

## Math 525: Assignment 5

1. Let  $X$ ,  $Y$ , and  $Z$  be square integrable random variables such that  $X$  and  $Y$  are independent. Show that

$$\mathbb{E}[XYZ] \leq \sqrt{\mathbb{E}[X^2] \mathbb{E}[Y^2] \mathbb{E}[Z^2]}.$$

2. Let  $X$  be an integrable random variable with moment generating function  $M$ . Show that  $M(\theta) \geq e^{\theta \mathbb{E}[X]}$  whenever  $e^{\theta X}$  is integrable.
3. Let  $p$  be a positive integer and  $(X_n)_n$  be a sequence of random variables satisfying

$$\mathbb{E}[|X|^p] \leq f(n)$$

where  $f$  satisfies  $\sum_n f(n) < \infty$ . Show that

$$\lim_n X_n = 0 \text{ a.s.}$$

**Hint:** use Markov's inequality and the Borel-Cantelli lemma.