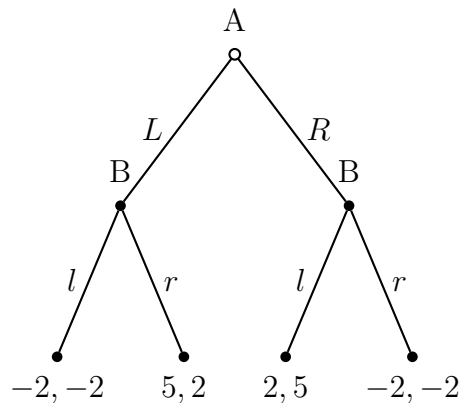


總分 41 分。答題皆須附說明，未做解釋的答案概不計分。

1. (4 points) Consider the following game.  $s_i$  denotes the row player's pure strategy,  $i = 1, 2$ , and  $t_i$  denotes the column player's pure strategy,  $i = 1, 2$ . In the payoff vector, the first element is the row player's payoff and the second element is the column player's payoff. Please find all the Nash equilibrium.

	$t_1$	$t_2$
$s_1$	1, 2	2, 0
$s_2$	0, 2	3, 4

2. In the following game, A could choose the  $L$  branch or the  $R$  branch, and in either case, B could choose the  $l$  branch or the  $r$  branch. In a payoff vector, the first element is A's payoff, and the second element is B's payoff.



- (a) (3 points) Please find the subgame perfect equilibrium.
  - (b) (1 point) In a strategic form (or a normal form, or a game matrix) to describe this game, how many pure strategies does B have?
  - (c) (2 points) Please find a Nash equilibrium of this game which is not a subgame perfect equilibrium.
3. There are only two firms in the market of good X. The market demand is:

$$q = 100 - p,$$

where  $q$  is the quantity, and  $p$  is the price. Firm 1 has a constant unit cost of \$10, and firm 2 has a constant unit cost of \$16.

- (a) (2 points) In a Cournot equilibrium, how many units will each firm produce?
  - (b) (3 points) Suppose firm 1 is a leader, and firm 2 is a follower. In a Stackelberg equilibrium, how many units will each firm produce?
4. Suppose that a monopoly steel producer produces steel at zero marginal costs and sells to a monopoly automaker at a price  $p_s$ . The automaker has to use one unit of steel and one unit of labor to produce one car. The labor market is competitive and the automaker could employ any amount of labor at a unit cost of \$20, so his marginal cost to produce one car is:

$$MC_c = p_s + 20.$$

The demand for cars is:

$$q_c = 100 - p_c,$$

where  $q_c$  is the number of cars, and  $p_c$  is the price of a car.

- (a) (2 points) Please derive the automaker's demand for steel.
  - (b) (2 points) What is the optimal  $p_s$  for the steel producer?
  - (c) (2 points) If the steel producer acquires ownership of the automaker, how many cars are produced?
5. A is a monopolist who has no production cost and simply wishes to maximize his revenue. A has two customers, B and C, whose demand functions for A's products are as follows:

$$D_B : q = 80 - p,$$

$$D_C : q = 60 - p,$$

where  $q$  is the quantity and  $p$  is the price.

- (a) (2 points) If A practices the first-degree price discrimination, how much will A receive from B and how much will A receive from C?

- (b) (4 points) Suppose A only knows the demand functions of two customers as above, and A does not know a customer's identity when the customer places an order. In this case, A will practice the second-degree price discrimination. A customer could choose to purchase  $n_1$  units with a total payment of  $p_1$ , or he could choose to purchase  $n_2$  units with a total payment of  $p_2$ . Please calculate  $n_i$  and  $p_i$ ,  $i = 1, 2$ .
- (c) (4 points) Suppose different from the original setting, A has a marginal cost function  $MC(q)$ :

$$MC(q) = q,$$

where  $q$  is the total output for B and C. Other things remain the same thing. Now A considers to practice the third-degree price discrimination by charging B a unit price of  $p_b$  and charging C a unit price of  $p_c$ . Please solve for  $p_b$  and  $p_c$ .

