classifier + Activation Function" is written in red, with the word "Linear Classifier" underlined. To the right of this equation, there are two double quotes (""). Below the entire diagram, the text "Activation Function을 통해, 출력 신호의 강도 조절" is written in red, with "Activation Function" underlined.

Perceptron = Linear Classifier + Activation Function

Activation Function을 통해, 출력 신호의 강도 조절

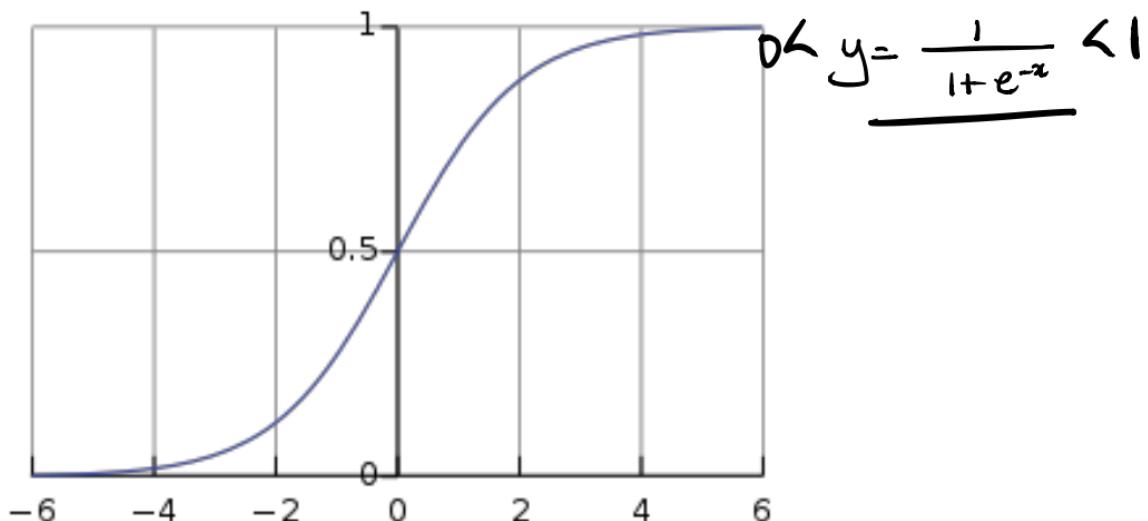
Perceptron : Activation Function

1. RELU



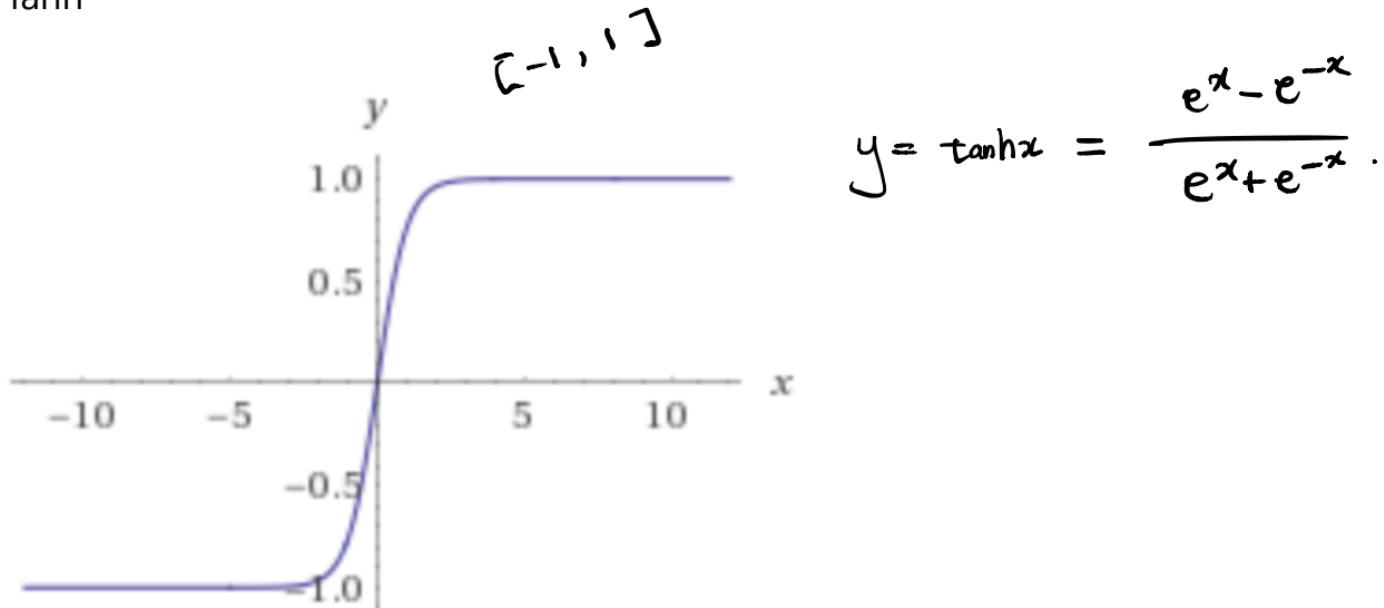
Perceptron : Activation Function

2. Sigmoid



