Micro2 CH11

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Q1

N1. Suppose that a monopoly steel producer produces steel at zero marginal costs and sells to a monopoly automaker at a price P_{steel}. The automaker has no costs other than the cost of steel, which is converted into cars at the rate of one ton of steel to one car. There is no other way to produce a car than to use a ton of steel. The demand for cars is given by $Q_{cars} = 100 - P_{cars}$.

a. For a given price of steel, what quantity of cars will the automaker produce in order to maximize profits? (*Hint:* The function $-Q_2 + kQ$, with k constant, is maximized at $Q = k/2$.)

b. What is the equation for the automaker's demand curve for steel?

c. How much steel is produced? At what price? How many cars are produced? At what price?

d. If the steel producer acquires ownership of the automaker, how many cars are produced? At what price?

(a) Given P_{steel}, the profit function

$$\pi = P_{\text{cars}}Q_{\text{cars}} - P_{\text{steel}}Q_{\text{cars}} = -Q_{\text{cars}}^2 + (100 - P_{\text{steel}})Q_{\text{cars}}$$

is maximized at $Q_{\text{cars}} = 50 - \frac{P_{\text{steel}}}{2}$.

(b) The demand curve for steel is therefore $Q_{\text{steel}} = 50 - \frac{P_{\text{steel}}}{2}$.

(c) Steel is produced at MR=MC. First note that $P_{\text{steel}} = 100 - 2Q_{\text{steel}}$, so

$$MR = Q\frac{dP}{dQ} + P = 100 - 4Q.$$

Hence, $Q_{\text{steel}} = 25$ tons, at $P_{\text{steel}} = 50$. $Q_{\text{cars}} = 25$ cars are produced, at $P_{\text{cars}} = 75$.

(d) Now, $P_{\text{cars}} = 100 - Q_{\text{cars}}$, so $MR = -2Q + 100$, so $Q_{\text{cars}} = 50$ cars are produced, at $P_{\text{cars}} = 50$.

Q2

N2. Suppose that Microsoft is the only producer of operating systems and Netscape is the only producer of Web browsers. Suppose also that nobody wants an operating system without a Web browser and nobody wants a Web browser without an operating system. Suppose that both firms produce at zero marginal cost and that the demand for a package consisting of an operating system and a browser is given by $Q = 100 - P$.

a. Suppose that Microsoft and Netscape take each others' prices as given. What is the price of an operating system? What is the price of a browser?

b. Suppose instead that Microsoft first announces a price for its operating system; then Netscape takes this price as given and sets a price for its browser. Now what is the price of an operating system? What is the price of a browser?

c. Suppose that Microsoft merges with Netscape. Now what is the price for a package consisting of an operating system and a browser?

d. Suppose instead that Microsoft sells consumers a package consisting of a operating system and a Netscape browser and pays Netscape a royalty for each package that it sells. What royalty does Netscape charge and what price do consumers pay for the package?

(a) Suppose Microsoft sets price at p_1, and Netscape sets at p_2. The demand function is $Q = 100 - p_1 - p_2$. For Microsoft, the profit function is maximized at

$$MR = -Q + p_1 = 0.$$

For Netscape, the profit function is maximized at

$$MR = -Q + p_2 = 0.$$

Solving the equations, one could obtain $p_1 = \frac{100}{3}$, $p_2 = \frac{100}{3}$, and $Q = \frac{100}{3}$.

(b) By part (a), Netscape will choose at $p_2 = Q$. Taken this into account, Microsoft will set the price p_1 that satisfies

$$MR = -2Q + p_1 = 0.$$

Hence, $p_1 = 50$, $p_2 = 25$, and $Q = 25$.

(c) Merge Company faces the demand function of $Q = 100 - p$, and

$$MR = -Q + p = 0.$$

Hence, $p = 50, Q = 50$.

(d) Suppose the royalty price is set at r. The marginal cost is r, so the package price p should satisfy

$$MR = -Q + p = -2Q + 100 = r.$$

Taken this into account, r must satisfy

$$MR = -2Q + r = 0.$$

Hence, $r = 50$, $Q = 25$, and $p = 75$.

