Asymmetry, Partial Moments, and Production Risk

John M. Antle

Partial-moment functions are proposed as a flexible way to characterize and estimate asymmetric effects of inputs on output distributions. Methods for econometric estimation of partial-moment functions, and tests for input symmetry and location-scale distributions, are presented. A Monte Carlo study demonstrates properties of proposed tests. A study of Ecuadorian potato production illustrates the methods. Hypotheses of input symmetry and location scale are rejected. A risk-value model based on partial moments implies that fertilizer is risk increasing and fungicides and labor are risk reducing in potato production, whereas an expected utility model based on full moments has the opposite implications.

Key words: asymmetry, Monte Carlo, partial moments, potatoes, production risk.

JEL Classification: C2, C5.

The objective of this article is to propose the use of partial-moment functions as a flexible way to characterize, estimate, and test asymmetric effects of inputs on output as deviations from a reference value, such as the mean or a behaviorally determined threshold. This research contributes to the growing body of literature on methods to characterize agricultural output as a random variable determined by complex interactions between management decisions and exogenous random events such as weather and pests (e.g., Antle 1983; Antle and Goodger 1984; Nelson and Preckle 1989; Ker and Goodwin 2000; Ker and Coble 2003; Hennessy 2009a, b). This article demonstrates how standard econometric methods can be used to estimate and test partial-moment functions, by generalizing the moment-based methods of Antle (1983).

An important motivation for the partial-moment approach, as compared with other approaches in the literature, such as parameterization of a beta or other flexible distribution function, is to link empirical representations of output distributions to decision models that utilize partial moments. There is a long history of research on decision making under uncertainty that takes asymmetry of distributions into account, using partial moments as well as third and higher-order full moments. The relevant literature includes research on safety-first preferences (Roy 1952; Atwood 1985; Bigman 1996); mean-semivariance models (Markowitz 1959; Hazell 1971); target return models (Fishburn 1977; Holthausen 1981; Tauer 1983); models that incorporate downside risk aversion (Anderson, Dillon, and Hardaker 1980; Menezes, Geiss, and Tressler 1980; Antle and Goodger 1984; Antle 1987, Di Falco and Chavas 2006, 2009; Groom et al. 2008; Koundouri et al. 2009); and the finance literature on value-at-risk (Jorion 1996; Rockafellar and Uryasev 2000) and studies emphasizing asymmetric risk (Černý 2004; Lence 2009). Another relevant branch of literature presents alternatives to and generalizations of expected utility theory, including prospect theory (Khaneman and Tversky 1979; Tversky and Khaneman 1992), regret and disappointment models (Bell 1982, 1985), and the more recent generalization of those concepts in risk-value models (Jia, Dyer, and Butler 2001; Butler, Dyer, and Jia 2005; Delquié and Cillo 2006). The objective functions used in risk-value models are functions of partial moments, and I show in this article that they provide a natural way to link general decision models with a partial-moment representation of production technology.

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The next section of this paper discusses the processes generating asymmetric output distributions, and shows how partial moments can be used to represent output distributions and their relationship to input choice. The following sections present methods for the econometric estimation and testing of partial-moment functions. Monte Carlo simulation is used to investigate the properties of the proposed tests for statistical significance and symmetry of partial-moment functions. These methods are then illustrated with the example of Ecuadorian potato production, an interesting case to test the partial-moment model because farmers use large amounts of fertilizer to achieve high yields and apply large amounts of fungicides to control the late blight fungus, a potentially catastrophic risk to potato production. The concluding section discusses implications of these methods for research on production risk.

Output Distributions, Asymmetric Effects of Inputs, and Partial Moments

In this section I explore the processses generating output distribution asymmetry and its relation to input use. The goal here is to show that the properties of output distributions, and in particular the effects of inputs on the distribution's shape, can be understood to be the consequence of complex interactions between the biophysical and management processes that jointly determine production outcomes. This understanding is useful in formulating models and in interpreting the econometric estimates and test results presented below.

Economic theory posits a production function $q = f(x)$ where q is output and x represents inputs (treated here for simplicity as a scalar). To translate this deterministic concept into a stochastic model, many researchers have recognized that in agriculture there is a set of random factors not under the control of the decision maker, including weather (temperature, rainfall, solar radiation, hail, wind), other events that may be weather related such as pest infestations, and conditions not readily observed by the manager that affect productivity, such as soil chemistry and microbiological conditions. Thus the agricultural production function can be written as $q = f(x, w)$ where w represents these stochastic factors affecting production. Figure 1 illustrates the implications of this stochastic production function model for the properties of output distributions. The right horizontal axis represents the stochastic input w which follows

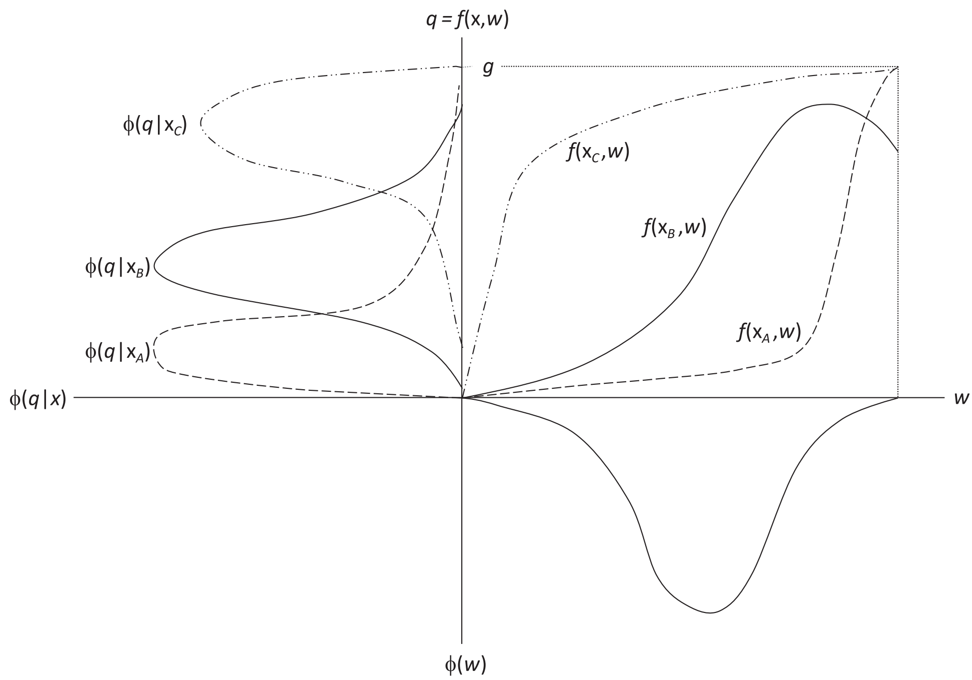

Figure 1. Output distribution properties determined by production functions with management inputs (x) and random inputs (w)

the distribution $\phi(w)$. The positive vertical axis measures output per unit of land (yield) with the upper bound g defined as the maximum output per hectare determined by the crop's genetic potential. The negative horizontal axis measures the probability density of output given inputs x, denoted as $\phi(q|x)$.