Perceptron : Activation Function

3. Tanh



Perceptron : Activation Function

..
RELU . Sig . Tanh . h

They are also ... **Hyperparameters!!!**

R - L1 / L2 .

λ

lr

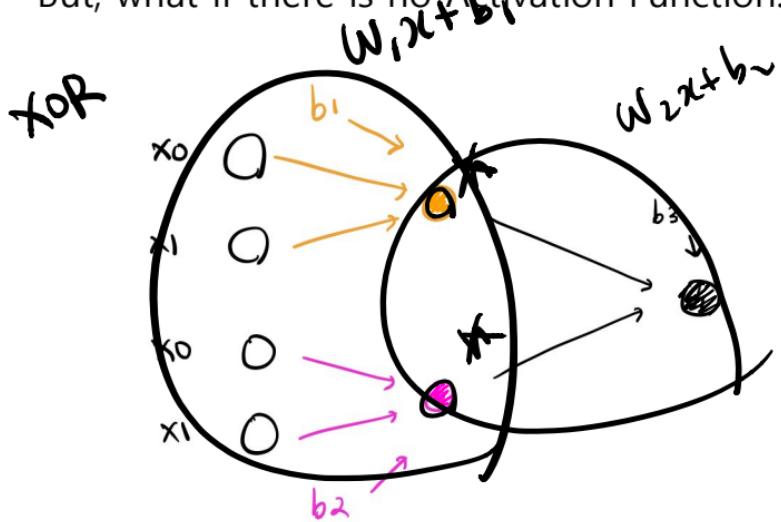
A . F

Perceptron : Activation Function

$$w_2(w_1x + b_1) + b_2$$

$$= w'x + b'$$

But, what if there is no Activation Function...?



Perceptron : Activation Function

Activation Function gives the capacity for Linear Classifiers to,
handle Nonlinear Classification Problems

MLP: Idea

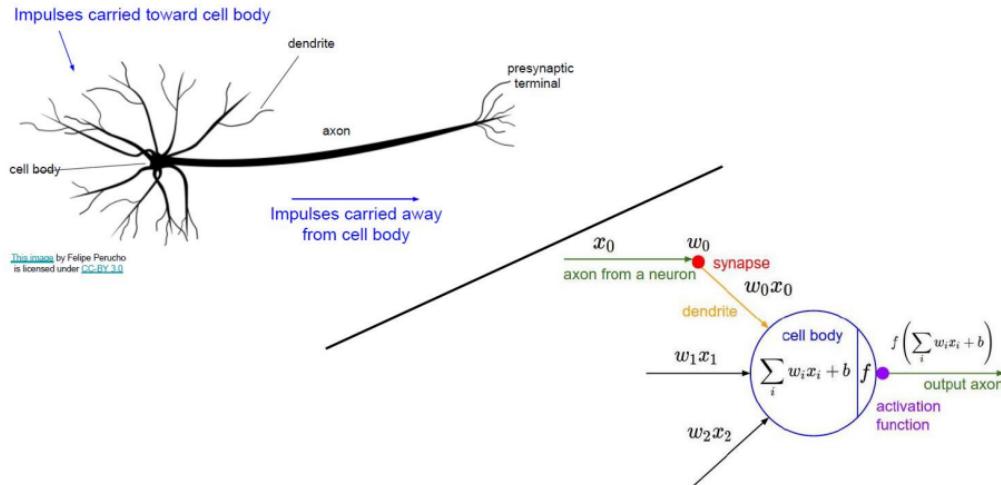
XOR

" Human Neural Network "



