Micro2 CH12

Shen, r10323001

Q1

(5pts, solution 3 + show 2)

A \ B	$B1$	$B2$	$B3$
$A1$	$3, 7$	$2, 8$	$3, 9$
$A2$	$4, 3$	$5, 5$	$5, 7$
$A3$	$5, 10$	$9, 6$	$4, 8$

(a) 請問在 Nash 均衡中, A 是否可能用 $A1$ 的策略?

(b) 請寫出所有的 Nash 均衡。

(a) No, since A2 strictly dominates A1, A will not play A1 in Nash equilibrium.

(b) We can eliminate the dominated strategies repeatedly to exclude the impossible pure strategies.

A \ B	$B1$	~~$B2$~~	$B3$
~~$A1$~~	~~$3, 7$~~	~~$2, 8$~~	~~$3, 9$~~
$A2$	$4, 3$	~~$5, 5$~~	$5, 7$
$A3$	$5, 10$	~~$9, 6$~~	$4, 8$

We first eliminate $A1$ since it is dominated by $A2$. Second, eliminate $B2$ since it is dominated by $B3$.

Now assume the mixed strategy $\sigma = (pA2 + (1-p)A3, qB1 + (1-q)B3)$ is a NE. We could compute the utilities for both players.

$$
\begin{aligned}
U_A(\sigma) =& 4pq + 5p(1-q) + 5(1-p)q + 4(1-p)(1-q) \\
=& (-2q+1)p + (q+4)
\end{aligned}
$$

$$
\begin{aligned}
U_B(\sigma) =& 3pq + 7p(1-q) + 10(1-p)q + 8(1-p)(1-q) \\
=& (-6p+2)q + (8-p)
\end{aligned}
$$

The best response correspondings are

$$
\mathcal{B}_1(q) = \begin{cases} p = 1, \text{ if } q < \frac{1}{2} \\ p \in [0,1], \text{ if } q = \frac{1}{2} \\ p = 0, \text{ if } q > \frac{1}{2} \end{cases} \quad \mathcal{B}_2(p) = \begin{cases} q = 1, \text{ if } p < \frac{1}{3} \\ q \in [0,1], \text{ if } p = \frac{1}{3} \\ q = 0, \text{ if } p > \frac{1}{3} \end{cases} .
$$

The intersection of the best responses are $(p=0, q=1)$, $(p=\frac{1}{3}, q=\frac{1}{2})$, $(p=1, q=0)$, so the NEs are $(A3, B1)$, $\left(\frac{1}{3}A2 + \frac{2}{3}A3, \frac{1}{2}B1 + \frac{1}{2}B3\right)$, $(A2, B3)$.

Q2

(5pts)

		Column player		
		a	b	c
Row player	d	$6, 10$	$0, 0$	$3, 3$
	e	$0, 0$	$4, 10$	$3, 3$

(a) 請找出 column player 的 dominant strategy (包括混合策略在內)。

(b) 利用 (a) 的結果來簡化賽局, 並寫出簡化賽局中的 Nash 均衡。

(a)[1] There is no pure dominant strategy[2] for Column player.

(b) Consider the mixed strategy $pa + (1-p)b$ for Column player. The expected pay-off is $10p$ when Row player plays d; and is $10 - 10p$ for e. Therefore, c is dominated by $\frac{3}{10} < p < \frac{7}{10}$. After eliminating c, we can assume the mixed strategy $\sigma = (pd + (1-p)e, qa + (1-q)b)$

[1] Will not be graded since the hint is somehow misleading.

[2] A pure strategy s_i is strictly dominant if every other pure strategy s_i' is strictly dominated by s_i, that is, $u_i(s_i', s_{-i}) < u_i(s_i, s_{-i}) \quad \forall s_{-i} \in \mathcal{S}_{-i}$. Note that a mixed strategy will never be a dominant strategy.

is a NE, and compute the utilities for both players.

$$\begin{aligned} U_1(\sigma) =& 6pq + 4(1-p)(1-q) \\ =& (10q-4)p + (4-4q) \end{aligned}$$

$$\begin{aligned} U_2(\sigma) =& 10pq + 10(1-p)(1-q) \\ =& (20p-10)q + (10-10p) \end{aligned}$$