Figure 1 shows three examples of possible production relationships that generate three different output distribution shapes. Distribution A occurs with a low level of an input (x_A) which must be applied at a high level to realize the crop's yield potential. This could be the case of an improved crop variety that responds strongly to high nutrient and water inputs (interpreting w as rainfall). Alternatively, w could be defined as pest absence, and x could be interpreted as a pesticide that prevents pest damage. In this case, the mass of the output distribution is concentrated at low output levels and is right-skewed because there is a small probability of obtaining a relatively large output. At the other extreme, distribution C is the case where a protective input is used at a high level (x_C), so that the output distribution is concentrated near the upper bound, and is left skewed. Similarly, Hennessy (2009b) shows that a left-tail skew may occur if the production function is concave in the random variable w, as is the case with C, and cites a result by van Zwet (1964) that a transformed random variable $q = f(x, w)$ is more (less) skewed than w whenever $f(x, w)$ is increasing and concave (convex), as illustrated by Figure 1. Hennessy (2009a) shows that both positive and negative skewness is possible using a law-of-the-minimum production technology. Distribution B represents a case where the density of w is concentrated on an approximately linear segment of the production function. In this situation, it is possible for the distribution to be approximately symmetric and may even be well approximated by a normal distribution.

Several facts can be inferred from this characterization of output distributions. First, output distributions cannot be normally distributed, due to the fact that they are derived from non-linear functions defined on a finite interval of the real line and can only appear to be symmetrically distributed over a limited range of inputs.[1] Second, changes in inputs are likely to have different effects on the positive and negative tails of distributions, as shown by distributions A and C. Third, inputs may have various qualitative and quantitative effects on the shape of the output distribution, depending on the type of input, the level of input use, and interactions between inputs. For example, as input use increases and the shape of the output distribution goes from positively skewed to a more symmetrical shape with a higher mean, it is possible for the variance to increase or decrease.These observations suggest that a flexible model is needed to quantify the effects of inputs on output distributions, where flexibility means that inputs may have different effects on the positive and negative tails of the distribution.

To demonstrate the empirical relevance of the hypothetical distributions in figure 1, figure 2 presents output distributions derived from the Ecuadorian potato production data discussed in more detail below. The data were stratified into groups with low and high levels of two key inputs, fertilizer and fungicides. Figure 2 shows histograms of the residuals from each group, centered on the group means. With low inputs, the output distribution is positively skewed, with the mass of the distribution concentrated at low outputs. The high-input distribution shows that the mass of the distribution has shifted rightward toward the production frontier and has a higher variance, lower skewness, and higher kurtosis than the low-input case. While suggestive, this graphical analysis cannot distinguish the effects of the individual inputs. To do that, we need an econometric model that is flexible enough to differentiate effects of individual inputs on the properties of the distribution.

Measuring Effects of Inputs on Output Distributions Using Partial Moments

To simplify the following discussion, production is defined as a single-period process in which variable inputs are chosen and applied, random events then occur, and output is realized, so that inputs can be treated as statistically independent of output; this assumption can be relaxed using the methods in the literature for models with endogenous inputs. As in Figure 1, production is a stochastic process and output q

[1] Some researchers have argued that aggregate yields should be normally distributed based on the central limit theorem (Just and Weninger 1999). Others have argued that statistical properties such as spatial dependence are likely to invalidate the central limit theorem (Ker and Goodwin 2000). Koundouri and Kourogenis (2010) argue that the central limit theorem will fail for aggregate yields if the number of observations being aggregated, i.e., the number of acres in production, is a random variable.

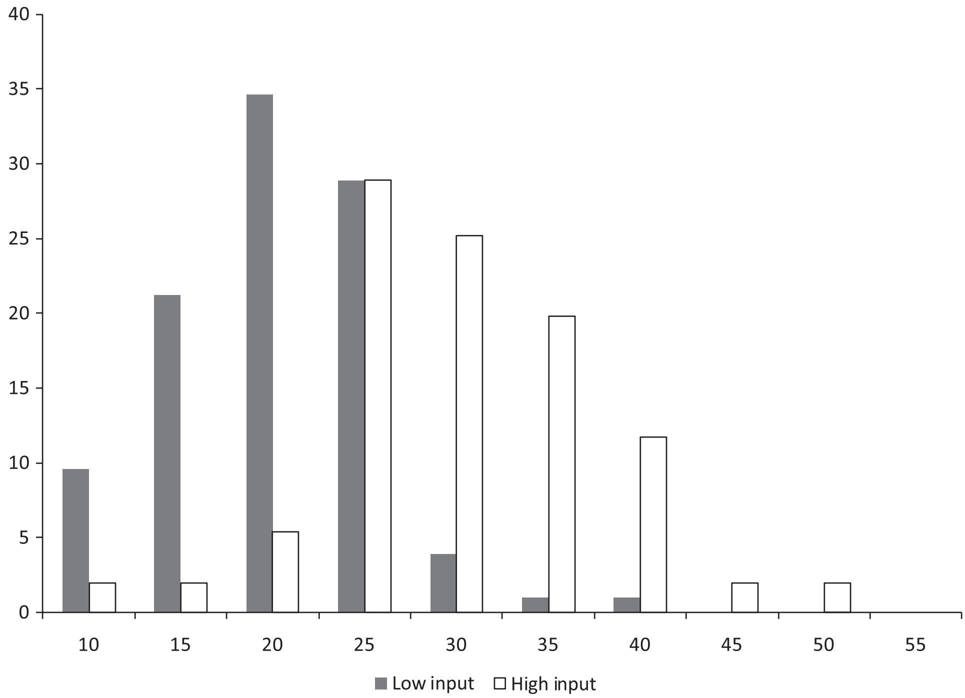

Figure 2. Frequency distribution of Ecuadorian potato production with low and high input levels (horizontal scale in tonnes/ha, vertical scale in percent)

is characterized by a probability density $\phi(q|x)$ defined on a finite interval of the real line $(0, g)$. If q is defined as average product per unit of a fixed input (e.g., crop yield, milk production per animal), then g can be defined as a constant representing the absolute upper bound determined by the genetic potential of the species. Under these conditions, the moments of q provide a unique representation of its distribution (Kendall and Stuart 1977; Antle 1983). The operator $\mu_1(x)$ represents the expectation of q given x, and the operator $\mu_i(x)$, $i > 1$, represents the ith central moment of q conditional on x. The central moments of output are therefore functions of x:

$$(1) \qquad \mu_1(x) \equiv \int_0^g q\phi(q|x)\,\mathrm{d}q$$

$$(2) \qquad \mu_i(x) \equiv \int_0^g \{q - \mu_1(x)\}^i \phi(q|x)\,\mathrm{d}q \qquad i = 2, 3, \ldots$$