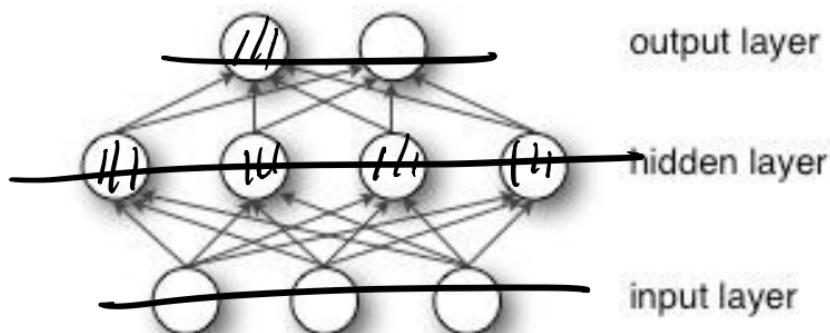
MLP : Idea

“Neuron and Perceptron”

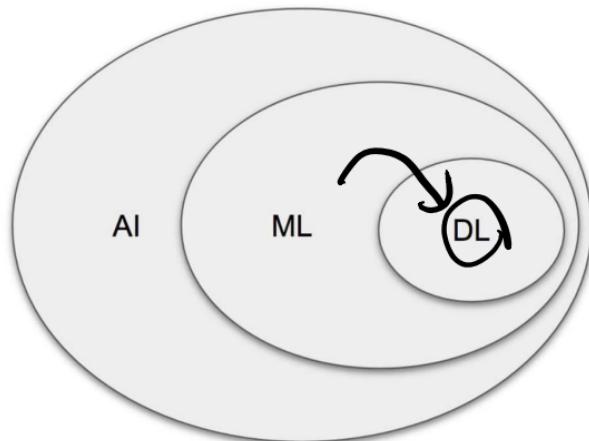


MLP : Idea

m v l n N
Multilayer Perception / Artificial Neural Network



MLP : Intro to DL



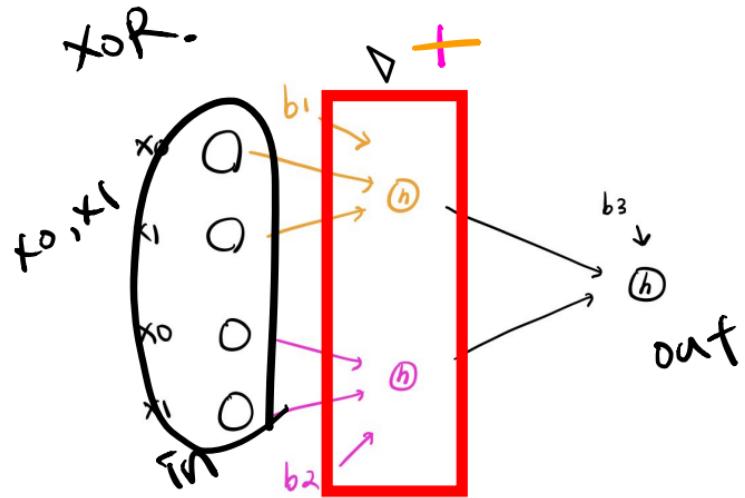
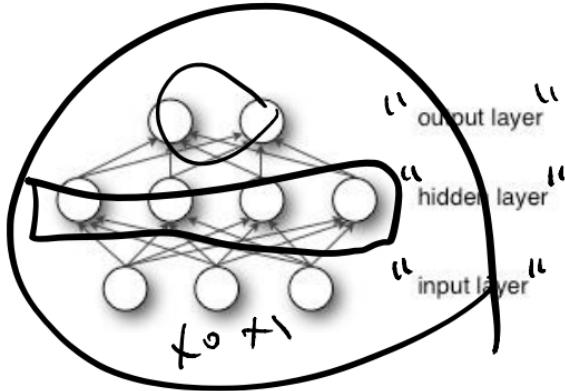
"Deep" learning.

복잡한 Dataset 처리

via ANNs

MLP : Hidden Layer

Multilayer Perceptron / Artificial Neural Network



MLP : Hidden Layer

What does a Hidden Layer do?

Map the Input Feature into other dimension, so that it becomes Linearly Separable

STRUCTURE	TYPES OF DECISION REGIONS	EXCLUSIVE OR PROBLEM	CLASSES WITH MESHED REGIONS	MOST GENERAL REGION SHAPES
SINGLE-LAYER	HALF PLANE			
TWO LAYER	TYPICALLY CONVEX			
THREE LAYER	ARBITRARY			

MLP : Idea

However, can we guarantee that
the MLP will eventually give the correct classification?

MLP : The Universal Approximation Theorem

“ 1개의 hidden layer를 가진 MLP를 이용해 어떠한 함수도 근사할 수 있다. ”

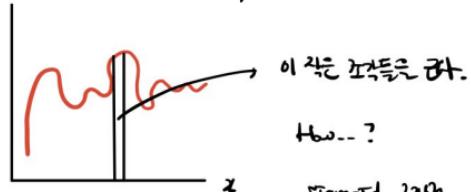
○ when Activation Function is nonlinear

○ when Hyperparameters are appropriately chosen

MLP : The Universal Approximation Theorem

Idea of Proof (Optional)

NN은 아주(특정한 경우) 근사하는 것.

