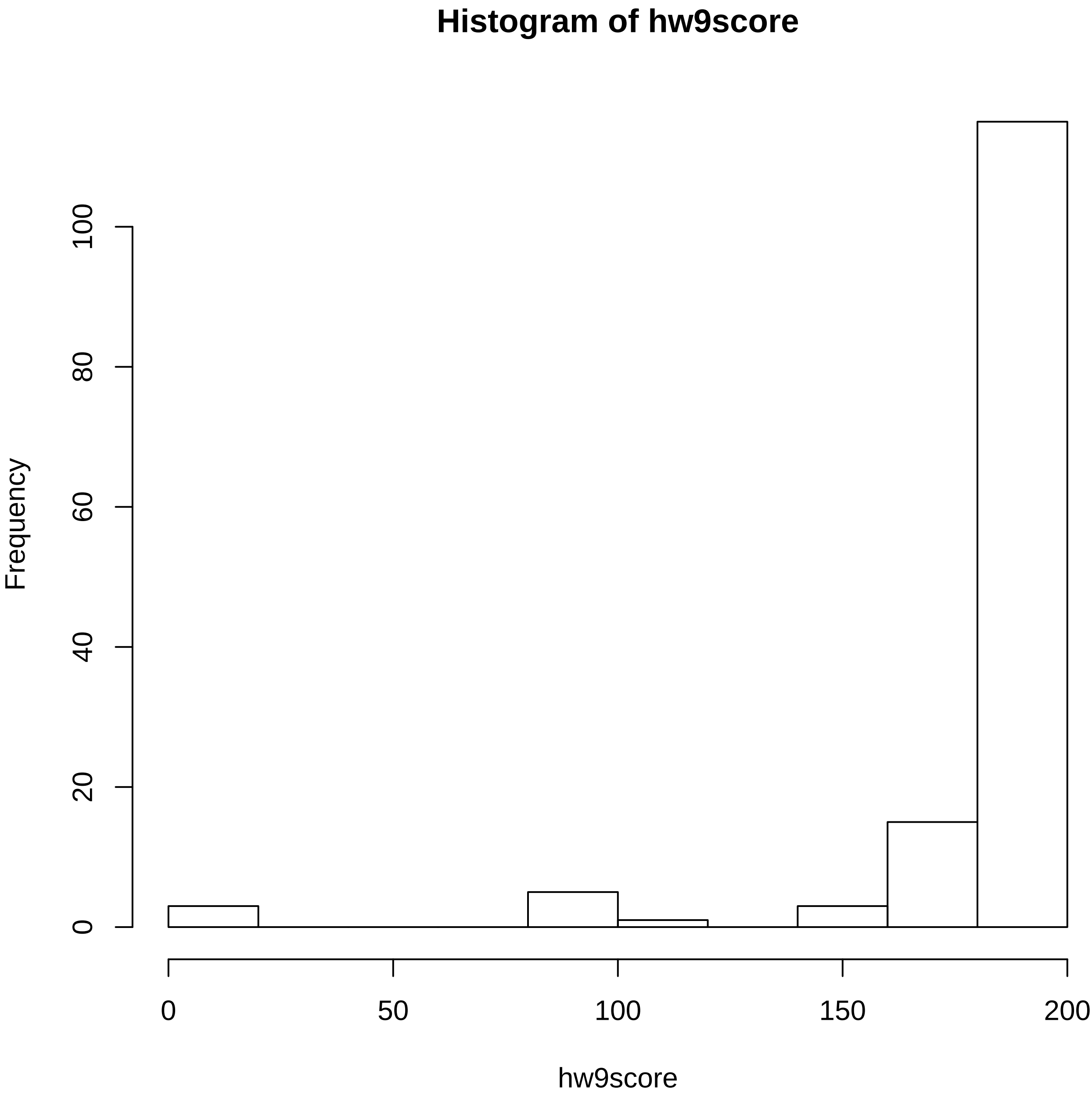
Histogram of hw9score
Frequency
0
20
40
60
80
100
0
50
100
150
200
hw9score

Homework 9.

