「Electromagnetism」Homework Sheet No. 6

Issued 22 Dec. 2021

1. Calculate the torque exerted on the square loop shown in Fig. 1, due to the circular loop (assume r is much larger than a and b).
If the square loop is free to rotate, what will its equilibrium orientation be?
(Textbook, p. 269, Problem 6.1).

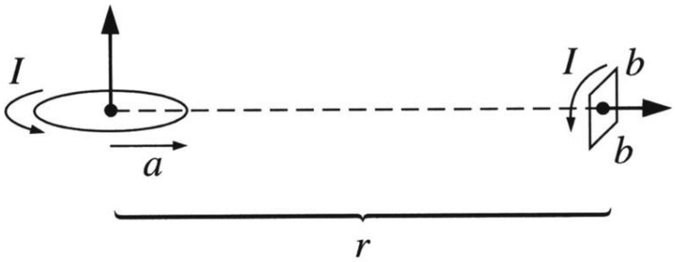


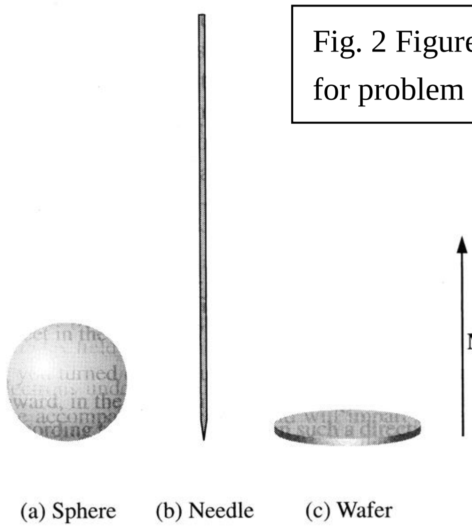
Fig. 1 Figure for problem 1.

2. A uniform current density $\mathbf{J} = J_0\hat{\mathbf{z}}$ fills a slab straddling the yz plane, from $x = -a$ and $x = +a$. A magnetic dipole $\mathbf{m} = m_0\hat{\mathbf{x}}$ is situated at the origin.
(a) Find the force on the dipole, using $\mathbf{F} = \nabla(\mathbf{m}\cdot\mathbf{B})$.
(b) Do the same for a dipole pointing in the y direction: $\mathbf{m} = m_0\hat{\mathbf{y}}$.
(c) In the *electrostatic* case, the expressions $\mathbf{F} = \nabla(\mathbf{p}\cdot\mathbf{E})$ and $\mathbf{F} = (\mathbf{p}\cdot\nabla)\mathbf{E}$ are equivalent (prove it), but this is *not* the case for the magnetic analogs (explain why). As an example, calculate $(\mathbf{m}\cdot\nabla)\mathbf{B}$ for the configurations in (a) and (b).
(Textbook, p. 270, Problem 6.5).

3. A long cylinder, of radius R, carries a "frozen-in" magnetization, parallel to the axis, $\mathbf{M} = k\rho\hat{\mathbf{z}}$, where k is a constant and ρ is the distance from the axis; there is no free current anywhere. Find the magnetic field inside and outside the cylinder by two different methods:
(a) As in Sect. 6.2, locate all the bound currents, and calculate the field they produce.
(b) Use Ampere's law ($\oint \mathbf{H}\cdot d\mathbf{l} = I_{f_{enc}}$) to find $\mathbf{H}$, and then get $\mathbf{B}$ from $\mathbf{H} = \mathbf{B}/\mu_0 - \mathbf{M}$.
(Textbook, p. 282, Problem 6.12).

4. Suppose the field inside a large piece of magnetic material is $\mathbf{B}_0$, so that $\mathbf{H}_0 = \mathbf{B}_0/\mu_0 - \mathbf{M}$, where $\mathbf{M}$ is a "frozen-in" magnetization.
(a) Now a small spherical cavity is hollowed out of the material (Fig. 2). Find the field at the center of the cavity, in terms of $\mathbf{B}_0$ and $\mathbf{M}$. Also find $\mathbf{H}$ at the center of the cavity, in terms of $\mathbf{H}_0$ and $\mathbf{M}$.
(b) Do the same for a long needle-shaped cavity running parallel to $\mathbf{M}$.

Fig. 2 Figure for problem 4.



(c) Do the same for a thin wafer-shaped cavity running parallel to **M**.
(Textbook, p.282, Problem 6.13)

5. A current I flows down a long straight wire of radius a. If the wire is made of linear material (copper, say, or aluminum) with susceptibility χ_m, and the current is distributed uniformly, what is the magnetic field a distance ρ from the axis? Find all the bound currents. What is the *net* bound current flowing down the wire?
(Textbook, p. 287, Problem 6.17).

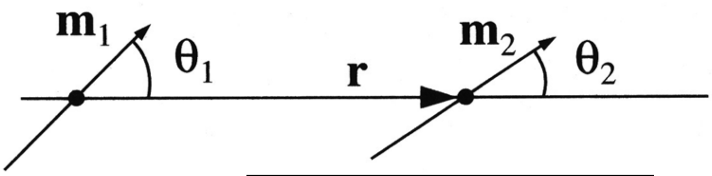


Fig. 3 Figure for problem 6.

6. (a) Show that the energy of a magnetic dipole in a magnetic field **B** is $U = -\mathbf{m}\cdot\mathbf{B}$. [Assume that the magnitude of the dipole moment is fixed, and all you have to do is move it into place and rotate it into its final orientation.] Compare $U = -\mathbf{p}\cdot\mathbf{E}$.

(b) Show that the interaction energy of two magnetic dipoles separated by a displacement **r** is given by $U = \frac{\mu_0}{4\pi r^3}[\mathbf{m}_1\cdot\mathbf{m}_2 - 3(\mathbf{m}_1\cdot\hat{\mathbf{r}})(\mathbf{m}_2\cdot\hat{\mathbf{r}})]$. Compare $U = \frac{1}{4\pi\varepsilon_0 r^3}[\mathbf{p}_1\cdot\mathbf{p}_2 - 3(\mathbf{p}_1\cdot\hat{\mathbf{r}})(\mathbf{p}_2\cdot\hat{\mathbf{r}})]$.

(c) Express your answer to (b) in terms of the angles θ_1 and θ_2 in Fig. 3, and use the result to find the stable configuration two dipoles would adopt if held a fixed distance apart, but left free to rotate.

(d) Suppose you had a large collection of compass needles, mounted on pins at regular intervals along a straight line. How would they point (assuming the earth's magnetic field can be neglected)?
(Textbook, p. 291, Problem 6.21).