13.2 Geometrical Optics

13.2.1 Reflection

Rule of Reflection

The continuity condition at the interface where reflection occurs implies the rule of reflection: Incident angle = reflection angle.

The same continuity condition also leads to the rule of refraction (Snell's law) as we have seen above.

The continuity condition also implies that the frequencies of the incident wave, the reflected wave and the transmitted wave must be the same.

Q 13.8: In the reflection of a mirror, why is left turned into right, but up is not turned into down?

Q 13.9: How do we know how far something (including a virtual image) is?

Focus

For a (small) spherical mirror of radius R, we have the focus

$$f = \frac{R}{2}, \tag{13.5}$$

and

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{f}, \tag{13.6}$$

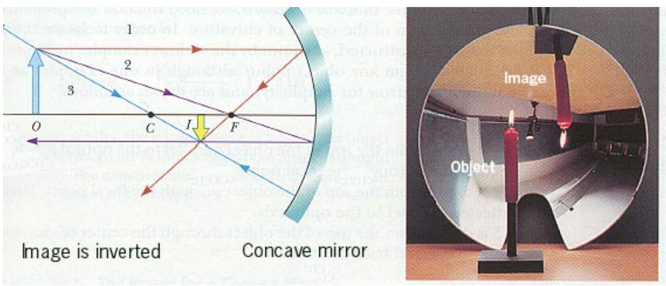
where p and q are given by the locations of the object and the image.

The focus is defined as the point where all light from infinity at normal incidence converge.

One can derive this equation from simple geometric relations for the concave mirrors and convex mirrors separately. (See the textbook pp.723 – 725 for details.)

Ideal mirrors with a well defined focus should be parabolic. For a sufficiently small surface, the difference between the spherical surface and the parabolic surface is small, and the focus is approximately a fixed point. The formula above (13.5) is defined for an infinitesimal spherical mirror.

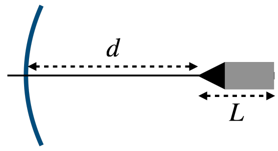
Concave Mirror



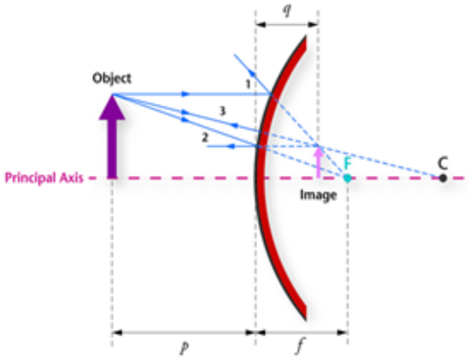
Q 13.10: Why does the image in the spoon look like it is behind the spoon? How to verify that it is actually in front of the spoon?

Q 13.11: How to design the statues in the "Haunted Mansion" so that the heads are always facing the riders?

HW: (4-4) A pencil of length L is placed horizontally at a distance d from a concave mirror of focus f. What is the length of the pencil's image?



Convex Mirror



13.2.2 Refraction

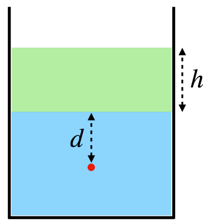
Snell's Law

The Snell's law is imposed by hand in geometric optics.

$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \tag{13.7}$$

Q 13.12: How does a spoon look when half of it is immersed in water?

HW: (4-5) Two types of liquid are separated by an interface at a distance h from the surface between air and the liquid on the top. Looking down from above, what is the apparent depth of a point at a distance d below the interface between the two types of liquid?



Q 13.13: What happens when there is no solution of θ_2 for given n_1, n_2, and θ_1?

total internal reflection

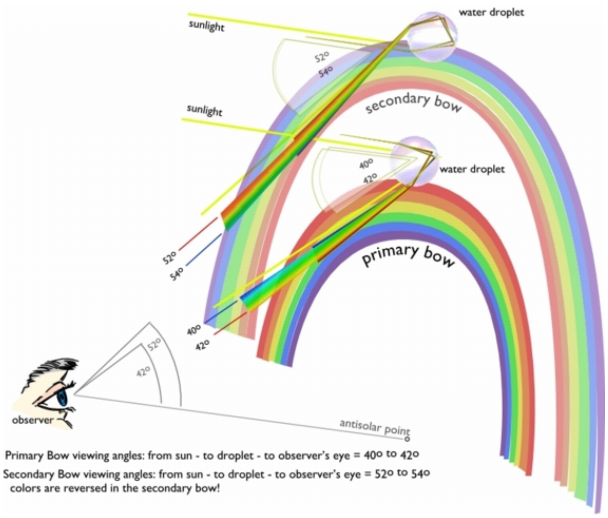
Rainbow

Q 13.14: Why is the sky blue?
Q 13.15: Why is the sunset red?
Q 13.16: Why is there sometimes a "green flash" at sunset or sunrise?
Q 13.17: How do we perceive color?
Q 13.18: Are there 7 colors in the rainbow?

> On a monitor/screen, the colors "add" up. On the paper, the colors are "subtracted".
> There are different models of color to describe the different colors we perceive: RGB/RYB/CMYK (cyan, magenta, yellow, key). They have different definitions of the primary colors. In addition to colors, we also perceive hue, tints, shades and tones as related aspects of color.

The rainbow is an effect due to the refraction of light in suspended droplets in the air. There is one internal reflection and two refractions in one droplet for the primary bow. There is also a secondary bow, corresponding to two internal reflections in a droplet.

Taiwan shatters world record for longest-lasting rainbow: nearly 9 hours on Nov 30, 2017.



Q 13.19: Why is the outermost ring red for the primary bow?
Q 13.20: Why is innermost ring red for the secondary bow?
Q 13.21: Why is the background brighter "inside" the rainbow?

Lenses

In the derivation below, we use the approximation $\tan\theta \simeq \theta \simeq \sin\theta$.

At a spherical interface of radius r, when the center of the sphere is on the side of the transmitted wave, the indices of refractions n_i, n_t and the distances of the object and the image from the interface d_o, d_i satisfy the relation

$$\frac{n_i}{d_o} + \frac{n_t}{d_i} = \frac{n_t - n_i}{r}. \tag{13.8}$$

Q: What does a negative radius mean in eq.(13.8)?

Here we assume that the incident light only hits on a small area on the spherical interface around the center. (See Benson Sec.36.6 on p.753 for the derivation of eq.(13.8).)

Ex 13.2: Derive the thin lens formula

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \tag{13.9}$$

from eq.(13.8).

Solution:
Think of the thin lens as a composition of two spherical interfaces. For the 1st interface, we have

$$\frac{1}{d_o} + \frac{n}{d_1} = \frac{n-1}{r_1}, \tag{13.10}$$

and for the 2nd interface,

$$\frac{n}{-d_1} + \frac{1}{d_i} = \frac{1-n}{-r_2}. \tag{13.11}$$

Q: What does a negative d_o mean in eq.(13.8)?

Combining these two equations, we find

$$\frac{1}{d_o} + \frac{1}{d_i} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right), \tag{13.12}$$

from which we deduce that the focus f is related to the radii r_1, r_2 via

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2}\right). \tag{13.13}$$

Incidentally, for thick lens, a better formula for the focus is

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{(n-1)d}{nr_1r_2}\right). \tag{13.14}$$

Ex 13.3: Prove that the focus is the same regardless of which side of a lens is facing the incident wave.

Q: How do you deal with a sequence of lenses?