「Electromagnetism」Homework Sheet No. 5

Issued 8 Dec. 2021

1. For a configuration of charges and currents confined within a volume V, show that

$$\int_V \mathbf{J} d\tau = \frac{d\mathbf{p}}{dt}$$ where $\mathbf{p}$ is the total dipole moment. [Hint: evaluate $\int_V \nabla \cdot (x\mathbf{J}) d\tau$.]

(Textbook, p. 223, Problem 5.7).

2. (a) Find the magnetic field at the center of a square loop, which carries a steady current I. Let R be the distance from the center to any side (Fig. 1).

(b) Find the field at the center of a regular n-sided polygon, carrying a steady current I. Again, let R be the distance from the center to any side.

(c) Check that your formula reduces to the field at the center of a circular loop, in the limit $n \to \infty$.

(Textbook, p. 228, Problem 5.8).

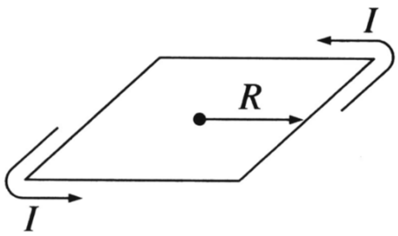

Fig. 1 Figure for problem 2.

3. Find the magnetic field at point P on the axis of a tightly wound solenoid consisting of n turns per unit length wrapped around a cylindrical tube of radius a and carrying current I (Fig. 2). Express your answer in terms of θ_1 and θ_2. Consider the turns to be essentially circular. What is the field on the axis of an infinite solenoid?

(Textbook, p.229, Problem 5.11)

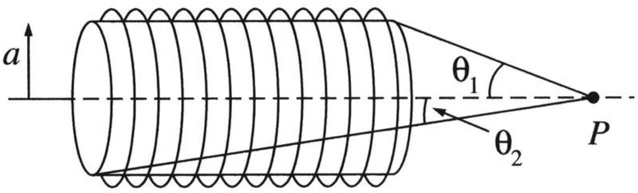

Fig. 2 Figure for problem 3.

4. A steady current I flows down a long cylindrical wire of radius a (Fig. 3). Find the magnetic field, both inside and outside the wire, if

(a) The current is uniformly distributed over the outside surface of the wire.

(b) The current is distributed in such a way that J is proportional to ρ, the distance from the axis.

(Textbook, p. 239, Problem 5.14).

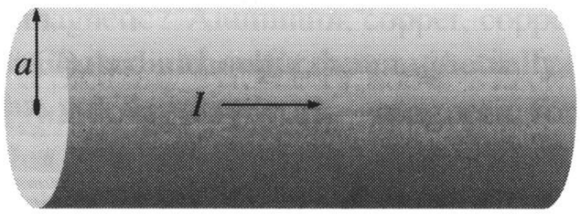

Fig. 3 Figure for problem 4.

5. A large parallel-plane capacitor with uniform surface charge σ on the upper plate and $-\sigma$ on the lower is moving with a constant speed v (see Fig. 4).

(a) Find the magnetic field between the plates and also above and below them.
(b) Find the magnetic force per unit area on the upper plate, including its direction.
(c) At what speed v would the magnetic force balance the electrical force?
(Textbook, p. 240, Problem 5.17).

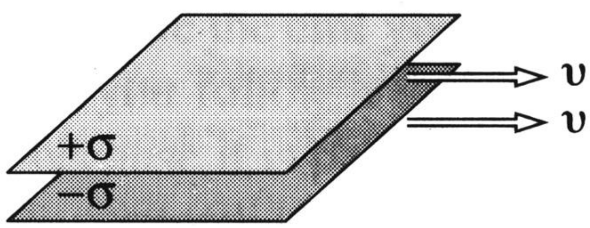

Fig. 4 Figure for problem 5.

6. (a) By whatever means you can think of (short of looking it up), find the vector potential a distance ρ from an infinite straight wire carrying a current I. Check that ∇·A = 0 and $\nabla\times\mathbf{A} = \mathbf{B}$.
(b) Find the magnetic potential inside the wire, if it has radius R and the current is uniformly distributed. (Textbook, p. 248, Problem 5.26).

7. The multipole expansion for the vector potential of a line current was worked out in the class because that is the most common type, and in some respects the easiest to handle. For a *volume* current **J**:
(a) Write down the multipole expansion, analogous to

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi}\sum_{n=0}^{\infty}\frac{1}{r^{n+1}}\oint (r')^n P_n(\cos\alpha)\,d\mathbf{l}'.$$

(b) Write down the monopole potential, and prove that it vanishes.
(c) Using $\int_S d\mathbf{a} = \frac{1}{2}\oint \mathbf{r}\times d\mathbf{l}$ and $\mathbf{m} = I\int_S d\mathbf{a}$, show that the dipole moment can be written

$\mathbf{m} = \frac{1}{2}\int (\mathbf{r}\times\mathbf{J})\,d\tau.$ (Textbook, p. 256, Problem 5.38).

8. A thin uniform donut, carrying charge Q and M, rotates about its axis [Fig. 5].
(a) Find the ratio of its magnetic dipole moment to its angular momentum. This is called the gyromagnetic ratio (or magnetomechanical ratio).
(b) What is the gyromagnetic ratio for a uniform spinning sphere?
(c) According to quantum mechanics, the angular momentum of a spinning electron is $(1/2)\hbar$, where $\hbar$ is Planck's constant. What, then, is the electron's magnetic dipole moment, in A(·m^2? (Textbook, p. 263, Problem 5.58).

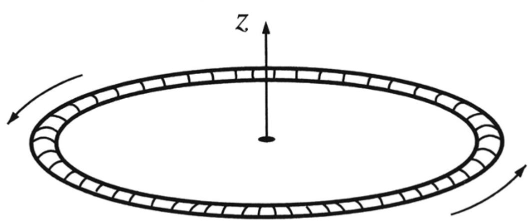

Fig. 5 Figure for problem 8.