CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

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Midterm 1 reminder

- Midterm exam 10-06 (Thursday), B204, 12h30pm-1h50pm
- Open books, open notes, open Internet. Individual work
- A physical copy of the exam will be provided. But bring your scanning device to upload answers to Blackboard

Finish around 1h50

Different styles for designing an exam

- ? to make everyone pass
- ? to make someone suffer
- ? to provide a fair evaluation

How?

- Breadth: Covering most points
- Depth: Variate difficulty, but no more difficult than homework

Points covered in Midterm 1

1. Propositional statements
2. Negation
3. Inference rules
4. Truth tables (tautology, validity, equivalence)
5. Direct proof
6. Proof by dividing into cases
7. Proof by contradiction/contraposition
8. Application (Disproof etc, or a real-world scenario)

Mock midterm 1

(problems are taken from final of 2022 Spring)

To finish around 1h50pm

Problem 1. Propositional statements (points = 8)

Determine if the following statements are true or false. No explanation is needed.

1. $\forall\, x \in R, x^2 > 0$.
2. If $x,y \in R$, then $|x + y| = |x| + |y|$.
3. For every natural number n, the integer $n^2 + 17n + 17$ is prime.
4. If $a,b \in N$, then $a + b < ab$.

Solution

Determine if the following statements are true or false. No explanation is needed.

1. $\forall\, x \in R,\, x^2 > 0$.
2. If $x, y \in R$, then $|x + y| = |x| + |y|$.
3. For every natural number n, the integer $n^2 + 17n + 17$ is prime.
4. If $a, b \in N$, then $a + b < ab$.

- 1. False (Let x=0)
- 2. False
- 3. False (Let n = 17)
- 4. False

Problem 2. Negation (points = 8)

Negate the following statements.

1. The numbers x and y are both odd.
2. If x is prime, then x is not a rational number.
3. There exists a real number r for which $r + x = x$ for every real number x.
4. For every positive number ε, there is a positive number δ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.

Solution

1. The numbers x and y are both odd.
2. If x is prime, then x is not a rational number.
3. There exists a real number r for which $r + x = x$ for every real number x.
4. For every positive number ε, there is a positive number δ such that $|x - a| < \delta$ implies $|f(x) - f(a)| < \varepsilon$.

- 1. The number x is even, or the number y is even
- 2. x is prime and x is a rational number
- 3. For any real number r, there exists x such that r+x!=x
- 4. There exists a positive number \epsilon, such that for all positive number \delta, |x -a| <\delta and |f(x) - f(a)|>=\epsilon

Problem 3. Inference rules (points = 12)

Fill in the missing "- - - -" parts following the inference rule mentioned in the text.

a.

1. $\sim(\sim p \vee q)$ Premise
2. - - - - De Morgen with 1

b.

1. $(p \to q) \vee r$ Premise
2. $\sim r$ Premise
3. - - - - Elimination with 1, 2

c.

1. $(p \wedge r) \to \sim q$ Premise
2. $\sim q \to s$ Premise
3. $\sim s$ Premise
4. - - - - Transitivity with 1, 2
5. - - - - Modus Tollens with 3, 4

d.

1. $(p \wedge q) \to r$ Premise
2. p Premise
3. q Premise
4. - - - - Conjunction with 2, 3
5. - - - - Modus Ponens with 1, 4

Solution

a.

1. ~(~p ∨ q) Premise
2. - - - - De Morgen with 1

b.

1. (p -> q) ∨ r Premise
2. ~r Premise
3. - - - - Elimination with 1, 2

c.

1. (p ∧ r) -> ~q Premise
2. ~q -> s Premise
3. ~s Premise
4. - - - - Transitivity with 1, 2
5. - - - - Modus Tollens with 3, 4

d.

1. (p ∧ q) -> r Premise
2. p Premise
3. q Premise
4. - - - - Conjunction with 2, 3
5. - - - - Modus Ponens with 1, 4

- a. ~~p ∧ ~q
- b. p->q
- c.(p∧r) -> s; ~(p∧r)
- d. p∧q; r

Problem 4. Truth table (points = 10)

