

# Micro2 CH17

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For simplicity, all calculations assume to be only 2 periods, similar results as the infinite horizon model.

## Q1

6. Suppose that apartments in San Francisco typically sell for \$300,000 and rent for \$1,500 a month. The market interest rate is 10%. **True or False:** The market must be anticipating a rise in apartment rentals at some time in the future.

**TRUE/FALSE.** One invests 300,000 dollars in the rental market can earn  $1500 \times 12 = 18,000$  a year, whereas saves those money into the financial market can earn  $300,000 \times 10\% = 30,000$  a year. Hence, the apartment rentals must rise in the future. (Or, there is another possibility that the market anticipates a fall in the interest rate.)

## Q2

15. **True or False:** If a monopolist owned an exhaustible resource, he would control its availability so that the price rose faster than the rate of interest.

**FALSE.** Consider the two period model. The monopolist who faced a linear demand  $q = a - bp$ , aims at

$$\max_{p_1, p_2} p_1(a - bp_1)(1 + r) + p_2(a - bp_2).$$

subject to the resource constraint  $a - bp_1 + a - bp_2 = S$ . Solving the optimization problem, we have

$$p_1 = \frac{4a + ar - 2S}{2br + 4b}$$

$$p_2 = \frac{4a + 3ar - 2S - 2Sr}{2br + 4b}.$$

The price rises at the rate

$$\pi = \frac{p_2 - p_1}{p_1} = \frac{2r(a - S)}{4a + ar - 2S} < \frac{2r(a - S)}{2a - 2S} = r.$$

Intuitively, the monopolist wants his marginal revenue to grow at the rate of interest. Because the marginal revenue curve is steeper than the demand curve, a given rise in marginal revenue corresponds to a smaller rise in price.

### Q3

- 20.** Suppose that the interest rate is 12% and that the representative agent's tastes are such that the interest rate would have to rise to 20% to get him to voluntarily cut current consumption by \$1,000. Suppose now that there is a war that destroys \$1,000 worth of consumption goods for every agent in the economy. **True or False:** The interest rate must rise to 20% to restore equilibrium.

**FALSE.** The representative agent is poorer now, and has a lower demand curve for current consumption. If his current consumption quantity is  $Q$ , then  $Q - 1000$  corresponds to 20% on the old demand curve. It must correspond to something less than 20% on the new demand curve.

## Q4

4. In Ch'ing dynasty Taiwan, a tenant usually paid his landlord a fixed amount of rental (定額租) every year. Suppose the rental for a given piece of land is  $R$  for a year. It's now the beginning of a year, and a landlord will receive the rental only at the end of year. The annual market interest rate is  $i$ ,  $i > 0$ . What is the equilibrium market price for this piece of land?

Suppose a piece of land costs  $M$  dollars. A person possesses such wealth can invest either in the rental market or the financial market. In equilibrium, both market should have the same return, that is,

$$R = Mi \Rightarrow M = \frac{R}{i}.$$

## Q5

5. It's interesting to note that not all pieces of land were sold unconditionally (絕賣) in Ch'ing dynasty Taiwan. Quite often, a seller of a piece of land had an option of repurchasing (or redeeming) it at the same price in the future. The custom was named Dian (典賣). Researchers find that subjecting a parcel of land to Dian reduced its market value to 60 to 80 percent of its value without Dian. Suppose a landlord wishes to raise  $\$K$ . There are 3 options: (1) To Dian his land at a price of  $\$K$  with the option that he could purchase it back 10 years later at the same price. (2) To sell the same piece of land at a full price of  $\$5K/4$ . (3) To borrow  $\$K$  for 10 years and the market interest rate is  $i$ ,  $i > 0$ . We assume the market is perfect and one could always sell/buy anything he wishes at the market price. Which option is the most appealing financially?

Here I assume only two periods. Compare the future values:

(1)  $K(1 + i) - K = Ki$ .

$$(2) \frac{5}{4}K(1+i) - \frac{5}{4}K = \frac{5}{4}Ki$$

(3) By Q4, the revenue for owning a house in the next period is the rental  $R = \frac{5}{4}Ki$ .

Hence,  $(2) = (3) > (1)$ .