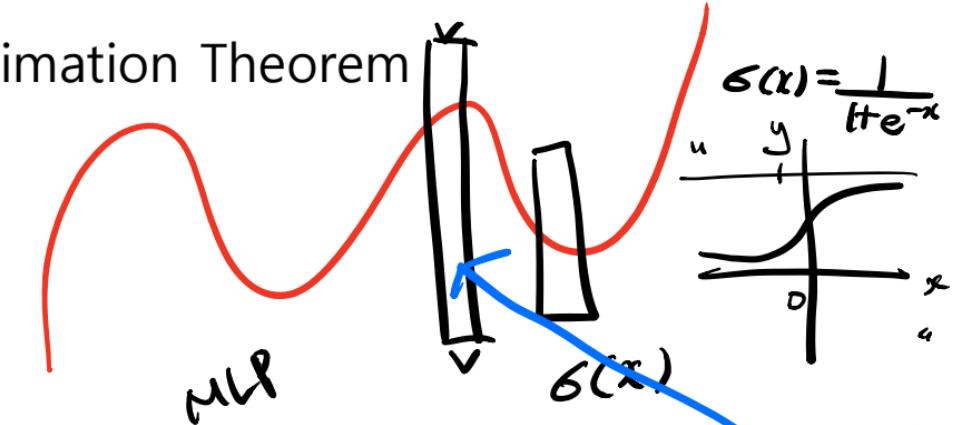


Sigmoid $\sigma(wx+b)$, tower 구조를 만들 가능.

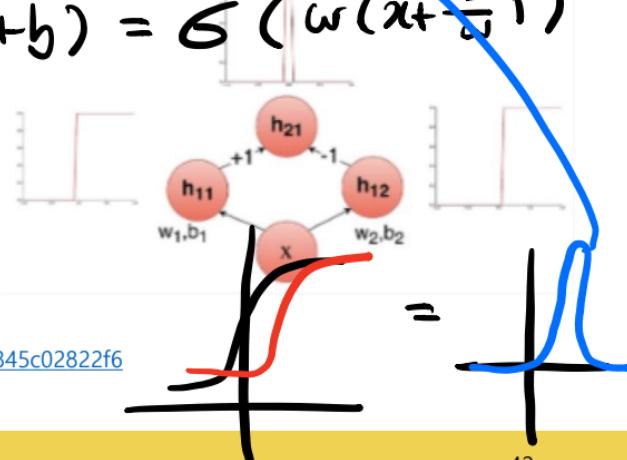
t

weight는 sigmoid 구조 조절, (into hidden layer)

두 sigmoid 막대 tower 형성, (out of hidden layer)



$$\sigma(wx+b) = \sigma(w(x+\frac{b}{w}))$$



For more detail, <https://hackernoon.com/illustrative-proof-of-universal-approximation-theorem-5845c02822f6>

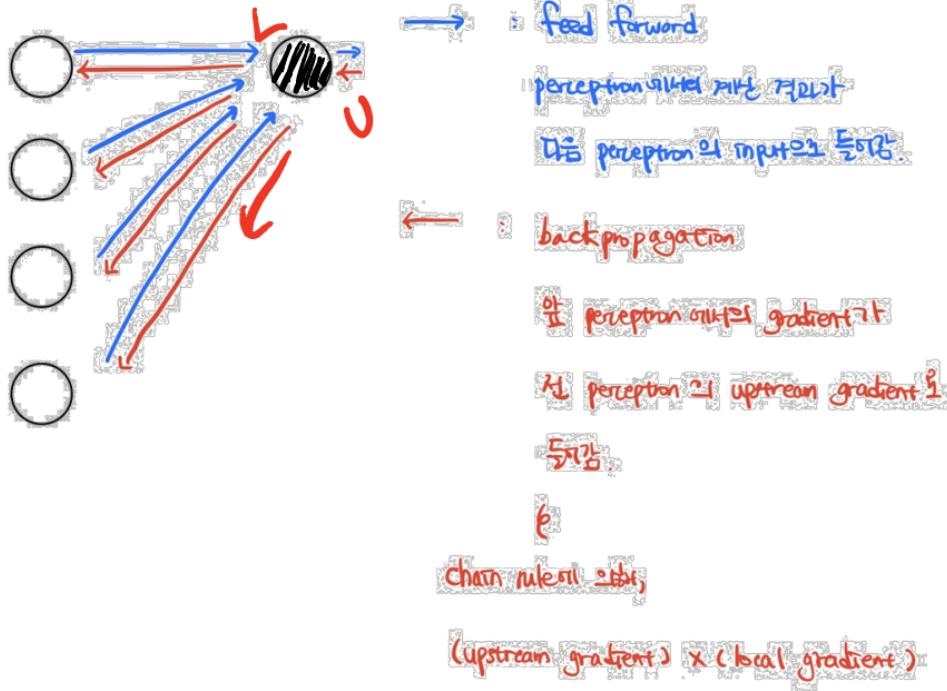
MLP: Looking Back at Backpropagation “ ω_1 , ω_2

