Models and the Real World

Reference:
Sheldon Natenberg, "Option Volatility & Pricing" 2ed. 2015, Ch. 23

Risks in Using Models

- A trader who uses a theoretical pricing model is exposed to two types of risk
- The risk that the trader has the wrong inputs into the model
- The risk that the model itself is wrong because it is based on false or unrealistic assumptions

Important Assumptions

- Markets are frictionless
- Interest rates are constant over the life of an option
- Volatility is constant over the life of an option
- Trading is continuous, with no gaps in the price of an underlying contract
- Volatility is independent of the price of the underlying contract
- Over small periods of time, the percent price changes in an underlying contract are normally distributed, resulting in a lognormal distribution of underlying prices at expiration

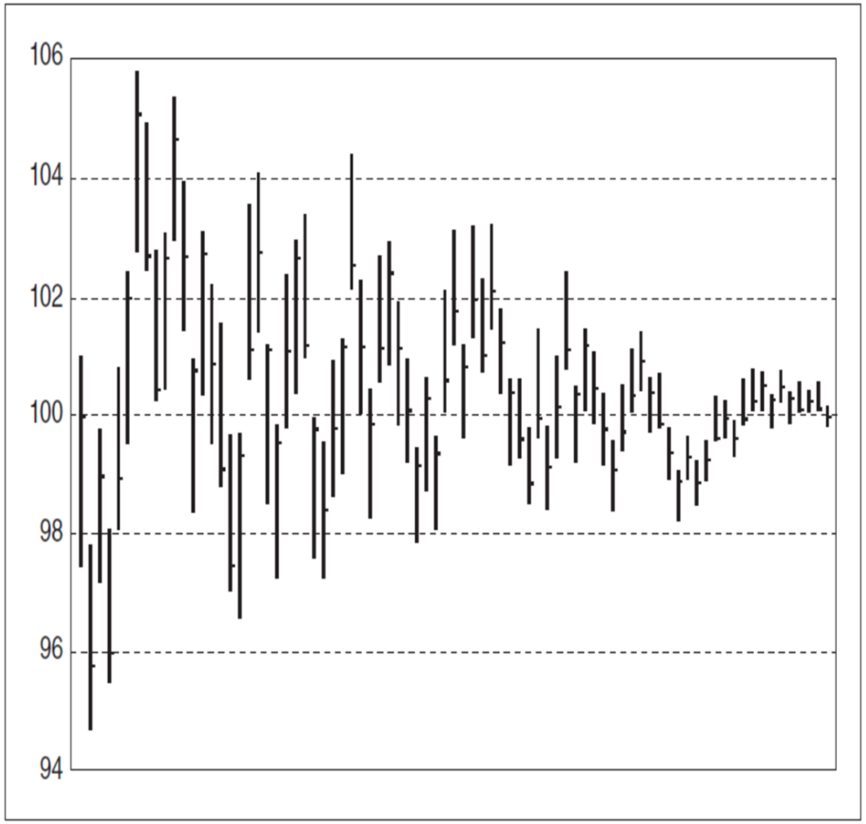
Markets Are Frictionless

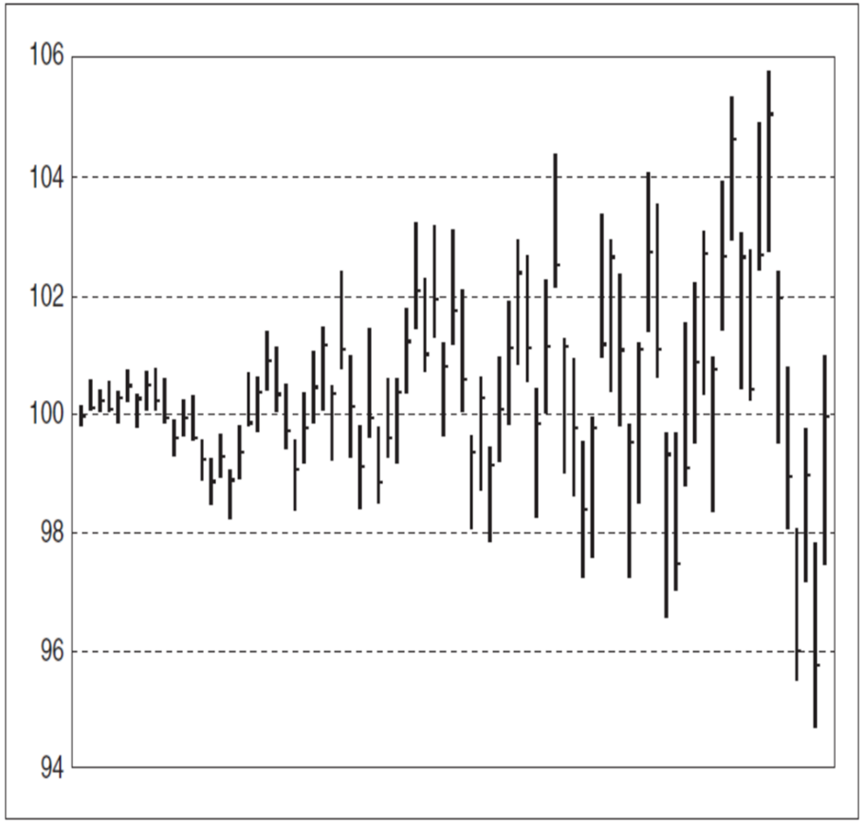
- The underlying contract can be freely bought or sold, without restriction
  - daily price limit
  - circuit breakers
- Unlimited money can be borrowed or lent, and the same interest rate applies to all transactions.
- There are no transaction costs.
  - Significant impact on model-generated values.
- There are no tax consequences.

Interest Rates Are Constant over the Life of an Option

- Although changing interest rates will cause the value of a trader's option position to change, interest rates tend to be a lesser risk for most traders, at least for short-term option strategies
- The impact of changing interest rates is a function of time to expiration
- For stock options especially, raising interest rates raises the forward price, which raises the value of calls and lowers the value of puts
- Deep ITM is most sensitive

Volatility Is Constant over the Life of the Option





Even though the volatility unfolded in two completely different scenarios, in both cases, a pricing model will use **the same volatility, 28 percent**, to make all calculations.

Volatility Is Constant over the Life of the Option

The value of the option is in fact **path dependent**

Figure 23-5 Option values under three different volatility scenarios.

Underlying price = 100
Time to expiration = 80 days
Interest rate = 0
Volatility = 28%