(Due 6 PM on May. 19)

10.1.4 A new radar system is being developed to detect packages dropped by airplane. In a series of trials, the radar detected the packages being dropped 35 times out of 44. Construct a 95% lower confidence bound on the probability that the radar successfully detects dropped packages.

Solution. *Since* $n - x = 9 > 5$ *and* $n = 44 > 5$, *we can use normal approximation. (+3) The formula for the answer is*

$$\left(\frac{x}{n} - \frac{Z_{0.05}}{n}\sqrt{\frac{n(n-x)}{n}}, 1\right),$$

which is

$$\left(\frac{35}{44} - \frac{1.645}{44}\sqrt{\frac{35 \cdot 9}{44}}, 1\right) = (0.70, 1).$$

(+7)

□

10.1.8 In trials of a medical screening test for a particular illness, 23 cases out of 324 positive results turned out to be "false-positive" results. The screening test is acceptable as long as p, the probability of a positive result being incorrect, is no larger than 10%. Calculate a p-value for the hypotheses

$$H_0 : p \geq 0.1 \qquad \text{versus} \qquad H_A : p < 0.1$$

Construct a 99% upper confidence bound on p. Do you think that the screening test is acceptable?

Solution. $n - x, n > 5$ *so the normal approximation can be done. (+2) Our observed statistic is*

$$\frac{x - np_0 + 0.5}{\sqrt{np_0(1-p_0)}} = -1.65$$

where $+0.5$ *of the numerator is due to the continuity correction.(+5)*

Now, p-value is

$$P(Z \leq -1.65) = 0.049.$$

(+2)

And the upper confidence bound is

$$\left(0, \frac{x}{n} + \frac{Z_{0.01}}{n}\sqrt{\frac{x(n-x)}{n}}\right) = (0, 0.104).$$

(+1)

Since p-value of our test is larger than 0.01, *we can say that the screening thest is non-acceptable.*

□

10.1.18 The dielectric breakdown strength of an electrical insulator is defined to be the voltage at which the insulator starts to leak detectable amounts of electrical current, and it is an important safety consideration. In an experiment, 62 insulators of a certain type were tested at 180°C, and it was found that 13 had a dielectric breakdown strength below a specified threshold level.

(a) Conduct a hypothesis test to investigate whether this experiment provides sufficient evidence to conclude that the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level is larger than 5%.

(b) Construct a one-sided 95% confidence interval that provides a lower bound on the probability of an insulator of this type having a dielectric breakdown strength below the specified threshold level.

Solution. *(a)* $n = 62$, $x = 13$ *and* $p_0 = 0.05$. *Note that the normal approximation is applicable for our data. Set the null hypothesis* $H_0 : p \leq 0.05$ *and the alternative* $H_1 : p > 0.05$. *Now observed test statistic is*

$$\frac{x - np_0 - 0.5}{\sqrt{np_0(1 - p_0)}} = 5.48$$

by normal approximation and continuity correction. Then p-value is

$$P(Z > 5.48),$$

which is quite smaller than usual confidence level($0.05, 0.01$*). So we can reject the null under* $\alpha = 0.01, 0.05$.*(+5)*

(b) By using the same formula which is used in the previous problem, the answer is

$$\left(\frac{13}{62} - \frac{Z_{0.01}}{62}\sqrt{\frac{13(62-13)}{62}}, 1\right) = (0.125, 1).$$

(+5)

□

10.2.2 Suppose that $x = 261$ is an observation from a $B(302, p_A)$ random variable, and that $y = 401$ is an observation from a $B(454, p_B)$ random variable.

(a) Compute a two-sided 99% confidence interval for $p_A - p_B$.

(b) Compute a two-sided 90% confidence interval for $p_A - p_B$.

(c) Compute a one-sided 95% confidence interval for $p_A - p_B$. Repeat (a) and (b) with the data in (c).

(d) Calculate the p-value for the test of the hypotheses

$$H_0 : p_A = p_B \qquad \text{versus} \qquad H_A : p_A \neq p_B$$

Solution. $(n, m, x, y) = (302, 454, 261, 401)$.

