HW2 STA521

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Due September 12, 2019 10am

Exploratory Data Analysis

0. Preliminary read in the data. After testing, modify the code chunk so that output, messages and warnings are suppressed. *Exclude text from final*

```
library(alr3)
data(UN3, package="alr3")
#help(UN3)
library(car)
library(dplyr)
library(knitr)
library(ggplot2)
library(GGally)
```

1. Create a summary of the data. How many variables have missing data? Which are quantitative and which are qualtitative?

```
str(UN3)
```

```
## 'data.frame':    210 obs. of  7 variables:
##  $ ModernC  : int  NA NA 49 NA NA NA 51 NA 22 NA ...
##  $ Change   : num  3.88 0.68 1.67 2.37 2.59 3.2 0.53 1.17 -0.45 2.02 ...
##  $ PPgdp    : int  98 1317 1784 NA 14234 739 8461 7163 687 NA ...
##  $ Frate    : int  NA NA 7 42 NA NA 63 44 51 53 ...
##  $ Pop      : num  23897 3167 31800 57 64 ...
##  $ Fertility: num  6.8 2.28 2.8 NA NA 7.2 NA 2.44 1.15 NA ...
##  $ Purban   : int  22 43 58 53 92 35 37 88 67 51 ...
```

```
smry_UN3 <- summary(UN3)
smry_UN3
```

```
##     ModernC          Change           PPgdp           Frate      
##  Min.   : 1.00   Min.   :-1.100   Min.   :   90   Min.   : 2.00  
##  1st Qu.:19.00   1st Qu.: 0.580   1st Qu.:  479   1st Qu.:39.50  
##  Median :40.50   Median : 1.400   Median : 2046   Median :49.00  
##  Mean   :38.72   Mean   : 1.418   Mean   : 6527   Mean   :48.31  
##  3rd Qu.:55.00   3rd Qu.: 2.270   3rd Qu.: 8461   3rd Qu.:58.00  
##  Max.   :83.00   Max.   : 4.170   Max.   :44579   Max.   :91.00  
##  NA's   :58      NA's   :1        NA's   :9       NA's   :43     
##       Pop             Fertility         Purban      
##  Min.   :      2.3   Min.   :1.000   Min.   :  6.00  
##  1st Qu.:    767.2   1st Qu.:1.897   1st Qu.: 36.25  
##  Median :   5469.5   Median :2.700   Median : 57.00  
##  Mean   :  30281.9   Mean   :3.214   Mean   : 56.20  
##  3rd Qu.:  18913.5   3rd Qu.:4.395   3rd Qu.: 75.00  
##  Max.   :1304196.0   Max.   :8.000   Max.   :100.00  
##  NA's   :2           NA's   :10
```

```
na_count <- smry_UN3[7,]
na_count
```

```
##        ModernC         Change          PPgdp          Frate            Pop
## "NA's   :58  "  "NA's   :1  "  "NA's   :9  " "NA's   :43  "  "NA's   :2  "
##      Fertility         Purban
## "NA's   :10  "             NA
```

answer: As we can see in outlook of data.frame UN3, there are all quantative variables. Except for variable named Purban, those of variables including ModernC, Change, PPgdp, Frate, Pop, Fertility have at least one missing data.

2. What is the mean and standard deviation of each quantitative predictor? Provide in a nicely formatted table.

```
mn_st_table <- matrix(rep(0,3*length(UN3)),nrow = length(UN3))
for(i in 1:length(UN3)){
  mn_st_table[i,] <- c(colnames(UN3)[i],
                       round(mean(UN3[,i],na.rm = T),3),
                       round(sd(UN3[,i],na.rm=T),3))
}
rownames(mn_st_table) <- 1:length(UN3)
colnames(mn_st_table) <- c("variable","mean","stand deviation")
kable(mn_st_table)
```

variable	mean	stand deviation
ModernC	38.717	22.637
Change	1.418	1.133
PPgdp	6527.388	9325.189
Frate	48.305	16.532
Pop	30281.871	120676.694
Fertility	3.214	1.707
Purban	56.2	24.11

3. Investigate the predictors graphically, using scatterplots or other tools of your choice. Create some plots highlighting the relationships among the predictors. Comment on your findings regarding trying to predict `ModernC` from the other variables. Are there potential outliers, nonlinear relationships or transformations that appear to be needed based on your graphical EDA?

```
for(i in 1:length(UN3)){
  print(ggplot(data=UN3, mapping=aes(x=1:nrow(UN3),y=UN3[,i]))+
          geom_point()+
          geom_hline(yintercept = mean(UN3[,i],na.rm = T),color="red")+
          ylab(colnames(UN3)[i]) + xlab("index")+
          labs(title=paste("distribution of",colnames(UN3[i])),
               caption = paste("fig ",i,if(i==3) {
                 ". distribution is right skewed"
               } else if(i==5){
                 ". There are potentially influential points"
               } else ". randomly distributed")))
```

```
}
```

