「Electromagnetism」Homework Sheet No. 3

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1. Find the general solution to Laplace's equation in spherical coordinates, for the case where V depends only on r. Do the same for cylindrical coordinates, assuming V depends only on s.
(Textbook, p. 119, Problem 3.3).

2. Prove that the field is uniquely determined when the charge density ρ is given, and either V or the normal derivative $\partial V/\partial n$ is specified on each boundary surface. Do not assume the boundaries are conductors, or that V is constant over any given surface.
(Textbook, p. 124, Problem 3.5).

3. A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)
(a) Find the potential in the region above the plane.
(b) Find the charge density σ induced on the conducting plane.
(Textbook, p. 130, Problem 3.10).

4. Two semi-infinite grounded conducting planes meet at right angles. In the region between them, there is a point charge q, situated as shown in Fig. 1. Set up the image configuration, and calculate the potential in this region. What charges do you need, and where should they be located? What is the force on q? How much work did it take to bring q in from infinity? Suppose the planes met at some angle other than 90°; would you still be able to solve the problem by the method of image? If not, for what particular angles does the method work?
(Textbook, p.130, Problem 3.11)

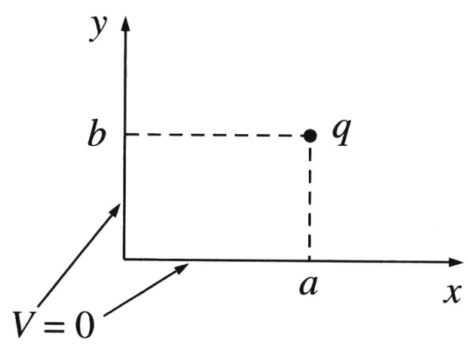

Fig. 1 Figure for problem 4.

5. A rectangular pipe, running parallel to the z-axis (from $-\infty$ to $+\infty$), has three grounded metal sides at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at specified potential $V_0(y)$.
(a) Develop a general formula for the potential inside the pipe.

(b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).
(Textbook, p. 140, Problem 3.15).

6. The potential at the surface of a sphere (radius R) is given by $V_0 = k \cos 3\theta$, where k is a constant. Find the potential inside and outside the sphere, as well as the surface charge density $\sigma(\theta)$ on the sphere. (Assume there is no charge inside or outside the sphere.)
(Textbook, p. 149, Problem 3.19).

7. A sphere of radius R, centered at the origin, carries charge density $\rho(r,\theta) = k\dfrac{R}{r^2}(R-2r)\sin\theta$, where k is a constant, and r, θ are the usual spherical coordinates. Find the approximate potential for points on the z axis, far from the sphere.
(Textbook, p. 154, Problem 3.27).

8. A stationary electric dipole $\mathbf{p} = p\hat{z}$ is situated at the origin. A positive point charge q (mass m) executes circular motion (radius ρ) at constant speed in the field of the dipole. Characterize the plane of the orbit. Find the speed, angular momentum and total energy of the charge. $\left[\text{Answer}: L = \sqrt{qpm/3\sqrt{3}\pi\varepsilon_0}\right]$
(Textbook, p. 166, Problem 3.57).