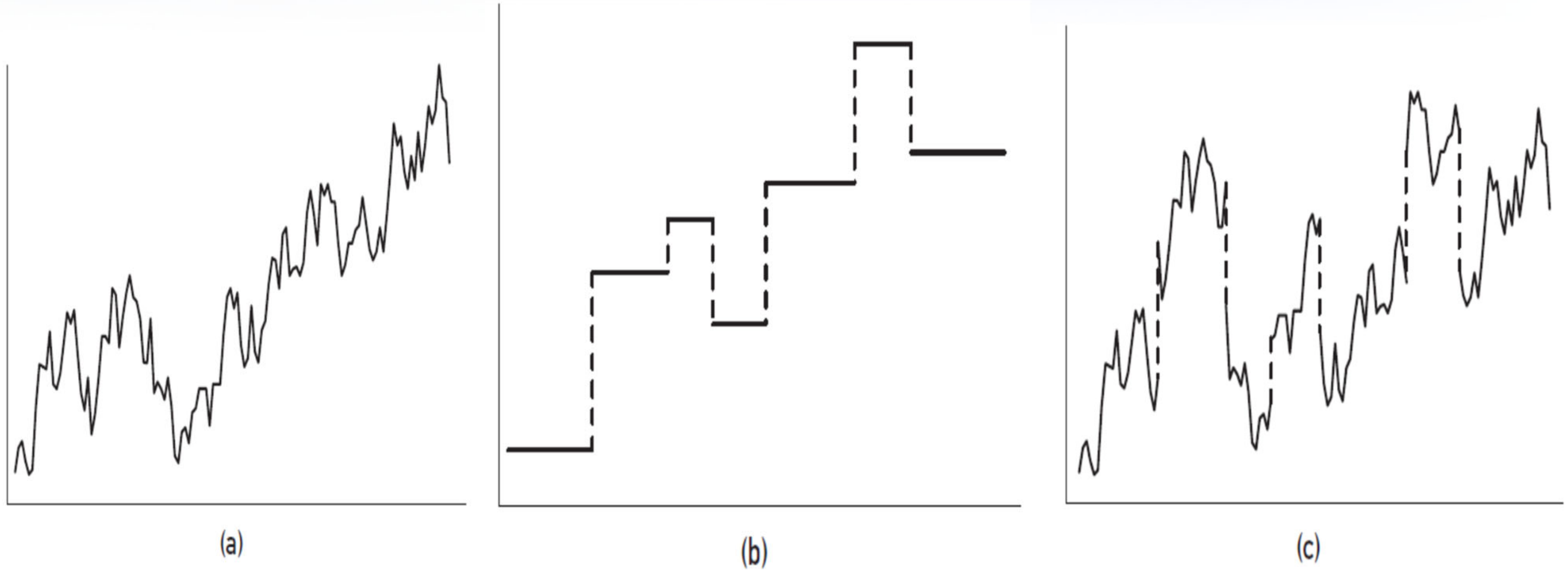
	Exercise price:	70	75	80	85	90	95	100	105	110	115	120	125	130
Constant volatility (Black-Scholes)	Calls:	30.01	25.06	20.21	15.62	11.48	7.98	5.23	3.22	1.87	1.03	0.54	0.27	0.13
	Puts:	0.01	0.06	0.21	0.62	1.48	2.98	5.23	8.22	11.87	16.03	20.54	25.27	30.13
	Straddle:	30.01	25.12	20.42	16.24	12.96	10.96	10.46	11.44	13.74	17.06	21.08	25.54	30.26
Falling volatility (Figure 23-3)	Calls:	30.04	25.15	20.44	16.00	11.79	7.59	2.97	2.69	2.10	1.43	0.89	0.51	0.27
	Puts:	0.04	0.15	0.44	1.00	1.79	2.59	2.97	7.69	12.10	16.43	20.89	25.51	30.27
	Straddle:	30.08	25.30	20.88	17.00	13.58	10.18	5.94	10.38	14.20	17.86	21.78	26.02	30.54
Rising volatility (Figure 23-4)	Calls:	30.00	25.01	20.05	15.21	10.75	8.97	6.41	3.36	1.29	0.41	0.14	0.05	0.02
	Puts:	0	0.01	0.05	0.21	0.75	3.97	6.41	8.36	11.29	15.41	20.14	25.05	30.02
	Straddle:	30.00	25.02	20.10	15.42	11.50	12.94	12.82	11.72	12.58	15.82	20.28	25.10	30.04

Using an interest rate of zero, the time premium for a call and put with the same exercise price must be identical. The value of the call and put will differ only by intrinsic value.

Volatility Is Constant over the Life of the Option

- Because the value of an option seems to be path dependent, one might conclude that the Black-Scholes model is unreliable
- But the Black-Scholes model is a probabilistic model. A given volatility will, on average, result in a given value for the option
- Stochastic volatility models
- Special models to evaluate interest-rate instruments.

