「Electromagnetism」Homework Sheet No. 1

Issued 29 Sept. 2021

1. The height of the Taipei Elephant Mountain (in feet) is given by $h(x, y) = 10(2xy - 3x^2 - 4y^2 - 18x + 28y + 12)$, where y is the distance (in miles) north, x is the distance east of Taipei 101 Tower.

(a) Where is the top of the Elephant Mountain located?

(b) How high is the mountain?

(c) How steep is the slope (in feet per mile) at a point 1 mile north and one mile east of Taipei 101 Tower? In what direction is the slope steepest, at that point?

(Textbook, p. 15, Problem 1.12).

2. Calculate the divergence of the following vector functions:

(a) $\mathbf{v}_a = x^2\hat{\mathbf{x}} + 3xz^2\hat{\mathbf{y}} - 2xz\hat{\mathbf{z}}$.

(b) $\mathbf{v}_b = xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3zx\hat{\mathbf{z}}$.

(c) $\mathbf{v}_c = y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}$.

(Textbook, p. 18, Problem 1.15).

3. Calculate the curls of the vector functions in Problem 2.

(Textbook, p. 20, Problem 1.18).

4. Calculate the line integral of the function $\mathbf{v} = x^2\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + y^2\hat{\mathbf{z}}$ from the origin to the point (1,1,1) by three different routes:

(a) (0,0,0) $\rightarrow$ (1,0,0) $\rightarrow$ (1,1,0) $\rightarrow$ (1,1,1).

(b) (0,0,0) $\rightarrow$ (0,0,1) $\rightarrow$ (0,1,1) $\rightarrow$ (1,1,1).

(c) The direct straight line.

(d) What is the line integral around the closed loop that goes out along path (a) and back along path (b).

(Textbook, p.28, Problem 1.29.)

5. Calculate the volume integral of the function $T = z^2$ over the tetrahedron with corners at (0,0,0), (1,0,0), (0,1,0), and (0,0,1).

(Textbook, p.28, Problem 1.31.)

6. Check corollary 1 of the Stoke's theorem (namely, $\int_S (\nabla \times \mathbf{V}) \cdot d\mathbf{a}$ depends only on the

boundary line) by using the same function and boundary line as in Example 1.11 in the textbook, but integrating over the five faces of the cube in Fig. 1. The back of the cube is open.
(Textbook, p. 35, Problem 1.35).

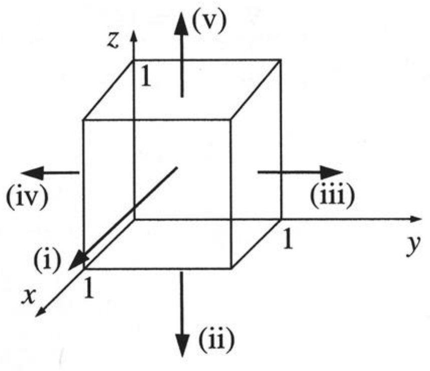

Figure 1.

7. Prove the following equations:

(a) $\int_S f(\nabla\times\mathbf{A})\cdot d\mathbf{a} = \int_S [\mathbf{A}\times(\nabla f)]\cdot d\mathbf{a} + \oint_P f\mathbf{A}\cdot d\mathbf{l}.$

(b) $\int_V \mathbf{B}\cdot(\nabla\times\mathbf{A})d\tau = \int_V \mathbf{A}\cdot(\nabla\times\mathbf{B})d\tau + \oint_S (\mathbf{A}\times\mathbf{B})\cdot d\mathbf{a}.$

(Hints: Read textbook Sec. 1.3.6)
(Textbook, p. 38, Problem 1.36).

8. (a) Find the divergence of the function

$\mathbf{v} = s(2+\sin^2\phi)\hat{\mathbf{s}} + s\sin\phi\cos\phi\hat{\boldsymbol{\phi}} + 3z\hat{\mathbf{z}}.$

(b) Test the divergence theorem for this function, using the quarter-cylinder (radius 2, height 5) shown in Fig. 8.
(c) Find the curl of **v**.
(Textbook, p. 45, Problem 1.43).

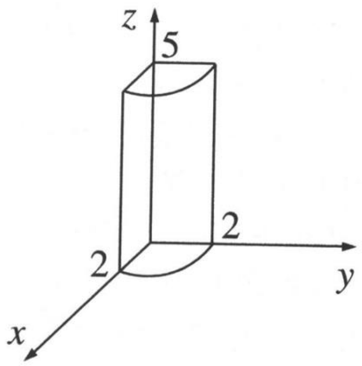

Fig. 2 Figure for problem 8.