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1. **Plane Wave:** Below is the electric field solution for a free-space plane wave:

$$\tilde{\vec{E}}(\vec{r}) = 10\,[3\hat{x} - 4\hat{y} + 5\hat{z}]\exp\left(-j\,[2\hat{x} - \hat{y} - 2\hat{z}]\cdot\vec{r}\right)\ \mathrm{mV/m}$$

(a) What is the corresponding $\tilde{\vec{H}}(\vec{r})$ that accompanies this electric field? What is the wavelength of this plane wave?

For planewave in freespace

$$\vec{H}(r) = \frac{1}{\eta_0}\hat{k}\times\vec{E} \qquad \eta_0 = 120\pi\,\Omega,\quad \hat{k} = \frac{k}{\|k\|}$$

$$\|k\| = \sqrt{2^2+(-1)^2+(-2)^2} = 3\ \mathrm{rad/m}$$

$$\hat{k} = \frac{1}{3}(2,-1,-2)$$

$$\hat{k}\times(3,-4,5) = \left(-\frac{13}{3}, -\frac{16}{3}, -\frac{5}{3}\right)$$

10mV/m = 0.01V/m

$$\vec{H}(r) = \frac{10\times10^{-3}}{\eta_0}\left[-\frac{13}{3}\hat{x}, -\frac{16}{3}\hat{y}, -\frac{5}{3}\hat{z}\right]e^{-jk\cdot r}\ \mathrm{A/m}$$

$$\boxed{\vec{H}(r) = [-1.15, -1.41, -0.442]\times10^{-4}e^{-jk\cdot r}\ \mathrm{A/m}}$$

$$\boxed{\lambda = \frac{2\pi}{|k|} = \frac{2\pi}{3} \approx 2.094\mathrm{m}}$$

(b) If $\hat{z}$ corresponds to the vertical direction, $\hat{x}$ corresponds to due east along the horizon, and $\hat{y}$ corresponds to due north along the horizon, what look angle (azimuth and elevation) should you use to point a dish antenna for receiving this wave?

$k = (2,-1,-2)$

Dish aims to the "coming" wave. $\longrightarrow$ look vector : $-k = (-2, 1, 2)$

Azimuth (north = 0°, east = 90°)

Azimuth = atan2 $(-2, 1) = -63.434°$

$360 - 63.434 = \boxed{296.6°}$ (NW)

$$\text{Elevation} = \arctan\left(\frac{2}{\sqrt{(-2)^2+1^2}}\right) = \boxed{41.8°}$$

2. **Mission to Saturn:** The planet Saturn is 1.2×10^9 km from Earth at the time a NASA space probe must communicate back to an earth station using a 28 GHz carrier with a minimum received power of -105 dBm. If the satellite's transmit amplifier maximum output power is 500 W and the Earth station's receiver dish antenna must be 20 times larger in electromagnetic area than the transmitter antenna (implying 13 dB greater antenna gain), at least how much gain must the satellite dish antenna have?

(R) Distance : 1.2×10^9 km $= 1.2 \times 10^{12}$ m

Carrier : 28 GHz $\Rightarrow \lambda = \dfrac{c}{f} \approx 0.0107$ m

P_r^{min} : -105 dBm

P_t : 500W

$10 \log 20 \approx 13$ dB

$G_r = G_t + 13$

$P_t(\text{dBm}) = 10 \log (500\text{W} \cdot 1000 \text{mW/W})$

$= 10 \log (5 \cdot 10^5) \approx 57$ dBm

Free space path loss.

$$L_{fs} = 20 \log \left(\frac{4\pi R}{\lambda}\right) \approx 20 \log \left(\frac{4\pi \cdot 1.2 \cdot 10^{12} \text{m}}{0.0107 \text{m}}\right) \approx 303 \text{ dB}$$

Gain of receiver (link budget)