The standard definition of partial moments is based on deviations above and below a reference point r. Although many studies utilize the mean as the reference, and I will do that in this study as well, some studies of decision models (e.g., Butler, Dyer, and Jia 2005) suggest that other reference points may be used by decision makers. In the case of output distributions defined as above, for $r \in (0, g)$ the lower or negative partial absolute moments are $\mu_i^-(x, r) \equiv \int_0^r \{q - r\}^i \phi(q|x)\,\mathrm{d}q$, and the upper or positive partial moments are $\mu_i^+(x, r) \equiv \int_r^g \{q - r\}^i \phi(q|x)\,\mathrm{d}q$, so that $\mu_i(x) = \mu_i^-(x, r) + \mu_i^+(x, r)$. For reasons that will become apparent below, it will be useful to utilize an alternative definition of the partial moments here, in which the moments are defined relative to the truncated distributions of q defined on $(0, r)$ and (r, g). Define the probability that output is less than r as

$$(3) \qquad \Phi(x, r) \equiv \int_0^r \phi(q|x)\,\mathrm{d}q.$$

Accordingly, the probability density for q less than r is $\phi(q|x)\Phi(x, r)^{-1}$ and the density for q greater than r is $\phi(q|x)(1 - \Phi(x, r))^{-1}$. It will also be useful to define the partial moments in absolute terms. Thus, the absolute partial moments for the distributions truncated below and above the mean are defined as

$$(4) \qquad \eta_i(x, r) \equiv (-1)^i \int_0^r \{q - r\}^i \phi(q|x) \times \Phi(x, r)^{-1}\,\mathrm{d}q$$

$$= (-1)^i \Phi(x,r)^{-1} \mu_i^-(x,r)$$

$$(5) \qquad \varphi_i(x,r) \equiv \int_r^g \{q-r\}^i \phi(q|x) \times (1-\Phi(x,r))^{-1}\,\mathrm{d}q = (1-\Phi(x,r))^{-1}\mu_i^+(x,r).$$

In th remainder of this article I use the mean as the reference point; so to simplify notation, I use $\Phi(x) \equiv \Phi(x,\mu_1), \eta_i(x) \equiv \eta_i(x,\mu_1)$ and $\varphi_i(x) \equiv \varphi_i(x,\mu_1)$. From equation (2) it follows that

$$(6) \qquad \mu_i(x) = (-1)^i \Phi(x)\eta_i(x) + (1-\Phi(x))\varphi_i(x), \quad i = 2,3,\ldots$$

If the output distribution is symmetric, then $\Phi(x) = 0.5$, and $\eta_i = \varphi_i$, implying that $\mu_i = \eta_i = \varphi_i$ for i even; for i odd, symmetry implies $\eta_i = \varphi_i$ and $\mu_i = 0$ by equation (6). Thus, rejection of the null hypothesis of equal partial moments implies rejection of symmetry of the output distribution.

Using equation (6), the marginal effect of an input on the ith moment is

$$(7) \qquad \frac{\partial \mu_i}{\partial x} = (-1)^i \Phi \frac{\partial \eta_i}{\partial x} + (1-\Phi)\frac{\partial \varphi_i}{\partial x} + ((-1)^i \eta_i - \varphi_i)\frac{\partial \Phi}{\partial x}, \quad i = 2,3,\ldots$$

Define μ_i^* as the elasticity of μ_i with respect to x, and similarly define η_i^*, φ_i^* and Φ^* as elasticities. Setting $n_i = (-1)^i \frac{\eta_i}{\mu_i}$ and $p_i = \frac{\varphi_i}{\mu_i}$, equation (7) in elasticity form is

$$(8) \qquad \mu_i^* = \Phi n_i \eta_i^* + (1-\Phi)p_i\varphi_i^* + (n_i - p_i)\Phi\Phi^*, \quad i = 2,3,\ldots$$

Equation (6) shows that $\Phi n_i + (1-\Phi)p_i = 1$, thus equation (8) shows that elasticities of full moments with respect to inputs are approximately weighted averages of the partial-moment elasticities when $(n_i - p_i)\Phi^*$ is small. Symmetry implies $\Phi = 0.5$ and $\Phi^* = 0$, and thus $\mu_i^* = \eta_i^* = \varphi_i^*$ for i even. For i odd, symmetry implies $\eta_i^* = \varphi_i^*$ and $\mu_i^* = 0$.

Asymmetry, Partial Moments, and Input Choice

Much of the literature on production risk and input use has been based on the expected utility maximization paradigm, with production risk represented by the variance of output which may be a function of inputs (e.g., Just and Pope 1978; Love and Buccola 1991; Saha 1997; Isik and Khanna 2003). Of course, the limitations of variance as a measure of risk have been discussed extensively in the literature. Some studies have utilized the concept of downside risk, also within the expected utility framework, as noted in the introduction. Other researchers have pointed out the limitations of expected utility and have introduced more general frameworks such as prospect theory and the risk-value model. Here I compare the expected utility model to the risk-value model to demonstrate the fact that the two can lead to substantially different behavioral implications with the same data. This point is then illustrated in the case study presented later.

To simplify the presentation, define expected net returns as $\mu_1(x) - cx > 0$, where c is the price of x normalized by the nonstochastic output price. Let the decision maker's expected utility function be $\mathrm{U}[\mu_1(x) - cx, \mu_2(x)]$, and define $\mathrm{U}_i = \frac{\partial \mathrm{U}}{\partial \mu_i}$. The decision maker chooses x to satisfy $\frac{\partial \mu_1}{\partial x} - c = -\frac{\mathrm{U}_2}{\mathrm{U}_1}\frac{\partial \mu_2}{\partial x}$, which can be written in elasticity form as

$$(9) \qquad \mu_1^* - cx/\mu_1 = R_2 s_2 \mu_2^*$$

where $\mu_i^* \equiv \partial \ln \mu_i / \partial \ln x$, $s_i \equiv \mu_i/(\mu_1(\mu_1 - cx)^{i-1})$, and $R_2 \equiv -(\mu_1 - cx)\frac{\mathrm{U}_2}{\mathrm{U}_i}$ is approximately one-half of the Arrow-Pratt relative risk aversion coefficient (Antle 1987). Following Pope and Kramer (1979), the the right-hand side of equation (9) is interpreted as the marginal risk effect (MRE) of the input, which in this case is positive (negative) when the variance is increasing (decreasing) in x. The MRE is a convenient way to compare the behavioral implications of alternative decision models without solving for optimal inputs, because it is defined in percentage terms; and in the multiple input case, other inputs are held constant.

As noted in the introduction, another strand of the risk literature has recognized that decision makers may respond to downside risk. The analysis of downside risk can be implemented with an expected utility function of the form $\mathrm{U}[\mu_1(x) - cx, \mu_2(x), \mu_3(x)]$, implying a first-order condition

$$(10) \qquad \mu_i^* - cx/\mu_1 = R_2 s_2 \mu_2^* - R_3 s_3 \mu_3^*$$

where μ_i^*, s_i and R_2 are defined as in equation (9), and $R_3 \equiv (\mu_1 - cx)^2 \frac{U_3}{U_1}$ is approximately one-sixth times the relative downside risk aversion coefficient (Antle 1987). In this model, input use depends on both Arrow-Pratt risk aversion (R_2) and downside risk aversion (R_3), thus the risk effects of inputs, i.e., the value of MRE, cannot be inferred from the variance alone.

