FBA | QUANTITATIVE FINANCE RESEARCH GROUP

Financial Engineering

Homework 3

Due at 07:00 pm (Korea Standard Time) on Saturday, March 4.

Submit one file: written solutions with executable Python code

Problem 1. Build a 15-period binomial model whose parameters should be calibrated to a Black-Scholes geometric Brownian motion (GBM) model with: $T = .25$ years, $S_0 = 100$, $r = 2\%$, $\sigma = 30\%$ and a dividend yield of $c = 1\%$. Hint: Your binomial model should use a value of $u = 1.0395$. Now answer the following questions:

(a) Compute the price of an American call option with strike $K = 110$ and maturity $T = .25$years.

(b) Compute the price of an American put option with strike $K = 110$ and maturity $T = .25$years.

(c) Is it ever optimal to early exercise the put option of part (b)?

(d) If your answer to part (c) is "Yes", when is the earliest period at which it might be optimal to early exercise?

(e) Do the call and put option prices of parts (a) and (b) satisfy put-call parity? Why or why not?

Problem 2. Consider the following two period binomial tree. Suppose you purchase an European call option with maturity $T = 2$, Strike $= 100$. Suppose $R_f = 1.0001$, that is, \$1 invested at $T = 0$ is worth 1.0001^t at time t. At each time step, the stock can increase by a factor of 1.05 or decrease by a factor of 1/1.05. Assume there is no arbitrage.

(a) Compute the price of the option.

(b) Repeat the problem above for an American option

Problem 3. Suppose that $u = e^w$, where w is normal with expected value $\bar{w}$ and variance σ^2. Then

$$\bar{u} = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^w e^{-(w-\bar{w})^2/2\sigma^2} dw$$

Show that

$$w - \frac{(w-\bar{w})^2}{2\sigma^2} = -\frac{1}{2\sigma^2}[w - (\bar{w} + \sigma^2)]^2 + \bar{w} + \frac{\sigma^2}{2}$$

Use the fact that

$$\frac{1}{\sqrt{2\pi\sigma^2}} \int_{-\infty}^{\infty} e^{-(x-\bar{x})^2/2\sigma^2} dx = 1$$

to evaluate $\bar{u}$.

Problem 4. You are considering an investment in a tree farm. Tree grow each year by the following factors:

Year	1	2	3	4	5	6	7	8	9	10
Growth	1.6	1.5	1.4	1.3	1.2	1.15	1.1	1.05	1.02	1.01

The price of lumber follows a binomial lattice with u = 1.20 and d = .9. The interest rate is constant at 10%. It costs $2 million each year, payable at the beginning of the year, to lease the forest land. The initial value of the trees is $5 million (assuming they were harvested immediately). You can cut the trees at the end of any year and then not pay rent after that. (For those readers who care, we assume that cut lumber can be stored at no cost.)

(a) Argue that if the rent were zero, you would never cut the trees as long as they were growing.

(b) With rent of $2 million per year, find the best cutting policy and the value of the investment opportunity.

Problem 5. A gambler starts with an initial fortune of i dollars. On each successive game, the gambler wins \$1 with probability p, $0 < p < 1$, or loses \$1 with probability $q = 1 - p$. He will stop if he either accumulates N dollars or loses all his money. What is the probability that he will end up with N dollars?

Problem 6. Monty Hall problem is a probability puzzle based on an old American show Let's Make a Deal. The problem is named after the show's host. Suppose you're on the show now, and you're given the choice of 3 doors. Behind one door is a car; behind the other two, goats. You don't know ahead of time what is behind each of the doors. You pick one of the doors and announce it. As soon as you pick the door, Monty opens one of the other two doors that he knows has a goat behind it. Then he gives you the option to either keep your original choice or switch to the third door. Should you switch? What is the probability of winning a car if you switch?