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#1. Let $\hat{x} = \langle x_1, x_2 \rangle$
 $\hat{y} = \langle y_1, y_2 \rangle$.

$$\hat{x} - \hat{y} = \langle x_1 - y_1, x_2 - y_2 \rangle$$

$$\therefore \|\hat{x} - \hat{y}\| = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}$$

$$\|\hat{x} - \hat{y}\|^2 = (x_1 - y_1)^2 + (x_2 - y_2)^2.$$

prove $(x_1 - y_1)^2 + (x_2 - y_2)^2 = 2(1 - \cos \theta)$.

$$x_1^2 - 2x_1y_1 + y_1^2 + x_2^2 - 2x_2y_2 + y_2^2 = 2(1 - \cos \theta)$$

Use identity $\cos \theta = \hat{x} \cdot \hat{y}$

$$= \langle x_1, x_2 \rangle \cdot \langle y_1, y_2 \rangle$$

$$= x_1y_1 + x_2y_2.$$

$$x_1^2 - 2x_1y_1 + y_1^2 + x_2^2 - 2x_2y_2 + y_2^2$$

$$= x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2(x_1y_1 + x_2y_2)$$

$$= x_1^2 + y_1^2 + x_2^2 + y_2^2 - 2(\cos \theta)$$

$$= (x_1^2 + x_2^2) + (y_1^2 + y_2^2) - 2\cos \theta$$

$$= \|\hat{x}\|^2 + \|\hat{y}\|^2 - 2\cos \theta$$

$$= 1^2 + 1^2 - 2\cos \theta$$

$$= 2(1 - \cos \theta).$$