Kuramoto-Sivashinsky Equation

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Kuramoto Sivashinsky Equation.

The equation reads:

$$u_t = -u_{xx} - u_{xxxx} - uu_x,$$

where $(t, x) \in (0, 150] \times (0, 32\pi)$ with the periodic boundary condition. This equation is classified into stiff PDEs, since it has a form of

$$u_t = \mathcal{L}u + \mathcal{N}[u],$$

where $\mathcal{L}$ is a linear operator with high order (spatial) derivatives, and $\mathcal{N}$ is a nonlinear operator with lower order derivatives. A lot of studies has been performed to analyze equations of such formulation. Please refer to Kassam & Trefethen 2005, SISC.

In fact, the equation can be solved within 1 second in my laptop, by combination of fast fourier transform and exponential time differencing Runge-Kutta time-stepping of order 4. However, it is notoriously difficult to solve with Physics-informed neural networks (PINNs). There was a result in "Causality PINN" by S. Wang, yet the computational cost was exceedingly high compared to other problems. He used 10-layer (modified) multi-layer perceptron with 256 hidden units, decomposed time domain into 10 sub-pieces, and utilized multiple GPUs.