「Electromagnetism」Homework Sheet No. 4

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1. A dipole **p** is situated a distance *z* above an infinite grounded conducting plane (Fig. 1). The dipole makes an angle θ with the perpendicular to the plane. Find the torque on **p**. If the dipole is free to rotate, in what orientation will it come to rest?
(Textbook, p. 172, Problem 4.6).

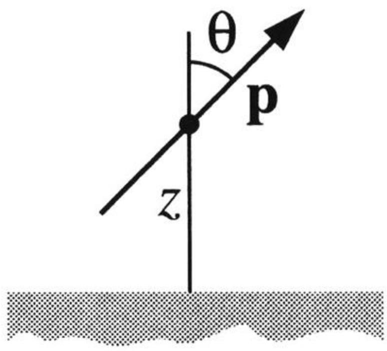

Fig. 1 Figure for problem 1.

2. A sphere of radius *R* carries a polarization $\mathbf{P}(\mathbf{r}) = k\,\mathbf{r}$, where *k* is a constant and **r** is the vector from the center.
(a) Calculate the bound charges σ_b and ρ_b .
(b) Find the field inside and outside the sphere.
(Textbook, p. 176, Problem 4.10).

3. A very long cylinder of radius *a*, carries a uniform polarization **P** perpendicular to its axis. Find the electric field inside the cylinder. Show that the field outside the cylinder can be expressed in the form

$$\mathbf{E}(\mathbf{r}) = \frac{a^2}{2\varepsilon_0 \rho^2}\left[2(\mathbf{P}\cdot\hat{\boldsymbol{\rho}})\hat{\boldsymbol{\rho}} - \mathbf{P}\right].$$

(Textbook, p.179, Problem 4.13)

4. Suppose the field inside a large piece of dielectric is $\mathbf{E}_0$, so that the electric displacement is $\mathbf{D}_0 = \varepsilon_0 \mathbf{E}_0 + \mathbf{P}$.
(a) Now a small spherical cavity (Fig. 2a) is hollowed out of the material. Find the field at the center of the cavity in terms of $\mathbf{E}_0$ and **P** . Also find the displacement at the center of the cavity in terms of $\mathbf{D}_0$ and **P** . Assume the polarization is "frozen in", so it doesn't change when the cavity is excavated. (b) Do the same for a long needle-shaped cavity running parallel to **P** (Fig. 2b). (c) Do the same for a thin wafer-shaped cavity running perpendicular to **P** (Fig. 2c). Assume the cavities are small enough that **P**, $\mathbf{E}_0$ and $\mathbf{D}_0$ are essentially uniform. [Hint: Carving out a cavity is the same as superimposing an object of the same shape but opposite polarization.]
(Textbook, p. 184, Problem 4.16).

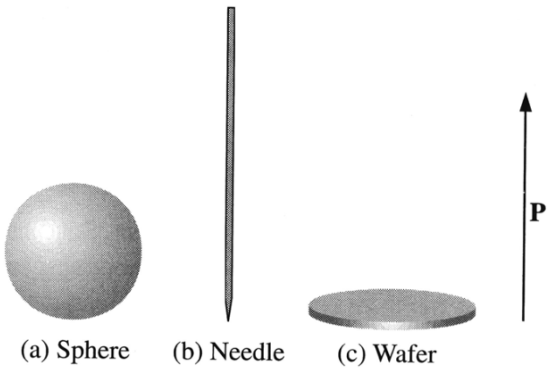

Fig. 2 Figure for problem 4.

5. A certain coaxial cable consists of a copper wire, radius *a*, surrounded by a concentric copper tube of inner radius *c* (Fig. 3). The space between is partially filled (from *b* out to *c*) with material of dielectric constant ε_r. Find the capacitance per unit length of this cable.
(Textbook, p. 192, Problem 4.21).

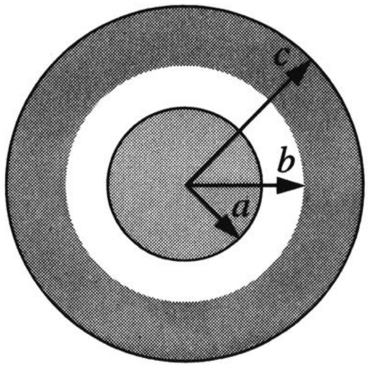

Fig. 3 Figure for problem 5.

6. A point dipole **p** is imbedded at the center of a sphere of linear dielectric material (with radius *R* and dielectric constant ε_r). Find the electric potential inside and outside the sphere.

$$\left[\text{Answer: } \frac{p\cos\theta}{4\pi\varepsilon r^2}\left(1+2\frac{r^3}{R^3}\frac{(\varepsilon_r-1)}{(\varepsilon_r+2)}\right),\ (r\le R);\ \frac{p\cos\theta}{4\pi\varepsilon_0 r^2}\left(\frac{3}{\varepsilon_r+2}\right),\ (r\ge R)\right]$$

(Textbook, p. 207, Problem 4.37).

7. Calculate *W*, using both $W=\frac{\varepsilon_0}{2}\int E^2 d\tau$ and $W=\frac{1}{2}\int \mathbf{D}\cdot\mathbf{E}\,d\tau$, for a sphere of radius *R* with frozen-in uniform polarization $\mathbf{P}$.
Comment on the discrepancy. Which (if either) is the “true” energy of the system?
(Textbook, p. 202, Problem 4.27).

8. The space between the plates of a parallel-plate capacitor is filled with dielectric material whose dielectric constant varies linearly from 1 at the bottom plate ($x = 0$) to 2 at the top plate ($x = d$). The capacitor is connected to a battery of voltage V. Find all the bound charge, and check that the total is zero.
(Textbook, p. 206, Problem 4.34).