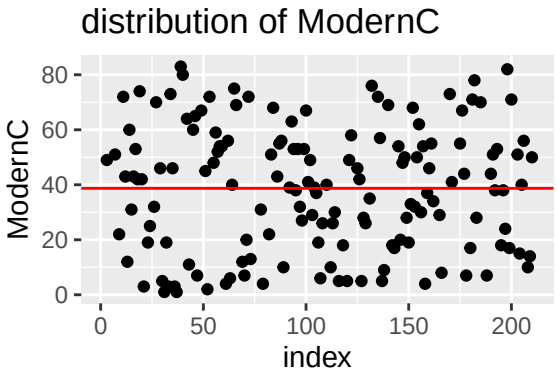


fig 1 . randomly distributed

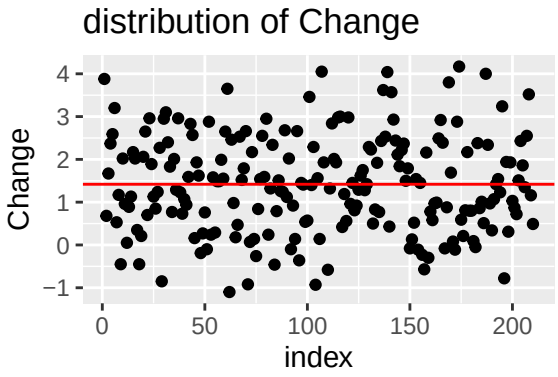


fig 2 . randomly distributed

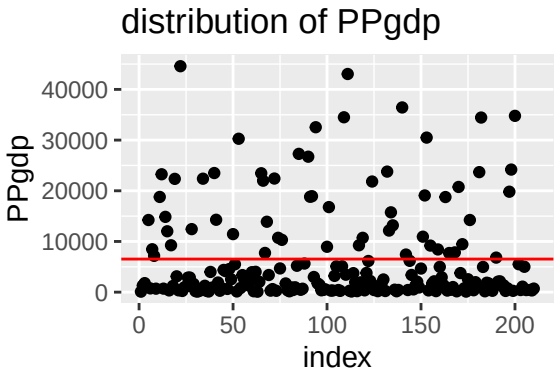


fig 3 . distribution is right skewed

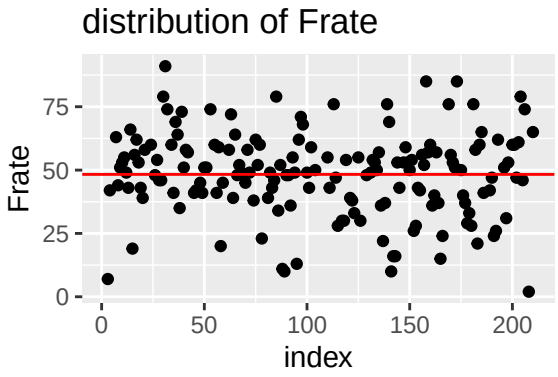


fig 4 . randomly distributed

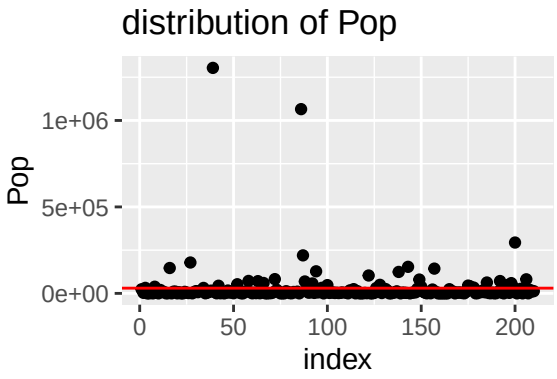


fig 5 . There are potentially influential points

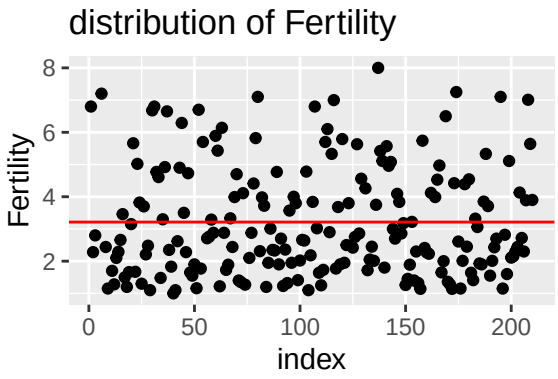


fig 6 . randomly distributed

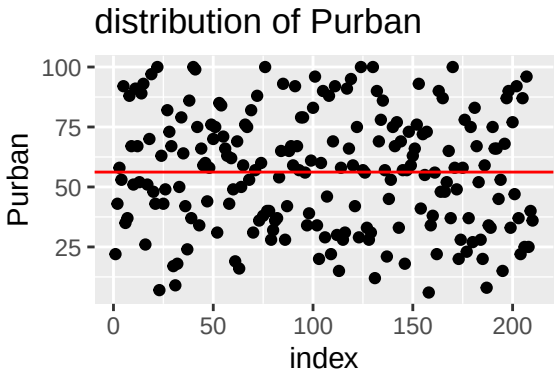


fig 7 . randomly distributed

When we inspect scatterplots of predictors, most of them are distributed randomly from their mean. But in case of PPgdp, they are skewed right. Thus I think it needs to be transformed. Furthemore, Pop seems to have some potential outliers. Therefore, we should be cautious dealing with Pop variable.

```
ggpairs(UN3) +
  labs(title = "pairwise relationship of predictor",
       caption = "fig7. ModernC has relationships with change,PPgdp,Fertility,Purban")
```

