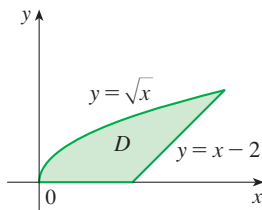




Section 15.2 Double Integrals over General Regions

9. (a) Express the double integral $\iint_D f(x, y) dA$ as an iterated integral for the given function f and region D .
 (b) Evaluate the iterated integral.



Solution:

(a) We express the iterated integral as a Type II. A Type I would require the sum of two integrals. The curves intersect when

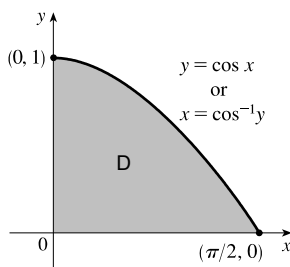
$\sqrt{x} = x - 2 \Rightarrow x = x^2 - 4x + 4 \Leftrightarrow 0 = x^2 - 5x + 4 \Leftrightarrow (x - 4)(x - 1) = 0 \Leftrightarrow x = 1 \text{ or } x = 4$. The point for $x = 1$ is not in D . Thus, the point of intersection of the curves is $(4, 2)$ and the integral is $\int_0^2 \int_{y^2}^{y+2} xy \, dx \, dy$.

$$\begin{aligned} \text{(b)} \quad \int_0^2 \int_{y^2}^{y+2} xy \, dx \, dy &= \int_0^2 y \left[\frac{x^2}{2} \right]_{x=y^2}^{x=y+2} dy = \frac{1}{2} \int_0^2 y[(y+2)^2 - (y^2)^2] dy = \frac{1}{2} \int_0^2 [y^3 + 4y^2 + 4y - y^5] dy \\ &= \frac{1}{2} \left[\frac{1}{4}y^4 + \frac{4}{3}y^3 + 2y^2 - \frac{1}{6}y^6 \right]_0^2 = \frac{1}{2} \left(4 + \frac{32}{3} + 8 - \frac{32}{3} \right) = 6 \end{aligned}$$

21. Set up iterated integrals for both orders of integration. Then evaluate the double integral using the easier order and explain why it's easier.

$$\iint_D \sin^2 x \, dA, \quad D \text{ is bounded by } y = \cos x, \quad 0 \leq x \leq \frac{\pi}{2}, \quad y = 0, \quad x = 0$$

Solution:



If we describe D as a type I region, $D = \{(x, y) \mid 0 \leq x \leq \pi/2, 0 \leq y \leq \cos x\}$

and $\iint_D \sin^2 x \, dA = \int_0^{\pi/2} \int_0^{\cos x} \sin^2 x \, dy \, dx$. As a type II region,

$D = \{(x, y) \mid 0 \leq x \leq \cos^{-1} y, 0 \leq y \leq 1\}$ and

$$\iint_D \sin^2 x \, dA = \int_0^1 \int_0^{\cos^{-1} y} \sin^2 x \, dx \, dy. \text{ Evaluating } \int_0^{\cos^{-1} y} \sin^2 x \, dx \text{ will}$$

result in a very difficult integral. Therefore, we evaluate the iterated integral that

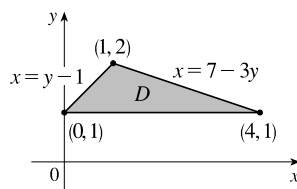
describes D as a type I region because integrating $\sin^2 x$ with respect to y is easy.

$$\begin{aligned} \int_0^{\pi/2} \int_0^{\cos x} \sin^2 x \, dy \, dx &= \int_0^{\pi/2} \sin^2 x \left[y \right]_{y=0}^{y=\cos x} dx = \int_0^{\pi/2} \cos x \sin^2 x \, dx \\ &= \int_0^1 u^2 \, du \quad \left[\begin{array}{l} u = \sin x, \\ du = \cos x \, dx \end{array} \right] = \left[\frac{u^3}{3} \right]_0^1 = \frac{1}{3} \end{aligned}$$

25. Evaluate the double integral.

$$\iint_D y^2 \, dA, \quad D \text{ is the triangular region with vertices } (0, 1), (1, 2), (4, 1)$$

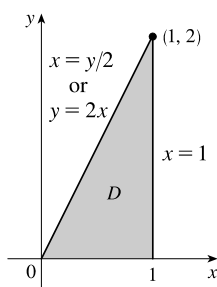
Solution:



$$\begin{aligned} \iint_D y^2 \, dA &= \int_1^2 \int_{y-1}^{7-3y} y^2 \, dx \, dy = \int_1^2 [xy^2]_{x=y-1}^{x=7-3y} dy \\ &= \int_1^2 [(7-3y) - (y-1)] y^2 \, dy = \int_1^2 (8y^2 - 4y^3) \, dy \\ &= \left[\frac{8}{3}y^3 - y^4 \right]_1^2 = \frac{64}{3} - 16 - \frac{8}{3} + 1 = \frac{11}{3} \end{aligned}$$

64. Evaluate the integral by reversing the order of integration. $\int_0^2 \int_{y/2}^1 y \cos(x^3 - 1) dx dy$

Solution:



$$\begin{aligned} \int_0^2 \int_{y/2}^1 y \cos(x^3 - 1) dx dy &= \int_0^1 \int_0^{2x} y \cos(x^3 - 1) dy dx \\ &= \int_0^1 \cos(x^3 - 1) \left[\frac{1}{2} y^2 \right]_{y=0}^{y=2x} dx \\ &= \int_0^1 2x^2 \cos(x^3 - 1) dx = \left[\frac{2}{3} \sin(x^3 - 1) \right]_0^1 \\ &= \frac{2}{3} [0 - \sin(-1)] = -\frac{2}{3} \sin(-1) = \frac{2}{3} \sin 1 \end{aligned}$$