Micro2 CH18

Shen, r10323001

Q1

1. A has \$100 and if he becomes sick, he needs to spend \$36 on medical treatment. The probability for A to fall sick is 1/2. Let x denote A's wealth when he is sick, and y denote A's wealth when he is healthy. A's preference could be described by the following utility function:

$$u(x, y) = 0.5\sqrt{x} + 0.5\sqrt{y}.$$

   (a) Please show that the marginal rate of substitution is: $|dy/dx| = \sqrt{y/x}$.

   (b) Does A have a convex preference? Why?

   (c) Is A risk-neutral, risk-loving, or risk-averse? Why?

   (d) What is A's willingness to pay to purchase a full medical insurance?

   (e) What is the insurance fee for a fair full insurance in A's case?

(a)

$$\left|\frac{dy}{dx}\right| = \frac{\frac{\partial u}{\partial x}}{\frac{\partial u}{\partial y}} = \frac{\frac{1}{4}x^{-\frac{1}{2}}}{\frac{1}{4}y^{-\frac{1}{2}}} = \sqrt{\frac{y}{x}}$$

(b)

$$\frac{d}{dx}\sqrt{\frac{y}{x}} = \frac{1}{2}\sqrt{\frac{x}{y}}\frac{\frac{dy}{dx}x - y}{x^2} = \frac{1}{2}\sqrt{\frac{x}{y}}\frac{1}{x^2}(-\sqrt{xy} - y) < 0$$

Hence, A has a convex preference.

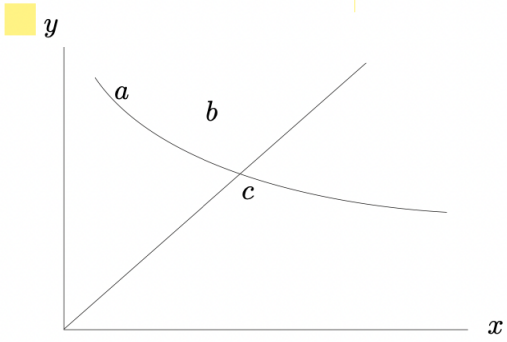
(c) Risk-averse, since A has a convex preference.

(d) The endowment is at $(64, 100)$. $2\sqrt{k} = \sqrt{64} + \sqrt{100} = 18 \Rightarrow k = 81$. So A's willingness to pay to purchase a full medical insurance is $100 - 81 = 19$.

(e) Because $\frac{p}{1-p} = 1$, the intersection with the 45 degree line is $(82, 82)$. The insurance fee for a fair full insurance is $100 - 82 =$ 18 dollars.

Q2

2. A 有0.1的機率會遭竊, 令 x、y 分表其遭竊與未遭竊的財富。下圖中 a 爲未投保時的原賦點, b 爲購買保險後的財富狀況。曲線爲過 a 之 A 的無異曲線, 其與 45° 線的交點爲 c。假設保險公司爲風險中立。



(a) 請繪圖說明 A 繳了多少保費。

(b) 請問 c 點之 MRS 爲何?

(c) 請繪出通過 b 之保險公司的無異曲線, 並計算其斜率。

(d) b 點背後的保險契約是否有 Pareto 效率性?

(e) 保險公司的最適保約爲何?

(a) The insurance fee is the difference between the y coordinates of point a and b.

(b) $\frac{p}{1-p} = \frac{1}{9}$

(c) The indifference curve is a straight line with negative slope $\frac{1}{9}$.

(d) No, since the core area is still bigger than zero.

(e) The optimal for the insurance company is at point c.

Q3

3. 英國某知名教授 B 專注於「黑洞」現象之探索。美國一大牌學者 A 則窮盡心力宣揚「黑洞」其實並不存在。物理界新銳 C 正絞盡腦汁驗證 A 之「不存在定理」。若 A 之假說被證實, B 之理論將爲空中樓閣, B 再也申請不到研究獎助, 財務損失慘重。反之, 由於 A 的工作僅限於假說之提出, 而未有理論的後續發展, A 的聲譽與財富將不受 C 結論影響。B 與 A 分別是「花花公子」與「國家地理」的忠實讀者。B 與 A 相約: 若 A 之假說被證明爲真, A 將爲潦倒的 B 續訂「花」雜誌 10 年。反之, 若 C 推翻 A 的理論, B 會爲 A 訂「國」雜誌 10 年。已知 A 爲風險厭惡者。(本題引自《時間簡史》。《不只一點瘋狂》(數學家 Paul Erdos 的傳記) 提供了另一類似的故事。長久以來, 數學家絞盡腦汁想證明「雷曼假說」, 成功者將享受榮耀至尊。英國數學家哈代懼怕船難, 有一回不得不上船遠行, 行前發信給友人吹噓他已證明「雷曼假說」爲真。英國學者真是精通保險學!)

   (a) 此約定會致 B 之預期財富增加還是減少?

   (b) B 爲風險愛好者嗎?

(a) A is risk averse, so A will not accept any gamble unless it improves A's expected wealth. The sum of their expected wealth is fixed, hence B's expected wealth decreases.

