

# Micro2 CH12

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## Q1

(5pts, solution 3 + show 2)

		B		
		B1	B2	B3
A	A1	3, 7	2, 8	3, 9
	A2	4, 3	5, 5	5, 7
	A3	5, 10	9, 6	4, 8

(a) 請問在 Nash 均衡中, A 是否可能用 A1 的策略?

(b) 請寫出所有的 Nash 均衡。

(a) No, since A2 strictly dominates A1, A will not play A1 in Nash equilibrium.

(b) We can eliminate the dominated strategies repeatedly to exclude the impossible pure strategies.

		B		
		B1	B2	B3
A	A1	3, 7	2, 8	3, 9
	A2	4, 3	5, 5	5, 7
	A3	5, 10	9, 6	4, 8

We first eliminate A1 since it is dominated by A2. Second, eliminate B2 since it is dominated by B3.

Now assume the mixed strategy  $\sigma = (pA2 + (1 - p)A3, qB1 + (1 - q)B3)$  is a NE. We could compute the utilities for both players.

$$\begin{aligned}
 U_A(\sigma) &= 4pq + 5p(1 - q) + 5(1 - p)q + 4(1 - p)(1 - q) \\
 &= (-2q + 1)p + (q + 4)
 \end{aligned}$$

$$\begin{aligned}
U_B(\sigma) &= 3pq + 7p(1 - q) + 10(1 - p)q + 8(1 - p)(1 - q) \\
&= (-6p + 2)q + (8 - p)
\end{aligned}$$

The best response correspondings are

$$\mathcal{B}_1(q) = \begin{cases} p = 1, & \text{if } q < \frac{1}{2} \\ p \in [0, 1], & \text{if } q = \frac{1}{2} \\ p = 0, & \text{if } q > \frac{1}{2} \end{cases} \quad \mathcal{B}_2(p) = \begin{cases} q = 1, & \text{if } p < \frac{1}{3} \\ q \in [0, 1], & \text{if } p = \frac{1}{3} \\ q = 0, & \text{if } p > \frac{1}{3} \end{cases} .$$

The intersection of the best responses are  $(p = 0, q = 1)$ ,  $(p = \frac{1}{3}, q = \frac{1}{2})$ ,  $(p = 1, q = 0)$ , so the NEs are  $(A3, B1)$ ,  $(\frac{1}{3}A2 + \frac{2}{3}A3, \frac{1}{2}B1 + \frac{1}{2}B3)$ ,  $(A2, B3)$ .

## Q2

(5pts)

		Column player		
		a	b	c
Row player	d	6, 10	0, 0	3, 3
	e	0, 0	4, 10	3, 3

- (a) 請找出 column player 的 dominant strategy (包括混合策略在內)。  
(b) 利用 (a) 的結果來簡化賽局, 並寫出簡化賽局中的 Nash 均衡。

(a)<sup>1</sup> There is no pure dominant strategy<sup>2</sup> for Column player.

(b) Consider the mixed strategy  $pa + (1 - p)b$  for Column player. The expected pay-off is  $10p$  when Row player plays d; and is  $10 - 10p$  for e. Therefore, c is dominated by  $\frac{3}{10} < p < \frac{7}{10}$ . After eliminating c, we can assume the mixed strategy  $\sigma = (pd + (1 - p)e, qa + (1 - q)b)$

<sup>1</sup>Will not be graded since the hint is somehow misleading.

<sup>2</sup>A pure strategy  $s_i$  is strictly dominant if every other pure strategy  $s'_i$  is strictly dominated by  $s_i$ , that is,  $u_i(s'_i, s_{-i}) < u_i(s_i, s_{-i}) \quad \forall s_{-i} \in \mathcal{S}_{-i}$ . Note that a mixed strategy will never be a dominant strategy.

is a NE, and compute the utilities for both players.

$$\begin{aligned} U_1(\sigma) &= 6pq + 4(1-p)(1-q) \\ &= (10q - 4)p + (4 - 4q) \end{aligned}$$

$$\begin{aligned} U_2(\sigma) &= 10pq + 10(1-p)(1-q) \\ &= (20p - 10)q + (10 - 10p) \end{aligned}$$

The best response correspondings are

$$\mathcal{B}_1(q) = \begin{cases} p = 1, & \text{if } q > \frac{2}{5} \\ p \in [0, 1], & \text{if } q = \frac{2}{5} \\ p = 0, & \text{if } q < \frac{2}{5} \end{cases} \quad \mathcal{B}_2(p) = \begin{cases} q = 1, & \text{if } p > \frac{1}{2} \\ q \in [0, 1], & \text{if } p = \frac{1}{2} \\ q = 0, & \text{if } p < \frac{1}{2} \end{cases} .$$

The intersections of the best responses are  $(p = 0, q = 0)$ ,  $(p = \frac{1}{2}, q = \frac{2}{5})$ ,  $(p = 1, q = 1)$ , so the NEs for this game are  $(e, b)$ ,  $(\frac{1}{2}d + \frac{1}{2}e, \frac{2}{5}a + \frac{3}{5}b)$ ,  $(d, a)$ .

