CSci 4270 and 6270 Computational Vision, Spring Semester, 2021 Lecture 03 Exercise

Due: Wednesday, February 3, 2021 at 5 pm EST

Problems

1. For unit vectors \hat{x} and \hat{y} in N dimensional space, the square Euclidean distance between the vectors is $\|\hat{x} - \hat{y}\|^2$ while the cosine distance is $\cos \theta = \hat{x} \cdot \hat{y}$. Give a derivation showing that

$$\|\hat{x} - \hat{y}\|^2 = 2(1 - \cos \theta).$$

Note: all submissions must be pdf files. You may write the solution by hand but you must scan it or take a low-res (but still clear) picture. Do NOT submit a high resolution photo! For homework assignments, written solutions must be typeset, preferably with LaTeX (get an Overleaf account).

2. The supplied code reads in a list of points in two dimensions and returns it as an array. Complete the code by finding and printing the first principle component vector. Use either np.linalg.eigh or np.linalg.svd to do so. Print the vector as two values on two separate lines, accurate to three decimal places. Note that there is a sign ambiguity to the vectors, such that x_1, y_1 and $-x_1, -y_1$ are equally correct. Resolve this ambiguity by ensuring that x_1 is positive (and don't worry about the case of $x_1 \approx 0$).

The outputs from my code on the two example files are

% python prob2_sol.py pts_v1.txt

0.454

0.891

and

% python prob2_sol.py pts_v2.txt

0.962

-0.272