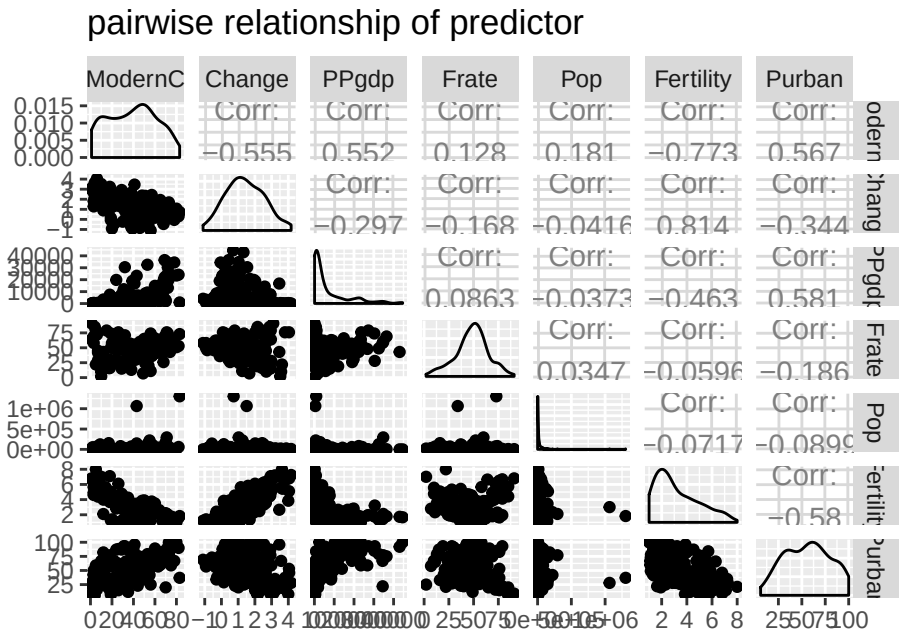


fig7. ModernC has relationships with change,PPgdp,Fertility,Purban

```
ggpairs(UN3[,c(1,2,3,6,7)]) +
  labs(title = "ModernC's relationship with predictors",
       caption = "fig8. ModernC has nonlinear relationship with PPgdp")
```

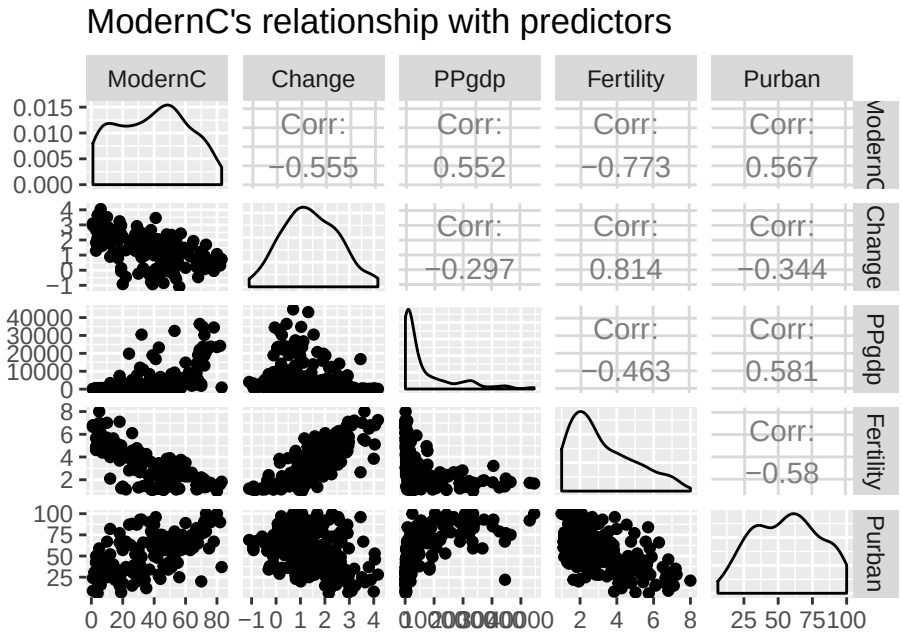


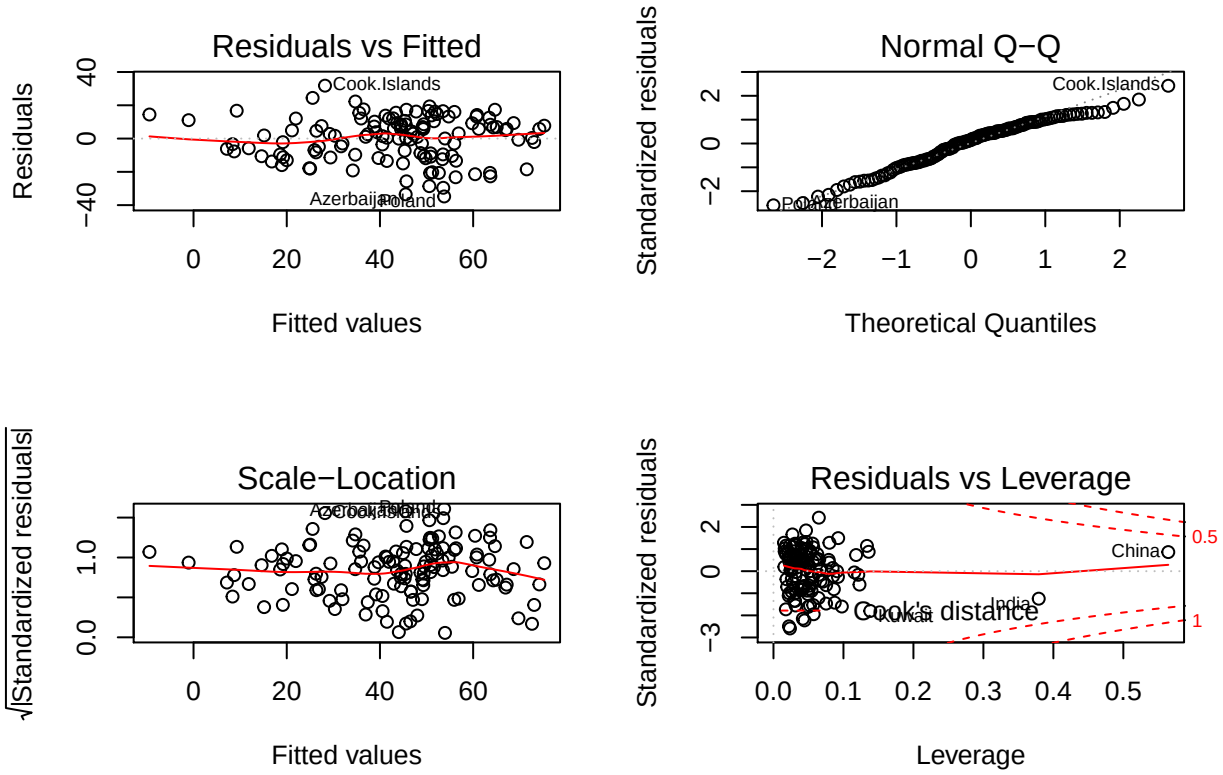
fig8. ModernC has nonlinear relationship with PPgdp

