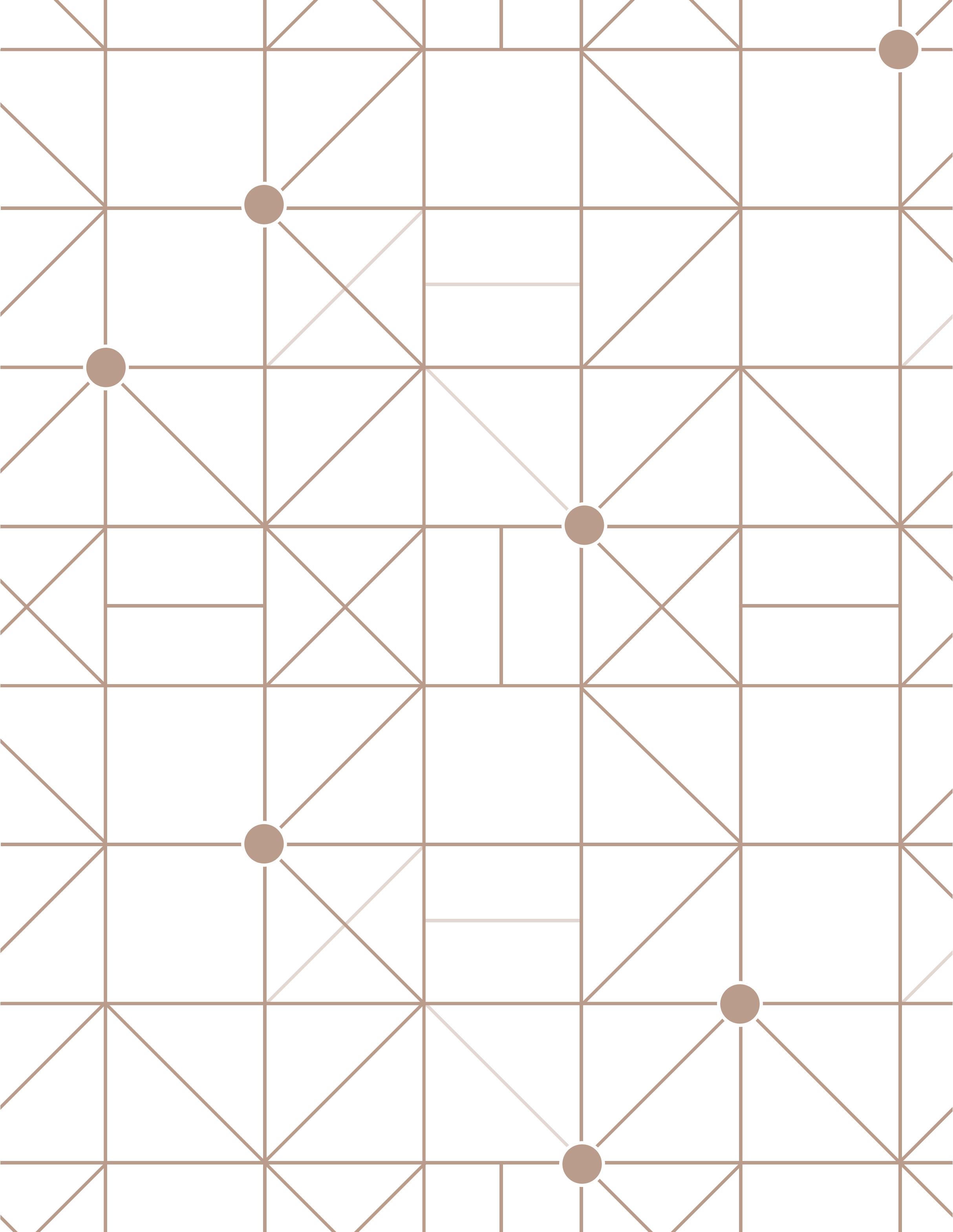
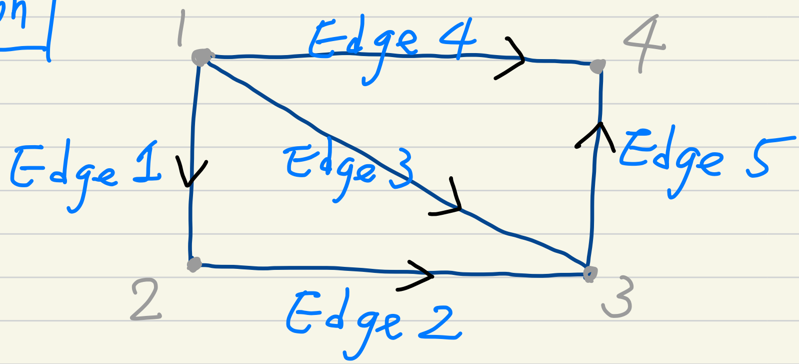


