CSE215: Lecture 02
Foundations of Computer Science

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Propositional Logic

Predicate Logic

Proof

Why does a computing system fail (or work)?

Sequences

Sets

Functions

Relations

Today's objectives

Know a list of key things that will be covered in the exams

Today's work

book chapter	Topics	Exam problems
2	Propositional logic	2021-final, pb 1
3	Predicate logic	2021-midterm1, pb3
4	Proof	2021-final, pb4
5	Sequences	2021-final, pb7
6	Sets	2021-midterm2, pb2
7	Functions	2021-final, pb9
8	Relations	2021-final, pb11

How we proceed today:

- We first go over SBU exam problems to highlight the "key" concepts.
- We will then go over the solutions.
- No worry if you do not understand the details.

Key concepts

Propositional Logic
Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Key: Truth Table

Truth table for p ^ q

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

Key: Negation & quantifiers

- $\sim(\forall \boldsymbol{x}, P(\boldsymbol{x})) \equiv \exists \boldsymbol{x}, \sim P(\boldsymbol{x})$
- $\sim(\exists \boldsymbol{x}, P(\boldsymbol{x})) \equiv \forall \boldsymbol{x}, \sim P(\boldsymbol{x})$

Proof — Final 2021

Problem 4. [5 points]
Prove that the sum of the squares of any two consecutive odd integers is even.

Key: Prove things about integers from basic facts

Example of basic facts:

- an even integer can be written as 2*n;

- (x+y)^2= x^2 + 2xy + y^2

Sequences - Final 2021

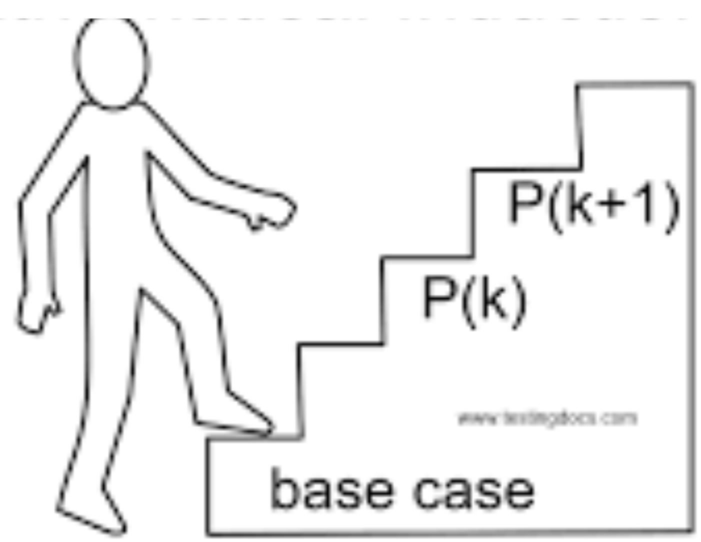
Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

Key: Use Mathematical Induction to show facts about integers



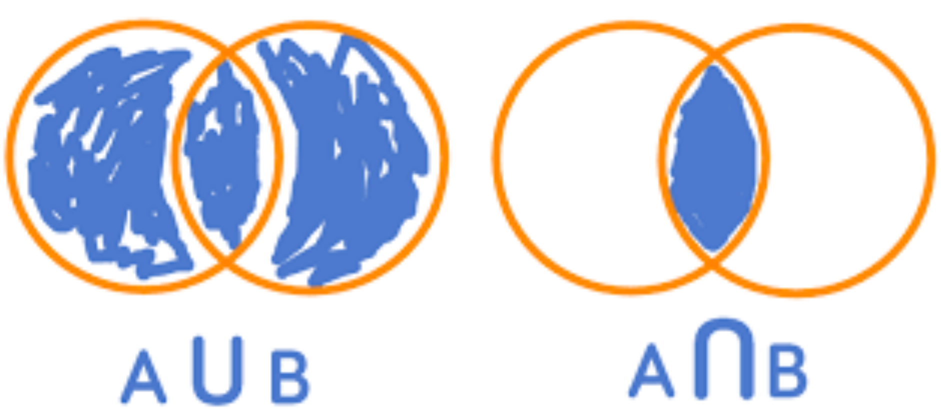
Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) **[1 point]** $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

