Chapter 13

Optics

13.1 Wave Optics

The polarization will be ignored in this chapter.

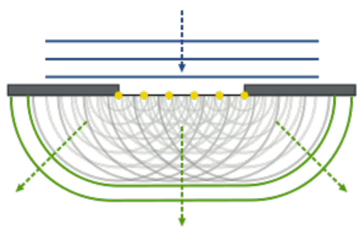
13.1.1 Huygens Principle

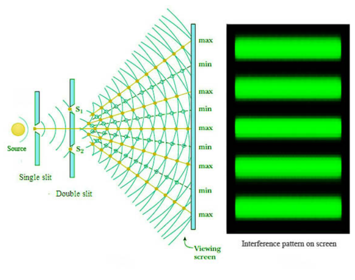
The Huygens principle is an approximate description of the propagation of waves. The result of Huygens principle is in general different from exact solutions to the Maxwell equations. But it often provides us with a good qualitative description of the waves.

In the application of the Huygens principle, it is often convenient to use the phasor representation of waves, and consider the superposition of waves as the addition of vectors on the complex plane.

13.1.2 Interference

Coherence is a necessary condition to observe the pattern of interference. Coherent sources of waves have constant phase differences. (They necessarily have the same frequencies by assumption.)





Q 13.1: What is a "coherent" source of waves? Is it necessary to have a single slit in front of the double slits to observe the interference pattern?

Young's Experiment (Double-Slit Experiment)

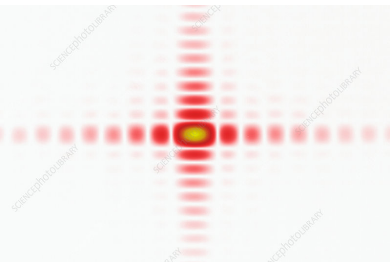
Q 13.2: For a coherent source of monochromatic light of wavelength λ, where are the peaks of the interference pattern?

Q 13.3: What kind of double-slit pattern would you see for a uniform spectrum of light waves?
Q 13.4: What would happen if the incoming wave does not have normal incidence on the slits?
Q 13.5: Why by experience it is often just two bright strips on the screen behind two slits as if the light is composed of particles (as if there is no interference)?

Multiple-Slit Experiment

HW: (4-3) Plot the intensity versus position diagram (interference pattern) for the light on a screen that is at a distance L away from a set of 5 slits. The monochromatic light has wavelength λ, and is incident on the slits at normal angle. The neighboring slits are separated by a distance d. Make sure that you have all the small local maxima and minima of light intensity in your plot in the range $(0, 10L\lambda/d)$. (Assume that $L \gg d \gg \lambda$.)

13.1.3 Diffraction



Single-Slit Experiment

Q 13.6: What is the difference between an n-slit experiment with the separation d and the superposition of n single-slits of width w separated by the distance d? (Why did we not consider the width of each slit in the double-slit or multiple-slit experiments? What is the condition for this ignorance to be justified?)

13.1.4 Refraction

The velocity v of light in matter is typically different from that in vacuum c. The ratio

$$n \equiv \frac{c}{v} \tag{13.1}$$

is called the *index of refraction.*

In general, the phase velocity $v(\omega)$ depends on the frequency of the light. This is the effect of **dispersion** we observe on prisms.

Here v refers to the *phase velocity*. It can be different from the group velocity, or other definitions of the velocity of light in media.

In the glass, $n(\omega)$ is larger for larger ω for visible-light frequencies.

At the interface of two media of different index of refraction, the phase of the wave must match on both sides of the interface (the wave — the EM fields — must be continuous at every point on the interface). This continuity condition leads to Snell's law

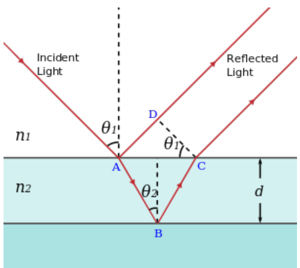
$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \tag{13.2}$$

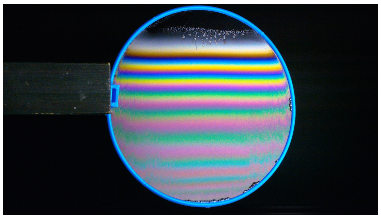
Thin Films

When the light coming from a medium with a lower index of refraction is incident upon another medium with a higher index of refraction (such as air to water), the reflected wave undergoes a relative 180-degree change of phase with respect to the incident wave at the interface.

On the other hand, if the light coming from a medium with a higher index of refraction is incident upon another medium with a lower index of refraction (such as water to air), the reflected wave is in phase with the incident wave on the interface.

Q 13.7: Why does it have to be a *thin* film in order to see the interference? Is it possible to observe the interference pattern for a thick film? How thin is thin enough?





Ex 13.1: Show that the condition for bright fringes is

$$2\frac{n_2}{n_1} d\cos\theta_2 = m\lambda \qquad \text{for} \quad m \in \mathbb{Z}, \tag{13.3}$$

where λ is the wavelength in media 1, assuming that $n_1 > n_2 > n_3$.

Solution:

The distance of the path ABC is $\frac{2d}{\cos\theta_2}$. The distance of the path AD is $2d\tan\theta_2\cos\theta_1$. Let the wavelength of the wave in media 1 be denoted λ. The wavelength in media 2 is then $\frac{n_1}{n_2}\lambda$. When $n_1 > n_2 > n_3$, the difference in phase divided by 2π in the two different paths is

$$\begin{aligned}
\Delta &\equiv \frac{2d}{\frac{n_1}{n_2}\lambda\cos\theta_2} - \frac{2d\tan\theta_2\sin\theta_1}{\lambda} \\
&= \frac{2d}{n_1\lambda\cos\theta_2}(n_2 - n_1\sin\theta_2\sin\theta_1) \\
&= \frac{2d}{n_1\lambda\cos\theta_2}(n_2 - n_2\sin^2\theta_2) \\
&= \frac{2n_2 d}{n_1\lambda}\cos\theta_2,
\end{aligned} \tag{13.4}$$

where we have used the Snell's law in the 2nd equality. We get constructive interference when this number is an integer.

The ideal notion of a monochromatic wave can only be approximated by a narrow bandwidth of frequencies in reality. What we see is the superposition of the interference patterns for all frequencies within the bandwidth.

Newton Rings, Interferometers

For the Newton rings, the dark rings have radii $r = \sqrt{nR\lambda}$ for $n = 1, 2, 3, \cdots$.