$P_r(\text{dBm}) = P_t(\text{dBm}) + G_t + G_r - L_{fs}$

$-105 = 57 + G_t + G_t + 13 - 303$

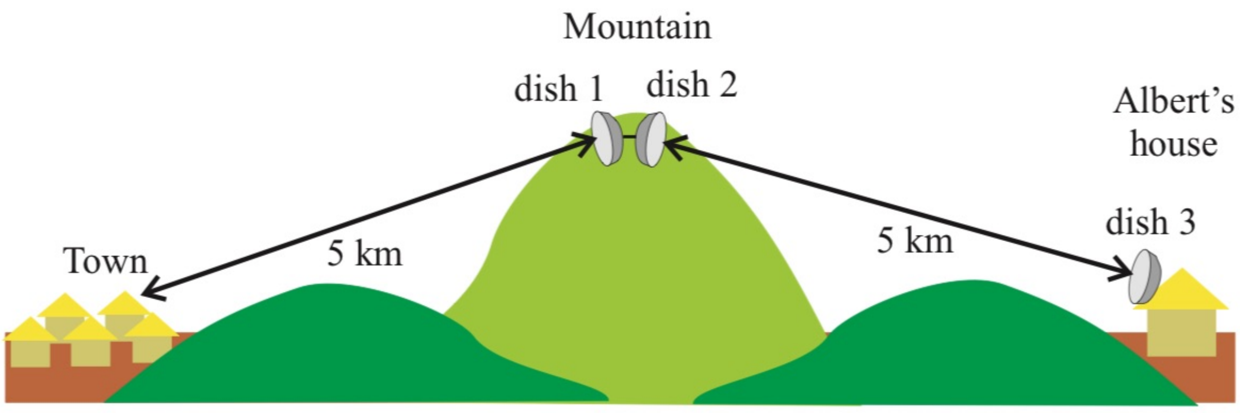
$-105 \text{ dBm} = 2G_t - 233 \text{ dBm}$

$2G_t = 128$

$G_t = 64$

$\boxed{G_t \geq 64 \text{ dBi}}$ minimum

3. **Stealing WiFi:** Albert lives in a quiet valley in North Georgia where he operates a winery and vineyard. Set apart from civilization, he has no access to cable, DSL, phone lines, or any other wired conduit of internet access. But Albert is a crafty graduate of the Georgia Institute of Technology and devises a clever way to steal WiFi service from the nearby town of Unprotectedlinksville, population 53. This town is 10 kilometers away from Albert, on the other side of a large mountain, and has several unprotected home WiFi servers broadcasting local internet service. His plan is to purchase 3 **identical** dish antennas that operate at 2.45 GHz and arrange them in the following configuration:



With this set-up, a signal will propagate from the town to the first dish in the link, which is pointed toward the town. The received power of this dish is piped directly to another dish which is pointed towards Albert's vineyard. Thus, this pair of dish antennas acts like a passive repeater that does not require any power or maintenance. A third dish is mounted on top of Albert's home, where a minimum value of -95 dBm must be received in order to maintain a wireless internet link on his home computer. Answer the following questions assuming matched and lossless cables. Assume that the antenna gain of the WiFi access point in town is 5 dBi, that the transmit power of this link is 30 dBm, and that both links are essentially free space.

(a) What is the minimum gain in dBi of these antennas to make this system work?

$f = 2.45\,GHz \rightarrow 2450\,MHz$

$d = 5\,km$ (each)

$G_a = 5\,dBi$

$P_t = 30\,dBi$

Required $P_r = -95\,dBm$

For 5km

$L_{fs} = 32.44 + 20\log(2450) + 20\log(5) \approx 114.2\,dB$

from town → mountain

$P_r = P_t + G_a + 3G_{dish} - 2L_{fs}$

$-95 = 30 + 5 + 3G_{dish} - 2(114.2)$

$-130 = 3G_{dish} - 228.4$

$98.4 = 3G_{dish}$

$\boxed{G_{dish} \geq 32.8\,dBi}$ minimum

(b) If these are ideal circular dishes with 100% efficiency, what is the minimum dish radius based on your answer in part (a)?

$G = \left(\frac{\pi D}{\lambda}\right)^2, \ (\eta = 1)$

$\lambda = \frac{c}{f} = \frac{3\times10^8}{2.45\times10^9} \approx 0.12245\,m$

$G_{dBi} = 32.8 \rightarrow G = 10^{\frac{32.8}{10}} \approx 1.91\times10^3$

$D = \frac{\lambda}{\pi}\sqrt{G} = \frac{0.11245}{\pi}\sqrt{1.91\times10^3} \approx 1.7\,m$

for radius $\approx \frac{1.7}{2}$ $\boxed{\approx 0.85\,m}$