Chapter 14

Special Relativity

Special Relativity is concerned with the description of physical laws in all inertial frames. (General reference frames will be considered in General Relativity.)

Physical laws can be expressed in general frames regardless of whether they are non-relativistic or relativistic. But they will look different in a general reference frame.

14.1 Historical Note

According to Maxwell's equations, the speed of light in vacuum is

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}. \tag{14.1}$$

The natural interpretation of this result at Maxwell's time is the same as how we interpret the speed of sound. That is, it is the speed of light with respect to the medium in which electromagnetic waves propagate. (Recall that sound waves are fluctuations of the material in which they propagates. It is natural to assume that EM waves are similar.) Although the medium of EM waves had never been observed in any way other than its fluctuations (as the EM waves), the medium was postulated and called *aether or ether*.

The mechanical properties of ether was not known.

Both *aether* and *ether* have other meanings.

However, the experiments of Bradley and the experiments of Michelson and Morley showed that the speed of light is independent of the speed of ether, if ether exists. (Assuming that EM waves are oscillations of ether, Bradley's experiment showed that ether does not move along with the earth, and the Michelson-Morley experiment showed that the motion of ether does not change the speed of light. That is, the speed of light is the same in all inertial frames.)

In contrast, the flow of air changes the speed of sound.

Some physicists proposed that, actually the motion of ether does change the speed of light, just like the motion of air changes the speed of sound, but the motion of ether also changes how atoms interact with each other in materials, while their interactions are also mostly electromagentic. As a result, it is possible that the length of the ruler changes in motion and our measurement of distance is wrong, such that we mistakenly think that the speed of light is not changed. Similarly, it is possible that the period of the physical process in a clock changes when the clock is in motion, so that the reading on the clock is not the actual time. While such speculations are not totally unreasonable, we find Einstein's explanation more elegant and concise, although more revolutionary at the same time.

Einstein proposed that, instead of imagining that there is an absolute reference frame and the actual length (time) of something is not correctly measured by a ruler (clock), we can try simply defining physical quantities by how they are measured, and build physical theories on top of that. (We should also avoid imagining things we cannot measure, or making assumptions without evidence.)

> Einstein and other physicists all thought that the rulers and clocks probably work differently in motion. The difference between Einstein and others is whether the readings of certain rulers and clocks should be considered special (as the "actual" or "real" length or time).
>
> If there is an absolute frame, we can define the length and time measured by rulers and clocks at rest with respect to the absolute frame as the "actual" length and time. If there is no absolute frame, we have to accept measurements in all inertial frames on equal footing, and it should be possible to develop equally good physical laws directly in terms of measurements in any inertial frame. (See Sec.14.9 on *Covariance/Invariance*.)
>
> **Q 14.1:** Why don't we take the rest frame of the universe as the absolute frame? What is the difference between a special frame in theory and a special frame in observation?

It is natural to imagine the existence of the "absolute frame of reference", which is "truly" at rest with respect to the Universe.

Q 14.2: If there is an absolute reference frame, how do you find it?
Q 14.3: What is the biggest problem if there is no absolute frame?

There have been many mistaken absolute frames: the earth, the sun, the Milky Way, ...

On the other hand, even if there is no ether, i.e., the EM waves are not oscillations of any particle or substance, it is still hard to accept that the speed of light c is not referring to a particular frame of reference. If c is the speed of light for a particular reference frame, shouldn't it appear to be different in another frame?

"Conceptually concise" does not imply "easy to understand".

Einstein proposed that we change our understanding of the length and the time (defined by rulers and clocks). We used to think that the length and the time of something are the same for all reference frames (cf. Galilean transformation). Now we have to accept that, if length and time are defined in terms of how we measure, their values for a given object or process depend on the choice of a reference frame.

More precise understanding of the length and the time will be explained below.

Physical laws are about reproducing the same result every time after following exactly the same procedure. It is not completely necessary for length and time to have a universal meaning independent of reference frames.

> Historically, it was the measurement of the speed of light that triggered the discovery of Special Relativity, but conceptually, all we need for Special Relativity is the existence of a universal speed in all inertial frames.

The most important quality of physics is *predictability*.

