

# Outline of Study Groups : EM & CM

## 1. Electromagnetism

### a. Maxwell Equations:

- i Differential and Integral form
- ii Derive scalar potential  $V$  and vector potential  $\vec{A}$  and discuss their dynamics.
- iii Gauge transformation, Coulomb and Lorentz Gauge

*Ref: Jackson - Classical Electrodynamics Ch6.1, Ch6.6(Macroscopic Maxwell Equations), Ch6.2(Vector and Scalar Potentials), Ch6.3(Gauge Transformation)*

*Griffith - Introduction to Electrodynamics Ch7.3.3(conclusion), Ch2.2~Ch2.3(Guass's Law and Electrostatic potential), Ch5.3~5.4(Ampere's Law and no-name Law and vector potential for magnetostatics case), Ch7 (for the remaining), Ch10.1(gauge, gauge transformation)*

### b. Solving Maxwell Equations (static case):

- i Some Examples for Maxwell Eqs + B.C.
- ii Solving Laplacian and Poisson Equation and their properties (uniqueness)
- iii Green's Function & Mirror charges methods

*Ref: Jackson - Classical Electrodynamics Ch1.5(Boundary Condition at Interface Between Different Media), Ch1.9(Uniqueness of the Solution with Boundary Conditions), Ch2.9, Ch3.1, Ch3.7(Laplace equation in Rectangular, Spherical and Cylindrical Coordinates), Ch2.6(Green Function for the Sphere)*

*Griffith - Introduction to Electrodynamics Ch2.3.5(Electrostatic B.C), Ch3.1 ~ Ch3.3 (Laplace Equation and mirror charge), Ch5.4(solving magnetic vector potential)*

### c. EM waves with or without source

- i Poyting Vector & the energy conservation
- ii Electric & Magnetic Dielectric & B.C.
- iii Multiple Expansion (optional)
- iv EM Wave in Dielectric & Snell Eq. & Fresnel Eq.

*Ref: Jackson - Classical Electrodynamics Ch6.7(Poynting's Theorem and Conservation of Energy and Momentum ), Ch4.1(Multipoles Expansion), Ch4.4(Boundary-Value Problems with Dielectrics )*

*Griffith - Introduction to Electrodynamics Ch2.4(energy density for electric field), Ch7.2.4(energy density for magnetic field), Ch8.1(poynting theorem), Ch9 (em wave), Ch3.4(multiple expansion), Ch4(Dielectric), Ch6(Magnetic Field in Matter)*

## 2. Classical Mechanics

### a. Lagrangian Formalism

- i Generalized coordinates, degree of freedom and constraints (holonomic, nonholonomic, nonintegrable, rheonomous, scleronomous)
- ii Using virtual displacement to derive d'Alembert's principle (no constraint = Newton 2nd law) and Euler Eq. (E.O.M.)
- iii The principle of least action, using variation method to derive Euler Eq.
- iv Some examples : pendulum, free fall, a ball on a bowl,... (optional)
- v What is field? Lagrangian formalism for fields (optional)

*Ref: Goldstein - Classical Mechanics Ch1.4, Ch1.6(d'Alembert's principle, a first taste of Lagrangian), Ch2.1 ~ 2.5 (Least Action Principle, variation)*

*Marion - Classical Dynamics of Particles and Systems Ch6.2, Ch6.3 (the easier version for variation method)*

*David Tong, "Lectures on Quantum Field Theory" Ch1.1 (Classical Field (advanced))*

*Jorge V. Jose Classical Dynamics - A Contemporary approach Ch2,3*

### b. Noether Theorem

- i. Infinitesimal and finite transformation (this is why  $\omega$  is a vector while  $\theta$  is not)
- ii. The def of symmetry and the statement of the Noether Theorem
- iii. Noether current and the conserved charge
- iv. Some examples : translation, rotation, internal,... (optional)

*Ref: Goldstein - Classical Mechanics Ch2.6, Ch2.7(Noether Theorem)*

*David Tong, "Lectures on Quantum Field Theory" Ch1.3.1 ~ Ch1.3.3 (Noether Theorem for Classical Field (advanced))*

*A short review on Noether's theorem, gauge symmetries and boundary terms 1601.0361 p5~p26*

### c. Hamiltonian Formalism

- i. Legendre transformation, canonical relation, physical meaning ( $H=T+V$  holds iff system have some restrictions), Liouville Theorem
- ii. Canonical transf, generating function and Hamilton-Jacobi Eq.
- iii. Action-angle coordinates and adiabatic invariance (optional)
- iv. Poisson bracket (Lie bracket structure) and canonical invariant
- v. Some examples : charged particle in an EM field (optional)
- vi. Primary, Secondary constraints & First, Second class constraints & Dirac bracket : Gauge Invariance
- vii. Hamiltonian in field language, canonical conjugate field (optional)

*Ref: Goldstein - Classical Mechanics Ch8 (Hamilton E.O.M)*

*Shankar - Principles of Quantum Mechanics Ch2.5~2.8*

*Jorge V. Jose - Classical Dynamics - A Contemporary approach Ch5*

### d. Gauge Theory (optional)

- i. Gauge transformation, gauge symmetry and gauge invariance of the action

- ii. Noether Identity
- iii. generating sets and Lie algebras
- iv. Counting degree of freedom, gauge fixation

*Ref: Marc Henneaux and Claudio Teitelboim - Quantization of Gauge Systems Ch1*

*R.Jackiw - (Constrained) Quantization Without Tears*

*L.D.Faddeev - Gauge Fields Introduction to Quantum Theory*

*David Bleecker - Gauge Theory and Variational Principles*

*A short review on Noether's theorem, gauge symmetries and boundary terms*

*1601.0361 - section 3*

e. Rotation

- i. Velocity and acceleration in rotation frame
- ii. Coriolis force
- iii. Euler Rotation and Euler angles
- iv. Quaternion and rotation,  $SU(2)$ ,  $SO(3)$  and  $Sp(1)$  (optional)

*Ref: Goldstein - Classical Mechanics Ch4*

f. Central Force field

- i. Differential and integration Eqs, effective potential
- ii. Kepler laws, Relations between some physical quantities : Energy, Eccentricity, Angular Momentum, ...
- iii. Laplace-Runge-Lenz (LRL) Vector,  $SO(4)$  (optional)
- iv. Scattering, Differential Cross-Section

*Ref: Goldstein - Classical Mechanics Ch3*

*L. D. Landau and E. M. Lifshitz Ch18~Ch20*

- Recommended books for E& M :
  - a. David J. Griffiths, "Introduction to electrodynamics" 4th ed.
  - b. John David Jackson, "Classical Electrodynamics" 3rd ed.
  - c. Roald K. Wangsness, "Electromagnetic Fields" 2nd ed.
- Recommended books for Classical Mechanics:
  - a. Vladimir I. Arnold, "Mathematical Methods of Classical Mechanics"
  - b. "Classical Mechanics" by H. Goldstein, C.P. Poole, JR. J.L. Safko, 3rd edition
  - c. L.D. Landau and E.M.Lifshitz , "Mechanics"
  - d. L.D. Landau and E.M.Lifshitz, "The Classical Theory of Fields"
  - e. David Tong, "Lectures on Quantum Field Theory", section 1
  - f. Marc Henneaux and Claudio Teitelboim, "Quantization of Gauge Systems"