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HW 2

1. At one large midwest university, about 40% of the college seniors have a social science major. Five seniors will be selected at random. Let X denote the number that **don't** have a social science major.

(a) List the probability distribution

Assuming sampling with replacement,

$p = 0.6 \qquad P(X=k) = {}_nC_k \cdot 0.6^{k} \cdot 0.4^{n-k}$

	$P(X=0)$	$P(X=1)$	$P(X=2)$	$P(X=3)$	$P(X=4)$	$P(X=5)$
probs	0.0102	0.0767	0.2304	0.3455	0.2592	0.0777
	× 0	× 1	× 2	× 3	× 4	× 5

⊕ sum

(b) Calculate mean and variance from the entries in the list from part (a).

$\rightarrow$ mean $= \sum x \cdot P(X=x) = \underline{3}$

Variance $= \sum P(X=x)(x-\bar{x})^2 \quad = \underline{1.2}$

(c) Calculate E(X)=np and Var(X)=np(1-p) and compare your answer with part (b).

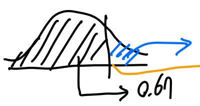
$E(X) = n \cdot p = \underline{3}$, $Var(X) = np(1-p) = \underline{1.2}$ the same

$X \sim N(100, 5^2)$, $Z = \frac{X-100}{5} \sim N(0,1)$

2. If X has a normal distribution with $\mu = 100$ and $\sigma = 5$, find b such that

(a) P(X<b)=.67

$P(X<b) = P(Z < \frac{b-100}{5}) = 0.67$



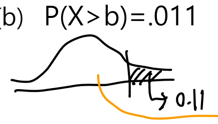
0.33

cdf(?) = 0.33

$\hookrightarrow Z = 0.44$

$0.44 = \frac{b-100}{5}$, $\underline{b = 102.2}$

(b) P(X>b)=.011

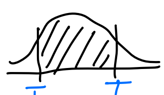


$P(X>b) = P(Z > \frac{b-100}{5}) = 0.011$ $\quad \underline{b = 111.4518}$

cdf(?) = 1 − 0.011 = 0.989

(c) $P(|X-100| < b) = .966$

$P(-b < X-100 < b) = P(-b+100 < X < 100+b) = 0.966$



89.399 $\quad$ 110.600

$\underline{b = 10.6}$

(d) P(X<110)=b

$\underline{b = 0.977}$

(e) P(X>95)=b

$\underline{b = 0.8413}$

3. Suppose the amount of sun block lotion in plastic bottles leaving a filling machine has a normal distribution. The bottles are labeled 300 milliliter(ml) but the actual mean is 302 ml and the standard deviation is 2 ml.

$X \sim N(302, 4)$

(a) What is the probability that an individual bottle will contain less than 299 ml?

$P(X<299) = P(Z < \frac{299-302}{4}) = 0.0668$

(b) If you pick up 5 bottles and check the amount of sun block lotion in each bottle, what is the probability that all of 5 bottles contain less than 299 ml?
(Assume that they are independent.)

$p = 0.0668$

$\binom{5}{5} p^5 \cdot (1-p)^0 = 1.330411 \cdot 10^{-6}$

4. The number of complaints per day, X, received by a cable TV distributor has the probability distribution

x	0	1	2	3
$P(X=x)$ f(x)	.4	.3	.1	.2

(a) Find the expected value and the standard deviation of the number of complaints per day.

$E(X) = 0 \cdot 0.4 + 1 \cdot 0.3 + 2 \cdot 0.1 + 3 \cdot 0.2 = 1.1$

$std(X) = \sqrt{0.4 \cdot (0-1.1)^2 + 0.3 \cdot (1-1.1)^2 + 0.1 \cdot (2-1.1)^2 + 0.2 \cdot (3-1.1)^2} = 1.1357$

(b) What is the approximate probability that the distributor will receive more than 2 complaints in average during 100 days?

$n=100$

$\bar{X} = \frac{X_1+X_2+\cdots+X_{100}}{100}$, $E(\bar{X}) = 1.1$, $Var(\bar{X}) = \frac{\sigma^2}{100}$, $std(\bar{X}) = 0.11357$

$P(\bar{X} > 2) = 1.110223 \times 10^{-15}$

(c) If we observe for five days, what is the probability that the distributor will receive exactly 1 complaint only for two days?

↳ 0+1
or
1+0

$p = \binom{5}{2} (0.3)^2 \cdot (0.7)^3$

$(0.3) = P(X=1)$, $(0.7) = P(X \neq 1)$

$= 0.30869$

(d) If we observe for 100 days, what is the approximate distribution of the number of days that the distributor will not receive any complaint?

X : no complaint → $P(X=0) = 0.4$

$n=100$,

$X \sim Binom(40, 24)$

(e) Using the approximate distribution in (d), what is the approximate probability that the number of days that the distributor will not receive any complaint is at most 30 days?

since $n=100$, we can approximate $X \sim N(40, 24)$

$P(X < 30) = 0.0206$

5. The probability that a voter will believe a rumor about a politician is .3. $p = 0.3$

(a) Find the probability that the first 3 voters don't believe the rumor but the 4th voter believe it.

$$p = (1-0.3)^3 \cdot 0.3 = 0.1029$$

(b) Find the probability that the exactly one person believe the rumor if 5 voters are told individually.

$$p = \binom{5}{1} 0.3^1 \cdot (1-0.3)^4 = 0.3601$$

6. Here is the assignment of probabilities that describes the age (in years) and the gender of a randomly selected American College student.

	At least 14 and less than 18	At least 18 and less than 25	At least 25 and less than 35 (B)	At least 35
Male	0.01	0.28	0.13	0.04
Female (A)	0.02	0.3	0.14	0.08

A college student will be selected at random. Let

A=[student is Female] and B=[student is at least 25 but less than 35 years old].

Find,

(a) P(A) and P(B)

$P(A) = 0.54$ $\qquad$ $P(B) = 0.27$

(b) P(A or B)

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= 0.54 + 0.27 - 0.14 = 0.67$$

7. Show the following statement:

If $X \sim t[k]$, then $X^2 \sim F[1, k]$.

↳ student T's dist w/ k d.f $\qquad$ ↳ F statistics

if $X \sim t[k]$,

$X = \frac{Z}{\sqrt{\frac{\chi^2}{k}}}$ w/ 1 and k d.f

then,

$$X^2 = \frac{\boxed{\frac{Z^2}{1}}}{\boxed{\frac{\chi^2}{k}}} \quad \frac{Z^2}{1} \sim \chi^2(1), \quad \frac{\chi^2}{k} \sim \chi^2(k) \quad = \frac{\frac{U}{k_1}}{\frac{V}{k_2}}$$

Z^2 and χ^2 are independent.

so, $X^2 \sim F(1, k)$

previous attempts

(b) What is the approximate probability that the distributor will receive more than 2 complaints in average during 100 days?

CLT?

$n=100$

$\hookrightarrow$ >200 complaints

$E(100X) = 110$

$Std(100X) = 100 \cdot Std(X) = 113.59$

$P(100X > 200) = 0.21406$

(c) If we observe for five days, what is the probability that the distributor will receive exactly