mas651 exercises

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Problem (8.1.1)**.**

By theorem 8.1.1, there is $T_{U,V}$ *stopping time such that*

$$B(T_{U,V}) =_d X \text{ and } EX^2 = ET_{U,V}.$$

By exercise 7.5.4,

$$ET_{U,V}^2 \le 4EB(T_{U,V})^4 = 4EX^4.$$

$\square$

Problem (8.1.2)**.**

Let $\psi(w) = \max\{w(t) : 0 \le t \le 1\} - \min\{w(t) : 0 \le t \le 1\}$*. Then*

$$\frac{R_n}{\sqrt{n}} = \frac{1}{\sqrt{n}} + \psi\left(\frac{S_{nt}}{\sqrt{n}}\right).$$

Now $1/\sqrt{n} \to 0$ *as* $n \to \infty$ *in probability, and by theorem 8.1.5, the last term goes to*

$$\psi(B_t) = \max_{0\le t\le 1} B_t - \min_{0\le t\le 1} B_t$$

in distribution.

By Slutsky's eqn, we can say that

$$\frac{R_n}{\sqrt{n}} \to_d \max_{0\le t\le 1} B_t - \min_{0\le t\le 1} B_t.$$

$\square$