FBA | QUANTITATIVE FINANCE RESEARCH GROUP

Financial Engineering

Homework 8

Due at 11:59 pm (Korea Standard Time) on Friday, June 30.

Submit one file: written solutions with executable Python code

Problem 1. Suppose you are a portfolio manager and you are going to use derivatives to construct certain portfolio. The current stock price for AAPL is \$171.01 per share, at 10 am, March 8, 2019. You want to sell 1 unit of European Call on AAPL, with strike of \$180 and maturity of 1 year. Suppose the annualized interest rate is 3%, the annualized drift for AAPL is 5%, no dividend, and the annualized volatility is 10%. In order to hedge the potential risk of your option, you are going to calculate several Greeks based on Black-Merton-Scholes model.

(a) Calculate the Call option price, at 10 am, March 8, 2019.

(b) Calculate the Delta, Gamma, Vega, Theta at 10 am, March 8, 2019. Also, give your thoughts about why Delta is positive or negative.

(c) Suppose you are doing a monthly Delta-Hedge, that is, you re-hedge per month. After one month, at 10 am, April 8, 2019, the stock price is \$180.2, and you want to re-hedge right now. Calculate the additional amount of shares of AAPL you should buy (positive for buying, negative for selling) at 10 am, April 8, 2019, in order to re-hedge. Also, give your thoughts about why you are selling or buying stocks when the stock price goes up from the meaning of Delta Hedging.

(d) Calculate the change of your portfolio value (i.e. the net cashflow). Note that your answer should be in \$ with new portfolio value minus previous value, in present value at 10 am, April 8, 2019.

Problem 2. Assume the same situation as Problem 1. Back to at 10 am, March 8, 2019. Suppose you are constructing another portfolio by buying a European Call on AAPL with strike 180 and selling the European Call on AAPL with strike 185.

(a) Without volatility skew, that is, volatility is always 10% for different strikes. Calculate your portfolio value V, at 10 am, March 8, 2019.

(b) Suppose the market actually has a volatility skew: $\sigma(K) = min\{1, 18K^{-1}\}$. Calculate your portfolio V' again, at 10 am, March 8, 2019. Also, give your thought about why the portfolio value goes higher or lower after we assume the volatility skew in the market.

Problem 3. Assume the same situation as Problem 1. Now you are focusing on portfolios with digital options on AAPL, at 10 am, March 8, 2019. All the information about the stock stays the same, and we are back to a world of constant volatility 10% again.

(a) Note that the payoff of this option at maturity is: $1_{S_T \geq K}$
Calculate the price of digital call option on AAPL with strike 180 and maturity of 1 year, at 10 am, March 8, 2019. Also, give your thoughts about the proper range for the digital option's price, from the Put-Call Parity. And compare this number with the Delta of European Call in Problem 1.

(b) Suppose you are going to sell this digital option at the price you calculated at (a). Meantime, you want to do Delta-Hedging to lowered potential risk. Calculate the amount of shares of AAPL you should buy or sell (positive for buying, negative for selling), based on Black-Scholes model, at 10 am, March 8, 2019. Again, compare this number with the Gamma of European Call in Problem 1.

(c) Suppose you are constructing another portfolio by buying a digital European Call on AAPL with strike 180 and selling a digital European Call on AAPL with strike 185. Calculate your portfolio value W, at 10 am, March 8, 2019.

(d) Now we assume the market has a volatility skew: $\sigma(K) = min\{1, 18K^{-1}\}$.
Note that here we should use the pricing formula based on chain rule:

$$C_{Digital}(K) = -\frac{dC\big(K, \sigma(K)\big)}{dK} = -\frac{\partial C(K, \sigma(K))}{\partial K} - \frac{\partial C(K, \sigma(K))}{\partial \sigma} \cdot \frac{\partial \sigma(K)}{\partial K}$$

Calculate your portfolio W' again, at 10 am, March 8, 2019. And compare the difference of portfolio value W' - W with the previous one V' - V in Problem 2. Also, give your thoughts about the impact of volatility skew on different portfolios.

Problem 4. Solve the corresponding leetcode problem below and register the solution on GitHub.

https://leetcode.com/problems/increasing-triplet-subsequence/

https://github.com/fbaquant/leetcode-challenge/issues

Problem 5. Solve the corresponding leetcode problem below and register the solution on GitHub.

https://leetcode.com/problems/factorial-trailing-zeroes/

https://github.com/fbaquant/leetcode-challenge/issues