When I see pairwise plot among predictor variables, I can find that ModernC has quite strong relationship with variables named change,PPgdp,Fertility,Purban. Three of them have linear relationship with ModernC. But PPgdp seems to have non-linear relationship with ModernC. I think this phenomenom stem from skewness of PPgdp. Thus I should recheck after taking transformation on PPgdp.

Model Fitting

4. Use the `lm()` function to perform a multiple linear regression with `ModernC` as the response and all other variables as the predictors, using the formula `ModernC ~ .`, where the `.` includes all remaining variables in the dataframe. Create diagnostic residual plot from the linear model object and comment on results regarding assumptions. How many observations are used in your model fitting?

```
Fm <- lm(ModernC~.,data = UN3)
par(mfrow=c(2,2))
plot(Fm,
     sub.caption = "fig9. potentially influential point detected at leverage plot")
```



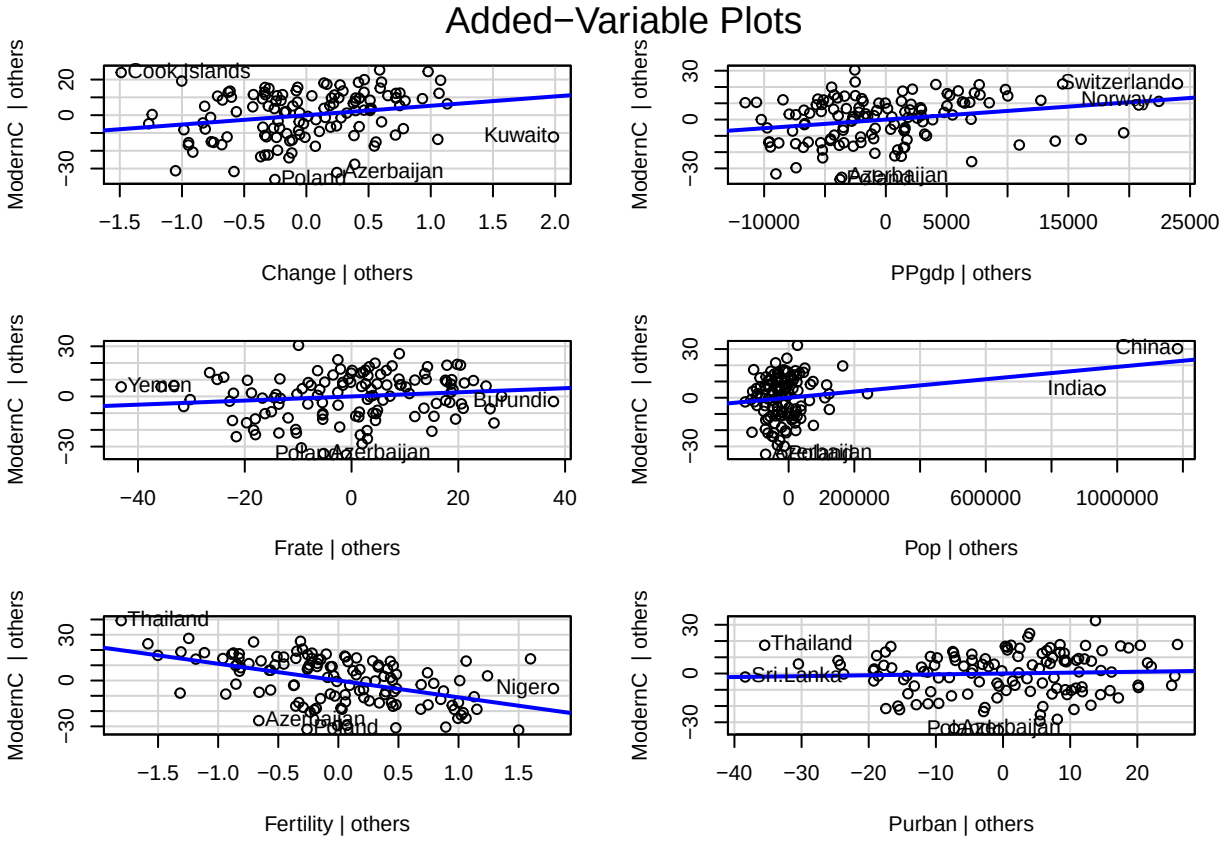

```
summary(Fm)
```

```
##
## Call:
## lm(formula = ModernC ~ ., data = UN3)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -34.781  -9.698   1.858   9.327  31.791
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5.529e+01  9.467e+00   5.841 4.69e-08 ***
## Change       5.268e+00  2.088e+00   2.524  0.01294 *
## PPgdp        5.301e-04  1.770e-04   2.995  0.00334 **
## Frate        1.232e-01  8.060e-02   1.529  0.12901
## Pop          1.899e-05  8.213e-06   2.312  0.02250 *
## Fertility   -1.100e+01  1.752e+00  -6.276 5.96e-09 ***
## Purban       5.408e-02  9.285e-02   0.582  0.56134
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 13.58 on 118 degrees of freedom
##   (85 observations deleted due to missingness)
## Multiple R-squared:  0.6183, Adjusted R-squared:  0.5989
## F-statistic: 31.85 on 6 and 118 DF,  p-value: < 2.2e-16
```

When it comes to residual vs fitted value plot, there isn't any violation sign such as non-linear relationship between them. However, although it it not severe, I can find out normality assumption is violated at margin of normal q-qplot. In scale-location plot, they are randomly distributed forming straight band. Thus there is no evidence that homongenuity assumption is violated. But in leverage vs residual plot, there are some potential influencial point. Therefore we should pay attention to those observation. In model fitting, 125 observations are used and 85 observations are omitted because of their missingness.