MLP를 통해, 복잡한 Classification Function을 근사하고 있다.

Thus, we should utilize **Backpropagation** to compute the gradient

MLP : Looking Back at Backpropagation

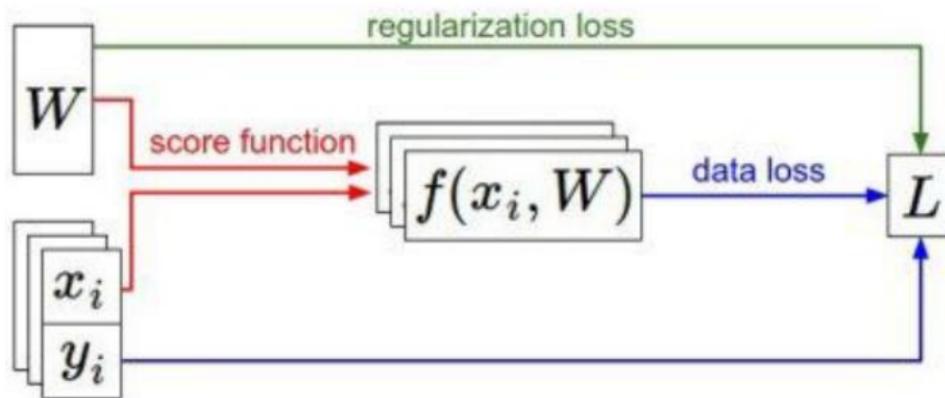
With Backpropagation,



MLP : Looking Back at Backpropagation

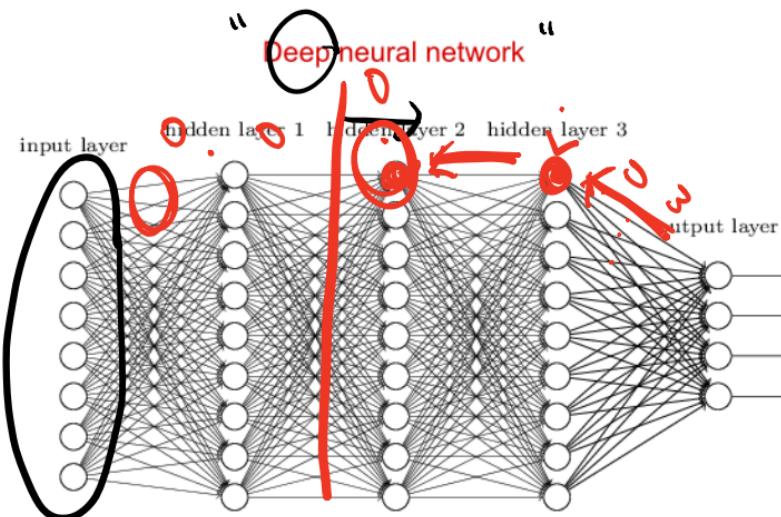
지난 Lecture까지는,

$$L = \frac{1}{N} \sum_{i=1}^N L_i + \lambda R(W)$$

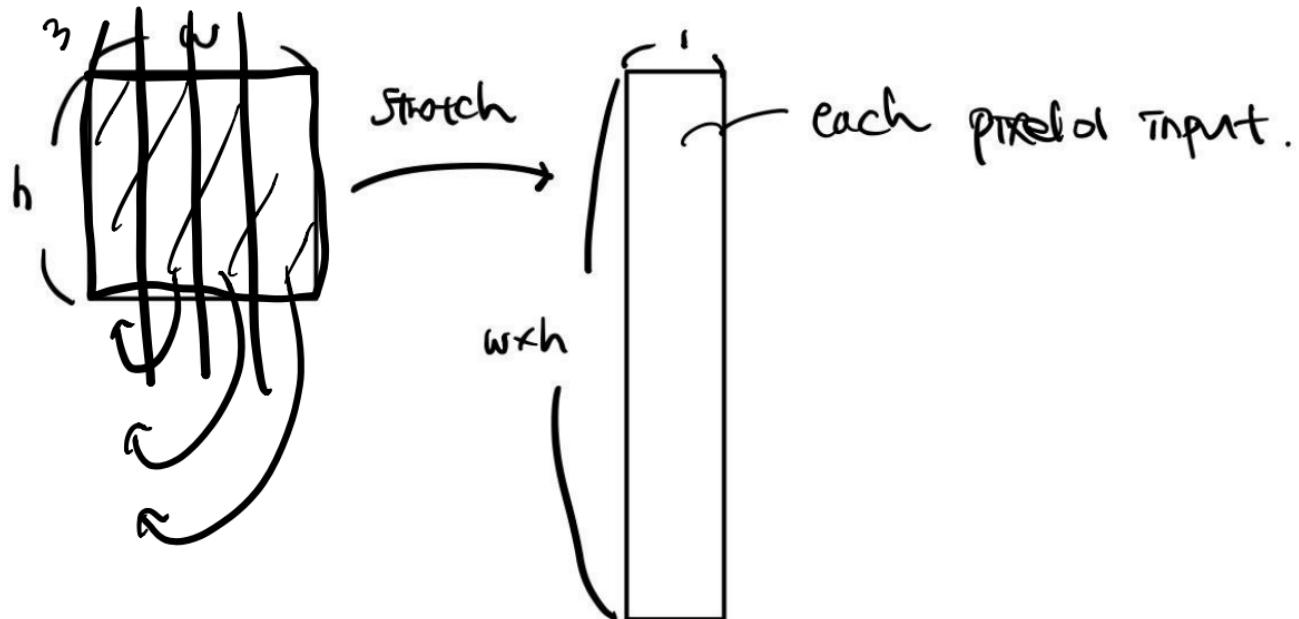


“ MLP : Looking Back at Backpropagation”

