

總分 40 分。答題皆須附說明, 未做解釋的答案概不計分。

1. A has \$100 and if he becomes sick, he needs to spend \$64 on medical treatment. The probability for A to fall ill is  $1/2$ . Let  $x$  denote A's wealth when he is ill, and  $y$  denote A's wealth when he is healthy. A's preference could be described by the following utility function:

$$u(x, y) = xy.$$

- (a) (2 points) Is A risk-neutral, risk-loving, or risk-averse? Why?
  - (b) (2 points) What is the insurance fee for a fair full insurance in A's case?
  - (c) (2 points) What is A's willingness to pay to purchase a full medical insurance?
2. Reconsider the previous insurance problem. Half of the population are like A and half of the population are like B. The situation of B is almost the same as the situation of A except for two things: (1) The probability for B to fall ill is  $1/4$ . (2) Let  $x$  denote B's wealth when he is ill, and  $y$  denote B's wealth when he is healthy. B's preference could be described by the following utility function:

$$u(x, y) = \frac{1}{4}\sqrt{x} + \frac{3}{4}\sqrt{y}.$$

The insurance industry is perfectly competitive, and all owners of insurance companies are risk neutral. There is no other cost to run an insurance company, so a company's profit is simply insurance fees collected minus its insurance payments.

Every one knows his probability to fall ill, but an insurance company could not tell if a customer is A-typed or B-typed.

- (a) (2 points) Is it possible that in equilibrium all companies offer a fair full insurance for a type A customer, and a fair full insurance for a type B customer?
  - (b) (2 points) Let  $\$F$  be a full insurance fee: after paying  $\$F$ , a customer will have his medical bill of \$64 completely covered by the insurance company when he is ill. Let  $\$F$  be such that if

the whole population purchase medical insurance, the expected profit of an insurance company is zero. Calculate  $F$  and determine who will purchase such a full insurance if it exists.

- (c) (3 points) Consider an equilibrium in which there are two kinds of insurance policies in the market: (1) a full insurance for a A-typed customer and (2) a partial insurance for a B-typed customer who pays an insurance fee of  $\$G$  and receives  $\$H$  ( $H < \$64$ ) from the insurance company if he falls ill. Given the market conditions, list two equations to solve for  $G$  and  $H$ . You don't need to solve them.
3. (2 points) In Qing Taiwan, a land owner could pawn his land for  $n$  years at a price of  $P_d$ . The pawnee who had paid  $P_d$  could collect annual rents  $R$  from the land owner's tenant  $n$  times starting one year from the pawning trade. At the end of the  $n$ -th year, after the pawnee collected the last rents, the pawner would redeem the land at the original price of  $P_d$ . Let  $i$  denote the annual interest rate. What is the equilibrium pawning price  $P_d$ ? Please express it in terms of  $R$ ,  $i$  and  $n$ .
4. (2 points) Which interest rate in the following two scenarios is higher?  
 (a) A year (1st period) of bad weather resulting in low agricultural productivity. (b) Nuclear contamination permanently lowers agricultural productivity. The damage each year is the same as in (a). Use a model of a representative agent to analyze this problem. Consider only two periods, and assume that both current consumption and future consumption are normal goods.
5. There are two aquariums side by side in a city. In the weekend, a person could choose either to visit one aquarium or to picnic. Every one enjoys a visit to an aquarium in the same manner. Let  $AV_i^a$  denote the value of a visit to aquarium  $i$ , and let  $n_i$  denote the number of visitors in aquarium  $i$ .

$$AV_i^a(n_i) = 120 - n_i.$$

On the other hand, people value picnics differently. If we sort their value from low to high, the  $n$ th person's picnic value is:

$$MV^p(n) = n.$$

We shall treat  $n$  as a continuous variable.