Challenge:

The new understanding of length and time will have to explain why the speed of light measured in any inertial frame is the same.

Regardless of whether physical laws turn out to comply with Lorentz symmetry (the symmetry of Lorentz transformations), one should appreciate the brilliance of proposing the seemingly impossible possibility that the length and time can be just "relative", in the absence of an absolute frame, while there are so many obvious paradoxical problems.
As an example of the paradoxes, if the clocks in S run slower than the clocks in S' when S' is moving with respect to S, should we not also conclude that the clocks in S' run slower than the clocks in S (assuming parity/rotation symmetry)? There seems to be the logical inconsistency of a clock A running slower *and* faster than another clock B at the same time.
The remarkable ingenuity of Einstein is not the guts to challenge a deep-rooted belief (the absolute frame), but the ability to demonstrate rigorously the falsehood in those paradoxes, with a seemingly impossible alternative logical possibility.

14.2 Principle of Relativity

In Newtonian mechanics, Newton's laws of motion are obeyed in all inertial frames, while different inertial frames can differ by (1) translations in space, (2) translations in time, (3) rotations in space, and (4) boosts (relative velocity). We focus on *boosts* in this section.

It is called Galilean relativity in the textbook.

The transformation law for a shift of the origin is given by

$$\mathbf{r}' = \mathbf{r} - \mathbf{r}_0, \qquad t' = t. \tag{14.2}$$

The Galilean transformation law for a boost is

$$\mathbf{r}' = \mathbf{r} - \mathbf{v}t, \qquad t' = t. \tag{14.3}$$

A boost in Lorentz transformation obeys a different transformation laws.

Ex 14.1: Check that Newton's laws of motion are invariant under a boost.
Q 14.4: Are all physical laws supposed to be invariant under a boost?

In principle, we need to specify the transformations of not only the coordinates of particles, but also their velocities and forces, etc, to completely transform the description of a physical system from one reference frame to another. But we often write down just the transformation of the coordinates, as the transformations of other physical quantities are typically determined accordingly. For instance, in order for Newton's 2nd law to hold in all inertial frames, it is obvious that we need $\mathbf{F}' = \mathbf{F}$ together with eq.(14.3).
The symmetry of boosts for Newton's laws of motion was considered as a coincidence at Newton's time. Some other laws of physics (e.g. the universality of the speed of light) do not have to respect the boost symmetry.

There are two logically possible interpretations of Galilean relativity:

1. There is an absolute reference frame, but some physical laws (e.g. Newton's laws of mechanics) happen to look the same in other frames that move at constant velocity. (This is the traditional view of classical mechanics at Newton's time.)

2. There is no absolute reference frame. All inertial frames are on equal footing. But due to the Galilean transformation law (14.3), no speed has the same value in all inertial frames. (This is the $c \to \infty$ limit of special relativity, and a logically possible interpretation of Newtonian physics.)

The difference between the two viewpoints is whether some physical laws (e.g. Maxwell equations) look different in different inertial frames.

The first interpretation of Galilean symmetry is a weaker condition than the latter interpretation. The principle of relativity in Special Relativity shares the stronger interpretation that all inertial frames are equally good. The only difference is the existence of a (finite) universal speed that has the same value in all inertial frames.

Newtonian = Einsteinian with $c \to \infty$.

Q 14.5: Is it possible to imagine another type of principle of relativity in which there are more constraints in addition to the existence of a universal speed, e.g. a universal length?

Often we say that special relativity follows from two postulates. The first postulate is the stronger version of the *principle of relativity*. It states that all physical laws apply equally well in all inertial frames. This principle is also obeyed by Newton's laws if we adopt the 2nd interpretation above for Galilean relativity. Thus it should be obvious that one can not deduce Special Relativity only from the principle of relativity. We must assume other postulates. Einstein chose the 2nd postulate to be the existence of a *universal speed* — a (finite) speed at which all inertial frames agree on its value.

If all physical laws look the same in all inertial frames, there is no way to distinguish which is the absolute frame.

On the other hand, are these two postulates sufficient to logically deduce special relativity uniquely? Strictly speaking, this is not true.
We will see in Sec.14.4 a derivation of the Lorentz transformation law from the two postulates together with a set of additional assumptions. The choice of assumptions is not unique.