The best response correspondings are

$$\mathcal{B}_1(q) = \begin{cases} p = 1, \text{ if } q > \frac{2}{5} \\ p \in [0,1], \text{ if } q = \frac{2}{5} \\ p = 0, \text{ if } q < \frac{2}{5} \end{cases} \quad \mathcal{B}_2(p) = \begin{cases} q = 1, \text{ if } p > \frac{1}{2} \\ q \in [0,1], \text{ if } p = \frac{1}{2} \\ q = 0, \text{ if } p < \frac{1}{2} \end{cases} .$$

The intersections of the best responses are $(p=0, q=0)$, $(p=\frac{1}{2}, q=\frac{2}{5})$, $(p=1, q=1)$, so the NEs for this game are (e, b), $(\frac{1}{2}d + \frac{1}{2}e, \frac{2}{5}a + \frac{3}{5}b)$, (d, a).

Q3

(2pts, must say he bids at his evaluation)

3. 德國詩人歌德於1797/1/16寫信給一出版商 Vieweg, 商談其作品 *Hermann and Dorothea* 之交易, 英譯如下:

> I am inclined to offer Mr. Vieweg from Berlin an epic poem, *Hermann and Dorothea*, which will have approximately 2000 hexamesters... Concerning the royalty we will proceed as follows: I will hand over to Mr. Counsel Böttiger a sealed note which contains my demand, and I wait for what Mr. Vieweg will suggest to offer for my work. If his offer is low than my demand, then I take my note back, unopened, and the negotiation is broken. If, however, his offer is higher, then I will not ask for more than what is written in the note to be opened by Mr. Böttiger.

歌德認爲 Mr. Vieweg 會提出什麼樣的價格?

Let G denote the bid made by Goethe, and V the bid made by Mr. Vieweg. Also let θ be the Mr. Vieweg's evaluation for the poem. We claim that Mr. Vieweg will not bid below his evaluation, $V < \theta$. Here are three possible cases with respect to the bid made by Goethe.

1. If $G < V$, in which case Mr. Vieweg wins and pay G. If instead of bidding V, Mr. Vieweg would have bid θ, then he would still win and pay the same price, so in this case bidding his valuation is as good as bidding V.

2. If $G > \theta$, in which case Mr. Vieweg loses. If instead of bidding V, Mr. Vieweg would have bid θ, then he would still lose, so bidding his valuation is as good as bidding V.

3. If $V < G < \theta$, then Mr. Vieweg loses. If instead of bidding V, Mr. Vieweg would have bid θ, then he would have won the auction and receive a pay-off of $\theta - V$, making this a profitable deviation, so bidding his valuation is strictly better than V.

It is similar to show that bidding above his valuation $V > \theta$ is worse than bidding his valuation. Hence, V is a dominant strategy[3], and thus we conclude that Mr. Vieweg might

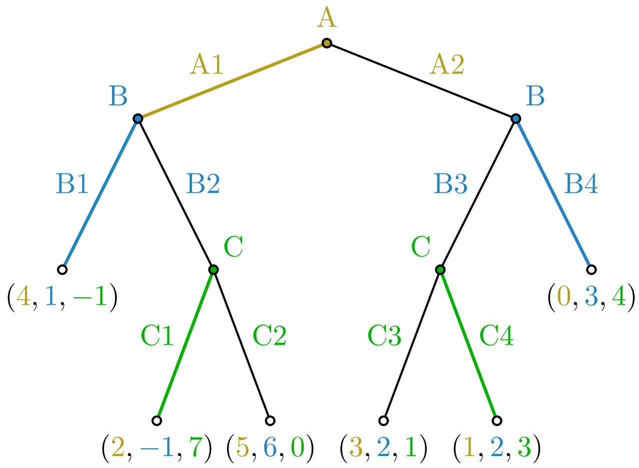
[3]The strategy is a weakly dominant strategy. Although one may find a lot of NEs in the original game, we must further consider that players will not play dominated strategies under the rationality assumption.

offer at his evaluation of Goethe's work.[4]

Q4

(2pts)

4. 請考慮 A、B、C 的三人賽局。枝幹旁的字標示的是各人的策略, 如 A 有策略 A1、A2 等。報酬向量中的三個數字挨序爲 A、B 與 C 的報酬。請找出此賽局的 subgame perfect equilibrium。