Risk-value models provide an alternative approach to decision making in which the objective function is defined as $V[\mu_1(x), \eta_i(x), \varphi_i(x)]$. This model is motivated by research in psychology and economics, suggesting that decision makers respond differently to positive and negative deviations from a reference point such as expected value, experiencing "disappointment" or disutility from negative deviations and "elation" from positive deviations (Jia, Dyer, and Butler 2001). Many decision models in the literature can be defined as special cases of the risk-value model, including models based on prospect theory and expected utility. For example, for the risk-value model $V[\mu_1(x), \eta_2(x), \varphi_2(x)]$, the first-order condition for input choice is

$$\text{(11)} \quad \mu_1^* - cx/\mu_1 = s_2(V_\eta \eta_2^* - V_\varphi \varphi_2^*)$$

where s_2 is defined above, $V_\eta \equiv -(\mu_1 - cx)\frac{V_{\eta_2}}{V_{\mu_1}}$, $V_\varphi \equiv (\mu_1 - cx)\frac{V_{\varphi_2}}{V_{\mu_1}}$, $\eta_2^* \equiv \partial \ln \eta_2 / \partial \ln x$ and $\varphi_2^* \equiv \partial \ln \varphi_2 / \partial \ln x$. In this model, $V_\eta > 0$ is interpreted as *disappointment* and $V_\varphi > 0$ is interpreted as *elation*. If inputs have symmetric effects on output variance, i.e., if $\eta_2^* = \varphi_2^* = \mu_2^*$, then setting $R_2 = V_\eta - V_\varphi$ shows that equations (9) and (11) are equivalent and that risk aversion implies $V_\eta - V_\varphi > 0$. However, if inputs have asymmetric effects on $\eta_2(x)$ and $\varphi_2(x)$, then the two models do not necessarily have the same risk implications, as represented by the MRE. For example, in the mean-variance model with symmetric effects of inputs, the variance could be increasing in x, implying that the MRE in equation (9) is positive, but x also could be downside-risk reducing, so that the MRE in equation (11) could be negative. The risk-value model also can represent other types of preferences—for example, setting $V_\varphi = 0$, the risk-value model can take the form of a mean-negative semivariance model. Both the expected utility model in equation (10) and the risk-value model (11) can be defined with respect to third and higher-order moments. Koundouri, Nauges, and Tzouvelekas (2006) provide an analysis using expected utility with four moments.

Estimating and Testing Partial-Moment Functions

In this section, methods are presented for estimating partial-moment functions and testing the null hypothesis that the effects of inputs on moments are symmetric, by testing for equality of parameters of the negative and positive partial-moment functions. The goal is to specify the partial-moment functions in a way that does not impose arbitrary restrictions on the form of the distribution. Define the first moment model

$$\text{(12)} \quad q = \mu_1(x) + e, \quad E(e|x) = 0$$

where $E(e|x)$ is the expectation of e given x. Note that we could set $\mu_1(x) = r$. Applying the procedures discussed here would result in estimates of partial moments with respect to the reference point r. To simplify the discussion, it is assumed that observations are independent; generalizations for dependent observations are possible with suitable data to estimate off-diagonal elements of the error covariance matrix. Using equations (2) and (12), the higher central moments can be specified as

$$\text{(13)} \quad e^i = \mu_i(x) + v_i, \quad E(v_i|x) = 0, \quad i = 2, 3, \ldots$$

The consistency of the residuals from least squares estimation of equation (12) can be used to show that least squares estimation of equation (13) implemented with $\hat{e}^i$ produces consistent estimates of the parameters (Antle 1983). The error e is generally heteroskedastic, so use of weighted least squares, following procedures outlined in Antle (2010), or a heteroskedastic-consistent estimator, is appropriate. The errors v_i are correlated across equations, so a heteroskedasticity-corrected seemingly unrelated regression (SUR) estimator is more efficient than a single equation estimator.

Using equations (4), (5), and (13), the partial-moment functions can be specified as:

$$\text{(14)} \quad |e|^i = \eta_i(x) + v_{in}, \quad E(v_{in}|x) = 0, \quad i = 2, 3, \ldots, \text{ for } e < 0$$

$$\text{(15)} \quad |e|^i = \varphi_i(x) + v_{ip}, \quad E(v_{ip}|x) = 0, \quad i = 2, 3, \ldots, \text{ for } e > 0$$

where the expectation operators are defined with respect to the truncated distributons for $e < 0$ and $e > 0$. Equations (14) and (15) comprise a system of two equations similar to a switching regression model. In most switching regression models, the econometrician does not observe which regime an observation belongs to, but in this case it is possible to classify observations into the two regimes because we can observe the residuals which converge in distribution to e. Therefore, in large samples, the methods described above for the estimation of full-moment functions can be applied to the partial-moment equations (14) and (15) individually, and under standard regularity conditions, the linear or nonlinear least squares estimates will be consistent and asymptotically normal. However, as with the full moments, the errors generally are not independent and identically distributed (i.i.d.), so correction for heteroskedasticity is appropriate. To test the hypothesis of symmetric effects of inputs, equations (13), (14), and (15) can be estimated separately, and a Chow test can be used to test for the equality of the parameters of equations (14) and (15).

Alternatively, equations (14) and (15) can be combined into one equation following the approach suggested by Quandt (1972). Define the indicator variable δ such that $\delta = 1$ if $e < 0$ and $\delta = 0$ otherwise. From equations (14) and (15) it follows that

$$(16) \quad |e|^i = \delta\eta_i(x) + (1-\delta)\varphi_i(x) + \delta v_{in} + (1-\delta)v_{ip}, \quad i = 2, 3, \ldots$$

Equation (16) can be estimated using the same procedures described above for the full-moment model. Standard asymptotic procedures can be used to test for symmetry by testing for the equality of the parameters of the partial-moment functions. As an alternative to asymptotic tests, nonparametric bootstrap methods suitable for heteroskedastic and skewed error distributions (the so-called wild bootstrap) can be used to construct tests, as discussed further below.

In concluding, it is useful to observe that the full-moment specification equation (13) has both similarities and differences with the partial-moment specification equation (16). First, if the same functional form is used for the full- and partial-moment functions, then equation (16) has twice as many parameters as equation (13). Thus, the flexibility that the partial-moment specification affords, by allowing differential effects of inputs on negative and partial moments, does come at a statistical cost. Second, it should be noted that apart from this increase in the number of parameters, the partial-moment model in equation (16) is statistically equivalent to the full-moment specification in equation (13) for even-order moments and is identical under the symmetry restriction. However, for odd-order moments such as the third, equations (13) and (16) are not equivalent, because the dependent variable in equation (13) has values that are both positive and negative, whereas the dependent variable in equation (16) is always positive. This fact means that equation (16) will fit the data differently than equation (13) and suggests that the flexibility afforded by the partial-moment specification may be particularly important for the estimation of odd-order moments, as borne out in the case study presented below.