Q3

N3. Dr. Miles is a monopolist who sells a type of patent medicine through competitive retailers. The demand curve for this patent medicine is given by $Q = 100 - 2P$, where P is the price and Q is the number of bottles sold.

a. If Dr. Miles has zero marginal cost, how many bottles of medicine will she sell and at what price? Calculate the consumers' surplus. Calculate Dr. Miles's producer's surplus.

b. Now suppose that retailers are able to provide their customers with valuable services by explaining how the medicine is to be used, what ailments it is effective against, and so on. By incurring a cost of C in time and effort per bottle sold, the retailer can provide services that consumers value at V per bottle sold, where V is given by $V = 5C - C^2$. What is the socially optimal amount of service per bottle for retailers to offer? What is the cost of this service?

c. Now suppose that retailers who offer services do not sell any additional medicine, because customers accept the services and then shop elsewhere, buying from a cut-rate supplier who offers no services. To combat this, Dr. Miles institutes a fair trade agreement under which she will sell at a wholesale price of P_0 but retailers must charge a retail price of P_1. Retailers have no other costs. Explain why retailers will incur costs of service equal to $C = P_1 - P_0$. What is the socially optimal value for C?

d. Taking C as given, write the equation of the new demand curve that retailers face after Dr. Miles institutes fair trade. Write the equation of the new demand curve Dr. Miles faces. In view of her wanting to face the highest possible demand curve, what value will Dr. Miles choose for C?

e. Using your answers to part (d), calculate the new price P_0 that Dr. Miles will charge, the new quantity sold, the new consumers' surplus, and the new producer's surplus.

(a) First note that $P = 50 - \frac{1}{2}Q$, so the marginal revenue facing Dr. Miles should be

$$MR = -\frac{1}{2}Q + P = 50 - Q = 0.$$

Dr. Miles will provide $Q = 50$, at price $P = 25$. The consumer surplus and the producer surplus are

$$CS = \int_0^{50} \left(50 - \frac{1}{2}Q - 25\right) dQ = 625$$

$$PS = \int_0^{50} 25dQ = 1250.$$

(b) The social surplus is $5C - C^2 - C$, which is maximized at $C = 2$. The optimal amount of service per bottle for retailers to offer is 6, and the cost of this service is 2.

(c) If C is less than $P_1 - P_0$, retailers make a positive profit on each bottle, so it is worth increasing C to lure competitors' customers away. Thus C rises to $P_1 - P_0$. The socially optimal value for C, by part b, is 2 per bottle.

(d) Following from c, at a wholesale price of P_0 dollars per bottle, the retail price will be $P_0 + C$ dollars per bottle. However, consumers, who receive V worth of services, feel as though they are paying $P_0 + C - V = P_0 - 4C + C^2$ dollars per bottle. At this price they purchase $Q = 100 - 2(P_0 - 4C + C^2)$ bottles. Thus demand is given by

$$Q = 100 - 2P_0 + 8C - 2C^2.$$

Dr. Miles sets $C = 2$ per bottle. That is, Dr. Miles requires retailers to sell for 2 per bottle over the wholesale price.

(e) The demand curve facing Dr. Miles is now $Q = 108 - 2P_0$, so $P_0 = 54 - \frac{1}{2}Q$. The marginal revenue is

$$MR = -\frac{1}{2}Q + P_0 = 54 - Q.$$

Dr. Miles will sold $Q = 54$, set price at $P_0 = 27$. The demand function retailers faced will be $Q = 112 - 2P_1$, so the inverse function $P = 56 - \frac{1}{2}Q$. The new consumer surplus and the producer surplus are

$$CS = \int_0^{54} \left(56 - \frac{1}{2}Q - 29\right) dQ = 729$$

$$PS = \int_0^{54} 27dQ = 1458$$

respectively.

Q4

15. Suppose that there are exactly *N* identical firms in an industry, all with flat marginal cost curves. Industry demand is linear. How much does each firm produce, compared with the competitive quantity, under the Cournot assumption that each takes its rivals' outputs as given? How much does the industry produce? What happens to industry output as *N* gets large? (*Hint:* Follow carefully the argument that is given in the text for the case $N = 2$.)

