complex integral formulae

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Theorem 1 *If $f : [a,b] \to \mathbb{C}$ is continuous and if there exists a function $F(x)$ such that $F' = f$ on $[a,b]$, then*

$$\int_a^b f(x)dx = F(x)|_a^b = F(b) - F(a) \tag{1}$$

Theorem 2 (Green's Theorem) *Let P, Q be continuous with continuous partials in a simply connected closed region R whose boundary is the contour C. Then*

$$\int_C Pdx + Qdy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dxdy \tag{2}$$

where C is traversed in the positive sense.

Theorem 3 (Cauchy's Weak Theorem) *If $f(z)$ is analytic (with a continuous derivative) in a simply connected domain D, and C is closed contour lying in D, then we have $\int_C f dz = 0$*

Necessity of continuous derivative of f can be removed later by considering rectangle, closed ball, . . . etc.

Theorem 4 (corollary 7.34) *Under the conditions of above, let C_1, C_2 be any contours in the domain with the same initial and terminal points. Then*

$$\int_{C_1} f dz = \int_{C_2} f dz \tag{3}$$

Theorem 5 *Suppose that f is analytic (with a continuous derivative) in a multiply connected domain and on its boundary C. Then we have $\int_C f dz = 0$, where the integration is performed along C in the positive sense.*

Theorem 6 (Cauchy's theorem for a rectangle) *Let f be analytic in a domain containing a rectangle C and its interior. Then $\int_C f dz = 0$.*

Proof is similar to proof of 'Heine-Borel' thm. Divide given rectangles evenly and so on.

Theorem 7 (Corollary 7.40) *Let f be continuous in a domain D containing a rectangle C and its interior. Suppose that f is analytic in $D \setminus \{a\}$ for some point $a \in D$. Then $\int_C f dz = 0$*

Theorem 8 (Fundamental Theorem of Integration) *Let f be continuous in a domain D, and suppose there is an antiderivative F of f in D. Then for any contour C in D parameterized by $z(t), a \leq t \leq b$, we have*

$$\int_C f dz = F(z(b)) - F(z(a)) \tag{4}$$

In particular, if C is closed then $\int_C f dz = 0$.

Theorem 9 (Cauchy's theorem for a disk) *Let f be analytic in a domain containing the closed disk $|z - z_0| \leq r$. Then $\int_{|z-z_0|=r} f dz = 0$.*

Theorem 10 (Corollary 7.46) *Let f be analytic for $|z - z_0| < r$ except at some point a inside the disk and continuous for $|z - z_0| \leq r$. Then $\int_{|z-z_0|=r} f dz = 0$.*

Theorem 11 (Cauchy's Theorem) *If f is analytic in a simply connected domain D and C is a closed contour lying in D, then $\int_C f dz = 0$.*

Theorem 12 (Cauchy's Theorem for multiply connected domains) *Let D be a multiply connected domain bounded externally by a simple closed contour C and internally by n simple closed nonintersecting contours $C_1, C_2, \cdots, C_n$. Let f be analytic on $D \cup C \cup C_1 \cup \cdots \cup C_n$. Then*

$$\int_C f dz = \sum_{k=1}^{n} \int_{C_k} f dz \tag{5}$$

where C is taken counterclockwise around the external boundary C and clockwise around the internal boundaries $C_1, C_2, \cdots, C_n$.

Theorem 13 *If f is analytic and nonzero in a simply connected domain D, then there exists a function g, analytic in D, such that $e^{g(z)} = f(z)$.*

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