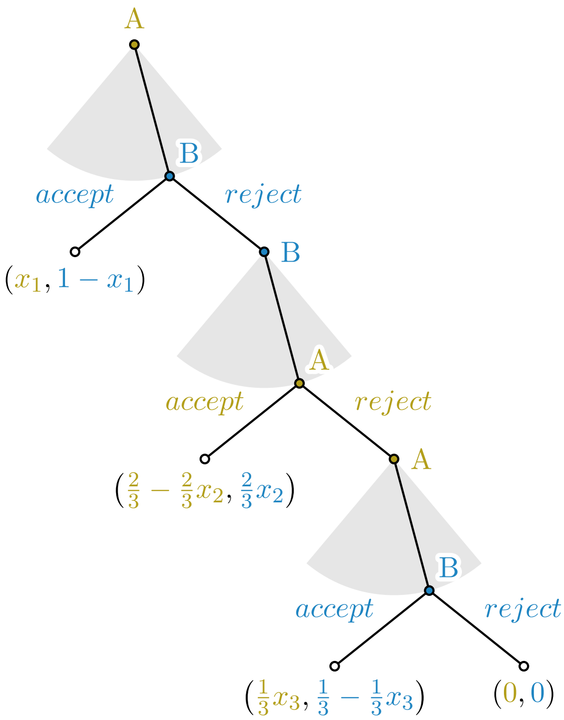
After drawing the possible paths using backward induction. The SPNE is $(A1, B1B4, C1C4)$.

Q5

(3pts)

5. A、B 分食冰淇淋, A 先提議兩人分食的比例, 若 B 接受, 則按 A 提議方式分配。若 B 反對, 輪到 B 提案, 只是當 B 提案時, 冰淇淋融化只剩原有的2/3。A 若能接受 B 的提案, 兩人將分食剩下的2/3。A 若反對 B 的意見, 再換回 A 提案, 此時 A 將考慮如何分配融化中只餘原先大小1/3的冰淇淋, 若 B 再表反對, 等不及新提案, 冰淇淋便化光了, 誰都沒得吃。A、B 希望自己吃到的越多越好; 兩人精打細算, 完全理性。請問 A 最先開始時, 會如何提議分食的比例?

[4] To see more information about this story, please read Moldovanu, B., & Tietzel, M. (1998). Goethe's Second-Price Auction. Journal of Political Economy, 106(4), 854–859. https://doi.org/10.1086/250032



Through backward induction, B will accept the offer in the third stage. Taken this into account, A will offer $x_3 = 1$. For A in the second stage, he will accept the offer if and only if $\frac{2}{3} - \frac{2}{3}x_2 \geq \frac{1}{3}$, if and only if $x_2 \leq \frac{1}{2}$. Taken this into account, B will offer $x_2 = \frac{1}{2}$. For B in the first stage, he will accept the offer if and only if $1 - x_1 \geq \frac{1}{3}$, if and only if $x_1 \leq \frac{2}{3}$. Taken this into account, therefore, A will offer $x_1 = \frac{2}{3}$ at the beginning.

Q6

(3pts)

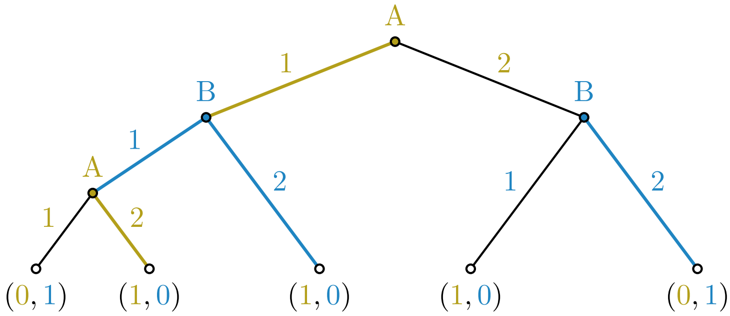
6. A、B 兩人玩搶4的遊戲: 兩人從0開始, 輪流報加數, 加數可爲1或爲2 (不能 pass), 誰將累加的和湊爲4, 誰就是贏家。A 先開始報。

   (a) 請繪此遊戲之 game tree。

   (b) Subgame perfect equilibrium 中, 誰會贏?

   (c) 假設將搶4改爲搶20, 其他遊戲規則相同, 如果你來玩, 你可選先報或讓對手先報, 你要選先報嗎? 爲什麼? (你的目標是贏。)

(a)



(b) The SPNE for this game are (12, 12) and (12, 22) . Both on-path solutions suggest that A will win the game.

(c) Using backward induction, we know that the one who first reaches 17 wins. Recursively, we conclude that the one who first reaches 2 wins. Hence, the first player always wins the game.