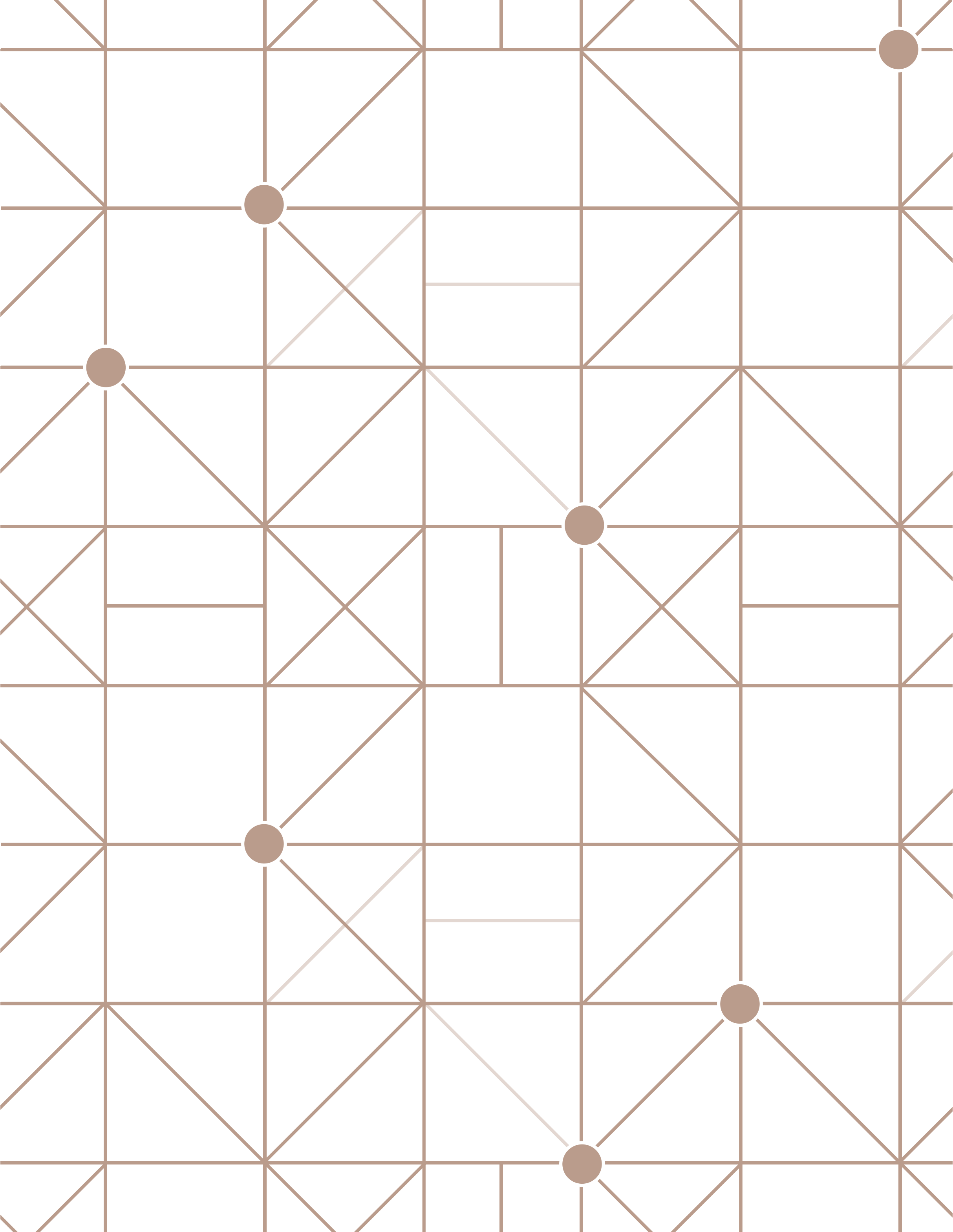


Ch. 2.3

$\begin{cases} \text{Linear independence} \\ \text{Spanning a Space} \\ \text{Basis \& Dimension} \end{cases}$ (LD/LI)

[A] Linear independence:

1) Suppose $\boxed{c_1 \vec{v}_1 + \cdots + c_k \vec{v}_k = 0}$ only happens when $c_1 = \cdots = c_k = 0$.

Then $\vec{v}_1 \cdots \vec{v}_k$ are LI.

If any c's are nonzero, then $\vec{v}$'s are LD.

2) $A = \begin{bmatrix} 2 & 1 & 2.5 \\ 1 & 2 & -1 \end{bmatrix}$

$\boxed{AC = 0}$ $\begin{array}{c} \begin{array}{ccc} v_1 & v_2 & v_3 \end{array} \\ \begin{bmatrix} \boxed{\begin{matrix} 2 \\ 1 \end{matrix}} & \boxed{\begin{matrix} 1 \\ 2 \end{matrix}} & \boxed{\begin{matrix} 2.5 \\ -1 \end{matrix}} \end{bmatrix} \end{array} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$

- v_1, v_2, v_3 are columns of A.
- They are $\boxed{\text{indep. columns}}$ if the nullspace of A is

$$\boxed{\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.}$$

- They are $\boxed{\text{dep. columns}}$ if $AC = 0$ for some nonzero C.

3) $A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 5 \\ 0 & 0 & 2 \end{bmatrix}$

Solve $\boxed{AC=0}$

$$c_1 \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix} + c_2 \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 5 \\ 2 \end{bmatrix} = 0$$

$$c_1 = c_2 = c_3$$

The columns of A are indep.

$\Rightarrow$ exactly when $\boxed{N(A) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}$.

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}.$$

[B] Span:

1) Vectors V_1, V_2, V_3 span a space

$\Rightarrow$ The space contains of all combinations of those vectors.

$\Rightarrow$ Column Space.

[C] Basis:

1) Basis for a space is a sequence of vectors $V_1 \cdots V_d$ with 2 properties:

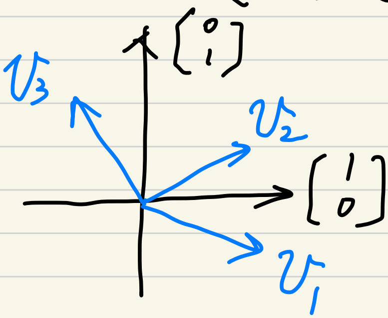
① $V_1 \cdots V_d$ are indep.

② $V_1 \cdots V_d$ span the space.

2) Space is $\mathbb{R}^2$.

One basis is $\begin{bmatrix}1\\0\end{bmatrix} \begin{bmatrix}0\\1\end{bmatrix}$.

But,



Any 2 of these vectors, they span and they are indep.

[D] Dimension :

1) Dimension of a Vector space is the "degree of freedom" of the space.

2) The dim. of R^{n} is n.

3) Every basis for the space has the same # of vectors.

4) If $\boxed{v_1 \cdots v_m}$ and $\boxed{w_1 \cdots w_n}$ are both bases for the same vector space, then $m = n$.

[E]

① The $\boxed{\text{rank}}$ of A

= # of pivot columns

= dim. of the Column Space

② $A_{m \times n} = \begin{bmatrix} 1 & 2 & 3 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \end{bmatrix}_{3 \times 4}$

pivot columns

$\therefore$ rank of $A = 2$

$\therefore$ dim. of $C(A) = 2$

$\therefore$ dim. of $N(A)$ = # of free variables

= $(n-r)$ free columns

dim. of $N(A) = 4 - 2 = 2$ #