

CSE215: Lecture 02

Foundations of Computer Science

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**Propositional
Logic**

**Predicate
Logic**

Proof

**Why does a computing system
fail (or work)?**

Sequences

Sets

Functions

Relations

Today's objectives

Know a list of **key things**
that will be covered in the
exams

Today's work

book chapter	Topics	Exam problems
2	Propositional logic	2021-final, pb 1
3	Predicate logic	2021-midterm1, pb3
4	Proof	2021-final, pb4
5	Sequences	2021-final, pb7
6	Sets	2021-midterm2, pb2
7	Functions	2021-final, pb9
8	Relations	2021-final, pb11

How we proceed today:

- We first go over SBU exam problems to highlight the “key” concepts.
- We will then go over the solutions.
- No worry if you do not understand the details.

Key concepts

Propositional Logic

Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

Key: Truth Table

Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

Key: Negation & quantifiers

$$\blacksquare \sim(\forall \mathbf{x}, P(\mathbf{x})) \equiv \exists \mathbf{x}, \sim P(\mathbf{x})$$

$$\blacksquare \sim(\exists \mathbf{x}, P(\mathbf{x})) \equiv \forall \mathbf{x}, \sim P(\mathbf{x})$$

Proof — Final 2021

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

**Key: Prove propositions about integers
using basic facts**

Example of basic facts:

- an even integer can be written as $2n$;
- $(x+y)^2 = x^2 + 2xy + y^2$

Sequences - Final 2021

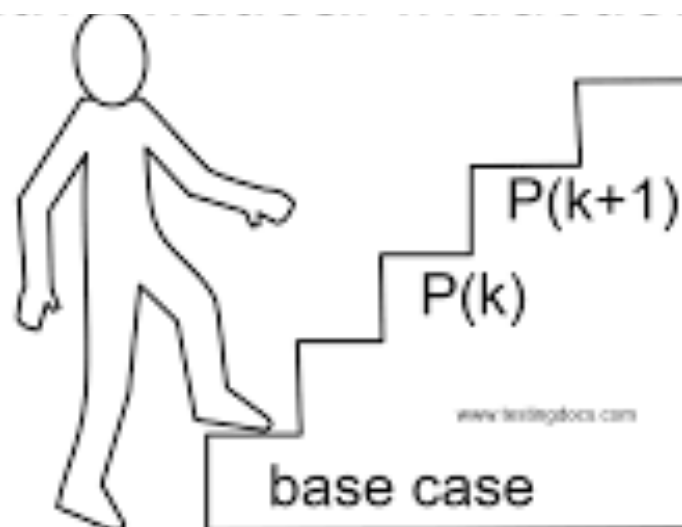
Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^n i(i!) = (n+1)! - 1.$$

Key: Use Mathematical Induction to show facts about integers



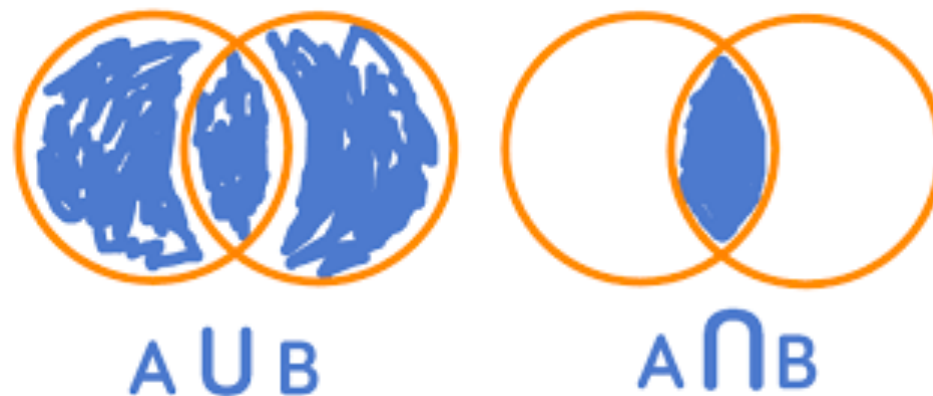
Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U .