Monte Carlo Simulation

The econometric procedure for estimation and testing of partial moments involves splitting the sample into subsamples above and below the reference point, so it is important to consider the performance of the tests proposed for statistical significance of partial moments and for symmetry. To this end, Monte Carlo simulations were performed using combinations of three data-generating processes (DGPs) and two functional forms. I use the notation $t(m)$ for a variate following a standard triangular distribution on $[0, 1]$ with mode m, and z for a standard normal variate. Exogenous variables are generated as $x_1 = 1 + \exp(1 + \log t(.5))/1.4$, $x_2 = 1 + \exp(1 + z)/3.8$, producing right-skewed distributions typical of input data from a sample of farms. The production function is specified with an additive error defined according to one of the following DGPs:

DGP1: $e_1 \sim 3(t(0.5) - 0.5)$.
DGP2: $e_2 = e_1 - \delta\eta + (1-\delta)\varphi$, $\delta = 1$ if $z < 0$
$\delta = 0$ otherwise,
$\eta^2 = 5 + 0.5\log(x_1) + 0.5\log(x_2)$, $\varphi^2 = 5 + \log(x_1) + \log(x_2)$
DGP3: DGP2 with $\eta^2 = 5 + 0.5\log(x_1) + 1.5\log(x_2)$, $\varphi^2 = 5 + 1.5\log(x_1) + 0.5\log(x_2)$

Table 1. Size and Power of Tests for Partial Second-Moment Functions

	Model						
	1	2	3	4	5	6	7
DGP Function	1 SLF	2 SLF	2 CEF	3 SLF	3 CEF	4 SLF	4 CEF
			Sample Size = 100				
$\varphi=0$	0.07	**0.99**	**0.77**	**0.98**	**0.70**	**0.64**	**0.85**
$\eta=0$	0.06	**1.00**	**0.80**	**1.00**	**0.91**	**0.56**	**0.71**
$\eta=\varphi$	0.07	0.04	0.22	**0.66**	**0.54**	**0.58**	**0.71**
			Sample Size = 300				
$\varphi=0$	0.05	**1.00**	**1.00**	**1.00**	**0.98**	**0.85**	**0.93**
$\eta=0$	0.04	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**	**0.95**
$\eta=\varphi$	0.05	0.03	0.25	**1.00**	**0.90**	**0.98**	**0.91**
			Sample Size = 500				
$\varphi=0$	0.05	**1.00**	**1.00**	**1.00**	**1.00**	**1.00**	**0.97**
$\eta=0$	0.04	**1.00**	**1.00**	**1.00**	**1.00**	**0.99**	**0.98**
$\eta=\varphi$	0.04	0.03	0.19	**1.00**	**0.98**	**1.00**	**0.95**

Note: $\varphi=0$ and $\eta=0$ are tests for zero slope coefficients, $\varphi=\eta$ is test for equality of negative and positive partial moment coefficients. Figures are the percent of likelihood ratio statistics exceeding the nominal 5% critical value. Bold indicates cases where the null hypothesis is false. Data-generating processes (DGPs) are defined in the text. CEF = constant elasticity functional form; SLF = semi-log functional form.

DGP4: $e_3 = q_2 - \mu_1(x_1, x_2), q_2 = q_1^{.5}x_2^{.5} + \delta q_1 + (1-\delta)x_2 + t(0.1), q_1 = x_1 + t(0.1)/x_1$
$\delta = 1$ if $t(.9) < 0.5, \delta = 0$ otherwise, $\mu_1(x_1, x_2) =$ quadratic function of x_1 and x_2.

The two functional forms used for estimation are the constant elasticity function CEF = $\exp(\gamma_0 + \gamma_1 \ln x_1 + \gamma_2 \ln x_2)$ and the semi-log function SLF = log(CEF). The model parameters and error variances were scaled to produce model fits that are similar to the case study of Ecuadorian potato production presented in the next section.

Table 1 presents the results of the Monte Carlo analysis of seven models replicated 1,000 times for sample sizes of 100, 300, and 500. Likelihood ratio test statistics were computed, based on SUR estimates using the procedure described above. Model 1 represents the case of an additive-error production function with an i.i.d. error. The table shows the results from the SLF, but very similar results not shown here were obtained with the CEF function, producing test sizes very close to the nominal significance level for all sample sizes. Models 2 and 3 represent SLF and CEF functions used to estimate DGP2, a symmetric heteroskedastic process. The SLF function produces tests with high power and sizes slightly less than the nominal test size of 5% at all sample sizes, even for the small sample size of 100. This shows that when the functional form is a good approximation to the true function, the test performs extremely well. This was also confirmed by examining the distributions of the parameter estimates, which were symmetrically distributed around the true values. Model 3 shows that when the functional form is different than the true one, the power is high for sample sizes of 300 and 500, but the size of the test is in the range of 20–25%, indicating that the risk of a Type I error is higher than indicated by the nominal size of the test. This finding suggests that in the realistic case of some specification error, tests will tend to reject the null too often, and conservative test sizes should be used. Models 4 and 5 repeat the tests using SLF and CEF with the parameters of the DGP modified so that the distribution is asymmetric and heteroskedastic. The results show that the tests for slope significance are quite powerful even with the small sample size of 100, and the symmetry tests are powerful for the 300 and 500 sample sizes. Finally, Models 6 and 7 are designed to represent a DGP similar to those illustrated in figure 1, with a nonlinear production function and stochastic inputs generating skewed output distributions similar to those in figure 2. The results show that the slope and symmetry tests are powerful for the 300 and 500 sample sizes. We can conclude that the proposed tests may lack adequate power for small sample sizes, but power increases rapidly with sample size, and is high for samples of 300 or more.

Asymmetric Effects of Inputs on Output Distributions: The Case of Ecuadorian Potato Production

In this section the methods presented above are applied to data collected in the Carchi Province of northern Ecuador in a study of economic, environmental, and health effects of pesticide use in potato production (Antle, Capalbo, and Crissman 1998). These data provide a useful test of the effects of inputs on the output distribution because farmers face a substantial degree of production risk in Carchi potato production—they invest heavily in seed and fertilizer and face potentially catastrophic losses from the late blight disease, which is prevalent in the Carchi region. Carchi farmers typically apply a combination of fungicides and insecticides more than seven times during the growing season to control late blight and important insect pests such as the Andean weevil. In addition to the methodological interest, the issue of the risk effects of inputs is of interest in its own right. In fact, there do not appear to be any studies of the risk effects of fungicide use in potato production, even though late blight is an important global disease. Moreover, studies of fertilizers have produced mixed results, and empirical generalizations are lacking despite the obvious importance of nutrients to crop agriculture (e.g., Isik and Khanna 2003).

Another advantage of using this case study to demonstrate the proposed econometric methods is data quality. These data were collected using a dynamic survey methodology with quality control to ensure data accuracy. Skilled enumerators interviewed farmers on a monthly basis to collect information about input use, output was measured in the field at harvest, and data quality was assured using follow-up to investigate observations that appeared to involve respondent error or coding error. This type of procedure is particularly important for accurate measurement of input use such as fungicides and insecticides, because farmers use many different commercial formulations and often combine them into a mixture of several products. In addition, the fungicide and insecticide data were quality adjusted using hedonic methods to account for the various types of chemicals used. This quality-adjustment issue is critical for many types of pesticides, because recommended application rates can differ in some cases by orders of magnitude across different chemical types used to treat a given pest. Many pesticide productivity studies do not address the quality-adjustment issue; furthermore, many farm surveys rely on farmer recall (as opposed to periodic interviews), which increases the likelihood of measurement errors in input quantities. Accurate output measurement is important for risk analysis, because errors in output measurement result in errors in residuals, which are then squared or cubed, thus magnifying errors and distorting inferences.

