Algo. Trading and Quant. Trading (2024 Fall)

Homework #1

Due date: 11/28 18:59

1. (Optimal execution under stock price with drift)

Let the stock midprice dynamics satisfy

$$dS_t = \mu dt + \sigma dW_t$$

where $\sigma > 0$, μ is a constant and W_t is a standard Brownian motion. The agent wishes to liquidate N shares and his trades create a temporary adverse move in prices so that the price at which he transacts is

$$\hat{S}_t^v = S_t - kv_t$$

with $k > 0$ and the inventory satisfies

$$dQ_t^v = -v_t dt$$

where v_t is the liquidation rate. Any outstanding inventory at time T is liquidated at the midprice and picks up a penalty of αQ_T^2 where $\alpha \geq 0$ is a constant.

(a) Denote the agent's value function as $H(t, S, Q)$. Write down $H(t, S, Q)$ using parameters above.

(b) Show that the optimal liquidation rate in feedback form is

$$v^* = \frac{\partial_Q H - S}{-2k}$$

(c) Use the trial solution $H(t,S,Q) = QS + h(t,Q)$ to show that the optimal liquidation rate is given by

$$v_t^* = \frac{Q_t^{v^*}}{(T-t)+\frac{k}{\alpha}} - \frac{1}{4k}\mu(T-t)\frac{(T-t)+\frac{2k}{\alpha}}{(T-t)+\frac{k}{\alpha}}$$

Discuss the relation between μ and the liquidation rate v_t^*.

(d) Let $\alpha \to \infty$ and show that the inventory along the optimal strategy is given by

$$Q_t^{v^*} = (T-t)\left(\frac{N}{T} + \frac{\mu}{4k}t\right)$$