Week-2

王弘禹

June 29, 2022

2.3

11.

(a)

Consider two vectors v_1, v_2 in V

$$(cv_1 + v_2) \in V, \quad c \in R \tag{1}$$
$$\Longrightarrow \text{V is a vector space over real numbers.} \tag{2}$$

(b)

There are six components in the basis:

$$\beta_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \tag{3}$$
$$\beta_3 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \qquad \beta_4 = \begin{bmatrix} 0 & i \\ 0 & 0 \end{bmatrix} \tag{4}$$
$$\beta_5 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \qquad \beta_6 = \begin{bmatrix} 0 & 0 \\ i & 0 \end{bmatrix} \tag{5}$$

(c)

The proof that W is a subspace of V is same method like (a). There are four components in the basis:

$$\beta_1 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad \beta_2 = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} \tag{6}$$

$$\beta_3 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad \beta_4 = \begin{bmatrix} 0 & i \\ i & 0 \end{bmatrix} \tag{7}$$