Ch. 2.5

Graphs and Networks
Incidence Matrices
Kirchhoff's Edges

Graph



$n = 4$ nodes (columns)
$m = 5$ edges (rows)

Incidence Matrix

$$
\begin{array}{c|c}
A = \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} & \begin{array}{c} \text{Edge} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{array} \\
\hline
\text{Node} \quad 1 \quad\quad 2 \quad\quad 3 \quad\quad 4 &
\end{array}
$$

另) $\begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ pivots

$\therefore$ rank $\boxed{r=3}$

[A] Nullspace $Ax=0$

$$\begin{bmatrix} & & \\ & A & \\ & & \end{bmatrix}_{5\times4} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}_{4\times1} = \begin{bmatrix} x_2-x_1 \\ x_3-x_2 \\ x_3-x_1 \\ x_4-x_1 \\ x_4-x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$X = x_1, x_2, x_3, x_4$

(X: $\boxed{\text{potentials at nodes}}$)

$\boxed{(x_2-x_1)}$, etc. : $\boxed{\text{potential difference}}$

$$\therefore X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = C \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$$

[physical meaning]:

X: potentials

$$\dim. N(A) = 1$$

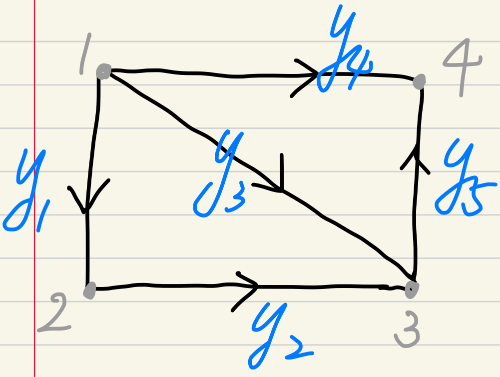
[B] Left Nullspace : $N(A^T)$

pivot column

$$A^T_{\substack{n\times m \\ 4\times 5}} = \begin{bmatrix} -1 & 0 & -1 & -1 & 0 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{A^T y = 0}$$

$$\begin{bmatrix} A^T \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$



$$\begin{cases} -y_1 - y_3 - y_4 = 0 \\ y_1 - y_2 = 0 \\ y_2 + y_3 - y_5 = 0 \\ y_4 + y_5 = 0 \end{cases}$$

$\boxed{\text{net current flow} = 0}$

$(x_2 - x_1)$ etc. $\longrightarrow$ y_1, y_2, y_3, y_4, y_5

(potential difference) (currents on edges)

$\boxed{A^T y = 0}$ Kirchhoff's Current Law

1) Basis for $N(A^T)$

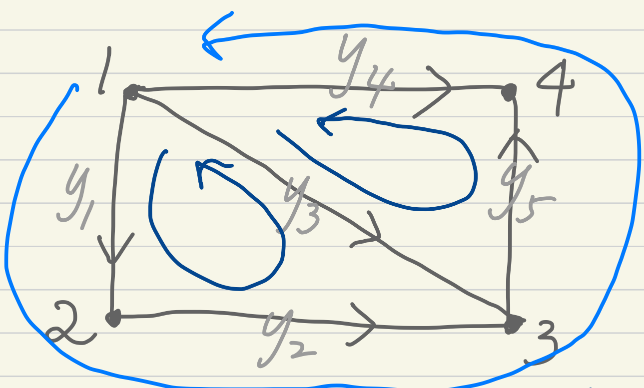
$\because$ rank of $A^T = 3$

$\therefore$ dim. of $N(A^T) = m - r$

$= 5 - 3 = 2$

$\therefore$ Basis for $N(A^T)$

$$\begin{array}{c} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \end{array} \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \\ 0 \end{bmatrix} \quad \begin{bmatrix} 0 \\ 0 \\ 1 \\ -1 \\ 1 \end{bmatrix} \xRightarrow{\text{combinations}} \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \\ 1 \end{bmatrix}$$



indep. dep.

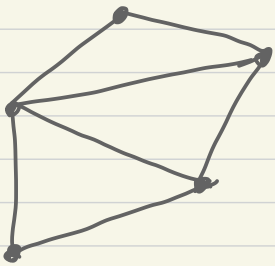
$$\dim N(A^T) = m - r$$

$$\boxed{\#\text{ loops} = \#\text{ edges} - (\#\text{ nodes} - 1)}$$

$(\text{rank} = n-1)$

$$\boxed{\underbrace{(\#\text{ nodes})}_{0\text{-dim.}} - \underbrace{(\#\text{ edges})}_{1\text{-dim.}} + \underbrace{(\#\text{ loops})}_{2\text{-dim.}} = 1}$$

Euler's Formula



\# nodes : 5

\# edges : 7

\# loops : 3

Euler's Formula: $5 - 7 + 3 = 1$