14.3 About Reference Frames

A general reference frame is a system of labeling every point in space by a set of spatial coordinates (x, y, z) (How do we do that?), together with a clock at every point to define the time t at that point. Whenever we observe an *event* at a point (x, y, z), the time t of the event is defined by the clock at that point.

An *event* = a point in spacetime.

Here we are concerned with inertial frames in flat (Minkowski) spacetime.

Q 14.6: What is an inertial frame?

Assuming that the space is a 3D Euclidean space, we assign spatial coordinates (x, y, z) to each point in space. We also install infinitely many *synchronized* clocks at every point in space to define the time t at each point (x, y, z). It is an inertial frame if the Law of Inertia is always satisfied.

Notice that there is no unique way of identifying spatial points at different times.

Q 14.7: How do you make sure that all the grids in the x, y, z directions are equally spaced, and that the x, y, z axes are orthogonal?

Q 14.8: How do you synchronize all the clocks in an inertial reference frame?

The synchronization of clocks is not assumed for a general non-inertial frame.

> It seems rather unnecessary to imagine infinitely many clocks everywhere in space. But imagination is free and it is conceptually convenient.

Notice that such a setup to record physical events relies on physical laws. First, the Cartesian coordinates (x, y, z) rely on a ruler, which should be as rigid as possible. If physical laws only allow slimy things, we may not be able to define the Cartesian coordinates with reliability.

We will see that in Special Relativity there can be no ideal rigid body.

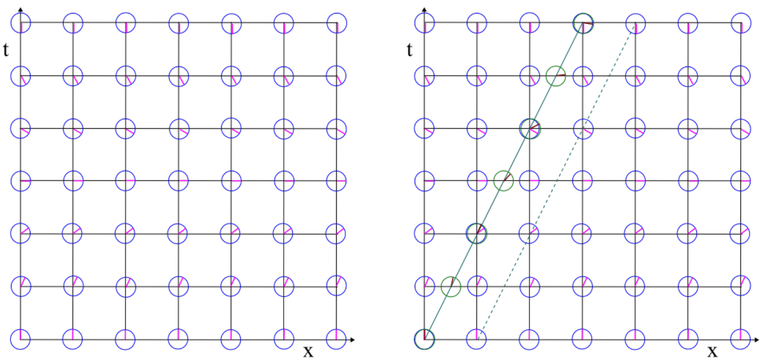
Secondly, the clocks normally include physical systems whose change of states involve well-defined periods. This also relies on physical laws.

Therefore, when we talk about the relations among (the coordinates of) different inertial frames, we are actually talking about properties of physical laws.

Assume that two professors moving at non-zero velocities with respect to each other are leading their students to set up the rulers and clocks to define their own inertial frames. (They use the same type of rulers and clocks bought from the same company.) Now the question is this: would the physical laws based on which the rulers and clocks work have the properties that (1) the spatial distance between any two given events measured by the rulers be the same for both reference frames, and (2) the time difference between any two given events measured by the clocks be the same for both reference frames?

The professors are always at rest in their own inertial reference frames.

Q 14.9: How would you measure the distance and the time difference between two given events in an inertial reference frame?



Q 14.10: Set up an inertial reference frame S and observe a clock in motion. Should the moving clock always be in sync with every stationary clock it passes by? If there is another clock moving at the same velocity, and if it is synchronized with the first moving clock by an observer moving at the same velocity, would the two clocks be considered synchronized in the frame S?

It is important to note that one needs to refer to more than one clocks to claim that a moving clock runs slower (or faster).

Q 14.11: If a clock in motion appears to run slower than the clocks at rest, does it imply that the clocks at rest would appear to run slower than the clock in motion?

Everything is possible unless it is incompatible with logic.

The Principle of Relativity demands that physical laws are the same in all inertial frames. More precisely, there is a way to express physical laws such that, when all physical quantities are properly transformed, they look the same in all inertial frames.

The transformation law relating different inertial frames includes a universal property of physical laws that can be attributed to the spacetime. This is the basis for the Lorentz symmetry.