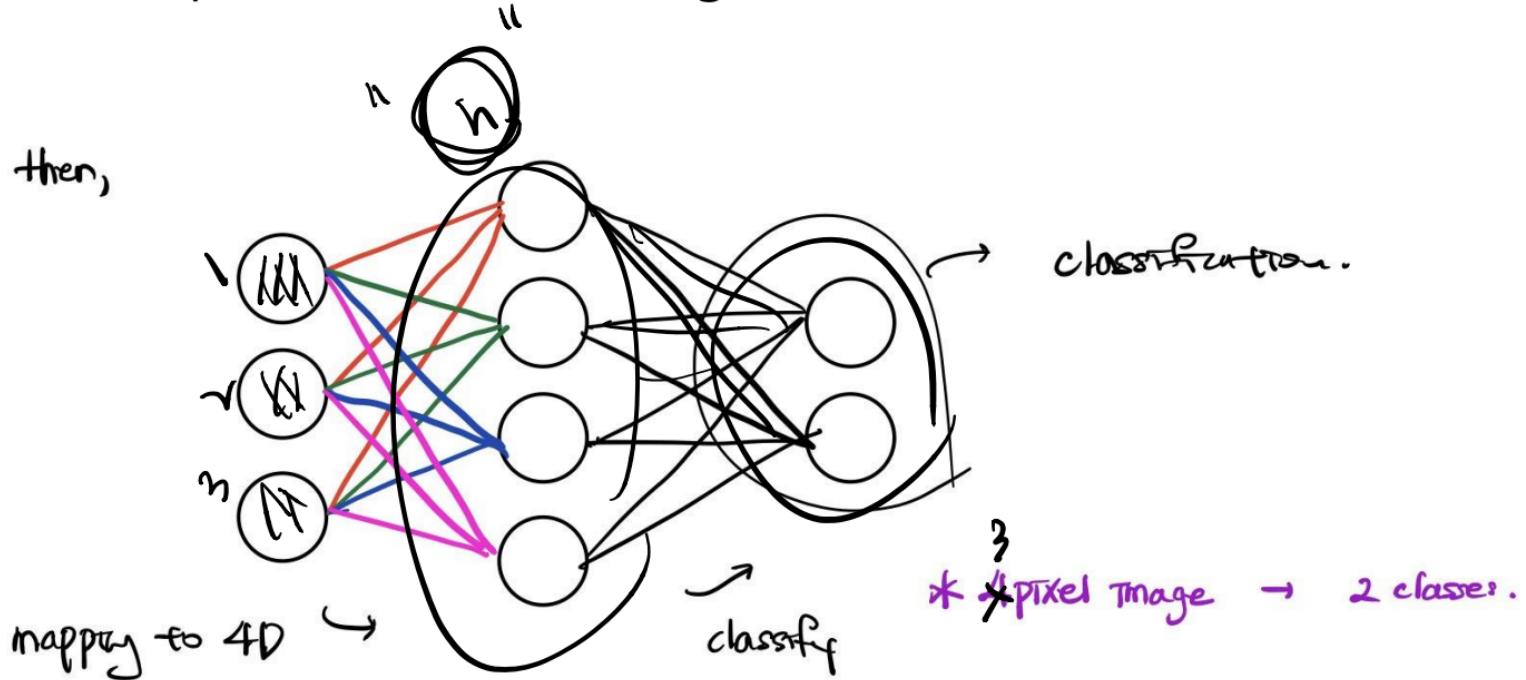
but now,