(a) *The formula is*

$$\left(\hat{p}_A - \hat{p}_B - Z_{\alpha/2}\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n} + \frac{\hat{p}_B(1-\hat{p}_B)}{m}}, \hat{p}_A - \hat{p}_B + Z_{\alpha/2}\sqrt{\frac{\hat{p}_A(1-\hat{p}_A)}{n} + \frac{\hat{p}(1-\hat{p}_B)}{m}}\right).$$

Realized value is $(-0.083, 0.045)$.*(+2)*

(b) *Procedure is the same. (+2)*

(c) *Procedure is the same. (+2)*

(d) *Our test statistic is*

$$z = \frac{\hat{p}_A - \hat{p}_B}{\sqrt{\hat{p}(1-\hat{p})\,(1/n + 1/m)}} = -0.78$$

where $\hat{p}$ *is the pooled one. Since we are doing two-sided test, our p-value is*

$$P(|Z| \geq 0.78) = 0.44.$$

(+4)

□

10.2.12 Recall from Problem 10.1.16 that in a particular day, 22 out of 542 visitors to a website followed a link provided by an advertiser. After the advertisements were modified, it was found that 64 out of 601 visitors to the website on a day followed the link. Is there any evidence that the modifications to the advertisements attracted more customers? ($\alpha = 0.05$)

Solution. $(n, m, x, y) = (542, 601, 22, 64)$. *Set the null* $H_0 : p_A \geq p_B$ *and the alternative* $H_1 : p_A < p_B$.*(+3) Use test statistic* z *used in the above. Then observed test statistic is* -4.22. *So, p-value is pretty smaller than 0.05 since p-value is* $P(Z \leq -4.22)$, *and* $-4.22 < -1.64$. *Thus we can reject the null, so there is evidence of the claim.(+7)*

□

10.1.16 [Not a homework problem] In a particular day, 22 out of 542 visitors to a website followed a link provided by one of the advertisers. Calculate a 99% two-sided confidence interval for the probability that a user of the website will follow a link provided by an advertiser.

10.3.6 **Taste Tests for Soft Drink Formulations**
A beverage company has three formulations of a soft drink product. DS 10.3.6 gives the results of some taste tests where participants are asked to declare which formulation they like best. Is it plausible that the three formulations are equally popular? ($\alpha = 0.05$)

Solution. *Since we are using the approximated statistic, checking the approximation condition is always important. And note that* χ^2, G^2 *are asymptotically chi-squared. So if* n *is sufficiently large, then you can use anything. I'll use the first one.*

Set the null $H_0 : p_i = 1/3, i = 1,2,3$ and the alternative $H_1 :$ not H_0.(+3) The total number of participants is 600, so the expected frequencies are $e_i = 200$ each.(+3) Our test statistic is

$$\sum_{i=1}^{3} \frac{(x_i - e_i)^2}{e_i} = 17.29$$

where x_i is the frequency of each cell.(+2) Now p-value is

$$P(\chi_2^2 \geq 17.29) = 0.00018$$

so we can reject the null under $\alpha = 0.05$.(+2)

□

10.3.14 A survey is performed to test the claim that three brands of a product are equally popular. In the survey, 22 people preferred brand A, 38 people preferred brand B, while 40 people preferred brand C. The Pearson goodness of fit can be used to test whether the survey provides sufficient evidence to conclude that the three brands are not equally popular. ($\alpha = 0.05$)

Solution. *The total number of participants is 100. Let $(x_1, x_2, x_3) = (22, 38, 40)$. Set the null $H_0 : p_i = 1/3$ and the alternative $H_1 :$ not H_0.(+3) Then the expected frequency is $100/3 = e_i$ each.(+3) Our test statistic is*

$$\sum_{i=1}^{3} \frac{(x_i - e_i)^2}{e_i} = 5.84.$$

(+2) So, p-value is

$$P(\chi_2^2 \geq 5.84) = 0.054,$$

thus we cannot reject the null under $\alpha = 0.05$.(+2) (False: the survey does not provide sufficient evidence.)

