個體經濟學期末考 (2019.6)

總分43分。答題皆須附說明, 未做解釋的答案概不計分。

1. A firm which is a price taker has the following production function:

$$q = (kl)^{1/3},$$

where q is the output and k and l are quantities of two factors. The price of output is \$2 and the price of k is \$1.

   (a) (2 points) Are k and l substitutes or complements?

   (b) (2 points) In the short run, l is a variable factor and k is a fixed factor and $k = 27$. Let w denote the price of l. Please derive the firm's demand for l: $l(w)$.

   (c) (2 points) In the long run, both factors are variable factors. Please derive the firm's demand for l: $l(w)$.

2. Jack can work up to 8 hours a day at a wage rate of w and as much more as he wants at the higher overtime rate of w'. He chooses to work 10 hours. Jane can work as many hours as she wants at a wage of w''. Jack and Jane have the same tastes, the same assets ($A > 0$), and are equally happy.

   (a) (2 points) What can you conclude about the size of w'' compared with w and w'? What can you conclude about the number of hours Jane works? Please explain your answer using a graph with labor at the horizontal axis and consumption at the vertical axis.

   (b) (2 points) If w'' increases, how will Jane change her working hours? Decompose the change into the substitution effect and the income effect in a graph.

3. Each potential user of the Bay Bridge is willing pay up to \$29 per crossing during rush hours, provided there are no other cars to slow him down. When there are more cars, willingness to pay goes down. When there are n cars on the bridge, each user is willing to pay up to \$(30-n)

1

to cross. When a user is indifferent between crossing and not crossing, we shall assume that he will choose to cross the bridge.

(a) (1 point) If there is no bridge toll, how many people will cross the bridge during rush hours?

(b) (1 point) What is the optimal number of bridge crossing during rush hours?

(c) (1 point) What is the optimal bridge toll to control the traffic flow?

4. In a village there are two farmers. Each farmer has the same daily production function of rice:

$$r_i = \sqrt{c_i w}, \quad i = 1, 2$$

where c_i is farmer i's cultivation time spent on his field and w is the quality of their communal irrigation facility, $w \geq 0$. Each farmer has 6 working hours per day to spend on cultivating his own field and maintaining the communal irrigation facility. Let w_i denote the time farmer i spends on maintaining irrigation facility,

$$w_i + c_i = 6,$$

and

$$w = w_1 + w_2.$$

Each farmer wishes to maximize his output of rice.

(a) (1 point) Let r^* denote the maximum of their joint rice output. Is it Pareto efficient when $r_1 + r_2 < r^*$? State your argument clearly.

(b) (2 points) When the joint output of r^* is achieved, is it possible that $r_1 > r_2$? Prove your statement rigorously.

(c) (3 points) Calculate r^*.

5. Farmers A, B and C live in a small village. Their harvests of apples in two years are as follows:

year	A	B	C
1	15	15	30
2	25	25	10

Apples will be rotten after one year so all the apples harvested in the first year cannot be saved for next year's consumption. However, a farmer could lend his unconsumed apples to others and charge an interest rate. The representative agent's utility function is:

$$u = c_1^3 c_2^2,$$

where c_i is the amount of apples consumed in year i, $i = 1, 2$.

(a) (3 points) Please calculate the equilibrium interest rate.

(b) (3 points) Farmer A has the same preference as the representative agent. In equilibrium, how many apples will farmer A lend (or borrow) in the first year?

6. Mr. Z is risk averse and he visits a casino. He is invited to bet on whether a randomly drawn card from a newly opened pack is not a heart. If he bets m and the card turns out to be a heart, Mr. Z will lose his m. If the card turns out to be any other suit, the casino will return m back to him and in addition, he wins $0.4m$ extra. When entering the casino, Mr. Z has $100 in his pocket. Please draw a graph with Mr. Z's wealth when a heart is drawn on the x-axis and Mr. Z's wealth when a heart is not drawn on the y-axis. In this problem, m is a continuous variable.

(a) (2 point) Calculate Mr. Z's $MRS \equiv |dy/dx|$ at (100,100).

(b) (2 points) Will Mr. Z gamble?