## Q6

6. X 獨居荒島兩期，島上有果樹1株，第一期結果8個，第二期結果6個。X 可以將第一期的果子收藏起來，第二期再享用。令  $c_1, c_2$  表第一、二期 X 的消費數量，X 之跨期效用函數為：

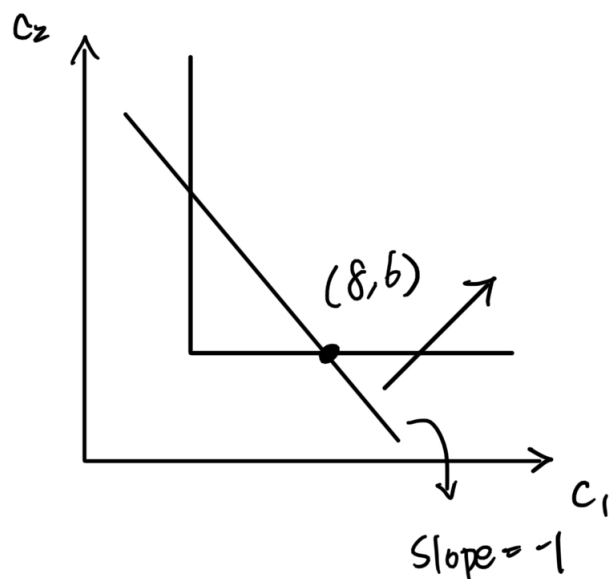
$$u = \min\{c_1, c_2\}$$

- (a) 請在  $c_1 - c_2$  平面上繪出 X 的無異曲線。(可先嘗試比較  $(c_1, c_2)$  為  $(3,2)$ 、 $(2,3)$ 、 $(2,2)$  的效用。)

- (b) 請在  $c_1 - c_2$  平面上繪出 X 的預算線，並標明斜率。

- (c) X 第一期會吃幾個果子？

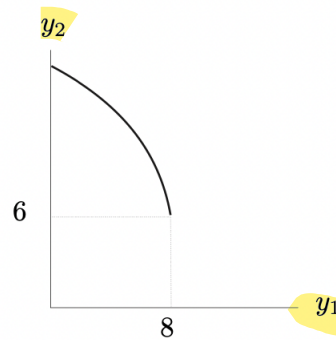
(a)(b)



(c) 7

## Q7

7. 承前題，除了將果子存於儲藏室外，X 發現若第一期的果子不消費而埋在地下，第二期地下的果子會長出新果子。令  $y_1$  為第一期吃掉的果子，埋了  $8 - y_1$  的果子，第二期的總收穫 (包括原先自然生長的6個) 量為  $y_2$ 。  $y_2(y_1)$  如下圖粗線所示：

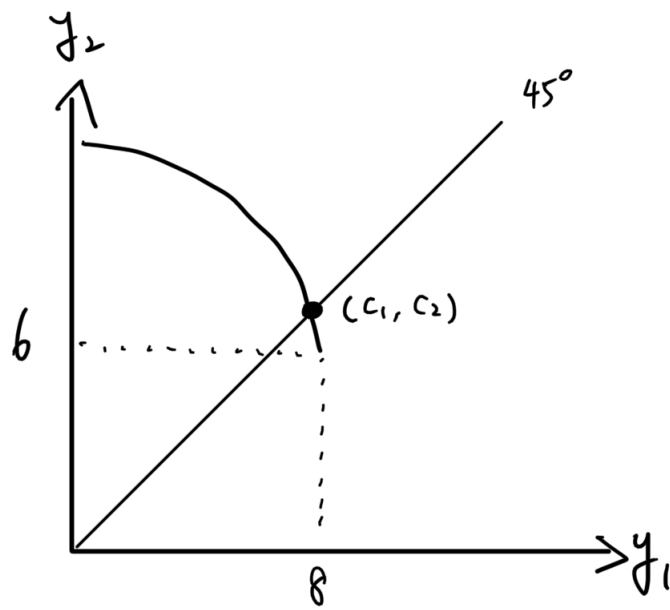


已知

$$\frac{dy_2}{dy_1}\bigg|_{y_1=8} = -2, \quad \frac{dy_2}{dy_1}\bigg|_{y_1=0} = -1.1$$

- (a) 請繪圖說明 X 第一期的消費  $c_1$  為何。
- (b) 若耕種以外，X 尚有借貸機會，跨期利率為 20%。請繪圖決定 X 第一期的消費  $c_1$ 。

(a)



(b)