5. Examine added variable plots `car::avPlot` or `car::avPlots` for your model above. Are there any plots that suggest that transformations are needed for any of the terms in the model? Describe. Is it likely that any of the localities are influential for any of the terms? Which terms?

```
avPlots(Fm)
```



Among these variables, there are two variables, PPgdp, Pop, which need to be transformed. As mentioned before, PPgdp is right skewed and Pop has potentially influential points which are China and India.

6. Using the multivariate BoxCox `car::powerTransform` or Box-Tidwell `car::boxTidwell` find appropriate transformations of the predictor variables to be used as predictors in the linear model. If any predictors are negative, you may need to transform so that they are non-negative. Describe your method and the resulting transformations.

```
summary(UN3)
```

```
##     ModernC          Change            PPgdp            Frate
##  Min.   : 1.00   Min.   :-1.100   Min.   :   90   Min.   : 2.00
```

```
##  1st Qu.:19.00   1st Qu.: 0.580   1st Qu.:  479   1st Qu.:39.50
##  Median :40.50   Median : 1.400   Median : 2046   Median :49.00
##  Mean   :38.72   Mean   : 1.418   Mean   : 6527   Mean   :48.31
##  3rd Qu.:55.00   3rd Qu.: 2.270   3rd Qu.: 8461   3rd Qu.:58.00
##  Max.   :83.00   Max.   : 4.170   Max.   :44579   Max.   :91.00
##  NA's   :58      NA's   :1        NA's   :9       NA's   :43
##       Pop              Fertility         Purban
##  Min.   :      2.3   Min.   :1.000   Min.   :  6.00
##  1st Qu.:    767.2   1st Qu.:1.897   1st Qu.: 36.25
##  Median :   5469.5   Median :2.700   Median : 57.00
##  Mean   :  30281.9   Mean   :3.214   Mean   : 56.20
##  3rd Qu.:  18913.5   3rd Qu.:4.395   3rd Qu.: 75.00
##  Max.   :1304196.0   Max.   :8.000   Max.   :100.00
##  NA's   :2           NA's   :10
```

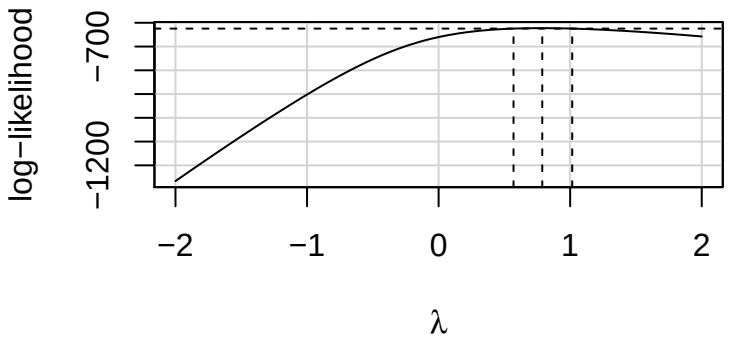
```
UN <-UN3 %>%
      mutate(Change_add = Change+1.2) %>%
      select(ModernC,Change_add,PPgdp,Frate,Pop,Fertility,Purban)
powerTransform(UN,family="bcPower")
```

```
## Estimated transformation parameters
##     ModernC  Change_add       PPgdp       Frate         Pop   Fertility
##  0.87069484  0.93338011 -0.15621030  1.09144998  0.06285445  0.18829460
##      Purban
##  0.92703643
```

Checking summary of UN3, I can find out that Change variable has minimum negative value -1.1. Thus I decide to add 1.2 on Change. Since ModernC, Frate, Purban, Change_add have optimal value for lamda which is approximately 1, they don't need to be transformed. However, in the case of Pop, PPgdp, and Fertility, they have optimal value for lamda which is approximately 0. Thus they are required to be log transformed.

7. Given the selected transformations of the predictors, select a transformation of the response using `MASS::boxcox` or `car::boxCox` and justify. Do you need to do this if you used `car::powerTransform` above? Explain.

```
boxCox(lm(ModernC~.,data=UN))
```