Key: Union and intersection on Sets



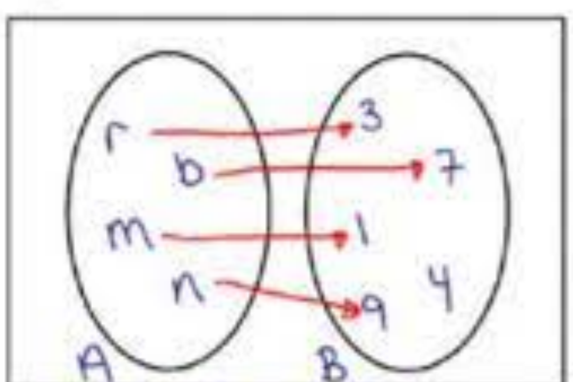
Functions — Final 2021

Problem 9. [5 points]

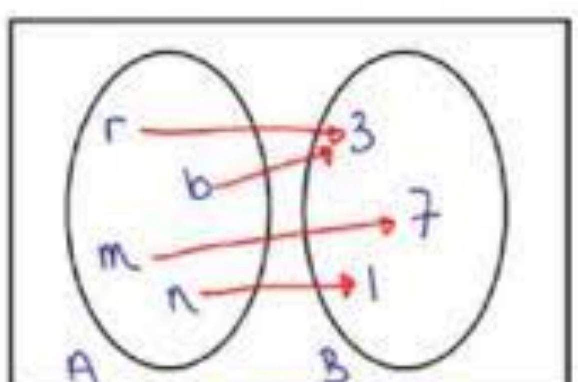
Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		

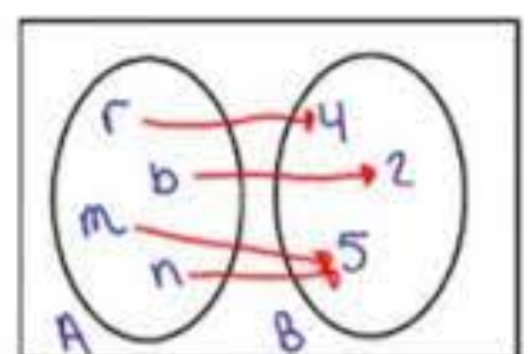
Key: One-to-one and onto functions



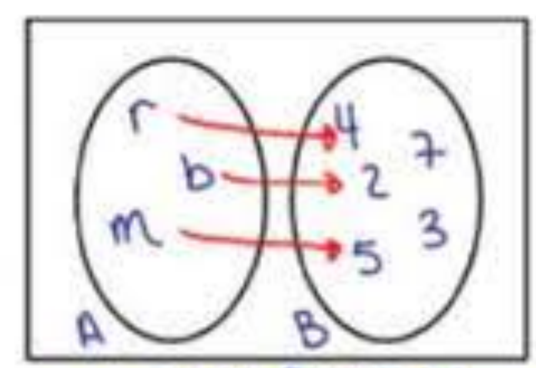
"one-to-one"



NOT "one-to-one"



"ONTO"
(since all elements in B are used)



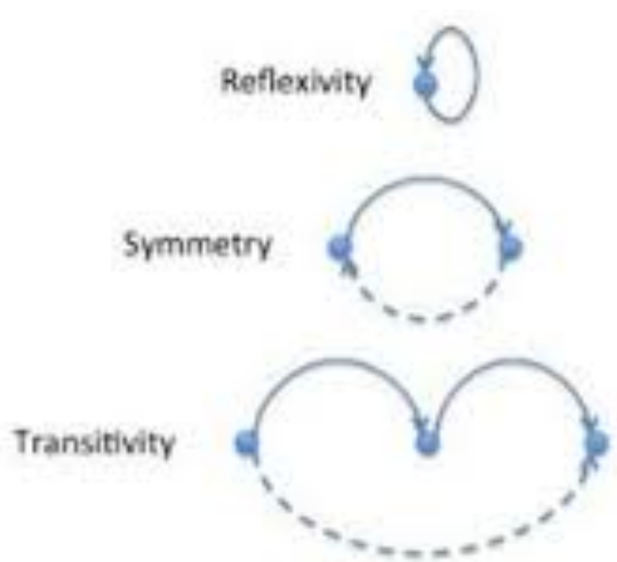
Not "ONTO"
(the 7 and 3 in B are not used)