Trading Is Continuous



(a) Diffusion process. (b) Jump process. (c) Jump-diffusion process

Trading Is Continuous

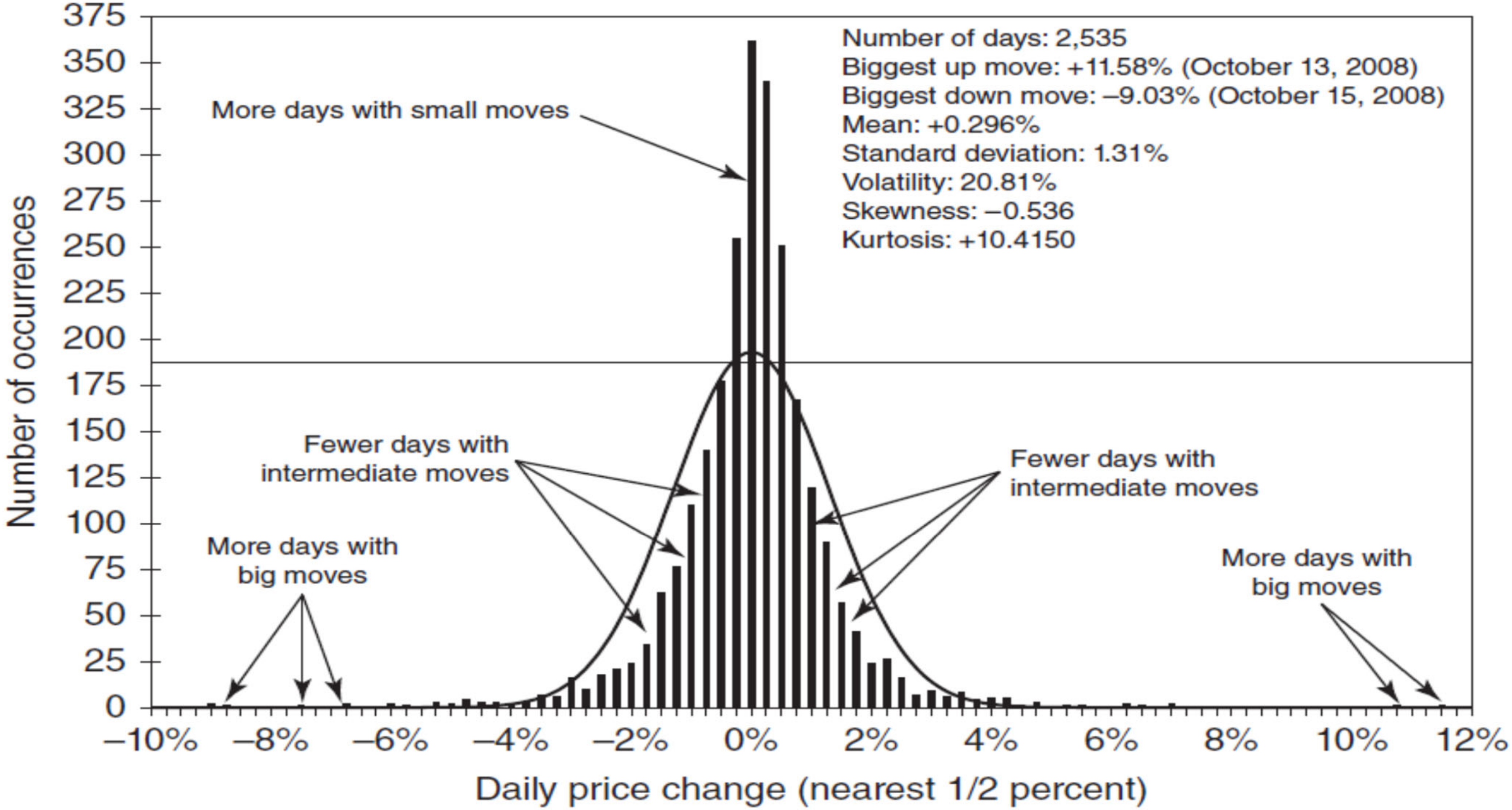
Figure 23-7 Effect of a gap on the value of a 100 straddle.

Underlying price = 100						
Time to Expiration	1 Day	1 Week	1 Month	3 Months	6 Months	1 Year
Implied volatility = 15%						
Initial straddle value	0.63	1.66	3.45	5.98	8.46	11.96
Straddle gamma	101	38	18	11	8	5
Straddle value after a gap from 100 to 105	5.00	5.01	5.58	7.39	9.57	12.90
Increase in value	4.37	3.35	2.13	1.41	1.11	0.94
Implied volatility = 25%						
Initial straddle value	1.04	2.76	5.76	9.97	14.09	19.90
Straddle gamma	61	23	11	6	4	3
Straddle value after a gap from 100 to 105	5.00	5.25	7.20	10.98	14.98	20.78
Increase in value	3.96	2.49	1.44	1.01	0.89	0.88