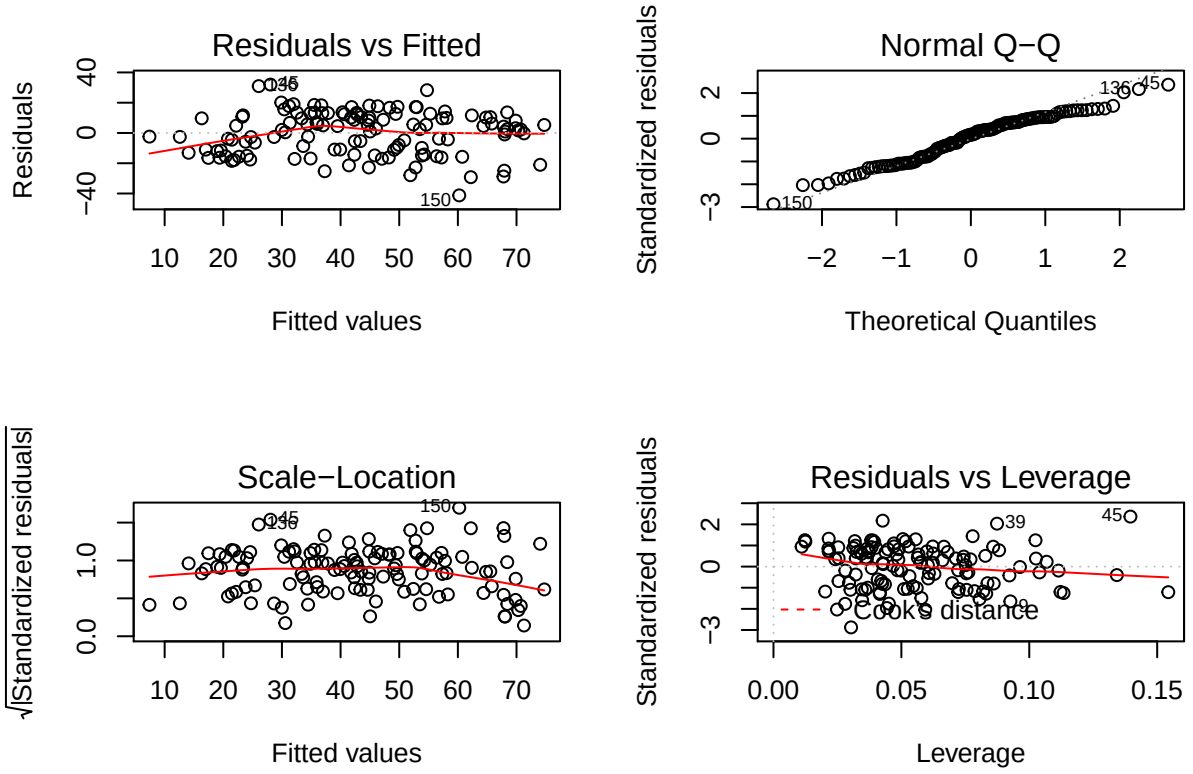


As we can see at above plot, value 1 is included in 95% confidence interval of lambda which is same result from Q6. Thus we don't need to transform. However, even though we have checked through 'powerTransform' function, we should check again with boxcox function for potential disctintion between results.

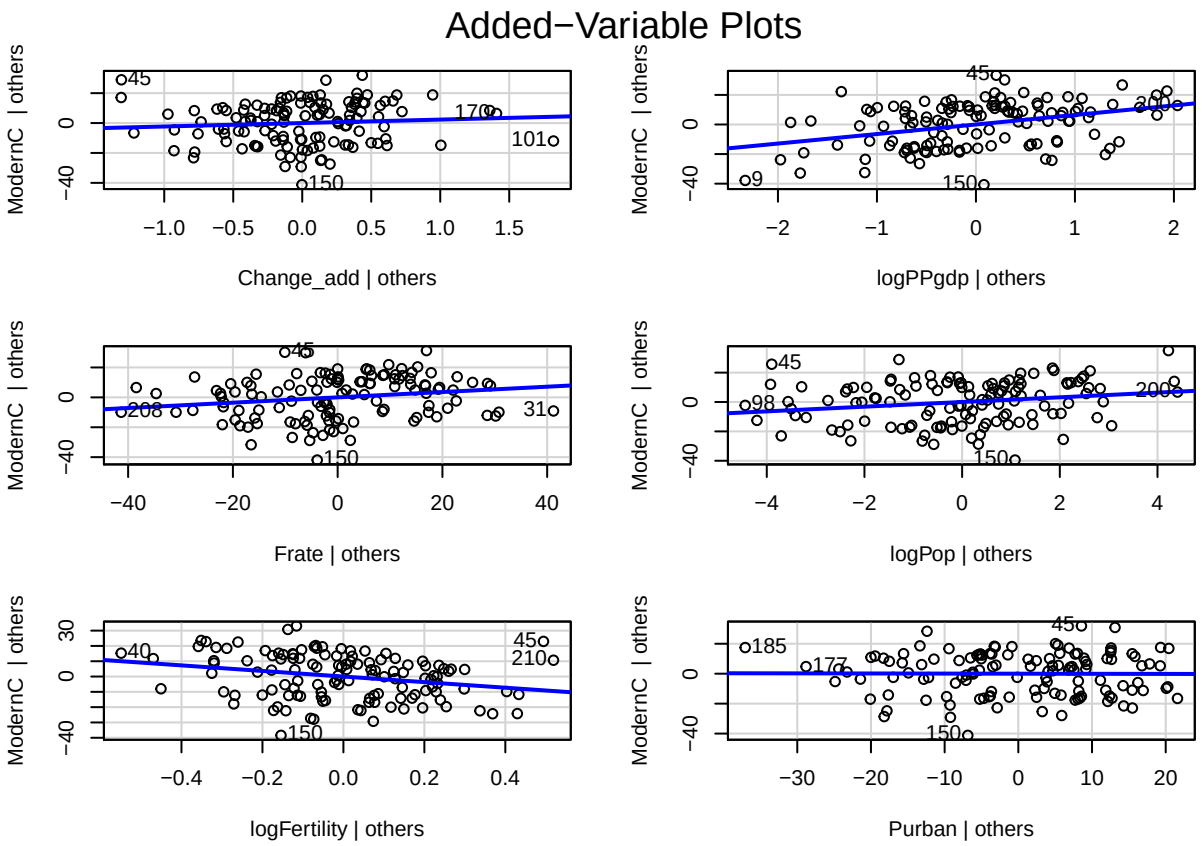
8. Fit the regression using the transformed variables. Provide residual plots and added variables plots and comment. If you feel that you need additional transformations of either the response or predictors, repeat any steps until you feel satisfied.

```
library(dplyr)
UN2 <- UN %>%
  mutate(logPPgdp = log(PPgdp),
         logPop = log(Pop),
         logFertility = log(Fertility))  %>%
  select(ModernC,Change_add,logPPgdp,Frate,logPop,logFertility,Purban)

Cm <- lm(ModernC~.,data=UN2)
par(mfrow=c(2,2))
plot(Cm,
     sub = "fig10. every plot don't show any violation of assumption")
```




```
avPlots(Cm)
```



In both diagnostic plot and added variable plots, I cannot find severe violation of assumption. The problems we could find in previous models are improved.