A priori, the spacetime is just the arena in which events happen. It does not need to have any structure or property.

Tips for understanding reference frames in special relativity:

- Think of everything in terms of "events".
  When you think of everything in spacetime as sequences of events, it is easier to understand special relativity and Lorentz transformation.

The existence of an event is universal to all observers.

- Each inertial frame is composed of infinitely many detectors at every point with their own clocks.
  Each detector is given a spatial label (its spatial coordinates) and they are relatively at rest. Any two clocks can be synchronized by an observer at equal distance from the clocks.

- The spacetime coordinates of an event observed in an inertial frame is determined by the spatial label on the coincident detector and the reading on its clock.
  For any given event (at a given point in spacetime), there is a detector coincident with the event for every inertial frame, and each of them records the event according to its spatial label and clock.

14.4 Lorentz Transformations

We make the following assumptions.

1. The space is 3D Euclidean space (hence there are translation symmetry and rotation symmetry).

2. The time has translation symmetry.

3. There is a universal speed c that is the same in all inertial frames.

The universal speed is now understood as the speed of light.

4. No inertial frame is special.

5. If inertial frame S moves at velocity $\mathbf{v}$ with respect to S', then S' moves at the velocity $-\mathbf{v}$ with respect to S.
   Q 14.12: Is this an independent assumption?

See the derivation below.
The 4th assumption is needed to claim that Λ is a function of $\mathbf{v}$ only.
The 5th assumption is needed to claim that $\Lambda(-\mathbf{v}) = \Lambda^{-1}(\mathbf{v})$.

Using Cartesian coordinates in the 3D Euclidean space and the time coordinate, we consider two inertial frames S and S' with the coordinates (x, y, z, t) and (x', y', z', t'), respectively. The translation symmetries of S and S' imply that x', y', z', t' are linear functions of x, y, z, t. Therefore, we have

$$x'^{\mu} = \Lambda^{\mu}{}_{\nu} x^{\nu} + a^{\mu}, \tag{14.4}$$

where

$$x^0 = ct, \quad x^1 = x, \quad x^2 = y, \quad x^3 = z, \tag{14.5}$$

for a 4×4 matrix Λ which depends on the velocity $\mathbf{v}$ of S relative to S', and a set of constants a^{μ}.

Einstein's summation convention: When an index, say μ, is repeated on the same side of an equation, a symbol for sum $\sum_{\mu=0}^{3}$ is omitted but implied.

According to the 4th assumption, Λ is completely determined by the relative velocity $\mathbf{v}$.

Without loss of generality, one can choose the origin $x = y = z = t = 0$ of S and the origin $x' = y' = z' = t' = 0$ of S' to coincide. This implies that $a^{\mu} = 0$.

In order for the speed of light to be the same, the matrix Λ has to make sure that whenever

$$dx^{\mu} \eta_{\mu\nu} dx^{\nu} = 0, \tag{14.6}$$

we have

$$dx'^{\mu} \eta_{\mu\nu} dx'^{\nu} = 0, \tag{14.7}$$

where

$$\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, 1). \tag{14.8}$$

That is, we need

$$\Lambda^T(\mathbf{v}) \eta \Lambda(\mathbf{v}) = \lambda(\mathbf{v}) \eta \tag{14.9}$$

for a certain function $\lambda(\mathbf{v}) \in \mathbb{R}$.

Here, η and Λ are understood as matrices and matrix multiplication is implied.

Q 14.13: Can you find any Λ satisfying eq.(14.9) with $\lambda = -1$?

Ex 14.2: Prove that all pure spatial rotations have $\lambda = 1$.

The 5th assumption above imply that

$$\Lambda^{-1}(\mathbf{v}) = \Lambda(-\mathbf{v}), \tag{14.10}$$

and thus

$$\lambda^{-1}(\mathbf{v}) = \lambda(-\mathbf{v}). \tag{14.11}$$

It is possible to have $\lambda = -1$ in 1+1 dimensions, e.g. $\Lambda = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

Due to rotation symmetry, [1] $\lambda(\mathbf{v})$ must only depend on $|\mathbf{v}|$, so $\lambda(-\mathbf{v}) = \lambda(\mathbf{v})$, and the equation above implies that $\lambda(\mathbf{v}) = \pm 1$.