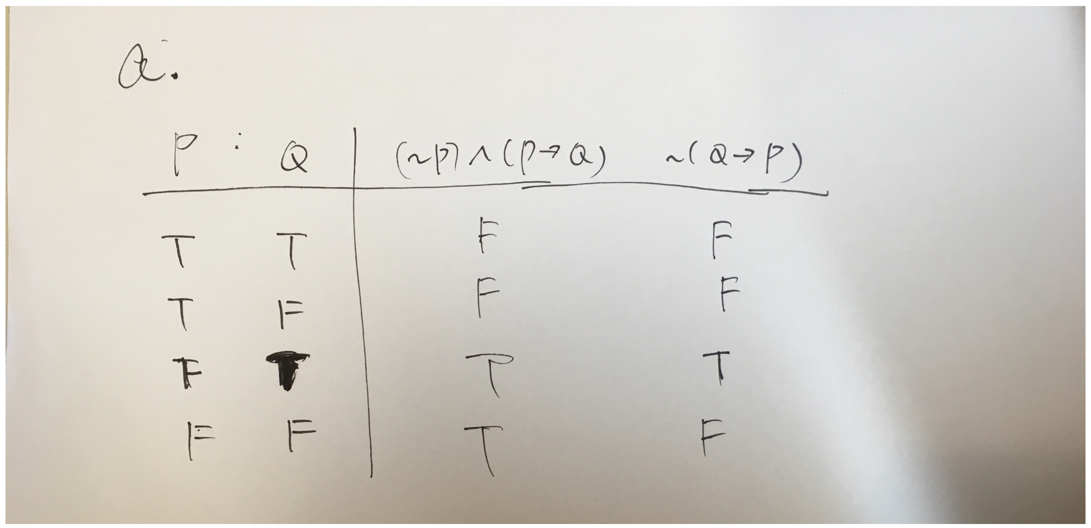
a. Determine if the following two statements are logically equivalent using a truth table.

- (~P) ∧ (P -> Q)
- ~(Q -> P)

b. Determine if the following logic inference is valid using a truth table.

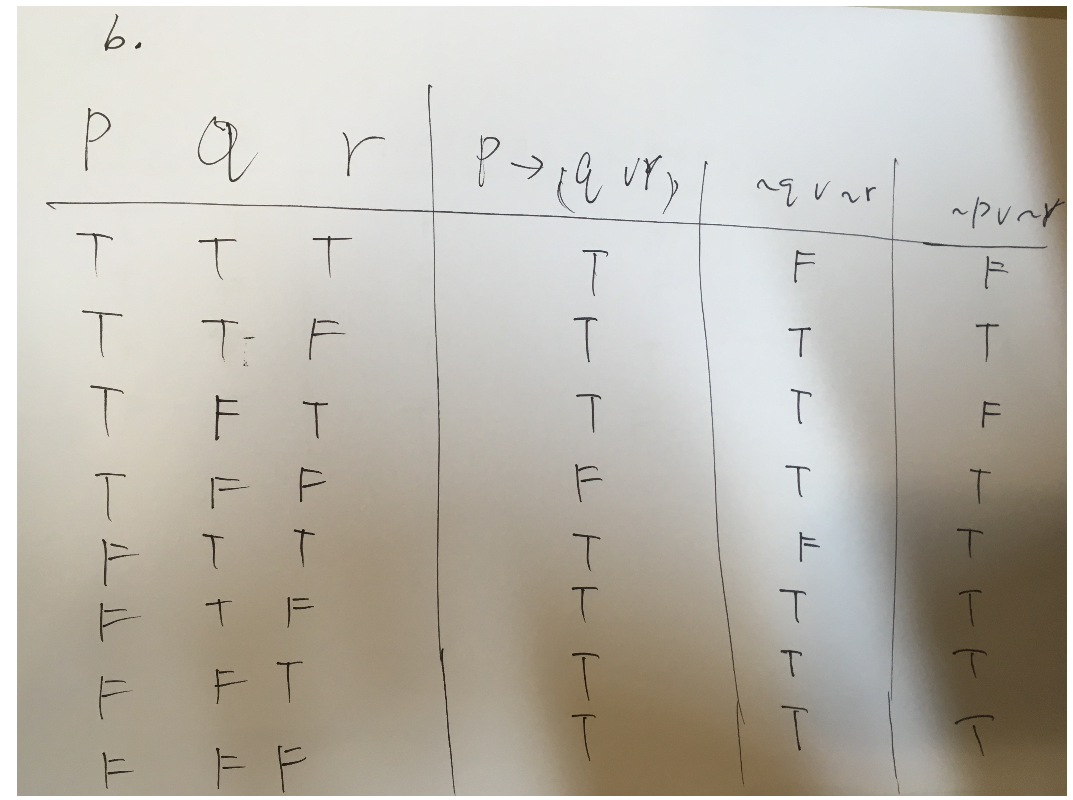
- premise p -> q ∨ r
- premise ~q ∨ ~r
- conclusion ~p ∨ ~r