9. Are there any outliers or influential points in the data? Explain. If so, refit the model after removing any outliers/influential points and comment on residual plots.

```
abs.ti <- abs(rstudent(Cm))
pval <- 2*(1-pt(abs.ti, Cm$df -1))
min(pval) < .05/(Cm$df + ncol(UN))
```

```
## [1] FALSE
```

When executing outlier test with Bonferonni correction, I cannot find any outliers. Even though there are some suspects for outlier, their leverage changed after taking power transform.

Summary of Results

10. For your final model, provide summaries of coefficients with 95% confidence intervals in a nice table with interpretations of each coefficient. These should be in terms of the original units!

```
sum_Cm <- cbind(summary(Cm)$coefficient[,1],
                confint(Cm,c('(Intercept)',colnames(UN2)[-1]),level = 0.95))
sum_Cm <- round(sum_Cm,3)
interpret <- c("When predictor variables are 0, ModernC has -17.879 on average","Increase in 1 unit of C
```

```r
colnames(sum_Cm) <- c("coefficient","2.5%","97.5%")
names(interpret) <- rownames(sum_Cm)
kable(sum_Cm)
```

	coefficient	2.5%	97.5%
(Intercept)	-17.879	-48.229	12.471
Change_add	2.310	-2.761	7.381
logPPgdp	6.446	3.459	9.432
Frate	0.178	0.013	0.344
logPop	1.596	0.211	2.980
logFertility	-18.238	-30.786	-5.689
Purban	-0.007	-0.218	0.204

```r
kable(interpret)
```

	x
(Intercept)	When predictor variables are 0, ModernC has -17.879 on average
Change_add	Increase in 1 unit of Change_add makes increase 2.31 unit of ModernC on average
logPPgdp	Increase in 1 unit of PPgdp makes increase 6.44*log(1+1/PPgdp) unit of ModernC on average
Frate	Increase in 1 unit of Frate makes increase 0.17 unit of ModernC on average
logPop	Increase in 1 unit of Pop makes increase 1.59*log(1+1/Pop) unit of ModernC on average
logFertility	Increase in 1 unit of Fertility makes decrease -18*log(1+1/Fertility) unit of ModernC on average
Purban	Increase in 1 unit of Purban makes decrease 0.007 unit of ModernC on average

Every interpretation assumes that other predictors are remain constant.

11. Provide a paragraph summarizing your final model and findings suitable for the US envoy to the UN after adjusting for outliers or influential points. You should provide a justification for any case deletions in your final model.

```r
any(cooks.distance(Cm)>1)
```

```
## [1] FALSE
```

```r
summary(Cm)
```

```
##
## Call:
## lm(formula = ModernC ~ ., data = UN2)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -41.235 -11.589   2.498  10.748  31.954
##
## Coefficients:
##               Estimate Std. Error t value Pr(>|t|)
## (Intercept)  -17.878819  15.326291  -1.167  0.24575
```

```
## Change_add     2.310274   2.560728   0.902  0.36879
## logPPgdp       6.445713   1.508057   4.274 3.91e-05 ***
## Frate          0.178242   0.083567   2.133  0.03500 *
## logPop         1.595611   0.699289   2.282  0.02430 *
## logFertility -18.237639   6.336680  -2.878  0.00475 **
## Purban        -0.007352   0.106591  -0.069  0.94513
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 14.55 on 118 degrees of freedom
##   (85 observations deleted due to missingness)
## Multiple R-squared:  0.5615, Adjusted R-squared:  0.5392
## F-statistic: 25.19 on 6 and 118 DF,  p-value: < 2.2e-16
```

As we can see summary of final model, using predictors Change_add, logPPgdp, Frate, logPop, logFertility, Purban, my model explain 56% of variation in ModernC. Among 210 observations, only 125 observations are used to construct model because of some observations' missingness. On the course of EDA, I detected some potentially influential points(China and India) at Pop variable but after taking log transformation, their influence was reduce. Although there is remaining risk to includes that observations, but It would be better to include them rather than excluding them because it can contain some important information.

Methodology

12. Prove that the intercept in the added variable scatter plot will always be zero. *Hint: use the fact that if H is the projection matrix for X which contains a column of ones, then $1_n^T(I-H)=0$ or $(I-H)1_n=0$. Use this to show that the sample mean of residuals will always be zero if there is an intercept.*

answer: Added scatter plot draw linear relationship of model: $e_Y = \beta_0 + \beta_p e_{x_p}$. X^* is set of predictors except pth variable x_p where e_Y is residual of fitted model on Y using X^* and e_{x_p} is residual of fitted model on x_p using X^*. By simple regression estimate result, estimate for intttercept is

$$\hat{\beta}_0 = \overline{e_Y} - \hat{\beta}_p \overline{e_{x_p}}$$

and

$$\overline{e_Y} = \frac{\sum_{i=1}^n e_{Y_i}}{n} = \frac{1}{n} \times 1_n^T (I - H^*) Y$$

where $H^* = X^*(X^*X^*)^{-1}X^{*T}$. But H^* is projection matrix of X which include one's column. According to fact that if H^* is the projection matrix for X which contains a column of ones, then $1_n^T(I-H)=0$ or $(I-H)1_n=0$, $\overline{e_Y}=0$

$$\overline{e_{x_p}} = \frac{\sum_{i=1}^n e_{X_{pi}}}{n} = \frac{1}{n} \times 1_n^T (I - H^*) x_p$$

By same logic, $\overline{e_{x_p}} = 0$.

