CSE215
Foundations of Computer Science

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September 22, 2022

Today

Homework 02

Review about facts used in proof techniques

Proof exercises for clarification (if time allows)

- To finish by 4h25

* Exercise 1 (score = 20)

Use truth tables to determine whether the argument form below is valid

(1)

- premises: p -> q, q
- conclusion: p

(2)

- premises: p-> q, ~p
- conclusion: ~q

(3)

- premises: p-> q, p
- conclusion: q

(4)

- premises: p-> q, ~q
- conclusion: ~p

Solution 1.1

(1)

- premises: p -> q, q
- conclusion: p
- **Not valid**

p	q	p->q	q	p
T	T	T	T	T
T	F	F	F	T
F	T	T	T	F
F	F	T	F	F

Solution 1.2

(2)

- premises: p-> q, ~p
- conclusion: ~q
- **Not valid**

p	q	p->q	~p	~q
T	T	T	F	F
T	F	F	F	T
F	T	T	T	F
F	F	T	T	T

Solution 1.3

(3)

- premises: p-> q, p
- conclusion: q
- **Valid**

p	q	p->q	p	q
T	T	T	T	T
T	F	F	T	F
F	T	T	F	T
F	F	T	F	F

Solution 1.4

(4)

- premises: p-> q, ~q
- conclusion: ~p
- **Valid**

p	q	p->q	~q	~p
T	T	T	F	F
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

* Exercise 2 (score = 60)

Use truth tables to determine whether the argument form below is valid

(1)
- Premises: p -> q, ~p -> ~q
- Conclusion: p \/ q

(2)
- Premises: p \/ q, p -> ~q, ~r -> ~p
- Conclusion: r

(3)
- Premises: p, ~q -> ~p, ~q \/ r
- Conclusion r

(4)
- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r

(5)
- Premises: p -> r, q -> r
- Conclusion: (p \/ q) -> r

(6)
- Premises: p -> (q \/ r), ~q \/ ~r
- Conclusion: ~p \ / ~r

Solution 2.1

(1)

- Premises: p -> q, ~p -> ~q
- Conclusion: p ⋁ q
- **Not valid**

p	q	~p	~q	p->q	~p->~q	p ⋁ q
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	T	F	T
F	F	T	T	T	T	F

Solution 2.2

(2)

- Premises: p \/ q, p -> ~q, ~r -> ~p
- Conclusion: r
- **Not valid**

p	q	r	~p	~q	~r	p \/ q	p->~q	~r->~p	r
T	T	T	F	F	F	T	F	T	T
T	T	F	F	F	T	T	F	F	F
T	F	T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T	F	F
F	T	T	T	F	F	T	T	T	T
F	T	F	T	F	T	T	T	T	F
F	F	T	T	T	F	F	T	T	T
F	F	F	T	T	T	F	T	T	F

Solution 2.3

(3)

- Premises: p, ~q -> ~p, ~q \/ r
- Conclusion: r
- **Valid**

p	q	r	~q	~p	~q->~p	~q \/ r	p	r
T	T	T	F	F	T	T	T	T
T	T	F	F	F	T	F	T	F
T	F	T	T	F	F	T	T	T
T	F	F	T	F	F	T	T	F
F	T	T	F	T	T	T	F	T
F	T	F	F	T	T	F	F	F
F	F	T	T	T	T	T	F	T
F	F	F	T	T	T	T	F	F

Solution 2.4

(4)

- Premises: p/\q -> ~r, p \/ ~q, ~q -> p
- Conclusion: ~r
- **Not valid**

p	q	r	~q	~r	p /\ q	p/\q ->~r	p\/~q	~q->p	~r
T	T	T	F	F	T	F	T	T	F
T	T	F	F	T	T	T	T	T	T
T	F	T	T	F	F	T	T	T	F
T	F	F	T	T	F	T	T	T	T
F	T	T	F	F	F	T	F	T	F
F	T	F	F	T	F	T	F	T	T
F	F	T	T	F	F	T	T	F	F
F	F	F	T	T	F	T	T	F	T

Solution 2.5

(5)

- Premises: p -> r, q -> r
- Conclusion: (p ⋁ q) -> r
- **Valid**

p	q	r	p ⋁ q	p -> r	q -> r	p ⋁ q -> r
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	T	T	T	T
T	F	F	T	F	T	F
F	T	T	T	T	T	T
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	F	T	T	T

Solution 2.6

(6)

- Premises: p -> (q V r), ~q V ~r
- Conclusion: ~p V ~r
- **Not valid**

p	q	r	~p	~q	~r	q V r	p->(qVr)	~q V ~r	~p V ~r
T	T	T	F	F	F	T	T	F	F
T	T	F	F	F	T	T	T	T	T
T	F	T	F	T	F	T	T	T	F
T	F	F	F	T	T	F	F	T	T
F	T	T	T	F	F	T	T	F	T
F	T	F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T	T	T
F	F	F	T	T	T	F	T	T	T

* Exercise 3 (score = 20)

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Check if the two statement form are equivalent, and expaian why:

