RCA SOLUTIONS

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CHAPTER 1

Problem 10.

$|f_n| \leq c_n$ for each $n \in \mathbb{N}$ since each f_n is bounded. Let ϵ be given positive real number. Then there exists positive integer N such that $|f - f_N| \leq \epsilon$ since f_n uniformly converges to f. So, we can assert that f is bounded since $|f| \leq |f_N - f| + |f_N| \leq c_N + \epsilon = C'$. Therefore, $|f_n| \leq \sum_{n=1}^{N} c_n + \epsilon + C' = C$ for all positive integer n. So, $|f_n| \leq C \in \mathcal{L}^1(\mu)$ because $\int_X C d\mu = C\mu(X) < \infty$. Also, $f = \lim f_n$ is measurable since limit of measurable function is measurable. By dominated convergence thm, $\lim_{n\to\infty} \int_X f_n d\mu = \int_X f d\mu$.

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