Week-1

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1.5

8.

$$C_{ij} = (AB)_{ij} - (BA)_{ij} \tag{1}$$

$$= \sum (A_{ir}B_{rj} - B_{ir}A_{rj}) \tag{2}$$

$$= \sum_{r} (A_{ir}B_{rj} - B_{ir}A_{rj})$$

$$\sum_{i} C_{ii} = \sum_{i} \sum_{r} (A_{ir}B_{ri} - B_{ir}A_{ri})$$

$$(3)$$

$$= \sum_{(i,r), i \le r} (A_{ir} B_{ri} - B_{ri} A_{ir}) = 0$$
(4)

1.6

9.

Assume $A_{kk} = 0$ and $A_{ii} \neq 0$, $i \neq k$, then, we can reduce A to the form below.

$$A'_{ij} = \delta_{ij}, \ j \neq k \tag{5}$$

In this form, the linear combination of rows can't be the k-th row of identity matrix.

11.

Trivial problem. We can first do row operation to make PA be a row-reduced echelon, and then do column operations to make PAQ be column-reduced echelon matrix.