□

A. True B .False

10.4.2 **Fertilizer Comparisons**

Seedlings are grown without fertilizer or with one of two kinds of fertilizer. After a certain period of time a seedling's growth is classified into one of four categories, as given in DS 10.4.2. Test whether the seedlings' growth can be taken to be the same for all three sets of growing conditions. ($\alpha = 0.05$)

Solution. *Sort the data by the matrix(= A), where the same row indicates fertilizer and the same column indicates growth. (row1 = no fer, row2 = fer1, row3 = fer2, col1 = dead, col2 = slow, ...)*

Set the null $H_0 :$ two categories are independent and the alternative $H_1 :$ not H_0.(+3) Let $x_{ij} = (A)_{i,j}$ be the (i,j) entry of our matrix A. Then expected frequency is

$$e_{ij} = \frac{x_{i\cdot} x_{\cdot j}}{n}$$

where $x_{i\cdot} = \sum_{j=1}^4 x_{ij}$*.(+3)*
Then our test statistic is

$$X^2 = \sum_{i=1}^{3}\sum_{j=1}^{4} \frac{(x_{ij} - e_{ij})^2}{e_{ij}} = 13.66.$$

(+2) So p-value is

$$P(\chi_6^2 \geq 13.66) = 0.034.$$

Thus we can reject the null.(+2)

□

10.4.6 Show that for a 2×2 contingency table the Pearson chi-square statistic can be written

$$X^2 = \frac{n(x_{11}x_{22} - x_{12}x_{21})^2}{x_{1\cdot}x_{\cdot 1}x_{2\cdot}x_{\cdot 2}}$$

Solution. *Note that*

$$x_{ij} - e_{ij} = x_{ij} - \frac{(x_{i1} + x_{i2})(x_{1j} + x_{2j})}{\sum_{i,j} x_{ij}} = \frac{x_{ij}x_{i'j'} - x_{ij'}x_{i'j}}{n} = \frac{x_{11}x_{22} - x_{12}x_{21}}{n}$$

where $i \neq i'$ *and* $j \neq j'$*.*
Thus

$$\sum_{i,j} \frac{(x_{ij} - e_{ij})^2}{e_{ij}} = \frac{(x_{11}x_{22} - x_{12}x_{21})^2}{n^2} \sum_{i,j} \frac{1}{e_{ij}}.$$

But

$$\frac{1}{e_{ij}} = \frac{n}{x_{i\cdot}x_{\cdot j}} = \frac{n x_{i'\cdot}x_{\cdot j'}}{x_{1\cdot}x_{\cdot 1}x_{2\cdot}x_{\cdot 2}}.$$

Also,

$$\sum_{i,j} x_{i\cdot}x_{\cdot j} = n^2.$$

Combinining the above,

$$X^2 = \frac{(x_{11}x_{22} - x_{12}x_{21})^2}{n^2} \frac{n}{x_{1\cdot}x_{\cdot 1}x_{2\cdot}x_{\cdot 2}} n^2.$$

(+10)

□

10.4.10 **Asphalt Load Testing**
An experiment was conducted to compare three types of asphalt. Samples of each type of asphalt were subjected to repeated loads at high temperatures, and the resulting cracking was analyzed. For type A, 57 samples were tested, of which 9 had severe cracking, 17 had medium cracking, and 31 had minor cracking. For type B, 49 samples were tested, of which 4 had severe cracking, 9 had medium cracking, and 36 had minor cracking. For type C, 90 samples were tested, of which 15 had severe cracking, 19 had medium cracking, and 56 had minor cracking. Does this experiment provide any evidence that the three types of asphalt are different with respect to cracking?

Solution. *row1 = severe, row2 = medium, row3 = minor, col1 = A, col2 = B, col3 = C.*

Set the null and the alternative hypothesis.(+3)

Expected frequency(+3)

*Test statistic and the observed value(*5.02*)(+2)*

*p-value with chi-squared distribution, dof 4(*0.28*), conclusion(+2)*

The procedure is the same as 10.4.2, so I omitted calculations.

□