Solution



a.

p	Q	$(\sim p)\wedge(p\to Q)$	$\sim(Q\to p)$
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F



b.

p	q	r	$p\to(q\vee r)$	$\sim q\vee\sim r$	$\sim p\vee\sim r$
T	T	T	T	F	F
T	T	F	T	T	T
T	F	T	T	T	F
T	F	F	F	T	T
F	T	T	T	F	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	T	T

- a. Not equivalent, shown in the last row
- invalid shown in critical row p=T, q=F, r=T

Problem 5. Direct proof (points = 5)

Suppose a, b and c are integers. If $a^2|b$ and $b^3|c$, then $a^6|c$.

Solution

Proof. Since $a^2 \mid b$ we have $b = ka^2$ for some $k \in \mathbb{Z}$. Since $b^3 \mid c$ we have $c = hb^3$ for some $h \in \mathbb{Z}$. Thus $c = h(ka^2)^3 = hk^3a^6$. Hence $a^6 \mid c$. ■

Problem 6. Proof by dividing into cases (points = 10)

Consider the expression $1 + (-1)^n (2n - 1)$. below is a table showing its values for various integers $n > 0$. Notice that $1 + (-1)^n (2n - 1)$ is a multiple of 4 in every line.

n	$1 + (-1)^n(2n - 1)$
1	0
2	4
3	-4
4	8
5	-8
6	12

1. Prove that $1 + (-1)^n(2n - 1)$ is a multiple of 4 for every integer n.

Solution

Proposition If $n \in \mathbb{N}$, then $1+(-1)^n(2n-1)$ is a multiple of 4.

Proof. Suppose $n \in \mathbb{N}$.
Then n is either even or odd. Let's consider these two cases separately.

Case 1. Suppose n is even. Then $n = 2k$ for some $k \in \mathbb{Z}$, and $(-1)^n = 1$.
Thus $1+(-1)^n(2n-1) = 1+(1)(2\cdot 2k-1) = 4k$, which is a multiple of 4.

Case 2. Suppose n is odd. Then $n = 2k+1$ for some $k \in \mathbb{Z}$, and $(-1)^n = -1$.
Thus $1+(-1)^n(2n-1) = 1-(2(2k+1)-1) = -4k$, which is a multiple of 4.

These cases show that $1+(-1)^n(2n-1)$ is always a multiple of 4. ■

Problem 7. Proof by contraposition/contradiction (points = 15)

Prove the following statements:

1. The number $\sqrt{2}$ is irrational.
2. If r is a non-zero rational number, then $r/\sqrt{2}$ is an irrational number. [Hint: You could use the conclusion from 1.]
3. Every non-zero rational number can be expressed as a product of two irrational numbers. [Hint: You could use the conclusions from 1 and 2.]

Solution for 7.1

- Proof. We want to prove sqrt(2) is irrational.
  - Proof by contradiction. Assume sqrt(2) is rational. Namely, sqrt(2)=m/n for some integers m, n having no common factors.
  - Thus m^2=2n^2. We have m must be even. Thus, m = 2k for some integer k. Thus, n^2 = 2k^2. Thus n must be even. The fact that m and n are both even contradicts with the assumption above.
- QED.

Solution 7.2

- Proof. We want to prove: for any nonzero rational r, r/sqrt(2) is irrational.
    - Proof by contradiction. Assume there exists a nonzero rational r, r/sqrt(2) is rational
    - Then r/sqrt(2) = m/n for some integers m, n where m!=0. Thus sqrt(2) = r n / m. Since r is rational, r = a/b for some integers a, b. Thus sqrt(2) = an/(mb) which contradicts with the fact sqrt(2) is irrational (7.1).
- QED.

Solution 7.3

- Proof. We need to show, for any rational number r, there exists irrational numbers ir1, ir2, such that r =ir1 * ir2
  - Suppose r is rational.
  - Let ir1=sqrt(2), ir2= r/sqrt(2)
  - We have r = ir1*ir2 and ir1 and ir2 are irrational following 7.1 and 7.2.
- QED

Problem 8 Application

- Determine if the following is valid argument or not. Explain with inference rules or truth table.
- Premise 1: If the instructor is sick, the class will be canceled.
- Premise 2: If the class is cancelled, the students are happy
- Conclusion: If the instructor is sick, the students are happy