12.2 DC Circuits

We define the following quantities on each element in a circuit.

- **Current**
$$I \equiv \frac{dQ}{dt}, \tag{12.47}$$
where Q is the charge that passes through a fixed cross section on the circuit.

  The cross section is defined with the choice of a normal direction $\hat{\mathbf{n}}$, and the current is positive or negative if it flows in the $\hat{\mathbf{n}}$ or $(-\hat{\mathbf{n}})$ direction.
  Q 12.61: As the current flows in a conducting wire in a DC circuit, what makes the field line bend to follow the wires?

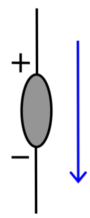
  We assume that, except on capacitors, there is no charge accumulation on a circuit, so that the current is everywhere the same regardless of how the cross-sectional surface is defined.

  There are actually small amount of charges accumulated at various places on a circuit.

  Just like how we have defined charge density ρ, surface charge density σ, and line charge density λ for the electric charge, for the electric current I, we define current density $\mathbf{J}$, and surface current density $\mathbf{K}$. The current density $\mathbf{J}$ is the current per unit area, and the surface current density $\mathbf{K}$ is the current per unit length.
  Q 12.62: What is the current I along a strip of conductor of width L with a uniform surface current density K?

- **Voltage**

  The voltage across a circuit element is also defined with the choice of a direction. It is conventionally defined together with the current as follows (with the exception of batteries). We label the two endpoints of a circuit element by $+$ and $-$. The voltage across the element is defined to be $V = V(+) - V(-)$, and the current is positive if it flows from the end labelled $+$ to the end labelled $-$.



  In the figure on the right, the arrow is used to define the reference direction. It does not have to agree with the actual direction of the current, as I can be both positive and negative. Similarly, the voltage of the point with the $+$ sign is not necessarily higher than the point with the $-$ sign.

- **Resistance**
$$R \equiv \frac{V}{I}. \tag{12.48}$$

  R is normally positive for resistors if we define V and I in the way described above.

- **Ohm's Law**
  R is (often, roughly) approximately a constant in a certain range of voltage/current for a resistor. (There is almost no information in this statement

except that it appears to hold in all experiments people have done during Ohm's time.)

- **Resistivity**
  The *resistivity* (which is unfortunately also conventionally denoted as ρ) of a material is defined by $\rho \equiv RA/\ell$, where R is the resistance of the resistor made of this material as a tube of length ℓ and cross-sectional area A.
  Q 12.63: Why is ρ expected to be independent of A and ℓ?

- **Power**
  $$P \equiv \frac{dE}{dt} = IV, \tag{12.49}$$
  where E is the energy dissipated over an element.

- **DC** $\equiv$ **Direct Current**, **AC** $\equiv$ **Alternating Current**.

  We will first discuss DC circuits.

It refers to the source of electric power.

- **Kirchhoff's Rules**:

  Kirchhoff's Junction Rule: $\sum_i I_i = 0$.

  Kirchhoff's Loop Rule: $\sum_a V_a = 0$.

  Q 12.64: Why do Kirchhoff's laws hold?

  > The assumptions behind Kirchhoff's laws are that there is no time-dependent net charges accumulated anywhere on the circuits and that there is no time-dependent magnetic field passing through the circuit.

- **Series And Parallel Connections**

  Two elements in series have the same current; two elements in parallel have the same voltage.

Superposition Principle

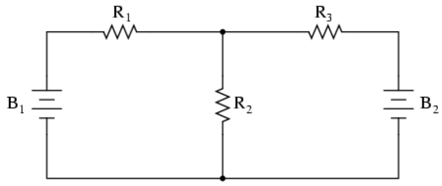
The circuit elements R, C, L are linear elements, because when the current $I_1(t)$ is associated with the potential $V_1(t)$, and $I_2(t)$ with $V_2(t)$, we immediately know that the current $I(t) = aI_1(t)+bI_2(t)$ is associated with the potential $V(t) = aV_1(t)+bV_2(t)$.

The Kirchhoff's rules are linear. Hence, a circuit with only linear elements obey superposition principle.

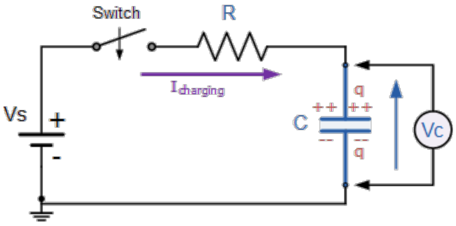
The batteries are not linear because its voltage is a fixed number. But we can imagine superposing a battery V_1 with another battery V_2 to obtain a new battery $V = V_1 + V_2$.

A conducting wire is the same as a battery with $V = 0$.

Ex 12.45: Find the current on the resistors for (1) $V_{B_1} = V_1$ and $V_{B_2} = 0$, and (2) $V_{B_1} = 0$ and $V_{B_2} = V_2$. Show that the current on the resistors is a superposition of these two problems when $V_{B_1} = V_1$ and $V_{B_2} = V_2$.



12.2.1 RC Circuits



Ex 12.46: For the circuit above, if the circuit is closed at $t = 0$, and the capacitor has a constant voltage V_0 for $t < 0$, what is the voltage on the capacitor $V_C(t)$ for $t > 0$? (The resistance of the resistor is R and the battery has a constant voltage V_S.)

HW: (2-3) See the figure below. For $t < 0$, the voltage across C_1 is a given number V_1. The circuit is closed at $t = 0$. Find the voltage $V(t)$ across C_1 for $t > 0$. For simplicity, let $C_1 = C_2 = C$ and $R_1 = R_2 = R$.

Define $V(t)$ such that it is positive when the upper plate of C_1 has a higher voltage than the lower plate.