The data used in the analysis are summarized in Table 1. The output variable is potato yield, adjusted for quality using a hedonic procedure (Antle, Capalbo, and Crissman 1998). Explanatory variables are land (size of field), hired labor (land preparation and preharvest labor for input application), mineral fertilizer, quality-adjusted quantities offungicide and insecticide, and a dummy variable indicating whether the previous crop was potato. Two dummy variables were also included in the model, representing the mid- and high elevation agro-ecological zones.

An important issue in production function estimation is the potential endogeneity of inputs that are selected during the growing season, as is typical of pesticides and the labor inputs used for their application. Labor for land preparation and fertilizer applications occur before production shocks occur and are clearly predetermined relative to output. However, an average of about seven applications of fungicides and insecticides are made during the growing season, so pesticides could be endogenous. To test for input endogeneity, the mean production function was estimated using two-stage least squares, and a Hausman test for endogeneity of fungicides, insecticides, and labor was carried out, using output and input prices as instruments. The resulting chi-square test statistic (4.79, with 9 degrees of freedom) did not reject input exogeneity at any conventional significance level. This finding is consistent with evidence indicating that farmers spray regularly on a schedule, because a late blight infestation is a potentially catastrophic event and late blight is highly prevalent in this region of Ecuador (Antle, Capalbo, and Crissman 1998).

Specification of the mean function is important to the properties of the higher moments estimated with residuals, so the mean function was specified as $q = g[(f(x)] + e$ for the additive-error model, where $f(x)$ is a function that is quadratic in the logs of the continuous variables (i.e., a translog function), and $g[.]$ was specified as either the identity or the exponential function. The Ramsey reset test was applied to the linear form and the quadratic specification was not rejected. Both forms produced similar results, and the results based on the exponential specification which fit slightly better are presented here.The Jarque-Bera test applied to the residuals from the mean function rejects normality at better than 1%, and skewness and kurtosis coefficients are individually significant at 1% and 5% levels.

The second full and partial moments, and the third partial moments, were specified in constant-elasticity form. A constant-elasticity functional form cannot be used for the full third moment because the dependent variable is the cubed residual, which has both positive and negative values. Other studies have used a quadratic function which allows for both positive and negative values of the dependent variable (e.g., Antle and Goodger 1986; Groom et al. 2008), but the quadratic is parameter intensive and prone to muticollinearity problems, so the SLF form was used with the dependent variable scaled by its mean so the parameters are elasticities.

The previous section showed that the cumulative probability function $\Phi(x)$ is a function of x, and derivation of full-moment elasticities from partial moments requires estimates of this function's derivatives (equation (8)). This cumulative function can be estimated using several specifications. For this case it was specified as a linear probability model $\delta = h(x) + u$ and as a logistic regression $\delta = 1/(1 + \exp(h(x)) + u$, where h($x$) is a polynomial in x and $\delta = 1$ if $e < 0$ and $\delta = 0$ otherwise. However, the results were statistically insignificant and are not presented here. As a result, in constructing tests and in the analysis of the model, Φ was set equal to its sample mean value of 0.42.

In addition to the tests for their significance and symmetry, the partial-moment functions can be used to test for restrictions implied by location-scale distributions. This is of interest because location-scale distributions have been used in a number of empirical production studies, following the model with multiplicative heteroskedasticity proposed by Just and Pope (1978). Antle (1983) observed that this model imposes restrictions across second and all higher-order moments, of the form $\mu_i^* = (i/2)\mu_2^*$, where μ_i^* is the elasticity of the ith moment with respect to an input. Following Antle (2010), the full- and partial-moment functions in equations (13), (14), and (15) can be specified in multiplicative error form, thus across-moment restrictions apply to both full and partial moments in multiplicative error models. Using the constant-elasticity functional form for partial moments, these restrictions can be tested as across-equation restrictions on the parameters using standard test statistics. Antle (2010) shows that location-scale distributions imply the equality of parameters from additive-error and multiplicative error models of even-order moments such as the variance. These within-moment restrictions were tested by jointly estimating additive-error and multiplicative-error versions of equation (16) using heteroskedasticity-corrected SUR, where the multiplicative error model is implemented by regressing $\ln |e|^i$ on $\delta \ln \eta_i(x)$ and $(1 - \delta) \ln \varphi_i(x)$. Note that these within-moment tests can be implemented for only even-order full moments but can be implemented for both even- and odd-order moments using the absolute partial moments.

Empirical Results

Table 2 presents estimates of the mean, full second and third moments, and second- and third-order partial-moment functions for the Ecuadorian potato data. To simplify the presentation, Table 2 presents elasticities of the mean function estimates from a constant-elasticity model; these are similar to the elasticities implied by the quadratic-in-logs function that was estimated to construct the residuals used to estimate the higher moments. The second and third full-moment functions (equation (13)) and the additive-error second and third partial-moment functions (equation (16)) were estimated as a two-equation SUR,

Table 2. Summary Statistics for Potato Production, Carchi, Ecuador

	Mean	Std. Dev.
Yield (mg/ha)	24.93	10.99
Area (ha)	0.62	0.56
Previous crop (1 = potato, 0 = other)	0.55	0.50
Fertilizer (kg/ha)	631.02	246.82
Insecticide	1.00	0.61
Fungicide	1.00	0.75
Labor (days/ha)	33.06	24.08
Mid-zone	0.34	0.46
High-zone	0.46	0.50

Note: Number of observations = 316. Insecticide and fungicide in quality-adjusted units normalized to have a mean of 1.0.

weighted by their respective variance functions to correct for heteroskedasticity. Likelihood ratio statistics were calculated for tests of slope parameters, the symmetry test for the equality of the partial-moment parameters, and location-scale tests.

The Monte Carlo results presented above indicate that the sample size of 316 should provide reasonable power for the hypothesis tests of the partial-moment functions. Another concern with using these tests is whether the asymptotic distributions of the test statistics are reasonable approximations to the actual finite-sample distributions. To address this concern, a variety of simulation-based (bootstrap) procedures have been developed. An appropriate method for the partial-moment models, which are expected to have residuals with non-i.i.d. and skewed distributions, is the "wild bootstrap" (Davidson and Flachaire 2008; Mackinnon 2009). This procedure was used to replicate *p*-values for the tests presented in table 2, by comparing the statistics in table 2 to statistics generated with residuals derived from the corresponding restricted regressions with the wild bootstrap adjustment. As shown in table 2, the empirical distributions generally result in higher *p*-values. Nevertheless, all but one of the *p*-values for tests of significance of partial-moment function slope coefficients, for symmetry, and for between-moment restrictions were less than 0.05, so the inferences drawn from the statistics in table 2 based on asymptotic distributions are the same as those based on the empirical distributions.