Thus estimated intercept $\hat{\beta}_0 = 0$ for every added variable plot.

13. Exercise 9.12 from ALR

Using $X^TX = X_{(i)}^T X_{(i)} + x_i x_i^T$ where the subscript (i) means without the ith case, show that

$$(X_{(i)}^T X_{(i)})^{-1} = (X^TX)^{-1} + \frac{(X^TX)^{-1} x_i x_i^T (X^TX)^{-1}}{1 - h_{ii}}$$

where h_{ii} is the ith diagonal element of $H = X(X^TX)^{-1}X^T$ using direct multiplication and simplify in terms of_ h_{ii}.

answer: If above statement is true then their multiplication will be I. Thus if we multiply $(X_{(i)}^T X_{(i)})$ on both side then

$$I = (X_{(i)}^T X_{(i)})[(X^TX)^{-1} + \frac{(X^TX)^{-1}x_i x_i^T (X^TX)^{-1}}{1-h_{ii}}]$$

we have to prove that right side is also I

$$\begin{aligned}
&= (X_{(i)}^T X_{(i)})[(X^TX)^{-1} + \frac{(X^TX)^{-1}x_i x_i^T (X^TX)^{-1}}{1-h_{ii}}] \\
&= (X^TX - x_i x_i^T)[(X^TX)^{-1} + \frac{(X^TX)^{-1}x_i x_i^T (X^TX)^{-1}}{1-h_{ii}}] \\
&= I - x_i x_i^T (X^TX)^{-1} + \frac{x_i x_i^T (X^TX)^{-1}}{1-h_{ii}} - \frac{x_i x_i^T (X^TX)^{-1} x_i x_i^T (X^TX)^{-1}}{1-h_{ii}} \\
&= I - x_i x_i^T (X^TX)^{-1} + \frac{x_i x_i^T (X^TX)^{-1}[1 - x_i (X^TX)^{-1} x_i^T]}{1-h_{ii}} \\
&= I - x_i x_i^T (X^TX)^{-1} + \frac{x_i x_i^T (X^TX)^{-1}[1 - h_{ii}]}{1-h_{ii}} \\
&= I - x_i x_i^T (X^TX)^{-1} + x_i x_i^T (X^TX)^{-1} = I
\end{aligned}$$

Thus suggested statement

$$(X_{(i)}^T X_{(i)})^{-1} = (X^TX)^{-1} + \frac{(X^TX)^{-1}x_i x_i^T (X^TX)^{-1}}{1-h_{ii}}$$

is true.

14. Exercise 9.13 from ALR. Using the above, show

$$\hat{\beta}_{(i)} = \hat{\beta} - \frac{(X^TX)^{-1}x_i e_i}{1-h_{ii}}$$

answer:

$$\hat{\beta}_{(i)} = (X_{(i)}^T X_{(i)})^{-1} X_{(i)}^T Y_{(i)}$$

as shown previous question,

$$(X_{(i)}^T X_{(i)})^{-1} = (X^TX)^{-1} + \frac{(X^TX)^{-1}x_i x_i^T (X^TX)^{-1}}{1-h_{ii}}$$

and

$$X_{(i)}^T Y_{(i)} = \sum_{j=1}^{n} x_j y_j - x_i y_i = X^TY - x_i y_i$$

Thus

$$
\begin{aligned}
\hat{\beta}_{(i)} &= [(X^TX)^{-1} + \frac{(X^TX)^{-1}x_ix_i^T(X^TX)^{-1}}{1-h_{ii}}][X^TY - x_iy_i] \\
&= [(X^TX)^{-1} + \frac{(X^TX)^{-1}x_ix_i^T(X^TX)^{-1}}{1-h_{ii}}]X^TY - [(X^TX)^{-1} + \frac{(X^TX)^{-1}x_ix_i^T(X^TX)^{-1}}{1-h_{ii}}] \\
&= (X^TX)^{-1}X^TY + \frac{(X^TX)^{-1}x_ix_i^T}{1-h_{ii}}(X^TX)^{-1}X^TY - (X^TX)^{-1}x_iy_i - \frac{(X^TX)^{-1}x_ix_i^T(X^TX)^{-1}x_iy_i}{1-h_{ii}} \\
&= \hat{\beta} + \frac{(X^TX)^{-1}x_ix_i^T}{1-h_{ii}}\hat{\beta} - (X^TX)^{-1}x_iy_i - \frac{(X^TX)^{-1}x_ix_i^T(X^TX)^{-1}x_iy_i}{1-h_{ii}} \\
&= \hat{\beta} + \frac{(X^TX)^{-1}x_i\hat{y_i}}{1-h_{ii}} - (X^TX)^{-1}x_iy_i - \frac{(X^TX)^{-1}x_iy_i}{1-h_{ii}}h_{ii} \\
&= \hat{\beta} + \frac{(X^TX)^{-1}x_i\hat{y_i}}{1-h_{ii}} - \frac{(X^TX)^{-1}x_iy_i}{1-h_{ii}} \\
&= \hat{\beta} - \frac{(X^TX)^{-1}x_i}{1-h_{ii}}(y_i - \hat{y_i}) = \hat{\beta} - \frac{(X^TX)^{-1}x_ie_i}{1-h_{ii}}
\end{aligned}
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