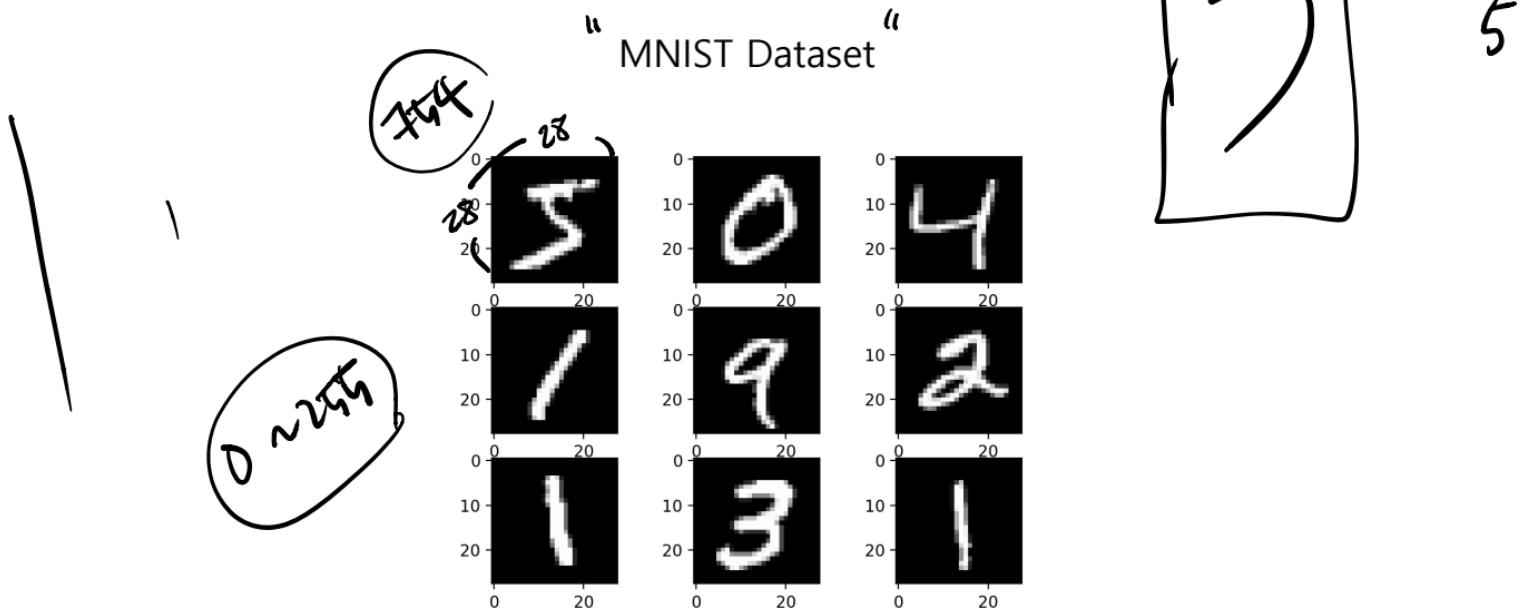
MLP : Implementation on Image Classification



