$$y_t = F_t \beta_t + v_t$$
$$\beta_t = G_t \beta_{t-1} + \varepsilon_t$$

Let $\beta_{t-1} \mid D_{t-1} \sim N(m_{t-1}, \frac{v_t}{s_{t-1}} C_{t-1})$

$$\beta_t \mid D_{t-1} \sim N(a_t, \frac{v_t}{s_{t-1}} R_t)$$

$$a_t = G_t m_{t-1} \quad R_t = G_t C_{t-1} G_t' + W_t$$

$$\underset{\sim}{y_t} \mid D_{t-1} \sim N(\underset{\sim}{f_t}, \frac{v_t}{s_{t-1}} q_t)$$

$$f_t = F_t a_t, \quad q_t = F_t' R_t F_t + v_t$$

For FFBS, we need to specify G_t.

In Var(1) model for β $\quad G = \begin{pmatrix} \phi_{11} & \phi_{12} & \phi_{13} & \phi_{14} \\ \phi_{21} & \phi_{22} & & \\ & & \ddots & \phi_{44} \end{pmatrix}$

$$\begin{cases} \beta_{0,t} = \phi_{11}\beta_{0,t-1} + \phi_{12}\beta_{1,t-1} + \phi_{13}\beta_{2,t-1} + \phi_{14}\beta_{3,t-1} \\ \beta_{0,t-1} = \phi_{11}\beta_{0,t-2} + \phi_{12}\beta_{1,t-2} + \phi_{13}\beta_{2,t-2} + \phi_{14}\beta_{3,t-2} \\ \end{cases}$$

$$\beta_{0,2} = \phi_{11}\beta_{0,1} \cdots \qquad \phi_{14}\beta_{3,1}$$

$$\Rightarrow \begin{pmatrix} \beta_{0,t} \\ \vdots \\ \beta_{0,2} \end{pmatrix} = \begin{pmatrix} \beta_{0,t-1} & \cdots & \beta_{3,t-1} \\ & \vdots & \\ \beta_{0,1} & \cdots & \beta_{3,1} \end{pmatrix} \begin{pmatrix} \phi_{11} \\ \phi_{12} \\ \phi_{13} \\ \phi_{14} \end{pmatrix}$$

$$Y \quad = \quad X\beta$$

$$\hat{\beta} = (X^T X)^{-1} X^T Y$$