### Q3

(2pts, must say he bids at his evaluation)

3. 德國詩人歌德於1797/1/16寫信給一出版商 Vieweg, 商談其作品 *Hermann and Dorothea* 之交易, 英譯如下:

I am inclined to offer Mr. Vieweg from Berlin an epic poem, *Hermann and Dorothea*, which will have approximately 2000 hexameters... Concerning the royalty we will proceed as follows: I will hand over to Mr. Counsel Böttiger a sealed note which contains my demand, and I wait for what Mr. Vieweg will suggest to offer for my work. If his offer is low than my demand, then I take my note back, unopened, and the negotiation is broken. If, however, his offer is higher, then I will not ask for more than what is written in the note to be opened by Mr. Böttiger.

歌德認為 Mr. Vieweg 會提出什麼樣的價格?

Let  $G$  denote the bid made by Goethe, and  $V$  the bid made by Mr. Vieweg. Also let  $\theta$  be the Mr. Vieweg's evaluation for the poem. We claim that Mr. Vieweg will not bid below his evaluation,  $V < \theta$ . Here are three possible cases with respect to the bid made by Goethe.

1. If  $G < V$ , in which case Mr. Vieweg wins and pay  $G$ . If instead of bidding  $V$ , Mr. Vieweg would have bid  $\theta$ , then he would still win and pay the same price, so in this case bidding his valuation is as good as bidding  $V$ .
2. If  $G > \theta$ , in which case Mr. Vieweg loses. If instead of bidding  $V$ , Mr. Vieweg would have bid  $\theta$ , then he would still lose, so bidding his valuation is as good as bidding  $V$ .
3. If  $V < G < \theta$ , then Mr. Vieweg loses. If instead of bidding  $V$ , Mr. Vieweg would have bid  $\theta$ , then he would have won the auction and receive a pay-off of  $\theta - V$ , making this a profitable deviation, so bidding his valuation is strictly better than  $V$ .

It is similar to show that bidding above his valuation  $V > \theta$  is worse than bidding his valuation. Hence,  $V$  is a dominant strategy<sup>3</sup>, and thus we conclude that Mr. Vieweg might

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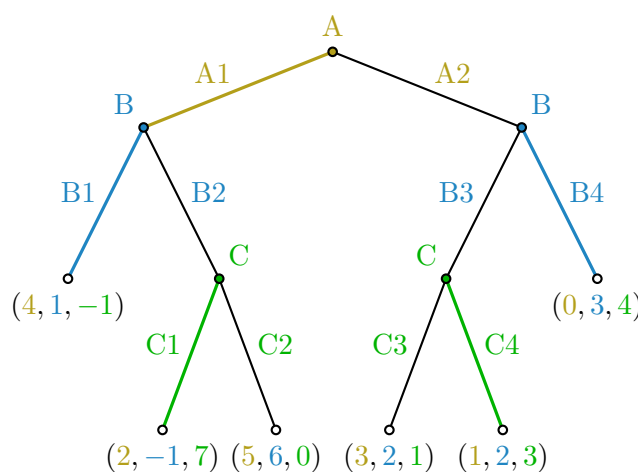
<sup>3</sup>The strategy is a weakly dominant strategy. Although one may find a lot of NEs in the original game, we must further consider that players will not play dominated strategies under the rationality assumption.

offer at his evaluation of Goethe's work.<sup>4</sup>

## Q4

(2pts)

4. 請考慮 A、B、C 的三人賽局。枝幹旁的字標示的是各人的策略，如 A 有策略 A1、A2 等。報酬向量中的三個數字挨序為 A、B 與 C 的報酬。請找出此賽局的 subgame perfect equilibrium。



