

Financial Engineering

Homework 6

Due at 07:00 pm (Korea Standard Time) on Saturday, May 6.

Submit one file: written solutions with executable Python code

Problem 1. Consider a 10-year zero-coupon bond with face value \$100. The interest rate is fixed at 5%. The credit spread for the bond is estimated to be 1% (except in part(a)). Calculate:

- (a) The bond value if there is no possibility of default
- (b) The probability of default
- (c) The value if there is no recovery
- (d) The value if there is 50% recovery at maturity
- (e) The value of \$100 that is paid at default

Problem 2. Consider the interest-only (IO) and principal-only (PO) securities in a deterministic world without prepayments and defaults. These securities have time k cash-flows of $P_k := B - cM_{k-1}$ and $I_k := cM_{k-1}$, respectively, for $k = 1, \dots, n$ and where M_k (and all other notation) is defined in Coursera.

- (a) Compute the present value, V_0 , of the PO security
- (b) What happens to V_0 as $n \rightarrow \infty$?
- (c) Compute the present value, W_0 , of the IO security
- (d) Which of the two securities do you think has the longer duration? Justify your answer.

Problem 3. We can define the default intensity, $\lambda(t)$ to be

$$\lambda(t) := \frac{f_{\tau}(t)}{P(\tau > t)}$$

where τ is the time of default and $f_{\tau}(\cdot)$ is the PDF of τ . It therefore follows that $\lambda(t)dt$ is equal to the probability of defaulting in the interval $(t, t + dt)$ given that default has not occurred in $[0, t]$. Show that

$$P(\tau > s) = e^{-\int_0^s \lambda(t)dt}$$

Problem 4. Let X_1 and X_2 be IID random variables with uniform distribution between 0 and 1, $Y = \min(X_1, X_2)$ and $Z = \max(X_1, X_2)$. What is the probability of $Y \geq y$ given that $Z \leq z$ for any $y, z \in [0, 1]$? What is the correlation of Y and Z ?

Problem 5. Solve the corresponding leetcode problem below and register the solution on GitHub.

<https://leetcode.com/problems/move-zeroes/>

<https://github.com/fbaquant/leetcode-challenge/issues>

Problem 6. Solve the corresponding leetcode problem below and register the solution on GitHub.

<https://leetcode.com/problems/longest-common-prefix/>

<https://github.com/fbaquant/leetcode-challenge/issues>