# DQ - Probability

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**Problem** (2012.3)**.**

*(a)* $(\tau = n) = \{S_n > 0, S_k \leq 0 \ \forall k < n\} \in \mathcal{B}$. *And* $\tau < \infty$. *Thus the result follows.*

*(b)* *Note that* $S_\tau > 0$ *and* $\tau < \infty$ *for all* $\omega \in \Omega$. *Thus, to* $S_\tau$ *be random variable, it is enough to check that* $(S_\tau < a)$ *is measurable for* $a > 0$.

$$(S_\tau < a) = \bigcup_{n=1}^{\infty} \{\tau = n\} \cap \{S_n < a\}$$

*Therefore* $S_\tau$ *is a random variable.*

□

**Problem** (2012.9)**.**

*Since* $P(N < \infty) = 1$, $X_{n\wedge N} \to X_N$ *almost surely. Since* $X_{n\wedge N}$ *is uniformly bounded, bounded convergence theorem implies* $X_{n\wedge N} \to X_N$ *in* $L_1$. *Thus*

$$EX_{n\wedge N} \to EX_N.$$

*But* $EX_0 = EX_{n\wedge N}$ *since* $X_{n\wedge N}$ *is a martingale. Therefore the result follows.*

□