Micro2 HW1

Q1, 6pts

a-d 1pt, e 2pts

N2. Suppose that a monopolist sells in two markets with the following demand curves:

$$Q_A = 100 - 10P_A$$
$$Q_B = 8 - 2P_B$$

a. Show that for any given quantity, demand is more elastic in market A than in market B.

b. Suppose that the monopolist produces at zero marginal cost. How much does he supply in each market, and what prices does he charge? (*Hint:* Use the formula for marginal revenue from the preceding problem.)

c. Suppose that the monopolist's marginal cost curve is given by the following equation:

$$MC = Q/21$$

How much does he supply in each market, and what prices does he charge?

d. Reconcile your answers to parts a, b, and c with the statement in the text that the group with more elastic demand is always charged the lower price.

e. Suppose that the monopolist's marginal cost curve is given by

$$MC = Q/3$$

What will the monopolist do?

(a) Given any quantity Q, the demand elasticity for both markets are

$$\eta_A = -\frac{P_A}{Q}\frac{dQ}{dP_A} = 10\frac{100-Q}{10Q} = \frac{100}{Q} - 1$$
$$\eta_B = -\frac{P_B}{Q}\frac{dQ}{dP_B} = 2\frac{8-Q}{2Q} = \frac{8}{Q} - 1$$

respectively. Hence easy to see that $\eta_A > \eta_B$ for any Q.

(b) For $Q = a - bP$, $P = \frac{a-Q}{b}$

$$\text{MR} = \frac{dPQ}{dQ} = \frac{a-2Q}{b}$$

So the marginal revenues for both markets are $MR_A = \frac{100-2Q_A}{10}$ and $MR_B = \frac{8-2Q_B}{2}$ respectively. To satisfy MR=MC, the monopolist will charge A market at $P_A = 5$, and B market at $P_B = 2$. The quantities he will supply are $Q_A = 50, Q_B = 4$, respectively.

(c) Both markets should yield the same marginal revenue

$$\frac{100-2Q_A}{10} = \frac{8-2Q_B}{2} = \frac{Q_A+Q_B}{21} \Rightarrow Q_A = 40, Q_B = 2 \Rightarrow P_A = 6, P_B = 3.$$

(d) What (a) says is in the condition that both markets are given the same quantity. Nonetheless, (b) and (c) are having different quantities in both markets, so there's no conflict between the statements. Note that for part (c), demand in market A at the quantity 40 is less elastic than the demand in market B at the quantity 2 and $P_A > P_B$. But for (b), while both markets have the same elasticity of 1, their prices are different, and this is because $MC = MR = P_A \times 0 = P_B \times 0$.

(e)

First, check the condition $MR_A = MR_B = MC$

$$\frac{100-2Q_A}{10} = \frac{8-2Q_B}{2} = \frac{Q_A+Q_B}{3} \Rightarrow Q_B = -2, Q_A = 20,$$

we find a corner solution. Thus, the optimal case should be reasonably quit the B market. We could set $Q_B = 0$ and then check $MR_A = MC$ condition

$$\frac{100-2Q_A}{10} = \frac{Q_A}{3} \Rightarrow Q_A = \frac{75}{4} \Rightarrow P_A = \frac{65}{8}.$$

Q2, 3pts

N3. A monopoly barber sells haircuts to adults for \$30 and to children for \$10. Let η_A represent adults' elasticity of demand for haircuts and let η_C represent children's elasticity of demand.

a. Explain why $|\eta_A|$ and $|\eta_C|$ must both be greater than 1.

b. Find a formula for η_A in terms of η_C.

c. What is the largest possible value for $|\eta_A|$?

(a) Since

$$MR = \frac{dPQ}{dQ} = P\frac{dP}{dQ} + P = P(1 - \frac{1}{|\eta|}).$$

The monopolist will choose Q at MR=MC to maximize the profit. Since $MC > 0$, $|\eta|$ must be greater then 1 to make $MR > 0$ too.

(b) Assume MC are the same for cutting both adults and children,

$$P_A(1 - \frac{1}{|\eta_A|}) = P_C(1 - \frac{1}{|\eta_C|}) \Rightarrow \frac{3}{|\eta_A|} - \frac{1}{|\eta_C|} = 2.$$

(c)

$$\frac{3}{|\eta_A|} - \frac{1}{|\eta_C|} = 2 \Rightarrow \frac{3}{|\eta_A|} > 2 \Rightarrow |\eta_A| < \frac{3}{2}$$

Q3, 2pts

18. Many hotels allow children to stay in their parents' rooms for free. Why?

The key is that families with children should have higher elasticity demand than other travellers, and thus will be charged for lower fee. One answer might be that families with children are far less likely to be on business trips than other travellers, and hence more apt to change their travel plans if they find the hotels are too expensive. Another is that families travelling with children tend to have more expenses (because they have children) and/or to be less well off (because they can't afford to leave the children home with a sitter), and are consequently more sensitive to price than other travellers.