[1] Notice that this rotation symmetry is not available in 1+1D, so it is possible to have more general solutions of Λ in 1+1D such that $\lambda^{-1}(v) = \lambda(-v)$.

Example: We will see below that a boost in the x-direction has

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\frac{v}{c} & 0 & 0 \\ -\gamma\frac{v}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{14.12}$$

corresponding to the Lorentz transformation

$$x' = \gamma(x - vt), \tag{14.13}$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right), \tag{14.14}$$
$$y' = y, \tag{14.15}$$
$$z' = z, \tag{14.16}$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}. \tag{14.17}$$

Lorentz transformations are defined to be the homogeneous (linear) part of the transformation (14.4) with Λ satisfying

$$\Lambda^T \eta \Lambda = \eta. \tag{14.18}$$

Lorentz tranformations plus the translations (14.4) are call Poincaré transformations. (Corresponding symmetries are called Lorentz symmetry and Poincaré symmetry.)

14.5 Minkowski Space

In Special Relativity, the spacetime is a **Minkowski space**. As a set of points, the Minkowski spacetime is the same as $\mathbb{R}^4$. The difference between the Minkowski spacetime and $\mathbb{R}^4$ is that the former has the line element ds defined by

$$\boxed{ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2 = \eta_{\mu\nu}dx^\mu dx^\nu,} \tag{14.19}$$

Q 14.14: What are the criteria for you to call something "*distance*"?

which defines the "distance" between two points ("events") in spacetime with coordinates x^μ and $x^\mu + dx^\mu$.

In the old days, people liked to view the time coordinate as an imaginary spatial coordinate and define $ds^2 = \sum_{i=1}^4 dx^{i2}$ with $dx^4 = icdt$. This is not a good convention in view of General Relativity.

The phrase "distance" here is obviously an extension of the usual meaning of the word "distance". We could have invented a different name for it. But, at least, under

It is non-trivial that there exists a unique notion of distance that respects the Poincaré symmetry.

certain circumstances, the "distance" here happens to agree with our usual notion of 3D distances.

The most important feature about the "distance" for the Minkowski space is that it respects the Poincaré symmetry, which is a generalization of the rotation and translation symmetry in 3D.

Physical laws in Special Relativity are found to observe Poincaré symmetry, and thus it is convenient to think about the Minkowskian structure as a fundamental property of the spacetime, although spacetime itself is just an arena in which things happen. All properties of the spacetime are really the properties of physical laws.

The *Poincaré symmetry* is composed of translations and Lorentz transformations

$$\boxed{x^\mu \to \Lambda^\mu{}_\nu x^\nu + a^\mu, \qquad (14.20)}$$

where $\Lambda^\mu{}_\nu, a^\mu \in \mathbb{R}$, and

$$\boxed{\eta_{\mu\nu}\Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta = \eta_{\alpha\beta}. \qquad (14.21)}$$

This condition is a symmetric equation of 4×4 matrices, and is thus composed of 10 independent equations. As a result, the matrix $\Lambda^\mu{}_\nu$ should have $4 \times 4 - 10 = 6$ independent free parameters.

Lorentz symmetry includes two types of transformations: rotations and boosts; each type of transformations is labelled by 3 independent parameters.

A rotation can be specified by giving a unit vector $\hat{\mathbf{n}}$ (a point on the unit sphere) as the axis of rotation and an angle θ of rotation along the $\hat{\mathbf{n}}$-axis. We can use the vector $\theta\hat{\mathbf{n}} = (\theta_x, \theta_y, \theta_z)$ to label such a rotation. Another way to specify rotation is to perform first a rotation along the x-axis by θ'_x, and then a rotation along the y-axis by θ'_y, followed by a rotation along the z-axis by θ'_z. Either way, it takes three independent parameters to specify a rotation.

Q 14.15: Given the same set of parameters $(\theta_x, \theta_y, \theta_z) = (\theta'_x, \theta'_y, \theta'_z)$, is the rotation according to the first definition the same as the second?