6. (6 points) A monopolist has the total cost function as follows:

$$TC(q) = q^2,$$

where  $q$  is his output. The market demand is:

$$q = 160 - p,$$

where  $p$  is the price. The monopolist considers to charge a single price  $p$  to maximize his profit. Unfortunately, his production pollutes local air and the harm to his neighbors expressed in monetary term increases with the output:

$$TH(q) = q^2,$$

where  $TH(q)$  is the total harm when  $q$  units are produced. The government wishes to maximize social welfare by taxing the monopolist an amount  $\$t$  for each unit he produces. Please solve for this  $t$ .

7. (4 points) Consider the same scenario as in the previous problem, but instead of taxing the monopolist, the government grants the pollution right to the monopolist. Now the neighbors come to negotiate with the monopolist to reduce his outputs so they will suffer less pollution

harm. Suppose the negotiation cost is zero, and they could reach an efficient deal which maximizes the welfare of two parties: the monopolist and neighbors. How many units will be produced according to their agreement?

## 解答

**1**  $(s_1, t_1), (s_2, t_2)$  (2 points)

$(1/2s_1 \oplus 1/2s_2, 1/2t_1 \oplus 1/2t_2)$  (2 points)

**2a** A chooses  $L$ ; (1 point)

B chooses  $r$  if A chooses  $L$  and B chooses  $l$  if A chooses  $R$ . (2 points)

**2b** 4

**2c** A chooses  $R$ ;

B chooses  $l$  no matter what A chooses.

**3a** Firm 1 wishes to maximize:

$$q_1(100 - q_1 - q_2 - 10),$$

So,  $q_1 = (90 - q_2)/2$  (1 point)

Similarly,  $q_2 = (84 - q_1)/2$ .

Solving two equations simultaneously,  $q_1 = 32, q_2 = 26$ . (1 point)

**3b** Firm 1 now wishes to maximize: (2 points)

$$q_1(100 - q_1 - \frac{84 - q_1}{2} - 10),$$

$q_1 = 48, q_2 = 18$  (1 point)

**4a** The marginal revenue of the automaker is:

$$MR_c = 100 - 2q_c.$$

When  $MR_c = MC_c$ ,  $100 - 2q_c = p_s + 20$ ,  $q_c = q_s = (80 - p_s)/2$ .

**4b**  $MR_s = 80 - 4q_s = 0$ ,  $q_s = 20$ ,  $p_s = 40$

**4c**  $MR_c = 20 = MR_c = 100 - 2q_c$ ,  $q_c = 40$ .

**5a** \$3,200 from B and \$1,800 from C

**5b** Drawing a graph of demand curves, we find  $n_1 = 40$ ,  $p_1 = (20+60)*40/2 = 1600$  (2 point), and  $n_2 = 80$ ,  $p_2 = 3200 - 40*20 = 2400$  (2 point),

**5c**  $MR_b = 80 - 2q_b = MR_c = 60 - 2q_c = MC = q_b + q_c$  (2 points)

$$q_b = 22.5, p_b = 57.5$$

$$q_c = 12.5, p_c = 47.5$$

**6** The private  $MC$  is:

$$MC = 2q.$$

The marginal harm is:

$$MH = 2q.$$

The social marginal cost is: (2 points)

$$SMC = 4q$$

To maximize social welfare,  $SMC = 4q = 160 - q$ ,  $q = 32$ . (2 points)

$$t = MR(32) - MC(32) = 32 \text{ (2 points)}$$

**7** To reduce 1 unit, the monopolist loses: (2 points)

$$MR(q) - MC(q) = 160 - 2q - 2q = 160 - 4q.$$

The agreed output is:

$$MH(q) = 2q = MR(q) - MC(q) = 160 - 4q,$$

$$q = 80/3.$$