Classical Mechanics (II)

Problem Set #1

1. (15 points) Ladder barn paradox: Suppose we have a ladder that is longer than the length of the garage, but it is traveling at a fast enough speed that, from the frame of reference of the garage, its length is contracted to less than the length of the garage. Then from the frame of reference of the garage, there is a moment in time when the ladder can fit completely inside the garage, and during that moment one can close and open both front and back doors of the garage, without affecting the ladder. However, from the frame of reference of the ladder, the garage is much shorter than the length of the ladder (it is already shorter at rest, plus it is contracted because it is moving with respect to the ladder). Therefore there is no moment in time when the ladder is completely inside the garage; and there is no moment when one can close both doors without it hitting the ladder. This is an apparent paradox. Use spacetime diagram to resolve this paradox.

2. (50 points) On their 21st birthday, one twin gets on a moving sidewalk, which carries her out to star X at speed $0.8c$; her twin brother stays home. When the traveling twin gets to star X, she immediately jumps onto the returning moving sidewalk and comes back to earth, again at speed $0.8c$. She arrives on her 39th birthday (as determined by her watch).

   (a) How old is her twin brother (who stayed at home)? How far away is star X? (Give your answer in light years.)

   Call the outbound sidewalk system $\bar{S}$ and the inbound one $\tilde{S}$ (the earth system is S). All three systems set their master clocks, and choose their origins, so that $x = \bar{x} = \tilde{x} = 0, t = \bar{t} = \tilde{t} = 0$ at the moment of departure.

   (c) Plot the spacetime diagram to describe the world lines of the twins.

   (d) What are the coordinates (x, t) of the jump (from outbound to inbound sidewalk) in S , $(\bar{x}, \bar{t})$ in $\bar{S}$, and $(\tilde{x}, \tilde{t})$ in $\tilde{S}$?

   (e) If the traveling twin wanted her watch to agree with the clock in S, how would she have to reset it immediately after the jump? If she did this, what would her watch read when she got home?

   (f) If the traveling twin is asked the question, "How old is your brother right now?', what is the correct reply (i) just before she makes the jump, (ii) just after she makes the jump?

3. (15 points) Show the wave equation

$$\nabla^2\Psi - \frac{1}{c^2}\frac{\partial^2\Psi}{\partial t^2} = 0,$$

is invariant under a Lorentz transformation but not under a Galilean transformation. (This is the wave equation that describes the propagation of light waves in free space.)

4. (15 points) A flash of light is emitted at an angle ϕ' with respect to the x' axis of the rocket frame, which is moving at a constant velocity v with respect to the laboratory frame. Show that the angle ϕ that the direction of this flash makes with respect to the x-axis of the laboratory frame is given by

$$\cos\phi = \frac{\cos\phi' + \beta}{1 + \beta\cos\phi'}; \qquad \beta = \frac{v}{c}$$

5. 14.7, .15, 14.24, 14.31, 14.35, 14.36

Note: Discussions are strongly encouraged, but please **no copying**. All parties involved in copying will get **zero** for their homework.