- (a) (2 points) If both aquariums are free, in equilibrium how many people will visit aquariums?
  - (b) (2 points) Who should be arranged to visit an aquarium if we wish to maximize total recreational value?
  - (c) (2 points) If we wish to achieve the goal in part (b) by selling tickets to aquarium visitors, what is the ticket price?
  - (d) (2 points) Let  $p_i$  be the ticket price at aquarium  $p_i$ ,  $i = 1, 2$ . Calculate  $n_1$  as a function of  $p_i$ ,  $i = 1, 2$ .
  - (e) (1 point) If we wish to maximize total ticket revenue instead, what will be the ticket prices at two aquariums?
6. Consider a perfectly competitive industry with many identical firms, each producing according to the production function:

$$q = \sqrt{l},$$

where  $l$  stands for labor. Labor is supplied to the industry according to the supply curve:

$$L = 100\sqrt{w},$$

where  $w$  is the wage rate of labor, and  $L$  is quantity of labor supplied. The total cost to a firm is:

$$wl + \$16,$$

where \$16 is an annual license fee to stay in the industry. The annual demand curve for the industry is:

$$Q = 116 - p,$$

where  $Q$  is the industry's total output and  $p$  is the price.

- (a) (1 point) Using symbols above to express a firm's marginal revenue product of labor.
- (b) (2 points) For a firm that already pays the license fee, please derive its demand for labor  $l(w)$ .

- (c) (4 points) Please calculate the product price  $p$  and the wage rate  $w$  in a long-run equilibrium..
7. There are two stocks in the financial market:  $x$  and  $y$ . The expected rate of returns is 12% from stock  $x$  and 8% from stock  $y$ . The standard deviation of the rate of returns is 4% for  $x$  and 3% for  $y$ . The correlation coefficient between two rates of returns is 0. Investors could lend or borrow at a risk-free rate of 1%.
- (a) (2 points) Consider a person who invests the same amount of money in  $x$  and  $y$ . Please calculate the mean and the standard deviation of his rate of return.
- (b) (1 point) Consider a person invests  $\$p$  in  $x$  and  $\$1 - p$  in  $y$ . Find the  $p$  to achieve the smallest standard deviation of the rate of returns.
- (c) (2 points) According to CAPM, which stock has a higher total market value?
8. (2 points) According to group 7's report, why is an e-book no longer cheap and at Amazon, the price of a Kindle version is sometimes about the same as the price of a paperback?

## 解答

**1a** Risk-averse because he has a convex preference.

**1b** The expected medical expense \$32.

**1c** Without the insurance, A's utility is  $100 \cdot 36 = 3600$ . After purchasing the full insurance, A's wealth is a constant  $z$  and his utility is  $z^2$ . Purchasing insurance is indifferent to A if  $z = 60$ , or if the insurance company asks for \$40.

**2a** All type A customers will purchase insurance for a type B customer. Companies have negative expected profit.

**2b**  $F = (64/2 + 64/4)/2 = (32 + 16)/2 = 24$  (1 point)

Type A will purchase insurance.

If a type B customer purchases insurance, his utility is:  $\sqrt{76}$ .

Without this insurance, his utility is:  $\sqrt{36}/4 + 3 \cdot \sqrt{100}/4 = 9 > \sqrt{76}$ . So a type B customer will not purchase this insurance. (1 point)

**2c** The market is competitive, so insurance will be fair, and

$$G = H/4. (1 \text{ point})$$

To sustain the separating equilibrium, An A-typed customer will be indifferent between 2 insurances. If he purchases the A-typed full insurance, he pays a fee of \$32, and has the utility:  $u = 68^2$ .

If he purchases the B-typed partial insurance, his utility is:

$$(36 + 3H/4)(100 - H/4) = 68^2.$$

**3**

$$P_d = \frac{R}{1+i} + \frac{R}{(1+i)^2} + \dots + \frac{R}{(1+i)^n} + \frac{P_d}{(1+i)^n}, 1 \text{ point}$$

$$P_d = R/i$$

**4** If we put the current consumption on the horizontal axis and the future consumption on the vertical axis,  $|MRS|$  at the agent's endowment is  $1+i$ .

