

# Chapter 1 矩陣與聯立方程組

# 1-1 矩陣的基本定義與分類

• 聯立方程組

$$\begin{array}{ccccccc} a_{11}x_1 & + & a_{12}x_2 & + & \cdots & + & a_{1n}x_n & = & b_1 \\ a_{21}x_1 & + & a_{22}x_2 & + & \cdots & + & a_{2n}x_n & = & b_2 \\ & & & & \vdots & & & & \\ a_{m1}x_1 & + & a_{m2}x_2 & + & \cdots & + & a_{mn}x_n & = & b_m \end{array}$$

聯立方程式用  $\mathbf{Ax} = \mathbf{b}$  來表示

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix}, \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix},$$

通常我們會稱矩陣  $\mathbf{A}$  為此聯立方程式的係數矩陣

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觀念

$(m \times n)$  Matrix:  $A=[a_{mn}]$

Diagram illustrating a  $3 \times 2$  matrix:

$$\begin{matrix} \text{Row 1} \rightarrow & \begin{pmatrix} 6 & 10 \end{pmatrix} \\ \text{Row 2} \rightarrow & \begin{pmatrix} 5 & 3 \end{pmatrix} \\ \text{Row 3} \rightarrow & \begin{pmatrix} 0 & 2 \end{pmatrix} \end{matrix}$$

Annotations:

- Column 1 (indicated by a green arrow pointing to the first column)
- Column 2 (indicated by a green arrow pointing to the second column)
- An element of the matrix (indicated by an orange arrow pointing to the element 3 in Row 2, Column 2)

Dimension of this matrix is  $3 \times 2$

• **Index of component:** the scalar in the  $i$ -th row and  $j$ -th column is called  $(i,j)$ -entry of the matrix

$$A = \begin{bmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$$

先 Row 再 Column

Diagram illustrating the  $(i,j)$ -entry notation for a  $3 \times 3$  matrix:

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 3 & 1 & -1 \\ -2 & 1 & 1 \end{bmatrix}$$

Annotations:

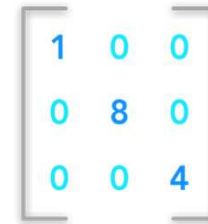
- $(1,2)$ -entry (indicated by a blue arrow pointing to the element 3 in Row 1, Column 2)
- $(3,1)$ -entry (indicated by a blue arrow pointing to the element -2 in Row 3, Column 1)
- $(3,3)$ -entry (indicated by a blue arrow pointing to the element 1 in Row 3, Column 3)

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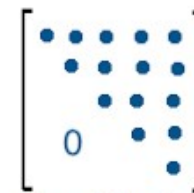
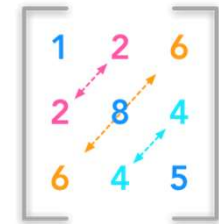
## 分類

- 1) 列矩陣 (row matrix)
- 2) 行矩陣 (column matrix)
- 3) 方陣 (square matrix)
- 4) 單位矩陣 (unit matrix)
- 5) 對角線矩陣 (diagonal matrix)
- 6) 上三角矩陣 (upper triangular matrix)
- 7) 下三角矩陣 (lower triangular matrix)

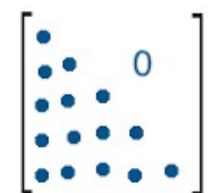
Diagonal matrix



Symmetric matrix



Upper Triangular Matrix



Lower Triangular Matrix

# 1-1 矩陣的基本定義與分類

## 定義

- 定義一：矩陣相等
- 定義二：矩陣相加
- 定義三：常數與矩陣相乘

設  $A$ 、 $B$  都是二階方陣

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \quad B = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$$

若  $A=B$  則  $a=p$ 、 $b=q$ 、 $c=r$ 、 $d=s$ 。

兩個矩陣相等不僅是行數列數要相等，而且所有互相對應的元素都要相等。

$$\begin{bmatrix} 1 & 3 \\ 1 & 0 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 7 & 5 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 \\ 1+7 & 0+5 \\ 1+2 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 8 & 5 \\ 3 & 3 \end{bmatrix}$$

$$2 \times \begin{pmatrix} 2 & 1 \\ 4 & 3 \end{pmatrix} = \begin{pmatrix} 4 & 2 \\ 8 & 6 \end{pmatrix}$$

# 1-1 矩陣的基本定義與分類

## 矩陣的變換 (transform of a matrix)

- 1) 轉置 (transpose):  $A^T$
- 2) 共軛 (conjugate):  $\bar{A}$
- 3) 共軛轉置:  $(\bar{A})^T (=A^H)$

A			$A^T$		
2	4	-1	2	-10	-18
-10	5	11	4	5	-7
18	-7	6	-1	11	6

Complex Conjugate	
$\begin{bmatrix} 3 & 3+i \\ 3-i & 2 \end{bmatrix}$	$\begin{bmatrix} 3 & 3-i \\ 3+i & 2 \end{bmatrix}$
A	$\bar{A}$

$$\begin{pmatrix} 3+i & 2 & 1-2j \\ 6-i & 4-i & 3-2i \\ 7+i & 4 & 1+2i \end{pmatrix}$$

→ 轉置 →  $\begin{pmatrix} 3+i & 6-i & 7+i \\ 2 & 4-i & 4 \\ 1-2i & 3-2i & 1+2i \end{pmatrix}$

→ 共軛 →  $\begin{pmatrix} 3-i & 6+i & 7-i \\ 2 & 4+i & 4 \\ 1+2i & 3+2i & 1-2i \end{pmatrix}$

## Matrix transpose

$$(AB)^T = B^T A^T$$

$$(ABC)^T = C^T B^T A^T$$

$$\mathbf{a}^T \mathbf{b} = \mathbf{b}^T \mathbf{a}$$

$$\|\mathbf{x}\|^2 = \mathbf{x}^T \mathbf{x}$$

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## 矩陣型式

- 1) 對稱矩陣 (symmetric matrix)
- 2) 斜對稱矩陣 (Skew-symmetric matrix)

### SYMMETRIC & SKEW SYMMETRIC MATRIX

Symmetric

$$A^T = A$$

$$\begin{bmatrix} 1 & 1 & -1 \\ 1 & 2 & 0 \\ -1 & 0 & 5 \end{bmatrix}$$

Skew-symmetric

$$A^T = -A$$

$$\begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & 3 \\ 2 & -3 & 0 \end{bmatrix}$$

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3) 赫米頓矩陣 (Hermitian matrix)  $A^H = A$

4) 斜赫米頓矩陣 (Skew- Hermitian matrix)  $A^H = -A$

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H.W.

17. The matrix  $\begin{pmatrix} 2 & 1+i & 3 \\ 1-i & 6 & i \\ 3 & -i & 4 \end{pmatrix}$  is

- (A) symmetric.
- (B) skew-symmetric.
- (C) Hermitian matrix
- (D) Skew-Hermitian matrix