7. Consider the capital asset pricing model which describes the equilibrium situation of the financial markets. There are only two stocks in the market: a and b. On the other hand, an investor could borrow/lend at a risk-free rate r_f. The risk-free loan will be enforced to be paid back. Let r_a and r_b denote the rate of return of stock a and the rate of return of

stock b, respectively. The expected value of r_a is 12%, and the standard deviation of r_a is 4%. The expected value of r_b is 8%, and the standard deviation of r_b is 3%. The coefficient of correlation of r_a and r_b is -1.

(a) (2 points) Consider a person who invests $\$p(p \in [0, 1])$ in stock a and $\$1 - p$ in stock b. Let r denote the rate of return. Please calculate the mean and the standard deviation of r (in terms of p).

(b) (2 points) What is the equilibrium risk-free rate r_f?

(c) (3 points) Let μ and σ denote the mean and the standard deviation of r (the unit being %). An investor's preference for the rate of return is characterized by the following utility function:

$$u = 7\mu - \sigma^2.$$

It is known that this investor spends all his money on stock a and stock b. Please calculate the proportion of his investment on stock a.

8. (3 points) Company X's stock is currently selling for $\$s$ per share that one year from today it will either go up to $\$u$ or down to $\$d$, and that the annual interest rate is r. A *put* option will allow a buyer to sell a share of Company X's stock next year at a price of $\$p$, where $d < p < u$. In terms of s, u, d, r and p, what is the equilibrium price of this put option?

9. (1 point) It is clear that we do not have an even income distribution. What did Milton Friedman suggest to alleviate poverty?

10. (1 point) Why did Milton Friedman criticize the American Medical Association?

11. (1 point) Did Milton Friedman think that business should contribute to the support of charitable activities?

12. (1 point) Fair employment practice commissions prevent discrimination in employment by reason of race, color, or religion. What did Milton Friedman think of these commissions?

解答

1a $MP_k = (1/3)k^{-2/3}l^{1/3}$

MP_k increases in l, so they are complements.

1b $MRP_l = 2 * MP_l = (2/3)l^{(}-2/3)k^{(}1/3) = 2 * l^{(}-2/3) = w$ (1 point)
$l = \sqrt{8/w^3}$

1c $MRP_l = 2MP_l = w$ and $MRP_k = 2MP_k = 1$. (1 point)
$MRP_l = (2/3)l^{(}-2/3)k^{(}1/3) = w$
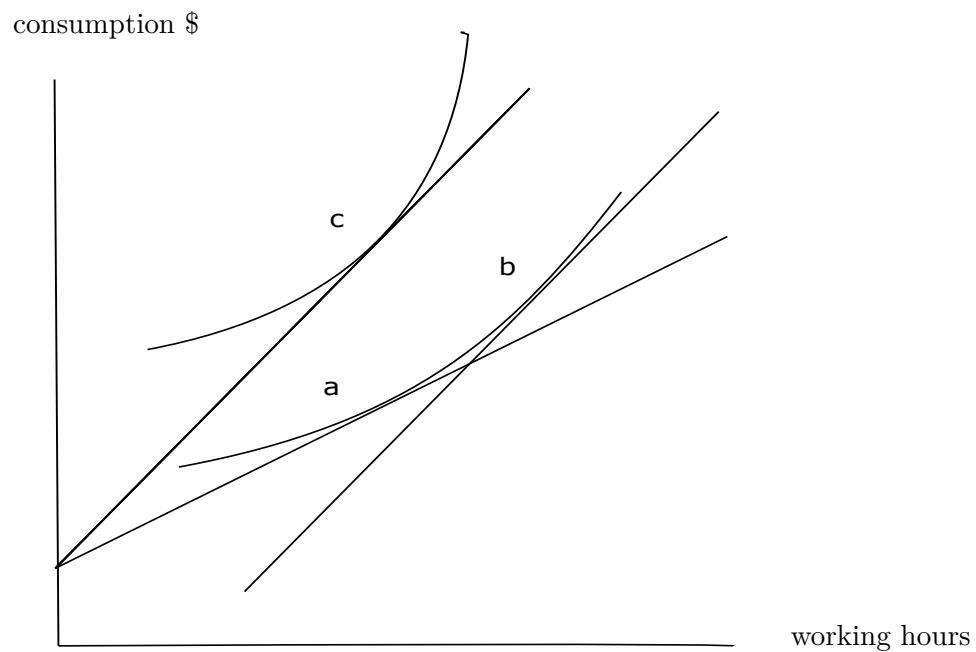$MRP_k = (2/3)k^{(}-2/3)l^{(}1/3) = 1$
Hence $k/l = w$, and $MRP_l = (2/3)l^{(}-2/3)(lw)^{(}1/3) = w$
So $l = 8/27w^2$