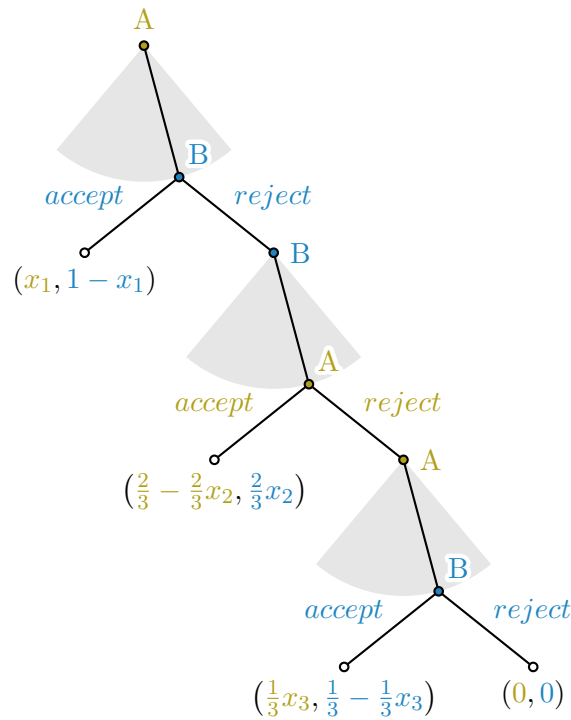
After drawing the possible paths using backward induction. The SPNE is (A1, B1B4, C1C4).

## Q5

(3pts)

5. A、B 分食冰淇淋，A 先提議兩人分食的比例，若 B 接受，則按 A 提議方式分配。若 B 反對，輪到 B 提案，只是當 B 提案時，冰淇淋融化只剩原有的  $\frac{2}{3}$ 。A 若能接受 B 的提案，兩人將分食剩下的  $\frac{2}{3}$ 。A 若反對 B 的意見，再換回 A 提案，此時 A 將考慮如何分配融化中只餘原先大小  $\frac{1}{3}$  的冰淇淋，若 B 再表反對，等不及新提案，冰淇淋便化光了，誰都沒得吃。A、B 希望自己吃到的越多越好；兩人精打細算，完全理性。請問 A 最先開始時，會如何提議分食的比例？

<sup>4</sup>To see more information about this story, please read Moldovanu, B., & Tietzel, M. (1998). Goethe's Second-Price Auction. *Journal of Political Economy*, 106(4), 854–859. <https://doi.org/10.1086/250032>



Through backward induction, B will accept the offer in the third stage. Taken this into account, A will offer  $x_3 = 1$ . For A in the second stage, he will accept the offer if and only if  $\frac{2}{3} - \frac{2}{3}x_2 \geq \frac{1}{3}$ , if and only if  $x_2 \leq \frac{1}{2}$ . Taken this into account, B will offer  $x_2 = \frac{1}{2}$ . For B in the first stage, he will accept the offer if and only if  $1 - x_1 \geq \frac{1}{3}$ , if and only if  $x_1 \leq \frac{2}{3}$ . Taken this into account, therefore, A will offer  $x_1 = \frac{2}{3}$  at the beginning.

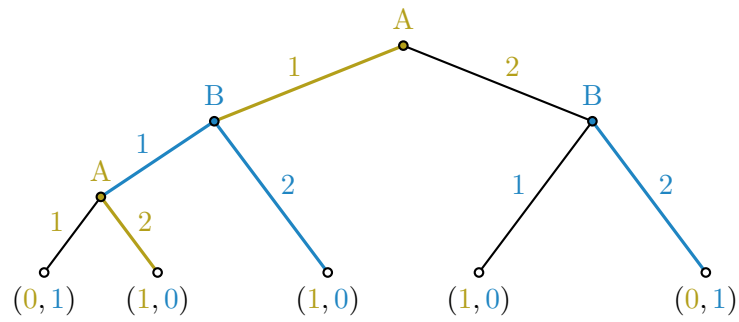
## Q6

(3pts)

6. A, B 兩人玩搶4的遊戲: 兩人從0開始, 輪流報加數, 加數可為1或為2 (不能 pass), 誰將累加的和湊為4, 誰就是贏家。A 先開始報。

- 請繪此遊戲之 game tree。
- Subgame perfect equilibrium 中, 誰會贏?
- 假設將搶4改為搶20, 其他遊戲規則相同, 如果你來玩, 你可選先報或讓對手先報, 你要選先報嗎? 為什麼? (你的目標是贏。)

(a)



(b) The SPNE for this game are  $(12, 12)$  and  $(12, 22)$ . Both on-path solutions suggest that  $A$  will win the game.

(c) Using backward induction, we know that the one who first reaches 17 wins. Recursively, we conclude that the one who first reaches 2 wins. Hence, the first player always wins the game.