From (a) to (b), the endowment point shifts down and  $|MRS|$  at the new endowment point is smaller, i.e.  $i$  decreases. So interest rate is higher in (a).

**5a** In equilibrium,  $AV_1^a = AV_2^a$ , so  $n_1 = n_2$ . (1 point)

$MV^p(2n_1) = 120 - n_1$ ,  $n_1 = 40$ , 80 people visit the aquarium.

**5b** It's clear that the number of visitors to two aquariums should be the same.

$MV_1^a(n_1) = 120 - 2n_1 = MV^p(2n_1) = 2n_1$  (1 point)

$n_1 = n_2 = 30$ .

60 people with the lowest picnic value should visit aquariums.

**5c**  $AV_1^a(30) - p = 90 - p = MV^p(60) = 60$ ,  $p = 30$ .

**5d** Due to consumers' choices,  $n_1 - n_2 = p_2 - p_1$ .

$n_1 + n_2 = 120 - n_1 - p_1$ .

From 2 equations above,

$$\begin{aligned} n_1 &= \frac{120 - 2p_1 + p_2}{3} \\ n_2 &= \frac{120 - 2p_2 + p_1}{3} \end{aligned}$$

**5e** To max  $n_1p_1 + n_2p_2$ ,  $p_1 = p_2 = 60$ .

**6a**  $p/2\sqrt{l}$

**6b**  $l = (p/2w)^2$

**6c**  $TC(q) = wq^2 + 16$

$AC(q) = wq + 16/q$ ,  $AC' = w - 16/q^2$

min of  $AC(q)$  occurs at  $q = \sqrt{16/w} = 4/\sqrt{w}$  and  $p = \min AC(q) = 8\sqrt{w}$  (2 points)

$Q = 116 - 8\sqrt{w}$

number of the firms  $n = Q/q = (116 - 8\sqrt{w})/(4/\sqrt{w}) = \sqrt{w}(29 - 2\sqrt{w})$

total labor used  $L = n * q^2 = \sqrt{w}(29 - 2\sqrt{w}) * 16/w = 16 * (29 - 2\sqrt{w})/\sqrt{w} = 100\sqrt{w}$ .

$w = 4$ ,  $p = 16$ .

**7a**  $\mu = 10\%$ ,  $\sigma = 2.5\%$

**7b**  $\sigma = \sqrt{p^2(4\%)^2 + (1-p)^2(3\%)^2}$

$d\sigma/dp = [(4\%)^2(2p) - 2(1-p)(3\%)^2]/2\sqrt{p^2(4\%)^2 + (1-p)^2(3\%)^2} = 0$   $p = 0.36$

**7c** We have to find if the line connecting (0,1%) and (2.5%, 10%) is tangent to the efficient frontier. First note this line has a slope of  $9/2.5=3.6$ . (1 point)

Consider to invest  $\$p$  in  $x$  and  $\$1 - p$  in  $y$ .

$$\mu = 12\%p + 8\%(1 - p)$$

$$d\mu/dp = 4\%$$

At  $p = 1/2$ ,  $d\mu/d\sigma = 4\%/1.4\% \simeq 2.9 < 3.6$ . So the line tangent to the efficient frontier should turn counter-clockwise. In the market portfolio,  $p < 0.5$ .  $y$  has a larger market value.

**8** At the beginning, when Amazon monopolized the retail market, it set cheap prices for e-book to promote e-books and kindles (wholesale model).

When Apple entered the market, it signed contracts of agency model with publishers. Publishers would decide prices of e-books which couldn't be too low that no readers would purchase paperbacks. More important, Apple asked publishers to sign contracts of agency model with other retailers. It was a blow to Amazon's whole model.

Group 7 also claims that publishers are collusive to set high prices.