2a $w' > w'' > w$ (1 point)
Jane works less then 10 hours. (1 point)

2b the substitution effect: $\vec{ab}$; the income effect $\vec{bc}$

3a 30

3b $\max n(30 - n)$, $n = 15$

3c $30 - t - 15 = 0$, $t = 15$

4a No, to produce r^* jointly makes it possible for farmer i to receive more than his previous r_i, $i = 1, 2$.

4b If $r_1 > r_2$, then $c_1 > c_2$.
$MP_c^i = \sqrt{w/c_i}/2$, so $MP_c^1 < MP_c^2$.
There exist $\Delta > 0$. To have farmer 1 spend Δ more (less) hours on maintaining irrigation facility (cultivation) and to have farmer 2 spend Δ less (more) hours on maintaining irrigation facility (cultivation) will increase the joint output.

4c Find out first how much a farmer is willing to give up c_i in order to have better water supply:
$|dc_i/dw| = c_i/w$. Together, they are willing to give up $(c_1 + c_2)/w$ which should be set optimally to 1 (from the budget constraint of time). (2 points) This implies
$$6 - w_1 + 6 - w_2 = w,$$
i.e. $w = 6$. It's already known that $w_1 = w_2$ in an optimal arrangement. So $w_i = 3$, $c_i = 3$, $r^* = 2 * \sqrt{6 * 3} = 6\sqrt{2}$.

5a The RA has 20 apples in the first year and 20 apples in the second year. (1 point)
His $MRS = |dc_2/dc_1| = 3c_2/2c_1$ which is 1.5 at the endowment point. (1 point)
The interest rate is hence 50%.

5b Farmer A's budget constraint is (1 point even when the interest rate is wrong):
$$c_1 + \frac{c_2}{1.5} = 15 + \frac{25}{1.5}$$
The optimal consumption is: $c_1 = 19$. (1 point if the budget constraint has the right format)
A will borrow 4 apples in the first year.

6a 1/3

6

6b The slope of the budget line is 0.4. So the budget line is steeper than the indifference curve at (100,100). Mr. will gamble.

7a $\mu = p * 12\% + (1 - p)8\%$
$\sigma = |p * 4\% + (1 - p)3\%|$

7b When $p = 3/7$, $\sigma = 0$ and $r = 68/7\% = r_f$.

7c $MRS = d\mu/d\sigma = 2\sigma/7$ (1 point)
His budget line connects (0, 68/7) and (4,12) with a slope of 4/7.(1 point)
So $\sigma = 2$ and $p = 5/7$.

8 The seller of the put option receives \$$y$ today and will lose \$$p-d$ if the price turns out to be \$$d$ next year.
If the seller borrows \$$b$ and keeps \$$y$ in his pocket today, he could then purchase $(b - x)/s$ shares of Company X's stock. He is indifferent between these two arrangements if (2 points)

$$
\begin{aligned}
u\frac{b - x}{s} &= b(1 + r) \\
d\frac{b - x}{s} &= b(1 + r) - (p - d)
\end{aligned}
$$

It could be solved that

$$
\begin{aligned}
b &= \frac{p - d}{1 + r}\frac{u}{u - d} \\
x &= \frac{p - d}{u - d}(\frac{u}{1 + r} - s)
\end{aligned}
$$

There is no arbitrage opportunity at equilibrium, so $y = x$.
1 point if the student endeavors to write something related to the first 2 equations.

9 adopting negative income tax

10 He considered it a labor union which successfully kept down the number of physicians and kept up the costs of medical care.

11 No, the corporation is an instrument of the stockers. If the corporation makes a contribution, it prevents the individual stockholder from himself deciding how he should dispose of his funds.

12 He was against these commissions because they interfered with the freedom of individuals to enter into voluntary contracts with one another. He considered discrimination as a taste and we should only try to change this taste.