- (p -> q)/\(q -> r)/\(r -> p)
- p /\ q /\ r
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- Intuition: #1 = p, q, r have the same truth value, either true, or false

Solution 3

p	q	r	~p	~q	~r	p->q	q->r	r->p	(p->q)/\(q->r)/\(r->p)	p /\ q /\ r
T	T	T	F	F	F	T	T	T	T	T
T	T	F	F	F	T	T	F	T	F	F
T	F	T	F	T	F	F	T	T	F	F
T	F	F	F	T	T	F	T	T	F	F
F	T	T	T	F	F	T	T	F	F	F
F	T	F	T	F	T	T	F	T	F	F
F	F	T	T	T	F	T	T	F	F	F
F	F	F	T	T	T	T	T	T	T	F

* Exercise 4 (points = 20)

Use inference rules to show the following argument is valid

Premises
- $p \lor q$
- $q \to r$
- $p \land s \to t$
- $\sim r$
- $\sim q \to u \land s$

Conclusion
- t

* Exercise 4 (points = 20)

Use inference rules to show the following argument is valid

Premises
- p ∨ q
- q -> r
- p ∧ s -> t
- ~r
- ~q → u ∧ s

Conclusion
- t

- From premises ~r and q->r, we have ~q using Modus Tollens
- From ~q and premise ~q -> u/\s, we have u and s following Modus Pollens and Conjunction
- From ~q and premise p\/q, we have p following Elimination
- From p, s, and p/\ s->t, we have t following conjunction and Modus Pollens

Proof techniques review

- To finish by 4h25

Define numbers precisely

- A number n is odd if ___
- A number r is rational if ___
- A number r is irrational if ___
- A number p is prime if ___
- A number p is composite if ___

expand-factorize

- Expand $(x+1)^2$
- Expand $(x-1)^2$
- Expand $(x+y)^2$
- Expand $(x+a)(x+b)$
- Factorize x^2+2x+1
- Factorize x^2+3x+2
- Factorize x^2+4x+3
- Solve this equation of real numbers: $x^2 - 2x + 1 = 0$
- Solve this equation of real numbers: $x^2 - 3x + 2 = 0$

True or false: odd, even, prime, composite

- if a and b are integers, is 10a^2+64b+7 is odd
- if a and b are integers, is a^2+64b+7 is even
- Write the first five prime numbers

True or false

- If a is odd and b is even, then, 2a+3b is even.
- For any nonnegative real numbers x, we have x^2 >x.
- For all integers m and n, if mn = 1 then m = n = 1.
- For all integers m and n, if m + n is even then m and n are both even.
- There exists an integer n such that 6n^2 + 27 is prime
- There exists an integer m>=3 such that m^2-1 is prime
- composite + composite = composite

Two proof exercises (if time allows)

1. How to write a proof
2. Different ways to address a proof

To finish by 4h25

Solution: How to write a proof: Prove rational + rational = rational

- Check whether the proposition is correct with intuition and examples.
- Begin a proof with “Proof.”
- State the proposition formally.
- Which type is the proposition — existential, universal?
- Which type of proof will you write? In particular, If existential, find an example; if universal, first fixing an arbitrary element in the domain set. Write “Suppose/Let…” to get started.
- Write the proof like an argument in an English essay; convince with rigor.
- End a proof with “QED.”

How to write a proof: Prove rational + rational = rational

- Proof.
  - We want to prove: for any rational numbers x and y, x+y is a rational number.
  - Suppose x and y are rational numbers. Then x can be written as a/b, and y can be written as c/d, and x+y=(ad+bc)/bd
  - Therefore x+y is a rational number
- QED

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

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Solution method 1

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

- Proof.
  - Proof by contraposition.
  - Instead of proving n^2+8n+20 is odd => n is odd. We prove
    - (G) n is even => n^2+8n+20 is even
  - That is,
    - for any natural number n, if n is even, then n^2+8n+20 is even
  - Suppose n is a natural number and n is even, we can immediately show n^2+8n+20 is even since it is a sum of three even numbers. Thus, (G) is proved.
- QED

Solution method 2

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

- Proof.
  - We want to prove
    - (G) for any natural number n, if n^2 + 8n + 20 is odd, then n is odd
  - Proof by contradiction. Namely, We negate (G) above, obtaining
    - (~G) there exists a natural number n, such that n^2+8n+20 is odd, and n is even.
  - The above (~G) is a contradiction since for any natural number n, n^2+8n+20 even whenever n is even. In other words, (~G) is false. Thus (G) is proved.
- QED

Solution method 3

Problem 6. [5 points]

Prove that if $n^2 + 8n + 20$ is odd, then n is odd for natural numbers n.

- Proof.
  - We want to prove
    - for any natural number n, if n^2 + 8n + 20 is odd, then n is odd
  - Suppose n is a natural number and n^2+8n+20 is odd. We need to prove
    - (G) n is odd
  - Proof by contradiction. Suppose n is even. Then n^2+8n+20 must be even, which contradicts with the assumption that n^2+8n+20 is odd. In other words, (~G) -> False. Thus (G) must be true.
- QED

That is all for today

- Homework 02
- Exercises on numbers
- Exercises on proof

Thank you for your attention!