BAF 640
Assignment 2

Today is August 3, 2020. You have 10,000,000 won in your brokerage account. You are considering a portfolio that invests in three individual KRX stocks and the KOSPI index with equal weights. Before making investments, you first want to understand how much risk you are taking.

From the previous analysis, you have a good understanding of the current stock market volatility.

Your next step is to assess the total risk of the specific portfolio you have in mind. In particular, you want to know the portfolio's one-day VaR and ES for tomorrow for a confidence level of 99%.

In answering the questions below, use the spreadsheet provided for the assignment in the course Dropbox folder.

Basic Historical Simulation Approach with 500 Days of Data

Estimate the VaR and ES of your portfolio for August 4, 2020 using 500 days of data.

1) Using the data you have collected in Assignment 1, generate 500 scenarios for one day loss of your portfolio. Note that you should use the daily observations over the most recent 500 days in your simulation.

   (Hint: The sample period is from July 23, 2018 to Aug 3, 2020. The fist observation on July 23, 2018 is scenario 0 and the last observation on Aug 3, 2020 is scenario 500.)

2) What is the one day 99% VaR and ES that are calculated using the basic historical simulation approach over the most recent 500 days?

Basic Historical Simulation Approach with 1000 Days of Data

Estimate the VaR and ES of your portfolio for August 4, 2020 using 1000 days of data.

3) Using the data you have collected in Assignment 1, generate 1000 scenarios for one day loss of your portfolio. Note that you should use the daily observations over the most recent 1000 days in your simulation.

   (Hint: The sample period is from July 7, 2016 to Aug 3, 2020. The fist observation on July 7, 2016 is scenario 0 and the last observation on Aug 3, 2020 is scenario 1000.)

4) What is the one day 99% VaR and ES that are calculated using the basic historical simulation approach over the most recent 1000 days?

Comparison between the Two Measures

5) You have two sets of VaR and ES measures, one based on 500 observations and the other based on 1000 observations. Discuss how and why the measures differ from each other.

Back-Testing

You are curious about how well the VaR measures generated from the basic simulation approach would have worked in the past. You back-test the first VaR measure from 500 observations using the full data you collected in Assignment 1.

6) Simulate scenarios for one day loss of your portfolio using the full sample from January 3, 2005 to August 3, 2020. You will end up with 3855 scenarios in total.

7) Calculate the portfolio's one-day 99% VaR from 500 observations for each day between January 9, 2007 and August 3, 2020. You will have 3355 number of VaR estimates in total.

   (Hint: For example, the one-day 99% VaR for January 19, 2009 can be calculated as the 5th largest simulated loss over the previous 500 days from January 9, 2007 to January 16, 2009. The VaR for the next day then can be computed by rolling the training window forward by one day. That is, the one-day 99% VaR for January 20, 2009 can be calculated as the 5th largest simulated loss over the 500 days from January 10, 2007 to January 19, 2009. Repeat this process until you reach August 3, 2020.)
   (Hint: Use Excel's "LARGE" function to get the 5th largest simulated loss.)

8) Compare the daily VaR with the actual portfolio loss on the corresponding day. On how many days does the actual loss exceed the VaR between January 9, 2007 and August 3, 2020? These observations are referred to as exceptions.

9) Plot the VaR and realized portfolio loss from January 9, 2007 to August 3, 2020. When do the exceptions tend to happen?

10) Let m the number of exceptions you get. Define $p = 1 - confidence\ level$. Recall that if the VaR model is accurate, that is, if the probability of an exception on any given day is p, then the probability of the VaR level being exceeded on m or more days out of a total of n days is

$$probability\ of\ m\ or\ more\ exceptions = \sum_{k=m}^{n} \frac{n!}{k!\,(n-k)!} p^k (1-p)^{n-k}$$

In Excel, you can calculate this probability using the BINOM.DIST function. In particular, for our VaR measure,

$$probability\ of\ m\ or\ more\ exceptions = 1 - BINOM.DIST(m-1, 3355, 0.01, TRUE).$$

Should we reject or accept the VaR model at a 5% significance level?

The following questions are independent of the previous questions.

Value at Risk and Expected Shortfall

11) Suppose that the change in the value of a portfolio over a one-day time period is normal with a mean of zero and a standard deviation of $2 million.

(Hint: Use the formula on slide 23 of lecture 6.)
(Hint: Use NORM.S.INV(probability) function to calculate the inverse of the standard normal cumulative distribution for a given probability level.)
(Hint: You can enter PI() in a cell to get the value of pi=3.14159265 in Excel.)

a) What is the one-day 99% VaR?
b) What is the five-day 99% VaR? Assume that the changes in the value of the portfolio on successive days are independent.
c) What difference does it make to you answer to (b) if there is first-order daily autocorrelation with a correlation parameter equal to 0.16?
d) What is the one-day 97.5% ES? Note that the one-day 99% VaR and one-day 97.5% ES are almost the same.

Extreme Value Theory

12) Recall the application of Extreme Value Theory in lecture 7, slide 36 through 40.

(Hint: Recall that $n = 500, n_u = 22,\ u = 160, \beta = 32.532, and\ \xi = 0.436$*.)*

a) What is the probability that the loss will exceed $400,000?
b) What is the one-day VaR with a confidence level of 97%?