Classical Mechanics (II)

Problem Set #3

1. Consider the following two-body relativistic reaction,

$$m_1(p_1) + m_2(p_2) \to m_3(p_3) + m_4(p_4).$$

   (a) Let s denote the square of the total energy in the CM system. Find the magnitude of three-momentum in the CM systems of particles 1 and 3, p_{1c} and p_{3c}, respectively, in terms of s.

   (b) Let θ_{3l} and θ_{3c} denote the scattering angle of the outgoing particle 3 after reaction in the laboratory and CM system, respectively. Find the relation between θ_{3l} and θ_{3c}.

2. A rocket traveling through the atmosphere experiences a linear air resistance $-k\mathbf{v}$. Find the differential equation of motion when all other external forces are negligible. Integrate the equation and show that if the rocket starts from rest, the final speed is given by

$$v = V\alpha \left[1 - (m/m_0)^{1/\alpha}\right]$$

   where V is the relative speed of the exhaust fuel, $\alpha = |\dot{m}/k| =$ constant, m_0 is the initial mass of the rocket plus fuel, and m is the final mass of the rocket.

3. Alpha Centauri is the nearest star system, about 4 light years from Earth.

   (a) Assume that an ion rocket has been built to travel to Alpha Centauri. Suppose the exhaust velocity of the ions is one-tenth the speed of light. Let the initial mass of the fuel be twice that of the payload (ignore the mass of the rocket, itself). Also, assume that it takes about 100 hours to exhaust all the fuel of the rocket. How long does it take the rocket to reach Alpha Centa (The speeds are small enough that you can neglect the effects of special relativity.)

   (b) Consider a chemical rocket whose exhaust velocity is $3\ \mathrm{km/s}$ is used instead. In the case of the ion rocket, $1\ \mathrm{kg}$ of fuel accelerates $1\ \mathrm{kg}$ of payload to a final velocity v_f. What fuel mass is required to accelerate the same payload to the same final velocity with the chemical rocket? (In each case, ignore the mass of the rocket.)

4. A fixed force center scatters a particle of mass m according to the force law $F(r) = k/r^3$. If the initial velocity of the particle is u_0, show that the differential scattering cross section is

$$\sigma(\theta) = \frac{k\pi^2(\pi - \theta)}{mu_0^2\theta^2(2\pi - \theta)^2 \sin\theta}$$

Note: Discussions are strongly encouraged, but please **do not copy**. All parties involved in copying will get **zero** for their homework. All problems will be graded.