Q4, 4pts

After consulting Professor Koo, there is no room for partial credit.

4. Suppose you are the monopoly owner of a movie theater. You can provide popcorn at a marginal cost of \$4 per bag. It costs you nothing to allow people to enter the theater. You have two customers, Gene and Roger. Gene is willing to pay up to \$28 to see the movie, and Roger is willing to pay up to \$10. Gene never buys popcorn under any circumstances. Roger's demand for popcorn in a theater is:

$$q = 12 - p,$$

where q denotes bag(s) of popcorn and p is the price of a bag of popcorn. (Both are allowed to be non-integers in this problem.) A strict rule is enforced to ban outside food in the theater. You have to decide how to charge for popcorn and the admission price to maximize profit.

(a) Suppose you charge \$8 for a bag of popcorn, what is the highest admission price you can charge if you're determined to keep both customers?

(b) At optimal, will you charge an admission price that drives Roger away? Why? Argue rigorously.

(c) At optimal, will you charge an admission price that drives Gene away? Why? Argue rigorously.

(d) Please solve for the optimal prices for popcorn and theater admission.

(a) Suppose the admission fee is for free, Gene's consumer surplus will be 28, and Roger's CS will be $10 + \frac{4\times(12-8)}{2} = 18$. So the highest admission price could be charged is at 18 dollars, so as to keep both customers.

(b) Keeping two customers can yield $18 \times 2 + (8-4)(12-8) = 52$ dollars, whereas the optimal price that drives Roger away can only obtain 28 dollars.

(c) To keep only Roger, we could adjust the popcorn price to increase his CS. Assume we set the price at p, then CS $= 10 + \frac{(12-p)^2}{2}$. Our goal now is to find p such that $CS \geq 28$ and also optimize our profit $(p-4)(12-p) + \frac{(12-p)^2}{2} + 10$. First solve the FOC of the profit function will obtain $p = 4$, and the corresponding $CS = 42 \geq 28$. We could then charge the admission price at 42 dollars and set the popcorn price at 4 dollars, yielding the total profit 42 dollars.

(d) Let p be the price of the popcorn. The optimal profit when both customers are in is to optimize the profit function

$$(p-4)(12-p)+2\times(\frac{(12-p)^2}{2}+10)=-8p+116$$

s.t. $\frac{(12-p)^2}{2}+10\leq 28$, or $6\leq p\leq 18$. We can see that $p=6$ will yield the maximum revenue at 68 dollars, better than both corner solutions. Thus, the optimal price for popcorn is set at 6 dollars, and the admission price is 28 dollars.

Q5, 5pts

5. 獨占廠商 A 有 B、C 兩位顧客。A 的成本爲0, A 求總收入之極大。B、C 的需求反函數如下, (本題考慮不連續的單位數):

	需求反函數 $(p(q))$	
數量 (q)	B	C
1	10	11
2	9	5

 (a) 若 A 採單一訂價, 他總共會賣幾單位?
 (b) A 考慮另一種定價方式: 買1件 $\$x$, 買2件共 $\$y$。
  i. 若要讓 B 選擇買2件, x、y 必須滿足那兩條限制式?
  ii. 若要讓 C 選擇買1件, x、y 必須滿足那兩條限制式?
  iii. 在讓 B 選買2件, C 選買1件的前提下, 最適的 x、y 爲何?
  iv. 可能安排出 B 選買1件, C 選買2件嗎?
  v. 請問最適的 x, y 爲何?

(a) (1pt)

- $p=5: PQ=5(2+2)=20$
- $p=9: PQ=9(2+1)=27$
- $p=10: PQ=10(1+1)=20$

- $p = 11 : PQ = 11(1) = 11$

So A will set the price at $P = 9$, and he will sell 3 units.

(b) Inequality 1pt, correctly specify consumer surplus 1pt, impossible reason 1pt, correct final number 1pt

(i) For B, buying one unit CS $= 10 - x$, and buying two units CS $= 19 - y$. To attract B to buy two units, it must satisfy

$$\begin{cases} 10 - x \leq 19 - y \\ 0 \leq 19 - y \end{cases}$$

(ii) For C, buying one unit CS $= 11 - x$, and buying two units CS $= 16 - y$. To attract B to buy two units, it must satisfy

$$\begin{cases} 11 - x \geq 16 - y \\ 11 - x \geq 0 \end{cases}$$

(iii)

$$\begin{cases} 10 - x \leq 19 - y \\ 0 \leq 19 - y \\ 11 - x \geq 16 - y \\ 11 - x \geq 0 \end{cases} \Rightarrow \begin{cases} x - y \geq -9 \\ y \leq 19 \\ x - y \leq -5 \\ x \leq 11 \end{cases}$$

The optimal price is set at $x = 11, y = 19$.

(iv) No, since then

$$\begin{cases} x - y \leq -9 \\ x - y \geq -5 \end{cases}$$

has no solution.

(v) The optimal price is to set at $y = 16, x \geq 11$, forcing both customers to buy two units.