CSE215
Foundations of Computer Science

Instructor: Zhoulai Fu

State University of New York, Korea

September 14, 2022

Today

Homework 01

Preparation exercises for Midterm1

- To finish by 4h25

```
* Exercise 1 (score = 10)
Construct the truth table for the following statement forms:

1. false \/ p
2. True /\ p
```

Solution

p	false \/ p
true	true
false	false

p	true \/ p
true	true
false	false

* Exercise 2 (score = 30)

Write truth tables for the following statement forms.

1. p -> q
2. ~p \/ q
3. q -> p
4. ~q \/ p
5. ~q -> ~p
6. ~p -> ~q

* Exercise 3 (score = 15)

  Among the six statement forms in Exercise 2, find at least five pairs that are equivalent? For example, if you believe statement forms 1 and 2 in Exercise 2 are equivalent, you have found a pair (1, 2).

Solution

Exercise 2

		1.	2.	3.	4.	5.	6.
P	q	$P \to q$	$\sim P \vee q$	$q \to P$	$\sim q \vee P$	$\sim q \to \sim P$	$\sim P \to \sim q$
T	T	T	T	T	T	T	T
T	F	F	F	T	T	F	T
F	T	T	T	F	F	T	F
F	F	T	T	T	T	T	T

Exercise 3

- 1, 2, 5 are equivalent
- 3, 4, 6 are equivalent

* Exercise 4. (score = 20)

Make a truth table for the prroposition ~P ∧ (Q → P). What can you conclude about P and Q if you know the statement is true?

Solution

P	Q	~P	Q->P	~P∧(Q->P)
T	T	F	T	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

If the whole statement is true, then P is false and Q is false.

* Exercise 5. (score = 15)
For each statement form below, use truth tables to determine if it is a tautology, contradiction, or neither.

1. (~p\/q)\/(p/\~q)
2. (p/\~q)∧(~p\/q)
3. (p/\q)\/(~p\/(p/\~q))

Solution

5.

P	Q	~P∨Q	P∧~Q	(~P∨Q) ∨(P∧~Q)
T	T	T	F	T
T	F	F	T	T
F	T	T	F	T
F	F	T	F	T

(~P∨Q)∨(P∧~Q) is a tautology.

P	Q	P∧~Q	~P∨Q	(P∧~Q) ∧(~P∨Q)
T	T	F	T	F
T	F	T	F	F
F	T	F	T	F
F	F	F	T	F

(P∧~Q)∧(~P∨Q) is a contradiction.

P	Q	P∧Q	P∧~Q	~P∨(P∧~Q)	(P∧Q)∨(~P∨(P∧~Q))
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	F	F	T	T

(P∧Q)∨(~P∨(P∧~Q)) is a tautology.

```
* Exercise 6. (score = 10)

  Check if the two statement forms below are logically equivalent using a truth table

-  p \/ q -> r
- (p -> r) /\ (q ->r)
```

Solution

6.

P	Q	R	PVQ	(PVQ)->R	P->R	Q->R	(P->R)∧(Q->R)
T	T	T	T	T	T	T	T
T	T	F	T	F	F	F	F
T	F	T	T	T	T	T	T
T	F	F	T	F	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	T	T
F	F	F	F	T	T	T	T

They are logically equivalent.

Midterm 1

01	08-30	Course overview	
	08-31	Recitation: Look and feel of a final exam	
	09-01	Propositional logic [homework01 announced]	Epp, Ch2
02	09-06	Propositional logic	Epp, Ch2
	09-07	Recitation	
	09-08	Propositional logic [homework01 due, homework02 announced]	Epp, Ch2
03	09-12	Predicate logic	Epp, Ch3
	09-14	Recitation	
	09-15	Proof techniques [homework02 due, homework03 announced]	Epp, Ch4
04	09-20	Proof techniques	Epp, Ch4
	09-21	Recitation	
	09-22	Proof techniques [homework03 due, homework04 announced]	Epp, Ch4
05	09-27	Proof techniques	Epp, Ch4
	09-28	Recitation	
	09-29	Midterm review [homework04 due]	
06	No class 10-04		
	10-05	Recitation	
	10-06	Midterm 1	

Exercise 1

Tautaulogy, contradition, or neither of them?

- (p XOR q) /\ (p <-> q)

- (p XOR q) \/ (p <-> q)

- (p -> q) /\ (~p -> ~q)

Solution

- Contradiction
- Tautology
- Neither contradiction, or tautology

* Exercise 2

Consider the following statement form: (p ⊕ q) → (~r → (p v q)).

1. How many rows would you need to construct its truth table?

2. How many columns would you need at least, to construct its truth table?

3. What is the truth value of the statement form when p = T, q = F, r = T?

Solution

- 9 rows including the header row
- 4 columns
- true

Exercise 3

Show (p -> q) -> r and p -> (q -> r) are **not** logical equivalent?

Solution

- Not equivalent. When p = false and r = false, the first proposition (p->q)->r is false whereas the second one, p->(q->r) is true.

* Exercise 4

Write a negation for the following statement:

∃x ∃y such that (0 < x <= y^2 < 100).

Solution

- ~ (∃x ∃y such that (0 < x <= y^2 < 100)) is the same as:

- ~(∃x ∃y such that (0 < x /\ x <= y^2 and y^2 < 100)), which becomes:

- \forall x \forall y, (0 >= x \/ x>= y^2 \/ y^2 >= 100),

* Exercise 5

Determine if the following argument is valid:

p ↔ q
q ⊕ r
———
∴ p \/ r

Solution

- It is valid. Below is the proof.
- Assume the premises are true.
- Namely, p<->q is true and q xor r is true.
- Since p <->q is true, we know p and q must be the same truth value.
- Since q xor r is true, we know q and r must be different truth values.
- Thus, p and r must be different truth values.
- Thus p and r can be either (true and false), or (false and true)
- Thus q ∨ r, namely, the conclusion, is true.