Volatility Is Independent of the Price of the Underlying Contract

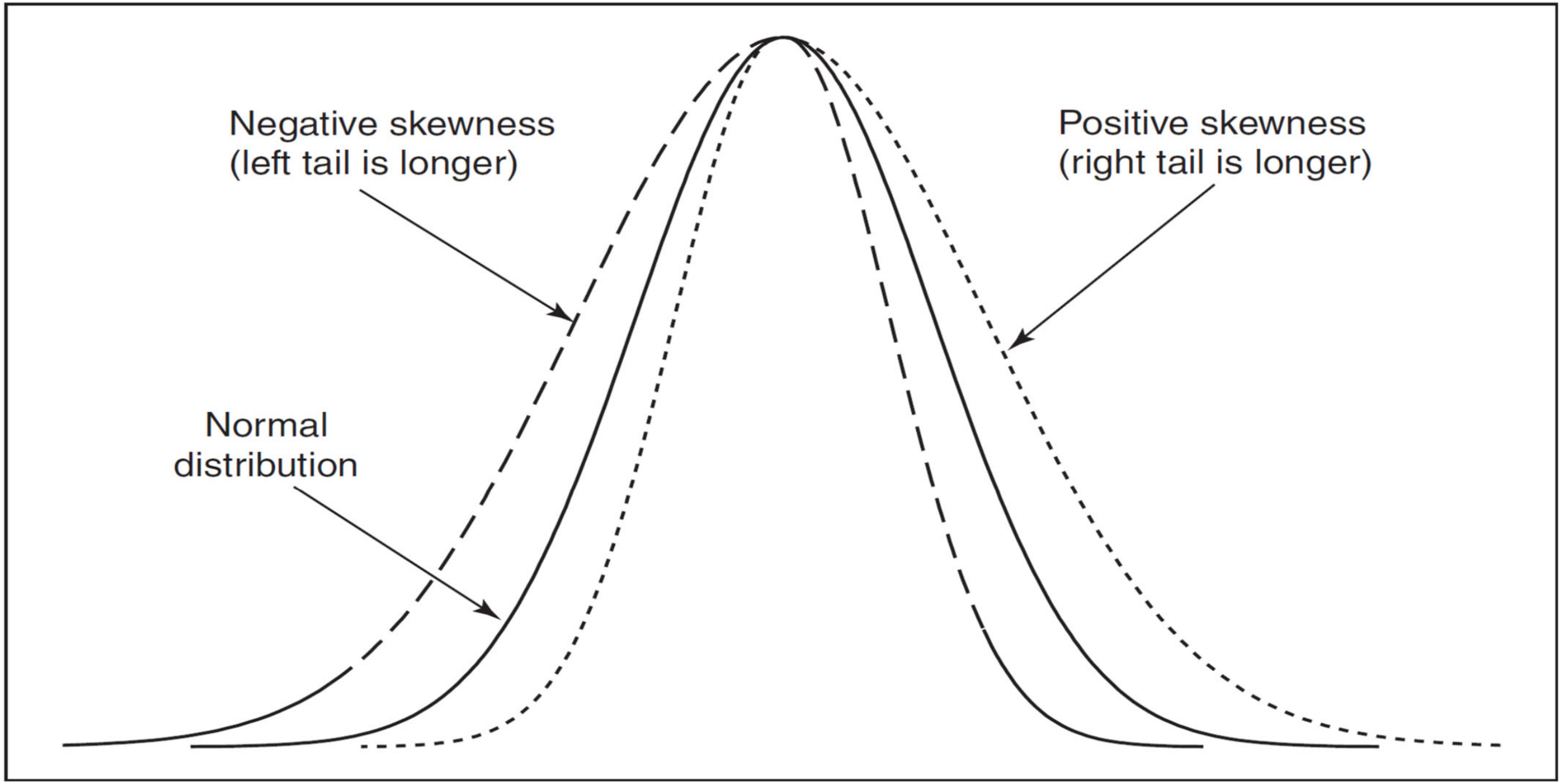
- In many markets, however, this assumption appears to be inconsistent with most traders' experience
- The volatility of a market is not independent of the price of the underlying contract

Underlying Prices at Expiration Are Lognormally Distributed



Skewness and Kurtosis

Figure 23-9 ***Skewness*—the degree to which one tail of a distribution is longer than the other tail.**



Skewness and Kurtosis

Figure 23-10 *Kurtosis*—the degree to which a distribution has a taller peak and wider tails.