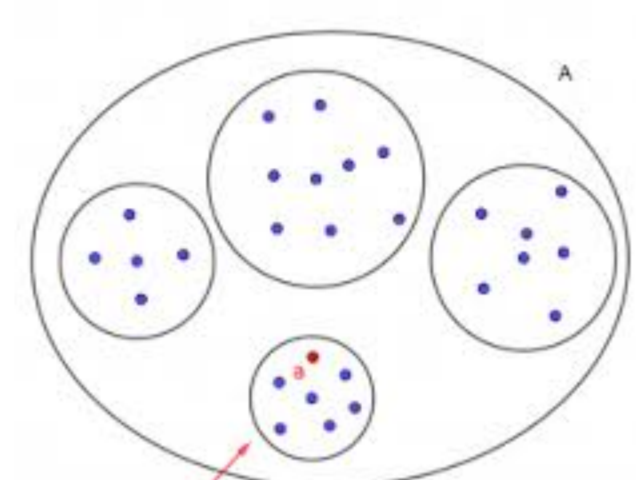
Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A, we have $p\ R\ q \Leftrightarrow p$ has the same birthday as q.
Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

Key: Equivalence relations and Equivalence classes





equivalence class

Break;

Solution overview

Propositional Logic
Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

p	q	r	p AND (q OR r)	p AND (q AND r)	p AND (q OR r) <==> p AND (q AND r)
T	T	T	T	T	T
T	T	F	T	T	T
T	F	F	F	F	T
T	F	T	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	T
F	F	T	F	F	T

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]
Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ **such that** $p(x, y)$

(g) [1 point] $\forall x, \exists y$ **such that** $p(x, y)$

- There exists x, there exists y, ~p(x,y)
- There exists x, for all y, ~p(x,y)

Proof — Final 2021

Problem 4. [5 points]
Prove that the sum of the squares of any two consecutive odd integers is even.

Proof

- We need to show: for any integer n, (2n+1)^2 + (2n+3)^2 is even.
- That is to say, we need to show the following proposition holds:
- for any integer n, 8n^2 + 16n + 10 is even.
- The formula above can be rewritten as 2 (4n^2 + 8n + 5) which must be even.

QED.

Sequences - Final 2021

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^{n} i(i!) = (n+1)! - 1.$$

- Proof.
  - Let P(n) be the predicate 1* 1! + 2 * 2! + … n* n! = (n+1)! -1
  - Base step: We prove P(1).
  - Inductive step: We prove for any integer k>=1, P(k) -> P(k+1)
    - Let k be an arbitrary integer and k>=1.
    - Assume P(k) holds. That is 1* 1! + 2 * 2! + … k* k! = (k+1)! -1
    - We need to prove P(k+1), namely, 1* 1! + 2 * 2! + … (k+1) * (k+1)! = (k+2)! -1
    - Following assumption P(k), Left-hand-side above = (k+1)! -1 + (k+1) * (k+1)! which equals to Right-hand-side above.
- QED.

Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U.

(a) **[1 point]** $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

- The statement is false. As a counter example:
  - A={1}, B={2}, C={1,2}.
  - Left-hand-side becomes empty set
  - Right-hand-side becomes {1}

Functions — Final 2021

Problem 9. [5 points]

Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \to \mathbb{Z}$		

- Yes: One to one funciton
- No: Onto function

Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A, we have $p \; R \; q \Leftrightarrow p$ has the same birthday as q.
Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

- R is an equivalence relation, as it is
  - reflective (p R p),
  - symmetric p R q <==> q R p
  - transititive p R q, q R r ==> p R r
- Equivalence classes is the set of the sets of people of A that have the same birthday.

Today's take-away

book chapter	Topics	Exam problems	Key
2	Propositional logic	2021-final, pb 1	truth table
3	Predicate logic	2021-midterm1, pb3	negation on quantifiers
4	Proof	2021-final, pb4	facts about integers
5	Sequences	2021-final, pb7	math induction
6	Sets	2021-midterm2, pb2	unions and intersections
7	Functions	2021-final, pb9	1-1 and onto
8	Relations	2021-final, pb11	equiv. rel. and classes

Thank you for your attention!