Table 1 shows that the symmetry restrictions are rejected for both second and third moments. The within-moment tests for location-scale are not significant, but the across-moment restrictions are rejected. The failure to reject the within-moment restrictions implied by location-scale distributions is explained by the poor fit and low statistical significance of the full-moment functions, and also by the relatively small sample size. In this case there are 133 observations for the negative partial moments and 183 observations for the positive partial moments. The Monte Carlo analysis by Antle (2010) showed that the power of the within-moment test was low for a sample size of 100 but increased substantially with sample size and was high for a sample size of 500. Furthermore, it is apparent from the R^2 statistics and tests for slope significance that the partial-moment functions fit the data better than the full moments. Most notably, the full third-moment model is not statistically significant and has a very poor fit, whereas the third partial-moment functions have much better fit and are highly significant. This result confirms that the partial-moment specification is particularly valuable for estimating odd-order moments, as hypothesized earlier in the discussion of the partial-moment specification. We can conclude that inputs do have asymmetric effects on moments, as hypothesized, and that the output distribution is not likely to be a member of a location-scale family.

Fertilizer's effects on the output distribution are likely to depend on the conditions under which it is used. In the case of northern Ecuador, where climate variability is low and water-holding capacity of the soil is high, farmers use relatively high rates of fertilizer application (table 1). Under these conditions, an increase in fertilizer use can be expected to shift the yield distribution toward the production frontier and decrease the positive skew or increase the negative skew of the distribution (see figure 2). However, in shifting the distribution away from zero toward the frontier, it is not clear what effect an increase in fertilizer use would have on the variance. Pesticides—particularly fungicides—are expected to be risk reducing, but there are several ways that these effects could change the shape characteristics of the output distribution. For an input to be risk reducing in the three-moment expected utility model and in the risk-value model, it need not be variance decreasing, as shown above (see equations (10) and (11)). If pesticide inputs interact with other inputs such as fertilizer so that higher potential yields are realized, then variance could increase while the skewness increases, but the input could still be risk reducing overall. Similarly, human labor used for input application, weeding, and cultivation should be risk reducing, but effects on variance and skewness are not clear *a priori*.

With respect to the full moments, the mean function shows that fertilizer and insecticides have positive, statistically significant effects, as expected. Both the fungicide and hired labor elasticities are small and close to zero, suggesting that they are used beyond profit maximizing levels, as would be the case if they were risk-reducing inputs. The full variance function is decreasing in fertilizer and increasing in fungicides and labor. The full third moment is statistically insignificant overall, and none of the individual coefficients is statistically significant; the third moment was also tested for input interactions which were also statistically insignificant. Thus, from the perspective of the symmetric mean-variance risk model (equation (9)), these results would be interpreted as indicating that relationships between inputs and production risk are weak, with labor the only variable that is statistically significant, and is risk increasing, contrary to expectations. Fungicides are found to be weakly risk increasing but not statistically significant, thus suggesting that fungicides have no clear effect on production risk, contrary to expectations. Combined with the fact that the mean production elasticity of fungicides is near zero, the full-moment model indicates that fungicides have little productive value to farmers, contrary to expectations in this system where late blight is a major production risk and farmers apply large quantities of fungicides.

The partial-moment functions present a very different picture of the risk effects of inputs. Symmetry of the variance and third-moment functions is strongly rejected, and both second and third positive partial moments are statistically significant. In interpreting the negative third partial moment, keep in mind that it is estimated as an absolute moment, so the sign of the parameters in table 2 are the opposite of the effect on the full moment (see equations (7) and (8)). Examination of the parameters shows that fertilizer has large and statistically significant effects on the positive and negative tails of the distribution, resulting in a large reduction in skewness, consistent with the output distribution shifting toward the frontier, similar to case C illustrated in figure 1. Similarly, the partial third moments show that fungicides and labor strongly increase the positive skew of the distribution. These effects are very different from those implied by the full-moment model. Referring back to figure 2, we can see that the low-input and high-input distributions reflect these two effects. On the one hand, fertilizer shifts the mass positively but maintains some mass in the lower tail, thus reducing skewness; on the other hand, fungicides and labor extend the positive tail and truncate the negative tail, thus increasing skewness.

To investigate the behavioral implications of these results, table 3 presents the MREs of inputs calculated as percentages of expected returns, using equations (9), (10), and (11). For the calculations in table 3, the Arrow-Pratt partial risk aversion coefficient was set equal to 1, and the downside partial risk aversion coefficient was set equal to 2, consistent with estimates of these parameters in the literature (e.g., see Antle 1987; Di Falco and Chavas 2009). The risk-value model was specified with $V_\eta = 1$ and $V_\phi = 0.5$, thus implying the same level of risk aversion as the Arrow-Pratt coefficient in the expected utility model, if input effects on variance are symmetric (table 4).

When interpreting the implications of the expected utility model and the risk-value model, it is important to keep in mind that the two models attach value to asymmetry in different ways. In the case of the two-moment expected utility model, the estimates in table 3 imply that fertilizer is necessarily a risk-reducing input because it reduces variance. However, in the case of the risk-value model, the value of this variance reduction depends on the behavior of the positive and negative partial second moments. The data show that fertilizer reduces variance primarily by reducing the positive partial moments and increasing the negative partial moments, thus having a net risk-increasing effect (third column of table 3). The risk-value model also shows that fungicides and labor are risk reducing, as expected, whereas the mean-variance expected utility model implies that they are risk increasing. Insecticides show small MREs for both models. Field size (area) and previous crop have small negative risk effects according to the risk-value model, whereas the expected utility model implies that area is risk increasing and previous crop (indicating that previous crop is potato) is risk reducing.

Another way to check for the consistency of the decision models with theory is to compare the signs of the mean production elasticities with the MREs. The model shows that mean production elasticities of fertilizer and insecticides are positive and statistically significant, whereas the mean elasticities of fungicides and labor are near zero. From a behavioral perspective, this situation is consistent with fertilizer and insecticides being either marginally risk neutral or marginally risk increasing and with fungicides and labor being marginally risk reducing. The data in table 3 show that the signs of the MREs derived from the expected utility models are inconsistent with the signs of the mean production elasticities for all of the inputs, whereas the risk value model is consistent with them. An interesting implication of these findings is that the expected utility model implies that farmers are overusing fungicides, in the sense that the mean marginal product is near zero and the MRE of fungicides is positive. In contrast, the risk-value model is consistent with rational use of fungicides by farmers, in the sense that a negative MRE is consistent with a low mean marginal product.