(b) The gamble works as a partial insurance for B. It reduces the variation of B's future wealth and it lower B's expected wealth. Only a risk-averse person will accept such a gamble. So B is risk averse.

Q4

4. Consider the capital asset pricing model which describes the equilibrium situation of the financial markets. There are only two stocks in the market: a and b. On the other hand, an investor could borrow/lend at a risk-free rate r_f. The risk-free loan will be enforced to be paid back. Let r_a and r_b denote the rate of return of stock a and the rate of return of stock b, respectively. The expected value of r_a is 12%, and the standard deviation of r_a is 4%. The expected value of r_b is 8%, and the standard deviation of r_b is 3%. The covariance of r_a and r_b is 0. The market value of all stocks a and the market value of all stocks b are equal. Let r_m denote the rate of return of the *market portfolio*.

   (a) Please calculate the expected value of r_m.

   (b) Please calculate the standard deviation of r_m.

   (c) Please list the equations to solve for r_f without solving them.

(a) Since both a and b have the same market value, the market portfolio should contain $\frac{1}{2}$ a and $\frac{1}{2}$ b.

$$\mu_m = \frac{1}{2} \times 12\% + \frac{1}{2} \times 8\% = 10\%$$

(b)

$$\sigma_m = \sqrt{\frac{1}{4} \times 4^2 + \frac{1}{4} \times 3^2} = 2.5\%$$

(c) The parametric equation for the efficient frontier is given by

$$\sigma(p) = \sqrt{p^2\sigma_a^2 + (1-p)^2\sigma_b^2}$$
$$\mu(p) = p\mu_a + (1-p)\mu_b.$$

The tangent slope at $p = \frac{1}{2}$ is

$$\frac{d\mu}{d\sigma}(\frac{1}{2}) = \frac{\frac{d\mu}{dp}}{\frac{d\sigma}{dp}}(\frac{1}{2}) = \left.\frac{\mu_a - \mu_b}{\frac{p\sigma_a^2 - (1-p)\sigma_b^2}{\sqrt{p^2\sigma_a^2 + (1-p)^2\sigma_b^2}}}\right|_{p=\frac{1}{2}} \approx 2.86,$$

which should equal to the slope of the straight line that connected risk free bond and the market portfolio

$$\frac{10 - r_f}{2.5} = 2.86 \Rightarrow r_f = 2.85\%.$$

Q5

5. Suppose that it is known for certain that the demand for wheat this year is identical to the demand for wheat next year. This year's wheat crop of 100 tons has just been harvested. Everybody believes that next year's wheat crop which has already been planted, will also be 100 tons. Now a speculator arrives on the scene, convinced that next year's crop will be only 80 tons.

   (a) If wheat can be stored costlessly, what will the speculator do? What happens to this year's wheat supply and to next year's? (Assuming an interest rate of 0%)

   (b) How long does the speculator continue this activity? What is this year's wheat supply when he is finished? What is next year's wheat supply when he is finished if he turns out to be right? What is it if he turns out to be wrong?

   (c) Use a graph to show the social gains with and without a speculator, on the assumption that the speculator is right. If he is right, does he improve social welfare?

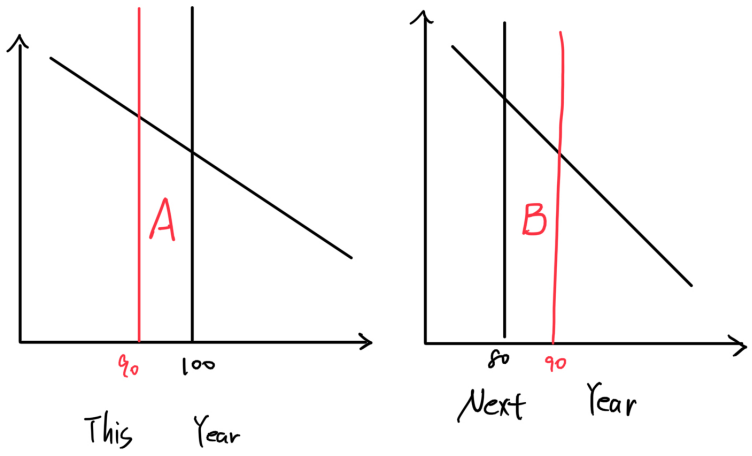
   (d) Use a graph to show the social gains with and without a speculator on the assumption that the speculator is wrong. If he is wrong, does he improve social welfare?

(a) Since the supply will decline next year, the price of wheat will rise. Thus, the speculator will store some wheat this year and sell them next year.

(b) They will continue to store wheat until the market price this year equal to the market price next year which is accustomed to the new supply provided by the speculator. The next year's wheat supply is 90 provided that the guess is right. If the speculator turns out to be wrong, the next year's wheat supply will be 110.

(c) The difference of the social welfare in both situations is $B - A$. Since the demand

curve is negative sloping, the difference must be positive. Hence, the speculator improves the social welfare if he is correct.



(d) The difference of the social welfare in both situations is $B - A$. Since the demand curve is negative sloping, the difference must be negative. Hence, the speculator worsen the social welfare if he is incorrect.