

HW 3

1. If X_1, X_2, \dots, X_n are i.i.d. random variables with mean μ and variance σ^2 , calculate the covariance \bar{X} and $X_i - \bar{X}$ for any $i = 1, \dots, n$.

2. A commercial for a manufacturer of household appliance claims that 3% of all its product require a service call in the first year. A consumer protection association wants to check the claim by surveying 400 households that recently purchased one of the company's appliances. What is the probability that more than 5% require a service call within the first year?

(a) Calculate the probability using the approximate normal approach.

(b) Calculate the probability using the binomial distribution.

3. Suppose that X has normal distribution with mean $\mu = 10$ and standard deviation $\sigma = 2$. Doing the following parts:

(a) Calculate $P(6 < X < 14)$.

(b) Find the percentile c such that $P(X \leq c) = 0.95$.

(c) A random sample of size 4, $\{X_1, \dots, X_4\}$, is taken from the normal distribution with mean = 10 and standard deviation = 2. Find the probability that the average of this sample is at most 12.

4. Bits are sent over a communications channel in packets of 160. If the probability of a bit being corrupted (one error) over this channel is 0.2 and such errors are independent. Let X denotes the number of bits that are corrupted over this channels.

(a) What is the distribution of X ? Can it be approximated as normal distribution?

(b) Approximately, what is the probability that more than 50 bits in a packet are corrupted?

5. A large population is described by the probability distribution

| x | f(x) |
|---|------|
| 0 | 0.1 |
| 1 | 0.2 |
| 2 | 0.7 |

Let X_1 and X_2 be a random sample of size 2 from the distribution

(a) Determine the sampling distribution of $\max(X_1, X_2)$?

(b) Determine the sampling distribution of $X_1 + X_2$?