(a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

Key: Union and intersection on Sets



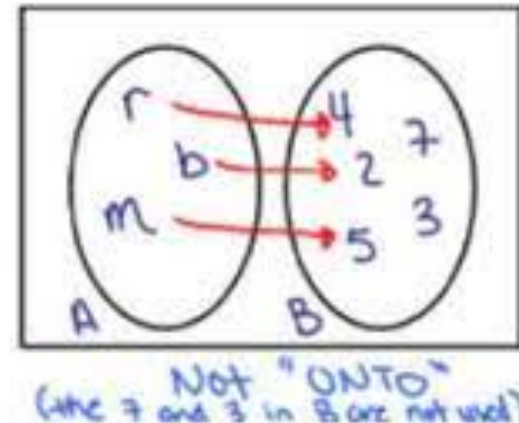
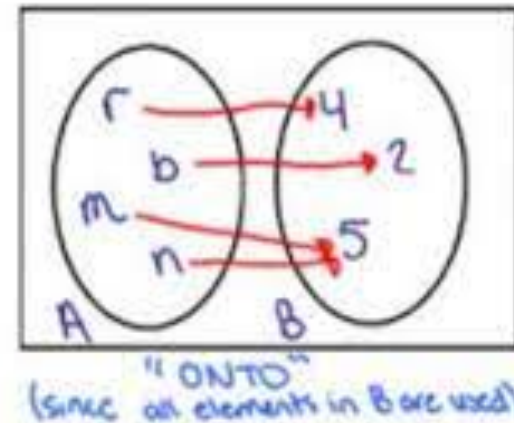
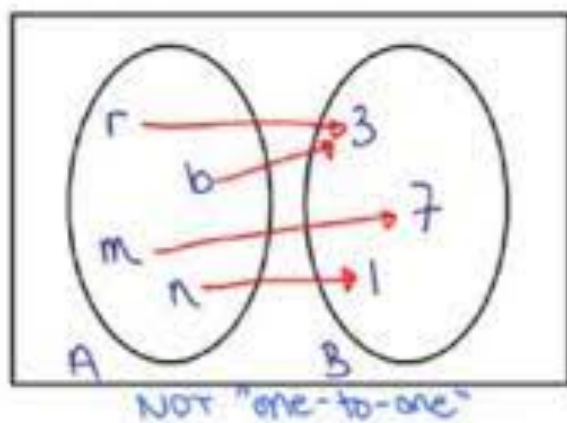
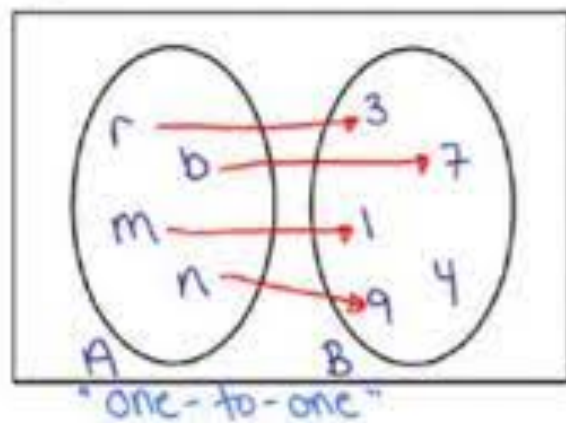
Functions — Final 2021

Problem 9. [5 points]

Write and fill the table with \checkmark or \times . If a function is one-to-one or onto, then use \checkmark . On the other hand, if a function is not one-to-one or not onto, then use \times .

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \rightarrow \mathbb{Z}$		

Key: One-to-one and onto functions



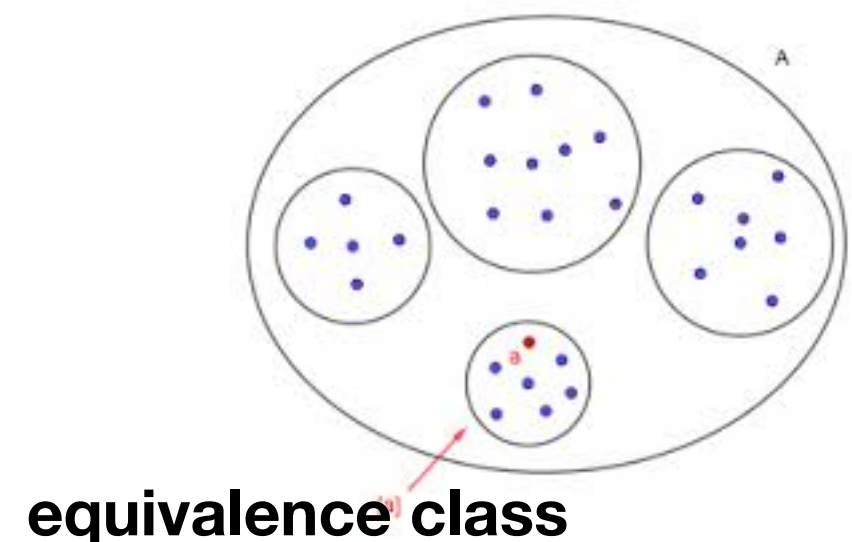
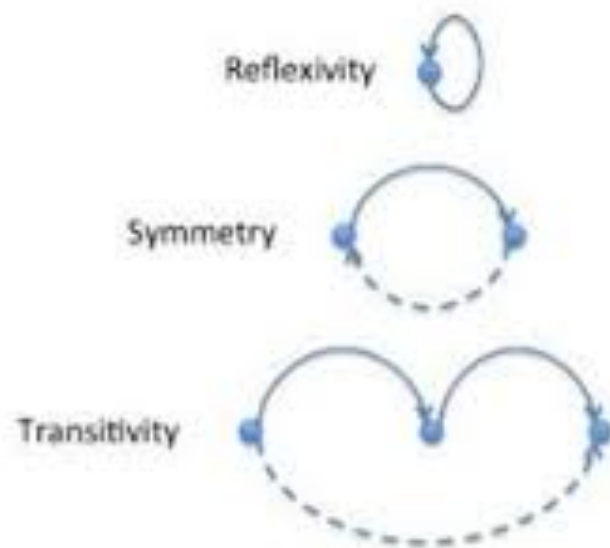
Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A , we have $p R q \Leftrightarrow p$ has the same birthday as q .

Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

**Key: Equivalence relations and
Equivalence classes**



Break;

Solution overview

Propositional Logic

Final 2021

Problem 1. [5 points]

Construct a truth table for the following statement form: $p \wedge (q \vee r) \leftrightarrow p \wedge (q \wedge r)$.

p	q	r	p AND (q OR r)	p AND (q AND r)	p AND (q OR r) <==> p AND (q AND r)
T	T	T	T	T	T
T	T	F	T	F	F
T	F	F	F	F	T
T	F	T	T	F	F
F	T	T	F	F	T
F	T	F	F	F	T
F	F	F	F	F	T
F	F	T	F	F	T

Predicate Logic — Midterm 1, 2021

Problem 3. [10 points]

Give negations of the following statements. Reasoning is not required.

(f) [1 point] $\forall x, \forall y$ such that $p(x, y)$

(g) [1 point] $\forall x, \exists y$ such that $p(x, y)$

- There exists x , there exists y , $\sim p(x, y)$
- There exists x , for all y , $\sim p(x, y)$

Proof — Final 2021

Problem 4. [5 points]

Prove that the sum of the squares of any two consecutive odd integers is even.

Proof

- We need to show: for any integer n , $(2n+1)^2 + (2n+3)^2$ is even.
- That is to say, we need to show the following proposition holds:
 - for any integer n , $8n^2 + 16n + 10$ is even.
- The formula above can be rewritten as $2(4n^2 + 8n + 5)$ which must be even.

QED.

Sequences - Final 2021

Problem 7. [10 points]

Use mathematical induction to prove the following identities.

(a) [5 points] For all integers $n \geq 1$,

$$\sum_{i=1}^n i(i!) = (n+1)! - 1.$$

• Proof.

- Let $P(n)$ be the predicate $1 * 1! + 2 * 2! + \dots n * n! = (n+1)! - 1$
- Base step: We prove $P(1)$.
- Inductive step: We prove for any integer $k \geq 1$, $P(k) \rightarrow P(k+1)$
 - Let k be an arbitrary integer and $k \geq 1$.
 - Assume $P(k)$ holds. That is $1 * 1! + 2 * 2! + \dots k * k! = (k+1)! - 1$
 - We need to prove $P(k+1)$, namely, $1 * 1! + 2 * 2! + \dots (k+1) * (k+1)! = (k+2)! - 1$
 - Following assumption $P(k)$, Left-hand-side above = $(k+1)! - 1 + (k+1) * (k+1)!$ which equals to Right-hand-side above.

• QED.

Sets — Midterm 2, 2021

Problem 2. [5 points]

Mention whether the following statements are true or false without giving any reasons. Assume all sets are subsets of a universal set U .

(a) [1 point] $(A \cap B) \cap (A \cap C) = A \cap (B \cup C)$

- The statement is false. As a counter example:
 - $A=\{1\}$, $B=\{2\}$, $C=\{1,2\}$.
 - Left-hand-side becomes empty set
 - Right-hand-side becomes $\{1\}$

Functions — Final 2021

Problem 9. [5 points]

Write and fill the table with ✓ or ✗. If a function is one-to-one or onto, then use ✓. On the other hand, if a function is not one-to-one or not onto, then use ✗.

Function	Domains	One-to-one function?	Onto function?
$f(x) = 3x$	$f : \mathbb{Z} \rightarrow \mathbb{Z}$		

- Yes: One to one function
- No: Onto function

Relations - Final 2021

Problem 11. [5 points]

Let A be the set of all people. Let R be the relation defined on A as follows: For persons p and q in A , we have $p R q \Leftrightarrow p$ has the same birthday as q .

Is R an equivalence relation? Prove your answer. If R is an equivalence relation, what are the distinct equivalence classes of the relation?

- R is an equivalence relation, as it is
 - reflective ($p R p$),
 - symmetric $p R q \Leftrightarrow q R p$
 - transitive $p R q, q R r \Rightarrow p R r$
- Equivalence classes is the set of the sets of people of A that have the same birthday.

Today's take-away

book chapter	Topics	Exam problems	Key
2	Propositional logic	2021-final, pb 1	truth table
3	Predicate logic	2021-midterm1, pb3	negation on quantifiers
4	Proof	2021-final, pb4	facts about integers
5	Sequences	2021-final, pb7	math induction
6	Sets	2021-midterm2, pb2	unions and intersections
7	Functions	2021-final, pb9	1-1 and onto
8	Relations	2021-final, pb11	equiv. rel. and classes

Thank you for your attention!