A boost is also specified by three parameters. You can boost by a velocity vector $\mathbf{v} = (v_x, v_y, v_z)$, or you can first boost along the x-axis by v'_x, and then along the y-axis by v'_y, followed by a boost along the z-axis by v'_z.

A generic Lorentz transformation is a combination of rotation and boost. You can first rotate by $\theta\hat{\mathbf{n}}$ and then boost by $\mathbf{v}$, or vise versa. Either way, you need 6 free parameters to specify a generic Lorentz transformation.

Q 14.16: Is the boost by (v_x, v_y, v_z) equivalent to the boost by $(v'_x, v'_y, v'_z) = (v_x, v_y, v_z)$?

With a suitable choice of the x axis, we can always have $\mathbf{v} = \hat{\mathbf{x}}v$ for the boost. The y and z coordinates are invariant under the boost in the x-direction, and so the corresponding matrix $\Lambda^\mu{}_\nu$ is of the form

The existence of a solution to eq.(14.21) in this form is the proof that we can assume $y' = y$ and $z' = z$.

$$\Lambda^\mu{}_\nu = \begin{pmatrix} A & B & 0 & 0 \\ C & D & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{14.22}$$

or more conveniently, we can omit the y, z coordinates and simply write

$$\Lambda^\mu{}_\nu = \begin{pmatrix} A & B \\ C & D \end{pmatrix}. \tag{14.23}$$

There are 4 elements (A, B, C, D) in Λ subject to 3 independent conditions (**??**) and so there is only one free parameter in the boost, and the general solution is

$$\boxed{\Lambda^\mu{}_\nu = \begin{pmatrix} \gamma & \beta\gamma \\ \beta\gamma & \gamma \end{pmatrix},} \tag{14.24}$$

where

$$\boxed{\beta = v/c, \qquad \gamma = \frac{1}{\sqrt{1-\beta^2}}.} \tag{14.25}$$

Equivalently,

$$\Lambda^\mu{}_\nu = \begin{pmatrix} \cosh\chi & \sinh\chi \\ \sinh\chi & \cosh\chi \end{pmatrix}. \tag{14.26}$$

χ is called the *rapidity*.

Q 14.17: How is χ related to γ or β?

Q 14.18: What is the 4×4 matrix Λ representing a boost in the y-direction?

The idea of Special Relativity is that the spacetime, as a 4-dimensional "space" (set of points) probed by all sorts of objects available in the universe, has the geometric property of a Minkowski space mentioned above. More precisely, there is a universal feature shared by all sorts of physical phenomena (Poincaré symmetry) that we can choose to attribute to the geometric nature of spacetime. The notion of distance stands out naturally as an invariant quantity of the Poincaré transformations.

14.6 Spacetime Diagrams

We often draw the t-axis at right angle with respect to the x-axis. But this angle of 90° has no physical meaning. One can equally well present the set of points in spacetime with tilted axes at an arbitrary angle.

For a diagram with axes at an arbitrary angle, the t-axis is the trajectory of the point $x = 0$, and the x-axis is the set of points at the same time $t = 0$. Similarly, the trajectory of a point with a given value of the x-coordinate is a straight line parallel to the t-axis. Any line parallel to the x-axis is the set of spacetime points at the same time.

Q 14.19: How is the "length" of the line connecting a generic point (x^0, x^1) to the origin related to the lengths of its projection on the x^0 and x^1 axes?

In terms of the coordinates $x^0 = ct$ and $x^1 = x$, the trajectory of a particle moving at the speed of light (in the x-direction) must have

Q 14.20: What is the "distance" between any two points on the trajectory of particle moving at the speed of light?

$$\frac{dx^1}{dx^0} = \pm 1. \tag{14.27}$$

The line with the slope $dx^1/dx^0 = 1$ passing through the origin always bisects the angle between the x^0-axis and the x^1-axis.

Ex 14.3: Plot the x^0-axis and the x^1-axis of another frame S' on top of the spacetime diagram for an inertial frame S, assuming that S' is moving at velocity v with respect to S. Consider both possibilities $v > 0$ and $v < 0$.

14.7 Examples

In the following, we consider two inertial frames S and S', with S' moving with respect to S at the speed v in the x-direction. (Assume that the x, y, z directions of the two systems are parallel.)