Table 3. Heteroskedasticity-Corrected SUR Estimates of Full- and Partial-Moment Function Parameters for Ecuadorian Potato Production

	μ_1	μ_2	μ_3	η_2	φ_2	η_3	φ_3
Intercept	0.053	−4.12	−9.73	−6.88	−4.96	−10.53	−5.76
	(0.19)	(−2.33)	(−0.60)	(−4.10)	(−3.22)	(−4.67)	(−3.35)
Area	0.010	0.166	0.122	0.085	0.231	0.177	0.267
	(0.61)	(1.58)	(0.13)	(0.86)	(2.36)	(1.44)	(2.46)
Previous crop	−0.010	−0.156	−0.163	−0.278	−0.330	−0.438	−0.460
	(−0.31)	(−0.93)	(−0.09)	(−1.55)	(−1.92)	(−1.78)	(−2.25)
Fertilizer	0.189	−0.056	0.734	1.00	−.537	1.820	−0.699
	(5.16)	(−0.23)	(0.37)	(4.40)	(−2.36)	(5.59)	(−2.64)
Insecticide	0.142	−0.036	−1.01	−0.139	−0.342	−0.177	−0.454
	(6.49)	(−0.28)	(−0.81)	(−1.17)	(−2.77)	(−1.20)	(−3.22)
Fungicide	0.002	0.181	0.351	−0.125	0.427	−0.306	0.536
	(0.11)	(1.75)	(0.43)	(−1.82)	(4.07)	(−4.42)	(4.18)
Hired labor	−0.014	0.358	−0.055	0.078	0.479	0.120	0.543
	(−0.64)	(2.60)	(−0.04)	(0.59)	(3.69)	(0.70)	(3.65)
Mid-zone	0.052	0.264	−1.072	0.193	−0.082	0.370	−0.225
	(1.07)	(1.15)	(−0.68)	(0.72)	(−0.44)	(0.82)	(−1.08)
High-zone	0.189	0.282	−2.153	0.762	−0.130	1.205	−0.251
	(4.06)	(1.24)	(−1.15)	(2.94)	(−0.71)	(2.78)	(−1.25)
R^2	0.266	0.080	0.012	0.084	0.144	0.143	0.150
Likelihood ratio parameter tests							
Slopes = 0	129.6	18.4	2.9	32.9	36.0	75.5	35.8
	(<.01/<.01)	(.02/.07)	(.94/.96)	(<.01/0.10)	(<.01/.04)	(<.01/<.01)	(<.01/.05)
Symmetry				54.0		45.8	
				(<.01/.02)		(<.01/.02)	
Location-scale							
Within-moment				14.7	2.9	14.9	4.2
				(.06/.30)	(.94/.96)	(.07/.37)	(.83/.88)
Between-moment				66.6	64.3		
				(<.01/.04)	(<.01/.02)		

Notes: t-Statistics are in parentheses below parameters. Asymptotic/bootstrapped p-values for likelihood ratio tests are in parentheses below slope, symmetry and location-scale test statistics. Number of observations for full moments = 316; for negative partial moments = 133; and for positive partial moments = 183. SUR = seemingly unrelated regression.

In conclusion, these results show that the hypothesis of symmetric input effects is strongly rejected, as are the between-moment restrictions implied by a location-scale distribution. The results also show that the partial-moment model does a better job of representing the effects of inputs on the changes in the symmetry of the output distribution, in terms of both second and third moments. The partial-moment models provide greater insight into the relationship between inputs and the shape characteristics of the output distribution and show that inputs often have different effects on positive and negative tails of the distribution that are obscured by the full-moment models. Moreover, the expected utility model and the risk-value model have opposite implications for the risk effects of fertilizer, fungicides, and labor inputs. The risk-value moment model provides an explanation for the counter-intuitive results produced by the expected utility model, and is consistent with the hypothesis that fertilizer is risk increasing and that fungicides and labor are risk reducing.

Table 4. Marginal Risk Effects (MREs) of Inputs for Ecuadorian Potato Production Implied by the Expected Utility Model and the Risk-Value Model (percent)

	Expected Utility		
	MRE2	MRE23	Risk-Value
Area	6.3	5.8	−0.8
Prev crop	−5.9	−6.6	−0.4
Fertilizer	−2.1	−5.8	14.0
Insecticide	−1.4	2.7	1.1
Fungicide	6.9	5.5	−4.4
Labor	13.6	13.8	−2.7

Notes: MRE2 = mean-variance model; MRE23 = mean-variance-skewness model.

Conclusions and Implications

This article discusses the conceptual basis for asymmetric effects of inputs on output distributions and how those effects can be related to models of decision making. The conceptual analysis of the relationship between inputs and the shape of output distributions showed that inputs are likely to have different effects on positive and negative tails of distributions. Moreover, the risk-value model shows that the effects of inputs on partial moments of output distributions can result in different marginal risk effects of inputs than full-moment models. Econometric methods are presented for estimation of partial-moment functions, and partial-moment tests for input symmetry and for restrictions implied by location-scale distributions are developed. A Monte Carlo study indicated that these tests may be powerful in sample sizes of 300 or larger.

Data from a potato producing region of northern Ecuador were used to illustrate the application of these methods and to investigate the risk effects of inputs in this system. The results reject the restrictions of partial-moment symmetry and the between-moment restrictions implied by location-scale distributions. The results also show that inputs can have substantially different effects on positive and negative tails of distributions and that the full third moment may not provide an accurate representation of the effects of inputs on the asymmetry of the output distribution. This result is likely due to the fact that the full third moment cannot effectively represent both positive and negative deviations from the mean in one functional relationship. Thus, the econometric results confirm that the partial-moment specification provides a better statistical representation of the output distribution than a full-moment specification, particularly for odd-order moments such as the third. The case study also showed that the expected utility model based on full moments may have different risk implications than a risk value model based on negative and positive partial moments.

In this article asymmetry of second and third moments was considered. More generally, the fourth moment (kurtosis) also could be considered. Indeed, the statistics for the Ecuadorian potato yield distribution showed that the degree of kurtosis increased with the levels of input use (see figure 2), suggesting that if decision makers are averse to extreme outcomes, it may be useful to extend the model to the fourth moment. Another example where fourth moments may be relevant is in research on changes in environmental conditions, such as climate change, which have been hypothesized to lead "fat-tailed" distributions. However, to reliably study the properties of the tails of distributions, large samples of high-quality data are likely to be necessary. This remains a potentially important topic for further research, perhaps using both Monte Carlo methods as well as new and better data.

Just and Pope (2003) observed that a limitation of many econometric models of production risk is that restrictive assumptions about the form of the output distribution and producers' objective functions are needed to secure identification of technology parameters and risk attitude parameters. The partial-moment model, by providing a flexible and empirically tractable way to estimate the effects of inputs on asymmetric output distributions, presents an opportunity to advance the empirical understanding of how management decisions affect production risk. The risk-value model also appears to provide a simple but flexible way to represent risk preferences with asymmetric distributions using partial moments. Further research is needed to determine which partial moments are important to decision makers, and whether the mean is the appropriate reference point. It may also be possible to utilize the moment-based econometric methods to estimate parameters of a risk-value model along the lines of the approach developed by Antle (1987) for full moments. In doing so, researchers should note the results presented by Lence (2009), suggesting that econometric estimation of risk attitudes may be unreliable unless data represent sufficiently large and asymmetric risks. The results reported here indicate that yield risk caused by the late blight fungus in potato production is an example of this type of risk.

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