Assume the demand curve is of the form $Q = aP + b$, and the marginal cost that every firm faced is c.

First, we consider the competitive case. Since $P = c$, the industry will produce the total amount $ac + b$, and each firm contributes $\frac{ac+b}{N}$.

Second, consider the Cournot competition case. For firm one,

$$MR = q_1 \frac{dP}{dq_1} + P = \frac{1}{a}(q_1 + (q_1 + \cdots + q_N - b)) = c,$$

and for other $N - 1$ firms, we will obtain similar results due to symmetry. After combining the N equations

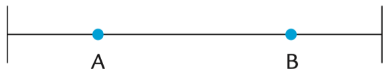
$$\begin{cases} 2q_1 + q_2 + \cdots + q_N = ac + b \\ q_1 + 2q_2 + \cdots + q_N = ac + b \\ \vdots \\ q_1 + q_2 + \cdots + 2q_N = ac + b \end{cases} \Rightarrow (N+1)(q_1 + \cdots + q_N) = N(ac + b),$$

we obtain that $q_1 = q_2 = \cdots = q_N = \frac{ac+b}{N+1}$, and the industry will produce the total amount $\frac{N(ac+b)}{N+1}$. As $N \to \infty$, the total industry output will converge to $ac + b$, tantamount as the total output of the competitive market.

Q5

18. Suppose there are four ice cream vendors on the beach depicted in Exhibit 11.11. How will they locate themselves in equilibrium? What can you say if there are five vendors? What if there are more than five?

EXHIBIT **11.11** **Ice Cream Vendors on a Beach**



If the vendors start out in the locations shown, each will move toward the center in an attempt to gain more customers. The equilibrium is reached when they are located right next to each other and can move no farther.

Given that 4 vendors at $x_1 \leq x_2 \leq x_3 \leq x_4$ respectively. The left most vendors payoff is $\frac{x_1+x_2}{2}$ if $x_1 < x_2$. Therefore, x_1 is willing to approach to x_2, so does x_4 is willing to approach x_3. To make sure that both 2-2 overlapping configuration is stable, the payoff for x_2 moving to the middle region is $\frac{x_4-x_1}{2}$, and the payoff for x_2 moving to the left region is infinitely close to x_1, which cannot be greater than the payoff at x_1, i.e. $\frac{x_1+x_4}{4}$. Similar discussion for x_3, so we have two inequalities

$$
\begin{cases}
x_1 \leq \frac{x_1+x_4}{4} \\
1 - x_4 \leq \frac{x_1+x_4}{4} \\
\frac{x_4-x_1}{2} \leq \frac{x_1+x_4}{4} \\
\frac{x_4-x_1}{2} \leq \frac{2-x_1-x_4}{4}
\end{cases}
\Rightarrow
\begin{cases}
x_4 = 3x_1 \\
3x_4 - x_1 = 2
\end{cases}
\Rightarrow x_1 = \frac{1}{4} \quad x_4 = \frac{3}{4}
$$

The equilibrium for the 4 vendors is to form a 2-2 group, occupying $\frac{1}{4}$ and $\frac{3}{4}$.

Given 5 vendors at x_1, x_2, x_3, x_4, x_5, respectively. As discussed above x_1 will approach to x_2, so does x_5 to x_4. To check is the system stable, the payoff for x_2 moving to the right is $\frac{x_3-x_1}{2}$, and left is x_1.Similar discussion is applied for x_4. Note that x_3 has to be $\frac{1}{2}$ to make the whole system symmetric.

$$\begin{cases} x_1 \leq \frac{x_1+x_3}{4} \\ 1 - x_5 \leq \frac{2-x_3-x_5}{4} \\ \frac{x_3-x_1}{2} \leq \frac{x_1+x_3}{4} \\ \frac{2-x_3+x_5}{2} \leq \frac{2-x_3-x_5}{4} \end{cases} \Rightarrow \begin{cases} 3x_1 = x_3 \\ 2 = 3x_5 - x_3 \end{cases} \Rightarrow x_1 = \frac{1}{6}, \quad x_3 = \frac{1}{2}, \quad x_5 = \frac{5}{6}$$