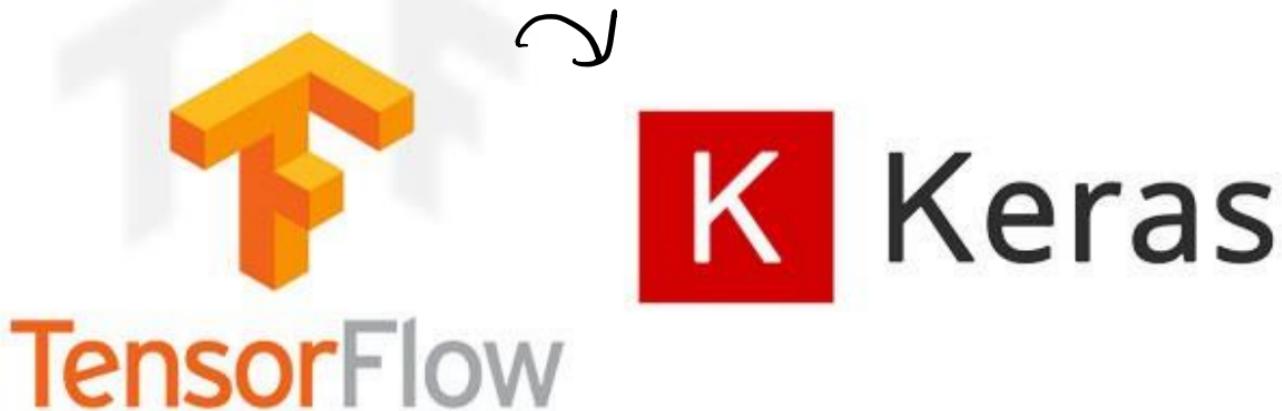
MLP : Implementation on Image Classification



MLP : Implementation on Image Classification



MLP : Implementation on Image Classification



4. Limitations of MLP

MLP + Backpropagation 이론이 나온 것은, 1980s

However, 2010s부터 본격적으로 쓰이기 시작



Limitations of MLP

1. Needs a lot of Labeled Data

2. Vanishing Gradient

3. Overfitting (Reg.)

4. Gets stuck in the Local Minima / Saddle Point

t_{gt}

Loss

G.M.

$w_n = w_0$

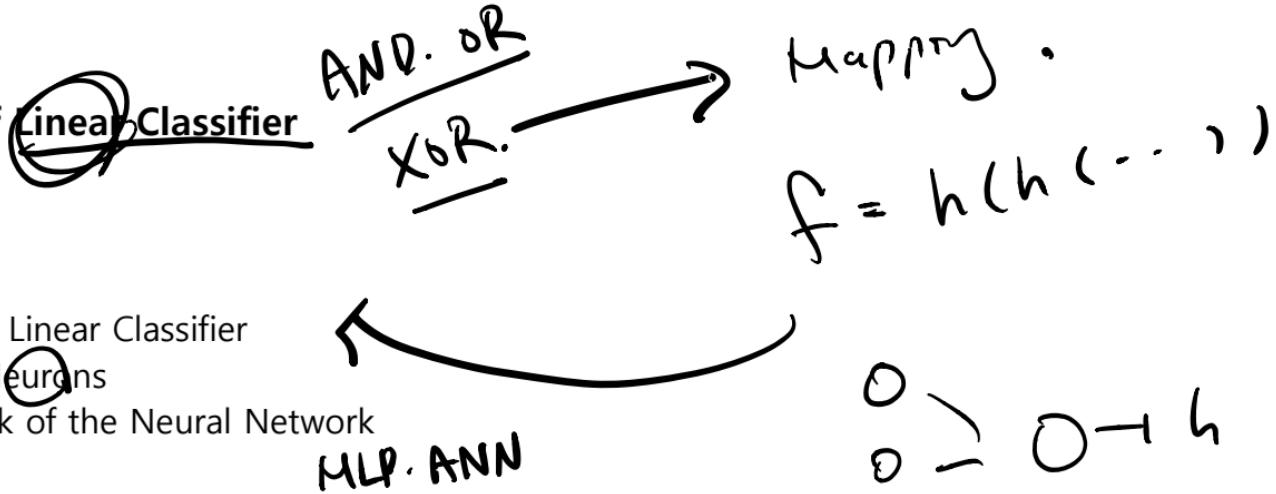
$$\frac{\partial L}{\partial w} \Big|_{w=w_0}$$



"Review"

1. Limitation of ~~Linear~~ Classifier

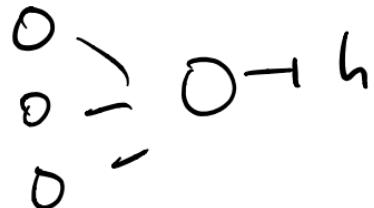
- XOR Gate



3. MLP

- MLP and the Neural Network
- The Universal Approximation Theorem
- Where Backpropagation becomes Important

4. Limitations of MLP



Preview on Next Lecture(s)

wolo



Limitations of MLP

- 1. Needs a lot of Labeled Data
- 2. Vanishing Gradient
- 3. Overfitting
- 4. Gets stuck in the Local Minima

How to overcome these limitations

: Dropout, Adam, Ensemble, Batch Normalization...

From 2010s, NNs became widely used

: CNN for Image Classification

4/5