It will be helpful to keep in mind that an inertial frame consists of as many clocks as it takes. The time of an event is recorded according to the clock coinciding in space with the event.

14.7.1 Simultaneity

In general, t' is a linear combination of t, x, y, z, in a boost transformation. As a result, we no longer have consensus on simultaneity among inertial frames.

Q 14.21: Two events happen at $x = 0$ and $x = \Delta L$, both at $t = 0$ in S. What are the spacetime coordinates of these two events in S'? Do they have the same value of x'^0?

Q 14.22: Do two (synchronized) clocks in S look synchronized in S'? How is the time difference between these two clocks determined in S'?

14.7.2 Time Dilation

The time separation between two events A_1 and A_2 in a reference frame S is by definition $(T_2 - T_1)$, where T_i $(i = 1, 2)$ is the time recorded by the clock coincident with A_i in space. Unless A_1 and A_2 occur at the same point in space, T_1 and T_2

are recorded by different clocks, which are synchronized in S but not synchronized in another reference frame.

(Everyone else looks dumb to you.)

A clock in S' appears to tick slower with respect to S.

We will omit the phrase "synchronized" in the following.

Q 14.23: How much time does it take for clocks in S to see a clock in S' to go through a time change of $\Delta T'$?

Q 14.24: How much time does it take with respect to S' to see a clock in S to go from $t = 0$ to $t = T$?

Q 14.25: Explain time dilation by (1) drawing a spacetime diagram, and (2) carrying out algebraic calculation using Lorentz transformation law.

14.7.3 Length Contraction

The length of a moving object in S is by definition the distance between two events A_1 (head) and A_2 (tail) at the same instant of time with respect to S.

(Running helps you look slim.)

Q 14.26: A train has length L in the rest frame. How long is the train when it moves at the velocity v?

Q 14.27: Find two events according to which the length of a moving train is defined. How would the observers at rest with respect to the train feel about this measurement?

Q 14.28: Explain length contraction by (1) drawing a spacetime diagram, and (2) carrying out algebraic calculation using Lorentz transformation law.

14.7.4 The Universal Speed

Ex 14.4: A photon travels from the origin ($x^0 = x^1 = x^2 = x^3 = 0$) of an inertial frame S at the speed of light c in the x^1-direction. Check that the speed of the photon is still c in the inertial frame S'.

Ex 14.5: Repeat the same question above, but with the photon moving in the x^2-direction in S.

Ex 14.6: Repeat the same question again, but with the photon moving in the direction $\hat{\mathbf{x}} \cos\theta + \hat{\mathbf{y}} \sin\theta$ for a given angle θ.

14.7.5 Addition of Velocities

Ex 14.7: A particle is moving with the velocity $\mathbf{u} = \hat{\mathbf{x}}u$ in S. What is its velocity $\mathbf{u}'$ in S'?

HW: (5-1) A particle is moving with the velocity $\mathbf{u} = \hat{\mathbf{y}}u$ in S. What is its velocity $\mathbf{u}'$ in S'?

14.7.6 Observer vs. Inertial Frame

An observer sees an event a bit later after it happens because it takes time for the light to travel from the event to the eyes of the observer.

HW: (5-2) For a clock moving at the velocity $\mathbf{v} = \hat{\mathbf{x}}v$ in S, how much change in time does itself read over a lapse of time of Δt in S? For an observer at rest in S, when the observer sees the face of the clock, how fast is the clock ticking in comparison with the observer's own watch? Consider separately the two possibilities: (a) the clock is moving towards the observer, and (b) the clock is moving away from the observer.

Ex 14.8: A clock moves at the speed u towards an observer at rest in S until it moves a distance L (according to the inertial frame S), and then it immediately returns to the starting point at the same speed but in the opposite direction. Answer the following questions. (a) How much time does it take for the clock to make the whole trip with respect to S? (b) How much time does the clock record over the whole trip? (c) According to the eyes of the observer, how much time does it take for the clock to move from the starting point to the turning point? (d) According to the eyes of the observer, how much time does it take for the clock to move from the turning point back to the starting point?